Croatian Journal of Education
Vol: 14 (2/2012), pages: 307-357
Original scientific paper
Paper submitted: 23rd December 2011
Paper accepted: 17th February 2012

Comparing the probability-related misconceptions of pupils at different education levels

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Abstract

The aim of the paper is to compare and evaluate the probability-related misconceptions of pupils at different education levels. A cross-sectional/age study was thus conducted with 540 pupils in 5th-8th grades. An instrument, comprising six questions on the concepts of compound events, probability of an event and probability comparisons, was used. Data were analyzed using descriptive statistical techniques (SPSS 15.0) and the chi-square test. This study showed that the percentage of correct answers increased when the level of education increased, whereas the misconceptions about the concept of compound events I decreased; also, the percentage of correct answers decreased and the misconceptions about the concept of compound events II increased. In the questions related to the concept of probability of an event and probability comparisons, as the level of education increased both the percentage of correct answers and the misconceptions increased. Therefore, it can be suggested that pupils' misconceptions regarding probability concepts vary, which depends on the characteristics of such concepts and pupils' age and education level.

Key words: *mathematics education, primary school pupils, probability, misconceptions, cross-age study*

Introduction

Mathematics has played a strategic role in human development and has kept a vital and unique place in many societies throughout history. After the mother tongue, mathematics has been given the greatest priority in the primary and secondary school curricula of most countries. Mathematics, as a universal language, is thus the common thinking means. Learning mathematics is viewed as an obligation because it can contribute to higher level skills, such as analyzing, reasoning, communicating, generalizing, creative and reflective thinking, which are necessary for everybody in daily life (Fast, 2001; Gürbüz, 2010). The necessities in work and daily lives have played a significant role in creating awareness of that obligation and the interest given to mathematics throughout history.

Since prehistoric times people have faced random events, e.g. unpredictable natural events and games of chance, but the birth of probability theory and its turning into a branch of mathematics did not occur until the middle of 17th century. Although probability is a very common concept in games of chance, it has been used in many areas, such as science, genetics, industry, management, economics, banking and insurance, kinetic theory of gases, statistical mechanics, and quantum mechanics.

The term 'probability learning' is associated with a specific experimental paradigm within which a person is presented with successive trials, each one having two possible outcomes, and, at each trial, is required to predict which will happen before being shown the outcome. In the standard form, the sequence is randomly determined by some process, with the probability of each outcome being fixed. In these circumstances, a phenomenon known as 'probability matching' is very frequently observed, whereby the relative frequencies of the person's predictions, over a series of trials, come to approximate the probabilities of the respective outcomes (Greer, 2001).

According to Fischbein (1975), probability matching is observed in children as young as 3-4, and generally appears to be well-established by the age of 6. He suggested that 'when, without special instructions, the probabilities of the responses approximate the probabilities of the events, it is possible to assume that the subject possesses some particular intuition of chance and probability'. Two explanations are usually used regarding the subject of probability. These explanations - the former derived from the earlier theories of conditioning and reinforcement, and the latter from the later cognitive theories of pattern induction – account for probability matching in terms of cognitive processes that do not depend upon any implicit or explicit representation of two outcome probabilities. Fischbein (1975) shows many interesting phenomena, such as negative recency effect and many developmental changes, such as that "older children increasingly seek more sophisticated strategies, based on the conviction that there are rules determining random sequences" (p. 56). Furthermore, a small number of studies showed positive effects of formal instruction in probability in the sense that the responses of children given such instruction approximated maximizing rather than probability matching (Fischbein, 1975).

Misconceptions/Difficulties and Probability

Various terms are used in order to define the difficulties faced in mathematics learning/teaching, namely, "difficulty", "misconception" and "error". These terms are used mostly interchangeably. "Difficulty" is a comprehensive term used to describe the difficulties faced by pupils in mathematics learning in general (Bingölbali and Özmantar, 2009). Taken this way, the term falls short of defining and solving the learning difficulties of pupils. Because of the above-mentioned characteristic of the term, the difficulties faced by pupils are mostly handled using the term "misconception". It is defined as "perceptions or conceptions that are far from the meaning agreed upon by the experts" (Zembat, 2008). In other words, it can be defined as the perceptions that are diverging from the view of experts in a field or subject (Hammer, 1996). According to Hammer (1996), misconceptions affect pupils' perceptions and understandings.

For the past 30 years or so, scholars have studied pupils' misconceptions regarding mathematics. Studies have shown that pupils' conceptions of scientific issues are often not in line with the accepted scientific thinking; that is, they have misconceptions about various notions. Cornu (1991) claimed that the mathematical difficulties and misconceptions faced by pupils might stem from three main reasons. These can be listed as: *epistemological reason* (the difficulties arise from the concept nature), *psychological reason* (the pupil's difficulty or misconception in learning a concept stems from his/her personal development, prior knowledge, and mathematical comprehension skill) and *pedagogical reason* (the pupil's difficulty in learning a concept stems from factors such as the means, content and method of instruction). Shaugnessy (1977) and Gürbüz (2007) have suggested that some misconceptions stem from the subject nature or pupils' prior theoretical knowledge about probability. In other words, these types of misconceptions might have epistemological or psychological roots and teaching probability rules might not be sufficient on their own to remedy these misconceptions. For example, pupils believe that, after six heads, the seventh roll will be tails. This thought is known as "Gambler's Fallacy" and, in fact, people decide whether a coin is fair or not by the equality of the emergence of likely results (Fast, 2001).

Probability concepts are widely used in decision-making processes related to uncertain situations we encounter in our daily lives. In spite of this importance, due to several reasons, probability is not being taught as effectively in Turkey as it is in many other countries. The most important reason for this lies in the misconceptions related to this subject. Such misconceptions make it difficult to understand the subject. Indeed, many studies were conducted into the misconceptions in teaching mathematics in general and, specifically, in the subject of probability (Barnes, 1998; Batanero and Serrano, 1999; Çelik and Güneş, 2007; Dooren et al., 2003; Fast, 1997; 2001; Fischbein and Schnarch, 1997; Fischbein, Nello and Marino, 1991; Gal-Ezer and Zur, 2004; Gürbüz and Birgin, 2012; Hope and Kelly, 1983; Huang, Liu and Shiu, 2008; Jacobsen, 1989; Kahneman and Tversky, 1972; Konold et al., 1993; Konold, 1989; Lecoutre, 1992; Perkins and Simmons, 1988; Polaki, 2002; Pratt, 2000; Shaughnessy, 1977; Stavy and Tirosh, 1996a; 1996b; Watson and Moritz, 2002).

Kahneman and Tversky (1972) are among the first few researchers who investigated the misconceptions related to probability. They found that individuals used a general approach when deciding on the approximate result of an event. Moreover, they argued that individuals with insufficient probability knowledge used intuitive strategies when determining the probability of complex events and this might lead to misconceptions. For example, when pupils were asked to find out the probability of their presence in a car accident, they stated that the implications of such events would affect their estimations. In other words, individuals believed that the probability of such an accident would increase if the number of accidents they remembered was large. The misconceptions formed after such a process are generally resistant to change (Fast, 2001; Konold, 1988). Therefore, individuals show a great unwillingness to abandon their misconceptions, despite the evidence contradicting their existing knowledge (Brown and Clement, 1987).

Literature Framework Regarding Probability (Cross-Age Comparison)

Since prehistoric times people have faced random physical events, e.g. unpredictable natural events and games of chance, but the birth of probability theory and its turning into a branch of mathematics did not occur until the middle of 17th century. Although probability is a very common concept in games of chance, it has been used in many areas, such as science, genetics, industry, management, economics, banking and insurance, kinetic theory of gases, statistical mechanics, and quantum mechanics. However, probability as a subject started to appear in school curricula after 19th century and, since then, cognitive psychologists and mathematics educators examined pupils' misconceptions and misunderstandings about probability in different age groups (Çelik and Güneş, 2007; Dooren et al., 2003; Fast, 2001; Fischbein and Schnarch, 1997; Fischbein et al., 1991; Gal-Ezer and Zur, 2004; Kahneman and Tversky, 1972; Konold et al., 1993; Lecoutre, 1992; Perkins and Simmons, 1988; Pratt, 2000; Stavy and Tirosh, 1996a; 1996b). For example, Fischbein and Schnarch (1997), who explored the changes in misconceptions of 5th, 6th, 7th, 9th, and 11th graders and college students who had not previously attended any probability course, reported that an increase in the students' level of education decreased or increased or did not change their misconceptions about some concepts. In parallel to this research, Fischbein et al. (1991) administered two different tests, consisting of 7 parallel and open-ended questions, to pupils having been and having not been taught about probability. As a result of their analysis, researchers generally found out that the more the pupils' learning level increases, the more the percentage of correct answers increases. But in case of two questions in every test (2, 2) they found out that as pupils' learning level increased, their correct answers' percentage decreased. When conceptual mistakes across groups were compared, in some questions it was found out that as learning level increased, conceptual mistakes also increased but in some questions they decreased. Similarly, Dooren et al. (2003), who compared misconceptions in 10th and 12th graders, implied that there was no significant difference between the groups despite an increase in the level of education decreasing their misconceptions. Stavy and Tirosh, (1996a, 1996b) examined common thinking patterns underlying misconceptions in different content areas. They found that many misconceptions in mathematics and science stemmed from a small number of intuitive rules. They identified three different intuitive rules: 'more of A, more of B', 'everything can be divided by two' and 'the same A, the same B'. Pupils consider these rules to be self-evident, and apply them with great confidence. By creating logical environmental relations, they appear to enable pupils to understand a given situation to a certain extent without having any real understanding of the concept. Watson and Kelly (2004), who used a test based on a spinner divided into two identical parts (50-50) to determine 3rd, 5th, 7th and 9th graders' understanding of statistical variation in a chance setting, identified that there was a steady increase in conceptual development in the whole process in 3rd, 5th, and 9th grades, except for 7th graders. Çelik and Güneş (2007) conducted a study to investigate the conceptions and misconceptions of 7th, 8th and 9th grade pupils related to the concept of probability. As a result of this study, it was determined that misconceptions on "representativeness" and "negative and positive recency effect" decreased by grade level, while misconceptions on "simple and compound events", "conjunction fallacy" and "the effect of sample size" remained stable across all levels among most of the pupils.

