FRACTANGI: A TANGIBLE LEARNING ENVIRONMENT FOR LEARNING ABOUT FRACTIONS WITH AN INTERACTIVE NUMBER LINE

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ABSTRACT

Tangible user interfaces (TUIs) are frequently used to teach children abstract concepts, in science and mathematics. TUIs offer a natural and immediate form of interaction that promotes active and hands-on engagement and allows for exploration and reflection. Tangible objects are representational artifacts in their essence, and they increase the representational power, which is a much needed quality in fraction teaching. By exploiting research into the external representations for learning, tangibles and embodied learning, we have designed an interactive tangible number line, named Fractangi, as a conceptual metaphor for helping students to understand and exploit fractions by acting with their hands. A pilot study was conducted in a Department of Primary Education with 65 undergraduate students in order to extract preservice teachers' views on the usability and learning effectiveness of the proposed tangible interface. Preservice teachers' views were extremely positive, in regards both to the context of usage (tangibility, gamification), and to the content of the interaction (learning interactions were considered better than traditional forms of teaching). Fractangi seems able to transform an unpopular learning topic to an enjoyable learning experience.

KEYWORDS

Tangible User Interface, Fractions, Number Line, Embodied Learning, Ubiquitous Computing

1. INTRODUCTION

Many researchers have suggested that tangible user interfaces (TUIs) have a great potential for supporting children's informal and formal learning because they leverage both familiar physical artifacts and digital computation. Tangibles are frequently used to teach children abstract concepts, in science and mathematics (Manches et al. 2010, McNeil et al. 2009). For example, Button Matrix (Cramer & Antle, 2015) uses coupled tactile, vibration and visual feedback to highlight features of a physical experience with arithmetic concepts and cue reflection on the links between the physical experience and mathematical symbols. Tangibles are able to offer a natural and immediate form of interaction that is accessible to learners, promotes active and hands-on engagement, encourages exploration and reflection, provides learners with tools to think with, enables learning abstract concepts through concrete representations while also offering collaborative opportunities for learners (Antle & Wise, 2013). Research has highlighted that technological tools for children need to be exciting, support exploration, inspire creativity, grow curiosity, stimulate interaction, and collaboration with peers, while being intuitive to use (Sylla et al. 2015). These are exactly the opportunities offered by TUIs for education through their natural support for collaborative activities, physical interactions, and external representations (Schneider et al. 2011).

While similar efforts have been made in many fields of mathematics, just few educational applications aim at learning about fractions. Fractions knowledge is a difficult to grasp domain, given the complex conceptual content and the frequent students' misconceptions. In this study, we have created a tangible learning environment called Fractangi, which aims to help students understand fractions with the help of a tangible interactive gamified number line. After a brief literature review, we will describe the design and operation of the tangible explorative interface, and then the results of a pilot study.

2. LITERATURE REVIEW

2.1 Learning about Fractions

Fractions instruction is usually a challenge for the teachers, since it is an object with complex conceptual content. Main fraction knowledge include the facts that fractions represent a part of an object or parts of a set of objects, they can be represented by fractional symbols, and that they are numbers that reflect numerical magnitudes (Jordan et al, 2013).

However, students often meet difficulties, as they cannot overcome the belief that the whole number properties are not applicable for all numbers in all cases and, even in high-school, many students are unable to understand that there are infinite numbers between any two fractions (Vamvakoussi and Vosniadou, 2010). In addition, many students encounter fractions as "meaningless symbols" or view numerator and denominator as separate numbers and not as a unified whole (Fazio and Siegler, 2011). Apart from these, some weaknesses in conceptual understanding can also be identified by the variability of strategies used by students, even in the case of the same problems and procedures (Lemonidis, 2015; Siegler et al, 2013). Due to such misconceptions, many teachers tend to work with fractions entirely as part-whole concepts (Lee, 2012). However, it is difficult for students to reconcile the part-whole model with the fact that fractions are continuous and infinitely divisible (Riconscente, 2013) and, consequently, more learning obstacles appear.

Among various teaching approaches suggested in contemporary bibliography, the most effective method to understand that fractions are numbers seems to be their representation on a number line. As Siegler et al, (2011) state "numerical development involves coming to understand that all real numbers have magnitudes that can be ordered and assigned specific locations on number lines”. As a result, the number line model is suitable for fractions comprehension, as it offers significant benefits compared to other representational models. For instance, it offer advantages over the area models (such as pizza and rectangles) since it is much easier to divide the whole into equal parts; only length is involved, and hence addition and subtraction of fractions are more easily modeled on the number line (Wu, 2011).

Consequently, the number line is an advantageous tool for the understanding fractions as number magnitudes and that’s why we selected this representational approach to become the basis of our tangible environment.

