Spatial Reasoning Influences Students' Performance on Mathematics Tasks

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Although the psychological literature has demonstrated that spatial reasoning and mathematics performance are correlated, there is scant research on these relationships in the middle years. The current study examined the commonalities and differences in students' performance on instruments that measured three spatial reasoning constructs and two mathematics content areas. There were no gender differences in terms of performance on the three constructs that measured students' spatial visualisation, mental rotation and spatial orientation. There were strong positive relationships between the students' spatial reasoning and mathematics performance (r=0.66), with over 44% of shared variance between the two dimensions. Our study highlights the importance of spatial reasoning in the mathematics curriculum and the necessary promotion of this dimension as a general numeracy capability.

For more than 40 years, spatial "ability" was regarded as an innate intelligence, with such views still prevalent in recent research (Johnson & Bouchard, 2005). Nevertheless, a growing body of cognitive psychologists have argued that spatial ability is malleable and transferrable (Uttal et al., 2013; Wai, Lubinski, Benbow & Steiger, 2010). Moreover, improvement on spatial tasks is found to be transferable to novel stimuli within the same task or to the tasks of the same type (Samsudin, Rafi, & Hanif, 2011). The role of spatial ability has been afforded heightened attention in Australasian curricula in recent years. For example, it has been identified as one of the general numeracy capabilities in the Australian Curriculum (Australian Curriculum, Assessment and Reporting Authority, 2015) and an explicit learning process in the Singaporean curriculum (Ministry of Education, 2006). Nevertheless, we prefer to use the term spatial reasoning to describe the construct, rather than spatial ability since spatial capacities are indeed malleable and can be developed. To some degree, the term "ability" conjures notions of fixed intelligence.

Spatial Reasoning in Mathematics

Spatial reasoning supports our understanding of our geometric world (National Council of the Teacher of Mathematics, 2000). Such reasoning allows us to navigate our surroundings, position furniture in a room, and visualise a diagram when solving a mathematics problem. Spatial thinking is positively associated with mathematics thinking (Battista, 1990); in the sense that those students who perform better on spatial tasks tend to do well on mathematics tests (Rasmussen & Bisanz, 2005). Although the performance relationships are apparent, there also appears to be process relationships that have some overlap. For example, the capacity to locate, orientate, and visualise objects; navigate paths; decode information graphics; and use and draw diagrams are critical to success in mathematics and related fields. Such activities require spatial reasoning, which can be defined as the process of recognising and manipulating spatial properties of objects and the spatial relations among those objects (Mulligan, 2015).

Although there seems to be strong consensus regarding the nexus between spatial reasoning and mathematics in both the psychology and education literature, some

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important questions remain unanswered. First, there is lack of consensus about the elements of spatial reasoning; in terms of the constructs that form spatial reasoning. Second, little is known about the developmental progression of spatial ability from the start of adolescence to adulthood. Some attention has been afforded to the early years of schooling (Sinclair & Bruce, 2014; Davis, 2015) with much more afforded to students at an undergraduate level (Hegarty & Waller, 2005; Uttal et al., 2013). The present investigation is necessary, in order to capture data in the middle years of school.

Spatial Constructs and Differences in Performance

Spatial reasoning is generally regarded in terms of spatial visualization, mental rotation, and spatial orientation. However, these concepts are not always used with the same consistency, due in part to the complex relationships among them (Höffler, 2010). We consider *spatial visualization* to be the ability to "manipulate or transform the image of spatial patterns into other visual arrangements"; whereas *mental rotation* is the capacity to "solve simple mental rotation problems quickly, without imagining the oriented self." (Kozhevnikov, Hegarty, & Mayer, 1999, p.4). By contrast, *spatial orientation* is the capacity to understand and engage with relationships between the position of objects in relation to one's own position (Clements & Battista, 1992). Spatial orientation is also involved in situations where one has to navigate maps and mazes. The difference between mental rotation and spatial orientation is associated with the relationship between the observer and the object being manipulated. For the former the observer is fixed and the object is moved, for the latter the object is fixed and the observer has to move their perspective.

