Evaluation & Research Department

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Impact Evaluation

MIDDLE SCHOOL MATHEMATICS: 2006-07 to 2008-09

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Question 1: What need does the Algebraic Thinking approach to middle school mathematics address?

In 2006-07, seven Wake County Public School System (WCPSS) middle schools piloted Algebraic Thinking as an alternate approach to teaching middle school mathematics. Algebraic Thinking was originally developed to help students in grade 6 reach higher mathematics courses by combining the regular and advanced middle school mathematics courses into one heterogeneously-grouped class and differentiating instruction. WCPSS began Algebraic Thinking in response to research indicating heterogeneous grouping of students was beneficial to student achievement.

Question 2: What are the program goals and strategies?

The long-term goals of Algebraic Thinking are to increase both the percentage of students in grade 8 enrolled in Algebra I and the percentage of students reaching growth targets. To increase the likelihood of achieving these three-year goals, short-term and intermediate goals were established for grades 6 and 7. The short-term goals for 2006-07 were for students served in grade 6 to participate in heterogeneous mathematics classes and meet growth targets. The 2007-08 intermediate goal was to increase enrollment of grade 7 students in pre-algebra. The strategy for accomplishing these goals was heterogeneously grouping middle school mathematics students with a focus on differentiation and providing students with an advanced mathematics experience. The course material is not conceptually different from that covered in the standard mathematics curriculum. For further information on Algebraic Thinking's goals and strategies see Table 1.

Major Findings

- **Students Served:** In 2006-07, seven middle schools piloted Algebraic Thinking. Five schools that continued Algebraic Thinking through 2008-09 were selected for the study: Carroll, East Garner, Heritage, Martin, and Zebulon. Of the 1,493 grade 6 students enrolled in one of the five pilot schools, 1,413 participated in Algebraic Thinking and 1,087 were still enrolled in 2008-09 and are therefore included in the study. Five middle schools were selected for comparison—1,078 grade 6 students enrolled from 2006-07 to 2008-09 at the matched schools comprised the comparison cohort.
- **Implementation:** Schools implementing Algebraic Thinking in 2006-07 received a four-day summer training in 2006 for grade 6 mathematics teachers, plus some instructional resource teachers (IRTs), academically gifted (AG) teachers, special education teachers, and principals. While a more limited training was offered to grade 8 teachers in summer 2008, no training was offered to teachers at grade 7 since there was no changes made at this level.
- **Achievement:** While both cohorts of students improved academically, overall findings suggest that participation in Algebraic Thinking did not have a greater positive impact on students' mathematics achievement as measured by EOG proficiency or growth. However, a significantly higher percentage of Algebra I students at the Algebraic Thinking schools met their Algebra I growth target.
- **Enrollment in Higher Mathematics Courses:** A significantly higher percentage of students attending the comparison schools were enrolled in Algebra I in grade 8 as compared to Algebraic Thinking schools (37% and 30% respectively).
- **Recommendations:** Clarification and examination of goals, documentation and guidelines, and training needs could help improve the implementation of this initiative. Further, given the variation in school level results, exemplars should be used as guides.

Impact Evaluation reports provide basic evaluative outcome information on standard indicators. These reports may suggest a need for further study of the efficacy of a program, but should not be considered comprehensive enough to make funding decisions.

WAKE COUNTY PUBLIC SCHOOL SYSTEM

We acknowledge the contribution of the following WCPSS staff to this report (listed alphabetically): Aimee Lougee, Susan Fink Shell, and Christina Zukowski.

Goals and Strategies

Algebraic Thinking I combines regular 6th Grade Math and Advanced 6th Grade Math into one heterogeneous class (in other schools, students in grade 6 are enrolled in either 6th Grade Math or Advanced 6th Grade Math). There are no concepts particular to Algebraic Thinking; rather it is a difference in approach: heterogeneous grouping with a focus on differentiation. Algebraic Thinking I enables all grade 6 students to have the same advanced mathematical experiences typically available only in Advanced 6th Grade Math and is designed to provide them with the *opportunity* to move on to Pre-Algebra in grade 7. Algebraic Thinking was developed to improve learning opportunities in grade 6, give students an extra year to grow mathematically, and make it possible for more students to take advanced mathematics in grades 7 and 8. For consistency, within schools adopting this approach, the name of 7th Grade Math was changed to Algebraic Thinking II.[1] Similar to grade 6, 8th Grade Math and 8th Grade Math Plus were combined into Algebraic Thinking III. The program was rolled out gradually based on volunteer schools. The amount of training and support provided decreased over time. The logic model shown in Table 1 highlights the needs, resources, and desired goals of Algebraic Thinking.

Table 1
Algebraic Thinking Logic Model, 2008-09

Need: Algebraic Thinking I was originally developed to help students in grade 6 reach higher mathematics courses by combining the regular 6th Grade Math class and the Advanced 6th Grade Math into one heterogeneously grouped class and differentiating instruction.

INPUTS	STRATEGIES	OUTCOMES – IMPACT		
		Short-Term 2006-07	Intermediate 2007-08	Long-Term 2008-09
Math teachers are trained on content differentiation. The Middle School Math Team makes regular visits to the Algebraic Thinking Professional Learning Teams (PLTs) to offer support.	Heterogeneously grouped mathematics classes (normal offerings are just restructured). Differentiation (training and instructional delivery).	• Students are presented with the same mathematical opportunities in grade 6, therefore providing students the same opportunity to move on to higher-level classes. • Students allowed an extra year to grow mathematically before being considered for Pre-Algebra. • Students served in grade 6 in 2006-07 meet mathematics growth targets (academic change) on End-of-Grade (EOG).	• More students recommended for 7th grade Pre-Algebra. • Increase in grade 7 students enrolled in Pre-Algebra.	• Increase in students taking Algebra I in grade 8. • Students reach growth targets in grade 8. • All subgroups meet mathematics growth targets (academic change) on EOG.

Data Source: Program information provided by Curriculum and Instruction staff.

[1] This was a change in name only. Algebraic Thinking II is the same course as 7th Grade Math. Students are not heterogeneously grouped in grade 7; students are enrolled either in the regular 7th grade curriculum (7th Grade Math or Algebraic Thinking II) or Pre-Algebra.

Background

In 2006-07, seven middle schools elected to pilot Algebraic Thinking. These schools agreed to:

- provide administrative support for the program throughout the year;
- protect planning time for grade 6 mathematics teachers;
- encourage input from staff in AG, English as a Second Language (ESL), Special Education, and IRTs in differentiation planning;
- attend a four-day summer institute to better understand differentiation and how to plan for it;
- attend district Algebraic Thinking planning days once per quarter;
- accept district support every other week during planning time; and
- utilize enrichment units with students to work on the growth component.

In 2007-08, one new school elected to implement Algebraic Thinking. In 2008-09, three additional schools implemented Algebraic Thinking and one of the original pilot schools opted out of Algebraic Thinking due to scheduling issues.

Middle School Mathematics Course Sequence

Within the schools offering the traditional middle school mathematics alternatives, the vast majority of students in grade 6 are placed either in 6th Grade Math or Advanced 6th Grade Math. Sixth Grade Math is recommended for students who have mastered most of the elementary mathematics curriculum. Advanced 6th Grade Math emphasizes problem solving skills and the application of grade 6 mathematics topics as well as offering enrichment and extension activities focused on these topics. Students who have mastered all of the K-5 mathematics strands and have demonstrated a "desire and ability to accelerate in mathematics" are recommended for Advanced 6th Grade Math (Middle School Planning Program Guide: 2008-09).

Although Algebraic Thinking I is a combination of 6th Grade Math and Advanced 6th Grade Math, the 2008-09 and 2009-10 Middle School Planning Program Guides have a combined definition for Advanced 6th Grade Math and Algebraic Thinking I rather than two distinct definitions.[2] Table 2 displays the middle school mathematics course sequence for Algebraic Thinking and schools offering the traditional mathematics sequence.

[2] The definition for Algebraic Thinking appears under the heading Advanced 6th Grade Math/Algebraic Thinking I.

Table 2
Mathematics Course Sequence

The following mathematics sequences are available for middle school students.

<table>
<tr><th>Grade</th><th colspan="3">Course Options</th><th rowspan="4">Students can be accelerated beyond the middle school curriculum as appropriate</th></tr>
<tr><td rowspan="2">6</td><td colspan="2">6th Grade Math</td><td>Adv 6th Math</td></tr>
<tr><td colspan="3">Algebraic Thinking I</td></tr>
<tr><td rowspan="2">7</td><td>7th Grade Math</td><td colspan="2" rowspan="2">Pre-Algebra
(7th and 8th Grade Math in one year)</td></tr>
<tr><td>Algebraic Thinking II</td></tr>
<tr><td rowspan="2">8</td><td>8th Grade Math</td><td>8th Grade Math Plus</td><td rowspan="2">Algebra I</td></tr>
<tr><td colspan="2">Algebraic Thinking III</td></tr>
</table>

Data Source: Middle School Program Planning Guide 2008-09

National Research

A major goal of Algebraic Thinking—increasing the percentage of students enrolling in higher mathematics courses—is supported by research that suggests students who enroll in advanced mathematics in high school are more likely to attend college and subsequently earn a bachelor's degree (Burris, Heubert, & Levin, 2004).

The two major strategies utilized by Algebraic Thinking are heterogeneous grouping and differentiated instruction. Research suggests that grouping students heterogeneously into accelerated courses may improve their academic achievement and course-taking patterns. While some educators have expressed concern that high-achieving students may be negatively impacted by heterogeneous grouping, two longitudinal studies found that all student groups: low-, average-, and high-achieving were positively impacted. Thus, heterogeneous grouping did not appear to have a negative impact on initially high-achieving students (Burris et al., 2004; Laitsch, 2006).

While many teachers differentiate lessons within regular classrooms due to different student learning styles, differentiation of instruction is imperative within heterogeneously grouped classrooms. In a 1999 interview with Hess, Tomlinson described effective differentiating practices of successful teachers.[3] Successful teachers:

- Keep the focus on concepts, emphasizing understanding and sense-making, not regurgitation of fragmented facts.
- Use ongoing assessments of readiness and interests, and pre-assess to find students needing more support and those who can leap forward. They don't assume all students need a certain task.

[3] Carol A. Tomlinson, Ed.D. is an associate professor at the University of Virginia's Curry School of Education with more than 20 years of classroom experience. She is a proponent of mixed-ability classrooms.

- Make grouping flexible. They let students work alone sometimes and also in groups based on readiness, interests, or learning styles. They use whole-group instruction for introducing ideas, planning, or sharing results.
- See themselves as guides. They help students set goals based on readiness, interests, and learning profiles—and assess based on growth and goal attainment (Hess, 1999, p.2).

Another key element of providing effective differentiation is significant staff development. Hess defines "significant" as staff development that is more than a "one-shot" training. Teachers who lack appropriate support often teach to the middle rather than provide the range of instruction required in a mixed-ability classroom (Hess, 1999).

Methods

Student outcomes for schools with consistent implementation from 2006-07 to 2008-09 were examined and a comparison group was constructed from middle schools not offering Algebraic Thinking. The key questions of interest in this evaluation were:

- whether having Algebraic Thinking at grade 6 results in students showing a higher percentage meeting growth in middles school mathematics as measured by the Mathematics End-of-Grade (EOG) and Algebra I End-of-Course (EOC) tests, and
- whether students in grades 7 and 8 were more likely to enroll in the higher level mathematics courses.

The level of training and implementation was also examined. In order to address these questions 47 mathematics teachers currently teaching at one of the five pilot schools were surveyed in the spring of 2010. There was a 72% response rate with 34 of the 47 teachers responding to the survey. The 34 teachers represented six teachers from Carroll Middle, eight teachers from East Garner Middle**,** ten teachers from Heritage Middle, seven teachers from Martin Middle, and two teachers from Zebulon Middle.[4] Thirty-two of the teachers who responded reported that they taught Algebraic Thinking; thus, the majority of survey results are based on the responses of these 32 teachers.

