Changes in Teachers' Knowledge and Beliefs about Mathematics and Mathematics Teaching: A Case Study

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As teaching is a cultural activity embedded in a unique context, professional development opportunities for teachers may be best placed amid the relevance of everyday classroom practice, and supported by an innovative curriculum. This progress paper reports on data collected as part of doctoral research studying the changes in mathematical knowledge and beliefs of three year 5/6 teachers as they implemented a four-week, innovative curriculum unit. Early analysis of the case of Year 5 teacher Mark pointed to reflection on the pivotal influence of teachers' stated and inferred beliefs about mathematics and mathematics teaching on classroom practice, and the vagaries of didactic contracts in a change environment.

Literature and Theoretical Framework

There is widespread international acknowledgement that ongoing professional development of teachers is necessary to maintain a professional body with the knowledge and skills to improve student outcomes (National Council of Teachers of Mathematics, 2000; Sullivan, 2011). The lack of success of formulaic approaches to professional learning has resulted in calls for a change in perspective; from a focus on the innovations to looking primarily at the perspective of those *driving* them, including teachers and administrators (Doyle & Ponder, 1977; Guskey, 1986). Stigler and Hiebert (1999) referred to teaching as a "cultural activity" (p. 122). Attempts to develop skills and knowledge, then, must be done in the context of the classroom so teachers see changes as applicable to their everyday practice.

The knowledge and beliefs of teachers is an area of considerable current research activity (Beswick, 2011; Liljedahl, 2010). It includes modelling of Shulman's (1987) conceptualisation of teacher knowledge to the teaching of mathematics (Chick, Pham & Baker, 2006; Hill, Ball & Schilling, 2008), investigations into the reported connection between a teacher's beliefs and attitudes about, and of, mathematics, and their classroom practice (Anderson, White, & Sullivan, 2005, Thompson, 1992), and examination of the inconsistencies in teachers' espoused beliefs and enacted practices (Skott, 2001; Raymond, 1997). The relationship between beliefs and knowledge, and its congruent development in relation to curriculum reform has also been raised as an important area of future research (Beswick, 2011). The role of reflection is posited as being crucial for supporting teachers' changing beliefs and practices (Phillip, 2007; Stipek, 2001).

The focus of the present study is on *how* teachers hold mathematical knowledge for teaching. It seeks to illuminate the complexity of the relationship between a teacher's knowledge, their beliefs and extent to which the use of innovative materials yields changes in these. The major research question underpinning the study was: How do teachers' knowledge and beliefs about mathematics and mathematics teaching change as they teach an innovative mathematics curriculum? Interaction with curriculum and support materials can be thought of as an immersion experience (Loucks-Horsley, Love, Stiles, Mundry, & Hewson, 2010) that has the potential to build mathematical knowledge for teaching while in the *act of* teaching. Innovative curriculum tasks put extra demands on the teacher; they tend to be conceptually demanding and unpredictable in nature (no set path of solution). Such

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features require teachers to make connections between students' informal thinking and the formal knowledge of mathematics, possibly prompting high levels of anxiety in both students and themselves (Stein & Kim, 2010).

The Interconnected Model of Teacher Professional Growth (IMTPG, Clarke & Hollingsworth, 2002) supports interpretation of mechanisms that trigger change (or growth) in one or more of the four domains of a teacher's world; personal, external, of practice and of consequence in this study (see Fig. 1).

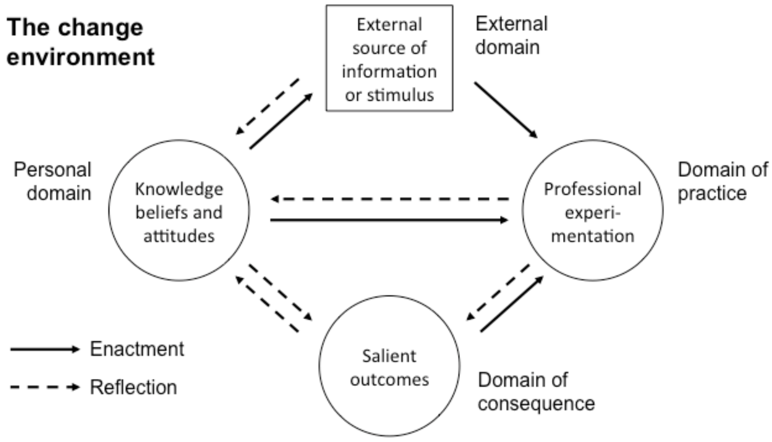

Figure 1. The Interconnected Model of Teacher Professional Growth (Clarke & Hollingsworth, 2002)

