Supporting Teachers Developing Mathematical Tasks With Digital Technology

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A crucial step towards improving the conceptual use of digital technology (DT) in the mathematics classroom is to increase teacher involvement in the development of tasks. Hence, this research considers some teacher factors that might influence DT algebra task development and implementation in secondary schools. We observed and assisted one group of three teachers as they designed and implemented DT tasks. Our preliminary analysis examines the richness of the two tasks produced by one group and seeks to explain the difference between them. The results suggest the intervention provided with respect to task design led to improved Pedagogical Technology Knowledge for the teachers, and hence a richer task. The delivery of the intervention could be of assistance in focusing professional development programs so they may better facilitate the training of teachers in the use of digital technology in teaching mathematics.

Introduction

The use of digital technology (DT) in the mathematics classroom has the potential to help students understand mathematical concepts meaningfully. However, the effectiveness of DT use depends on the tasks used (Leung & Bolite-Frant, 2015). Since the teacher plays a critical role in implementing DT tasks in the classroom (Sullivan et al., 2015), their relationship to the tasks requires careful consideration. In the light of this, this study seeks to identify ways in which teachers may be assisted with DT task development and implementation. In this paper we report the progress of a group of three teachers who developed two DT algebra tasks. In doing so we consider the possible role of teacher factors in the design and implementation of the tasks.

Background

One key decision for teachers wanting to use DT in their classroom is whether to use pre-existing tasks or to design their own. Clearly, pre-existing DT tasks may not be suitable for every class and teachers may “continuously have to adapt to the actual thinking and learning of his or her students” (Gravemeijer, 2004, p. 107). Further, the decisions made by teachers implementing tasks may not be what the designers expected (Sullivan et al., 2015). Hence, some studies suggest that teacher involvement in task design would help to improve the quality of task implementation (Kullberg et al., 2013; Sullivan et al., 2015) since they may be more sensitive to student learning. In one example, teachers who designed a mathematical task for Grade 8 and implemented it with their own students found what the learners encountered in the lesson varied from what was expected (Kullberg et al., 2013). It seems clear that it is difficult to predict the actual learning situation in the classroom since it depends on the manner of the teaching and the responses of the learners. However, these issues may be minimised when a teacher designs digital technology tasks for her own students given she has a good sense of the levels of understanding of her own students, their

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conceptions and misconceptions, their level of instrumental genesis (Artigue, 2002), their prior knowledge and the available resources. In other words, teachers are able to "make [their]…design decisions based on [their]…best guess [about] how learning might proceed" (Simon, 1995, p. 135) when designing DT tasks for their own students.

In order to consider the relationship between teacher resources and task development we employ the theoretical model of Pedagogical Technology Knowledge (PTK; Thomas & Hong, 2005). The focus of PTK is to place the enhancement of mathematical thinking at the centre of the processes of design and implementation of tasks, and hence consider mathematics principles, conventions, and techniques required to teach mathematics through the technology. Thus the focus is clearly on the mathematics and not on the technology, which has the role of an epistemic mediator. Clearly, both intrinsic and extrinsic factors may influence whether, and how, teachers use DT in their classroom mathematics teaching and the extent to which they need assistance to design suitable tasks. Examples of extrinsic factors that may limit or support such use have been well-reported and include a lack of time due to syllabus constraints, few opportunities for DT professional development (PD), poor access to technology hardware and software, limited availability of classroom teaching resource material, a lack of support from school colleagues, pressures of curriculum and assessment requirements, and inadequate technical support (Forgasz, 2006; Goos, 2005). Factors that may support effective use include working together collaboratively in teams and professional development of teachers' PTK (Oates, 2011; Thomas & Lin, 2013). However, the emerging PTK framework largely focuses on intrinsic factors, particularly a teacher's orientations and goals (Schoenfeld, 2010), including their perception of the nature of mathematical knowledge and how it should be learned as well as the value of DT in teaching; their instrumental genesis of the DT tools (Artigue, 2002); and their mathematical knowledge for teaching (MKT), which includes mathematical content knowledge (MCK) and Shulman's (1986) pedagogical content knowledge (PCK), as well as mathematics knowledge at the horizon (Hill & Ball, 2004).

Our conjecture, based on the PTK framework, is that to design rich tasks using DT, a teacher needs strong PTK, including good MKT as well as knowledge and experience in teaching with DT (Thomas & Lin, 2013). In addition, her beliefs and attitudes towards mathematics and technology will influence task design and implementation, while working collaboratively will add additional support to the whole process. Thus this paper discusses how an intervention that focussed on assisting teachers working in groups with rich task production may have influenced task design and implementation.