Several studies were conducted in order to determine how the meanings given by pupils to probability concepts changed with age or to reveal the effect of using different strategies in teaching statistics on pupils' thinking levels (Batanero and Serrano, 1999; Fast, 1997; Fischbein and Gazit, 1984; Jones et al., 1997; Pratt, 2000; Polaki, 2002; Sharma, 2006; Watson and Moritz, 2002; Weir, 1962). For example, Jones et al. (1997) developed a framework to assess pupils' considerations of some probability concepts. The framework, developed after different applications, was used in the evaluation of primary school pupils' considerations about some concepts of probability. In another work including 8 groups with two persons aged 10 or 11 in each, some computer-assisted experiments were conducted with materials prepared using a spinner, money, and dice, Pratt (2000) revealed a systematic bias about probability in pupils' daily lives. Then, since his pupils were exposed to numerous experiments by means of this material and different questions, he enabled them to overcome their systematic bias. Watson and Moritz (2002) conducted a study to investigate the development of pupils in answering questions regarding the probability of a single event, compound events and conditional events. For this aim, a total of 2615 pupils (aged 5-11) were asked 4 questions and their answers were evaluated in the context of these factors; *a) types of responses, b) cross-sectional comparisons, c) longitudinal change, d) cross-item comparisons and e) comparisons to other chance items*. As a result of the evaluations, when the ratio of groups' correct answers to the questions related to conditional probability was compared, it was found that the percentage of correct answers increased with the level of education. However, no correlation was found between the level of education and the ratio of correct answers to the questions related to the probability of complex events. Fischbein and Gazit (1984) carried out a study to investigate how teaching about probability affects probabilistic intuitions. Children in grades 5-7 (aged 10-13) in Israel were given 12 lessons covering certain concepts (e.g. certain, possible…) and their probabilities. It was considered that, while most of the ideas were too difficult for the grade 5 pupils, about 60-70% of the grade 6 pupils and 80-90% of the grade 7 pupils were able to understand and apply most of the concepts. Batanero and Serrano (1999) conducted a study with 277 pupils aged between 14 and 17 in Spain in order to investigate how the meaning given to the concept of "randomness" by the pupils changed with age. In this study, 8 test items (four referred to random sequences, and the remaining four referred to random two-dimensional distributions), developed by Green, were used. As a result of the study, it was revealed that age was not important in understanding the concept of "randomness"; it is a hard concept to understand and, in order to master it well, it is essential to understand many other probability concepts, such as sample space, probability of an event, probability comparisons and so on. Weir (1962) made a study to reveal how three different instructional practices based on the same material affected pupils' learning in age groups 5-7 and 9-13. For this aim, 96 pupils at different ages were divided into 6 groups of 16 pupils and these groups were exposed to different interventions. Three different instructional practices were applied to them, such as: *"without intervening to the group members", "by telling the group members that there is a solution for the question" and "by telling the group members that the question has no solution"*. As a result of the analysis, it was determined that: a) younger pupils preferred tips and encouragement more than older pupils, b) older pupils changed their initial answers after receiving tips more than younger pupils, c) different instructional practices had no effect on pupils' selection of situations or choices on which hints were given, d) it was determined that it was harder in older pupils to overcome prejudices. So, it was concluded that the pupils' prejudices or prior knowledge played an important role in their decisions about chance or probability concepts.

The reviewed literature showed that pupils' conceptions, errors and misconceptions varied depending on their age, level of education, language and culture. Moreover, the studies into the misconceptions of Turkish pupils on the subject of probability dependent on their age and level of education were found to be scarce. In this context, it is considered important to determine the conceptions, errors and misconceptions of Turkish pupils regarding probability concepts. Therefore, this study aims at comparing and evaluating misconceptions of pupils at various levels of education (5, 6, 7, and 8) and ages (11-14) about some probability concepts (*compound events, probability of an event, probability comparisons*).

Methods

Study Design

To determine pupils' conceptions related to their grade and understanding, cross-age and longitudinal studies are generally used. Despite the fact that the cross-age research involves different cohorts of pupils, it is more applicable than the longitudinal study when time is limited (Abraham, Williamson and Westbrook, 1994). These studies can be conducted within a relatively shorter time period and are usually conducted to provide comprehensive and deeper knowledge about the observed groups. Also, cross-age studies do provide an opportunity to observe shifts in concept development as a consequence of pupils' maturity, increase in their intellectual development, and further learning.

Sample

This study was conducted during the 2009 – 2010 school year, with a total of 540 pupils (aged 11-14) attending a primary school in the southeastern region of Turkey. The pupils participating in the study generally came from low or middle level socio-economic classes. The school of study is located in the province center and its physical infrastructure is sufficient. Table 1 shows the grades, ages and class sizes of pupils in the study group.

Table 1. *Distribution of pupils across groups according to their grade and age*

Grade	5th Grade	6th Grade	7th Grade	8th Grade
Age	11	12	13	14
Class Size (n)	131	147	134	128
(%)	24.3	27.2	24.8	23.7

Context of Education

In Turkey, education is structured into: preschool education (aged 3–6), basic education (primary and middle schools, aged 6–14), which is compulsory, secondary education (lycees or senior high schools, aged 14–17), and higher education (colleges and universities). All schools throughout the country must use the same curricula, developed and implemented by the National Ministry of Education. In addition, mathematics instruction has experienced a rapid change in the last few years embracing more pupil-centered teaching as a result of the constructivist approach.

Mathematics is a compulsory subject in Turkey and occupies a lot of space in primary and secondary education curricula. Its instruction is carried out by class teachers until the second phase (6-8th grades) of primary education. In this respect, pupils' mathematical competence in the first phase (1-5th grade) of compulsory education can be conjectured to be associated with class teachers' competence in mathematics. Thus, this fact should be considered in group comparisons. In the past, the concept of probability was covered only in 8th and 10th grade mathematics curricula. However, in 2005, the new curricula took effect and this issue was distributed to 4th, 5th, 6th, 7th and 8th grades at the primary level along with grade 10 in the secondary education. This implied that probability would be offered in almost every grade starting from the first phase of primary education (MEB, 2005). All of the pupil groups in the sample were exposed to formal teaching about probability. However, the amount of their exposure varied. When the 2005 primary education curriculum was examined, it was found that it involved 3 class hours (3*40 minutes) in 5th grade and 5 class hours (5*40 minutes) in 6th, 7th and 8th grades. According to this curriculum, the 5th grade objectives of learning about probability were that a pupil: "*makes estimations about the probabilities of events*", "*does experiments about the probability of a simple event and interprets the result*" and "*interprets whether an event is fair or not*". The learning objectives regarding probability in 6th grade were that a pupil: "*explains the terms, experiment, outcome, sample space, event, random selection and equally probable relation to a case*", "*explains a simple event and its probability*", "*solves the problems related to the probability of an event and interprets the result*", "*explains the possible range of the probability of an event*" and "*explains certain and impossible events*". The 7th grade learning objectives in terms of probability were that a pupil: "*determines the experiment, sample space and probability of discrete and indiscrete events*", "*explains discrete and non-discrete events*", "*calculates the probabilities of discrete and non-discrete events*" and "*calculates the probability of an event using geometry knowledge*". The learning objectives regarding probability in 8h grade were that a pupil: "*explains dependent and independent events*", "*calculates the probabilities of dependent and independent events*" and "*explains experimental, theoretical and subjective probabilities*".

The researchers observed the lessons of the intervention class in which the test was applied for one or two hours (1*40 or 2*40). It was noticed that, although slight differences existed in all groups, the instructions were generally conducted as teacher-centered and examination-oriented. The subjects were lectured according to the textbook and the teacher wrote necessary explanations on the board. Moreover, some problems such as "assume that we are rolling a dice..." or "assume that in a basket we have 4 red..." were first devised and then solved. The experiments were generally done and the results were found by imagination. In this process, the teachers answered the pupils' questions about the subject. In sum, approximately 70%-75% of the lesson included nothing but the teacher's talk. After lecturing, the teacher asked the pupils to answer the questions at the end of the unit.

Data Collection Instruments

An instrument comprising 6 items was prepared by making use of literature (Baker and Chick, 2007; Fischbein et al., 1991; Jones et al., 1997; Polaki, 2002). All of the items in this test are related to the concept of sample space which forms the basis of probability. Furthermore, the first three questions (Q1, Q2, Q3) are related to the concepts of probability of an event and probability comparisons, while the remaining three questions (Q4, Q5, Q6) are related to the concepts of compound events I and II. Three of the questions were open-ended, but the other ones (three) were two-leveled including multiple-choice and open-ended sections. In each question, the pupils were required to write their justifications to further investigate the factors which shaped their probability knowledge. Furthermore, a group of mathematics educators and mathematicians checked the test for validity and reliability, and then confirmed the content validity of the instrument. A pilot study was conducted with 40 pupils at different levels of education. The pilot study administration took about 25 minutes. The pilot study revealed that the questions on probability were understandable and clear for every level. Two mathematics educators independently graded the questions in this study to provide their scoring reliability. The Cronbach alpha coefficients calculated for each question were found to range between 0.78 and 0.93.

Procedure

The issue of probability was first taught in all groups and then the assessment instrument was implemented. However, the groups were not restricted with any time limit. The order of the questions was randomized so as to balance the effect of order.

Data Analysis

In data analysis, we were inspired by the study of Fischbein et al. (1991). Firstly, the pupils' answers to the questions were examined. They were categorized as "no answer", "correct answer" and "incorrect answer" (the sum of misconception types) and then assessed. When calculating the percentage of misconceptions in answers, their ratio to incorrect answers was taken into account. The correct answers were categorized as shown in Table 2. In this paper, the following symbols were used for indicating the grade to which the quoted subjects belonged - G5 means *Grade 5* and followed by 6, 7 or 8 indicates the respective grade. The percentages of correct answers falling into these categories were calculated as their ratio to all correct answers. Then, the data obtained for each question were presented in a percentage and frequency distribution table according to grades. Categorical variables were compared with the chi- square test.

Instead of repeating the misconceptions reported in literature, this paper deals only with the misconceptions revealed in the context of each question in this study and examines them within two main misconceptions regarding each question. Such an approach allowed the researchers to analyze pupils' answers without prejudices. Furthermore, since this study's aim was to investigate the change in misconceptions of pupils regarding probability instead of verifying or contradicting the previous findings on misconceptions, this kind of approach was found more appropriate.