The model provides a lens through which to describe interaction between domains in a particular change environment, the mediating processes of enactment (putting a new idea into action) and reflection, and the resultant (professional) growth. It aligns with an increasingly accepted view of change as professional growth, acknowledging teachers as active and reflective learners, socially situated within a learning environment. The IMTPG provided the theoretical framework for the present study. The external source of information or stimulus was the expectations (outcomes) of the Australian mathematics curriculum, and the innovative curriculum unit. Professional experimentation via the innovative curriculum unit drove the domain of practice.

Research Design

A theoretical perspective of interpretivism was employed as this study aimed to draw in-depth understanding of a teacher's world and the interpretations they made in this context (Merriam, 1998). Case study was the adopted methodology to best describe and understand the complex interactions and multiple realities of teachers as they implement curriculum in their mathematics classrooms.

Data Generation

Three Stage 3 (Year 5 and 6) primary teachers with at least five years of classroom experience, all at the same New South Wales government school agreed to participate in the study; a deliberate part of the research design to minimise differences caused by change of school context. The teachers agreed to teach a four-week rational number unit *Some of the Parts* (Mathematics in Context, 2006), based on the principles of Realistic Mathematics Education (see, e.g., Streefland, 1991).

One of the teachers, "Mark," is the focus of this paper. In the pre-intervention phase, Mark completed 2 questionnaires developed as part of the 17-country Teacher Education and. Development Study in Mathematics (TEDS-M) (Tatto et al., 2008). The first assessed Mathematics Content Knowledge (MCK) and Pedagogical Content Knowledge (PCK), measuring knowledge at least two years beyond the level the teachers were expected to teach, in the cognitive domains of knowing, applying and reasoning. The second was a beliefs questionnaire presented in a Likert scale of agreement in relation to beliefs about learning mathematics, about mathematics achievement, and the nature of mathematics. A background form was also completed to gain information about Mark's personal mathematics history, and professional development.

To gain a sense of current practice prior to the use of the innovative curriculum unit, there was an initial two-week lesson observation phase, in which three lessons were observed and detailed field notes taken. Semi-structured interviews followed each lesson to gauge how the Mark felt the lesson went, how it matched his intentions, moments he felt were significant, decision-making junctures, what he learned, and what he believed the students learned. Reflections on Mark's practice were made using a classroom observation record used by Copur-Gencturk (2015), addressing lesson design and implementation, mathematical discourse and sense-making, task implementation, and classroom culture.

During the five-week intervention phase, lesson observations included voice recording of the teacher. These audiotaped lessons were transcribed, supported by field notes. After each lesson, semi-structured interviews were conducted. An in-depth reflective interview was held with the teacher at the end of the five weeks.

Data Analysis

Making sense of the data in order to address the research question relied both on direct interpretation and categorical aggregation. Constant comparison began with initial observations, responses at interview, notes made through direct observation of lessons, and results of the TEDS-M instruments. Interviews were fully transcribed and coded using NVivo 11. Coding was guided by the overarching research question and connected "sensitising concepts" (Patton, 2002), informed by the relevant academic literature. Preliminary categories located via axial coding helped form a conceptual structural order that facilitated further analysis. These central themes form the basis for the research report "story" that attempts to address the research questions. In the following section, we report results from "pre-observations and interviews" prior to the use of the innovative curriculum, followed by results during its use.

