Abstract Title Page

Title: The Contribution of Domain-Specific Knowledge in Predicting Students' Proportional Word Problem Solving Performance

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Abstract Body

Background / Context: Proportional thinking, which requires understanding fractions, ratios, and proportions, is an area of mathematics that is cognitively challenging for many children and adolescents (Fujimura, 2001; Lamon, 2007; Lobato, Ellis, Charles, & Zbiek, 2010; National Mathematics Advisory Panel [NMAP], 2008) and "transcends topical barriers in adult life" (Ahl, Moore, & Dixon, 1992, p. 81). Problems associated with the failure to understand proportional concepts and operations, and difficulty distinguishing the multiplicative relationship between rational quantities from earlier learned additive arithmetic concepts, are key obstacles to progress in more advanced mathematics, including algebra (Boyer, Levine, & Huttenlocher, 2008; NMAP, 2008). In fact, "developing understanding of and applying proportional relationships" in Grade 7 is deemed to be one of the critical areas of focused instructional time (Common Core State Mathematics Standards, 2010).

In the current study, we aimed to develop greater understanding of why proportional thinking continues to be an area of great difficulty for students. In particular, it seems clear that solving proportion problems, whether they are presented in symbolic form or as word problems, require the coordination of a variety of domain-specific knowledge (including conceptual and procedural knowledge). Unfortunately, we know little about how these types of domain-specific knowledge interact and contribute to the development of proportional word problem solving. Identifying which of these variables is important in promoting the development of proportional thinking (and conversely which are implicated in those who have difficulty with proportional word problem solving) is critical to inform future interventions for improving students' performance in this area. We summarize prior research on domain-specific knowledge (i.e., fractions, proportions) as it provides the basis for our hypotheses that certain variables are important for proportional word problem solving.

Fractions. Given the fact that fractions are "a subset of the rational numbers" (Lamon, 2007, p. 635), understanding whether difficulties in conceptual and procedural fraction knowledge are associated with proportional word problem solving skills is important. Competence in fractions entails both conceptual and procedural knowledge. Following Rittle-Johnson and Alibali (1999), we define conceptual knowledge as "explicit or implicit understanding of the principles that govern a domain and of the interrelations between pieces of knowledge in a domain" (p. 175) and procedural knowledge as "action sequences for solving problems" (p. 175). Research has examined the influence of both types of fraction knowledge on fraction outcomes (Hallett, Nunes, & Bryant, 2010; Hecht, Close, & Santisi, 2003; Hecht & Vagi, 2012). Several studies indicate that conceptual knowledge is correlated with individual differences in basic fraction skills (Byrnes & Wasik, 1991; Hecht, 1998; Hecht et al., 2003). Nonetheless, there is some suggestive evidence that the lack of conceptual knowledge does not interfere with accurate implementation of the procedural steps to solve computational problems (see Hallett et al., 2010).

Results of studies examining the development of conceptual and procedural fraction knowledge are inconsistent (Hallett et al., 2010; Rittle-Johnson & Siegler, 1998); some research suggests that children develop conceptual knowledge before procedural knowledge (Byrnes & Wasik, 1991; Mack, 1990; Mix, Levine, & Huttenlocher, 1999), other researchers report that children learn procedural knowledge before they develop conceptual knowledge (Peck & Jencks, 1981). Still other researchers note that children develop both types of knowledge simultaneously (Hecht & Vagi, 2010; Rittle-Johnson, Siegler, & Alibali, 2001). Recent research suggests that individual differences in the extent to which students rely on either conceptual or procedural fraction knowledge or a combination of the two types of knowledge may explain the contradictory findings (Hallett et al., 2010; Hecht & Vagi, 2012). Results of these studies demonstrate different patterns of individual differences in conceptual and procedural knowledge, with an advantage on fraction outcomes for those children who rely more on conceptual knowledge compared to those who rely primarily on procedural knowledge. In light of this literature, we hypothesized that middle school students' difficulties in proportional word problem solving are associated with deficits in both conceptual and procedural fraction knowledge.