Method

This research uses a case study methodology implemented in three phases. Firstly, four groups, each comprising three Sri Lankan teachers, were engaged in designing and developing a DT algebra task for Grade 12 students (17-18 years old) studying Advanced Level (A-level) combined mathematics. Group A, discussed here, is from the Central province. We see from Table 1 that all Group A teachers were mostly experienced male teachers with at least a BSc degree comprising a substantial mathematics component, but they all had very limited, or no experience in using DT to teach mathematics. Note that, while not discussed here, other groups in the study did provide contrasting demographics, for example Group B who were all female teachers from the Western province. Teachers were free to select any topic from the A-level combined mathematics syllabus and their task development was video and audio recorded. Group A chose *graphs of quadratic functions* with GeoGebra as their choice of DT. None of the teachers had ever previously constructed a DT task in mathematics.

Table 1.

Group A Teachers' Demographics

Teacher	Gender	Age	Mathematics Qualifications	Years of Teaching	Use of DT in Teaching
A1	M	31-40	BSc, PGDE	10-15	Never
A2	M	31-40	BSc	<5	Never/ Rarely
A3	M	31-40	BSc, PGDE	10-15	Never

There were multiple aspects to the data collection process. Teachers first completed a questionnaire including a Likert-style attitude test with five subscales comprising: mathematics confidence, confidence teaching with DT, the value of DT in learning mathematics, confidence in DT task development, and attitude to teaching mathematics with DT. Space prevents reproducing all these here but examples of the questions from the subscale addressing confidence in DT task development subscale included: "I prefer to use digital technology tasks developed by other people"; "It is worth devoting time to task development with digital technology"; "More interesting tasks can be developed using digital technology"; and "I feel more comfortable in designing tasks using digital technology with other teachers who are good at it". The questionnaire was followed by a semi-structured interview, partially informed by, and seeking clarification of, responses to the questionnaire, where teachers talked about their experience in teaching, in teaching with DT, their knowledge, beliefs and confidence in using DT in teaching and any professional development learning experiences in relation to technology use.

Once they had completed their initial design the first-named author initiated a discussion with all three teachers of the tasks they had developed in the light of some theoretical principles of rich DT task design (e.g. Kieran & Drijvers, 2006). During this two-hour intervention she explained what a DT mathematical task might look like and most importantly what criteria comprise a rich mathematical task (Cennamo, Ross, & Ertmer, 2014), as seen in the list below. Secondly the participants were provided with an example of such a DT algebra task written by one of the researchers. Finally, they were given ideas on how to plan a lesson to implement a DT algebra task by considering what decision making might be needed and the role of resources, orientations and goals (ROG) in attaining these based on the theory of Schoenfeld (2010). In addition, teachers were introduced to a three point framework for lesson planning, delivery and review (Choy, 2014) that focuses teacher attention on the key concept, the possible point of difficulty and proposed course of action for the lesson, thus assisting with decision making that involves noticing student thinking. It was anticipated that this intervention would assist the teachers in the design of tasks suitable for their students.

Following this intervention the teachers were given an opportunity to modify their task and then they chose one of their number to implement it in their own classroom while the other group members observed the lesson, along with the same researcher. In Group A, this turned out to be one of the more experienced teachers. Following the intervention, the teachers took part in a group interview focused on their planning during task development, how it worked in practice, the modifications to the tasks they made after the discussion and their reflection on the factors that influenced their task development. The next phase was the task implementation with students in the classroom. Finally, a post implementation discussion was held with the researcher where the participant teachers reflected on their work and could then modify the task again based on the implementation. At this stage, Group A was satisfied with the task they implemented and made no changes. Copies of all the tasks were collected for analysis. All the teachers then answered a final questionnaire.

Results

Firstly, we describe the tasks Group A produced and examine them in the light of task design theory. We used a number of sources such as Kieran and Drijvers (2006) and Leung and Bolite-Frant (2015) as well as our own ideas to compile a list of features that might define a rich technology task. We then employed these principles to examine the changes in the tasks designed by the teachers in Group A following the intervention. The framework has twelve principles that are listed in the first column of Table 2, which summarises the richness of both Group A's tasks as measured against these criteria. Teachers' stated goals for the tasks were collected after the modification of the tasks.