There has been a sustained research base that highlights the fact that boys and girls differ in their spatial abilities (Voyer, Voyer, & Bryden, 1995). These patterns tend to emerge for spatial visualisation (Mayer & Massa, 2003), mental rotation (Maeda & Yoon, 2013) and spatial orientation (Bosco, Longoni & Vecchi, 2004). A range of factors have been presented to explain why boys and girls differ in spatial reasoning, highlighting both learner related factors (such as cognitive variables) and environmental factors (such as activities in which boys and girls are engaged in their daily life). With respect to learner-related factors, most attention has focused on the ways in which boys and girls encode and process information, what is commonly referred to as cognitive style (Arnup, Murrihy, Roodenburg, & McLean, 2013; Mayer & Massa, 2003). Environmental factors include social stereotypes and the different game playing and wayfinding experiences children are exposed to. It was important to consider the role of gender in performance within this study, given the ongoing debate in the psychology and education literature.

Purpose of the Investigation

The purpose of this investigation was to examine the relationship between students' spatial reasoning and their mathematics performance. Specifically, we focused on middle school students, since less attention has been afforded to students moving into adolescence than other categories along the developmental continuum. Two research foci were formulated, namely: (1) examining differences and commonalities in the students' performance across the spatial reasoning and mathematics content constructs, and (2) establishing the relationship between spatial reasoning and mathematics performance.

Method

Participants

Our investigation comprised 181 Year 5 and 6 students (91 boys and 90 girls) in four state primary schools in the Australian Capital Territory. The participants reflected the composition of the local area, typically middle-class families with a diverse cultural and ethnic composition.

Instrumentation

Two instruments were used to collect data for the investigation. One instrument measured three constructs within the spatial reasoning dimension; the second, two content and reasoning aspects of the mathematics curriculum.

Measurement of spatial reasoning. We used the Spatial Reasoning Instrument (SRI) to measure students' spatial reasoning. The Instrument is based on three constructs (spatial visualization, mental rotation, and spatial orientation) and is suitable for administration to students' in the primary grades. The 45-item multiple-choice instrument comprised fifteen items from each of the three constructs. The respective constructs have strong correlations with those commonly used in the cognitive psychology literature (for a detailed explanation of the Instrument see Ramful, Lowrie & Logan, in press). See Appendix, Figures 1, 2 and 3 for representations of spatial visualization, mental rotation, and spatial orientation respectively.

Measurement of mathematics performance. Mathematics performance measures were drawn from released items from Australia's National Assessment Program (NAP). The questions covered topics from number, geometry and measurement, in equal proportions. Six of the items contained geometry and measurement content, and required geometric reasoning to solve. The other six items contained number content, typically requiring number sense or algorithmic problem solving. Consequently, we gained performance measures for geometry and non-geometry tasks.

Procedure

Initial sample selection began with a thirty-minute information session conducted with the principals, randomly selected from across the state. The session provided an overview of the aims and scope of the project, the project procedure and a document that contained ethics approval and detailed information regarding the project.

The Instruments were administrated in two sessions, comprised the spatial reasoning instrument) and mathematics test. Both tests were untimed, however most students could complete each test within 50 minutes.

Results and Discussion

Our investigation was designed to investigate the influence of spatial reasoning on primary-aged students' mathematics performance. Data are analysed in relation to the two research questions.

Investigation 1: Differences and Commonalities in the Students' Performance

The first set of analyses examined: (a) student performance across the spatial reasoning and mathematics performance instruments, and (b) relationships among the constructs. Means and standard deviations for performance measures (see Table 1) and correlations between constructs (see Table 2) are presented below. Five Analysis of Variance (ANOVA) procedures were undertaken to determine whether there were any differences between the mean scores of boys and girls across the two mathematics and three spatial constructs. There were no statistical differences between the performance of boys and girls across the respective test constructs. These results are noteworthy, since there is a strong literature base that suggests boys outperform girls in spatial tests (Reilly & Neumann, 2013), especially within mental rotations tasks (Maeda & Yoon, 2013).

