PROSPECTIVE MATHEMATICS TEACHERS' DIFFICULTIES IN DOING PROOFS AND CAUSES OF THEIR STRUGGLE WITH PROOFS[1]

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This research aims to expose prospective mathematics teachers' difficulties while proving, as well as the reasons behind such difficulties. The research includes 121 second year undergraduate prospective teachers studying at the primary mathematics teaching department of a state university in Turkey. The study has found that prospective teachers had serious deficiencies in doing proof. The primary difficulty experienced by prospective teachers is expressing definitions. This difficulty is respectively followed by understanding theorem statement, using mathematical language and notations, selecting proper proof strategy and method, distinguishing concepts, creating a proof structure using definitions, and the difficulty of expressing thoughts. In addition, interviews have been conducted with seven prospective teachers representing each difficulty using semi-structured interview form. Interviews with the participants have shown that the major causes of such difficulties stem from prospective teachers having a negative attitude about proofs, and the various shortcomings in learning and teaching proofs.

Keywords: *Mathematical proof, difficulties in proof, prospective mathematics teachers, mathematics education*

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1. Introduction

In any discussion environment, people try to justify their thoughts with logical reasons in order to persuade their counter party. Logical reasons can vary depending on the matter at hand. For example, if the matter in question is in the field of law, the person will use constitutional or penal law articles as the basis to support and affirm his/her ideas. If history is the subject of discussion, however, people would feel the need to base their statements on historical documents widely accepted as accurate. Likewise, in mathematical discussions, proofs are used to validate mathematical statements. The person can persuade his opponent as long as he/she proves mathematical statements by use of mathematical facts, because mathematical proofs are used to validate a result, inform and persuade other people, find a result and put results into a deduction method (Almedia, 2003). People can effectively argue and persuade other communities with the use of well-constructed mathematical proofs (Almeida, 2003; Harel & Sowder, 1998).

As well as validating a result, a mathematical proof carries the function of explanation of the result and persuasion (Hanna, 2000). Therefore, mathematical proofs play an active role in establishing, developing, and transferring mathematical knowledge (Stylianides et al., 2007). Mathematical knowledge develops and matures with proofs (Kitcher, 1984). Mathematical proof is central to the discipline of mathematics (Ko, 2010). According to Weber (2001), proof is the objective of advanced mathematics, and according to Heinze & Reiss (2003) it is the building block of mathematics. Mathematical proofs are just as important in mathematics education as they are in the development and systematization of mathematics. Mathematics educators consider mathematical proof a significant part of mathematics education, and use mathematical proofs as a tool to help students learn mathematical concepts better (Hersh, 1993). Students get a better grasp of mathematical concepts with mathematical proof. And proofs also help in improving their critical thinking, reasoning, and mathematical thinking abilities (Dickersen, 2008; Fawcett, 1938; Hanna, 1991).

While mathematical proof is regarded highly in mathematics education and provides advantages to students, it is widely known that students at all educational levels from secondary to university, including prospective mathematics teachers, have difficulties in using mathematical proof, resulting in negativity towards it (Almeida, 2003; Coşkun, 2009; De Villiers, 1990; Moore, 1994; Moralı et al., 2006). Research in this field shows that difficulties experienced by students regarding mathematical proofs can be separated into two groups, namely, affective and cognitive. Negativity about proof manifests itself in the form of affective difficulties in mathematical proof. Students consider proving a hard, useless, and insignificant activity. Most students are biased against proof (Almeida, 2003; De Villiers, 1990; Doruk & Kaplan, 2013a; Raman, 2003). Negative attitudes toward mathematical proof negatively affect proving abilities of students (Almeida, 2000; Furinghetti & Morselli, 2009). Cognitive difficulties in proof are studied by various methods by researchers (Bayazit, 2009; Güler, 2013; Dreyfus, 1999; Moore 1994; Selden & Selden, 2007; Weber, 2006). Moore (1994) listed the proof difficulties experienced by students as: failing to express definitions, failing to garner an intuitive understanding of concepts, being unable to use concept images in proof, lack of generalizations and samples, not knowing how to make a proof structure using definitions, failing to understand mathematical language and notations, and not knowing how to begin the proof process. Weber (2006) lists the difficulties as insufficient conceptual information about mathematical proof, misunderstanding and misapplication of a concept or a theorem, and failing to develop proof strategies. Güler (2013) evaluates proofs of students in algebra classes under the following categories: expressing how to begin the proving process, expressing definitions, presentation of proof, use of mathematical language and notation, and use of logic and proof methods.

