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Title: A Replication Study to Evaluate the Effects of Schema-Based Instruction on Middle School Students' Proportional Problem-Solving Performance

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Abstract Body

Limit 4 pages single-spaced.

Background / Context:

Ratio and proportional relationships are of primary importance during the upper elementary and middle school grades (Kilpatrick, Swafford, & Findell, 2001; National Council of Teachers of Mathematics, 1989, 2000; National Mathematics Advisory Panel [NMAP], 2008). These relationships, along with the interrelated topics of fractions, decimals, and percent provide a critical foundation for algebra (NMAP, 2008). Proportionality involves the concept of ratio and is central to topics in mathematics such as linear functions, scale drawings, similarity, trigonometry, and probability. Proportionality is also at the core of many important concepts in the physical and social sciences. As such, the *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989) considered proportionality to be "of such great importance that it merits whatever time and effort must be expended to assure its careful development" (p. 82). In the Common Core State Standards (CCSS; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010), instructional time focused on proportionality occurs in middle school when students "develop understanding of proportionality to solve single and multi-step problems … solve a wide variety of percent problems, including those involving discounts, interest, taxes, tips, and percent increase or decrease" (p. 46).

Solving even simple proportion problems is challenging for many children and adolescents when they lack understanding of the problem situation and whether a solution strategy is applicable (Adjiage & Pluvinage, 2007; Fujimura, 2001; Lamon, 2007; Lobato et al., 2010; Miyakawa & Winslow, 2009; NMAP, 2008; Tourniaire & Pulos, 1985; Weinberg, 2002). Although students require instruction that supports the development of underlying concepts and flexible procedures to solve proportion problems (NMAP, 2008; Tourniaire & Pulos, 1985), few intervention studies have been conducted to improve students' learning of ratios and proportions. Most of the existing studies were short-term and did not focus on the broad domain of ratios and proportional relationships (Adjiage & Pluvinage, 2007; Fujimura, 2001; Myakawa & Winslow, 2009) or used quasi-experimental research designs or a teaching experiment, which limited causal inferences. Also, few studies have tested the effectiveness of a comprehensive curriculum package (e.g., Connected Mathematics Project; see Ben-Chaim, Fitzgerald, Benedetto, & Miller, 1998) or conducted randomized studies with teachers in various settings in which the intervention was implemented with fidelity.

The few randomized studies conducted to date have examined the efficacy of schema-based instruction (SBI), a multicomponent approach to teaching proportional problem solving using data from the upper Midwest of the U.S. (e.g., Jitendra et al., 2009; Jitendra, Star, Rodriguez, Lindell, & Someki, 2011; Jitendra, Star, Dupuis, & Rodriguez, 2013; Jitendra et al., in press). The SBI intervention is grounded in schema theory and research on expertise. Four major features underlie the SBI approach – priming the mathematical structure of problems, using visual representations, explicit teaching of problem solving heuristics, and developing procedural flexibility. Recently, Jitendra et al. (in press) described findings of a study in which teachers were randomly assigned to either a treatment condition (received professional development to implement SBI five days a week for approximately 6 weeks to teach problem solving involving ratio, proportion, and percent) or a control condition in which they taught the same topics from their district-adopted mathematics textbook. Results indicated that students in the SBI classrooms on average outperformed students in the control classrooms on a proportional problem solving measure and maintained their improved performance nine weeks later. Scores on the Process and Application subtest of the Group Mathematics Assessment and Diagnostic Evaluation (GMADE) were equivalent for the two groups. Thus there was evidence that the SBI treatment improves students' proportional thinking.

Purpose / Objective / Research Question / Focus of Study:

The purpose of this study was to replicate and extend the study of the SBI intervention conducted by Jitendra et al. (in press) that demonstrated impact in proportional problem solving for a homogeneous sample that was predominantly White. While the importance of replication in educational research is a staple of methods textbooks, replications are rare in practice (Duncan, Engel, Claessens, & Dowsett, 2015) but badly needed (Makel & Plucker, 2014).

