Looking at Students' Work: Assessment Strategies that Inform Instruction

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CollegeBoard

AP

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Assessment Strategies that Inform Instruction

High school graduates today are increasingly expected to …evaluate arguments, and understand and convey complex information in the college classroom, in the workplace…

The ability to reason allows for the systematic development of ideas, the ability to make sound choices, and the ability to make and understand persuasive arguments.

American Diploma Project, 2004, p. 29

Common Core State Standards in Mathematics

COMMON CORE
STATE STANDARDS INITIATIVE
PREPARING AMERICA'S STUDENTS FOR COLLEGE & CAREER

Overview of the Mathematics Standards

- Grade Level Standards
  - K-8, grade-by-grade standards organized by domain
  - 9-12 high school standards organized by conceptual categories

- Standards for Mathematical Practice
  - Describe mathematical "habits of mind"
  - Standards for mathematical proficiency:
    - reasoning, problem solving, modeling, decision making, and engagement
  - Connect with content standards in each grade

Organizational Structure

The High School Standards

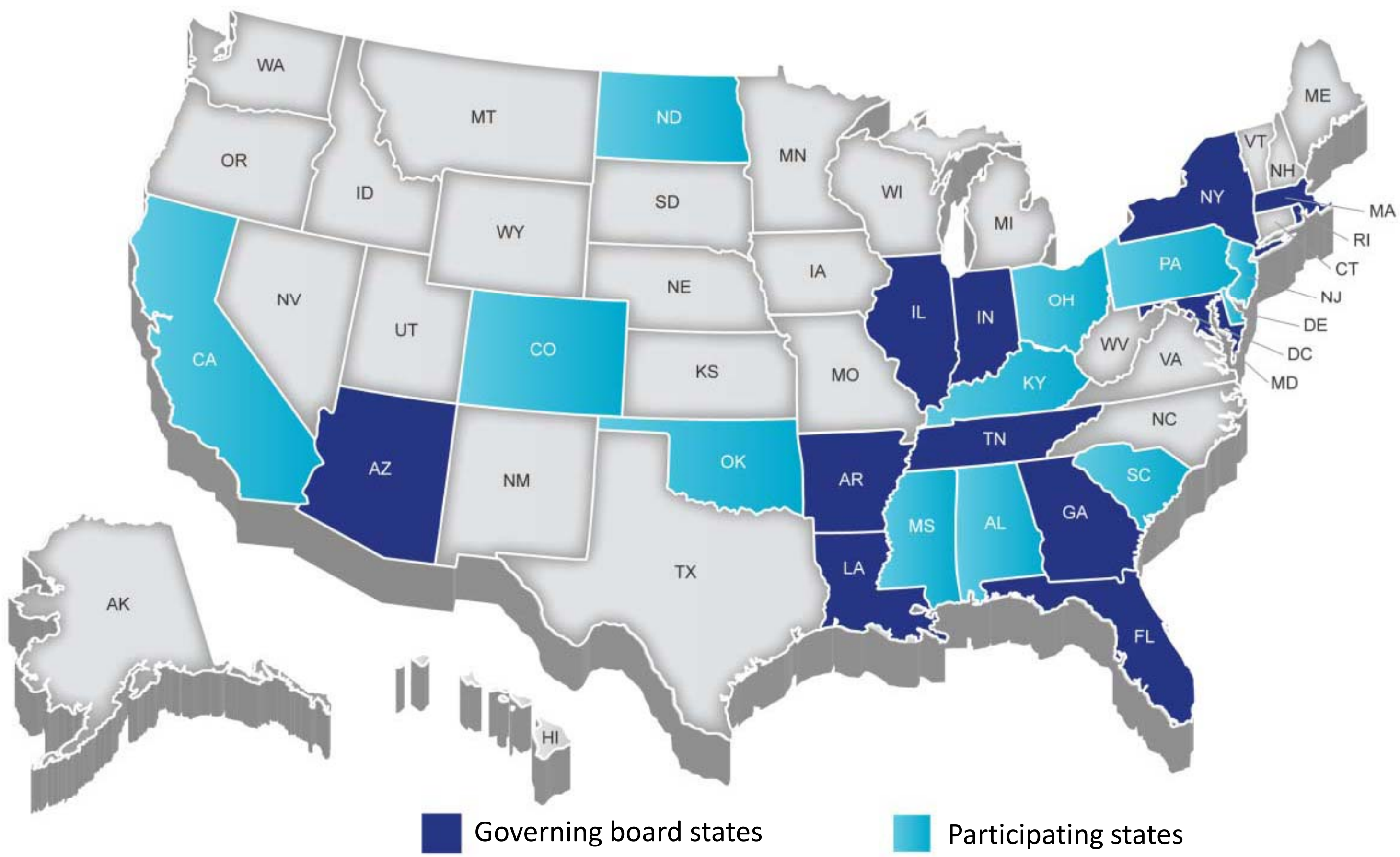
- Expect students to practice ***applying mathematical ways of thinking*** to real world issues and challenges
- Require students to develop a ***depth of understanding and ability to apply mathematics to novel situations***, as college students and employees regularly are called to do
- Emphasize ***mathematical modeling***, the use of mathematics and statistics to ***analyze empirical situations***, understand them better, and improve decisions
- Identify the mathematics that all students should study in order to be ***college and career ready***.