Teachers have a critical role in bringing students to sufficient proof levels and preventing them from developing negative attitudes about proof. Therefore, the above-mentioned qualifications of prospective teachers should be tested at teacher training institutions. To this end, research results can contribute to the training quality of prospective teachers. This research aims to reveal prospective mathematics teachers' difficulties in mathematical proofs. The study also involves prospective teachers' evaluations about the reasons for such difficulties. We are of the opinion that the results of this research can make valuable contributions by supplying information about the difficulties encountered by prospective mathematics teachers in mathematical proof, what they think about proof, and how they feel about proof teaching. In this regard, the results of this research can provide useful information to proof-oriented course lecturers of prospective teachers about difficulties in proof as well as reasons for these difficulties. Accordingly, the following questions are investigated.

- What are the difficulties experienced by prospective teachers in proof?
- What do prospective teachers think about the causes of such difficulties?

2. Method

The research aims to reveal the difficulties experienced while proving and the reasons behind these difficulties. Accordingly, the research was carried out using qualitative research method, because qualitative research method aims to thoroughly examine certain content (a culture, a school, a class, a social layer, a group of people etc.) (Yıldırım & Şimşek, 2008). The case study approach was considered as the most suitable research design for this study. In a case study, factors concerning certain conditions (environment, individuals, events, processes etc.) are researched with a holistic approach and the focus is on how they influence and get influenced by the determined conditions (Yıldırım & Şimşek, 2008).

2.2. *Research Group*

Research participants are 121 sophomore prospective teachers studying at a primary mathematics teaching department of a state university in the Eastern Anatolia Region of Turkey. In addition, semi-structured interviews have been held with seven prospective teachers representing each difficulty.

2.3. *Data Collection*

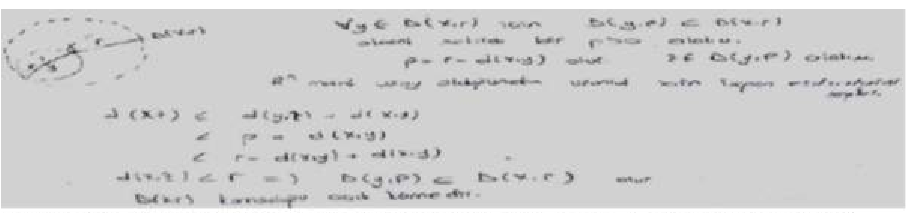
Two types of data collection tools were used in the research. To find an answer to the first research question, prospective teachers were asked the questions "Prove that every neighborhood in is an open set". This theorem was selected because theorem concepts had been taught just a week prior. The application was performed in the middle of the spring term of the 2012-2013 academic year. The said topological concepts in the theorem were thoroughly taught in an Analysis 2 course and the application was performed one week after teaching was completed. Accordingly, we can presume that the prospective teachers had sufficient knowledge about concepts in the theorem. Proofs of prospective teachers were received in written form. Documents obtained from 121 prospective teachers were first reviewed in terms of correctness of proof. This process was carried out together with an academician specialized in analysis and theory of functions. To find an answer to the first research question "What are the difficulties experienced by prospective teachers in proof?", proofs of 52 prospective teachers with invalid proofs were reviewed to see what difficulties they encountered. At the end of this stage, seven difficulties were found by the both authors. Lastly, to make a thorough analysis of difficulties experienced by prospective teachers in proof, a semi-structured Proof Difficulties Interview Form (PDIF) developed by researchers was applied to seven prospective teachers representing each proof difficulty. In PDIF, participants were asked about their opinions on proof, proof difficulties, and how proof is taught, and they were asked "Do you experience such a difficulty?" Then, questions such as "Why do you think you experience this difficulty?" and "What is the reason for this difficulty in a person?" were asked. Here, the aim was to assess whether participants were aware of the difficulties they experienced, and what they concluded about the reasons for such difficulties.