We chose to replicate this study in a geographically diverse location for several reasons: (a) there are few validated approaches to instruction in solving a wide range of single and multi-step problems involving ratios and proportional relationships, and (b) few replication studies are conducted in education (Yong, 2012) even though they are important to scientific inquiry and can enhance external validity. Specifically, we examined the following research questions: 1a. What are the effects of the SBI intervention on seventh-grade students' proportional problem solving immediately following the intervention and eight weeks following treatment? 1b. What are the effects of the SBI intervention on problem solving immediately following the intervention and eight weeks following treatment for students receiving special education services? 2a. What are the effects of the SBI intervention on seventh-grade students' general mathematical problem solving? 2b. What are the effects of the SBI intervention on general mathematical problem solving for students receiving special education services? 3a. Does the treatment moderate students' learning trajectory over time?

Setting and Population/Participants/Subjects:

Students from 20 seventh-grade classrooms ($N = 429$) and their teachers ($J = 20$) from 10 middle schools in an urban school district located in the Southeast region of the United States participated in the study. Based on available student demographic data for our sample (13% missing data), the majority of students were Hispanic (33%), with 27% Black, 21% White, 3% Asian, and 3% multiracial. The mean age of these students was 12 years 7 months ($SD = 5$ months). Approximately 63% of students were eligible for free or reduced price lunch (FRL), 21% were English language learners (ELL), and 5% were students receiving special education services (see Table 1 for teacher and student demographic information). In sum, 366 of the 429 students (85%) from all 20 classrooms provided data for three measurement occasions (i.e., pretest, posttest, retention test) on the proportional problem solving measure.

Intervention / Program / Practice:

The SBI instructional content consisted of ratio, proportion and percent topics. Within SBI, teachers used four instructional practices. First, teachers primed the mathematical structure of problems by focusing on a variety of problem types related to proportions (see Table 2). Teachers were encouraged to stimulate students' thinking about how problems within and across types are similar and different. Second, teachers visually mapped information in the problem using schematic diagrams. For example, to visually represent information in a proportion problem, teachers prompted students to identify the ratios in the problem and write them in the proportion diagram. Third, teachers provided explicit instruction on a problem-solving heuristic (DISC: D – Discover the problem type, I – Identify information in the problem to represent in a diagram, S – Solve the problem, C – Check the solution), with accompanying deep-level questions for each step in the heuristic (e.g., Why is this a *proportion* problem?). Finally, teachers worked to develop students' procedural flexibility, including explicit teaching of multiple solution methods for solving proportion problems and being cognizant of specific methods that are more efficient than others. Teachers modeled by thinking aloud as they engaged in these four practices and scaffolded by gradually shifting responsibility for problem solving to the students.

Research Design, Data Collection, and Analysis:

We used a randomized cluster design with longitudinal (repeated measures) data. For each of the 20 teachers, one class of students was randomly selected to participate in the study. All students in the participating class that were present at pretest or posttest were included in the student sample. Each of the 20 teachers and their participating class was randomly assigned to one of two conditions: treatment or control. In treatment classrooms SBI was implemented daily for 45-50 min class periods over 6 weeks beginning in late October of the school year. In the same time period students in the control condition were taught the same topics as in SBI classrooms but using their district-adopted textbook. All treatment teachers participated in 16 hours of professional development in early October of the school year, approximately three weeks before the study period began. The content and focus of the training was on the topics of ratio, proportion, and percent, particularly as they related to student understanding and implementation of SBI.

In early October of the school year pretest data were collected. For students this consisted of two measures of mathematical problem solving: a researcher-designed test of proportional problem solving (PPS) and the Group Mathematics Assessment and Diagnostic Evaluation (GMADE) Process and Applications subscale (Pearson Education, 2004), a standardized test of mathematical problem solving. Immediately after the intervention students completed the same two measures they had completed at pretest. Data were also collected approximately 11 weeks after the end of the intervention on the PPS test to measure retention of proportional problem solving skills. Reliability estimates for the PPS measure at pretest, posttest, and retention were 0.71, 0.78, and 0.76, respectively. Reliability estimates for the GMADE at pretest and posttest were 0.68 and 0.69, respectively. All student measures were group-administered by teachers.