The terms "correct" and "incorrect" are sometimes used in the text. We are aware that such a distinction is not an absolute one. Pupils possess representations and interpretations which have to be respected in their own right, which have their reasons, and may be adequate in certain situations. "Correct" means simply what is usually accepted in a standard probability textbook. On the other hand, one has to take into account that, sometimes, children give apparently "correct" answers for wrong reasons. In this study, some sample answers were included regarding each question. The paper copes with this difficulty by presenting the main types of pupils' justifications. Besides, if a pupil's answers fell out of these categories and were found interesting, they were also included to increase the readability of the paper.

Table 2. *Questions, categories and sample answers related to each category*

Question	Category	Sample Answers
Q1	CACJ	• The area covered by red color is equal in both spinners (G7; G8) • The probability in both spinners is 6/12 (G5; G6; G7; G8) • There are 12 sections in spinners A and B and 6 of them are red (G5; G6; G7)
	CANJ	○ Both spinners are equal (G5; G6; G7; G8)
	CAIJ	❖ Since both are turned at the same time, their probabilities to stop at the same color are equal (G5; G6; G7) ❖ The probabilities are equal because we do not know when the spinners will stop (G5; G6; G7) ❖ *P*(a)=6/12, *P*(b)=6/12 and *P*(c)=12/24 (G7; G8)
Q2	CACJ	• It does not matter whether coin or dice is used. If we choose a coin, the probability is ½ (G5; G6; G7; G8).
	CANJ	○ There is no difference between coin and dice (G5;G6;G7;G8)
	CAIJ	❖ Since this is a chance game, it does not matter whether it is with dice or coin (G5;G6) ❖ There is no difference between coin and dice. I have to be in my lucky day to win (G5;G7;G8) ❖ It does not depend on me. I know I lose no matter what I choose (G8)
Q3	CACJ	• The probabilities of all balls to be selected changed. Because sample space had changed (G7;G8) • The probability of red and blue to be selected decreased. However, the probability of green to be selected increased. Therefore, the probability of all balls to be selected changed (G5;G6;G7;G8)
	CANJ	○ The probabilities of all balls to be selected changed (G5;G6;G7;G8)
	CAIJ	❖ All changed, because everyone chose her/his favorite color (G7) ❖ The probability of red and blue decreased but that of green increased. Since red and blue are on the top, they have been chosen first (G6;G7)
Q4	CACJ	• The probability of selecting different numbers is bigger. Because the probability of selected numbers being the same is 3/9 whereas the probability of them being different is 6/9 (G5;G6;G7;G8) • The probability of selecting different numbers is bigger. Because although there are 3 pairs with the same numbers, there are 6 pairs with different numbers (G7;G8)
	CANJ	○ The probability of obtaining different numbers is bigger (G5;G6;G7;G8)
	CAIJ	❖ The probability of selecting different numbers is bigger. Because the probability of selected numbers being the same is 1/6 whereas the probability of them being different is 5/6 (G6) ❖ The probability of selecting different numbers is bigger. Because the starting points of spinners 1 and 2 are different (G6;G7;G8)
Q5	CACJ	• Choice a, because where *P*(5,6)=2/36, *P*(6,6)=1/36 (G6;G7;G8) • Choice a, because let us consider that we are tossing two coins instead of two dice, the probability of TH is higher than the probability of TT or HH. Where *P*(TT or HH)=1/4, *P*(TH)=2/4 (G5;G6)
	CANJ	○ Getting the pair 5-6 (G5;G6;G7;G8)
	CAIJ	❖ The probability of both dice coming 6 is low. Choice a is easier (G6;G7;G8) ❖ Choice a, because *P*(5,6)=(5+6)/12=11/12 (G7) ❖ Since the same person rolls the dice, they hit each other and so result will be as in choice a (G6)
Q6	CACJ	• Meryem is the favorite. Because *P*(a)=2/36 but *P*(b)=5/36 (G5;G6;G7;G8) • Meryem is the favorite. Because although there are 5 pairs with a sum of 5, there are 2 pairs with a sum of 2 (G5;G6;G7)
	CANJ	○ Meryem is the favorite (G5;G6;G7;G8)
	CAIJ	❖ Meryem is the favorite. Because the sum of two odd numbers is always even. For example, 9+7=16, 7+5=12, 5+3=8 (G5;G6) ❖ Meryem is the favorite. Because {1 5 2 4}, the probability of six is 4/6, {1 2}, the probability of three is 2/6 (G7)

CACJ = Correct answer, correct justification, CANJ = Correct answer, no justification, CAIJ = Correct answer, incorrect justification, G = Grade

Results an discussion

Findings Related to the Concepts of Probability of an Event and Probability Comparisons

The three questions (Q1, Q2, Q3) in the instrument are related to the concepts of probability of an event and probability comparisons. As seen in Table 3, there is no significant difference among groups in terms of the ratio of pupils' misconceptions in Q1 [χ^2 (3) = 4.871, p = .181]. Thus, it can be suggested that the ratio of pupils' misconceptions regarding Q1 did not vary according to the groups' level of education (36.6%, 32.7%, 29.1%, and 41.4% respectively). When their types of misconceptions here were compared, it could be seen that Type II misconception was more frequent than Type I misconception. Table 3 shows that, except for 8th grade, as the level of education increased, the number of correct answers increased slightly. However, there is no significant difference among the groups' correct answers to Q1 [χ^2 (3) = 4.33, p = .227]. According to Table 3, the CACJ ratio of the groups (28.3%, 40.7%, 56.4%, and 54.2% respectively) showed an increase in terms of the level of education, whereas the CANJ ratios (67.2%, 39.5%, 32.1%, and 30.5% respectively) decreased in terms of the level of education. The CAIJ ratios of the groups (4.5%, 19.8%, 11.5%, and 15.3% respectively) show an increase in terms of the level of education.

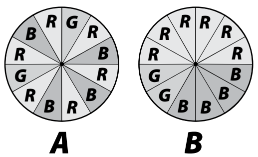

Figure 1a

On the spinners, R represents red; B represents blue and G represents green

Question 1 (Q1). A and B are two spinners in Figure 1a. *When these spinners are turned at the same time, which one is more likely to stop at red? Why?*

a) Spinner A

b) Spinner B

c) Both

Types of misconceptions for Q1
Type I: Because colors are mixed in spinner A or there are more red sections.
Type II: Because colors are together in spinner B or there are more red sections.

Figure 1. *Q1 and types of misconceptions for Q1*

Table 3. *Frequency and percentage-based distribution of answers to Q1*

Category	5th Grade (n=131)		6th Grade (n=147)		7th Grade (n =134)		8th Grade (n=128)	
	f	%	f	%	f	%	f	%
Correct Answer	67	51.2	81	55.1	78	58.2	59	46.1
CACJ	19	28.3	33	40.7	44	56.4	32	54.2
CANJ	45	67.2	32	39.5	25	32.1	18	30.5
CAIJ	3	4.5	16	19.8	9	11.5	9	15.3
Incorrect Answer	48	36.6	48	32.7	39	29.1	53	41.4
Type I [a]	18	37.5	15	31.3	5	12.8	16	30.2
Type II [b]	23	47.9	28	58.3	30	76.9	32	60.4
Other	7	14.6	5	10.4	4	10.3	5	9.4
No Answer	16	12.2	18	12.3	17	12.7	16	12.5

Note: CACJ = Correct answer, correct justification; CANJ=Correct answer, no justification, CAIJ=Correct answers, incorrect justification, [a, b]: Types of misconceptions for Q1

It was determined in this study that the pupils attempted to solve Q1 by using their intuition and informal calculation strategies. It should be noted that in several similar studies (Batanero and Serrano; 1999; Fischbein, Gal-Ezer and Zur, 2004; Gürbüz, 2007; Nello and Marino, 1991; Stavy and Tirosh, 1996a; 1996b) such kinds of strategies were used. On the other hand, in the solution of Q1 many pupils realized the importance of size but, focused on one size aspect, they gave a wrong answer to the justifications in *Type I* or *Type II* (Table 2). Some pupils answered, for instance, "*Choice b, because spinners were turned at the same time*" (G5; G6; G7), *"Choice b, because in spinner B, red color covers half of the shape"* (G5; G7), *"Choice b, because red, blue and green sections are together"* (G6; G7), *"Choice a, because in spinner A there is red everywhere"* (G5; G7). On the other hand, they gave answers to Q1 such as *"we can say nothing unless knowing who turns the spinners"* (G5; G8), *"my favorite color is red, so red"* (G5; G6)*", "it would be better if we divided these spinners into two or four equal parts"* (G7; G8), *"since we cannot know how many seconds or minutes the spinners will turn, we cannot know where they will stop"* (G6; G7), *"Choice b, because the blue color is on the lower and the red color is on the upper side"* (G8).

When Table 3 is examined, it can be seen that the ratio of wrong answers to Q1 showed no significant difference regarding the level of education and age. According to this finding, it is not possible to comment the positive or negative effect of age or the level of education on remedying pupils' misconceptions. However, aligning with the results of the study by Watson and Moritz (2002), it can be argued that the pupils gave more elaborate justifications to their answers as their age or educational level increased. This is thought to be associated with language development. Indeed, Ford and Kuhs (1991), Gibbs and Orton (1994), Kazıma (2006) and Tatsis et al. (2008) highlighted the importance of language development in understanding probability concepts.

Question 2 (Q2). Two players are playing a game. The first player tosses a coin and wins a point every time it turns up heads. The second player rolls a six-sided dice (123 456) and wins a point every time an even number (2, 4, or 6) comes up. If you were one of the two players, would you choose a coin or a dice? How did you decide? Please explain.

Types of misconceptions for Q2
Type I: I would choose a coin.
Type II: I would choose a dice.