Results and Discussion

Year 5 teacher Mark was very experienced, with 37 years of primary school teaching at the time of the study. Mark had only taught Years 5 and 6, apart from three years on Year 3 and 4; he had been on his current teaching level for the past 16 years. Mark said many times that he enjoyed mathematics and teaching mathematics. He had not sought further formal professional learning since his initial training in 1978. He had completed only 2 days of professional learning related to mathematics teaching in the previous five years. Mark reported on his background survey that he was highly confident in his knowledge of content and teaching mathematics, having taught this subject in middle school and tutored students studying mathematics in their HSC (Year 12). Mark was very self-assured about his ability to answer the questions in the TEDS-M assessment, and had allowed a short time frame to finish it. He was thoroughly engaged during the assessment, commenting intermittently that he liked a challenge: "this is difficult but it is fun" (11.6.15).

The Initial Observation Period

Two of the three initial observation lessons were directed by the use of the mathematics textbook favoured by Mark and his grade partner. He said he liked this textbook because it was "a higher level than a normal textbook… so the examples are more difficult" (18.6.15). Each unit in the textbook covered a different concept each day. Problem solving was presented in the textbook as an isolated lesson each week, rather than integrated. Lessons began with a 10 x 10 "drill grid," in which students combined the chosen operation. This activity was timed, with incentive given to improve on previous personal best times. The aim was "to get as many right as possible" in the fastest time. After whole class marking, comments related to time/error rate were sometimes made with suggestion that students not only need to be quick, but accurate. This part of Mark's regular lesson structure fitted with his professed beliefs about learning mathematics in relation to the TEDS-M beliefs survey prompt "to be good at mathematics you must be able to solve problems quickly," to which he indicated "slightly agree."

The purpose of the drill grid was to practise number facts and look for patterns, but was also for his "competitive" students – the students who were good at mental computation and could do it quickly. Mark said students liked the drill grid, "as old fashioned as it is" (18.6.15) because it is competitive, and asked for it if he missed it one week. "Boys like competition," he continued, "and the top kids in my class for maths are boys" (18.6.15).

Knowing the "bottom end was never going to finish whether I make it easy or hard." Mark set the same drill grid numbers for all students so at least they "feel like they are doing the same things, you know, as the clever kids, so they improve" (18.6.15). It was evident that there was also a school emphasis on grouping students into levelled categories and teaching to this level in mathematics.

The Copur-Gencturk (2015) classroom observation record suggested a class environment where an emphasis on investigative mathematics was low—one where students complete questions with proceduralised demands that promote little opportunity for in-depth investigation of "big ideas" in mathematics. The lack of context and absence of connection to ideas addressed in the previous lesson did not seem out of the ordinary. The discourse and management of Mark's class in these lessons was highly teacher-directed. Students were told to work on their own, not to discuss their work with others. When they had completed a question, they were to fold their arms and wait quietly for the next direction. Mark said he valued the process of working out problems, and wanted to be able to see the students' thinking in their working out (18.6.15). This was not obvious in his pre-intervention lesson practice. The hierarchy of gaining the right answer was high and opportunity to explore errors in thinking taken away from the students. Deep probing of mathematical ideas was driven by Mark's questioning, depending on what *he* wanted the students to focus on.

The didactical contract (Brousseau & Warfield, 1999), or reciprocal expectations of teacher and students, was evident from the first pre-observation lesson. According to Brousseau, this contract rules the relationship of students and teachers. How Mark's class interacted with the task was influenced by *his* perception of the difficulty of the task and how the students would cope with this. He adopted most of the responsibility for the task and highly-managed didactic interplay. The students' responsibility was to attempt the task presented in the given time frames, understanding that there would be little autonomy to control their own actions beyond completion of the set task. Students did scribe their working out and answers on their board, however the power in the collaboration still stood with Mark. It was Mark that pointed out errors in setting out, working out and answers. It was predominantly *his* reasoning that worked through corrections, taking over the mistakes of others to insert his own reasoning for the students to see. There were few surprises or challenging episodes in this predictable and supervised agreement across all stakeholders.

Mark described his management of tasks situations as "intuitive," with little planning needed (18.6.15). At first, the post-lesson reflective questions seemed to be quite uncomfortable for Mark and needed some rephrasing. He found the first prompt about significant moments in the lesson hard "because I don't really take much notice about what I am doing" (23.6.15). "It is not really significant, it is just what happens." By far the most difficult question for Mark was about the learning opportunities for himself. "For me?" he asked and paused, "I don't really learn many things new."