We gathered fidelity-of-implementation and quality of instruction data via classroom observations. We assessed procedural fidelity by observing videotaped lessons using a checklist developed to document the presence of the core features (e.g., identifies the problem type by focusing on the key problem features, connects the new problem to previously solved problems) of the SBI intervention. The same checklist was also used in the control condition to evaluate program differentiation and determine whether control teachers provided instruction that was similar to the key elements of SBI. The overall quality of instruction in both conditions was assessed using four items that focused on features such as the teacher's ability to clarify the lesson purpose, provide lesson closure, manage instructional time (i.e., how well the teacher managed student behavior), and minimize mathematical errors.

Both descriptive and inferential (multilevel) analyses were performed that analyzed the posttest and delayed posttest data separately (cross-sectional analyses) as well as the impact of treatment on scores across the three repeated measures. Naturally the use of multilevel models with 20 clusters raises concerns about whether there was adequate statistical power to detect treatment effects. We fitted multilevel models because: (a) previous SBI research has typically reported treatment effect sizes of at least .50 *SDs* for cross-sectional data which produces a power of .70 for an intra-class correlation of .15 and a within-cluster sample size of 25; for slightly larger effect sizes such as .60 *SDs* power is .84, (b) it is important to replicate SBI findings with a sample that enhances external validity.

Findings / Results:

Fidelity-of-implementation results indicate that the mean total score across the seven items on the fidelity checklist was 14.45 (SD = 4.13) for treatment teachers and 9.00 (SD = 4.18) for control teachers out of a possible 21 points (higher scores indicate implementation with greater fidelity). Results indicated statistically significant differences between the treatment and control groups on the total score for fidelity-of-implementation items, with treatment teachers implementing SBI elements more than control teachers, in general. With regard to quality of instruction, as expected, both treatment (M = 8.36) and control (M = 9.11) teachers were rated similarly and, on average, there were no differences between the two groups, $t(18) = -0.85$, p = .408.

Preliminary results of a series of multilevel (i.e., two-level, students within clusters/teachers) models for cross-sectional data with covariates at both levels indicated statistically significant differences favoring SBI on the PPS posttest (γ = 1.69, $t(15)$ = 2.96, p = .01, g = 0.44), but no differences on the PPS retention test (p = .13) or the GMADE posttest (p = .08) (see Tables 3, 4, and 5). For the third research question, there was evidence that linear slopes over time for PPS varied across students and that this variation was moderated by treatment (γ = .73, $t(327)$ = 3.36, p = .001). These findings suggested that the treatment group was associated with a larger (positive) linear slope over time compared to the control group.

Given the small numbers of clusters and its likely impact on power, we calculated effect sizes comparing the SBI group with the control group for the nonsignificant findings for the PPS retention test and GMADE, which were 0.27 and 0.25 SDs, respectively. Furthermore, results for students eligible for special education services indicated effects sizes of 0.56 on the immediate PPS posttest, 0.32 on the PPS retention test, and 0.34 on the GMADE. These effect sizes are not statistically different from zero but are at or above the .25 “substantively important” threshold used by What Works Clearinghouse (2014). Thus it seems likely that our nonsignificant findings for the PPS retention test and the GMADE are at least partly due to having only J = 20 clusters.

Conclusions:

The consistent effects in previous randomized studies suggest confidence in the positive effects of SBI in enhancing proportional problem solving for all students. Using a geographically diverse sample, this study replicated and extended earlier findings thus enhancing external validity. An SBI effect emerged for the immediate PPS test but unlike the original study (Jitendra et al., in press), participants did not maintain this learning over time for the PPS delayed posttest and the GMADE test. Future research, including a larger sample, is needed to confirm for whom and under what conditions the SBI intervention has positive effects. We are encouraged, however, about the positive effects of SBI compared to regular mathematics instruction for students eligible for special education on all measures.