Table 2.
The Framework Assessing the Richness of Group A's Tasks

Principles of Rich Tasks	First Task		Second Task	
	Evidence	Score (0-3)	Evidence	Score (0-3)
Focuses on mathematical ideas, e.g. epistemological obstacles	Implicit. For a function $y = ax^2 + bx + c$ effect of a on the shape of the graph.	1	Define quadratic function, completing the square, discriminant, roots of quadratics	3
Considers the role of language & discourse	Little: words such as variation and shape used without support.	1	Some words used instead of mathematical symbols. 'Compares Touches' and 'cuts'	1
Students written interpretations	Brief: Explain the observation; Conclude	2	State the behaviour of… (twice)	2
Goes beyond routine methods	Limited; looks at the role of a	1	Identify behaviour; observe how the graph depends on…	2
Encourages student investigation	Closely guided, limited investigation	1	Provides a table to be completed by investigation	2
Multi-representational aspects	Implicit: involves numbers and algebra linked to graphs	2	Explicit: links numbers and algebra to graphs	3
Appropriate for student instrumental genesis	Good. No instructions on function entry needed. 'Define the slider' sufficient for students.	3	Good. No instructions on function entry needed. 'Define the slider' sufficient. Independent slider use.	3
Instrumental feedback	Observe the graph's concavity and shape and make a conclusion related to a.	1	Observe the graph's concavity and relative position to axes and relate to a and D.	2
Integration of DT and by-hand techniques	Nothing explicitly mentioned.	0	Completing square by hand. Graphing by DT. Complete table by hand.	2
Aims for	Aim to generalise a>0 and	2	How graph depends on a	3

generalisation	a<0. Discriminant formula given but not used.		and discriminant D	
Students think about proof	No evidence	0	Not present	0
Develops mathematical theory	No evidence	0	Not present	0

Group A's first task constructed before the researcher intervention used GeoGebra to consider the effect of a on the graph of quadratic functions of the form $y = ax^2 + bx + c$ (see Figure 1). Although they intended sliders to be used for a, b and c we note they provided six sets each with three discrete values of a, b and c, three with a =1 (> 0) and three with a = –1 (< 0). The task concluded with the words "Explain the observation".

$y = ax^2 + bx + c$
Define the slider a, b, c
Check the variation of the graph according to the variation of a
Observe shape of the graph (i) $a > 0$
(ii) $a < 0$
$D = b^2 - 4ac$
Check the shape of the graph under the following conditions
(a) $a > 0$
(i) $a = 1, b = -4, c = 6$
(ii) $a = 1, b = -4, c = 4$
(iii) $a = 1, b = -4, c = 2$
(b) $a < 0$
(i) $a = -1, b = -4, c = -6$
(ii) $a = -1, b = -4, c = -4$
(iii) $a = -1, b = -4, c = -2$
Explain the observation
Conclude

Figure 1. Group A's first task.

In a consideration of this task using the task design principles above and attempting to produce a metric of 'richness', we assign a score of 0-3 for each of the 12 principles listed above. The scoring of these was done independently by each researcher and then agreed to ensure validity. Table 2 summarises the analysis of the richness of this first task and we can see that the richness metric gives a score of 14/36. We note that the mathematical sophistication of the content is not a factor considered in this analysis. Hence, for the generalisation criterion the mathematical level is not a factor, but the only consideration is the extent to which the opportunity to generalise is provided, or not.

Quadratic functions

There is a function as $y = ax^2 + bx + c$

1. Insert this in GeoGebra.
2. Define a, b, c as sliders
3. Draw the graphs for different values of a, b and c when $a \neq 0$
4. Identify and state the behaviour of the graph with the sign of "a": when a is positive and negative.
5. Rearrange $y = ax^2 + bx + c$ using completing the square method.
   $y = a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2-4ac}{4a}$
6. Insert $D = b^2 - 4ac$
7. Fill the following table

a	*b*	*c*	**D**			**Place above the *x*-axis**	**Touches the *x*-axis**	**Cuts the *x*-axis**
			+	–	0			

8. Observe and state how the behaviour of the graph of y depends on the values (+, –, 0) of a and D.

Figure 2. Group A's second task.

After the intervention, the role of a was still considered, but the task focused more on the effect of the sign of the discriminant $D = b^2 - 4ac$, as can be seen from Figure 2. Analysing this task according to the variables for rich tasks we can say that the richness metric for this second iteration of the task gives a score of 23/36 (see Table 2). Even though the difference in the mean score per principle was not quite significantly different statistically (m_1=1.18, m_2=1.82, t=1.45, p=0.08) we claim that the increase in score on seven of the principles provides evidence that the teachers' task design improved as a result of the intervention and the professional development strategies employed in this study. Further support for this claim can be found in responses to the questionnaires and the interviews discussed in the next section under *Teacher Factors*. In particular, Table 3 suggests teachers underwent a substantial improvement in their confidence with using DT, while comments in the interviews suggest teachers benefited from both the iterative and collaborative nature of the project.

Table 3.

Group A's Attitude Subscale Means

Subscale	Group A Pre	Group A Post
Maths Confidence	3.40	3.33
DT Confidence	3.89	4.11
Value of DT	3.33	3.47
Attitude to Teaching with DT	3.29	3.50
Confidence in Task Development	3.38	3.25

The Teacher Factors

Table 3 gives the means for Group A teachers with respect to the attitude subscales. A Cronbach's Alpha test of these subscales indicated that once one of the least reliable questions was removed, all the subscales showed acceptable internal consistency (Cronbach's alpha of >0.7), with the exception of the subscale *Value of DT* (0.61 & 0.66 respectively for each survey). The apparent improvement in *DT Confidence* suggested in Table 3 is supported by group interview comments, which also indicate improved PTK and the value of collaboration.

