EVALUATING THE INTERACTIVE LEARNING TOOL SIMREAL+ FOR VISUALIZING AND SIMULATING MATHEMATICAL CONCEPTS

Said Hadjerrouit
University of Agder, Kristiansand, Norway

ABSTRACT

This research study aims at evaluating the suitability of SimReal+ for effective use in teacher education. SimReal+ was originally developed to teach mathematics in universities, but it is has been recently improved to include school mathematics. The basic idea of SimReal+ is that the visualization of mathematical concepts is a powerful technique to enhance the understanding of mathematics. The study uses a set of quality criteria, and a survey questionnaire with open-ended questions to evaluate the suitability of SimReal+ in teacher education. The statements of the survey are categorized in five main groups of criteria that include technical usability, pedagogical usability, mathematical content, assessment issues, and teacher education considerations. Additional issues are programming mathematical visualizations, simulations using templates, and flipped classroom. Conclusions and some recommendations are drawn from the results to improve the design of SimReal+ to better suit the needs of students and teachers in mathematics education.

KEYWORDS

Mathematics, teacher education, SimReal+, simulation, usability, visualization.

1. INTRODUCTION

The Centre for Research, Innovation and Coordination of Mathematics Teaching (MatRIC) has been established in 2014 to create and disseminate knowledge and experience that will guarantee high quality learning opportunities in mathematics for Norwegian students. This work is situated within the simulation and visualization workgroup of MatRIC, which seeks to support a network of mathematics teachers collaborating in the production of digital simulations and visualizations by means of SimReal+ to support learning processes. SimReal+ is a digital tool that was developed to teach a wide range of mathematical topics in higher education. Recently, SimReal+ has been extended to include school mathematics, but the suitability of the tool has not yet been evaluated for use in teacher education. According to the New National Curriculum in Norway, being able to use digital tools in mathematics education involves using the tools for visualization, problem solving, simulation and modeling. However, mastering technical issues of digital tools does not automatically make SimReal+ pedagogical usable for teacher students in terms of motivation, variation, student autonomy, individualization. Moreover, the content provided by SimReal+ must be mathematically sound and correct, and help students gain knowledge that is otherwise difficult to acquire in a school environment. Furthermore, assessment issues in terms of feedback must be considered to support the learning process. Hence, adaptability of digital tools to a teacher education context is a complex issue that needs to be evaluated on the basis of criteria that are pertinent to pedagogical software. Clearly, success in mathematics-related subjects does not automatically guarantee success in teacher education. The paper uses an instrument to assess the suitability of SimReal+ for teaching and learning school mathematics. To achieve this goal, SimReal+ was used in a course on digital tools in mathematics education. The work was carried out in two steps. Firstly, teaching and exercise activities over a period of two weeks were performed. The activities included mathematical issues related to trigonometry, programming issues, and diverse teaching material made online. Secondly, the students were asked to evaluate SimReal+ using a set of criteria, and mixed data collection and analysis methods. The criteria were categorized in five groups that include technical usability, pedagogical usability, mathematical content, assessment issues, teacher education

ISBN: 978-989-8533-43-2 © 2015

considerations, and other issues such as programming and flipped classroom. Open-ended questions allowed the students to express in their own words what they think about different issues of SimReal+. Finally, the article suggests some recommendations that help placing SimReal+ as an integral part in teacher education.

2. SIMREAL+

SimReal+ is an interactive learning tool for teaching and learning mathematics for a range number of subjects related to mathematics such as engineering and physics. The basic idea of SimReal+ is that visualizations are powerful mechanisms for learning mathematics and explaining difficult topics. According to Arcavi (2003), visualization is the ability to use and reflect upon pictures, graphs, animations, images, and diagrams on paper or with digital tools with the purpose of communicating information, thinking about and advancing understandings. There is a huge interest in visualization in mathematics education (McKenzie, & Clements, 2014; Presmeg, 2014). Textbooks are filled with pictures, diagrams, and graphs. Graphing calculators have become integral part of mathematics education in secondary schools. Digital tools based on visualizations, such as GeoGebra and Geometer's Sketchpad, are in use in secondary and university mathematics classrooms. However, there is little empirical support for the use of visualizations in educational settings (Macnab, Phillips, & Norris, 2012). SimReal+ as a visualization tool uses a graphic calculator, video lectures, video streaming, video and interactive simulations to teach mathematics. It also provides exercises and applications in various areas of mathematics and physics at different educational levels (Brekke, & Hogstad, 2011). Figure 1 shows the main components of SimReal+.