Matched Schools

Five middle schools that began Algebraic Thinking in 2006-07 and continued through 2008-09 were selected for the study: Carroll, East Garner, Heritage, Martin, and Zebulon. Cluster analysis was conducted to select five middle schools as comparisons for Algebraic Thinking schools.[5] The analysis was run using centroid method: three variables (2005-06 performance composite, 2006-07 overall risk score, and 2006-07 days in membership on the 20th day of school) were included in the model.[6] The overall risk score is a school level score calculated based on the percentage of students at each school with academic risk factors, such as free or

[4] One teacher who responded to the survey did not report their school.
[5] Schools that implemented Algebraic Thinking in 2007-08 and 2008-09 were not included in this study.
[6] "*Centroid method*. The cluster to be merged is the one with the smallest sum of distances between cluster means (centroids) for all variables. The centroid method also weights for differences in cluster size" (Garson, 2010, p.8).

reduced-price lunch (FRL) status, limited English proficiency (LEP) status, and students with disability (SWD).

Table 3
Algebraic Thinking Schools and Matched Schools

Algebraic Thinking School	Matched School
Carroll	Reedy Creek
East Garner	North Garner
Heritage	Leesville
Martin	West Cary
Zebulon	East Wake

Data Source: 2006-07 Healthy Schools Indicators for Middle Schools

Study Participants

The 2006-07 cohort of grade 6 students who either attended one of the five Algebraic Thinking schools or one of the five comparison schools in 2006-07, 2007-08, and 2008-09 were included in this study. Of the 1,493 students in grade 6 enrolled at one of the five Algebraic Thinking schools, 1,413 participated in Algebraic Thinking. Of these students 1,087 were still enrolled in 2008-09 and are therefore included in the study. At the five comparison schools, 1,381 grade 6 students were enrolled in mathematics; 1,078 of these students remained enrolled from 2006-07 to 2008-09 and thus comprised the comparison cohort.

Matched Students

In order to verify findings among matched schools (Algebraic Thinking and matched comparison schools) a one-to-one matched group of students was generated from the 2,165 students included in this study. Students were matched on their prior (grade 5) mathematics EOG score (+ or – four points or one standard deviation) and several demographic characteristics—FRL, LEP, and SWD status. Of the 2,165 students included in the overall study, 982 were matched on a one-to-one basis. The additional analyses on the 982 one-to-one matched students (491 Algebraic Thinking students and 491 matched comparison students) were conducted to verify the study's findings. However, to maximize the group size and maintain large enough sample size in disaggregated comparisons, the majority of this study's comparisons were made utilizing the entire cohort (2,165 students) rather than the smaller one-to-one matched student groups. The one-to-one matched student comparisons were used to verify significant or notable findings.

Training

Schools implementing Algebraic Thinking in 2006-07 participated in a four-day summer training in 2006. This training was offered to grade 6 mathematics teachers, IRTs, AG teachers, and special education teachers. While mathematics teachers from each school attended the training, the attendance of the other specialists varied by school. An administrator from each school was invited to attend one day of the training. Central services staff provided training specifically

focused on Quarter 1 grade 6 material, deep understanding of content, focus on lesson design to reach all learners (differentiation of content), and various uses of enrichment units.[7] Teachers in other schools may have had brief training in differentiation, but not in the same depth and not specific to content.

Central services staff (middle school math senior administrator and coordinating teacher) conducted training for Algebraic Thinking III (grade 8 mathematics teachers) in summer 2008; however, this training was not as extensive as that offered to grade 6 teachers.[8] In 2008, the 6th grade summer training was offered to the mathematics teachers at the three new Algebraic Thinking schools and one school that began implementing in 2007-08.

The costs associated with Algebraic Thinking were not included in this report. Due to the similarity of approach at Algebraic Thinking schools and schools offering the traditional mathematics course sequence, the costs to the system were comparable. It should be noted that although Algebraic Thinking did not require material costs and was provided to students as their middle school mathematics course sequence (a cost that would have occurred in the absence of this program), there were opportunity costs associated with differentiation training provided to grades 6 and 8 teachers. The *opportunity* costs associated with training included both the teacher time out of the classroom and central services staff time required to provide training. By producing, providing, and participating in this training, WCPSS devoted staff time that would have otherwise been focused on an alternative approach (either teaching or supporting the traditional middle school mathematics program or developing, training, and learning an alternative method); thus, the training represented an *opportunity* cost to the district.

[7] The training was provided by a middle school math senior administrator and a contractor who filled in for the coordinating teacher who was on maternity leave.

[8] Because Algebraic Thinking II represents a change in name only from 7th Grade Math, no training was offered to teachers at this grade level.

Question 3: What are the characteristics of the students served?

Table 4 displays the demographic characteristics of the two cohorts of students enrolled in grade 6 in 2006-07: (1) those who participated in Algebraic Thinking and remained at one of the five Algebraic Thinking schools included in this study in 2007-08 and 2008-09, and (2) students enrolled in a comparison school from 2006-07 through 2008-09. Students participating in Algebraic Thinking and students enrolled in 6th Grade Math or Advanced 6th Grade Math at comparison schools were demographically similar (see Table 4). The one-to-one matched subgroup of students drawn from the study cohorts is not depicted in Table 4.

Table 4
2006-07 Students Characteristics of Grade 6 Students in Algebraic Thinking and Comparison Cohorts

	Algebraic Thinking		Comparison Students	
	Number	**Percent**	**Number**	**Percent**
FRL	358	32.9%	352	32.7%
SWD	163	15.0%	163	15.1%
LEP	57	5.2%	82	7.6%
Male	525	48.3%	547	50.7%
Female	562	51.7%	531	49.3%
American Indian	3	0.3%	4	0.4%
Asian	31	2.9%	67	6.2%
Black/African Am.	335	30.8%	286	26.5%
Hispanic/Latino	113	10.4%	134	12.4%
Multiracial	45	4.1%	51	4.7%
White	560	51.5%	536	49.7%
Total	**1,087**	**100%**	**1,078**	**100%**

Note: 1: Students will appear in more than one category: race and gender, FRL, SWD, and/or LEP.
2: Overall *N*=2,165. Table includes grade 6 students enrolled in 2006-07, 2007-08, and 2008-09 at the five Algebraic Thinking schools and students at the five comparison schools who took 6th Grade Math or Advanced 6th Grade Math.
3: Participants total *n*=1,087 and comparison students total *n*=1,078 within race and gender categories.
Data Source: June 2007 WCPSS Student Locator.
Interpretation Example: Of the 1,087 Algebraic Thinking participants, 163 (15.0%) were SWD students.

Question 4: What was the level of training and implementation?

In order to address this question, 47 mathematics teachers currently teaching mathematics in 2009-10 at the five Algebraic Thinking pilot schools were surveyed. Of the 34 teachers who responded to the survey, 32 reported that they taught Algebraic Thinking; thus, the majority of survey responses are based on the responses of these 32 teachers.

Training

Of the 32 teachers sampled, 26 taught grades 6 or 8 and were therefore offered training. Twenty of the 26 (77%) grades 6 and 8 Algebraic Thinking teachers surveyed reported receiving training. This means that more than 1 in 5 of the teachers surveyed did not receive training. It should be noted that grade 7 teachers were not offered training since there were no changes made at this grade level and those teachers are therefore not included in these percentages. Grade 6 teachers who entered an existing Algebraic Thinking school after the 2006-07 year were not offered centralized training. All three grade 6 teachers who reported receiving no training also reported teaching Algebraic Thinking for less than three years; thus, they would not have been present during the initial 2006 grade 6 training. One of the three grade 8 teachers who reported receiving no training also reported teaching Algebraic Thinking for one year. Since grade 8 teachers were trained in 2008-09, it is likely that either this teacher did not attend the training or began teaching Algebraic Thinking in 2009-10.

The vast majority 80% (16 of 20) of those trained, reported that the training was sufficient to allow them to implement Algebraic Thinking in their classroom.

Twelve of the 32 teachers surveyed had taught Algebraic Thinking for four years or since it began in 2006-07. Six teachers had only taught for one year, while seven teachers reported teaching two years, and another seven reported teaching three years.

Implementation

All 32 Algebraic Thinking teachers reported that their class included students of mixed mathematics ability. Thirty teachers reported that they had changed their classroom instruction to accommodate a mixed ability class.

- Nineteen teachers (59%) reported their students were heterogeneously grouped within the classroom most or all of the time. The remaining 13 (41%) teachers reported grouping their students heterogeneously only sometimes.

- Sixteen teachers (50%) reported that they differentiate instruction within their classroom most or all of the time while 14 teachers (44%) reported differentiating instruction sometimes and 1 teacher reported not differentiating at all.[9]

[9] One of the 32 teachers who taught Algebraic Thinking did not respond to this question, but is still counted in the denominator.

Thirty-one teachers responded to the open-ended question regarding how they differentiated within their classroom. Open-ended responses included changes such as:

- "Using pre-tests and grouping based on ability on some assignments. Providing enrichment for students who understand content."

- "I provide alternate assignments for students who are moving at a quicker pace. I offer lunch/encore help to students who need extra resources and practice. I provide multi-level tests in order to assess fairly."

There were some differences between the teachers' responses at schools in which their Algebraic Thinking cohort had high growth and those that did not. While teachers at both high and low growth schools reported differentiating instruction, the teachers at the schools with a high growth among their Algebraic Thinking cohort were more apt to differentiate assessment and more likely to mention providing enrichment opportunities.

Question 5: What facets of the project are viewed as most effective? Least effective?

Tables 5 and 6 present the teachers' perceived benefits and challenges associated with implementing Algebraic Thinking. Teachers were asked to rate the extent to which subgroups of students improved their learning due to Algebraic Thinking approaches of heterogeneous grouping and differentiation. For all students and each subgroup considered, the most common response given was that student learning was *somewhat* attributed to Algebraic Thinking—ranging from 81% for all students to 39% for Level IV and Academically Gifted students (see Table 5). The majority of teachers reported that Algebraic Thinking positively impacted student learning *A Great Deal* or *Somewhat f*or each student group considered—ranging from 91% for all students to 52% for AG and Level IV students.

Table 5
Perceived Improvement in Student Learning
by Student Group

Question: To what extent did the following groups of students improve their learning in your classroom due to Algebraic Thinking approaches of heterogeneous grouping and differentiation?

Student Group	A great deal	Somewhat	Not at All	I don't know	NA	Number of Responses
All students in my class	9.4%	81.3%	3.1%	6.3%	0.0%	32
Limited English Proficient (LEP)	15.6%	50.0%	15.6%	9.4%	9.4%	32
Students with disabilities (SWD)	12.5%	68.8%	6.3%	3.1%	9.4%	32
Academically Gifted	12.9%	38.7%	32.3%	6.5%	9.7%	31
Level I or II in 2008-09	6.5%	74.2%	3.2%	12.9%	3.2%	31
Level III in 2008-09	19.4%	58.1%	0.0%	19.4%	3.2%	31
Level IV in 2008-09	12.9%	38.7%	22.6%	16.1%	9.7%	31

Note: Level I or II = students who had an EOG Level I or II the previous year and were therefore considered below grade level. Level III = on grade level and Level IV = above grade level performance on the prior year's EOG.
Data Source: Algebraic Thinking Teacher Survey March 2010.

Teachers were split when asked if they believed Algebraic Thinking improved students' readiness for Pre-Algebra in grade 7. Grade 6 is the critical grade for Algebraic Thinking's impact on student growth in order to prepare them for more advanced work. Of the 34 teachers who responded to the survey:

- 12 teachers reported that they believed Algebraic Thinking did improve student readiness,
- 10 teachers reported it did not, and
- 12 teachers reported they did not know.

When asked about readiness for Algebra I in grade 8 the results were similar:

- 11 teachers reported that they did believe students were more prepared for Algebra I,
- 9 teachers felt students were not, and
- 14 teachers did not know if students were more ready for Algebra I.

Table 6 displays the perceived benefits and challenges associated with heterogeneous grouping, differentiation, and time as they apply to the implementation of Algebraic Thinking.

- While five of the six teachers who commented on the benefits of heterogeneous grouping discussed its positive effect on "lower ability" students (the sixth teacher mentioned the positive benefit for all students), the comments regarding the challenges associated with heterogeneous grouping referred to having too many Level I and II students or the negative impact on students performing at or above grade level (Level III and IV).

- While teachers had positive comments regarding the benefits of differentiation (i.e., the ability to provide enrichment, remediation, and cooperative learning opportunities), they were more likely to comment on the challenges associated with differentiating instruction. The challenges included: "Too many levels to address all at once" and "Keeping students working at grade level engaged."

