Abstract Title Page
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Title:
Algebra for all: the effect of algebra coursework and classroom peer academic composition on low-achieving students

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Abstract Body
Limit 4 pages single-spaced.

Background / Context:

A recent report by the Mathematics Advisory Panel (National Mathematics Advisory Panel, 2008) referred to algebra as a "gateway" to later achievement. To address low academic performance in algebra, an increasing number of states and school districts have begun requiring algebra for all students in ninth-grade or earlier. This policy is supported by a large body of research, which shows that students who enrolled in more rigorous coursework have better academic outcomes than students who did not do so. This includes research on Catholic versus public schools (Bryk, Lee, & Holland, 1993; Lee & Bryk, 1988), constrained curriculum (Lee, Croninger, & Smith, 1997), opportunities-to-learn (McKnight, Crosswhite, Dossey, Kifer, Swafford, Travers & Cooney, 1987;Westbury, 1992; Schmidt, Wang, & McKnight, 2005), tracking (Oakes, 2005; Gamoran & Mare, 1989; Lucas, 1999;), and curricular requirements (Attewell & Domina, 2008; Chaney, Burgdorf & Atash, 1997; Gamoran & Hannigan, 2000).

However, these studies typically compare outcomes among students who took different coursework, such as algebra vs. remedial math, controlling for students' background characteristics, including prior achievement and other socio-demographic characteristics. Such analysis may suffer from selection bias. Moreover, these studies do not necessarily inform us about the consequences of a systemic reform—a district-wide reform that mandates schools to expand algebra to many low-achieving students who would not likely to take the course otherwise.

More recent studies examined the policy of expanding algebra, using quasi-experimental design. These studies often use policy-induced variation in implementation to identify the policy effect on students who were affected by the policy. Unlike earlier studies on algebra course enrollment, these studies showed that expanding algebra to low-achieving students have no effect or negative effects (Allensworth, Nomi, Montgomery, and Lee, 2009; Nomi, 2012; Clotfelter, Ladd, & Vigdor, 2012). For example, a ninth-grade algebra-for-all policy, implemented in Chicago in 1997, successfully enrolled nearly all low-achieving students in algebra. However, the policy did not improve their math achievement, and their math course grades declined (Allensworth et al., 2009). Similarly, in North Carolina, the expansion of algebra in eighth-grade had no effects or negative effects on students' outcomes (Clotfelter et al., 2012).

In Chicago, the algebra-for-all policy also had an unintended consequence on high-achieving students whose algebra enrollment was not affected by the policy. This occurred because the policy led to detracking of math classes. For high achieving students who would always take algebra regardless of the policy, the policy resulted in declines in the average ability level of their classroom peers. Consequently, their math outcomes also declined (Nomi, 2012).

Prior studies on algebra expansion have not examined how the change in classroom peer ability affected the outcome of low-achieving students. Not only did the policy expanded algebra to low-achieving students, but also these students experienced a rise in classroom peer ability levels as schools tended to create more mixed-ability classrooms. Thus, the policy would affect their outcomes through two mechanisms—algebra enrollment and classroom peer ability.

The two effects may counteract with each other, resulting in no overall policy effect. For example, exposure to algebra and more challenging material may be beneficial to their learning, while improvement in classroom peer ability may negatively affect low-achieving students'

outcomes due to negative social comparison or fish pond effects. Alternatively, taking algebra may negatively affect their learning if the content is beyond their reach and they are no longer taught the foundational skills, while improvement in peer ability may benefit as higher ability classrooms are often less disruptive and more conducive to learning than low-ability classrooms.

Purpose / Objective / Research Question / Focus of Study:
        This study builds on an earlier study by Allensworth et al (2009) on the algebra-for-all policy in Chicago, showing no significant policy effect on low-achieving students. Extending this work, we examine how classroom peer ability and algebra enrollment may independently affect academic outcomes of low-achieving students.

Setting:
        Chicago has the third-largest school system in the United States. The student population is about 50% African-American, 38% Latino, 9% White, and 3% Asian. Approximately 85% of students are eligible for free/reduced-price lunch.

Population / Participants / Subjects:
        We use six cohorts of ninth-grade students and focus on students with low skills—three pre- and three post-policy cohorts from 1994 and 1999—in schools that existed in both pre- and post-policy periods (26,885 students in 59 schools). To define low-skill students, we first created a standardized measure of students' incoming abilities with a mean of zero and standard deviation of one based on the entire student population (see Table 1 for variable descriptions). For the purpose of this study, we define "low-skill" students as those whose incoming math abilities are below -0.5 SD. On average, 55 percent of low-skill students in the district took algebra prior to the policy, while nearly all of them took algebra post-policy.