2.2 Tangibles and Embodiment in Mathematics

Mathematical cognition is embodied in two senses: it is based on conception and action, and is founded on the natural environment (Alibali & Nathan, 2012). Original human experiences are with objects met in nature and not with symbols. So, from the perspective of embodied learning, people interactions with physical objects create the foundation for subsequent "construction" of non-physical entities, contained in formal mathematical definitions (Moore-Russo et al, 2014). For example, when students talk about the concepts they learn, they often express new knowledge with gestures and bodily expressions proving that gestures are an integral part of communication about mathematical concepts (Abrahamson et al, 2012). So, when a student "becomes a thing itself," researchers assume that has a completely different kind of experience from a student who just watches, because the embodied learning promotes links between physical activities and mathematics, in a way that observation cannot do. As Birchfield claimed (2015), embodied learning is kinesthetic, multimodal and collaborative. In the definition above, the term "kinesthetic" means that each student interacts with the physical space using movements of his entire body, while the term "multimodal" refers to the fact that students see, hear and touch their "experiences". According to Moore-Russo, Ferrara & Edwards (2014), the motor system is involved in learning in diverse ways such as whole body movements, gestures, gaze, head movements, body postures, object manipulation, rhythm, etc. Similarly, the means of interaction can be tangible, such as voice, hands and body, or external, such as a computer screen, pencil, electronic devices, manipulative material for mathematics etc.

Physical objects have been traditionally used in kindergartens and elementary schools to introduce young learners to abstract concepts such as quantity, numbers, base ten, etc. Interestingly, there is another stream of research which supports that physicality is not important and offer evidence where children do not transfer performance from physical to symbolic representations of problems (McNeil et al 2009). However, a recent meta-analysis found that the use of physical manipulatives in math education tends to improve retention, problem solving, and transfer (Carbonneau, Marley, & Selig, 2013). Additionally, the context of use seems also to have detrimental effects. For example, unconstrained physical manipulation is probably suboptimal for learning (Stull et al. 2013) or high interactivity may be overwhelming and also lower learning performance (Stull et al. 2012).

Summing up, tangibles seem to offer great opportunities for learning since they constrain input alternatives and thus reduce modality on the interface, they promote sensory engagement which is the natural way students learn, they facilitate spatial tasks through the inherent spatiality of their existence, they offer opportunities for coupling the control of the physical object and the manipulation of its digital representation and they promote group learning by providing a multi-hand interface that does not give control to one person.

3. FRACTANGI: INTERACTIVE NUMBER LINES

Tangible objects are representational artifacts in their core essence, and combine some form of external representation with physical objects. Hence, they increase the representational power which is a much needed quality in fraction teaching. By exploiting research into the external representations for learning, tangibles and embodied learning, we have designed an interactive tangible number line as a conceptual metaphor for helping students to understand and exploit fractions by acting with their hands.

The tangible interface consists of a rectangular wooden structure with a length of 130 cm and width of 30 cm. On the bottom side, there is a lane with metal points along its length and a little wooden car, which can be moved along the lane and is fastened with a strap that resists moving and always returns it to its initial position. All along the main lane there are 14 points, which act like "buttons" and can be clicked by players to indicate the respective answer during the game. Above the first lane, there are 3 colorful ones which correspond to number lines divided into 3, 4 or 5 parts. In this area there is also a vertical “marker” that can be moved right or left to enable the players to study the equivalences between the different number lines. The basic number line is located just above the first lane and ranges from 0 to 2. Similarly, under the main lane, the distance is also presented in meters, so as to offer various symbolic representations.

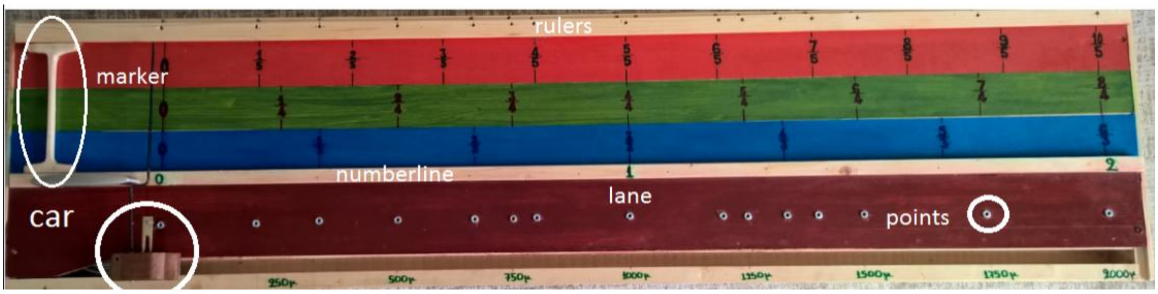

Figure 1. Fractangi

The game script describes that 20 runners from various countries had begun a race of 2 kilometers, which stopped due to heavy rainfall. At that time, each athlete had reached a specific position which was written down in cards. Students have to become the drivers of the wooden car and carry each athlete from the start to the point where they had stopped. Their runners’ last position is given as a fraction of the main lane or in some cases as a fraction in relation to the position of other runners.