Table 1

Means and Standard Deviations for Measures by Gender

Performance Construct	Means		S.D		*P* statistic
	Boys	Girls	Boys	Girls	*df*(1, 180)
Geometry Score	2.14	2.21	1.16	1.62	p=.745
Non-Geometry Score	2.46	2.24	1.31	1.24	p=.257
Visualization Score	6.16	6.26	2.74	2.38	p=.795
Mental Rotation Score	7.57	7.05	3.25	2.78	p=.254
Spatial Orientation Score	10.58	10.07	2.73	2.90	p=.230

Note. All ANOVAs were statistically insignificant (i.e. $p \geq .05$)

There were moderately high (positive) correlations between students' performance on the two types of mathematics content and the three spatial reasoning constructs. In fact, all of the correlations were statistically significant (at $p \leq .001$ level). It was unsurprising that the correlations between the three spatial constructs were moderately high (ranging from $r=.508$ to $r=.621$), since this has been identified elsewhere in the literature (Wai et al., 2010). The correlations between the geometric mathematics questions and the three spatial constructs were slightly higher (ranging from $r=.484$ to $r=.532$) than they were for the non-geometric mathematics questions (ranging from $r=.424$ to $r=.479$). These moderately strong relationships highlight the potential predictive dimension of spatial reasoning and mathematics performance (Uttal et al., 2013; Wai et al., 2010).

Table 2

Correlations Among Constructs

Performance Construct	1	2	3	4	5
1. Math Geometry Score	1	$.487^{**}$	$.484^{**}$	$.532^{**}$	$.515^{**}$
2. Math Non-Geometry		1	$.479^{**}$	$.444^{**}$	$.477^{**}$
3. Visualization Score			1	$.621^{**}$	$.508^{**}$
4. Mental Rotation Score				1	$.605^{**}$
5. Spatial Orientation Score					1

Note. ** Statistically significant at $p \leq 0.001$ level.

Investigation 2: Relationship between Spatial Reasoning and Mathematics Performance

This component of the study sought to determine the influence of spatial reasoning of mathematics performance. Since the three spatial constructs were moderately correlated, we produced a total spatial reasoning (TSR) measure by summing the student's scores on the spatial visualization, mental rotation and spatial orientation scores. We conducted multiple-regression analysis with the TSR score as the dependent variable. In the first model, we entered gendered as the first block, followed by the mathematics geometry score and the mathematics non-geometry score. This effectively treated gender as a covariate, which ensured that any shared variance was removed from the model. As anticipated, gender was not a significant contributor to the model. The two mathematics scores made a statistically significant contribution to the model, accounting for 44% of the variance (see Table 3). These results highlight the strong relationship between spatial reasoning and performance on mathematics items.

A second regression model was conducted to determine the extent to which the shared variance could be attributed to particular mathematics content. That is, whether the processing and content demands of the geometric mathematics items (in particular) attributed most variance. In this second model, the geometric mathematics score was entered as the first block in the equation, to remove any shared variance. Spatial reasoning performance accounted for 36.5% of the students' geometric mathematics performance. The non-geometric mathematics score made a statistically significant contribution to the model (a further 8%) even after accounting for the geometric mathematics score (see Table 3).

Table 3

Multiple-Regression Analysis Between Spatial Reasoning and Independent Variables

Model	ß	R^2	Change R^2	F (df 1, 180)	F test of change
Gender	-0.66	0.002	0.002	0.39	0.532
Geom. + non-Geom math		0.44	0.44	46.86	0.001
Geom. math	0.45	0.36	0.36	102.8	0.001
Non-Geom. math	0.32	0.44	0.08	70.4	0.001

Conclusions and Recommendations

Although there were statistically significant positive relationships between the three spatial constructs used to measure spatial reasoning, there was sufficient unexplained variance to suggest that each of the three constructs were individually important to defining the spatial dimension. For example, the strongest correlation among the constructs (between spatial visualisation and mental rotation) produced more than 60% of unexplained variance—highlighting the merit of promoting each of these constructs through classroom-based learning experiences. The relationships between each of these three spatial constructs and "non-geometry content" tasks were strong—and similar to that of the "geometry content" mathematics tasks. These data highlight the transferability of spatial reasoning beyond tasks that directly relate to the respective construct. Consequently, we recommend that teachers explicitly teach these constructs; such is their importance to mathematics.