Appendix A. References

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Appendix B. Tables and Figures

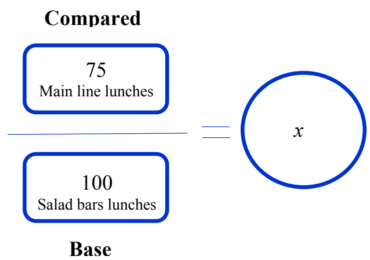
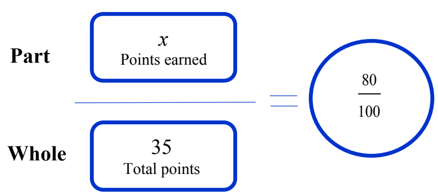
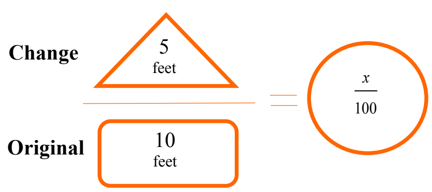
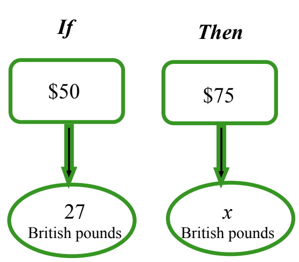
Table 1.
Participant Demographic Information by Treatment

		SBI				Control				Total			
		n	%	*M*	*SD*	*n*	%	*M*	*SD*	*n*	%	*M*	*SD*
Student Information													
Age				12.73	0.55			12.69	0.55			12.71	0.55
Sex	Female	116	47.2			75	41.0			191	44.5		
	Male	100	40.7			82	44.8			182	42.4		
Race	Asian	4	1.6			10	5.5			14	3.3		
	Black	73	29.7			44	24.0			117	27.3		
	Hispanic	78	31.7			65	35.5			143	33.3		
	Multiracial	8	3.3			3	1.6			11	2.6		
	White	53	21.5			35	19.1			88	20.5		
FRL	Yes	157	63.8			114	62.3			271	63.2		
	No	59	24.0			43	23.5			102	23.8		
ELL	Yes	50	20.3			39	21.3			89	20.7		
	No	166	67.5			118	64.5			284	66.2		
SWD	Yes	17	6.9			6	3.3			23	5.4		
	No	199	80.9			151	82.5			350	81.6		
Missing		30	12.2			26	14.2			56	13.1		
Teacher Information													
Sex	Female	7	63.6			7	77.8			14	70.0		
	Male	4	36.4			2	22.2			6	30.0		
Math courses taken				4.45	3.98			7.00	4.84			5.53	4.43
Years experience teaching math				6.45	5.28			7.78	5.83			7.05	5.42

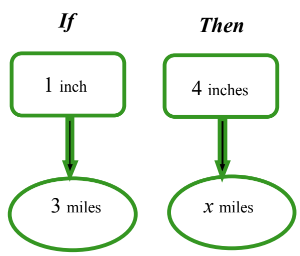
Note. SBI = schema-based instruction; FRL = students eligible for free or reduced priced lunch; ELL = English Language Learner; SWD = students with disabilities; Total student $N = 429$.

Table 2

Examples of Problem Types

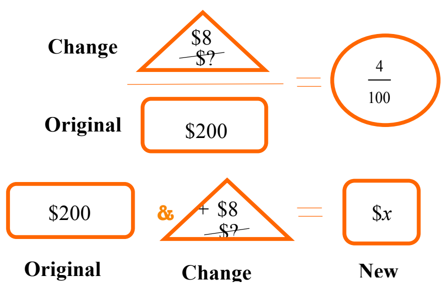
Problem Type	Example of Problem
Ratio 	On Thursday, the cafeteria at Osseo Middle School sold: 42 smoothies, 75 main line lunches, 80 cookies, 51 bags of chips, 100 salad bar lunches, and 26 breakfast bars. What is the ratio of the number of main line lunches sold to the number of salad bar lunches sold on Thursday?
Percent: Part-whole comparison 	On a chapter test, Janie got a grade of 80%. The test had a total of 35 possible points. How many points did Janie earn on the test?
Percent of change 	A tree that was 10 feet tall grew by 5 feet. What percent has it grown?
Proportion 	The Frank family from Minnesota, USA, is going to Britain for their summer vacation. They exchanged $50 for 27 British pounds. At that exchange rate, how many British pounds could they get for $75?