2.4. *Analysis of the data*

Content analysis method was employed in the analysis of prospective teachers' proofs as content analysis aims to reach concepts and relations to explain the collected data (Yıldırım & Şimşek, 2008). Codes and categories were designed for the data collected to answer the first research question. Difficulties obtained from the result of content analysis were determined by the both authors separately. They had an agreement about these difficulties after this process. Interviews held to find an answer to the second research question were recorded and audio records were transcribed after the interviews. Interview data were presented in a descriptive form. In order to improve the validity and reliability of the research, written and verbal statements of prospective teachers were transcribed without making any changes or corrections.

3. Results

Prospective teachers were asked to prove the posed theorem in order to determine their proving abilities. Then, proofs of participants were reviewed and separated under correct proof, invalid proof, incomplete proof and no-proof (no reply) categories. Review results show that prospective teachers with correct proofs constitute only 7% (n=9) of all prospective teachers, while prospective teachers with invalid proofs constitute 43% (n=52), prospective teachers with incomplete proofs constitute 6% (n=7), and prospective teachers with no reply constitute 44% (n=53) of all prospective teachers. It is striking to observe that only a few prospective teachers performed proving of the theorem correctly. With these results, we can conclude that prospective teachers are highly unsuccessful in proving. Figure 1 presents a sample of correct proofs of prospective teachers with its translation in English.



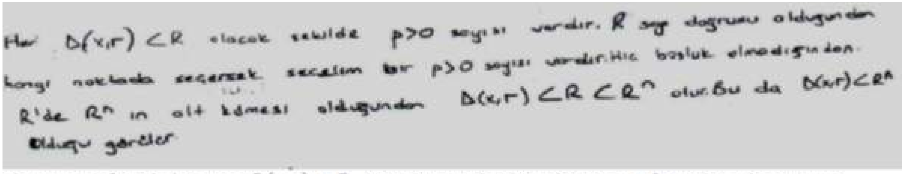
For $\forall y \in D(x,r)$, let we consider $p > 0$ such that $D(y,p) \subset D(x,r)$. $p = r - d(x,y)$. Let $z \in D(y,p)$.
Since R^n is a metrric space, triangle inequality for length is valid.

$$\begin{aligned} d(x,z) &\leq d(y,z) + d(x,y) \\ &< p + d(x,y) \\ &< r - d(x,y) + d(x,y) \\ d(x,z) &< r \Rightarrow D(y,p) \subset D(x,r) \end{aligned}$$

Thus, $D(x,r)$ neigbourhood is open set.

Figure 1. *Sample of Correct Proofs*

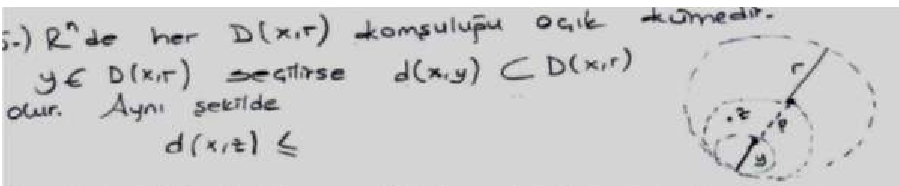
As mentioned above, 43% of prospective teachers performed invalid proofs. Prospective teachers in this category proved the theorem incorrectly or tried to prove other propositions instead of the given theorem. For this reason, proofs in this category were considered invalid. Accordingly, we can say that about half of the prospective teachers presented invalid proofs. Figure 2 presents a sample of invalid proofs of prospective teachers.



There is $p > 0$ such that every $D(x,r) \subset R$. Since R is number line, there is $p > 0$ number wherever we choose. Since there isn't any gap and R is a subset of R^n, then $D(x,r) \subset R \subset R^n$. Thus it is seen that $D(x,r) \subset R^n$.

Figure 2. *Sample of Invalid Proofs*

6% of prospective teachers failed to complete their proof. Figure 3 shows an incomplete proof example.



Every $D(x,r)$ neighborhood in R^n is open set. If it is choosen that $y \in D(x,r)$, then $d(x,y) \subset D(x,r)$. Similarly, $d(x,z) \leq \cdots$

Figure 3. *Sample of Incomplete Proofs*

Lastly, 44% of prospective teachers did not reply to the theorem proof question. Accordingly, we can say that a disconcerting percentage of prospective teachers do not have any measurable knowledge about the proof in question.