Intervention / Program / Practice:

        The intervention under the current study is the curriculum policy mandating Algebra I for all ninth-grade students. In 1997, the Chicago Public Schools (CPS) raised graduation requirements, eliminating the large array of remedial courses. In mathematics, students were required to take Algebra I in the ninth grade, followed by Geometry and Algebra II in the subsequent years. Prior to the policy, students were required to complete two to three years of mathematics in any subject and many students began high school with remedial coursework (pre-algebra or general mathematics).

Research Design:
        We use quasi-experimental design. Of 59 schools, 14 schools had already offered algebra to nearly all students, including low-achieving students, prior to 1997. These schools were unlikely to be affected by the algebra for all policy and serve as the comparison group. Our experimental schools consist of 45 schools that initially offered remedial math during pre-policy years. Figure 1 and Table 2 shows the overall change in algebra enrollment rates and classroom peer ability levels from pre to post-policy years.
        Importantly, there was considerable variation between experimental schools in both the degree to which they increased algebra enrollment, which was largely determined by algebra

enrollment rates pre-policy, and the degree to which they created mixed-ability classrooms when the policy was implemented. We use this between-school variation in policy-induced change to identify the independent effects of algebra enrollment and classroom peer composition on students' achievement.

Data Collection and Analysis:

This study draws from multiple data sources provided by CPS. Data on course transcripts and semester grade files are used to identify students' algebra enrollment and their teachers and classmates. Administrative records contain student demographic information, including gender, age, race/ethnicity, special-education status, and residential mobility prior to entering high school. Student socio-economic variables were constructed using the 2000 U.S. census block-level data linked to students' home addresses. Data on the Iowa Tests of Basic Skills in mathematics are used to measure students' incoming abilities and academic composition of students' algebra classes. High school achievement test scores come from the Tests of Academic Proficiency administered in the ninth-grade spring semester (see Table 1 for details).

We begin by estimating the ITT effect of the policy, using the following model;

$$Y_{dij} = B_j T_d * Post_{dij} + \alpha_d + \eta_j + \varepsilon_{dij} + r_{dj}, \tag{1}$$

where Y is an outcome for student i in year d in school j, T_j equals 1 for experimental schools and 0 for control schools , $Post_{dij}$ is an indicator variable for each post-policy year d, α_d is year fixed effects, η_j is school fixed effects, and ε_{dij} and r_{dj} are, respectively, individual and cohort (school-by-year) error terms. The coefficient, B_j provides an ITT estimate for school j; we are interested in the average causal effect, $E(B) = \beta$ and the variance in these effects $Var(B) = \tau_B^2$. Year fixed effects capture contemporaneous effects that apply to students in all schools, and school fixed effects account for all time-invariant school characteristics. We also consider including students-level covariates, such as prior math skills and socio-demographic characteristics and cohort-level covariates, including average students' incoming skills in year d in school j.

To estimate the effect of algebra enrollment and classroom peer ability, we adapt an instrumental variable approach. Specifically, to capture policy-induced variation in increases in algebra enrollment and peer ability change, we use interaction terms between school fixed effects for experimental schools and year fixed effects for post-policy years as our instruments. We estimate the following first-stage equation for algebra enrollment M and classroom peer ability C;

$$M_{dij} = \Gamma_{mj} T_d * Post_{dij} + \alpha_{md} + \eta_{mj} + \varepsilon_{mdij} + r_{mdj}$$
$$C_{dij} = \Gamma_{cj} T_d * Post_{dij} + \alpha_{cd} + \eta_{cj} + \varepsilon_{cdij} + r_{cdj}. \tag{2}$$

The coefficients, Γ_{mj} and Γ_{cj} are intended to capture the average causal effect on algebra enrollment M_{dij} and classroom peer ability C_{dij}, respectively, of attending an experimental school j in post-policy year d as compared to attending that school in the absence of the policy.

The second-stage equation takes the following form;

$$Y_{dij} = \delta_m \widehat{M}_{dij} + \delta_c \hat{C}_{dij} + \alpha_d + \eta_j + \varepsilon_{dij} + r_{dj,} \tag{3}$$

where $\widehat{M}_{dij}$ and $\hat{C}_{dij}$ represent, respectively, predicted probability of taking algebra and predicted classroom peer ability levels from Equation 2.