Assessment

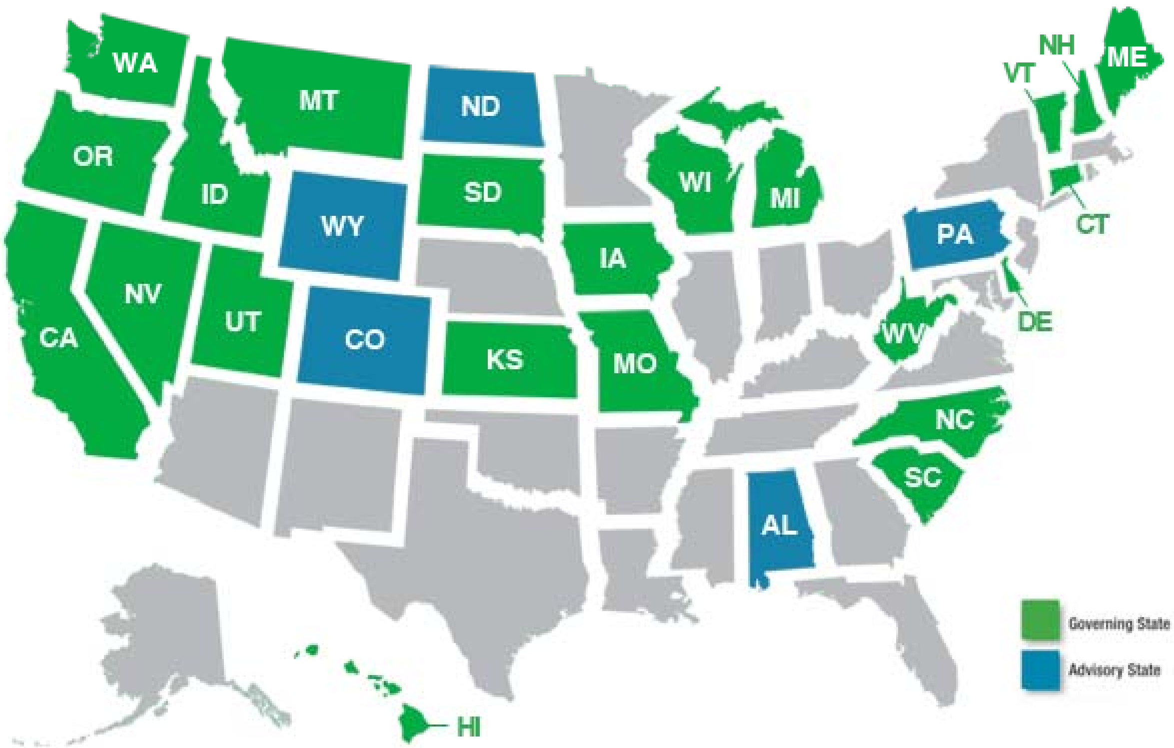
Race to the Top:

- PARCC Consortium
- Smarter Balanced Consortium

Partnership for Assessment of Readiness for College and Careers (PARCC) Assessment Consortium



Smarter Balanced Assessment Consortium



Building a pathway to college and career readiness

- Aligned to the college- and career-ready Common Core State Standards.
- Students who score proficient on the assessments will know they are on track for the next steps in their education, creating a more meaningful target.
- High School results will send an early signal about whether students are ready for entry-level, non-remedial courses at higher education institutions.
- Scores will provide reliable state-to-state comparability, with standards set against national and international benchmarks.

