A LEARNING PROGRESSIONS APPROACH TO EARLY ALGEBRA RESEARCH AND PRACTICE

Nicole L. Fonger
University of Wisconsin-Madison
nfonger@wisc.edu

Ana Stephens
University of Wisconsin-Madison
acstephens@wisc.edu

Maria Blanton
TERC
maria_blanton@terc.edu

Eric Knuth
University of Wisconsin-Madison
knuth@education.wisc.edu

We detail a learning progressions approach to early algebra research and how existing work around learning progressions and trajectories in mathematics and science education has informed our development of a four-component theoretical framework consisting of: a curricular progression of learning goals across big algebraic ideas; an instructional sequence of tasks based on objectives concerning content and algebraic thinking practices; assessments; and posited levels of sophistication in children's reasoning about algebraic concepts within big ideas of early algebra. This research balances the goals of longitudinal research on supporting students' preparedness for algebra while attending to the practical goals of establishing connections among curriculum, instruction, and student learning.

Keywords: Learning Trajectories; Curriculum; Algebra and Algebraic Thinking

Learning progressions and trajectories are currently receiving much attention in mathematics and science education, especially in advancing recommendations for standards, curriculum, assessment, and instruction (Daro, Mosher, & Corcoran, 2011). Some key issues within this domain of research include the use and meaning of terminology, methods of assessing sophistication in student thinking, and connections among curriculum, instruction, and student reasoning (Barrett & Battista, 2014; Ellis, Weber, & Lockwood, 2014). This paper addresses these issues, with particular attention to the ambiguous use of the term *learning progression*—"sometimes indicating developmental progressions, and at other times suggesting a sequence of instructional activities" (Clements & Sarama, 2014, p. 2). We take the stance that a learning progression includes both.

This research is situated within the Learning through an Early Algebra Progression (LEAP) project, which is grounded in a research agenda concerned with a fundamental question of how to prepare students in the elementary grades for success in middle grades algebra and beyond (Blanton, Stephens, Knuth, Gardiner, Isler, & Kim, 2015). The LEAP project builds on Kaput's (2008) framework for early algebra in documenting changes in students' learning of both algebraic content and algebraic thinking practices over time. Our purpose is to elaborate a theoretical framework for an *Early Algebra Learning Progression* (EALP), making progress in a program of research whose aim is to support an integrated system of curriculum, instruction, and student learning in early algebra.

Theoretical Framework

The EALP advanced in this research includes four components: (1) a curricular progression of learning goals across five big ideas and corresponding core concepts, (2) a sequence of instructional tasks based on objectives for content and algebraic thinking practices across big ideas, (3) assessments and coding schemes for analyzing student strategies, and (4) levels of sophistication in children's thinking about core concepts in early algebra. See Table 1.

Bartell, T. G., Bieda, K. N., Putnam, R. T., Bradfield, K., & Dominguez, H. (Eds.). (2015). *Proceedings of the 37th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. East Lansing, MI: Michigan State University.

Table 1: A Theoretical Framework for an Early Algebra Learning Progression (EALP)

EALP	Description and Sub-Components
Curricular Progression	The foundation from which a sequence of instruction and hypothesized paths of student thinking were built and investigated over time: • Big ideas and algebraic thinking practices (cf. Kaput, 2008) • Core concepts (Battista, 2004) or constructs (cf. Shin, Stevens, Short, and Krajcik, 2009) • Learning goals (cf. Clements & Sarama, 2004) or claims (cf. Shin et al., 2009)
Instructional Sequence	A sequence of instructional materials and tasks designed to guide activity around the Curricular Progression: • Lessons and lesson objectives • Tasks (jumpstarts, problem-solving tasks)
Assessment Items	The primary means to measure students' understanding of concepts within big ideas (cf. Battista, 2004) of early algebra. All items have multiple entry points; "anchor items" appear in multiple grades to track growth over multiple years.
Levels of Sophistication in Children's Thinking	Levels represent qualitatively distinct ways of thinking, capturing patterns in students' thinking and reasoning. For a core concept a level includes a description of a way of understanding (which could be a misconception), evidence (i.e., responses to assessment items), and an assessment item or subset of items.

This work builds on several related perspectives across "progressions" research in both mathematics and science education, elaborated next across each dimension of the framework.