Figure 2. *Q 2 and types of misconceptions for Q2*

Table 4. *Frequency and percentage-based distribution of answers to Q2*

Category	5th Grade (n=131)		6th Grade (n=147)		7th Grade (n=134)		8th Grade (n=128)	
	f	%	*f*	%	*f*	%	*f*	%
Correct Answers	23	17.6	27	18.4	36	26.9	25	19.5
CACJ	6	26.1	8	29.6	12	33.3	11	44.0
CANJ	8	34.8	9	33.3	13	36.1	5	20.0
CAIJ	9	39.1	10	37.1	11	30.6	9	36.0
Incorrect Answers	58	44.3	83	56.5	59	44.0	72	56.3
Type I [a]	15	25.9	33	39.8	24	40.7	39	54.2
Type II [b]	43	74.1	50	60.2	35	59.3	33	45.8
No Answer	50	38.1	37	25.1	39	29.1	31	24.2

CACJ=Correct answer, correct justification; CANJ=Correct answer, no justification,
CAIJ=Correct answer, incorrect justification, [a, b]: Types of misconceptions for Q2

As seen in Table 4, there is also no significant difference among groups in terms of the ratio of misconceptions in Q2 [χ^2 (3) = 8.053, p = .045]. According to this, 6th and 8th graders had more misconceptions in Q2 than 5th and 7th graders. When the types of misconceptions across groups were compared, it could be seen that *Type II* misconception was more frequent than *Type I* misconception. The correct answers of the groups (17.6%, 18.4%, 26.9%, and 19.5% respectively) show an increase in terms of the level of education as in the Fischbein and Gazit study results (1984). Moreover, when we examine Table 4, the ratio of correct answers with correct justifications shows an increase in the level of education (26.1%, 29.6%, 33.3%, and 44.0% respectively), unlike the ratio of correct answers with no justification (34.8%, 33.3%, 36.1%, and 20.0% respectively) and the ratio of correct answers with wrong justifications (39.1%, 37.1%, 30.6%, and 36.0% respectively). On the other hand, the ratio of blank answers to Q2 decreased substantially (38.1%, 25.1%, 29.1%, 24.2% respectively).

It was found that the pupils who had misconceptions regarding Q2 had different justifications for their wrong answers. They gave wrong answers either because they did not understand the problem or because they misunderstood it (they thought they should prefer either coin or dice) as found in the Fast study (1997). For example, "*I would choose coin because it is thrown*" (G5), "*I would choose coin because the probability of heads is higher in coin*" (G5; G7), "*I would choose dice because there is a number on every face of dice*" (G5), "*I would choose dice because I have 3 alternatives in dice whereas only 1 in coin*" (G6; G7; G8), "*I would choose dice because the number of even numbers in dice is bigger than the number of heads in coin*" (G6; G8), "*I would choose dice because a fair dice will always win*" (G8). Moreover, since some pupils related Q2 to ability or since they had deficiencies in the sample space concept, they gave answers based on these misconceptions. They answered showing misconceptions such as, "*I would choose coin because I can obtain heads in a shorter time*" (G5; G8), "*I would choose coin because it has 2 sides and the probability of heads is 1/2. However, dice has six sides and the probability of heads is 1/6*" (G6; G8), "*I would choose coin because the probability of* *heads is 50% whereas the probability of even outcome in dice is nearly 33%* (G6; G7; G8)", "*I would choose coin because I could cheat with it and always obtain heads but I could not do the same with dice*" (G6; G8), "*I would choose dice because the probability of even outcome is 80%* (G8)", "*it is like dice but I would make trials to be sure. I would choose the one occurring most frequently*" (G8).

It was found that the correct answers to Q3 with wrong justifications were ascribed mostly to chance and prior experiences. A number of researchers using the same type of question (Fischbein et al., 1991; Lecoutre, 1992, Pratt, 1998; Amir and Williams, 1999; Batanero and Serrano, 1999; Baker and Chick, 2007; Nilsson, 2007) reported similar conclusions in their studies.

As can be seen in Table 4, the CACJ ratio increase considering Q2 depended on the level of education. This finding is in line with the Polaki study results (2002). Here, we can say that the pupils in different learning settings gave similar answers because they had experienced similar mental processes. In this study, it was determined that the answers to Q2 involved some misconceptions because some pupils either could not understand the question or misunderstood it. It can be argued that misunderstanding the question is related to language development. Indeed, Ford and Kuhs (1991), Gibbs and Orton (1994), Kazıma (2006) and Tatsis et al. (2008) highlighted the importance of language development in understanding probability concepts. Moreover, some of the pupils' answers to this question involved misconceptions because either they could not define the sample space correctly or had misconceptions about fractions. This finding is in line with those reported by Carpenter et al. (1981), Jones et al. (1997), Ritson (1998), Polaki (2002) and Chernoff (2009). Furthermore, pupils generally preferred to use percentages (%) when solving these kinds of problems probably because they also prefer using percentages in daily life in some similar situations.

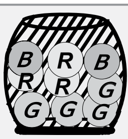

Figure 3a

On the balls,
R represents red;
B represents blue
and G represents
green respectively.

Question 3 (Q3). A red and a blue ball were picked from a basket (Fig. 3a) without being put back. Which color changed the probability of being selected? Why?

Types of misconceptions for Q3
Type I: Green, because the probability of selecting this color has increased.
Type II: Red/blue (red, blue, red and blue) because the probability of selecting these balls has decreased.

Figure 3. *Q 3 and types of misconceptions for Q3*

Table 5. *Frequency and percentage-based distribution of answers to Q3*

Category	5th Grade(n=131)		6th Grade(n=147)		7th Grade(n=134)		8th Grade (n=128)	
	f	%	*f*	%	*f*	%	*f*	%
Correct Answers	6	4.8	7	4.8	8	6.0	10	7.8
CACJ	2	33.3	2	28.6	3	37.5	4	40.0
CANJ	3	50.0	3	42.8	3	37.5	3	30.0
CAIJ	1	16.7	2	28.6	2	25.0	3	30.0
Incorrect Answers	71	54.2	108	73.7	94	70.1	94	73.4
Type I a	17	23.9	32	29.6	25	26.6	27	28.7
Type II b	45	63.4	64	59.3	62	65.9	57	60.6
Other	9	12.7	12	11.1	7	7.5	10	10.6
No Answer	54	41.2	32	21.8	32	23.9	24	18.8

CACJ: Correct answer, correct justification; CANJ: Correct answer, no justification, CAIJ: Correct answer, incorrect justification, a, b: Types of misconceptions for Q3

It can be understood from Table 5 that the misconception ratios of groups (54.2%, 73.7%, 70.1%, and 73.4% respectively) for Q3 show a significant difference in the level of education [χ^2 (3) = 15.502, p = .001]. When the types of misconceptions (Q3) across groups are compared, it can be seen that *Type II* misconception is more frequent than *Type I* misconception. The correct answer ratios of the groups (4.8%, 4.8%, 6.0%, and 7.8% respectively) were found to be quite low and they showed some slight increase in the level of education. On the other hand, the ratios of CANJs (50%, 42.8%, 37.5%, and 30% respectively) decreased regarding the level of education. Moreover, the ratio of CAIJs (16.7%, 28.6%, 25%, and 30% respectively) showed an increase related to the level of education whereas the ratio of blank answers (41.2%, 21.8%, 23.9%, and 18.8% respectively) showed a decrease in the level of education.

It was found that the pupils who had misconceptions regarding Q3 had different justifications for their wrong answers. It was determined that these pupils had misconceptions in their answers because they related it to the general chance factor and the favorite color concept, or because they focused on the location of the balls in the basket or their number in the basket. For example, "*we can say nothing without knowing the favorite color of the person who picks the balls*" (G5), "*red and blue because females prefer red and males prefer blue*" (G6; G7), "*green because the green balls at the bottom will go up when the red and blue balls are picked*" (G5; G6; G8), "*blue and green because they are at the bottom of the basket*" (G7), "*red because when we put our hand in the basket it touches directly the red ball*" (G6; G8), " *only red and blue because these balls are at the top* (G5; G7)", "*green, because the green ball will stay in the basket the longest*"(G5), "*red and blue because they decreased and their places changed*" (G6; G8), "*blue because there are the smallest number of blue ones in the basket*" (G7). Such approaches of the pupils who gave wrong answers are in line with the approaches in the studies of Gürbüz (2010), Gürbüz, Çatlıoğlu, Birgin and Erdem (2010), and Jones et al. (1997). Furthermore, the few pupils who made wrong connections with the situation dependent on this question gave answers involving their misconceptions. For example, "*green, because these are dependent events*" (G8) and "*red and blue because there is a dependent event here*" (G7). Moreover, a few pupils realized that they should make connections with the sample space. However, since they made deficient connections, they gave answers involving these misconceptions. For instance, "*red and blue, because the picked balls were not put into the basket again so the sample space reduced*" (G7), "*green because the probability of selecting the green ball increased due to reducing sample space*" (G7).

A few pupils who gave answers involving misconceptions either did not understand the question or misunderstood it, which is similar to the results of the study by Fast (1997). They generally found the probability of selecting each ball instead of what was really asked in the question. For example, "*P(b)=3/9, P(g)=4/9 and P(r)=2/9*" (G5; G7), "*P(g)=4/9 P(r)=1/7 and P(b)=2/7*" (G6; G8), "*P(g)=4/9, P(r)=1/9 and P(b)=2/9*" (G6; G7). Here the pupils made justifications parallel to the results of the studies by Gürbüz (2006), Gürbüz et al. (2010) and Jones et al. (1997) and related their answers to the favorite color or the location of the balls in the basket.

Q3 seems easy, but the pupils had difficulty solving it when compared to other questions. The pupils who had misconceptions in their answers to this question were found to make calculations ignoring the importance of sample space. This finding is in line with those reported by Carpenter et al. (1981), Jones et al. (1997), Ritson (1998), Polaki (2002) and Chernoff (2009). Furthermore, the answers to Q3 show that they could not understand this question well and focused on the location of the balls. This finding is in line with those reported by Jones et al. (1997) and Gürbüz (2007).

Findings and Discussion Regarding the Concepts of Compound Event I and II

Q4 (Question 4) and Q5 (Question 5) in this section are related to the concepts of "Compound Event I", while Q6 (Question 6) is related to the concept of "Compound Event II".

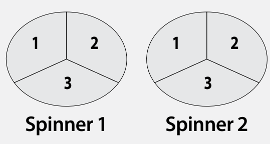

Figure 4a

Question 4 (Q4). When spinners (Figure 4a) are turned together, do you think the probability of getting the same numbers or different numbers is higher. Why?

Types of misconceptions for Q4
Type I: The probability of obtaining the same numbers is the same as obtaining different numbers.
Type II: The probability of selecting different numbers is higher.