Proportions. The relation between conceptual and procedural knowledge in the case of proportions is similar to the findings in the literature on fractions. One indicator of rational number sense is proportional reasoning (Howe, Nunes, & Bryant, 2011; Lamon, 2007), which we refer to here as conceptual proportion knowledge. According to Lamon (2007), conceptual proportion knowledge "means supplying reasons in support of claims made about the structural relationships among four quantities (say a, b, c, d) in a context simultaneously involving covariance of quantities and invariance of ratios or products; this would consist of the ability to discern a multiplicative relationship between two quantities as well as the ability to extend the same relationship to other pairs of quantities" (p. 638).

Researchers have examined how upper elementary- and middle-school students reason in various proportionality tasks and the extent to which developmental or instructional factors influence conceptual proportion knowledge (e.g., Bright, Joyner, & Wallis, 2003; Lamon, 1993, 2002; Lo & Watanabe, 1997; Post, Behr, & Lesh, 1988). It is known that children initially tend to apply additive reasoning to understand numerical relationships, which can either result in an inaccurate characterization of the relationship between quantities or result in "building-up" to the correct answer (Tourniaire & Pulos, 1985). At the same time, it is possible for students to rely on procedural proportion knowledge to obtain correct answers to proportion problems, even when they lack conceptual proportion knowledge. As Lamon notes, "the ability to give correct answers is no guarantee that proportional reasoning is taking place. Often, proportions may be solved using mechanical knowledge about equivalent fractions or about numerical relationships, or by applying algorithmic procedures (for example, the cross-multiply rule) that circumvent the use of the constant of proportionality" (Lamon, 2007, p. 638)

Overall, the application of procedures to solve proportional problems relates to the reasoning underlying those procedures (Berk, Taber, Gorowara, & Poetzl, 2009). For example, if a student knows to solve proportional problems using only the dominant step-by-step approach of cross-multiplying, the use of this procedure would result in the correct answer when applied correctly despite the fact that the procedure might not be understood. Whereas students having *deep procedural knowledge* would not only understand when to use cross-multiplication to find the exact answer, but also solve proportion problems by implementing alternate strategies that "exploit available integer multiplicative relationships and thereby reduce computational demands" (Berk et al., 2009, p. 116; Star, 2005). Given that incoming knowledge of proportional reasoning and procedures are likely to account for some variance in proportion outcomes, we included them as possible predictors of proportional word problem solving.

Purpose / Objective / Research Question / Focus of Study: The research literature has identified the cognitive resources associated with number sense (e.g., Jordan, Glutting, & Ramineni, 2010; De Smedt, Verschaffel, & Ghesquiere, 2009), computation with whole number (e.g., Fuchs et al., 2006) and rational number (Seethaler, Fuchs, Star, & Bryant, 2011), arithmetic and algebra word problem solving (e.g., Fuchs et al., 2008; Fuchs et al., 2010; Lee, Ng, & Ng, 2009; Swanson, 2006; Swanson, & Beebe-Frankenberger, 2004; Zheng, Swanson, & Marcoulides, 2011), and fraction skills such as computation, estimation, and word problem solving (e.g., Hecht et al., 2003; Hecht & Vagi, 2010). There are currently no studies that have examined the unique correlates of proportional word problem solving or identified the variables that may predict proportional word problem solving. As such, the research reported here takes the first exploratory step in assessing and establishing the contribution of variables associated with individual differences in the proportional problem-solving outcome. The following research question directed this study: Does domain-specific knowledge (conceptual and procedural fraction knowledge, conceptual and procedural proportion knowledge) predict proportional word problem solving? Based on the literature described above, we hypothesized domain-specific knowledge (including conceptual and procedural fraction and proportion knowledge) to be a significant predictor of individual differences in proportional word problem solving performance.

Setting/ Population / Participants / Subjects: All 429 seventh-grade students from 17 classrooms at three U.S. middle schools in two suburban Midwest school districts participated in the study. Of the 429 students, 411 students provided complete data to be included in the analysis. Of these 411 students, 195 (47.4%) were male, 192 (46.7%) were eligible to receive free or reduced lunch, 33 (8.0%) were English language learners, and 44 (10.7%) received special education services. In terms of ethnicity, 210 students (51.1%) were Caucasian, 118 (28.7%) were African American, 54 (13.1%) were Hispanic, 20 (4.9%) were Asian, and 9 (2.2%) were American Indian. The mean age of participants was 12.5 years (range 11.8–13.9 years).