- Teachers did not report any benefits regarding time, but did mention that pacing and time for planning were challenges.

Table 6
Perceived Benefits and Challenges of Algebraic Thinking

	Benefits	Challenges
Heterogeneous Grouping	"Allowing all students to be challenged at a higher level." "The fact that rigorous math is available to all students...that lower level students are exposed to the ideas/processes of higher level students." "Having students who are excellent students for low achieving students to look up to and try to emulate." "The facet that is most effective is allowing students time to work together in groups. I really like the teaching guides also." "Lower ability students appear to get the most out of the Algebraic Thinking model through the grouping strategies." "Heterogeneous grouping helps lower functioning students. Creates a nice environment in the classroom to have all levels together."	"Level III students are helping the [Level] IIs and Is but not the other way around—inequitable for all." "There are too many level I and II students in the class and not enough level III and IV to pair them together." "It is very hard when we have the extremely high kids mixed in with our [In-Class Resource] ICR math classes. I would prefer to have the High AG students spread between the other 3 classes."
Differentiation	"Giving students who already know how to do something an alternate activity--the blackboard site provided good ones in AT1 [Algebra Thinking I]. I wish there were more for AT2 [Algebra Thinking II], not just remediation activities." "Differentiating the instruction so all students receive the remediation or enrichment needed. Also, cooperative grouping used for concepts that all students struggle with so there is peer help for the students who really need the extra help."	"Some students (level 1 and low level 2) often do not have enough basic skills to be able to handle the material that is being presented/learned/taught." "Too many levels to address all at once." "DIFFERENTIATION! It is so difficult to meet all the needs of all students, when I am only one person!" "AT2 [Algebraic Thinking II] doesn't have the enrichment activities I need for the higher-level students like AT1 [Algebraic Thinking I] had." "Keeping students who are working at grade level engaged."
Time	*No benefits given related to time.*	"Time constraints when differentiating. It's difficult for the low level students to gain all the support needed in such a short period of time." "I don't feel as if the students get ample time to practice what they have learned before they are asked to move on to something new." "Time for planning." "The pacing can be challenging for lower level students. Some students may need 4 days for a concept that the pacing guide allows only 2. The challenge is moving on but finding the time to catch them up so as not to slow the entire class down." "I often feel like the AG students are lacking in my time."

Data Source: Algebraic Thinking Teacher Survey March 2010.

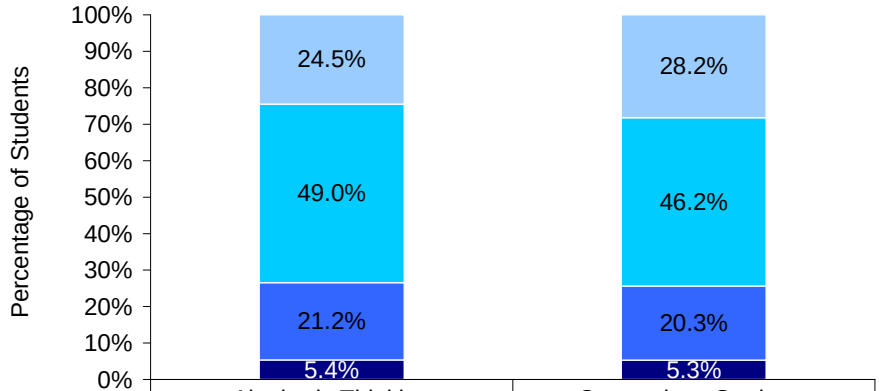
Question 6: Has the project improved students' academic achievement?

Overall, the findings suggest that participation in Algebraic Thinking generally did not improve students' mathematics achievement as measured by EOG proficiency or growth beyond that achieved by the traditional approach to middle school mathematics. Comparisons to the matched school cohort revealed that the short-term goal of an increase in the percentage of grade 6 students meeting mathematics growth and the long-term goal of an increase in the percentage of students meeting growth in grade 8 (as measured by mathematics EOG scores) were not met. However, Algebra I students at Algebraic Thinking schools had a significantly higher percentage meet the Algebra I growth target.

Academic Proficiency

The grade 5 mathematics EOG proficiency levels for students who participated in Algebraic Thinking and students at comparison schools are presented in Figure 1. The grade 5 EOG scores were used to capture student performance prior to program participation in 2006-07, 2007-08 and 2008-09. Overall proficiency (Level III and IV) for the two groups was similar. It should be noted that slightly more students (3.7%) at the comparison schools scored a Level IV on their grade 5 EOG.

Figure 1
Grade 5 (2005-06) Mathematics EOG Level for Algebraic Thinking and Comparison Students



	Algebraic Thinking n=1,010	Comparison Students n=944
Level IV	24.5%	28.2%
Level III	49.0%	46.2%
Level II	21.2%	20.3%
Level I	5.4%	5.3%

Note: 1. *n* = students with available grade 5 EOG mathematics scores.
2. Table includes grade 6 students enrolled in 2006-07, 2007-08, and 2008-09 at the five Algebraic Thinking schools and students at the five comparison schools who took 6th Grade Math or Advanced 6th Grade Math.

Data Source: 2008-09 End-of-Year Middle School Student Rosters

Interpretation Example: 24.5% of Algebraic Thinking participants scored a Level IV in 2005-06 compared to 28.2% of students at comparison schools.

Overall, the percentage of students proficient on the mathematics EOG at both the Algebraic Thinking and comparison schools increased significantly: approximately 10 percentage points between grade 5 in 2005-06 and grade 8 in 2008-09.[10] Table 7 depicts the percentage of students proficient on the mathematics EOG who were enrolled in 2006-07, 2007-08, and 2008-09 at each of the schools in the study. Improvement varied by school within both groups.

Table 7
Percentage of 2006-07 Cohort of Algebraic Thinking and Comparison Students Enrolled in 2006-07, 2007-08, and 2008-09 Proficient on Mathematics EOG

	Grade 5 2005-06	Grade 6 2006-07	Grade 7 2007-08	Grade 8 2008-09	Percentage Point Change 05-06 to 07-08
WCPSS	73.6%	74.4%	74.3%	82.1%	8.5**
Algebraic Thinking					
1. Carroll Middle	75.0%	75.4%	73.7%	81.6%	6.6
2. East Garner Middle	66.4%	58.8%	49.7%	64.5%	-1.9
3. Heritage Middle	69.3%	82.2%	86.5%	89.4%	20.1**
4. Martin Middle	88.4%	90.1%	90.6%	91.5%	3.1
5. Zebulon Middle	69.5%	67.2%	63.2%	83.9%	14.4**
Total	73.7%	76.4%	75.5%	84.2%	10.5**
Comparison Schools					
1. Reedy Creek Middle	65.0%	74.5%	77.4%	90.6%	25.6**
2. North Garner Middle	55.8%	55.1%	58.7%	73.1%	17.3**
3. Leesville Middle	83.5%	80.7%	79.4%	89.7%	6.2*
4. West Cary Middle	88.9%	90.5%	88.9%	93.4%	4.5*
5. East Wake Middle	71.3%	68.0%	71.8%	76.0%	4.7
Total	75.2%	75.7%	76.7%	85.4%	10.2**

Note: 1. State rules changed in 2008-09; thus, approximately 5% of the increase in 2008-09 may be attributed to the inclusion of retest scores in both cohorts and WCPSS overall.
2. z statistic was computed to test for significance. * indicates significance at the 0.05 level ** indicates significance at the 0.01 level. Shaded cells indicate greater change than individual matched school.

Data Source: 2008-09 End-of-Year Middle School Student Rosters and McMillen (2010) End-of-Grade Multiple-Choice Test Results, 2008-09.

Academic Growth

Increasing the percentage of students reaching growth targets is another way to gauge success in improving achievement, and is more sensitive to student gains even when growth was not sufficient to change level scores. The state's ABCs growth formula reflects approximately one year's growth for one year of instruction for each student. Schools are considered to show high growth if 60% of their students reach their growth target. For Algebraic Thinking, this would be

[10] Significance based on a z statistic. A z statistic was computed for Algebraic Thinking students and students at comparison schools to test if there was a significant percentage-point increase from grade 5 to grade 8 for both groups.

a sign that more students might be ready for higher level mathematics. Table 8 displays the percentage of students enrolled in 2006-07, 2007-08, and 2008-09 at each matched pair of schools who met North Carolina's ABCs mathematics growth targets by year.

Overall Growth

Overall the percentage of students who met growth in 2006-07 (grade 6), 2007-08 (grade 7) and 2008-09 (grade 8) at Algebraic Thinking schools was not significantly different from comparison schools (see Table 8). However, as shown in Table 8, there were significant differences between matched schools across the three years examined. The comparison schools showed higher percentages of students reaching growth targets in 7 out of 15 matched school comparisons; the Algebraic Thinking schools showed higher percentages of students reaching growth targets in four comparisons.

Table 8
Matched Pairs of Schools: Comparison of Mathematics Growth

Matched Pair	Significantly Higher Percentage of Students Who Met Growth		
	Grade 6 05-06 to 06-07	**Grade 7 06-07 to 07-08**	**Grade 8 07-08 to 08-09**
1	ns	Comparison	Comparison
2	ns	Comparison	Comparison
3	AT	AT	ns
4	Comparison	AT	ns
5	Comparison	Comparison	AT
Total	**ns**	**ns**	**ns**

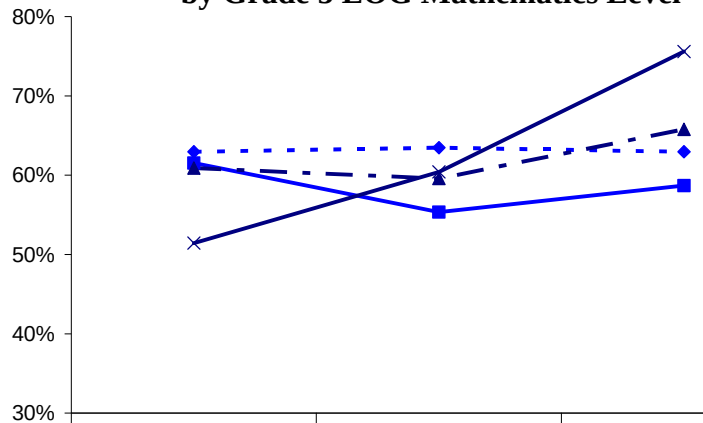
Note: 1. Comparison = comparison school. ns = not significant. Blue shaded cells indicate a significantly higher percentage of total students meeting EOG growth at the Algebraic Thinking school compared to the matched school (*p* is less than or equal to 0.05). Yellow shaded cells indicate a significantly higher percentage of total students meeting EOG growth at the comparison school (*p* is less than or equal to 0.05).
2. z statistic was computed to test for significance.

Growth by Level

In order to consider the impact of Algebraic Thinking on students who entered grade 6 with varying achievement levels, it is helpful to examine students in terms of their grade 5 EOG level. Figures 2 and 3 depict the percentage of students who met growth in 2006-07, 2007-08, and 2008-09 by mathematics EOG Level in 2005-06 (Grade 5).

- Among students at both the Algebraic Thinking and comparison schools, students who scored Level IV on their grade 5 EOG had the highest growth rate.
- While within both cohorts students who scored Level III on their grade 5 EOG increased in the percentage proficient from grade 6 to grade 8, the Algebraic Thinking students' increase was slightly higher compared to the students at comparison schools (4.9 and 2.5 percentage points respectively).
- Among students who scored a Level I or II in grade 5, the percentage that scored proficient either remained constant or declined from grade 6 to 8.