Creating better assessments …

The assessment systems will include:

- A mix of item types – multiple choice, short answer, longer open response and performance-based:
  - Encourage teachers to focus on helping each student develop a deep understanding of the subject matter, rather than just narrowing their instruction in order to “teach to the test”
- Testing at multiple points throughout the year
  - Provide teachers, parents and students information about whether students are “on track” or need some additional support in particular areas during the school year, not just at the end of the year

Knowledge and Skills that are important to measure:

Students can

- explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.
- frame and solve a range of complex problems in pure and applied mathematics.
- clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.
- analyze complex, real-world scenarios and use mathematical models to interpret and solve problems.

Looking at Students' Work-
How do we evaluate understanding and reasoning skills?

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Solve the following problem.

Show all work. Explain your answer.

A college sold tickets to a play at $4 per ticket in advance. Anyone who attended and purchased a ticket at the door had to pay $5 a ticket. A total of 480 people attended the play, and the revenue from the ticket sales was $2100.

How many people bought tickets in advance, and how many people bought tickets at the door?

Floodlights

Eliot is playing football.
He is 6 feet tall.
He stands exactly half way between two floodlights.
The floodlights are 12 yards high and 50 yards apart. The floodlights give Eliot two shadows, falling in opposite directions.

1. Draw a diagram to represent this situation. Label your diagram with the measurements.

2. Find the total length of Eliot’s shadows. Explain your reasoning in detail.

3. Suppose Eliot walks in a straight line towards one of the floodlights. Figure out what happens to the total length of Eliot’s shadows. Explain your reasoning in detail.

Using Assessments thoughtfully

Looking at students' work…

- Gives the ability to measure more complex skills that go beyond those that are measurable by multiple choice items.

- Multiple choice questions can determine whether students can solve many kinds of problems, but cannot determine if they can construct a mathematical proof.

- Constructed responses do not allow for "guessing the correct answer among the multiple choices presented"

Scoring the responses

Rubric

- a set of guidelines that tells the scorer what features of the response to focus on

- how to decide how many points to award to the response.

Scoring the responses

Analytic scoring

- lists specific features of the response
- specifies the number of points to award for each feature
- consistent from scorer to scorer
- explicitly specifies features of the response which should be awarded points, or lose points
- Important features of the response can be evaluated individually.
- Quality of the response does not depend on interactions among those features of the response. (Almost like a checklist)
- Some analytic scoring systems allow for “partial credit”

Scoring the responses

Holistic scoring

- The holistic rubric identifies characteristics of a typical response at each score level.
- A single judgement of the quality of the response assigns a numerical score.
- A single numerical score is given to the whole response
- Used in conjunction with exemplars (samples of responses for each possible score. Can include “borderline cases” –barely 5, almost 4…

New Jersey…

Scoring Rubrics

Rubrics - SCORING STUDENT RESPONSES

Rubric with Analytic Components According to Webster's: rubric, n. 5. An authoritative direction or rule.

Scoring rubrics provide the criteria for evaluating and scoring student performance. There is an item specific rubric supplied for each Open-Ended item in mathematics. These item specific rubrics are based upon the generic mathematics rubric and are developed by a committee of mathematicians and teachers. Rubrics ensure that there is consistency, fairness, and accuracy in scoring open-ended questions.

3 - Point Response	2 - Point Response	1 - Point Response	0 - Point Response
•complete understanding of the problem's essential mathematical concepts •executes procedures completely and gives relevant responses to all parts of the task. contains few minor errors, if any. •contains a clear, effective explanation detailing how the problem was solved so that the reader does not need to infer how and why decisions were made.	•nearly complete understanding of the problem's essential mathematical concepts. •executes nearly all procedures and gives relevant responses to most parts of the task. •response may have minor errors. explanation detailing how the problem was solved may not be clear, causing the reader to make some inferences.	•limited understanding of the problem's essential mathematical concepts. •response and procedures may be incomplete and/or may contain major errors. •incomplete explanation of how the problem was solved may contribute to questions as to how and why decisions were made.	•insufficient understanding of the problem's essential mathematical concepts. procedures, if any, contain major errors. no explanation of the solution or the reader may not be able to understand the explanation. •The reader may not be able to understand how and why decisions were made.