Scale Drawing



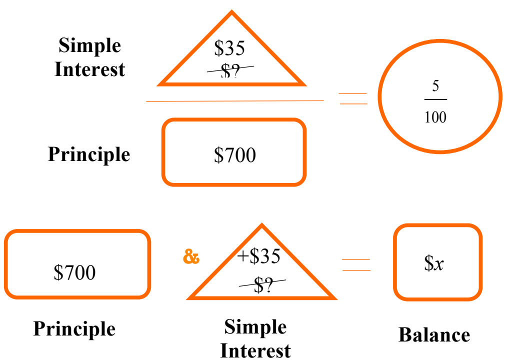
On the map of the Disney World Theme Parks, a scale of 1 inch represents 3 miles. What is the actual distance between Magic Kingdom and Animal Kingdom when it is 4 inches on the map?

Percent of change: Sales tax, tips, discounts, mark-ups



In New York, where the sales tax is 4%, what would be the total cost with tax of a leather coat that costs $200?

Simple interest



Anna deposits $700 in a savings account at the beginning of the year. The simple annual interest rate for the savings account is 5%. What will be the balance in Anna's account at the end of the year?

Table 3

HLM Results for PPS Posttest

Fixed Effect	*B*	*SE*	*t*-value	*df*	*p*
Between-Student Model					
Intercept	16.84	1.236	13.63	15	<.001
Gender	0.40	0.413	0.98	339	.330
Asian	-0.72	1.144	-0.63	339	.527
Black	-0.90	0.597	-1.52	339	.130
Hispanic	-1.05	0.553	-1.90	339	.058
Multiracial	-0.62	1.236	-0.50	339	.618
Pretest	0.58	0.055	10.57	339	<.001
Between-Classroom Model					
Treatment	1.69	0.571	2.96	15	.010
SWD	-0.03	0.041	-0.73	15	.480
ELL	-0.07	0.034	-2.17	15	.046
FRL	-0.05	0.019	-2.68	15	.018
Random Effect	*Variance*	*SD*	χ^2	*df*	*p*
Classroom	0.60	0.773	234.23	15	.003
Student	14.31	3.783			

Table 4

HLM Results for PPS Delayed Posttest

Fixed Effect	*B*	*SE*	*t*-value	*df*	*p*
Between-Student Model					
Intercept	17.09	1.294	13.20	15	<.001
Gender	0.09	0.401	0.22	323	.827
Asian	-0.27	1.096	-0.24	323	.809
Black	-1.55	0.578	-2.69	323	.008
Hispanic	-1.18	0.530	-2.24	323	.026
Multiracial	-2.44	1.173	-2.08	323	.038
Pretest	0.48	0.054	8.95	323	<.001
Between-Classroom Model					
Treatment	0.98	0.619	1.58	15	.135
SWD	-0.06	0.045	-1.42	15	.178
ELL	-0.03	0.037	-0.70	15	.495
FRL	-0.06	0.020	-3.28	15	.006
Random Effect	*Variance*	*SD*	χ^2	*df*	*p*
Classroom	0.90	0.951	43.76	15	<.001
Student	12.73	3.568			

Table 5

HLM results for GMADE Posttest

Fixed Effect	*B*	*SE*	*t*-value	*df*	*p*
Between-Student Model					
Intercept	16.55	1.032	16.03	15	<.001
Gender	0.93	0.389	2.40	332	.017
Asian	0.98	1.060	0.93	332	.355
Black	-0.78	0.562	-1.39	332	.166
Hispanic	-0.30	0.516	-0.58	332	.564
Multiracial	-0.32	1.197	-0.27	332	.788
Pretest	0.35	0.051	6.89	332	<.001
Between-Classroom Model					
Treatment	0.88	0.481	1.83	15	.086
SWD	-0.12	0.036	-3.46	15	.004
ELL	-0.04	0.029	-1.49	15	.158
FRL	-0.04	0.016	-2.55	15	.023
Random Effect	***Variance***	***SD***	χ^2	***df***	***p***
Classroom	0.29	0.538	28.34	15	.019
Student	12.40	3.523			