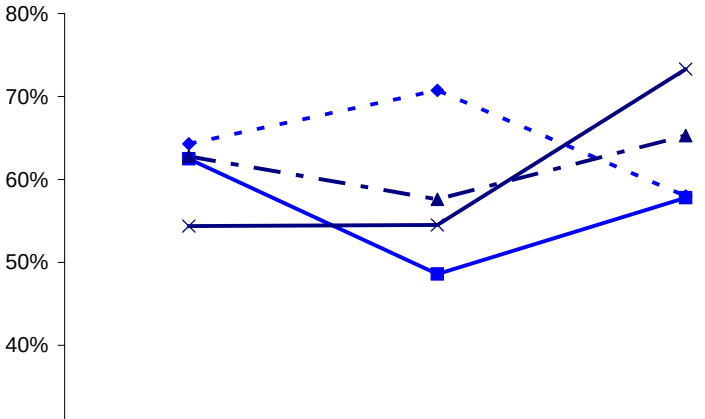
Figure 2
Percentage of 2006-07 Cohort of Algebraic Thinking Students Meeting Mathematics Growth Target by Year by Grade 5 EOG Mathematics Level



	Met Growth 06-07	Met Growth 07-08	Met Growth 08-09
Level I	62.96%	63.46%	62.96%
Level II	61.54%	55.34%	58.69%
Level III	60.91%	59.59%	65.79%
Level IV	51.43%	60.41%	75.61%

Note: 1. Students enrolled in 2006-07, 2007-08, and 2008-09 at Algebraic Thinking schools
2. The scale was truncated to show pattern.
Data Source: 2008-09 End-of-Year Middle School Rosters

Figure 3
Percentage of 2006-07 Cohort of Comparison Students Meeting Mathematics Growth Target by Year by Grade 5 EOG Mathematics Level



	Met Growth 06-07	Met Growth 07-08	Met Growth 08-09
Level I	64.29%	70.73%	58.00%
Level II	62.50%	48.59%	57.81%
Level III	62.79%	57.60%	65.29%
Level IV	54.37%	54.51%	73.31%

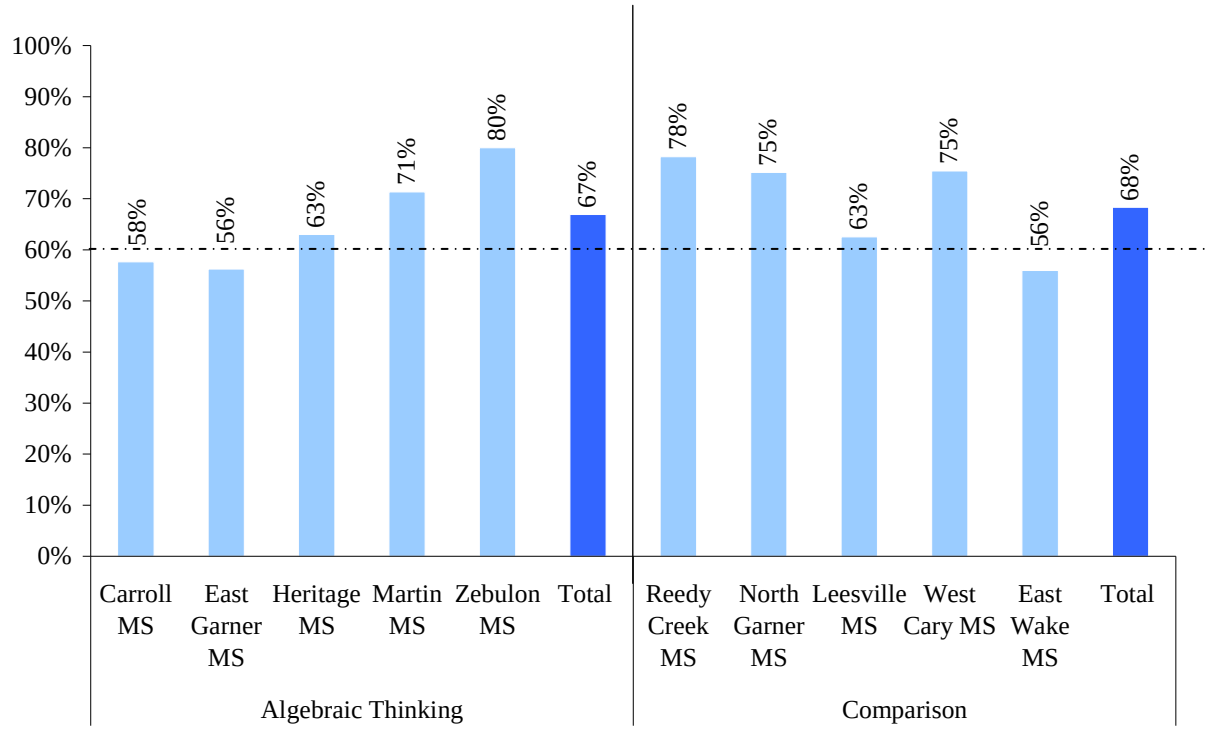
Note: 1. Students enrolled in 2006-07, 2007-08, and 2008-09 at comparison schools
2. The scale was truncated to show pattern.
Data Source: 2008-09 End-of-Year Middle School Rosters

Growth by School

Although the percentage of students who met growth in 2008-09 (grade 8) was almost identical for both groups of schools—67% for Algebraic Thinking schools and 68% for comparison schools—there was considerable variance by school among both the Algebraic Thinking and comparison schools. Figure 4 displays the percentage of students meeting mathematics growth targets in 2008-09 (grade 8) at each school in the study.

- Three of the five Algebraic Thinking schools and four of the five comparison schools had more than 60% of study participants who met growth targets on the EOG.
- The percentage of students who met growth ranged by school from 56% to 80% for Algebraic Thinking schools and from 56% to 78% for comparison schools.

Figure 4
Percentage of 2006-07 Cohort of Algebraic Thinking and Comparison Students Meeting Mathematics 2008-09 Growth Target

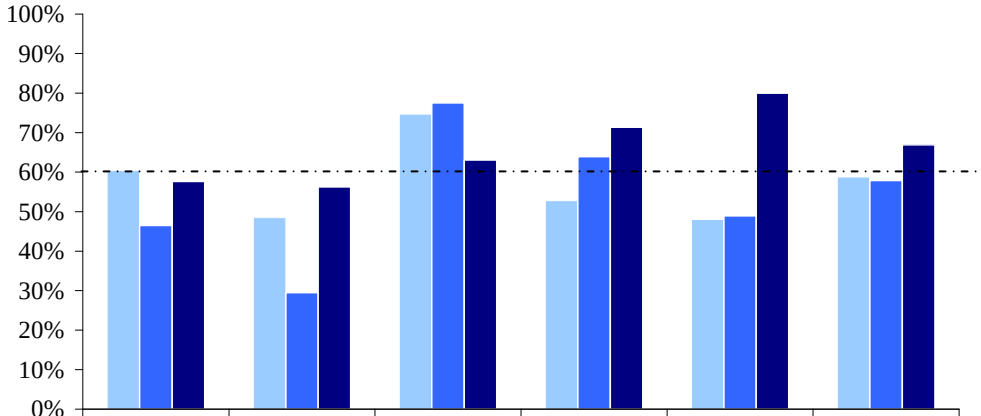


Note: 1. 1,070 Algebraic Thinking students and 1,042 students at the five comparison schools had complete data and thus are reflected in this figure.
2. Figure includes only students with valid pretests and 140 days in membership
Data Source: 2008-09 End-of-Year Middle School Student Rosters
Interpretation Example: 80% of Zebulon Middle students in grade 8 met growth in 2008-09, compared to 56% of East Wake Middle school students in grade 8.

Figures 5 and 6 depict the percentage of students who met growth in 2006-07, 2007-08, and 2008-09 by school.

- Six out of 10 schools experienced an increase in the percentage of students who met growth from 2006-07 (grade 6) to 2008-09 (grade 8).
- Only one school in each group (Heritage Middle and Reedy Creek) met the 60% target for high growth for each of the three years examined. However, in 2008-09 Heritage experienced a decrease in the percentage of students who met growth.
- Carroll Middle and East Garner Middle in the Algebraic Thinking group and West Cary Middle in the comparison group all showed a drop in the percentage of students who met growth in 2007-08 followed by an increase in 2008-09.

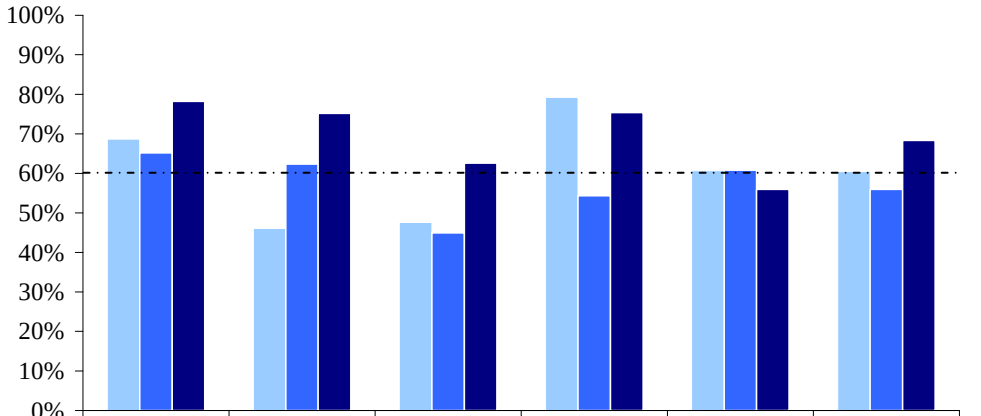
Figure 5
Percentage of 2006-07 Cohort of Algebraic Thinking Students Meeting Mathematics Growth Target by Year



	Carroll MS	East Garner MS	Heritage MS	Martin MS	Zebulon MS	Total
2006-07	60.38%	48.48%	74.68%	52.75%	48.02%	58.79%
2007-08	46.49%	29.49%	77.51%	63.91%	48.94%	57.89%
2008-09	57.63%	56.25%	63.03%	71.37%	80.00%	66.92%

Note: Students enrolled in 2006-07, 2007-08, and 2008-09 at Algebraic Thinking schools
Data Source: 2008-09 End-of-Year Middle School Rosters

Figure 6
Percentage of 2006-07 Cohort of Comparison Students Meeting Mathematics Growth Target by Year



	Reedy Creek MS	North Garner MS	Leesville MS	West Cary MS	East Wake MS	Total
2006-07	68.66%	46.03%	47.54%	79.21%	60.68%	60.42%
2007-08	65.16%	62.33%	44.93%	54.32%	60.83%	55.93%
2008-09	78.21%	75.17%	62.55%	75.41%	55.96%	68.33%

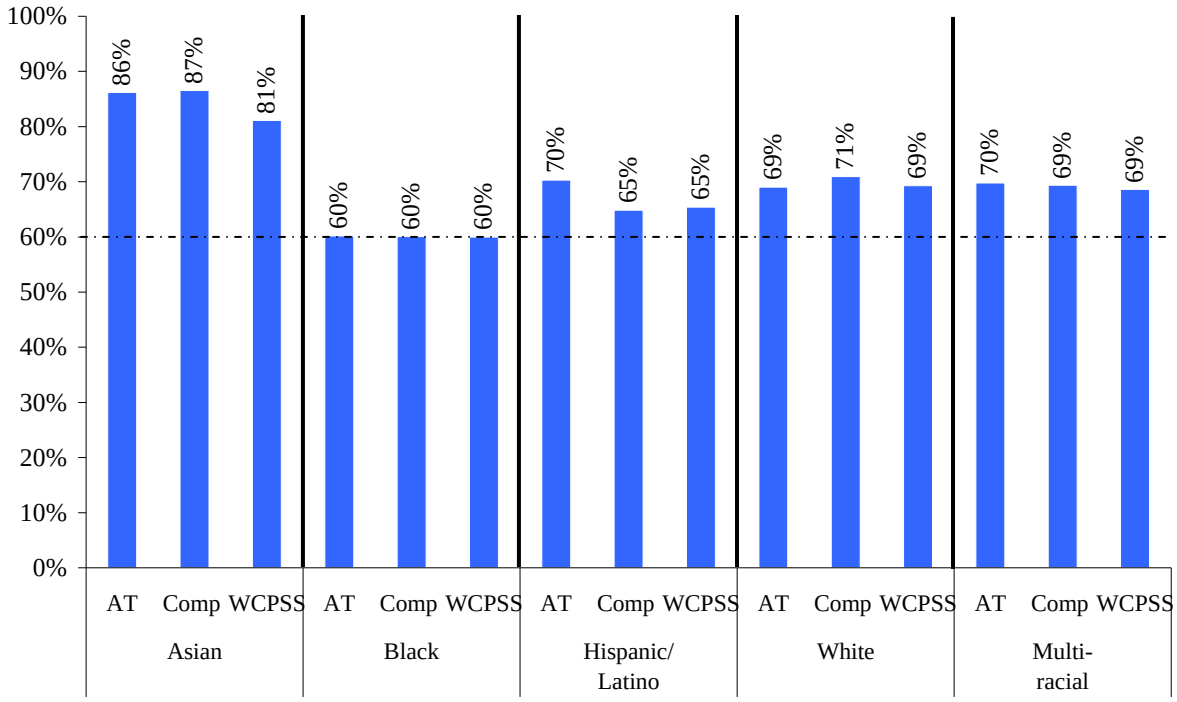
Note: Students enrolled in 2006-07, 2007-08, and 2008-09 at comparison schools
Data Source: 2008-09 End-of-Year Middle School Rosters

Growth by No Child Left Behind (NCLB) Subgroup

Figure 7 depicts the percentage of students in the Algebraic Thinking and comparison cohorts who met growth by racial group.