In the second section of the research, invalid proofs of 52 prospective teachers were analyzed in terms of difficulties experienced. Proof difficulties experienced by prospective teachers have been collected under the categories of the difficulty of expressing definitions, the difficulty of understanding theorem statements, the difficulty of using mathematical language and notations, the difficulty of selecting a proper proof strategy and method, the difficulty of distinguishing concepts, the difficulty of creating a proof structure from definitions, and the difficulty of expressing thoughts. Moreover, the difficulty of expressing definitions has been divided into three sub-categories, namely, the difficulty of expressing open set definition, the difficulty of expressing subset definition, and the difficulty of expressing metric definition. Table 1 presents these difficulties, the number of participants experiencing these difficulties, and indicators taken into account in creating the categories.

Table 1. *Difficulties Experienced by Participants in Proving*

Difficulties	f	Indicators
Expressing Definitions	34	Participants misstated or failed to state highly important definitions during the proving stage. This difficulty is grouped under three sub-categories, namely the difficulty of expressing open set definition, the difficulty of expressing subset definition and the difficulty of expressing metric definition. These difficulties prevented prospective teachers from proving the theorem.
Understanding Theorem Statement	24	Participants paid no attention to the hypothesis and rule of the theorem, and resorted to proving other propositions instead of the theorem in question.
Using Mathematical Language and Notations	13	Participants made mistakes in mathematical statements, and failed to use the mathematical language correctly. Moreover, they also made mistakes in using mathematical notations.
Selecting Proper Proof Strategy and Method	12	Participants didn't use a valid and proper proof method. Participants in this category have limited knowledge about proof methods.
Distinguishing Concepts	5	Participants confused proof concepts and definitions with other concepts and definitions. It is hard to say these participants can make a clear distinction between the said mathematical concepts.
Creating a Proof Structure From Definitions	5	Participants stated the definitions required for the proof, but couldn't reach a valid proof structure by organizing definitions.
Expressing Thoughts	2	Participants failed to design and write their thoughts despite having knowledge about proof methods with drawings and statements.

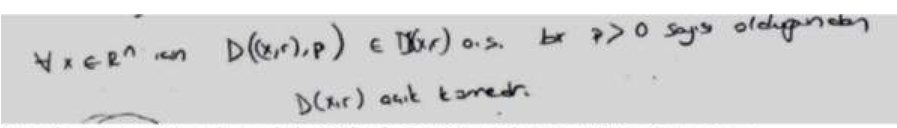
Table 1 shows that the difficulty of expressing definitions is the most common difficulty experienced by prospective teachers regarding proof. This difficulty was experienced by 34 participants. Consequently, it is clear that failure to express definitions is the most common difficulty of prospective teachers regarding proof. This difficulty is grouped under 3 sub-categories. From the highest error repeat rate to the lowest, these sub-categories are as follows:

a. failure to express open set definition (n=26),

b. failure to express subset definition (n=5),

c. failure to express metric definition (n=3).

More than half of the prospective teachers with invalid proofs experienced the difficulty of expressing definitions. Figure 4 shows an extract from the proof of Cengiz, a prospective teacher experiencing this difficulty. Cengiz failed to state the open set definition correctly as illustrated below.



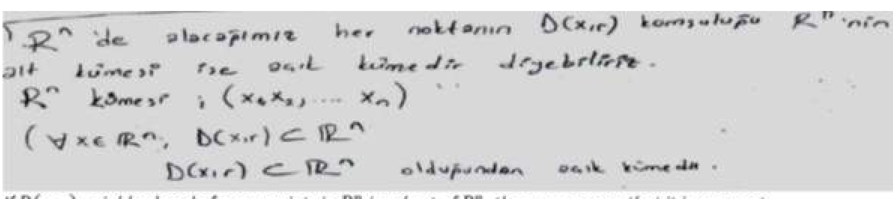
Since there is a $p > 0$ number such that $D((x,r),p) \in D(x,r)$ for $\forall x \in R$, $D(x,r)$ is open set.