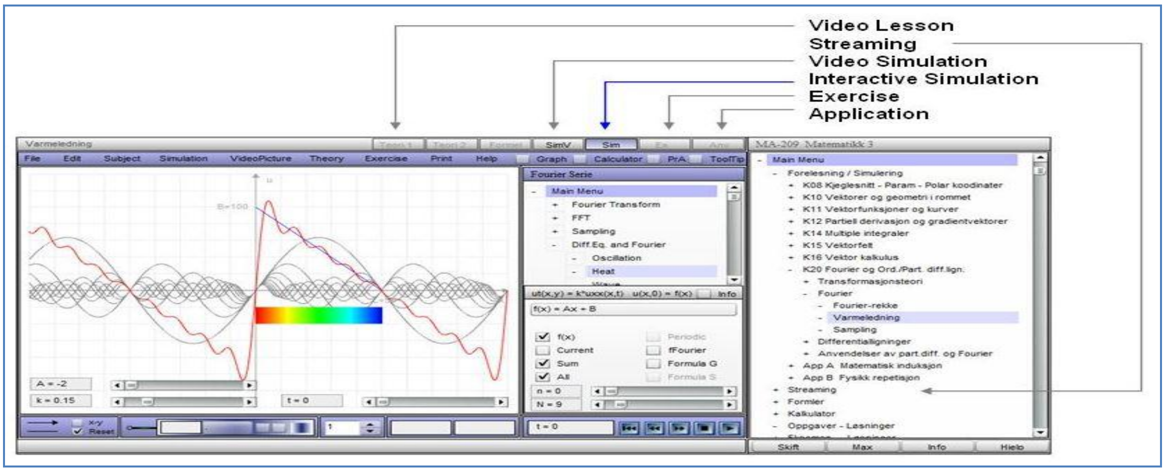

Figure 1. SimReal+ main components

There are three research studies on SimReal+ and its effect on teaching and learning mathematics. Two studies focused on teaching mathematics in higher education (Brekke, & Hogstad, 2010; Hogstad, 2012). These studies report on positive attitudes towards the use of SimReal+ and its suitability and usefulness in difficult and abstract mathematical areas. Students considered SimReal+ as a positive addition and supplement to ordinary teaching. Students encountered few challenges. The third study addresses the use of SimReal+ in teacher education (Cury, 2012). It reports on positive students' attitudes towards the use of the tool in an upper secondary school, but some students did not found visualizations very useful, and that integrating the tool into the educational curriculum was not simple. These studies used quantitative and qualitative methods to assess students' perceptions of SimReal+ such as interviews and surveys.

3. THEORETICAL FRAMEWORK

A wide range of theoretical approaches can be used to address the integration of digital tools in education (Drijvers et al. 2010). No one of these approaches is ready-made for the purpose of this work, but according to Cobb (2007), elements of different theoretical perspectives can be adapted for the concerns of a research study as a source of ideas. Hence, the framework used for evaluating SimReal+ is rooted in four approaches that are particularly pertinent to this study. The first one is the instrumental approach and the distinction between artefact and instrument, and how to transform an artifact to an instrument through the processes of instrumental genesis (Rabardel, 1995; Trouche, 2004). Second, the anthropological approach and the Task-Technique-Theory triad to capture the relationships between tool techniques and conceptual understanding, on the one hand, and tool techniques and paper-and-pencil techniques, on the other hand (Chevallard, 1995). Third, the Theory of Didactical Situations, and the tool's potential for providing feedback, which is an essential condition for supporting learning (Brousseau, 1997). The fourth theoretical background is based on usability issues and related research work on evaluating digital tools in mathematics education (Bokhove, & Drijvers, 2010; Hadjerrouit, & Bronner, 2014). As a result, five groups of evaluation criteria are derived from this theoretical background: Technical usability, pedagogical usability, assessment issues, mathematical content, and teacher education considerations (Hadjerrouit, & Bronner, 2014). Technical usability as defined by Nielsen (1993) is a self-evident requirement for any digital tool in mathematics education. In many cases, however, the impact of technical usability on learning is limited when it comes to pedagogical use of the tool. In addition, the functionality of a particular tool does not always result in pedagogical opportunities, which will only be visible when an explicit pedagogy guides the design and use of the tool in classroom (Burden & Atkinson, 2008). Nokelainen (2006) expanded the concept of technical usability to include pedagogical usability criteria, such as learner control, collaboration, variation, motivation, differentiation, flexibility, and feedback. Hadjerrouit (2010) stated that both aspects of usability are closely related to each other. In addition, the assessment dimension has emerged as one of the most important criteria for evaluating the quality of digital tools. Assessment is associated with the tool's feedback and builds the ground for formative assessment. Another criterion is the inclusion of mathematical content in terms of correctness of mathematical concepts, notations, and symbols (Artigue, et al, 2009; Bokhove, & Drijvers, 2010). Equally important is the congruence between the tool's features and paper-and-pencil techniques to facilitate mathematical reasoning. Finally, it is important to evaluate whether the tool is appropriate in teacher education, takes into account the requirement of adapted education, and enables the teacher to concretize the mathematical subject curriculum. The specificities of the criteria are described as follows:

a) Technical usability

- *Ease-of-use*: SimReal+ should be easy to use, to start, and to exit.
- *Accessibility:* SimReal+ should be accessible anytime and place.
- *Management facilities*: SimReal+ should provide management facilities, e.g., it should be possible to store answers given by students. The tool should also have readily available content that can be modified.

b) Pedagogical usability

- *Motivation*: Motivation measures the extent to which SimReal+ is attractive to use, adapted to the students' age, knowledge level, development, and interest, as well as tied to the students' other activities and tasks. Using SimReal+ should be a motivational factor for learning algebra.
- *Variation*: SimReal+ should be able to present the content in several ways, and facilitate various activities with students. SimReal+ should also be used as an alternative to achieve variation in teaching, and eventually in combination with textbooks and other study material and digital tools.
- *Student autonomy*: SimReal+ should enable a high degree of autonomy so that the students do not necessarily need teacher assistance for help. The knowledge provided by SimReal+ should be potentially powerful to enable students to become less dependent on teacher assistance.
- *Individualization*: SimReal+ should take into account adapted education, and different knowledge levels. The students should be able to work in their own pace, save their work and continue later. Disabled people should be able to work with the tool.
- *Differentiation:* Differentiation means that SimReal+ should provide multiple tasks with different levels of difficulty, and can be tailored to the students. In addition, the tool should provide opportunities for the teacher to make individual adjustments and customize the tool when needed.

c) Mathematical content

- *Quality of content*: Mathematical content (theory, exercises and problems) provided by SimReal+ should be of high quality. The content should be mathematically sound and faithful to the underlying mathematical properties. SimReal+ should provide opportunities to display formulas correctly, and help students to gain knowledge that is otherwise difficult to acquire.

- *Congruence with SimReal+ techniques*: SimReal+ should enable the student to apply his or her own paper-and-pencil technique reasoning steps and strategies, and express mathematical ideas, as well as facilitate students' mathematical activities.

d) Assessment

- *Formative assessment* is important to measure the extent to which SimReal+ provides several assessment and review modes, appropriate feedback in the process of problem solving, and use of several question types. Providing student profiles is important when it comes to adapt the questions to the student's knowledge level. The quality of interaction and feedback to students' actions are crucial for the learning process. In terms of *summative assessment*, the tool should provide teachers with quantitative data, statistics and results that help to evaluate students' performances. It should give scores and grading.

e) Teacher education

- Adaptability and suitability of SimReal+ in teacher education includes the appropriateness of the tool in teaching school mathematics. This criterion also measures the extent to which SimReal+ provides opportunities to concretize the curriculum as specified by the New National Curriculum, and whether the tool is tied to teaching and adapted to the curriculum.

4. RESEARCH DESIGN

This research study involved 22 teacher students taking the course on digital tools in mathematics education in the fall semester of 2014. The students had very different knowledge background both in mathematics and digital tools. None of the students had any prior experience with SimReal+. The work used a survey questionnaire with open-ended questions to gather empirical data. To explore the use of SimReal+, teaching activities over a period of two weeks were designed and analyzed according to the purpose of this work. The activities were mostly related to the use of SimReal+ to teach trigonometry, but also other mathematical concepts. Table 1 gives an overview of the teaching activities over a period of three weeks.