- All student racial/ethnic groups met high growth (60% or more of students met growth) in Algebraic Thinking schools, comparison schools, and the system overall.
- Within racial/ethnic groups, differences between Algebraic Thinking and comparison schools were not significant.
- The Algebraic Thinking cohort, the comparison cohort, and WCPSS overall had similar racial/ethnic patterns—Asian students had the highest percentages of students meeting growth targets and Black/African American students the lowest percentage. Although Black/African American students had the lowest percentage meet growth, 60% did meet growth.

Figure 7
Percentage of 2006-07 Cohort of Algebraic Thinking and Comparison Students Meeting Mathematics 2008-09 Growth Target by Racial/Ethnic Group



Note: 1. 1,070 Algebraic Thinking students and 1,042 students at the five comparison schools had complete data and thus are reflected in this figure.
2. Figure includes only students with valid pretests and 140 days in membership.
3. AT=Algebraic Thinking and Comp=comparison schools.

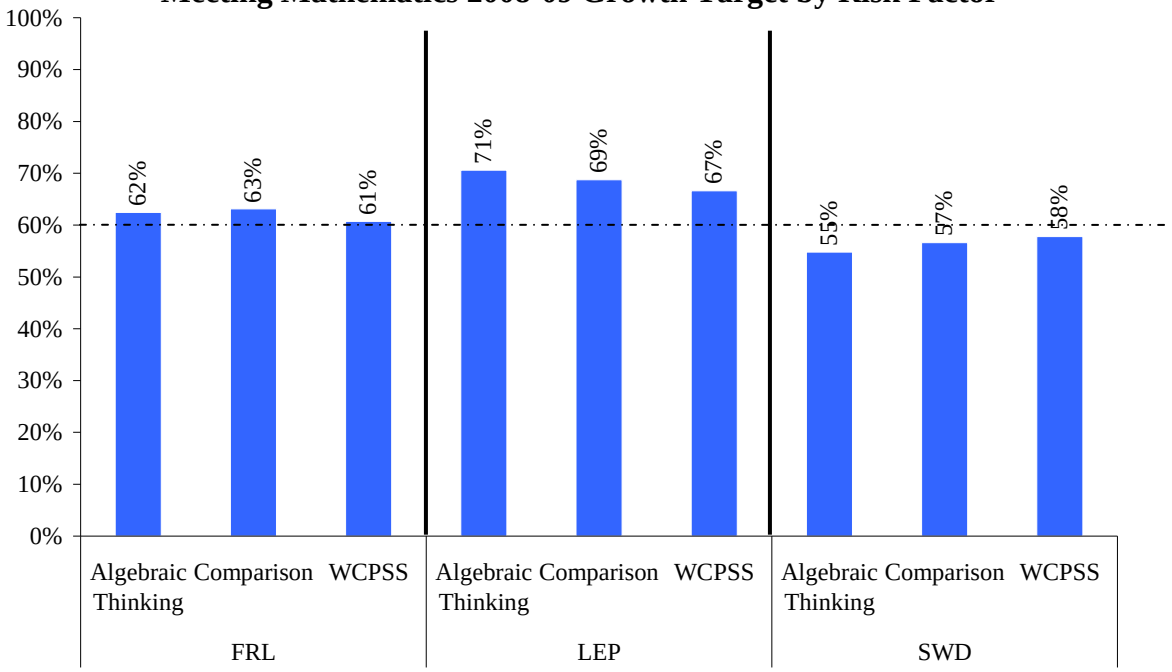
Data Source: 2008-09 End-of-Year Middle School Student Rosters

Interpretation Example: 70% of the Hispanic/Latino students at the Algebraic Thinking schools met growth in 2008-09, compared to 65% at comparison schools and in WCPSS overall.

Figure 8 illustrates the percentage of students in the Algebraic Thinking and comparison cohorts who met growth by FRL, LEP, and SWD status.

- While FRL and LEP students met high growth in Algebraic Thinking schools, comparison schools, and the system overall, fewer than 60% of SWD students met growth—55% of Algebraic Thinking students; 57% of comparison students; and 58% of WCPSS overall.
- Within category differences between Algebraic Thinking and comparison schools were small.

Figure 8
Percentage of 2006-07 Cohort of Algebraic Thinking and Comparison Students Meeting Mathematics 2008-09 Growth Target by Risk Factor



Note: Students enrolled in 2006-07, 2007-08, and 2008-09 at Algebraic Thinking and comparison schools.
Data Source: 2008-09 End-of-Year Middle School Rosters

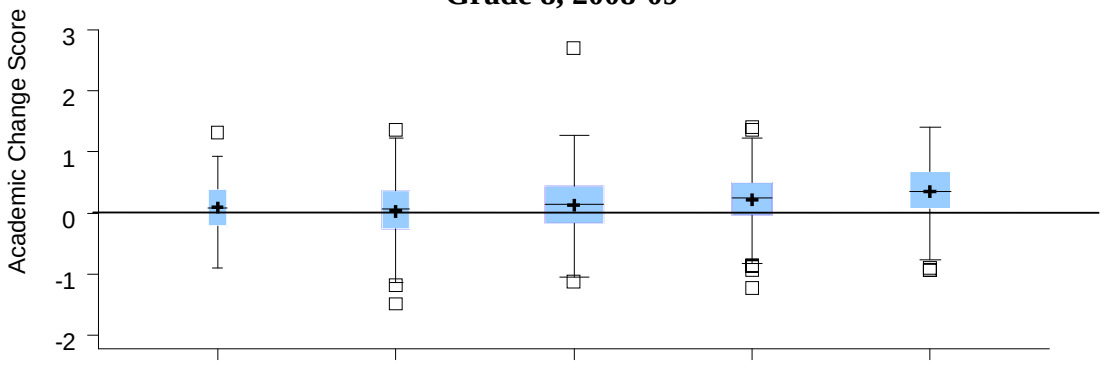
Academic Change

The state ABCs academic change score reflects whether students as a group grew more or less than the target projection. A growth score of zero means the target was met exactly. Figures 9 and 10 utilize a boxplot to depict the mean, median, and range of the academic change score for students enrolled in 2006-07, 2007-08, and 2008-09. The box represents the majority of student scores (25th to 75th percentile). The "whiskers," or vertical lines, extending from the box represent the range of scores, with the most extreme scores denoted by small boxes. Within each box, the mean is signified by a plus sign and the median by a horizontal line in the middle of the box.

For all Algebraic Thinking and comparison schools the average academic change scores in mathematics hovered close to zero, indicating performance close to what was expected (see Figure 9 and 10). While there were slight differences in the mean academic change score between matched schools examined, these differences were not statistically significant.[11] Thus, in actuality, the results should be considered approximately the same.

Figure 9 also illustrates the difference in the range of scores across schools as well, with East Garner Middle showing more variation in student results than Carroll Middle.

Figure 9
Mathematics EOG Academic Change Score
Algebraic Thinking Schools' 2006-07 Cohort
Grade 8, 2008-09

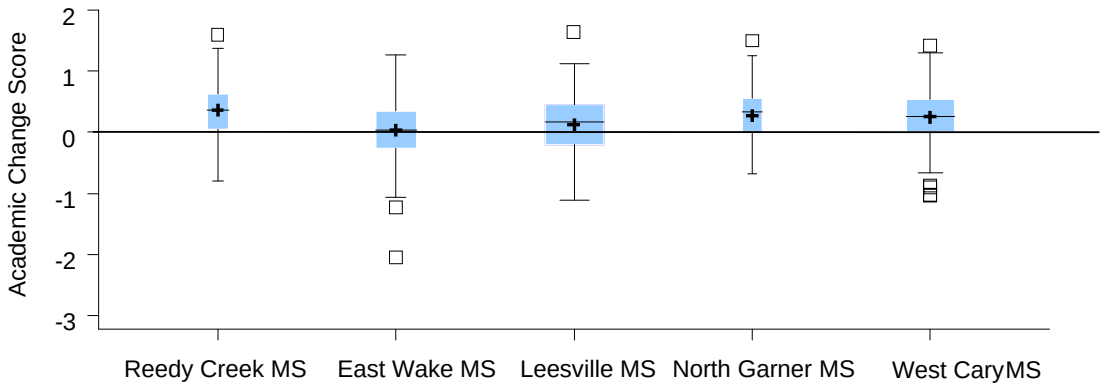


	Carroll MS	East Garner MS	Heritage MS	Martin MS	Zebulon MS
Min	-.89	-1.5	-1.1	-1.2	-.93
Mean	0.09	0.03	0.13	0.22	0.35
Max	1.32	1.36	2.70	1.41	1.40
Std Dev	0.37	0.48	0.46	0.43	0.47
N	119	160	330	227	236

Note: Wider boxes indicate more students in the group.
Data Source: 2008-09 End-of-Year Middle School Student Rosters
Interpretation Example: Students at each of the five Algebraic Thinking schools had an average academic change score greater than zero (signified by plus sign on blue bar).

[11] Significance based on a t-test on the difference of mean academic change score for Algebraic Thinking versus comparison group.

Figure 10
Mathematics EOG Academic Change Score
Comparison Schools' 2006-07 Cohort
Grade 8, 2008-09



	Reedy Creek MS	East Wake MS	Leesville MS	North Garner MS	West CaryMS
Min	-.80	-2.0	-1.1	-.68	-1.0
Mean	0.36	0.04	0.13	0.27	0.25
Max	1.60	1.26	1.64	1.50	1.42
Std Dev	0.44	0.46	0.43	0.41	0.43
N	157	219	275	150	244

Note: Wider boxes indicate more students in the group.
Data Source: 2008-09 End-of-Year Middle School Student Rosters
Interpretation Example: Students at each of the five Algebraic Thinking schools had an average academic change score greater than zero (signified by plus sign on blue bar).

EOG Goal Summary

Goal summary reports were examined in order to compare study participants at Algebraic Thinking and comparison schools on the five mathematics goals:

- Goal 1: Understand and compute with real numbers;
- Goal 2: Understand and use measurement concepts;
- Goal 3: Understand and use properties and relationships in geometry;
- Goal 4: Understand and use graphs and data analysis; and
- Goal 5: Understand and use linear relations and functions.

Two Goal Summaries were generated: one summary for the 1,074 students participating in Algebraic Thinking and one summary for the 1,034 students with complete data in the comparison group. There were no notable differences between Algebraic Thinking and comparison school participants across the five mathematics goals. Both student groups had a higher percentage of responses correct as compared to the state—ranging by goal from 4.5 to 8.1 percentage points higher than the state for Algebraic Thinking students and 3.7 to 8.1 percentage points higher than the state for comparison school students.

Algebra I EOC

For those students who took Algebra I in grade 8, nearly all students in the Algebraic Thinking and comparison schools (99% versus 96%) scored proficient (see Table 9). Algebraic Thinking students who were enrolled in Algebra I had significantly higher growth (Algebra I academic change scores) than comparison students. Since a slightly lower percentage of Algebraic Thinking students participated in Algebra I in grade 8 (30.1% versus 37.4%), an additional analysis was conducted to check for selection bias (i.e., whether Algebraic Thinking schools selected more qualified students). Even when students were matched individually, the findings were consistent with the overall group; Algebraic Thinking students enrolled in Algebra I had significantly higher growth than that of the one-to-one matched students.