Figure 4. *Q4 and types of misconceptions for Q4*

Table 6. *Frequency and percentage-based distribution of answers to Q4*

Category	5th Grade (n=131)		6th Grade (n=147)		7th Grade (n=134)		8th Grade (n=128)	
	f	%	*f*	%	*f*	%	*f*	%
Correct answers	32	24.4	48	32.7	48	35.8	53	41.4
CACJ	2	6.2	3	6.3	8	16.6	8	15.1
CANJ	18	56.3	25	52.1	21	43.8	19	35.9
CAIJ	12	37.5	20	41.6	19	39.6	26	49.0
Incorrect answers	43	32.8	45	30.5	33	24.6	40	31.3
Type I [a]	13	35.6	16	35.6	13	39.4	15	37.5
Type II [b]	23	53.3	24	53.3	17	51.5	21	52.5
Other	7	11.1	5	11.1	3	9.1	4	10.0
No Answer	56	42.8	54	36.7	53	39.6	35	27.3

CACJ=Correct answer, correct justification; CANJ=Correct answer, no justification, CAIJ=Correct answer, incorrect justification, [a, b]: Types of misconceptions for Q4

According to Table 6, the misconception ratios of the groups (32.8%, 30.5%, 24.6%, and 31.3% respectively) showed no significant difference in the level of education [χ^2 (3) = 2.461, p = .482]. When the types of misconceptions across groups were compared, it could be here seen that *Type II* misconception was more frequent than *Type I* misconception. The correct answers of the groups (24.4%, 32.7%, 35.8%, and 41.4% respectively) showed an increase in the level of education as in the results of the study by Fischbein and Gazit (1984). Similarly, the CACJ ratio of the groups (6.2%, 6.3%, 16.6%, and 15.1% respectively) slightly increased in terms of the level of education. Moreover, the ratio of CANJs (56.3%, 52.1%, 43.8%, and 35.9% respectively) showed a decrease in terms of the level of education, whereas the ratio of CAIJs (37.5%, 41.6%, 39.6%, and 49.0% respectively) showed an increase in terms of the level of education. In Q4, the highest number of correct answers were given by 5th grade (42.8%) and 7th grade (39.6%) pupils and the lowest number of correct answers by 8th grade (27.3%) pupils.

When the answers were examined, it was found that the pupils who had misconceptions regarding Q4 had different justifications for their wrong answers. Some of them were influenced by the general probability chance factor, and others by the perception that the probability of events would be equal. For example, "*I cannot comment on it because it depends on chance*" (G5; G6), "*The probabilities are equal. Because the numbers were written once and they are the same numbers*" (G5; G6), "*the probabilities are equal because the spinners are the same*" (G5), "*since all the numbers are the same, the probability of obtaining the same numbers is higher. Because the places of the numbers on the spinners had changed*" (G5; G6; G7; G8), "*the probability of obtaining the same numbers is higher because the spinners are turned at the same time*" (G6; G7), "*the probabilities are equal. Because it is different in (1,1), (2,2) and (3,3), whereas it is different in (1,2), (1,3) and (2,3)*" (G7). As mentioned before, here the pupils think that the probability of an event depends naturally on chance and, therefore, the events have equal probabilities. Fischbein et al. (1991), Lecoutre (1992), Pratt (1998), Amir and

Williams (1999), Batanero and Serrano (1999), Baker and Chick (2007), and Nilsson (2007) reported similar conclusions in their studies.

Similar to the Fast study results (1997), some of the pupils had misconceptions in their answers to Q4 either because they could not understand the question well or they focused on the skills of the persons who turned the spinners. For example, "*since they turn and stop at the same time, the probability of the event (3, 1) is higher*" (G7), "*since there are more odd numbers in spinners, the probability of an odd number is higher*" (G5), "*it is impossible to get the same numbers because the numbers are mixed*" (G5)", "*The probability of obtaining 2 is higher. Because the location of 2 did not change*" (G6; G8), "*I cannot say anything before seeing the person who turns the spinners*" (G5), "*if kids are turning the spinners it depends on chance. But if adults are turning it, it depends on which number they are aiming*" (G6; G8). Here, it can be seen that some pupils think that the outcomes are in control of those who turn the spinners. Amir and Williams (1999), Greer (2001) and Sharma (2006) reported similar conclusions in their studies.

In this study, most pupils gave wrong answers to Q4 because they focused on the location of the numbers on the spinners rather than related the question to the sample space. This finding is in line with those reported by Jones et al. (1997), Gürbüz (2007), and Gürbüz et al. (2010). A few pupils, who were not aware of a relation between probability and sample space, could not develop a systematical technique to produce all the possible outcomes. Thus, they could not find the correct result. For example, some of them thought that (1, 2) and (2, 1) were the same and so produced incorrect solutions. Baker and Chick (2007) and Gürbüz (2010) obtained similar findings with a parallel question. Most of the pupils who gave correct answers to this question either could not justify their reasons or, as can be understood from their answers, gave wrong justifications. The reason why the pupils could not provide correct reasons for their answers may be their lack of knowledge in sample space and fractions. This finding is in line with those reported by Carpenter et al. (1981), Jones et al. (1997), Ritson (1998), Polaki (2002) and Chernoff (2009).

Question 5 (Q5). Suppose one rolls two dice simultaneously. Which of the following has a greater chance of occurrence?
Getting the pair 5-6
Getting the pair 6-6
Both have the same chance

Types of misconceptions for Q5
Type I: There is a greater likelihood of obtaining (6, 6) than (5, 6).
Type II: The probability of obtaining (5, 6) and the probability of obtaining (6, 6) with two dice are the same.

Figure 5. *Q5 and types of misconceptions for Q5*

Table 7. *Frequency and percentage-based distribution of answers to question Q5*

Category	5th Grade (n=131)		6th Grade (n=147)		7th Grade (n=134)		8th Grade (n=128)	
	f	%	*f*	%	*f*	%	*f*	%
Correct Answers	21	16.0	24	16.3	27	20.1	26	20.3
CACJ	2	9.5	5	20.8	2	7.4	5	19.2
CANJ	14	66.7	8	33.3	19	70.4	17	65.3
CAIJ	5	23.8	11	45.9	6	22.2	4	15.3
Incorrect Answers	99	75.6	108	73.5	93	69.4	88	68.8
Type I[a]	32	29.3	20	16.7	17	17.2	12	10.2
Type II[b]	67	63.6	88	77.8	76	79.6	78	83.0
Other	7	7.1	6	5.6	4	4.3	6	6.8
No Answer	11	8.4	15	10.2	14	10.5	14	10.9

CACJ=Correct answer, correct justification; CANJ=Correct answer, no justification, CAIJ=Correct answer, incorrect justification, [a, b]: Types of misconceptions for Q5

According to Table 7, the misconception ratios of the groups (75.6%, 73.5%, 69.4%, and 68.8% respectively) showed a partial decrease in the level of education, but this change was not significant [χ^2 (3) = 2.461, p = 0.553]. Similarly, Dooren et al. (2003) found no correlation between the level of education and misconceptions in their study. When the types of misconceptions across groups are compared, it can be seen that Type II misconception is more frequent than Type I misconception. The correct answer ratios of the groups (16.0%, 16.3%, 20.1%, and 20.3% respectively) were quite low and they showed some slight increase in the level of education. However, no correlation was found between the CACJs (9.5%, 20.8%, 7.4%, and 19.2% respectively), CANJs (66.6%, 33.3%, 70.4%, and 65.3% respectively) and CAIJs (23.8%, 45.9%, 22.2%, and 15.3% respectively) ratios of the groups and the level of education.

It should be noted that the groups' correct answers showed a partial increase in the level of education but these ratios remained quite low. In addition, most of the pupils at all levels of education were found to have misconceptions. The most common misconception in any group was the second type of misconception. Several studies investigated similar misconceptions (Amir and Williams, 1999; Çelik and Güneş, 2007; Fischbein et al., 1971; 1991; Fischbein and Gazit, 1984; Lecoutre and Durand, 1988). For example, Lecoutre and Durand (1988) discovered that most of the participants answered, incorrectly, that the two events had the same probability and that this bias was very resistant (Cited in Fischbein et al., 1991). Lecoutre and Durand (1988) identified a number of "models" used by subjects in order to justify their answers. In case of the answer "the two probabilities are different", they found justifications, such as: "It is more seldom to obtain twice the same result". Fischbein et al. (1991) and Çelik and Güneş (2003) obtained the results similar with the Lecoutre and Durand results (1988). Here, it can be argued that learning about probability, being educated in different learning settings by different teachers, increasing age and enhancing mental development are not effective in remedying the stated misconceptions.

Analyzing the nature of the misconceptions, one can see that almost all of these answers confirm that the two outcomes have the same probability. The idea that the probability of 5-6 is twice that of 6-6 can be reached only by getting some representation of the corresponding sample space. The basic justification offered by the children is that, in both cases, one deals with chance events. Pupils' answers were, *"the probability is the same because one may obtain 6-6 and 6-5 or none of these results"* (G5; G6; G7), *"in my opinion, the probability is the same because the obtained number is a surprise"* (G5; G6; G8), *"the probability is the same because there is the same number of 6 and of 5 in both dice"* (G5; G6; G7), *"the probability is the same, because one cannot determine something which depends only on the motion of a small object like a dice, which is thrown by each person in a different manner"* (G6; G7; G8), "*because the dice are independent P(a) = 1/6.1/6=1/36, P(b) = 1/6.1/6 = 1/36*" (G7; G8), *"because the dice are independent P(6)=1/6, P(5)=1/6, hence the answer is c"* (G7; G8), *"since dice tends to yield bigger numbers, the answer is choice b"* (G5; G7), *"the outcome cannot be predicted because the dice hit each other when rolled"* (G7; G8). These pupils generally found the probability of selecting each ball instead of what was really asked in the question. For example, "*P(b)=3/9, P(g)=4/9 and P(r)=2/9*" (G5; G7), *"P(g)=4/9 P(r)=1/7 and P(b)=2/7"* (G6; G8), *"P(g)=4/9, P(r)=1/9 and P(b)=2/9"* (G6; G7).

When the wrong answers given to Q5 are examined, it can be seen that the general reason for the wrong approach is the pupils' lack of knowledge about the sample space concept and the types of events. Particularly, the two main ideas are then used to justify the equal probabilities of getting 6-6 and 5-6: (a) The more primitive idea that both events are the effect of chance and, therefore, there is no reason to expect one more than the other; and (b) The more sophisticated idea that 5 and 6 are equiprobable and, therefore, every event representing a binary combination of them has the same probability. Moreover, it was observed from the answers that the pupils were aware of the probability of event; yet, due to their wrong approaches, they could not produce correct solutions to determine the sample space and event. These findings generally corroborate those of Lecoutre and Durand (1988), Fischbein et al. (1991) and Çelik and Güneş (2007).

Question 6 (Q6). Musa and Meryem play with a pair of dice. If the sum of the points is 3, Musa is the winner. If the sum of the points is 6, Meryem is the winner. Which of the following answers seems to you to be the correct one? Why? a) Musa is the favorite b) Meryem is the favorite c) Musa and Meryem have the same chance
Types of misconceptions for Q6 Type I: The probability of obtaining sum 3 is higher than the probability of obtaining sum 6. Type II: The probability of obtaining sum 3 and that of obtaining sum 6 are equal.