To date, few studies have considered the influence of spatial reasoning of middle-school students' mathematics performance. We found strong relationships between the students' spatial reasoning and mathematics performance, with shared variance much higher than elsewhere in the literature (especially the prevailing studies with adults; Uttal et al., 2013; Wai et al., 2010). Our findings highlight the importance of promoting spatial reasoning in the curriculum, both in mathematics and beyond. Since spatial reasoning skills are developing at this age, and earlier (Davis, 2015), it is likely that such skill development will be malleable. The next challenge is to determine whether explicit training of these three spatial reasoning constructs lead to direct improvements in students' mathematics performance. This can only be achieved through a rigorous pre-post test experimental design (Cheng, Huttenlocher & Newcombe, 2013).

We found no performance differences between boys and girls in any cognitive aspect of this investigation. Given the long-held view that boys outperform girls of spatial tasks (Maeda & Yoon, 2013; Reilly & Neumann, 2013) such results are noteworthy. Elsewhere, we reported that there were no gender differences on Singapore students' spatial visualisation in a comprehensive study of 800 participants (Ramful & Lowrie, 2015). We attributed this to the fact that such reasoning is explicitly targeted in the curriculum—this is not the case in Australia (and therefore the present study). Consequently, we may well be experiencing a new phenomenon, with blurring of gender boundaries in recent years—especially in terms of young children's game play, and technology engagement. This may also point to the fact that differences in spatial performance may be associated with experiences and exposure to spatial constructs, and not some neurological disposition.

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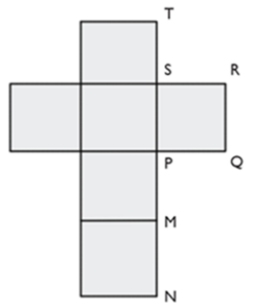
Appendix: Representations of the Three Spatial Constructs

The diagram below shows the net of a cube.



When it is folded to form a cube, which edge joins with edge MN?

A Edge QR

B Edge ST

C Edge MP

D Edge PQ

Figure A. Spatial Visualization

The figure below shows the picture of a bike.

Which one of the following shows a rotation of the picture?

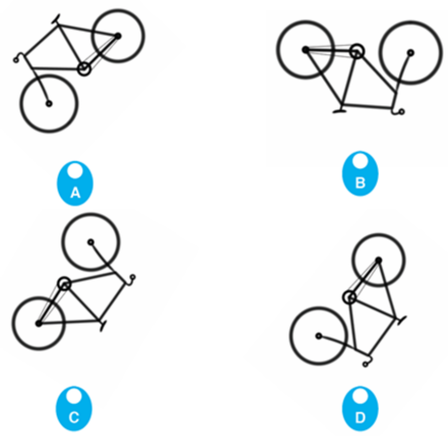
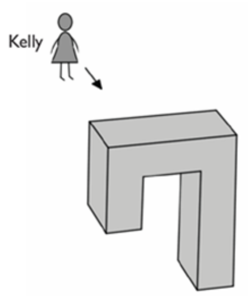


Figure B. Mental Rotation

Kelly was looking at the design below from the position indicated.



What does the front view of the design look like from Kelly's view?

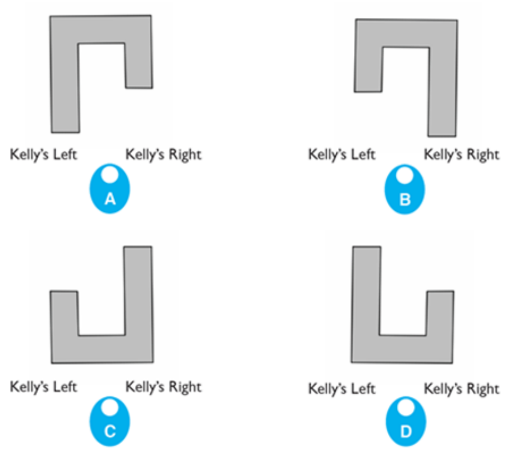


Figure C. Spatial Orientation