Table 1. Teaching activities

Teaching activity	Teaching aids	Subject	Date	Duration
Lecture	• Video lecture	• SimReal+ User Interface • 2D calculator	Week 33	45 min
Lecture on mathematical tasks with SimReal+	• Teaching material • SimReal+	• Introduction to SimReal+ • Visualizations and simulations of trigonometric functions and concepts	Week 34	3 hours
Exercises using SimReal+	• SimReal+	• Simulations of trigonometric functions and other similar tasks	Week 34	2 hours
Lecture on SimReal+ and programming	• SimReal+ • ActionScript	• Introduction to programming • programming mathematical functions using ActionScript	Week 35	3 hours
Tasks and exercises using SimReal+	• SimReal+ • ActionScript • Diverse teaching material made online	• Students explore the program *Moving a particle - Drawing a graph* • Students investigate the *Pythagoras theorem, conic section, properties of triangles, cycloid,* and *3D figures*	Week 35	2 hours
Lecture on evaluation issues	• SimReal+ • Survey questionnaire	• Evaluation of SimReal+ using a questionnaire with 73 statements grouped in 5 groups of criteria	Week 36	1-2 hours

The teaching activities included video lectures, simulations of mathematical trigonometric functions, programming visualizations using ActionScript, and diverse teaching material, such as Pythagoras theorem, conic section, properties of triangles, cycloid, 3D figures, and similar mathematical functions. To measure the students' responses, the survey questionnaire used a five-point Likert scale from 1 to 5, where 1 was coded as the highest and 5 as the lowest (1="Strongly Agree"; 2 = "Agree"; 3 = "Neither Agree or Disagree"; 4 = "Disagree"; 5= "Strongly Disagree"). The average score (MEAN) was calculated, and the responses to open-ended questions were analysed qualitatively. The survey included 73 items that were distributed as follows: Technical usability (12 items), pedagogical usability (24 items), mathematical issues (16 items); assessment (12 items), and teacher education (11 items). The students were asked to comment each of the statements in their own words. In addition, the students were required to address 5 open-ended questions to express in their own words what they think on different issues of SimReal+.

5. RESULTS

The students' responses to the survey questionnaire are described for the five broad categories that emerged from the evaluation criteria, and for supplementary issues that were addressed by open-ended questions.

5.1 Global Impression of SimReal+

First, the students were asked to express their opinion on SimReal+, and what they globally liked and disliked about the tool. Basically, most respondents reported both positive and negative aspects of SimReal+: "*It seems like a good program. I like that it is free and easy to use. So you can use it when you want. Sometimes it can be a bit hard to find your way around"*. Some students were more critical as the following comment indicates: *"I think it is confusing. It has too many options that are poorly linked together. It could learn a lot from "Mastering physics", in terms of more linear, logical structure and better feedback assessment. (...) I really like a teachers' opportunity to program (...) and add new graphics."*

5.2 Technical Usability

Most students were globally satisfied with the technical usability in terms of easy-to-start, management facilities, availability of mathematical content and exercises, quality of video streaming and calculator, and accessibility of the tool anytime and anywhere as this students' comment indicates: *"It is not all that difficult for first timer to use SimReal+ due to the information provided to make mathematics easy"*. In contrast, students were not satisfied with the technical usability in terms of ease-of-use or user friendliness, navigability, response time, as this comment clearly reveals: *"Everything does not work for me. I got confused. The long lists of lessons/exercises/ simulations in the rightmost window made it hard to focus attention to a specific task. Pre-made simulations and demonstrations worked well for illustration, but the visualizations that the activity should give, e.g. that Pythagoras' theorem is seen to be true, was not clear to me. The intention was clear, but I don't think the user will readily gain anything."*