Figure 6. *Q6 and types of misconceptions for Q6*

Table 8. *Frequency and percentage-based distribution of answers to question Q6*

Category	5th Grade (n=131)		6th Grade (n=147)		7th Grade (n=134)		8th Grade (n=128)	
	f	%	f	%	f	%	f	%
Correct Answers	50	38.2	45	30.6	47	35.1	38	29.7
CACJ	6	12.0	8	17.8	10	21.3	9	23.7
CANJ	32	64.0	10	22.2	21	44.7	11	28.9
CAIJ	12	24.0	27	60.0	16	34.0	18	47.7
Incorrect Answers	72	54.9	92	62.6	75	56.0	74	57.8
Type I[a]	19	22.2	16	15.2	15	16.0	15	17.6
Type II[b]	53	66.7	76	78.3	60	73.3	59	73.0
Other	8	11.1	6	6.5	8	10.7	7	9.4
No Answer	9	6.9	10	6.8	12	8.9	16	12.5

CACJ=Correct answer, correct justification, CANJ=Correct answer, no justification, CAIJ=Correct answer, incorrect justification, [a, b]: Types of misconceptions for Q6

According to Table 8, it can be observed that the misconception ratios across groups showed an increase (54.9%, 62.6%, 56.0%, and 57.8% respectively) in the level of education. However, as found by Dooren et al. (2003), this increase showed no significant difference [χ^2 (3) = 1.993, p = .574]. When the types of misconceptions across groups were compared, it could be seen that Type II misconception was more frequent than Type I misconception. The correct answer ratios of the groups (38.2%, 30.6%, 35.1%, and 29.7% respectively) were found to increase in relation to the level of education. However, the CACJ (12.0%, 17.8%, 21.3%, and 23.7% respectively) and CAIJ ratios of the groups increased (24%, 60%, 34%, and 47.7% respectively) with the level of education. The CANJ ratios of the groups (64%, 22.2%, 44.7%, and 28.9% respectively) showed a decrease in the level of education.

It was found that the pupils who had misconceptions regarding Q6 had different justifications for their wrong answers. For example, some of them used the general probability chance factor; the others used the perception that the probabilities would be the same and a small portion gave illogical justifications. *"The families of Musa and Meryem should be investigated. The one who was born to a luckier family will win"* (G7; G8), *"The one who is in his/her lucky day is the favorite because it depends on chance"* (G7; G8), *"Since 3 is the half of 6, both have the chance to win"* (G5), *"Musa and Meryem have the same chance. Because there is one 3 and one 6 on the dice"* (G6; G7; G8), *"3+6=9:2=4,5 both of their chance to win are equal"* (G6), *"Musa and Meryem have the same chance. Because there are 3 odd, 3 even numbers"* (G6; G8), *"Meryem and Musa have the same chance because one does not know which number one will get"* (G5; G7; G8), *"Meryem and Musa have the same chance because 3 is too low and 6 is too high"* (G5; G6), *"It can be understood at the end of the game. We cannot say anything at the moment"* (G8), *"Musa is my favorite. Because the probability of small outcomes is higher in dice"* (G6; G7). It should be noted that, as the pupils' age and level of education increased, they tended to answer the question by relating it to the chance factor. In other words, they thought that any event naturally depended on chance and, therefore, had an equal probability. Fischbein et al. (1991), Lecoutre (1992), Pratt (1998), Amir and Williams (1999), Batanero and Serrano (1999), Baker and Chick (2007), and Nilsson (2007) reported similar conclusions in their studies.

Some of the pupils having misconceptions gave their answers by using probable outcomes as reported by Amir and Williams (1999). For example, "*Musa is my favorite. Because when the outcomes are (1,2) and (2,1), their sum will be three. On the other hand, in order to obtain six, there is only (3,3)"* (G7), *"Musa and Meryem have the same chance. Because for 3, (1,2),(2,1) and for 6, (1,5),(5,1)"* (G7; G8). Such misconceptional answers can be argued to lack sufficient knowledge about the sample space concept. In parallel, Keren (1984), Fischbein et al. (1991), Polaki (2002), Baker and Chick (2007), Gürbüz (2007; 2010), Nilsson (2007) and Chernoff (2009) showed in their studies that pupils' knowledge about the sample space concept played an important role in their answers to probability-related questions. A few pupils gave misconceptional answers by considering the gender factor as reported by Amir and Williams (1999). For example, *"Musa is the favorite. Because, males are more lucky in this kind of chance games"* (G5; G7; G8), *"Meryem wins because girls are lucky"* (G6; G7). According to Table 8, it can be argued that probability instruction, age and mental development are not effective in remedying misconceptions related to this question.

Some of the correct answers to Q6, based on wrong reasons, are shown in Table 2. In this question (Q6), some pupils describe all the possible pairs but do not understand that each pair of different numbers has to be considered twice, i.e. the order should be considered, too. Similarly, some pupils focused on the magnitude only. For example, *"Meryem has advantage because she has the bigger number"* (G7), *"Meryem has more advantage because with it is more likely to obtain bigger numbers"* (G8), "*Meryem is the favorite because 6 is bigger than 3"* (G7; G8), *"Meryem has advantage because with two dice one obtains almost always numbers bigger than 3"* (G6). There seems to be only one explanation: some of the subjects chose the bigger number as the favorite one, regardless the sample space. In the literature, similar pupil approaches to similar questions were evaluated as rational reflection (Fischbein and Schnarch, 1997; Greer, 2001; Hope and Kelly, 1983; Kahneman and Tversky, 1972; Polaki, 2002; Shaughnessy, 1993). In Q6, the bigger number 6 is really more likely to win and their choice is apparently correct (though, in fact, based on a wrong reason). There are two main types of justifications for choosing the bigger number: a) Some of the subjects simply chose the bigger number because "it is bigger"; b) Others tried to identify the pairs which would yield 6 and 3 but they made serious mistakes in the selection of pairs whose sum yields 3 or 6. As in other questions, the pupils' justifications are thought to be affected by their individual learning, experiences, cultures and beliefs. Amir and Williams (1999), Fischbein et al. (1991), Shaughnessy (1993), Fischbein (1991; 1995) and Sharma (2006) reported similar results in their studies.

Generally, pupils are intuitively aware of a relation between the probability of an event and the sample space. However, they were found to have insufficient knowledge about the sample space in probability problems. Some subjects do not possess a systematic technique for producing all the possible outcomes related to an event. Generally, the size of sample space affects their evaluation in solving probability problems (Greer, 2001; Gürbüz, 2007; 2010; Gürbüz, Çatlıoğlu, Birgin, and Toprak, 2009; Stavy and Tirosh, 1994). Pupils were found to better calculate the probability of an event whose sample space was relatively small. For example, the correct answer percentages in Q1 and Q5 may be compared in order to verify this conjecture. The sample space concept is not a concept that can be developed per se. In order to develop this concept, different experiments should be used and sample spaces in these experiments should be highlighted as determined by Vidakovic et al. (1998), Speiser and Walter (1998) and Gürbüz et al. (2010).

General Discussion

The aim of the paper is to compare and evaluate the probability-related misconceptions of pupils at different levels of education. A test comprising 3 items about the concepts of compound events I and II and 3 items about the probability of an event and probability comparisons, (6 items in total), was developed and applied with the groups. When the blank answers to Q1, Q5, Q6 as well as to Q2, Q3, Q4 are compared, it is remarkable that the ratio of blank answers to Q1, Q5 and Q6 is considerably higher than that of Q2, Q3, Q4. This situation can be supported by the fact that Q1, Q5 and Q6 are more appropriate for the multiple-choice format. Almost all central examinations in Turkey are conducted in the multiple-choice format. Pupils at different levels had somewhat similar understanding of the same probability concepts. Parallel to the results of the work by Dooren et al. (2003), no significant difference was found among the misconceptions of pupils at different levels in some probability concepts. This is rather surprising, since we would expect that pupils' understanding would increase with their education level. Even more surprisingly, in Q3, misconceptions increased parallel to pupils' level of education (Table 5). The general reason lies in the fact that pupils learn about the probability concepts superficially or their learning is very theoretical. In fact, they have been learning about these concepts since 4th grade. It is difficult for individuals to change the knowledge they have obtained in daily life and formal education even if their level of education has increased. Indeed, many studies showed that individuals resisted a change in their existing knowledge and are hardly convinced to change it (Fast, 1997, 2001; Garfield and delMas, 1991; Green, 1982; Hope and Kelly, 1983; Kahneman and Tversky, 1972; Konold, 1988, 1989; Shaugnessy, 1981). Additionally, the method of instruction does not change even if the level of education increases.

The fact that teaching in Turkey is very teacher-dominant and heavily dependent on textbooks serves to encourage rote learning rather than conceptual learning. Pupils are, thus, compelled to memorize contents and regurgitate answers when asked questions in examinations. Therefore, they are probably unable to develop their scientific understanding of key concepts, i.e. probability concepts, or to represent their understanding when required in daily life or in scientific situations. Despite the fact that central entrance examinations concentrate on factual recall rather than conceptual understanding, it is not the only factor which inhibits the understanding of the concepts studied here. Of course, other factors, such as teaching–learning process, pupil's pre-existing knowledge, teachers and mathematics curricula influence their understanding. Teachers emphasize the coverage of contents and teach easy approaches or tricks for solving examination problems in Turkey. Although the new mathematics curricula, based on constructivism and pupil-centered learning, have been already released by the National Ministry of Education, it will take time to change the existing situation and to convince teachers about their effectiveness. In conclusion, we hope that the new attempt will be successful in time.

It is important to use the type of questions mentioned here in the instruction. From our results one may conclude that there are many children who do not detect the identical mathematical structure in practically different situations. What children do not understand is that situations have to be considered as mere embodiments for ideal experiments and that mathematics deals with ideal, abstract operations and entities. To teach a child that probability is a branch of mathematics and, consequently, that it has to do with abstract ideal and formally defined objects - like geometry or arithmetic - is one of the main instructional tasks. Comparing different embodiments of identical structures in mathematics is an effective procedure for reaching that aim, not only formally, but also intuitively. Mathematics has its historical character and mathematical concepts are, in fact, psychologically conditioned. Definitions and interpretations change. Yet, it is generally accepted today that every mathematical theory is based on a system of axioms that are explicitly formulated. Axioms may be replaced with the condition that they will not lead to contradictions (Fischbein et al., 1991). Therefore, mathematics teaching should be conducted on the basis of methods that would present this approach and use similar questions.