Assessing Levels of Expertise in Mathematics:

- Items and tasks will assess student knowledge and skill at the *novice, apprentice,* or *expert* level.
- The level of expertise refers to the student's ability to call upon and use the mathematics they understand in a range of contexts (*adaptive expertise*).
- Items will range from those with extensive scaffolding (novice) to those requiring the student to reason through a number of steps on their own and measure the ability to make strategic decisions on how to approach and solve the problem (expert).

Solve the following problem.

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How many people bought tickets in advance, and how many people bought tickets at the door?

Student Work

At your tables are 6 sample responses .

- Choose one of the samples and become familiar with the work the student did.
  - Was it correct?
- As an assessment, what does the work tell you?
- Within your group discuss all 6 samples.
- How would you assess the work shown in each sample?
- Explain what was done.
- How could this student work be improved?

Sample 1

Number of \$4 tickets	Number of \$5 tickets	Number of People Attended
0	420	= 420
525	0	= 525
100	340	= 440
200	260	= 460
☆ 300	180	= 480 ☆
400	100	= 500
275	200	= 475
150	300	= 450
250	220	= 470
350	140	= 490
337.5	150	= 487.5
212.5	250	= 462.5
87.5	350	= 437.5

300 people bought tickets in advance
180 people bought tickets at the door

Sample 2

Guess & Check table

guess # of people who bought ticket in advance	# of people who bought ticket at the door.	total people that bought tickets	cost of advance tickets ($4)	cost of tickets at the door ($5)	total cost of tickets
200	480-200=280	280+200=480	200·$4=$800	280·$5=$1400	$1400+800=2200
190	480-190=290	190+290=480	190·4=760	290·5=1450	1450+760=2210
220	480-220=260	220+260=480	220·4=880	260·5=1300	1300+880=2180
240	480-240=240	240+240=480	240·4=960	240·5=1200	1200+960=2160
* 300	480-300=180	300+180=480	300·4=1200	180·5=900	900+1200=2100

Check $2100
2,200 : too high
2210 : too high
2180 : too high
2160 : too high
2100 : correct

300 people bought tickets in advance and 180 people bought tickets at the door.

Sample 3

Extra Credit

A= advanced D= door

$4A+5D=\$2{,}100$

$A+D=480$

$$\begin{array}{r} 4A+5D=2100 \\ -A\quad -D=-480 \\ \hline 3A+4D=1620 \\ -A\quad -D=-480 \\ \hline 2A+3D=1140 \\ -A\quad -D\quad -480 \\ \hline A+2D-660 \\ -A\quad -D\quad -480 \end{array}$$

$D=180$

$$\begin{array}{r} 480 \\ -180 \\ \hline 300 \end{array}$$

$$\boxed{\begin{array}{l} A=300\times\$4 \\ D=180\times\$5 \end{array}}$$

Words:

First I made the problem into an equation. Then I subtracted 1 away from A and D everytime and 480, which was the amount of tickets sold from the amount of money there was. I keep doing this so I could get the D by itself to find out how much money the door tickets got. Then to find out the amount of the advanced tickets, I subtracted the amount of the door tickets from the original amount of tickets sold. Then I multiplied the amount of the advanced tickets by \$4 because that's how much they sold for and I multiplied the amount of the door tickets by \$5 because that's how much they sold for. 300 x \$4 is 1200 and 180 x \$5 is 900 and 1200 and 900 added together equals \$2,100, the original amount of the total price of all of the tickets.

Sample 4

My answer was: 300 people bought at the door ($5), + 180 people bought in advance ($4).

- First, I divided 480 (people) and 2100 (dollars) by 60 so I can calculate more easily. Now I got 8 (people) and 35 (dollars).
- Then I wrote a chart:

set	1	2	3	4	5	6
people who bought by $4	7	6	5	4	3	2
" $5	1	2	3	4	5	6

- If I calculate one set (the set I circled) by one, only the third one ($\frac{5}{3}$) equals to 35 dollars. The rest equals to: $33, $34, $36, $37, $38.
- So it means 5 people bought for $4, + 3 people bought for $5.
- Lastly, I multiplied 5 + 3 by 60, which equaled to: $5 \times 60 = 300$; $3 \times 60 = 180$.