Findings / Results:

We are currently analyzing our data. Our preliminary results showed that the average ITT effect of the policy was 0.07 SD and it was not statistically significant. The first-stage results showed that, on average, the policy increased algebra enrollment by 60 percent for low-achieving students in schools that had offered remedial math during pre-policy years. This suggests that the overall effect of taking algebra (i.e., a combined effect of taking algebra and peer ability change) among compliers is about 10% of a standard deviation (0.07/0.6=0.11). In comparison schools, 96 percent of low-achieving students took algebra and their enrollment rates remained the same from pre- to post-policy years. Our analysis also showed that during pre-policy years classroom peer ability levels were lower by about a quarter of a standard deviation for low-achieving students in experimental schools, many of whom took remedial math, than their counterparts in comparison schools. The policy raised the average classroom peer levels for low-achieving students in experimental schools by 0.2 SD (p<0.05).

The second stage result showed that there was no statistically significant effect of taking algebra on math achievement, controlling for classroom peer ability. However, the effect of classroom peer ability was positive and statistically significant; the model suggests that improvement in the average classroom peer ability by 1 SD would lead to improvement in students' math achievement by about 0.2 SD. This also suggests that the policy effect mediated by peer ability change is negligible; the policy increased the average peer ability levels by 0.25 SD, which translates into the policy effect through peer ability change of 0.2*0.25=0.05 SD.

We are currently analyzing the data to see whether the effect of algebra enrollment classroom peer ability may vary across post-policy years. We also plan to examine other outcomes, such as math course grades and failure rates.

Conclusions:

Algebra is often considered as a gateway for later achievement. A recent report by the Mathematics Advisory Panel (2008) underscores the importance of improving algebra learning in secondary school. Today, a growing number of states and districts require algebra for all students in ninth grade or earlier.

Chicago is at the forefront of this movement. Many low-achieving students took remedial math before 1997 and the algebra-for-all policy immersed these students in academic coursework for the first time. Moreover, these low-achieving students experienced a rise in the ability levels of their classroom peers. However, this study suggests that simply requiring algebra is insufficient to improve their outcomes, even though students may benefit from having higher achieving peers in their classrooms. Overall, taking Algebra, instead of remedial math, would have no significant effect and this may be because students lack sufficient skills to handle algebra. Chicago subsequently implemented double-dose algebra to address this problem, and research showed that offering extended instructional time and instructional supports to teachers was successful in improving algebra learning of low achieving students (Nomi and Allensworth, 2009; 2010).

Appendices
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Appendix A. References

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Appendix B. Tables and Figures
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Table 1. Variable descriptions

9th grade outcomes	

9th grade outcomes

Test scores — Math test scores measured in 9th grade spring semester with a mean of 0 and SD of 1. The test was last administered in spring 2000.

Student-level variables

Student incoming math ability — Students' incoming math ability was created using a vector of students' test scores on the Iowa Tests of Basic Skills from third through eighth grade. This "latent" math ability measure was then used to construct two sets of achievement variables, after being standardized across all cohorts with a mean of 0 and SD of 1.

Classroom ability — Average incoming math ability in a classroom

Special education — A dummy variable with 1= special education students and 0=otherwise

Gender — A dummy variable with 1=male and 0=female

Race/Ethnicity — A set of dummy variables indicating African-American (ref. group), Asian, Hispanic, and White.

SES — Two variables constructed from the U.S. census data on students' residential block groups:

1) Concentration of poverty (a composite of male unemployment rate and % families under the poverty line. 2) Social status (a composite of the median family income and the average educational attainment. Both were standardized to have a mean of 0 and SD of 1.

Mobility — A set of dummy variables: no moves, one move, two or more moves in the 3 yrs before high school

Age at HS entry — 1) Number of months old for high school (0 if a student is not older than the appropriate age), 2) a dummy variable indicating if students are slightly old, 3) a dummy variable indicating if students are young for starting high school

Cohort-level variables

Cohort year — A set of dummy variables distinguishing cohorts with the 1994 cohort as a reference group

Cohort average incoming ability — Average incoming ability of the entering ninth-grade cohort in a school

Table 2. Change in algebra enrollment before and after Algebra for All in experimental schools.

Student prior skills	Year					
	1994	1995	1996	1997	1998	1999
High	0.99	0.99	1.00	1.00	1.00	1.00
Medium high	0.97	0.96	0.97	1.00	1.00	1.00
Medium low	0.86	0.84	0.90	1.00	1.00	1.00
Low	0.53	0.52	0.66	1.00	1.00	1.00

Figure 1 Change in classroom peer ability before and after Algebra for All.