Athletes are depicted on the cards together with a name, the flag of their home country, and their last position as a fraction, a decimal number, and/or another virtual representation. The runners’ names rhyme humorously with the names of the countries, so as to offer a more appealing experience to the students. Students have to put the card on the back of the car, transfer the athletes to their last position and press the metal protrusion located in the side of the car in order to check their answers. The device either congratulates the student or gives her feedback for finding the right answer. When the students find the correct answer, they place the card in holes at the upper edge of the construction, so as to be able to compare it later with the other runners.

The construction exploits cheap prototyping hardware (Makey Makey) and is controlled by a Scratch 2.0 program so as to be easily replicated by school instructors.

Figure 2. Fractangi Playing

Figure 3. Samples of Cards with Runners

The game’s objective is to engage the students in a variety of actions and cognitive processes, such as placing simple and improper fractions on number lines, interacting with various representations of fraction magnitudes (areas, decimal numbers and bar charts), comparing fractions, converting fractions into other equivalent representations and, finally, making operations with fractions (mainly addition and subtraction).

4. METHOD

A study was conducted in order to extract preservice teachers’ views on the usability and learning effectiveness of the proposed tangible interface. The study was conducted in the context of a tangible and mixed reality educational spaces exhibition. Sixty five students of a Department of Primary Education participated in the study, 14 men and 51 women. The sampling was random and voluntary from the perspective of participants.

Every time a group of students approached the researchers, they explained the game rules and, then, asked them to get involved in the game. Participants usually played the game in groups of 3 to 4 people, but in few cases individually too. The mean usage of the environment was 20 to 30 minutes.

We employed both quantitative and qualitative measures. A questionnaire with 10 Likert type questions was used for evaluating the interface in 5 axes: usability, innovation, enjoyment, addressing learning objectives and future use. Ten of the participants voluntarily participated to a semi-structured interview process. The interview questions focused on the differences between the learning processes promoted by Fractangi, compared to traditional forms of teaching fractions.

5. RESULTS

5.1 Quantitative Data Analysis

The following table (Table 1) presents 65 students' responses in the closed-type questionnaire. The quantitative evaluation of the tangible environment was very positive. It can be concluded that the overwhelming majority of the students considered the tangible interface as useful, easy and enjoyable, both for themselves and their prospective students. And this becomes even more important, if we consider that fractions are a difficult and unpopular subject. Preservice teachers claimed that Fractangi is able to achieve its learning objectives, can help student exercise with fractions and fits well to the school's curriculum. The vast majority of the students also claimed that the interface offers interactions that were not possible in the past, provides understanding that is not easily achieved with traditional teaching means and, also, the feedback provided for the students' actions is satisfactory. It is particularly encouraging that students not only stated that they would like to test Fractangi in the classroom, but also that they would like to be in a position to create their own respective interfaces.

Consequently, preservice teachers' views for Fractangi were very positive, in regards both to the context of usage (tangibility, gamification), and to the content of the interaction (better learning interactions than traditional forms of teaching).

Table 1. Students' Responses to the Statements of the Questionnaire

Fractangi:	Totally Disagree	Disagree	Neither Agree Nor Disagree	Agree	Totally Agree
The use of the device was difficult.	2 (32,3%)	32 (49,2%)	7 (10,8%)	4 (6,2%)	1 (1,5%)
Students will find the interface enjoyable and they will be actively engaged with it.	0 (0%)	0 (0%)	0 (0%)	25 (38,5%)	40 (61,5%)
The learning objectives regarding fractions can be achieved through Fractangi.	0 (0%)	0 (0%)	1 (1,5%)	29 (44,6%)	35 (53,8%)
The device addresses the school's curriculum aims.	1 (1,5%)	0 (0%)	7 (10,8%)	34 (52,3%)	23 (35,4%)
The device allows students to make mistakes and gives them appropriate feedback.	0 (0%)	3 (4,6%)	4 (6,2%)	36 (55,4%)	22 (33,8%)
Students will probably prefer traditional teaching methods, as they will learn more that way.	35 (53,8%)	25 (38,5%)	1 (1,5%)	3 (4,6%)	1 (1,5%)
The device provides knowledge in such a way that could be hardly achieved in the classroom by traditional means.	0 (0%)	5 (7,7%)	11 (16,9%)	29 (44,6%)	20 (30,8%)

The interface is innovative and provides interactions that were unachievable in the past.	0 (0%)	0 (0%)	1 (1,5%)	31 (47,7%)	33 (50,8%)
I would like to experiment the device with students.	0 (0%)	0 (0%)	1 (1,5%)	27 (41,5%)	37 (56,9%)
I would like to be able to create such devices.	0 (0%)	0 (0%)	0 (0%)	20 (30,8%)	45 (69,2%)

5.2 Qualitative Data Analysis

The questions posed to the ten interviewees aimed at gathering qualitative data for the five axes of the questionnaire.