300 people bought for $5
180 " for $4

Sample 5

1. From the problem I know : → tickets in advance are 4 \$ → tickets at the door are 5 \$ → 480 people attended the play → the entire profit was 2100 \$

2. I am being asked to find out how many people payed at the door.

3. to get my solution I did the following:

① $\begin{array}{r} 480 \text{ people} \\ \times \quad 4\$ \\ \hline \$1{,}920 \end{array}$ ② $\begin{array}{r} \$2{,}100 \\ -\ 1{,}920\$ \\ \hline 180 \text{ people} \end{array}$ buys 5\$ ticket ③ $\begin{array}{r} 480 \\ -180 \\ \hline 300 \text{ people} \end{array}$ buy tickets in advance (4\$)

4. The answer is 180 people buy tickets at the door.

5. To check my answer I did the following:

① $\begin{array}{r} 300 \text{ people} \\ \times \quad 4\$ \\ \hline 1{,}200\$ \end{array}$ ② $\begin{array}{r} 180 \text{ people} \\ \times \quad 5\$ \\ \hline 900\$ \end{array}$ ③ $\begin{array}{r} 1{,}200\ \$ \\ +\ 900\ \$ \\ \hline 2{,}100 \end{array}$

	\$ spent	people
buy tickets at door	\$900	180
pay in advance	\$1200	300
total	\$2,100	480

Sample 6

First we wrote down the facts. x became the number of tickets sold in advance, and d became the number sold at the door. Equation number one was $4x + 5d = 2{,}100$. Two distinct variables in an equation is not something we know how to deal with yet, so we wrote a few more equations. Next equation was $x + d = 480$, which also means (by the subtraction property of equality) $480 - x = d$. Using that we substituted: $4x + 5(480 - x) = 2{,}100$. The Distributive property was what came next: $4x + 2{,}400 - 5x = 2{,}100$. Then we combined like terms $-x + 2{,}400 = 2{,}100$. Now we needed to isolate the variable, so we subtracted the constant 2,400 from both sides: $-x = -300$. Last we subtracted 300 from 480 which is 180. Our solution was found. 300 people bought tickets in advance and 180 bought tickets at the door.

There is another, more advanced way to do this equation. Taking the equation $x + d = 480$, we multiplied both sides by –4. Second we took the old equation; $4x + 5d = 2{,}100$, and added it to the revised equation of $-4x - 4d = -1920$. $-4x$ and $4x$ cancelled and $-4d$ plus $5d$ became d. $2{,}100 - 1920 = 180$. The reason for multiplying by 4 was to cancel out the $4x$ in the second equation and isolate the variable d. Both sides could be multiplied by –5 to cancel out $5d$, but either works.

Floodlights

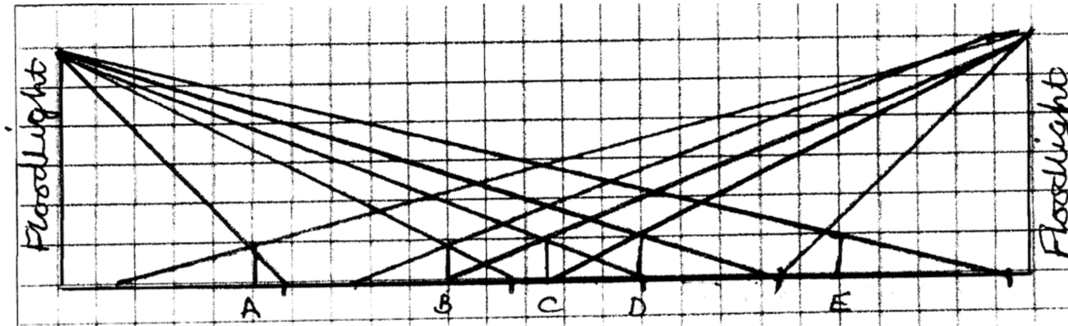
Eliot is playing football.
He is 6 feet tall.
He stands exactly half way between two floodlights.
The floodlights are 12 yards high and 50 yards apart. The floodlights give Eliot two shadows, falling in opposite directions.