The findings of this study showed that some pupils could not understand random events and that their reflections were based on subjective beliefs. One of the most important factors affecting their answers is that they have not fully developed their capacity to synthesize random occurrences and conditional occurrences (Gürbüz, 2007). Moreover, they based the answers on their subjective reasoning rather than on the true probability of an experiment. For example, their answers to Q3 were focused on the location of the balls in the basket and the ones to Q4 on the location of the numbers on the spinners. This subjective reasoning hindered their scientific answers. Ford and Kuhs (1991), Gibbs and Orton (1994), Kazıma (2006) and Tatsis et al. (2008) found in their studies that language development was important in teaching mathematics at the primary level. However, these pupils were found to have some difficulties in understanding what they read. Moreover, the findings showed that some pupils realized the importance of sample space when giving their answers. However, this awareness was determined to stem from intuitive evaluations and personal skills of the individual rather than on the education they obtained. This intuitive capacity can be said to improve spontaneously with age.

Some of the pupils in the study were found to relate probability with chance or their beliefs, so they searched for solutions by overlooking the particular conditions of experiments and demonstrated prejudices against this subject. These pupils stated that, due to their unwillingness about the probability concept, the skills and luck of players should be considered in a chance game and God's will should not be intervened. Their justifications confused chance concept, intuitive thought, personal skill and probabilistic thinking with beliefs that are neither scientific nor meaningful. This is in line with the findings reported by Amir and Williams (1994), Fischbein (1991), Greer (2001) and Sharma (2006). Amir and Williams (1994) claimed that such ideas were common in less competent individuals or in individuals raised in relatively more religious societies.

Conclusions and implications

The study showed that as the level of education increased, the percentage of correct answers increased whereas misconceptions decreased regarding the questions related to the concept of compound events I. However, it was also found that as the level of education increased, the percentage of correct answers decreased and misconceptions increased regarding the question related to the concept of compound events II. The increase in misconceptions related to this concept was insignificant, though. In Q2 and Q3, related to the concepts of probability of an event and probability comparisons, it was determined that the ratio of correct answers and misconceptions increased with the level of education and these increases were significant. In Q1, related to the concepts of probability of an event and probability comparisons, no correlation between pupils' level of education and correct answer percentages or misconceptions was found. In addition, it can be said that the number of justifications increased with the increase of educational level.

The concepts of probability seem to be much more complex, in many respects, than they are usually considered. If one intends to develop by instruction some strong, correct, coherent, formal and intuitive background for probabilistic reasoning, one has to cope with a large variety of misunderstandings, misconceptions, biases and emotional tendencies. Such distortions may be caused by linguistic difficulties, lack of logical abilities, difficulty to extract the mathematical structure from the practical embodiment, or difficulty to accept that chance events may be analyzed from a deterministic-predictive point of view. Similar difficulties were reported by Jones et al. (1997), Gürbüz (2007; 2010), and Gürbüz et al. (2010).

There is no natural understanding of the fact that, in a sample space, possible outcomes should be distinguished and counted separately if the order of their elementary components is different. One may immediately think: How important is intuitional knowledge in determining the probability of an event? Or, does the size of sample space play a role in determining the probability of an event? Fischbein et al. (1991), Vidakovic et al. (1998), Speiser and Walter (1998), Polaki (2002), and Gürbüz (2007) found in their studies that both intuitive knowledge and sample space knowledge play a role in determining the probability of an event. The findings of this paper also confirm this result.

In line with the results of Fischbein's work (1975), the capacity to synthesize the necessary and the random in the concept of probability are the two most important factors that influence some pupils when constructing their answers to the questions. The idea is that an outcome of a stochastic experiment depends only on chance - no matter what the given conditions are. This is a very important finding and it should be treated with much care by teachers and didacticians.

In traditional learning settings, learners are taught mostly by a teacher-centered approach through solving related examples and exercises. Pratt (2000), Gürbüz et al. (2009; 2010), and Gürbüz (2010) highlighted in their studies that misconceptions and incorrect intuitions considering probability should be remedied through material aided practices, because pupils are persuaded with experimental results more than with theoretical evidence. Using materials (dart, spinner, dice, coin, game card, etc.) in teaching probability and considering the likely misconceptions during the instruction are important for pupils' correct comprehension of the topic. Therefore, the material aided instruction in which pupils are active should be preferred when teaching probability.

In light of the current study results and the related literature, it can be concluded that, although the frequency of mistakes and misconceptions changed, the mistakes and misconceptions are very similar to each other. This shows that various teaching methods, learning theories, and infrastructure equipment used in learning environments are not effective enough to completely remedy all mistakes and misconceptions. Thus, to identify the factors that influence pupils' conceptual learning and misconceptions, apart from learning environments or materials, further studies should be conducted.

Finally, one point which we have not referred to at all in this paper, though it was among our findings, should be mentioned: the differences among teachers. We found that there were significant differences in the performance of pupils taught by different teachers. There are no important factors in schools that may increase the performance of teachers in Turkey. In this case, it can be argued that the most important factor in modeling their performance is their conscience. From this point of view, it can be concluded that teachers should be morally motivated to perform their tasks better and monitored with concrete performance evaluations.

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Usporedba pogrješnih predodžaba o konceptima vjerojatnosti kod učenika različitog stupnja obrazovanja

Sažetak

Cilj ovoga rada je usporediti i vrjednovati pogrješne predodžbe koje učenici na različitim obrazovnim razinama imaju kada je riječ o konceptima vjerojatnosti. Provedeno je stoga istraživanje na uzorku od 540 učenika od petog do osmog razreda. Korišten je instrument sa šest pitanja o konceptima složenih događaja, vjerojatnosti nekog događaja i usporedbama vjerojatnosti. Podatci su analizirani uz primjenu metoda deskriptivne statistike (SPSS 15.0) i hi-kvadrat testa. Istraživanje je pokazalo da se s povećanjem obrazovne razine povećava broj točnih odgovora, a smanjuje broj pogrješnih predodžbi o konceptu složenih događaja I, dok se smanjuje postotak točnih odgovora, a povećava broj pogrešnih predodžbi o konceptu složenih događaja II. Kada je riječ o pitanjima koja se odnose na koncepte vjerojatnosti nekog događaja i usporedbe vjerojatnosti, s povećanjem obrazovne razine dolazi do povećanja i postotka točnih odgovora i pogrješnih predodžbi. Ukratko, može se zaključiti da pogrješne predodžbe učenika o konceptu vjerojatnosti variraju ovisno o karakteristikama tih koncepata kao i učenikovoj dobi, odnosno obrazovnoj razini.

Ključne riječi: *nastava matematike, učenici osnovne škole, vjerojatnost, pogrješne predodžbe, međugeneracijsko istraživanje*

Uvod

Matematika ima strategijsku ulogu u ljudskom razvoju, a tijekom povijesti zadržala je vitalno i jedinstveno mjesto u mnogim društvima. Poslije materinskog jezika matematici se u većini zemalja daje najveće značenje u nastavnim planovima i programima na razini osnovnog i srednjeg obrazovanja. Drugim riječima, matematika je, kao univerzalni jezik, glavno sredstvo za razmišljanje. Usvajati gradivo iz matematike smatra se obveznim jer poboljšava sposobnosti više razine neophodne svima u svakodnevnom životu kao što su analiziranje, zaključivanje, komuniciranje, uopćavanje, kreativno i refleksivno razmišljanje (Fast, 2001; Gürbüz, 2010). Potrebe na poslu i u životu iz dana u dan imaju važnu ulogu pri stvaranju svijesti o toj nužnosti kao i zanimanja koje se u povijesti pokazivalo za matematiku.

Još od prapovijesnih vremena ljudi su se suočavali sa slučajnim događajima, na primjer nepredvidivim prirodnim pojavama i igrama na sreću, no rođenje teorije vjerojatnosti i njezino prerastanje u granu matematike dogodilo se tek sredinom XVII. stoljeća. Premda je vjerojatnost vrlo čest koncept u igrama na sreću, koristi se u mnogim područjima kao što su znanost, genetika, industrija, menadžment, ekonomija, bankarstvo i osiguranje, kinetička teorija plinova, statistička mehanika i kvantna mehanika.

Pojam ,,probabilnog učenja" povezuje se sa specifičnom eksperimentalnom paradigmom unutar koje se pojedincu prikazuje niz pokušaja od kojih svaki može imati dva ishoda te se od njega zahtijeva da predvidi što će se dogoditi prije nego što mu se pokaže ishod. U standardnom obliku, neki proces nasumce određuje taj niz, pri čemu je vjerojatnost za svaki ishod određena. Pojava poznata pod nazivom ,,usklađenost vjerojatnosti" primjećuje se vrlo često u takvim uvjetima, pri čemu se relativne frekvencije nečijih predviđanja, tijekom niza pokušaja, približavaju vjerojatnosti odgovarajućih ishoda (Greer, 2001).

Prema Fischbeinu (1975), usklađienost vjerojatnosti zapaža se kod trogodišnje i četverogodišnje djece, a općenito se dobro razvije do njihove šeste godine života. Navodi da ,,kada se vjerojatnosti odgovora, bez posebnih uputa, približe vjerojatnostima događaja, može se pretpostaviti da osoba ima određenu intuiciju za slučajnost i vjerojatnost". Obično se koriste dva objašnjenja kada je riječ o vjerojatnosti. Prvo nastalo iz ranijih teorija o uvjetovanju i potkrepljenju, a drugo proizašlo iz kasnijih kognitivnih teorija o induktivnom rezoniranju, tumače usklađenost vjerojatnosti u smislu kognitivnih procesa koji ne ovise o nekoj implicitnoj ili eksplicitnoj reprezentaciji vjerojatnosti dvaju ishoda. Fischbein (1975) ukazuje na niz zanimljivih pojava poput negativnog efekta recentnosti i mnoge razvojne promjene kao što je ona da „starija djeca sve više traže sofisticiranije strategije polazeći od uvjerenja da postoje pravila koja određuju slučajne nizove" (str. 56). Štoviše, mali je broj istraživanja ukazao na pozitivne učinke formalne poduke o vjerojatnosti tako da su se odgovori djece koja su prošla navedenu pouku približavala prije maksimiziranju nego usklađivanju vjerojatnosti (Fischbein, 1975).