Figure 4. *Difficulty of Expressing Definitions*

Cengiz stated that he was aware of the difficulty. He listed the reasons for this difficulty as not paying enough attention to definitions and theorems, studying relevant concepts for the exam just a little while before the exam, memorizing rather than studying, and inadequacy in proof teaching methods. Below are some of Cengiz's thoughts about his difficulty in expressing definitions.

Cengiz: *I don't think I have a good grasp of definitions because I don't study definitions and theorems. The only thing I pay attention to is mathematical operation, area and volume formulas. So I didn't spend much time on proof, definitions or theorems. Because I didn't think they would be asked in exams. I am responsible for this difficulty in the first place because I don't study definitions, theorems and I only review them just a couple of days before the exam. The same goes for everyone. The second reason is teachers. For instance, I repeat what I learn in class while walking on the street. If the subject is operations, I repeat in my mind the methods and ways of the operations. Teachers bombard us with readings and academic knowledge while teaching the subject of proof. When we think about what we did in the class, we can only say we have been given the subject of proof in lectures. We are left with nothing else to say. And we don't know how we performed in the proving process.*

One of the most common difficulties experienced by prospective teachers is the difficulty of understanding theorem statements. This was found in 24 prospective teachers. This corresponds to about half of prospective teachers with invalid proofs. These prospective teachers misunderstood the theorem statement, and tried to prove a different statement. Most of these prospective teachers tried to show that   is an open set instead of demonstrating that neighborhood is an open set. Figure 5 presents the proof of Gul, one of the participants experiencing the difficulty of understanding theorem statements.



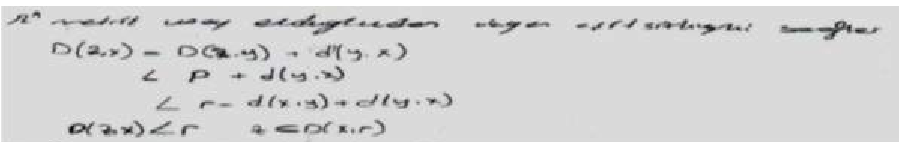
If $D(x,r)$ neighborhood of every points in R^n is subset of R^n, then we can say that it is open set.
R^n ; $(x_1, x_2, \dots x_n)$. $\forall x \in R^n$: $D(x,r) \subset R^n$. Since $D(x,r) \subset R^n$, it is open set.

Figure 5. *Difficulty of Understanding Theorem Statements*

Gül is not aware of this problem and thinks she does not have such a difficulty. As to the reasons for difficulties in general, she thinks that students do not pay attention to proof, that proof is memorized, and that the education system has faults. Gul's opinions on the matter are given below.

Gül: *I can understand theorems but can't construct proofs. I really do understand theorems and keep them in my head. I can keep the sequence of proofs in my mind. This difficulty might be caused by not having a clear understanding of concepts, not having a good grasp of properties or not listening to the teacher during the course. Normally, it is expected that first, the student, then the teacher is responsible for this; however, regarding proof, first, the teacher, then, the student is responsible for this difficulty.*

Another difficulty of prospective teachers regarding proof is the use of mathematical language and notations. A considerable part of prospective teachers had this difficulty (n=13). Figure 6 presents an extract from the proof of Mehmet, who had difficulties in using mathematical language and notations. The below figure shows that Mehmet had difficulties in using the "D" symbol which represents open ball, and the "d" symbol, which stands for metric. Moreover, Mehmet failed to use "=", "<" and " " symbols correctly.



Since R^n is a metrric space, triangle inequality is valid.

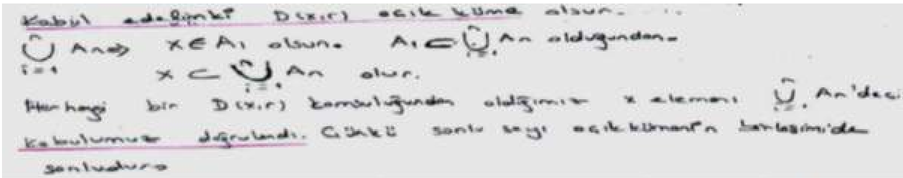
$$D(z,x) = D(z,y) + d(y,x)$$
$$< p + d(y,x)$$
$$< r - d(x,y) + d(y,x)$$
$$D(z,x) < r, \quad z \subset D(x,r)$$

Figure 6. *Difficulty of Using Mathematical Language and Notations*

Mehmet is aware of this difficulty. He thinks that not paying attention to proof, thinking that proof will not be useful in the future, memorizing proofs, accepting proofs without questioning, failing to grasp proof logic, and shortcomings in proof teaching are the reasons for this difficulty. Mehmet's thoughts about this difficulty and why he specifically is having this difficulty are given below.