1. Draw a diagram to represent this situation. Label your diagram with the measurements.

2. Find the total length of Eliot's shadows. Explain your reasoning in detail.

3. Suppose Eliot walks in a straight line towards one of the floodlights. Figure out what happens to the total length of Eliot's shadows. Explain your reasoning in detail.

Sample Response: Wendy



| ← Eliot

| square = 2 yards.

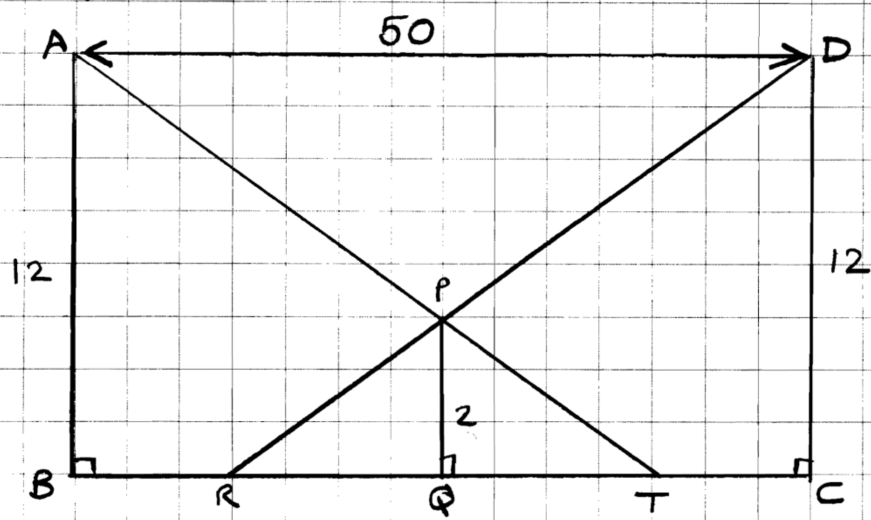
Eliot is 6feet = 2 yards tall.

Place Eliot stands	Left shadow	Right shadow	Total shadow
A	$3\frac{1}{2}$	1	$4\frac{1}{2}$
B	$2\frac{1}{2}$	$1\frac{1}{2}$	4
C	$2\frac{1}{2}$	$2\frac{1}{2}$	5
D	2	3	5
E	$1\frac{1}{2}$	$4\frac{1}{2}$	6

As Eliot goes to the right, the left shadow gets shorter and the right shadow gets longer. Total shadow stays about the same.

Sample Response: Tod

Tod



AB and CD are vertical. The football field is horizontal (BC) and so is Eliot. RQ and QT are shadows. I want to find QT.

$QT^2 = PT^2 - 4$ (Pythag).

$PT = AT - AP$

$AP^2 = 10^2 + 25^2$

$AT^2 = 12^2 + (25^2 + QT^2)$

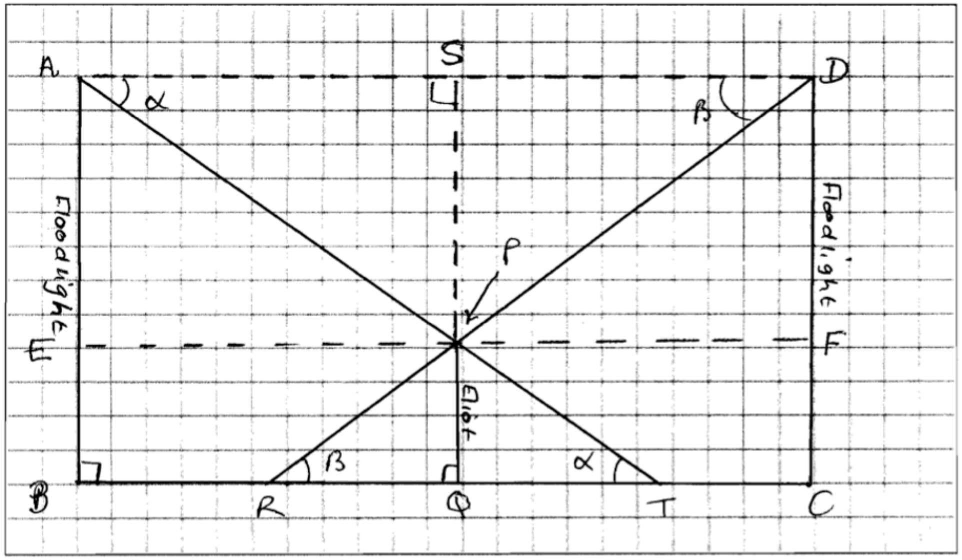
Triangle ABT and triangle PQT are similar so