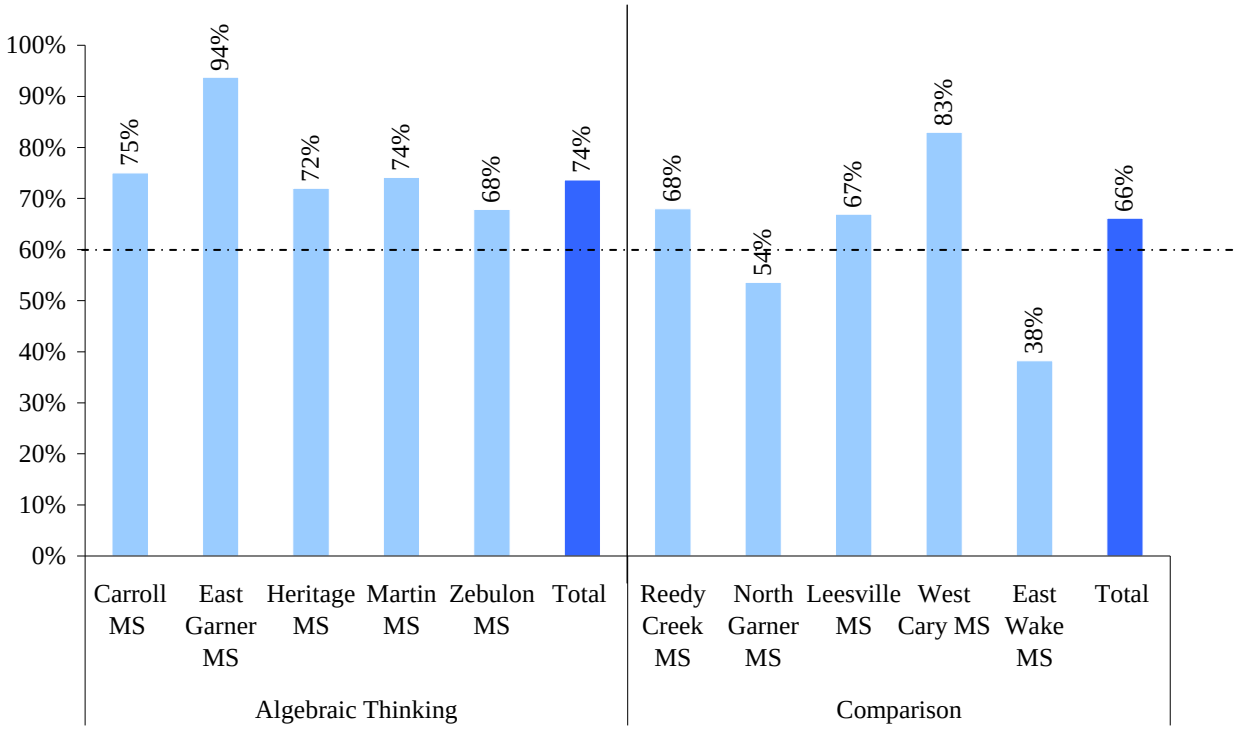
Table 9
Percentage of 2006-07 Cohort of Algebraic Thinking and Comparison Students Enrolled in 2006-07, 2007-08, and 2008-09 Proficient on Algebra I EOC

2008-09 Algebra I EOC Level	Algebraic Thinking		Comparison Students	
	Number	Percent	Number	Percent
Level I	0	0.0%	2	0.5%
Level II	2	0.6%	16	4.0%
Level III	37	11.3%	87	21.5%
Level IV	288	88.1%	299	74.0%
Total	**327**	**100%**	**404**	**100%**

Note: Cohort of students enrolled in 2006-07 who remained in an Algebraic Thinking or comparison school throughout the study (2006-07, 2007-08, and 2008-09).
Data Source: 2008-09 End-of-Year Middle School Student Rosters

A significantly higher percentage of grade 8 students at Algebraic Thinking schools met their EOC growth targets compared to the cohort of students at comparison schools. All five Algebraic Thinking schools and three out of the five comparison schools had greater than 60% of the 2006-07 cohort meet growth (see Figure 11). The percentage of students meeting growth ranged from 68% to 94% among Algebraic Thinking schools and from 38% to 83% among comparison schools.

Figure 11
Percentage of 2006-07 Cohort of Algebraic Thinking and Comparison Students Meeting 2008-09 Algebra I EOC Growth Target



Note:
1. Students enrolled in 2006-07, 2007-08, and 2008-09 at Algebraic Thinking and comparison schools.
2. This figure represents the 326 students at Algebraic Thinking schools and 402 students at comparison schools enrolled in Algebra I in grade 8 with valid pretests and 140 days in membership.

.
Data Source: 2008-09 End-of-Year Middle School Student Rosters

Question 7: Has Algebraic Thinking increased student participation in higher level courses?

Participation in Algebraic Thinking did not increase enrollment of students in higher mathematics courses. As compared to students in the comparison cohort, neither the intermediate goal of a higher percentage of students in grade 7 enrolled in Pre-Algebra, nor the long-term goal of a greater percentage of students enrolled in Algebra I in grade 8, were met. Figures 12 and 13 illustrate the mathematics course trajectory for Algebraic Thinking and comparison student cohorts. Figure 12 displays the primary mathematics courses taken by these students (courses with few students enrolled were grouped into the category *Other Math Course*). The weighted arrows indicate the number of students transitioning between courses.

- Most (77%) of the students enrolled in Algebraic Thinking I in grade 6, enrolled in Pre-Algebra (41%) or Algebraic Thinking II (36%) in grade 7.

- Among grade 7 students, similar percentages of Algebraic Thinking and comparison cohorts enrolled in Pre-Algebra in grade 7 (41% and 44% respectively).
- Among both Algebraic Thinking and comparison students who took Pre-Algebra in grade 7, most took Algebra I in grade 8 (81% and 77% respectively).
- Among grade 8 students, a significantly higher percentage of students in the comparison cohort enrolled in Algebra I than students in the Algebraic Thinking cohort (37% versus 30% respectively).

Figure 13 provides the course-taking sequence for Algebraic Thinking and comparison students in greater detail (depicting the specific courses for many of the students enrolled in *Other Math Course*). The figure also shows the course-taking pattern of the 80 students who were enrolled at one of the five Algebraic Thinking pilot schools, but who did not participate in Algebraic Thinking I in 2006-07. While the vast majority of students at Algebraic Thinking schools were enrolled in Algebraic Thinking I in grade 6, there were some students who were placed in another mathematics course. These students were included to capture the students at Algebraic Thinking schools who either were high-achievers (thus placed directly into Pre-Algebra or Algebra in grade 6) or students who were low-achievers (thus placed in a lower-level mathematics course).

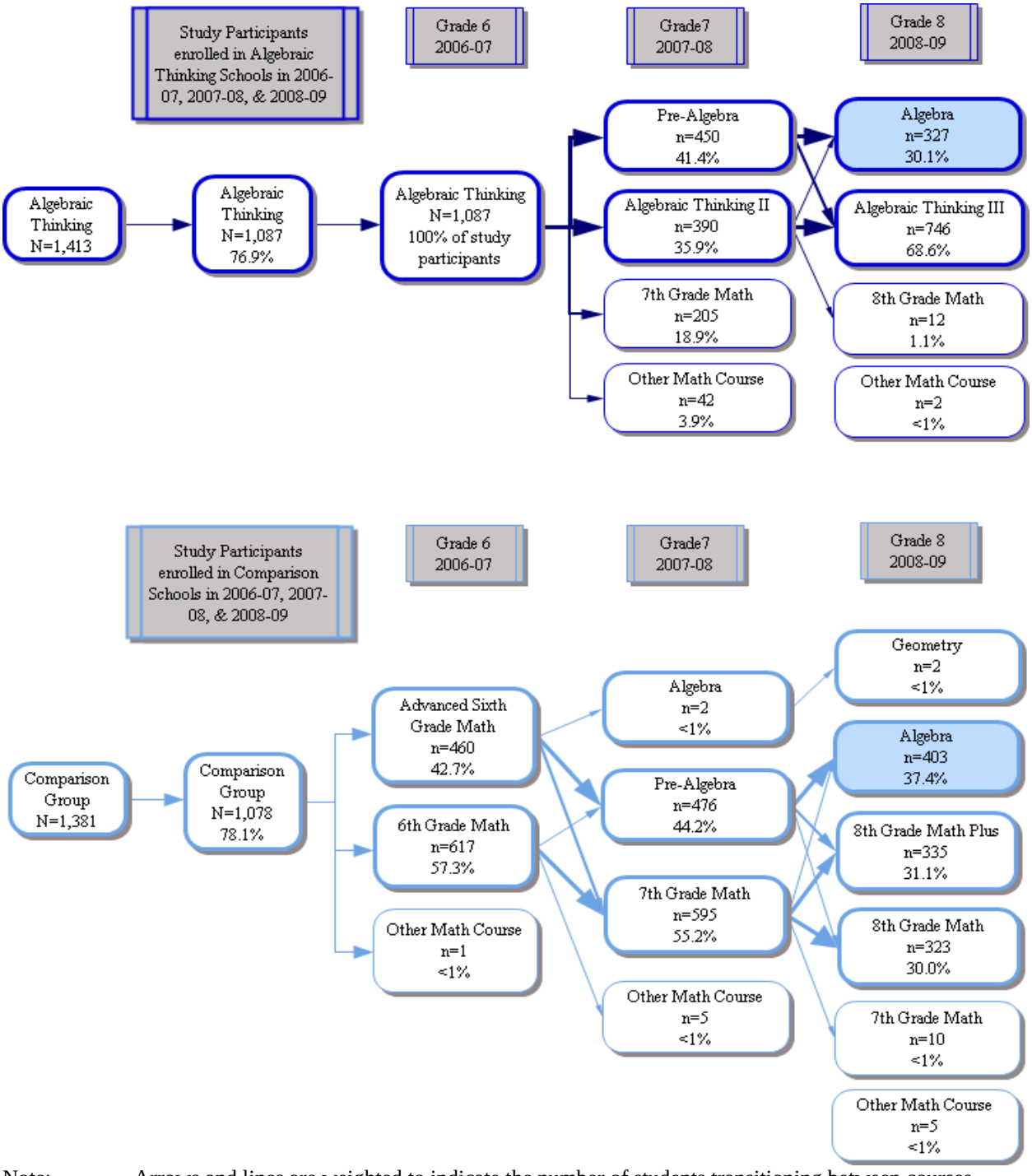
Appendices A, B, and C display the 2009-10 grade 9 mathematics courses for Algebraic Thinking and comparison students enrolled in Algebra I, Algebraic Thinking III, 8th Grade Math Plus, or 8th Grade Math in 2008-09. The grade 9 course selection of Algebraic Thinking III students more closely resembled students who took 8th Grade Math Plus than those in the regular 8th Grade Math. Appendixes A and B display the percentage of students enrolled by Grade 9 mathematic courses.

- About two-thirds (69.2%) of Algebraic Thinking III students enrolled in Algebra Plus or Algebra Part I and/or Part II compared to 78.5% of comparison students who took 8th Grade Math Plus, and 47.1% who took 8th Grade Math (see Appendix B).
- Nearly all Algebraic Thinking students (90.2%) and the vast majority of comparison (81.7%) and who took Algebra I in grade 8, enrolled in Geometry (Honors) or Geometry in grade 9 (see Appendix B).