5.3 Pedagogical Usability

In terms of pedagogical usability, the students agreed that SimReal+ provides various mathematical activities and multiple representation of mathematical content, and that SimReal+ can be used as a lecture and textbook supplement. Concerning motivational issues, many students think that the tool is motivating to do mathematics with, as this comment shows: *"Especially for mathematics interplay with physics, SimReal+ served as a good motivation and demonstration. However, motivation derives not only from an experience of «..., this is useful/beautiful», but also from one's own development of capabilities to make mathematics useful/beautiful. This latter element of motivation was not offered by SimReal+, so I only felt the brief sense of wonder, not of achievement and skillfulness"*. Furthermore, many students think that SimReal+ contains multiple levels of difficulty and provides opportunities for the teacher to make individual adjustments if necessary. However, the tool does not easily allow students to customize the tool. Nevertheless, the tool enables students to work at their own pace, which is a motivational factor in keeping students engaged in mathematics, as this comment reveals: *"I think this is the greatest advantage of SimReal+ the opportunities for adjustment and individualization. The teacher knowing a little programming (as should all upper secondary teachers in my opinion) can rather easily make small addition and alterations to a pre-programmed lesson, so it fits his needs: add a second particle, hiding/showing the kinematic (...) equations, etc."*. Moreover, most students think that SimReal+ does not fully allow to work independently from teacher assistance or fellow students. Also textbooks are still needed when using SimReal+ in classroom: *"Autonomous learning is not well-supported, only autonomous playing around without goal. And* *for the purpose of playing with mathematics, there were too few options for the student, (....), and mostly teacher-chosen and implemented possibilities/options"*. Furthermore, most students agreed that SimReal+ is not fully appropriate to use as an alternative to achieve variation in teaching mathematics as this comment clearly shows: "*The problem as I see it is that there is no clear-cut connection between the variations of a specific topic or notion. For example, changing the parameters in a wave function, (...) and its exact correlation with the graph, (....), but such things are not in any way implemented in SimReal+."*

5.4 Assessment Issues

Most students indicated that SimReal+ does neither provide a diagnosis of student's problem solving nor appropriate feedback that is adapted to the students' knowledge level. In addition, SimReal+ does neither build student profiles nor serve up appropriate questions or several questions to the students. Furthermore, SimReal+ does neither have a review mode showing what the student has done wrong or right, nor allow for the use of several question types. The lack of student profiles may prevent students from engaging in authentic learning activities with SimReal+. Hence, it is obvious that the role of the teacher is still important in assessing the students' learning of mathematics. The following comment summarizes the limitations of SimReal+ in terms of assessment: *"SimReal+ is very weak with respect to feedback. I experience no feedback or assessment of any work or exercises."*

5.5 Mathematical Issues

More than the majority of the students believed that SimReal+ has a good quality of mathematical content in terms of mathematical correctness and representation of mathematical properties and operations, e.g., formulas, functions, graphs, and geometrical figures. A typical comment was: *"SimReal+ is rich in mathematical content. It gives systematic procedure for solving a particular problem in mathematics. It is able to translate algebraic expressions into graphs or geometric figures, etc. The video and audio visual aspect of the software enhances deeper understanding of mathematical concepts"*. However, the practical applications and exercises are not fully well-designed as this comment indicates: *"The exercises were too general, not specifically focusing on some mathematical learning object. For example, the most focused exercise was about changing angles into radians and vice versa. The connection was very well visualized on the unit circle, but there weren't added any techniques to do the calculations. (...) The exercises were well-formulated but they were too focused on visualizations done by the program with no connection to paper-pencil work or text book theory"*. Furthermore, SimReal+ is useful when it combines video lessons, simulations, live streaming of lessons, and exercises. It helps to acquire new mathematical knowledge, and in a lesser degree, it is congruent with paper-pencil techniques. SimReal+ also provides opportunity to help students gain knowledge that is otherwise difficult to acquire. In addition, the tool's openness enables students to express mathematical ideas. The negative side of SimReal+ is that students do not like programming mathematical visualizations. They prefer focusing on the mathematical part of the program, which is more important than understanding programs written in Action Script as this comment expresses: *"Maybe for the students, but they don't need all the programming stuff, just an interface where mathematical properties can be approached. (...) For the teacher, I think it is essential that he/she understands the programming enough to make non-trivial changes such as adding (...) a button with a new function."*