$$\frac{QT}{PQ} = \frac{BT}{AB}$$

and

$$\frac{QT}{2} = \frac{BT}{12}$$

Sample Response: Uma



$\hat{TAD} = \hat{PTQ} = \alpha$

$\hat{ADP} = \hat{PRT} = \beta$

$\hat{APD} = \hat{RPT} = 180 - (\alpha + \beta)$

$\triangle RPT$ is similar to $\triangle APD$

SP is the perpendicular height of $\triangle APD$

PQ is the perpendicular height of $\triangle PRT$

$SP = 12 - 2 = 10$ yds, $PQ = 2$ yds

$SP : PQ = 10 : 2 = 5 : 1$

For More Information and General Resources

- Common Core State Standards: www.corestandards.org
- College Board Standards for College Success: http://professionals.collegeboard.com/k-12/standards
- College Completion Agenda Policy Guide: http://advocacy.collegeboard.org/sites/default/files/PolicyGuide_062810sm.pdf
- Smarter Balanced Assessment Consortia http://www.smarterbalanced.org/smarter-balanced-assessments/
- Partnership for Assessment of Readiness for College and Careers (PARCC Consortia) http://www.parcconline.org/parcc-assessment

For More Information and General Resources

- Mathematics Assessment Project (MAPS) http://map.mathshell.org/materials/index.php

- Exemplars: Standards Based Assessment and Instruction: www.exemplars.com

- Constructed-Response test questions: Why we use them; How we score them.   Samuel A. Livingston,ETS, R&D connections. No. 11, September, 2009. http://www.ets.org/Media/Research/pdf/RD_Connections11.pdf

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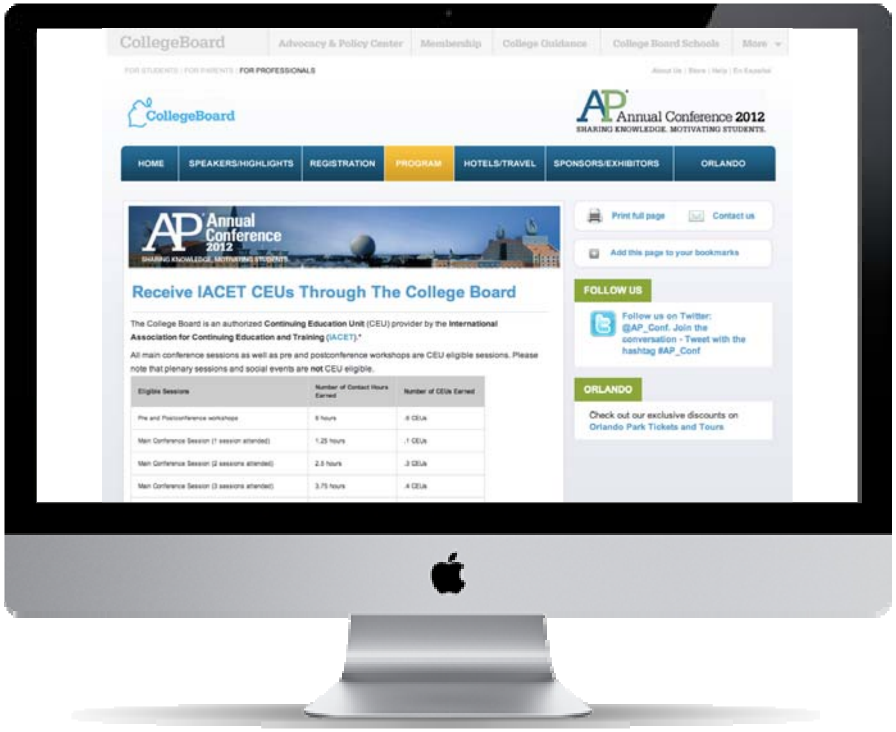

Thank You

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