PCK informs teacher decisions, in particular Knowledge of content and students (KCS) and Knowledge of content and teaching (KCT) (Hill et al., 2008). Results from Mark's TEDS-M assessment indicated that he was approximately at the 58th percentile for mathematical content knowledge, and the 96th percentile for pedagogical content knowledge. It seems he had the *knowledge* of how to evaluate the instructional advantages of particular representations (KCT) and *knew* which questions students may find difficult or easy (KCS) – this was demonstrated in his high PCK assessment, but observation of his classroom practice suggested that he did not give the power of such knowledge over to students in his own classroom. He knows what he *could* do in relation to scenarios given, like those in the TEDS-M MCK and PCK assessment, but chose not to in his daily practice. Knowledge and enaction seem to be quite separate. Such behaviour can be seen to have some ancestry in Mark's beliefs; on the TEDS-M survey he agreed with the statements "students learn best by attending to the teacher's explanations" and "students need to be taught exact procedures for solving mathematical problems."

Teaching the Innovative Curriculum: Fidelity, Engagement, and Change

When presented with the innovative curriculum unit for the first time, Mark poured carefully over each page. He took notice of detail like different units of measure, and tried to see how each lesson flowed on to the next. Mark was wary of how much content needed to be addressed over the four weeks and was concerned that his students would find it difficult. Mark made a considered attempt to maintain the pace the curriculum suggested: "People (curriculum designers) have done this and divided it up for specific reasons so I don't need to go and change the whole thing, so I don't mind following that" (20.7.15). This trusting relationship with the philosophy and lesson sequencing was maintained throughout.

While his fidelity to the content was apparent; his fidelity to the processes was not so consistent. There was lack of adherence to the underlying premise that students work from informal to formal representations by way of shared reasoning and problem solving. Each activity was sectioned off by Mark one at a time via whole-class direction; there was no self-pacing of activities as students' understanding emerged. The relevant, and provided, student book was not given to students. Only photocopies of selected pages were given out. At times, this led to a focus on constructing models (how to draw a rectangle) rather than using them as a thinking tool. To represent what the students had done, a 'booklet' was compiled of collected work. When asked why this work wasn't done in their mathematics grid book, Mark said it would get too mixed up with the other work they were doing. This indicated that Mark taught mathematics to the students outside of the allocated observation time. When asked about this, he said he needed to "cover" the rest of the curriculum outcomes.

While the didactic relationship of how he and the class would manage content delivery was maintained during the *Some of the Parts* unit, the change in expectation of how *problems* were presented and the concurrent *reasoning* required, created tension. The visual models featured in the unit promoted reasoning about relative sizes of fractions, equivalencies, and multiplicative relationships. Students were expected to fold rectangular models, and use relevant fraction strips to informally add and multiply fractions. The ratio table was then introduced to multiply and divide benchmark fractions. Realistic contexts were used to promote the connection of these *models of* situations to *models for* computational thinking. Mark was pleased with the engagement and increase in confidence of students "who don't usually get it" (20.7.15). He reflected: "It's something that they think they're not good at and all of a sudden they start to understand it and they think this is not as hard as I thought… so it's not like you whack a fraction up on the board and they go, Oh my God it's a fraction. You're talking about subs [sandwiches]…"

The difficulty that Harry, his "best mathematician" had with these models early in the unit was perplexing for Mark. Harry found the modelling difficult and was often off task. This irritated Mark. He put the behaviour down to laziness, and preference for mental calculations over hands-on activities as "it's not where he can use his mind, he has to physically do something and that's where he falls apart… and his understanding drops too" (21.7.15). Mark told Harry he needed to be more creative. Such tense interactions gave him "more of an insight into what my clever kids can't do," to look at more hands-on activities as well as abstract ones. He continued to push Harry to "think, act, participate, come up with some good solutions" (28.7.15). Mark had reduced the frequency with which he would call on Harry to answer questions in class, choosing another of his "clever" students – one that was quieter and complied with his requests for reasoned answers. Mark started to reflect on what he needed to do to engage Harry more. Initially he reverted to what he considered competition-like strategies (making up a hard question for another clever student to do) because Harry liked being the best. But by lesson 7, the contract agreement had shifted a little: Harry was involving himself more in reasoning about questions and Mark was praising his effort over speed. Such reconnection was not sustained; in lesson 12 Mark was admonishing Harry again for racing ahead and making errors, i.e., that $3/4 + 3/4 = 6/8$.