5.6 Adaptability of SimReal+ to Teacher Education

Most students think that SimReal+ could be an appropriate tool in secondary schools, but not in middle or primary schools. SimReal+ also enables teachers to concretize the curriculum. However, when asked whether they will continue using SimReal+ for teaching mathematics, most of them answered negatively as this comment clearly reveals: *"I will use SimReal+ to further develop an illustration of a phenomenon in my teaching but only for illustration, not interactive student activity."*

5.7 Flipped Classroom

The students were asked whether flipped classroom will be of interest and whether new digital tools can give students new possibilities in teaching mathematics. Most students agreed that flipped classroom is of huge interest, but *"(...) we need to focus and stop believing that just planning around will result in learning. We need strictly directed lessons that allow for continuous interchanging of visual illustrations, theoretical presentations, exercise solving, etc. In short, digital tools could help us in writing the classroom activities that have traditionally been separated in time and space. (...) Flipped classroom also is very interesting but needs highly motivated students.*

5.8 Programming Visualizations

The students were asked in what way do they think programming mathematical visualizations by their own will help them in understanding mathematics if they can use different templates so the concentration can be on mathematics and not on difficult details in the programming process. As mentioned earlier, students prefer focusing on the mathematical part of the program rather than programming. Most students agreed that templates can help to concentrate on the mathematical part, but *"(...), I need very specific templates focusing on illustrating a specific notion, e.g. the kinematic equations of cycloid (....), so I need to know in advance rather precisely which properties I am investigating.(...).* The developer of SimReal+ is planning to design such templates so that students can be able by their own to program mathematics (elementary and advanced visualizations and simulations) directly into their own Web pages without any special tool. They just need to write their own mathematical code. Most students like the idea of using templates, as this comment clearly highlights: *"I think it is great. I think there should be a clear distinction in the programming interface whether you are implementing graphics mathematical properties, exercises and theory (....). Then you can access more readily different settings that a visualization or simulation should offer to a student using it."*

5.9 Suggestions for Improvements

Finally, the students were asked to provide suggestions to make SimReal+ more appropriate for use in teacher education. Perhaps, the most interesting suggestion is expressed by the following comment: *"Focus. It tries to do everything, but really only presents a fragmentary list. Technologically, it needs a more directed structure such that one task leads to the next in a logical way, not just present an array of tasks. (...) Mathematically, it must connect the different settings graphical, algebraic, tabulator, arithmetic, geometric, etc. Assessment-wise, it must offer feedback to the implemented exercises."*

6. CONCLUSION AND RECOMMENDATIONS

The number of participants (*N=22*) may not be sufficient to adequately support the generalization of the results. Hence, new cycles of experimentations and evaluations of SimReal+ are warranted to generalize the findings of the present work to ensure more validity and reliability. However, despite the limitations of the study, it has been possible to make some reasonable interpretations of the results and draw some recommendations for using SimReal+ in teacher education. Firstly, the use of SimReal+ indicates that the tool shows potential for teaching mathematics that is suited to the students' knowledge level, although not all criteria are equally met. Secondly, SimReal+ is technically well designed in terms of accessibility and management facilities. Furthermore, SimReal+ covers a wide range of mathematical content with varied levels of difficulty, including school mathematics. Likewise, the content is mathematically correct and reflects the underlying properties of mathematics at different levels. However, mathematical and technical issues in themselves are not sufficient to make SimReal+ become an integral tool in learning and teaching mathematics in a school environment. SimReal+ is not pedagogically well designed to ensure a smooth integration of the tool in teacher education in terms of variation, student autonomy, differentiation, and individualisation, and assessment issues as well. Hence, SimReal+ in its present form is not fully appropriate for use in teacher education, unless didactical functionalities and pedagogical modalities of using SimReal+ are considered in future versions of the tool. Likewise, the user interface must be simplified to make SimReal+ more intuitive and easy to use in teacher education. Summarizing, the use of SimReal+ in teacher education needs to take into account the pedagogical dimension of teaching and learning mathematics in order to adapt the tool to the modalities of using digital tools in teacher education.

ACKNOWLEDGEMENT

This research was supported by a grant from MatRIC - The Centre for Research, Innovation and Coordination of Mathematics Teaching. Web site: http://www.matric.no/

REFERENCES

Arcavi, A. (2003). The Role of Visual Representations in the Learning of Mathematics. *Educational Studies in Mathematics*, *52*(3), pp. 215-241.