A2: Actually, my ideas have changed a lot when compared with what I had at the beginning. Now I have an idea about what a task is. And also I got an idea to use this sophisticated software which I was not aware of. Like that, I gain some knowledge on all of these.

A3: At the beginning we had a time where we were frightened to work with a computer…It's not a goal of our task to teach the definition. Therefore giving the definition, giving the definition will be done prior to DT work. We are not using DT to teach the definition but to make them understand the variations [this is the word they use].

A1: …we started the work with a doubt. When we working with (an) unfamiliar software and working with these two gents, solutions came from the discussions we had in the group… Firstly we chose the … Then we tried, we have to make this understand for the students….

A1 showed a particular shift in attitudes towards DT, confidence in its use and growing PTK. In his initial, individual interview, he expressed many doubts about the value of DT, continually emphasising the need for students to grasp definitions. When asked about using DT he said, "[b]ut there should be a change in our attitudes. We are still not ready". In the group interview after the intervention, he was enthusiastic about the possibilities of using DT to reinforce students' understanding in other topics, such as complex numbers and applications of derivatives. Similarly A2, when initially asked if he could use DT to design a task responded that "Can't say since I don't have any idea about a task." and A3 too was unclear on the value of DT in mathematics: "I have a neutral idea on that. Sometimes it may affect badly. Sometimes it may help also". Highlighting the value of group collaborative effort, in the interviews they spoke about how "…solutions came from the discussions we had in the group" (A1), "I think working in this group helped us a lot in this activity. If we work individually, even though we write three tasks none of them will be this task will it?" (A3), "[h]ere we all talk together, gave our ideas and we could come to one. It is better than we would do individually. I know that this is much better than I would do by myself. I think the other two also feel the same" (A3), and "I also think that it's better than doing it alone. Because it helps to minimise mistakes and we end up with a rich one, a perfect one" (A2). Prior to the intervention there was no indication that the teachers had goals for the DT task. Afterwards although they expressed these individually, there was close agreement and some goals for the students were: "To comprehend the definition of quadratic function" (A2); "To distinguish the shape of the curve when $a > 0$ and $a < 0$" (A1); "To rearrange the equation of the function using completing the square method" (A3); "To find the shape and the position of the graph when $\Delta > 0, \Delta = 0$ and $\Delta < 0$" (A1); and "To study the changes of the graph with changes of Δ" (A3). The teachers also showed improvements in their own instrumental genesis (Artigue, 2002). In one example of this they related how "we hadn't entered a '*' after 'a' the computer took it as the square of 'ax'. So when we realised that there must be an error we kept searching. We only checked with '*a, x squared*'. First we guessed and then once we entered '*' there it worked" (A3).

The teachers had few ideas before the intervention with regard to the three points framework of Choy (2014). Afterwards they were able to identify the key mathematical concepts of the lesson as the influence of a on concavity and the role of the discriminant in determining the number of zeroes. They also recorded possible points of difficulty, such as rearranging the equation using completing the square, and, most importantly, proposed courses of action to address these, deciding that "[t]he teacher would help in completing the square" and they would "use the words 'open upwards' and 'open downwards' at the beginning so that they can understand" concavity.

Conclusions

Overall we have suggested above that the intervention with the teachers helped to improve their task design. We note that each had a good mathematics content knowledge and as a group their pedagogical experience was also reasonable, with a mean of 10 years. In terms of their PTK what had been added to this? The evidence from the results is that a number of things helped the teachers to design a rich DT algebra task. In particular they:

- Focussed on the mathematics: "…we always tried to give the mathematical concept. We more focused on the mathematical concept. Sometimes when we use a tool the mathematical idea is hidden and the tool would highlight. So this probably would be a GeoGebra lesson."
- Progressed in their own instrumental genesis.
- Had a more positive attitude to using DT and were more confident in its use.
- Were able to focus on goals and to express these clearly. When setting the first task they did not give clear pedagogical goals for students.
- Paid attention to and were sensitive to the thinking of students and how to address their potential difficulties with the task, in response to the three points framework.
- Considered collaborative activity in the task design for a specific lesson very useful.

While this is only part of our research study and there is much more data to examine, it appears that paying attention to the factors outlined above could assist teachers in the DT task design process. Our teachers thought this approach was far better than traditional PD: "It is better to have PD programmes but those programmes must be based on these points. A training in the traditional method is useless, to be honest" (A2) since "the conventional method of conducting a training programme wouldn't work. We go there, get the hand outs, have a tea and come back at about 1.30pm." (A3). It remains to consider the role of the factors above in finer detail and to identify other factors that are important in teacher task design and implementation.

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