Curricular Progression

The multi-year scope of the LEAP project warrants attention to a continuum of levels of specificity in content across grades and within grades and lessons. Thus we define our *curricular progression* to encompass various grain sizes (from largest to smallest): big ideas, core concepts, and learning goals (or claims). The curricular progression establishes a foundation of targeted learning goals from which instruction, assessments, and levels of student thinking are based.

Big ideas are "key ideas that underlie numerous concepts and procedures across topics" (Baroody, Cibulskis, Lai, & Li, 2004, p. 24). Drawing from early algebra the big ideas of the EALP are: (a) equivalence, expressions, equations, and inequalities (EEEI), (b) generalized arithmetic, (c) functional thinking, (d) variable, and (e) proportional reasoning. The multi-year curricular progression is organized around these content strands and the *algebraic thinking practices* of generalizing, representing, justifying, and reasoning with mathematical relationships (Blanton et al., 2015; Kaput, 2008). A *core concept* is an idea critical to understanding a big idea. For the big idea of EEEI, a core concept is "The equal sign is used to represent the equivalence of two quantities or mathematical expressions." We take a *learning goal* (Clements & Sarama, 2014) to include claims about the nature of understandings or skills (Shin et al., 2009) expected of students regarding a concept. For example, in our work a learning goal for the big idea of EEEI is to "understand the equal sign as a relational (rather than operational) symbol," evidence of which is seen through students' actions in interpreting true/false and open equations.

Instructional Sequence

The EALP *instructional sequence* is defined to include a sequence of lessons that entail lesson objectives, jumpstarts, and problem-solving tasks designed to address both concepts and algebraic thinking practices. *Lessons* are defined as guides for an instructional intervention session (typically one 60-minute class period). *Lesson objectives* are defined as statements of targeted performances; they are derived from the curricular progression's learning goals and offer a systematic framework for designing or adapting tasks and allow for the revisiting and extending of algebraic ideas across the grades. Each lesson begins with a *jumpstart* designed to engage students in revisiting and strengthening their understanding of core concepts and algebraic thinking practices addressed in previous lessons. New concepts are introduced through *problem solving tasks*, or structured opportunities for students to build and extend understandings and practices around a goal-driven assignment. As an example for EEEI, students are asked to engage in tasks adapted from research (e.g., Carpenter, Franke, and Levi, 2003) that have been successful in supporting students' relational understanding of the equal sign.

Shin and colleagues (2009) note that learning progressions do not set forth a single, linear path to understanding, but a web of interconnected constructs within a big idea. We likewise acknowledge that our instructional sequence represents one possible path for supporting the development of algebra understanding, and different productive paths certainly exist.

Assessments

Written assessments for each of grades 3-7 were designed to elicit student reasoning across the big ideas and algebraic thinking practices. Assessment items were often adapted from those that had performed well in previous research (e.g., for equivalence items see Knuth, Stephens, McNeil, & Alibali, 2006) and were piloted and revised prior to administration. The assessments include several "anchor items" that appear in multiple grades to allow us to measure growth on the same item over multiple years. Assessment items offer multiple points of entry so that students at the very beginning of the progression as well as those more experienced in early algebra can demonstrate what they know regarding algebraic content and thinking practices. For example, the *True/False* task $57 + 22 = 58 + 21$is posed across grades 3-5 to elicit understandings of the equal sign and equation structure and can be solved in multiple ways.

Levels of Sophistication in Children's Thinking

The final piece of our approach to learning progressions in early algebra research concerns the documentation of changes in students' learning over time. Levels of sophistication are "benchmarks of complex growth that represent distinct ways of thinking" (Clements & Sarama, 2014, p. 14). We initially conjectured levels of sophistication in student thinking based on extant empirical research on student conceptions, misconceptions, and difficulties. We then refined these after analyzing students' responses to assessment items (i.e., student strategies)across grades to discern patterns in children's thinking. Each level of sophistication represents a level of understanding as evidenced in their responses to one or more assessment task(s).

For example, the levels of sophistication we conjectured and observed for students' developing understanding of the equal sign range from Level 1's "Student has operational view of the equal sign and inflexible view of equation structure" to Level 5's "Student has advanced relational-structural understanding of the equal sign and flexible view of equation structure and can consider relationships across equations." We view the levels of sophistication identified in our work as dependent on the learning goals and sequence of tasks that drive the intervention.