Artigue, M. et al (2009). Connecting and Integrating Theoretical Frames: The TELMA Contribution. *International Journal of Computers for Mathematical Learning 14*, pp. 217-240.

Burden, K., & Atkinson, S. (2008). Evaluating Pedagogical Affordances of Media Sharing Web 2.0 Technologies: A Case Study. *Proceedings of ascilite* 2008, Melbourne, Australia, pp. 121-125.

Bokhove, K., & Drijvers, P. (2010). Digital Tools for Algebra Education: Criteria and Evaluation. *International Journal of Mathematics Learning*, *15*, pp. 45-62.

Brekke, M., & Hogstad, P.H. (2010). New Teaching Methods – Using Computer Technology in Physics, Mathematics, and Computer Science. *International Journal of Digital Society* (IJDS), *1*(1), pp. 17-24.

Brousseau, G. (1997). *Theory of Didactical Situations in Mathematics*. Kluwer Academic Publishers.

Chevallard, Y. (1985). *La transposition didactique - Du savoir savant au savoir enseigné*. La Pensée sauvage, Grenoble (126 p.). Deuxième édition augmentée 1991.

Cobb, P. (2007). Putting Philosophy to Work: Coping with Multiple Theoretical Perspectives. In: F.K. Lester (Eds.). *Second Handbook of Research on Mathematics Teaching and Learning*, pp. 3–38. Reston: NCTM.

Curri, E. (2012). *Using Computer Technology in Teaching and Learning Mathematics in an Albanian Upper Secondary School. The Implementation of SimReal in Trigonometry Lessons*. Master thesis, University of Agder.

Drijvers et al. (2010). Integrating Technology into Mathematics Education: Theoretical Perspectives. In C. Hoyles & J.-B. Lagrange (Eds.). *Mathematics and Technology-Rethinking the Terrain*. Berlin: Springer.

Hadjerrouit, S. (2010). A Conceptual Framework for Using and Evaluating Web-Based Learning Resources in School Education. *Journal of Information Technology Education 9*, pp. 54-79.

Hadjerrouit, S., & Bronner, A. (2014). An Instrument for Assessing the Educational Value of Aplusix (a+x) for Learning School Algebra. In M. Searson & M. Ochoa (Eds.). *Proceedings of Society for Information Technology & Teacher Education International Conference 2014*, pp. 2241-2248. Chesapeake, VA: AACE.

Hogstad, N. M. (2012). Use of SimReal + in Mathematics at the University Level. A Case Study of Students' Attitudes and Challenges [Original title (in Norwegian): *Bruk av SimReal+ i matematiske fag på universitetsnivå. En case-studiet av studenters holdninger og utfordringer*]. Master thesis, University of Agder.

McKenzie, K., & Clements, A. (2014). Fifty Years of Thinking about Visualization and Visualizing in Mathematics Education: A Historical Overview. In M.N. Fried, & T. Dreyfus (Eds.). *Mathematics & Mathematics Education: Searching for Common Ground, Advances in Mathematics Education*, pp. 177-192. Berlin: Springer.

Macnab, J.S., Phillips, L.M., & Norris, S.P. (2012). Visualizations and Visualization in Mathematics Education. In S.P. Norris (Ed.). *Reading for Evidence and Interpreting Visualizations in Mathematics and Science Education*, pp. 103–122. Rotterdam: Sense Publishers.

Nielsen, J. (1993). *Usability Engineering*. Boston, MA: Academic Press.

Nokelainen, P. (2006). An Empirical Assessment of Pedagogical Usability Criteria for Digital Learning Material with Elementary School Students. *Educational Technology & Society*, *9*(2), pp. 178-197.

Presmeg, N. (2014). Visualization and Learning in Mathematics Education. In S. Lerman (Ed.). *Encyclopedia of Mathematics Education*, pp. 636-640. Berlin: Springer.

Rabardel P. (1995). *Les hommes et les technologies, approche cognitive des instruments contemporains*. Armand Colin.

SimReal+ Web site: http://grimstad.uia.no/perhh/phh/video/video.htm

Trouche, L. (2004). Managing the Complexity of Human/Machine Interactions in Computerized Learning Environments: Guiding Students' Command Process through Instrumental Orchestrations. *International Journal of Computers for Mathematical learning 9*, pp. 281-307.