Mehmet: *I don't find myself sufficient in terms of using mathematical language and notations. This is because I think like everyone else, I don't think proof will be useful for me in the future. I know this is a fallacy, but I resort to memorizing proofs. I don't question why. For instance, I don't question the teacher during courses, and I accept the proof as it is. The reason of these mistakes is the failure to get a good command of proof logic. For instance, I was puzzled over the proof question you asked about whether I should write d or D, whether there was a difference or not. The difference should be clearly stated while making proof argument. It looks like the same to me on the book. I think there are teaching problems, as well.*

Twelve prospective teachers could not determine a proper strategy or proof method, or used an invalid proof method. This difficulty might be caused by limited knowledge of prospective teachers about proof methods. Figure 7 shows an invalid proof method applied by Guven. And this difficulty inhibited the prospective teachers from constructing a correct proof.



Let we assume that $D(x,r)$ is open set. Let $\bigcup_{i=1}^{n} A_n \Rightarrow x \in A_1$. Since $A_1 \subset \bigcup_{i=1}^{n} A_n$, $x \subset \bigcup_{i=1}^{n} A_n$. x element chosen from any $D(x,r)$ neighborhood is in $\bigcup_{i=1}^{n} A_n$. Now our assumption is verified since the union of finite numbers of open set is finite.

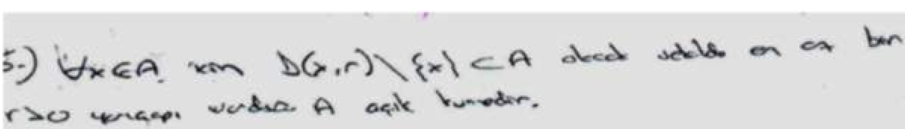
Figure 7. *Difficulty of Selecting Proper Proof Strategy and Method*

Güven is aware of the difficulty he is experiencing. According to Güven, this difficulty results from not paying attention to proofs and teaching methods used in proof teaching. Güven's ideas about this difficulty are summarized below.

Güven: *I don't have sufficient knowledge about proof methods. Proof methods should be applied to learn them. We should use each of these methods to get a good command. But that's not how we did it, we only learnt the method statements. Then, we put it off. But that's not how it should be. These methods should be applied. We don't do this and therefore do not learn the subject. I think everyone is responsible for this. For example, abstract mathematics course gives very little time (around 1 week) to teaching proof methods. And we didn't study and wonder about these methods.*

These prospective teachers' proofs show that, they experience the difficulty of making distinctions between mathematical concepts, the difficulty of creating a proof structure from definitions, and the difficulty of expressing thoughts. Participants stated they were aware of these difficulties. They consider paying little attention to proofs, and shortcomings in learning and teaching of proofs are the reasons for these difficulties. These participants were presented with the situation of participants who had difficulty in making distinction between concepts.

Participants who had difficulty in distinguishing concepts predominantly confused the cluster point concept with the open set or the related internal point concept. This difficulty influenced their ability to construct a correct proof. Figure 8 shows an extract from the proof of Arzu, who had this difficulty.



There is a $r > 0$ ratio such that $D(x,r)\setminus\{x\} \subset A$ for $\forall x \in A$. A is open set.

Figure 8. *Difficulty of Distinguishing Concepts*

Arzu is aware of this difficulty. She stated that this difficulty is caused by not paying attention to proof, memorizing proofs, and shortcomings in proof teaching. Below are Arzu's opinions about this difficulty.