Conclusion

The large-scale nature of the LEAP project and our desire to speak to both research and practitioner audiences led to practical decisions about our theoretical frame. This included clearly stated objectives and assessment tasks that integrate algebraic thinking across several grades. We also integrate several perspectives across learning progressions and learning trajectories. Science education literature on learning progressions (e.g., Shin et al., 2009) provided an initial frame for coordinating disciplinary and research-based perspectives on student thinking. It also led to our organizing the content of our EALP according to big ideas, core concepts, and claims. In mathematics education, our EALP parallels Battista's (2004) emphasis on connections among core concepts, assessment items, and levels of sophistication. We also emphasize that the observed levels of sophistication in student strategies are inseparable from the curricular and instructional context in which the learning was supported, yet given the large scale scope of the LEAP project, these connections are not as tightly linked as in some learning trajectories research (cf. Clements & Sarama, 2004).

Our continued research on a comprehensive approach to curriculum, instruction, and student learning is important work to share with the research community towards the goal of coordinating efforts to promote effective early algebra education and identifying important milestones in students' thinking. We also feel it is important that this work be available in a practical form for teachers (e.g., lesson plans and professional development) as they engage in the day-to-day and year-to-year work of developing students' algebraic reasoning. A feasible future direction of this work is to more closely examine paths of students' thinking across grades and in turn, to posit tighter links between tasks and instructional strategies that could be productive in supporting students' engagement in more sophisticated ways of reasoning.

Acknowledgments

The research is supported in part by the National Science Foundation under grants DRL-1207945, DRL-1219606, and DRL-1219605. The opinions expressed herein are those of the authors and do not necessarily reflect the views of the National Science Foundation.

References

Baroody, A. J., Cibulskis, M., Lai, M.-l., & Li, X. (2004). Comments on the use of learning trajectories in curriculum development and research. *Mathematical Thinking and Learning, 6*(2), 227-260.

Barrett, J. E., & Battista, M. T. (2014). Two approaches to describing the development of students' reasoning about length: A case study for coordinating related trajectories. In A. P. Maloney, J. Confrey, & K. H. Nguyen (Eds.) *Learning over time: Learning trajectories in mathematics education* (pp. 97-124): IAP.

Battista, M. T. (2004). Applying cognition-based assessment to elementary school students' development of understanding of area and volume measurement. *Mathematical Thinking and Learning, 6*(2), 185-204.

Blanton, M., Stephens, A., Knuth, E., Gardiner, A. M., Isler, I., & Kim, J.-S. (2015). The development of children's algebraic thinking: The impact of a comprehensive early algebra intervention in third grade. *Journal for Research in Mathematics Education, 46*(1), 39-87.

Carpenter, T. P., Franke, M. L., & Levi, L. (2003).*Thinking mathematically: Integrating arithmetic and algebra in the elementary school*. Portsmouth, NH: Heinemann.

Clements, D. H. & Sarama, J. (2004). Learning trajectories in mathematics education. *Mathematical Thinking and Learning, 6*(2), 81-89.

Clements, D. H., & Sarama, J. (2014). Learning trajectories: Foundations for effective, research-based education. In A. P. Maloney, J. Confrey & K. H. Nguyen (Eds.), *Learning over time: Learning trajectories in mathematics education* (pp. 1-30): Information Age Publishing.

Daro, P., Mosher, F. A., & Corcoran, T. (2011). Learning trajectories in mathematics: A foundation for standards, curriculum, assessment, and instruction: Consortium for Policy Research in Education (CPRE).

Ellis, A. B., Weber, E., & Lockwood, E. (2014). The case for learning trajectories research. In S. Oesterle, P. Liljedahl, C. Nicol & D. Allan (Eds.), *Proceedings of the Joint Meeting of PME 38 and PME-NA 36* (Vol. 3, pp. 1-8). Vancouver, Canada: PME.

Kaput, J. (2008). What is algebra? What is algebraic reasoning? In J. Kaput, D. W. Carraher & M. L. Blanton (Eds.), *Algebra in the Early Grades* (pp. 5-17). New York: Lawrence Erlbaum Associates/National Council of Teachers of Mathematics.

Knuth, E. J., Stephens, A. C., McNeil, N. M., & Alibali, M. W. (2006). Does understanding the equal sign matter? Evidence from solving equations. *Journal for Research in Mathematics Education, 37*(4), 297-312.

Shin, N., Stevens, S. Y., Short, H., & Krajcik, J. (2009). *Learning progressions to support coherence curricula in instructional material, instruction, and assessment design*. Paper presented at the Learning Progressions in Science, Iowa City, IA.