Intervention / Program / Practice: Teachers tested all students in their classrooms using standardized written directions. Trained research assistants provided support as needed and assessed reliability of test administration. In January of seventh-grade, students were assessed on measures of domain-specific knowledge in one whole-class testing session lasting approximately 50 minutes. In early March, students completed the proportional problem-solving test in one whole-class 50 min session.

Research Design and Data Collection: Hierarchical regression was used to investigate the domain-specific knowledge of seventh grade math that predicted proportional word problem solving performance. Predictor variables included measures of fraction concepts and procedures, and proportion concepts and procedures. The criterion variable was proportional word problem solving. A 12-item fraction knowledge measure adapted from Hecht and Vagi (2010) was used to assess conceptual and procedural fraction knowledge (see Figure 1). Conceptual proportion knowledge was measured using three short-answer items that required students to explain their reasoning in support of claims about the multiplicative relationship of quantities in the problems (see Figure 2 for a sample task). We assessed procedural proportion knowledge using two no-context missing-value proportion items (e.g., $\frac{3}{5} = \frac{x}{15}$). The criterion task was a 21-item proportion word problem-solving test used in Author (in press). Items for the problem solving measure were derived from TIMSS (Trends in International Mathematics and Science Study), NAEP (National Assessment of Educational Progress), and state assessments. Although the measure was not timed, all students were given about 50 minutes to complete it. To assess the reliability of this measure, we fit the data to three measurement models: the parallel model, the tau-equivalence model, and the congeneric model. The results indicated that the congeneric model fit the data best (GFI = .94; RMSEA = .03), and the reliability coefficient was .85.

Analysis/ Findings / Results: First, we conducted a descriptive analysis of the data to evaluate the distributional properties of the variables and calculated Pearson correlation coefficients to examine the inter-correlations among the variables (see Table 1). Our next step involved conducting a regression analysis to assess the contribution of the domain-specific knowledge variables in predicting students' proportional word problem solving performance. Preliminary analyses indicated no significant effect for any of the demographic variables on proportional word problem solving performance; for this reason and because the demographic variables were not of substantive interest, they were excluded from the final analyses. Therefore, we entered the set of predictors simultaneously into a multiple regression analysis to predict students' proportional word problem solving. To control for compounding Type I error rates, an adjusted alpha value of .008 (.10 divided by the total number of tests) was used for each statistical test.

Results showed that each measure was significantly related to all other measures (see Table 1). The correlations between proportional word problem solving and the domain-specific knowledge variables ranged from .37 to .45, with conceptual fraction knowledge ($r = .45$) and procedural proportion knowledge ($r = .43$) having the strongest relationship with proportional word problem solving. The associations between conceptual fraction knowledge and conceptual proportion knowledge ($r = .29$) and procedural proportion knowledge ($r = .30$) were stronger than the correlation between conceptual fraction knowledge and procedural fraction knowledge ($r = .21$). The associations between conceptual proportion knowledge and procedural fraction knowledge ($r = .32$) were stronger than those between conceptual proportion knowledge and procedural proportion knowledge ($r = .29$).

Results indicated that conceptual fraction knowledge, procedural fraction knowledge, conceptual proportion knowledge, and procedural proportion knowledge were significant predictors of proportional word problem solving performance. Together, these predictors explained 37% of the variance in proportional word problem solving, $R^2 = .37$, $F(4, 406) = 59.61$, $p < .001$. See Table 2 for the unstandardized and standardized beta coefficients, standard errors, and partial eta-squared (i.e., unique variance explained) for each predictor variable, with all others controlled.

Conclusions: Multiple regression analyses indicated that all four domain-specific knowledge variables (i.e., conceptual and procedural fraction knowledge, conceptual and procedural proportion knowledge) significantly predicted proportional word problem solving performance. Conceptual fraction and procedural proportion knowledge contributed the most unique variance (10.0 and 6.7 percent, respectively, of the total variance) to proportional word problem solving. Procedural fraction and conceptual proportion knowledge each also contributed significant unique variance to proportional word problem solving explaining 5.6 and 2.8 percent, respectively. The results support the notion that both conceptual fraction and proportion knowledge and procedural fraction and proportion knowledge play a major role in understanding individual differences in proportional word problem solving performance to inform interventions.