Block schedules give students the opportunity to complete more than one mathematics course in a year, potentially allowing students to take more advanced mathematics course (even if they were not enrolled in Algebra I in grade 8). Many students elected to take two semesters of mathematics in grade 9 (see Appendix C). The percentage of students who took two mathematics courses varied by course taken in grade 8:

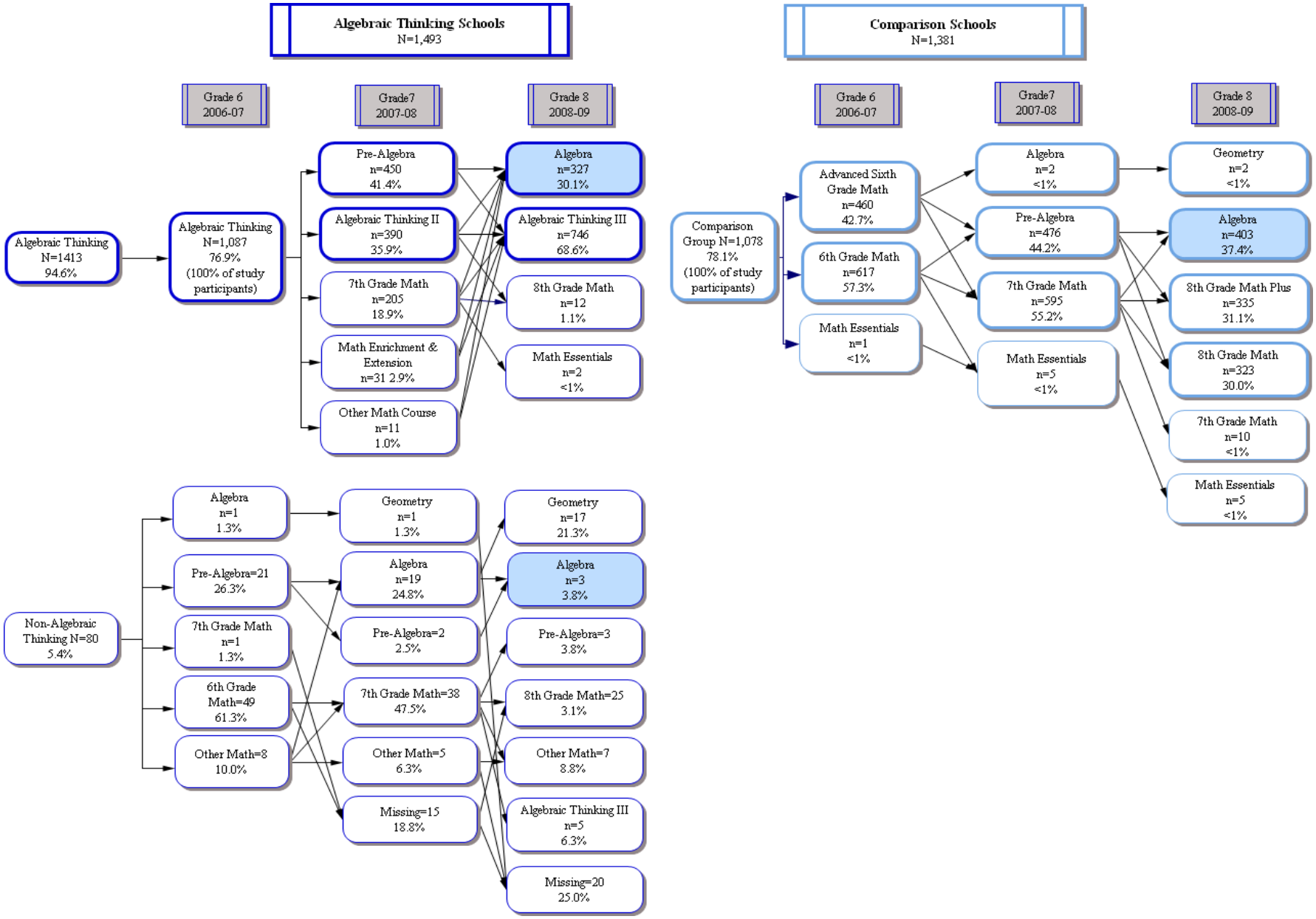
- Among students who took Algebra I in grade 8, 6.4% of Algebraic Thinking students and 31.7% of comparison students took two semesters of mathematics in grade 9.
- 57.6% of Algebraic Thinking III students took two semesters of mathematics, compared to 62.7% of students who took 8th Grade Math Plus and 57.3% of students who took 8th Grade Math at comparisons schools.

Figure 12
Mathematics Courses by Year
Students Enrolled in 2006-07, 2007-08, and 2008-09 at
Algebraic Thinking and Comparison Schools



Note: Arrows and lines are weighted to indicate the number of students transitioning between courses. Bolder lines and larger arrows signify larger numbers of students.
Data Source: SIGR1110 file obtained from the FTP interface on 3/15/2010

Figure 13
Mathematics Courses by Year
Students Enrolled in 2006-07, 2007-08, and 2008-09 at Algebraic Thinking and Comparison Schools



CONCLUSIONS

The long-term goals of Algebraic Thinking were to improve student achievement and increase the likelihood students would enroll in higher mathematics courses. To contextualize the results of Algebraic Thinking this evaluation has provided a comparison to matched schools. However, it is also helpful to consider how successful Algebraic Thinking was in meeting its goals independent of a comparison. Thus, Table 10 displays the status of each Algebraic Thinking goal in two ways:

1. independent of a comparison (i.e., if Algebraic Thinking met its stated goals) and
2. compared to the results of students receiving the standard mathematics sequence at the comparison schools.

Although independent of a comparison the outcomes for Algebraic Thinking students were relatively positive, the outcomes were similar to students receiving the traditional mathematics course sequence. While Algebraic Thinking schools met four of the five goals considered independent of a comparison, when compared to schools offering the standard mathematics course sequence there was no difference in student success—with the exception of Algebra I EOC growth where Algebraic Thinking students performed significantly better than the comparison cohort. However, Algebra Thinking students were less likely than the comparison cohort to enroll in Algebra I in grade 8, a primary goal of this initiative.

Table 10
Status of Algebraic Thinking Goals

Level	Goal	Status Independent of Comparison	Status Based on Comparison Schools
Short-term	Students presented with the same mathematical opportunities in grade 6, therefore giving students the same opportunity to move on to higher level classes.	Met	N/A
	Students served in grade 6 in 2006-07 meet mathematics growth targets (academic change) on EOG.	Not Met Overall & in 3 of 5 schools	Not Met
Intermediate	Increase in grade 7 students enrolled in Pre-Algebra.	N/A	Not Met
Long-term	Increase in the percentage of students reaching **EOG** growth targets at grade 8	Met Overall & in 3 of 5 schools	Not Met
	All sub-groups meet mathematics growth targets (academic change) on EOG at grade 8.	Met in 7 out of 8 subgroups	Not Met
	Increase in the percentage of students reaching **EOC** growth targets at grade 8.	Met Overall & in 5 of 5 schools	Met
	Increase in grade 8 students enrolled in Algebra.	N/A	Not Met

Interpretation Example: Students served in Algebraic Thinking schools in grade 6 in 2006-07 did not meet academic change growth targets overall or in 3 of the 5 schools. Growth was also not statistically different from that seen in comparison schools.

The goal of increased student achievement was measured by comparing the percentage of the Algebraic Thinking 2006-07 cohort who met EOG mathematics growth to the matched schools comparison cohort. This comparison generated mixed results. Both the grade 6 and grade 8 goals of an increased percentage of students making growth targets were not met. However, if we consider the Algebra I EOC growth results of the subpopulation of students who took Algebra I in grade 8, the long-term goal of grade 8 students who met growth becomes more complicated. For among this smaller portion of the Algebraic Thinking cohort, students were able to out-perform the comparison group on the Algebra I EOC (even when students were selected for a one-to-one matched sample group).

One of the primary goals was to increase the enrollment of grade 8 students in Algebra I. Based on the percentage of grade 8 students enrolled in 2008-09, this goal was not met. Fewer Algebraic Thinking students took Algebra I in grade 8 as compared to students in the comparison cohort. However, it should be noted that an examination of grade 9 course selection revealed that Algebraic Thinking III students' grade 9 course selections more closely resembled students who took 8th Grade Math Plus than those in the regular 8th Grade Math.

DISCUSSION

Algebraic Thinking was implemented to allow students an extra year of mathematics instruction prior to dividing them into regular and advanced mathematics. Through heterogeneous grouping and differentiated instruction an increased number of students were to be exposed to the possibility of moving to higher level mathematics in grades 7 and 8. There has been some concern that heterogeneous grouping would negatively impact high achieving students. This concern was evident in the surveyed teachers' perceptions that students performing below grade level benefited from Algebraic Thinking while student performing at or above grade level were negatively impacted. However, the teachers' perceptions were not supported by the data. In fact, consistent with national research (Laitsch, 2006), this study found that heterogeneous grouping did not appear to have a negative impact on initially high-achieving students. Rather, a higher percentage of students who scored a Level IV on the grade 5 mathematics EOG showed growth than did students who entered grade 6 with lower mathematics scores.

Moreover, fewer Algebraic Thinking students enrolled in Algebra I in grade 8 as compared to the cohort of students attending the comparison schools. However, Algebraic Thinking students who were enrolled in Algebra I had significantly higher growth (based on Algebra I academic change scores) than comparison students; this was true for both the cohort comparison and the one-to-one matched student comparison. This finding further strengthens the assertion that high-achieving students were not negatively impacted by their participation in Algebraic Thinking and may have actually benefited from attending an Algebraic Thinking school. On the other hand, the fact that a slightly lower percentage of students were placed in Algebra I in these schools was unexpected.

Researchers have found that students who participate in Algebra I in grade 8 are more likely to enroll in advanced mathematics in high school. It is important to recognize that this may be the result of selection bias toward more capable mathematics students, since these students are

selected to participate in Algebra I based on their prior mathematics ability (Burris, Heubert, & Levin, 2004). Thus, it should not be surprising that these students go on to take more advanced mathematics courses in high school and college.

Some educational researchers have found disadvantages to setting an educational goal to increase Algebra I enrollment in grade 8. This research suggests that simply targeting increased enrollment into Algebra I may encourage the placement of students who are not yet ready for this level of mathematics (Viadero, 2010). Although a primary goal of Algebraic Thinking was to increase the percentage of students enrolled in Algebra I in grade 8, this initiative was also designed to provide an advanced mathematics experience to all grade 6 students; thereby, preparing more students for placement in pre-algebra and algebra.

The heterogeneous grouping of mathematics students within Algebraic Thinking schools resulted in an unintended benefit by providing greater scheduling flexibility. Curriculum and Instruction staff reported anecdotal evidence from school staff regarding Algebraic Thinking's impact on course scheduling. Since mathematics was not tracked there was less tracking among other courses. In a school where mathematics courses are tracked, other core courses are tracked by default. Anecdotal data provided over the past few years to Curriculum and Instruction staff regarding the implementation of Algebraic Thinking supports this positive impact of Algebraic Thinking on core content teachers.

RECOMMENDATIONS

The Algebraic Thinking approach appears to lead to similar achievement outcomes as those produced by traditional approaches. It actually appears to benefit high achievers, which was one concern expressed. Although it does require some training, the cost is relatively minimal. Thus, school staff may elect to continue implementation of this approach based on the similarity of achievement outcomes. While it has not consistently met its goals, it also does not appear to negatively impact student outcomes. Therefore, the option of either approach might be offered to schools. Regardless of approach employed, improving student outcomes and increasing the number of students prepared for and successful in Algebra I should be emphasized.

Although Algebraic Thinking's approach of preparing students in grade 6 to be ready to take Algebra I in grade 8 is a laudable one, this goal was not realized. Given that the percentage of students who took Algebra I in grade 8 was significantly lower than at comparison schools (a primary program goal) implementation and support of this initiative should be strengthened or an alternative approach investigated. If schools elect to continue to implement Algebraic Thinking, implementation of the initiative could be improved through the clarification and examination of goals, documentation, and guidelines; and by meeting training needs. Furthermore, given the by school variation in results, exemplars should be used as guides. At this point, we have the following recommendations for improvement.

- ***Use schools with the strongest results as exemplars***. Given the large variation across schools, staff should study the schools with the greatest success more closely in terms of their level of implementation. The instructional differentiation and student grouping practices utilized by schools with higher achievement should be determined. These practices should

be compared to schools that did not have strong results. The differences in practices between the schools with stronger versus weaker achievement results can help isolate the key strategies that have the greatest likelihood of impacting student success. Once identified, these key strategies should be shared through videotaping model lessons, providing sample lessons, and/or encouraging mentoring pairs.

- ***Clarify goals***. The logic model presented in this report was produced for reporting purposes with assistance from Evaluation and Research staff—however, the logic model was drafted after the initiative began and training took place. The goals developed and presented here should be revisited in light of the findings of this report. For example, the student achievement goal was focused on EOG growth, but should perhaps be expanded to include Algebra I EOC growth. Additionally, annual growth targets should be set both overall and by subgroup. Finally, it is not clear whether seventh grade teachers understood the goal of moving more students into Pre-Algebra in grade 7 and Algebra I in grade 8 as a result of this new approach. Since they made those recommendations, greater involvement of those teachers could promote reaching this goal.