Arzu: *I have this difficulty. This might be because I study for exams during the last day. I confuse many concepts because I study on the final day. Another reason might be not carefully listening to the teachers during the course. It'd be better if the teacher gave the lesson in an easily comprehensible way for me. But it's boring for me, and I don't feel like listening. So, I study one day before the exam. And for this reason, I memorize concepts and definitions.*

4. Discussion and conclusion

Studies seeking an answer to the research questions showed that the valid proof rate of prospective teachers was 7%. 43% of prospective teachers constructed invalid proofs. 44% of prospective teachers did not construct a proof, and 6% failed to complete their proofs. Accordingly, we can clearly ascertain that prospective teachers have serious deficiencies in proof skills. This result parallels other research results positing that prospective teachers have difficulties and are unsuccessful in terms of proof and proving (Moralı et al, 2006; Sarı et al., 2007; Stylianides et al., 2007).

Regarding the first research question (What are the difficulties experienced by prospective teachers in proof?) the difficulty of expressing definitions is the most common. Prospective teachers with this difficulty could not state definitions correctly and, as a result, failed to form a correct proof. Most of the prospective teachers had difficulties in understanding the theorem statement. Prospective teachers with this difficulty misunderstood the theorem statement entirely and could not construct a valid proof. This is supported by Weber's (2006) finding that the difficulty experienced by students while demonstrating a proof is caused by the misunderstanding of a concept or a theorem and, accordingly, misapplication of the concept or theorem. A considerable number of prospective teachers had difficulties in using mathematical language and notations. Prospective teachers having difficulties in these areas particularly confused mathematical notations. A portion of prospective teachers could not determine a proper proof method and strategy. They identified an incorrect or unsuitable proof method and strategy leading to an invalid proof. These results also parallel similar studies in relevant literature (Bayazıt, 2009; Güler, 2013; Moore, 1994; Selden & Selden, 2007; Weber, 2006).

Interviews were held with prospective teachers to find answers to the second research question (What do prospective teachers think about the causes of such difficulties?). Interviews concluded that most of the prospective teachers were aware of the difficulties they confront in proofs and proving. Accordingly, we can conclude that prospective teachers have a high level of awareness about the difficulties they have regarding proofs. Taken as a whole, prospective teachers suggest that negative feelings about proof, deficiencies in proof learning, and shortcomings in proof teaching are the three major factors for said difficulties. Prospective teachers expressed their negative feelings about proof as the first cause. They stated that they do not pay attention to proving because they do not believe proofs will be useful to them in the future. This result supports other research result showing that prospective teachers think proof is not useful (Doruk & Kaplan, 2013a). In addition, negative feelings of students struggling with proving and who fail to construct a correct proof lead us to ascertain that there is a relation between opinions about proof and proving skills. This result supports research results claiming that opinions about proof influence proving ability (Almeida, 2000; Furinghetti & Morselli, 2009). From this point of view, it is reasonable to suggest that prospective teachers need to have positive attitudes toward proof. To help students develop positive feelings about proof, the role and importance of proof in mathematics and the mathematical benefits of proof should be taught to students in proof teaching courses.

Prospective teachers stated that proof learning methods were the second cause of the difficulties experienced in proofs and proving. They expressed that it was more practical to memorize proofs rather than understanding the subject, admitting they invest little study time to this subject before the exams. This result again parallels other research that cites students resort to memorization rather than understanding the logic of proof (Concradie & Frith, 2000; Doruk & Kaplan, 2013b). In order to eliminate this problem, teacher training institutions should give prospective teachers time to construct their own proofs during proof teaching. Students should test and experience proofs themselves. Students should be informed about their mistakes and encouraged to construct proofs. This will prevent students from memorizing proofs and lead them to employing proof methods according to proof logic.

Lastly, prospective teachers criticized the proof teaching method as an additional reason for difficulties in proving. They stated that the teaching methods applied in proof-oriented courses were not suitable for proof teaching. They mentioned that courses were taught with classical lecturing methods, and concepts could not be effectively taught. They asserted that proof-oriented courses should have an application basis. This research result concurs with the study result that classical lecturing method continues in our universities, and concepts are not retained in the minds of students (Yıldız, 2006). These criticisms by prospective teachers also correspond to results of the study showing that proof-oriented courses are taught with definition→theorem→proof method (Weber, 2004), and should be taught with examples→theorem→proof method (Almeida, 2003). Therefore, we are of the opinion that proof-oriented course instructors should re-evaluate their teaching methods by taking these outcomes into account.

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