A teacher's view of mathematics affects their espoused model of teaching and learning mathematics, and therefore their enacted model of the same (Thompson, 1992). Results from the TEDS-M beliefs survey suggest Mark held a problem solving view of mathematics. He agreed with such statements as "time to investigate is time well spent," "students can figure out a way to solve a problem without a teacher's help," and "teachers should encourage students to find their own solutions even if they are inefficient." Direct observation and lesson reflection data suggested his views are far closer to an instructor and explainer role in the classroom, associated with skills mastery with correct performance, and learning as the reception of knowledge. "Maths doesn't change, maths is maths," Mark reflected when discussing professional learning opportunities he found useful (14.8.15). This disparity between a teacher's espoused and enacted models of teaching and learning mathematics has been raised in other case studies (Cooney, 1985), citing the powerful influence of social context, and the teacher's level of consciousness about his or her own beliefs and the ensuing reflection on practice. In Mark's case it appears the school context complemented his beliefs about learning mathematics; students were grouped according to ability and textbooks were employed as the major presenter of content. Any challenges to pedagogical approaches in mathematics through professional learning were generally rejected. Reflection on knowledge was much more present than reflection on beliefs during the research period. Knowledge systems are more likely to be open to evaluation and critical examination than beliefs systems (Pajares, 1992). Mark reflected on his knowledge of different models to teach fractions during the innovative curriculum implementation, and was very positive about how they had contributed to his teaching of fractions: "This is definitely of assistance to me… I think I've probably done a better job in teaching fractions than I normally would have done without it" (19.8.15). Mark followed the content and modelling of the curriculum as closely as he could, making it "fit in with his teaching" (19.8.15). Being a teacher of such longevity, an embedded individual schema about how mathematics should be taught is unsurprising. Mark was open to change in relation to how to teach *fractions*, but was not open to suggestions about how to change his approach to *teaching in general*.

Conclusion

Analysing Mark's responses to the innovative curriculum through the IMTPG model (Clarke & Hollingsworth, 2002), it can be said that Mark adopted a growth perspective in relation to his practice and beliefs about how the teaching of fractions can be addressed in the classroom. Professional experimentation with innovative curriculum models gave Mark frequent positive feedback about how confident and engaged most of his students could be when addressing concepts such as equivalence. This was particularly salient for him. Through this experimentation, Mark felt his knowledge of students and teaching had increased. He was surprised (and delighted in this surprise) that students he thought were low mathematics achievers were participating during class conversations and demonstrating conceptual understanding about multiplicative relationships. He valued the use of concrete materials to assist this understanding. Such experiences also gave him insights into students he considered his "best mathematicians." Mark indicated he would like to use the curriculum in the future for these reasons. He saw the experience as an "individual journey;" there was little collaboration with the other teachers also using the curriculum. Opportunity for reflection beyond that directly addressed post-lesson was therefore narrowed. This was partly because the other case study teachers moved at such different rates through the curriculum unit – one much faster and the other much slower than the unit recommended.

Beliefs that touch on an individual's identity and self-efficacy in the classroom are tightly held and resistant to change (Phillip, 2007). Mark's belief in his role as instructor and explainer was robustly held; such positioning was well understood by his class. The benefits of the innovative curriculum content and models *were* assimilated into Mark's existing belief structure as this was an area about which he was open to new ideas. Mark made it clear that he did not respond to most professional learning opportunities offered as he was not prepared to try activities and advice given to him by those he considered less knowledgeable. Innovative curriculum that spoke *to* him as a practitioner rather than *through* him gained respect akin to a knowledgeable other, and offered him the opportunity to receive incremental and contextual challenges. Complemented with opportunities to reflect on the observable changes in student knowledge, understanding and engagement, the innovative curriculum connected what Mark knew to what he was learning, making it meaningful. Data analysis for Mark and the other two teachers continues.

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