Appendices

Appendix A. References

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Appendix B. Tables and Figures

Table 1

Mean Performance and Intercorrelations Among Variables

Variables	Mean	*SD*	Max	1	2	3	4	5
1. Conceptual Fraction Knowledge	4.20	1.26	6	–				
2. Procedural Fraction Knowledge	3.25	1.93	6	.208	–			
3. Conceptual Proportion Knowledge	3.05	2.29	12	.291	.323	–		
4. Procedural Proportion Knowledge	0.89	0.62	2	.298	.327	.286	–	
5. Proportional Word Problem Solving	14.46	3.87	21	.446	.397	.369	.433	–

Note. “Max” denotes the maximum possible score on each test. All correlations significant, $p < .001$.

Table 2

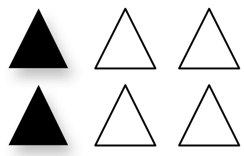
Multiple Regression Results in Predicting Proportional Word Problem Solving

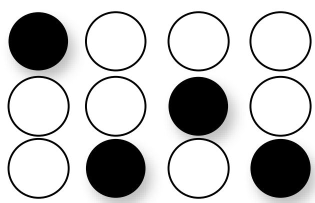
Variables	*B*	*SE(B)*	β	η^2	*t*	*p*
Constant	7.28	.553			13.17	<.001
Conceptual Fraction Knowledge	0.89	.130	.289	.102	6.80	<.001
Procedural Fraction Knowledge	0.42	.087	.212	.056	4.90	<.001
Conceptual Proportion Knowledge	0.25	.073	.150	.028	3.45	.001
Procedural Proportion Knowledge	1.47	.272	.235	.067	5.41	<.001

$R^2 = .37$, $F(4, 406) = 59.61$, $p < .001$.

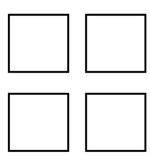
Figure 1. Items from Conceptual Fraction Measure.

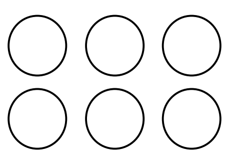
A. Write what fraction of the set of figures is shaded. For example, if $\frac{1}{4}$ of the set of figures is shaded, write $\frac{1}{4}$ in the blank provided.

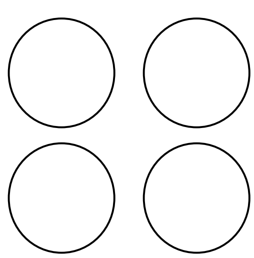
1.  ________

2.  ________

B. Shade each set of figures with the amount indicated by the fraction. For example, if the fraction is $\frac{1}{4}$, shade $\frac{1}{4}$ of the figure.

1. $\frac{1}{4}$ 

2. $\frac{2}{3}$ 

3. $\frac{3}{8}$ 

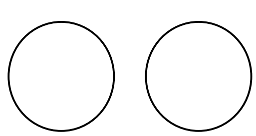
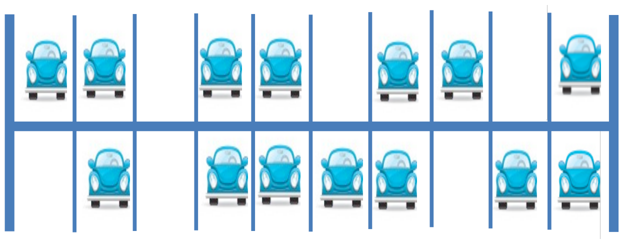
4. $\frac{5}{6}$ 

Figure 2. Sample Conceptual Item from Proportion Measure.

Below are pictures of two parking lots at the Mall of America showing which parking spaces are full and which are empty.

Parking Lot A



Parking Lot B

John says that parking lot A is emptier than parking lot B, because parking lot A has 6 empty spaces while parking lot B has only 4 empty spaces.

Do you agree or disagree with John?

☐ Agree

☐ Disagree

Why? Explain your answer in 1–2 sentences.