Regarding the innovation of the interface and its differences to traditional teaching methods, all students claimed that they find Fractangi very innovative. Also, they supported that Fractangi will be even more enjoyable for students thanks to its playful nature, which will undoubtedly stimulate children's interest, and will motivate them.

"It's very innovative and different in comparison to just listening to the teacher"

"It is innovative and quite helpful… it is a game, in which children will strive to participate, it will capture their interest and will become a creative activity"

"It is pleasant, interesting and has a lot of fun"

When participants were asked about whether a student could learn about fractions with Fractangi, everyone responded positively. Two undergraduates claimed that the tangible device probably is more intended for practicing in fractions, so they would use it complementary to conventional teaching in classroom.

"Students can obtain an enhanced sensitization of numbers, since they see the fractions in front of them"

"They see in front of them what the fractional unit is as well as its relationship with other fractions, so they will develop a better understanding"

"The game involves distances, which are familiar to students, and that will help them a lot to learn"

"It helps kids to learn, because although it is an embodied game, it is not abstract but focuses on specific learning objectives. I would definitely prefer Fractangi as a supplement to traditional teaching in the classroom"

It is important that although participants found the tangible device easy to use, interesting and playful at the same time, they said that the activities demanded a lot of cognitive effort.

"You stress your mind, but that's the point"

"There is an inherent difficulty in fractions, but that's the case. You aren't just placing the athletes; you have to work your mind in order to do it"

"…it cleverly asks students to simplify fractions and do fraction operations, since it asks them to calculate distances, by saying 'he is in front of somebody by a fraction'"

Some undergraduates confronted difficulties in putting the athletes in the right position, while during practicing, they also clarified issues around fractions that they had not comprehended in the past. The representations of the fractions were considered adequate and useful, both the ones that existed on the device and the ones presented on the cards.

"It offers good representations, you can see the equivalences and you can easily make conversions and simplifications."

"The cards are necessary and help a lot, because you can see the comparisons between runners and you can understand the relationship between fractions"

When asked to consider changes that should be made in order to improve the tangible device, all ten participants answered that they wouldn't change anything, while some argued that Fractangi should be even sold as a commercial game.

"It is very impressive, it has nice colors and it immediately caught my attention."

"It is robust because it is made of wood and you are not afraid to play. It could be sold as a toy. It is very well made"

A small concern stated was that the large size of the device may impede its usage for shorter kids. However, this can easily be overcome since two or more students can work together in choosing the position by just touching each other. As expected, a participant underlined that one device alone cannot address the needs of a classroom as its usage is intended for 4-6 individuals. He argued that if there are no other devices, that could be a deterring factor for using it in a classroom.

An important observation of the authors was that some participants, who faced considerable difficulties in carrying out the required operations with fractions, could continue its usage only with the explicit help of the researchers. Hence, if there was no presence of a researcher, some players would probably "stuck" at a point and would not manage to complete the game successfully. This means that despite the positive assertions for the feedback system, there is room for improvement. The device aims at letting the students to become autonomous in their play; the interaction should occur between them and the device, without requiring the researcher's or teacher's presence during the game play.

6. DISCUSSION

Low cost rapid prototyping hardware together with the uprising trend of arts and crafts fairs, tinkering and inventing have created a new trend of creating tangible technologies for learning. Researchers but also teachers and students can create or replicate tangible devices as Fractangi easily and, hence, the focus now is on the effective exploitation of physical interfaces or on identifying adequate embodied metaphors and realizing them into interaction models. Our pilot study indicated that Fractangi is a tangible environment that manages to transform exercising with fractions to an enjoyable and effective learning experience. Preservice teachers were enthusiastic with its instructional possibilities and also wanted to be able to create such devices by themselves.

Our study has several limitations. The most important one is that we present the perceived evaluations of preservice teachers. Although their views are a good indicator for the acceptability of the proposed interface from teachers, they cannot offer definite answers for the learning effectiveness of the device for students. Additionally, we do not analyze the underlying embodied mechanism for learning. Our hypothesis is that the Fractangi provokes students to develop a significant number of gestures that help them codify and understand better fractions and their equivalences and operations. This is the basic aim of our future research with Fractangi.

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