  While this report evaluated goals based on comparisons to matched schools, another method of gauging program success is by comparing the program outcomes across years. Thus, annual goals could be developed based on the current percentage of students meeting growth. For example, if the current percentage of grade 6 students meeting growth is 48.5%, then the short-term goal could be 55.5% in year 1 (a 7 percentage point increase) and the intermediate goal could be 62.5% in year 2 (an additional 7 percentage points). Given the mixed research on increasing the number of students enrolled in Algebra I in grade 8, it is our recommendation that the goal continue to focus on increasing the number of students *prepared for* and then *enrolled in* Algebra I and be worded in a manner to capture this progression.

  Furthermore, it is important to communicate to all stakeholders what defines the success of an initiative in order to have all participants working toward the same end. Thus, the goals should be further clarified and then communicated to those involved in this effort.

- ***Update Program Guide to reflect course sequence.*** Although Algebraic Thinking was piloted in 2006-07, the Middle School Planning Guide did not reflect this addition until 2008-09. While this information was verbally communicated to school staff, there was a two year lag in the updating of the system level reference material.

  Additionally, the description of Algebraic Thinking I is combined with Advanced 6th Grade Math and Algebraic Thinking III is combined with 8th Grade Math Plus. Algebraic Thinking is designed to combine 6th Grade Math with Advanced 6th Grade Math and 8th Grade Math with 8th Grade Math Plus; thus, a definition that only includes the advanced course seems incomplete. Therefore, the Middle School Planning Guide should include a description of Algebraic Thinking I and III separate from the other mathematic course descriptions.

- ***Offer training more consistently and review training content***. Of the 12 teachers who reported not receiving training, six reported teaching at grade 7 (no training was offered due

to the lack of programmatic change at this grade level). However, the other six teachers taught at grades 6 or 8. If grade 7 teachers are removed from consideration, this still leaves more than 1 in 5 teachers not having received training. Furthermore, of those who did receive training, 20% did not feel that the training was adequate to allow them to implement Algebraic Thinking in their classroom. Training new staff members was reportedly done using the train the trainer model; however, there may have been inconsistencies between schools that resulted in either no training or training that was considered inadequate. Considering the importance of training when implementing an effort, training both in terms of access for all teachers (including new teachers at existing Algebraic Thinking schools) and content of training should be reviewed. More consistent training and support across years (i.e., electronic resources and video presentations of actual training) could improve implementation.

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Appendix A

Grade 9 Mathematics Courses
Students Enrolled in 2006-07, 2007-08, and 2008-09 at
Algebraic Thinking and Comparison Schools

Mathematics Course Grade 9	Algebraic Thinking		Comparison		
	Algebra in Grade 8	Algebraic Thinking III	Algebra in Grade 8	8th Grade Math Plus	8th Grade Math
Geometry (Honors)	**79.2%**	1.7%	**72.5%**	0.3%	0.0%
Geometry	11.0%	0.7%	9.2%	0.6%	0.0%
Algebra II (Honors)	1.8%	0.0%	5.2%	0.3%	0.0%
Algebra II	0.0%	0.0%	0.5%	0.0%	0.0%
Algebra Plus	0.3%	2.9%	0.2%	1.2%	0.6%
Algebra I: Part I	0.3%	2.7%	1.2%	4.8%	10.8%
Algebra I: Part II	1.2%	**35.0%**	6.2%	**37.9%**	23.8%
Algebra I	0.3%	28.6%	0.2%	34.6%	11.8%
Foundations of Algebra	0.0%	0.5%	0.0%	0.9%	2.2%
Intro Math	0.0%	20.2%	0.3%	14.3%	**40.6%**
missing	5.8%	7.6%	4.5%	5.1%	10.2%
Total	100.0%	100.0%	100.0%	100.0%	100.0%

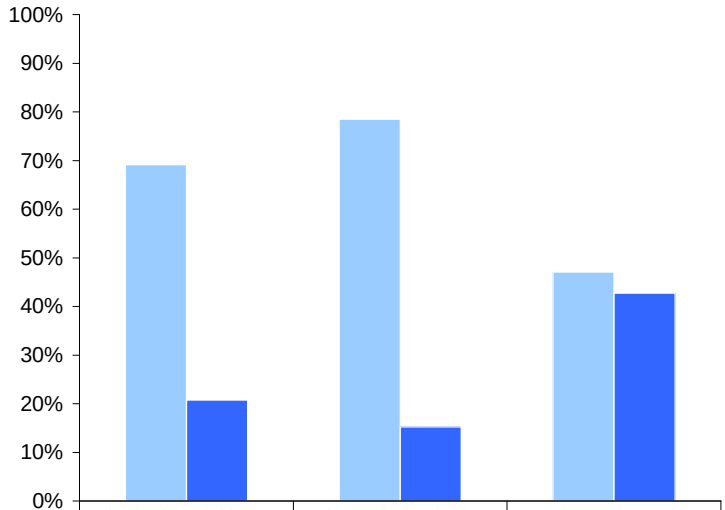
Note: 1. Bolded text indicates the most frequent course taken in grade 9.
2. The percentages represent students' highest mathematic course taken in grade 9 (2009-10 semester 1, 2 or 3).

Data Source: ACCMEM file from the first day of spring 2010.

Interpretation Example: Among students enrolled in Algebraic Thinking III in grade 8, 35.0% enrolled in Algebra I Part II in grade 9 compared to 37.9% of students at a comparison school who took 8th Grade Math Plus and 23.8% who took regular 8th Grade Math.

Appendix B

Percentage of Algebraic Thinking and Comparison Cohort Students Enrolled in Algebraic Thinking III, 8th Grade Math Plus, or 8th Grade Math by Grade 9 Mathematics Courses

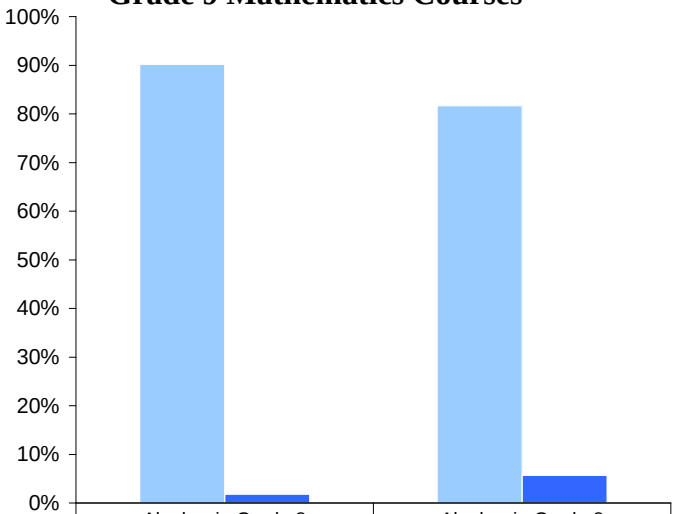


	Algebraic Thinking III	8th Grade Math Plus	8th Grade Math
Algebra (Plus, Part I & II)	69.2%	78.5%	47.1%
Foundation/Inro Math	20.8%	15.2%	42.7%

Data Source: ACCMEM file from the first day of spring 2010.

Interpretation Example: Among students enrolled in Algebraic Thinking III in grade 8, 69.2% enrolled in Algebra Plus or Algebra I Part I and/or Part II in grade 9 compared to 78.5% of students at a comparison school who took 8th Grade Math Plus and 47.1% who took regular 8th Grade Math.

Percentage of Algebraic Thinking and Comparison Cohort Students Enrolled in Algebra I in Grade 8 by Grade 9 Mathematics Courses



	Algebra in Grade 8	Algebra in Grade 8
Geometry (Honors)/ Geometry	90.2%	81.7%
Algebra II (Honors)/ Algebra II	1.8%	5.7%

Data Source: ACCMEM file from the first day of spring 2010.

Appendix C

Grade 9 Mathematics Courses
Students Enrolled in 2006-07, 2007-08, and 2008-09 at
Algebraic Thinking Schools

Algebra in Grade 8

		Sem 1	Sem 2	Sem 3	Sem 1 & 2	Sem 1&3	***N***	
Geometry (Honors)		91.4%	77.6%	89.2%	90.0%	100.0%	259	79.2%
Geometry		4.3%	17.8%	9.2%			36	11.0%
Algebra II (Honors)		2.9%	1.3%	1.5%	5.0%		6	1.8%
Algebra II							0	0.0%
Algebra Plus		1.4%					1	0.3%
Algebra I: Part I			0.7%				1	0.3%
Algebra I: Part II			2.0%		5.0%		4	1.2%
Algebra I			0.7%				1	0.3%
		100.0%	100.0%	100.0%	100.0%	100.0%	308	**94.2%**
	n=	70	152	65	20	1	308	
						missing	19	5.8%
							327	

Algebraic Thinking III

		Sem 1	Sem 2	Sem 3	Sem 1 & 2	Sem 1&3	***N***	
Geometry (Honors)		1.0%			2.8%		13	1.7%
Geometry					1.2%		5	0.7%
Algebra II (Honors)							0	0.0%
Algebra II							0	0.0%
Algebra Plus		5.8%	4.8%		2.6%		22	2.9%
Algebra I: Part I		8.7%	3.8%		1.6%		20	2.7%
Algebra I: Part II			11.4%		58.2%		261	35.0%
Algebra I		65.4%	42.9%	86.0%	13.3%		213	28.6%
Foundations of Algebra		1.9%			0.5%		4	0.5%
Intro Math		17.3%	37.1%	14.0%	19.9%	100.0%	151	20.2%
		100.0%	100.0%	100.0%	100.0%	100.0%	689	**92.4%**
	n=	104	105	50	428	2	689	
						missing	57	7.6%
							746	

Appendix C

Grade 9 Mathematics Courses
Students Enrolled in 2006-07, 2007-08, and 2008-09 at Comparison Schools

Algebra in Grade 8

		Sem 1	Sem 2	Sem 3	Sem 1 & 2	Sem 1&3	*N*	
Geometry (Honors)		91.6%	80.0%	100.0%	57.8%		293	72.5%
Geometry		5.6%	12.0%		10.2%		37	9.2%
Algebra II (Honors)		1.9%	3.3%		10.9%		21	5.2%
Algebra II					1.6%		2	0.5%
Algebra Plus		0.9%					1	0.2%
Algebra I: Part I			3.3%				5	1.2%
Algebra I: Part II			0.7%		18.8%		25	6.2%
Algebra I			0.7%				1	0.2%
Introduction to math					0.8%		1	0.3%
		100.0%	100.0%	100.0%	100.0%	0.0%	386	**95.5%**
	n=	107	150	1	128	0	386	
						missing	18	4.5%
							404	

8th Grade Math Plus

		Sem 1	Sem 2	Sem 3	Sem 1 & 2	Sem 1&3	*N*	
Geometry (Honors)		2.3%					1	0.3%
Geometry			1.6%		0.5%		2	0.6%
Algebra II (Honors)					0.5%		1	0.3%
Algebra II							0	0.0%
Algebra Plus		4.7%			1.0%		4	1.2%
Algebra I: Part I		4.7%	17.7%		1.4%		16	4.8%
Algebra I: Part II			1.6%		60.0%		127	37.9%
Algebra I		62.8%	53.2%	66.7%	25.7%		116	34.6%
Foundations of Algebra		4.7%			0.5%		3	0.9%
Intro Math		20.9%	25.8%	33.3%	10.5%		48	14.3%
		100.0%	100.0%	100.0%	100.0%	0.0%	318	**94.9%**
	n=	43	62	3	210	0	318	
						missing	17	5.1%
							335	

8th Grade Math

		Sem 1	Sem 2	Sem 3	Sem 1 & 2	Sem 2&3	*N*	
Geometry (Honors)							0	0.0%
Geometry							0	0.0%
Algebra II (Honors)							0	0.0%
Algebra II							0	0.0%
Algebra Plus		6.3%			0.6%		2	0.6%
Algebra I: Part I		12.5%	22.1%		7.9%		35	10.8%
Algebra I: Part II			8.1%		39.3%		77	23.8%
Algebra I		25.0%	8.1%	66.7%	14.0%		38	11.8%
Foundations of Algebra						100.0%	7	2.2%
Intro Math		56.3%	61.6%	33.3%	38.2%		131	40.6%
		100.0%	100.0%	100.0%	100.0%	100.0%	290	**89.8%**
	n=	16	86	3	178	7	290	
						missing	33	10.2%
							323	