Pogrješne predodžbe/poteškoće i vjerojatnost

Različiti se nazivi koriste da bi se definirali problemi pri usvajanju/poučavanju matematičkoga gradiva, kao što su „poteškoća", „pogrješna predodžba" i „pogrješka". Najčešće se primjenjuju naizmjence. ,,Poteškoća" je sveobuhvatan naziv i koristi se za opis problema s kojima se učenici općenito suočavaju kada uče matematiku (Bingölbali i Özmantar, 2009), pa nije dostatan da bi se odredili i riješili njihovi problemi s učenjem. Taj se problem onda uglavnom rješava korištenjem naziva „pogrješna predodžba". Definira se kao „percepcije ili koncepcije koje su daleko od značenja što su ga dogovorili stručnjaci" (Zembat, 2008). Drugim riječima, može se tumačiti kao percepcije koje odstupaju od stajališta stručnjaka u nekom području ili predmetu (Hammer, 1996). Prema Hammeru (1996), pogrješne predodžbe utječu na učenikove percepcije i razumijevanje.

Posljednjih tridesetak godina znanstvenici istražuju pogrješne predodžbe učenika o matematici. Istraživanja pokazuju da učenikove koncepcije o znanstvenim pitanjima često nisu usklađene s prihvaćenim znanstvenim mišljenjem, odnosno da imaju pogrješne predodžbe o raznim idejama. Cornu (1991) tvrdi da matematičke poteškoće i pogrješne predodžbe s kojima se učenici suočavaju imaju tri moguća glavna ishodišta. To su *epistemološko* (poteškoće potječu od prirode koncepta), *psihološko* (učenikova poteškoća ili pogrješna predodžba pri usvajanju nekoga koncepta rezultat je njegovog osobnog razvoja, prethodnog znanja i sposobnosti matematičkog razumijevanja) i *pedagoško* (učenikov problem pri usvajanju nekoga koncepta potječe od čimbenika kao što su nastavno sredstvo, sadržaj i metoda). Shaugnessy (1977) i Gürbüz (2007) sugeriraju da neke pogrješne predodžbe o vjerojatnosti imaju ishodište u prirodi predmeta ili učenikovu prethodno usvojenom teorijskom znanju o vjerojatnosti. Drugim riječima, oni navode da takve vrste pogrješnih predodžbi mogu imati epistemološki ili psihološki korijen te da poučavati učenike o pravilima vjerojatnosti ne bi moglo biti dostatno samo po sebi da bi se postojeće predodžbe ispravile. Učenici, na primjer, vjeruju da će se, nakon šest glava, kod sedmoga bacanja pojaviti pismo. Ta je misao poznata kao „kockarska zabluda" i ljudi zapravo odlučuju je li novčić pošten ili nije u pitanju jednakosti mogućih rezultata (Fast, 2001).

Koncepti vjerojatnosti uveliko se koriste u procesima odlučivanja o neizvjesnim situacijama na koje nailazimo u svakodnevnom životu. Unatoč toj važnosti, postoji nekoliko razloga zbog kojih se u Turskoj ne poučava o vjerojatnosti onako učinkovito kao u mnogim drugim zemljama. Najvažniji razlog leži u pogrješnim predodžbama o tom predmetu; one otežavaju njegovo razumijevanje. Doista, provedena su mnoga istraživanja o pogrješnim predodžbama u nastavi matematike općenito te specifično o predmetu vjerojatnosti (Barnes, 1998; Batanero i Serrano, 1999; Çelik i Güneş, 2007; Dooren i sur., 2003; Fast, 1997; 2001; Fischbein i Schnarch, 1997; Fischbein, Nello i Marino, 1991; Gal-Ezer i Zur, 2004; Hope i Kelly, 1983; Huang, Liu i Shiu, 2008; Jacobsen, 1989; Kahneman i Tversky, 1972; Konold i sur., 1993; Konold, 1989; Lecoutre, 1992; Perkins i Simmons, 1988; Polaki, 2002; Pratt, 2000; Shaughnessy, 1977; Stavy i Tirosh, 1996a; 1996b; Watson i Moritz, 2002).

Kahneman i Tversky (1972) su među prvima istraživali pogrješne predodžbe o vjerojatnosti. Otkrili su da se pojedinci koriste općim pristupom kada donose odluku o približnom rezultatu nekog događaja. Štoviše, tvrde da pojedinci koji raspolažu nedostatnim znanjem o vjerojatnosti primjenjuju intuitivne strategije kada određuju moguće složene događaje, a to pak može dovesti do pogrješnih predodžbi. Na primjer, kada su pitali učenike da procijene vjerojatnost svoje nazočnosti u nekoj automobilskoj nesreći, oni su odgovorili da će implikacije takvih događaja utjecati na njihove procjene. Drugim riječima, pojedinci su vjerovali da će se vjerojatnost za jednu takvu nesreću povećati ako je broj nesreća kojih su se sjećali velik. Pogrješne predodžbe, nastale nakon jednog takvog procesa, uglavnom su otporne na promjene (Fast, 2001; Konold, 1988). Pojedinci stoga pokazuju veliku nespremnost na odustajanje od svojih pogrješnih predodžbi unatoč dokazu koji je u suprotnosti s njihovim postojećim znanjem (Brown i Clement, 1987).

Vjerojatnost u stručnoj literaturi (usporedba po dobi)

Od prapovijesnih vremena ljudi se suočavaju sa slučajnim fizičkim događajima poput nepredvidivih prirodnih pojava i igara na sreću, ali do rođenja teorije vjerojatnosti i njezine pretvorbe u matematičku granu dolazi tek sredinom XVII. stoljeća. Iako je vjerojatnost vrlo čest koncept u igrama na sreću, koristi se u mnogim područjima kao što su znanost, genetika, industrija, uprava, ekonomija, bankarstvo i osiguranje, kinetička teorija plinova, statistička mehanika i kvantna mehanika. Međutim, vjerojatnost se kao predmet počinje pojavljivati u nastavnim planovima i programima poslije XIX. stoljeća te od tada kognitivni psiholozi i matematički pedagozi istražuju pogrešne predodžbe učenika I njihovo nerazumijevanje vjerojatnosti u raznim dobnim skupinama (Çelik i Güneş, 2007; Dooren i sur., 2003; Fast, 2001; Fischbein i Schnarch, 1997; Fischbein i sur., 1991; Gal-Ezer i Zur, 2004; Kahneman i Tversky, 1972; Konold i sur., 1993; Lecoutre, 1992; Perkins i Simmons, 1988; Pratt, 2000; Stavy i Tirosh, 1996a; 1996b). Na primjer, Fischbein i Schnarch (1997), koji su istraživali promjene pogrješnih predodžbi učenika petog, šestog, sedmog, devetog i jedanaestog razreda kao i studenata koji još nisu slušali o vjerojatnosti ni na jednom kolegiju, pokazali su da se njihovo nerazumijevanje nekih koncepata smanjilo ili povećalo na razini varijable obrazovanja ili je pak ostalo nepromijenjeno. Paralelno s ovim istraživanjem, Fischbein i sur. (1991) primijenili su s učenicima koji su prethodno poučavani o vjerojatnosti kao i onima koji nisu dva različita testa koja su sadržavala 7 paralelnih i pitanja otvorenog tipa. Polazeći od analize, utvrdili su da se postotak točnih odgovora povećava kako se povećava razina učenja. No u dva su slučaja (dva pitanja u svakom testu; 2, 2) utvrdili da se taj postotak smanjuje s porastom razine učenja. Kada su se usporedile konceptualne pogreške među skupinama, u slučaju nekih pitanja s porastom razine učenja došlo je do porasta konceptualnih pogrješaka, a u nekima pak do smanjenja. Slično tome, Dooren i sur. (2003), koji su uspoređivali pogrešne predodžbe učenika desetih i dvanaestih razreda, implicirali su nepostojanje značajne razlike među skupinama unatoč tome što porast obrazovne razine dovodi do smanjenja pogrješnih predodžbi. Stavy i Tirosh, (1996a, 1996b) istraživali su uobičajene misaone obrasce koji leže u osnovi pogrešnih predodžbi u različitim područjima. Otkrili su da mnoge pogrešne predodžbe u matematici i znanosti potječu od maloga broja intuitivnih pravila. Ustanovili su tri različite vrste intuitivnih pravila: „što je više A, tim je više B"; „sve je djeljivo s dva" i „isto A znači isto B". Učenici smatraju da su ta pravila evidentna sama po sebi te ih vrlo pouzdano primjenjuju. Stvarajući logične odnose u određenom okruženju, čini se da učenicima omogućuju donekle razumjeti datu situaciju, a da pritom zapravo ne razumiju koncept. Watson i Kelly (2004), koji su primjenjivali test s krugom podijeljenim na dva jednaka dijela (50-50) da bi odredili kako učenici trećeg, petog, sedmog i devetog razreda shvaćaju statističku varijaciju u slučajnom okruženju, otkrili su postojan napredak u konceptualnom razvoju tijekom čitavoga procesa u trećem, petom i devetom, ali ne i u sedmom razredu. Çelik i Güneş (2007) proveli su istraživanje da bi odredili točne i netočne predodžbe o vjerojatnosti što ih imaju učenici sedmih, osmih i devetih razreda. Kao rezultat njihova istraživanja proizašao je zaključak da se pogrješne predodžbe o „reprezentativnosti" i „negativnom/pozitivnom efektu recentnosti" smanjuju ovisno o razredu, ali pogrješne predodžbe o „elementarnim i složenim događajima", „zabludi o stjecaju okolnosti" i „učinku prostora elementarnih događaja" ostaju nepromijenjene na svim razinama kod većine učenika.

Provedeno je nekoliko istraživanja da bi se utvrdilo kako se s dobi mijenja značenje što ga učenici daju konceptu vjerojatnosti ili otkrio učinak uporabe različitih strategija u nastavi statistike na učenikove razine razmišljanja (Batanero i Serrano, 1999; Fast, 1997; Fischbein i Gazit, 1984; Jones i sur., 1997; Pratt, 2000; Polaki, 2002; Sharma, 2006; Watson i Moritz, 2002; Weir, 1962). Na primjer, Jones i sur. (1997) razvili su okvir za vrjednovanje razina na kojima učenici promišljaju neke koncepte vjerojatnosti. Taj se okvir, usavršen nakon različitih primjena, koristio pri vrjednovanju načina promišljanja nekih koncepata vjerojatnosti među učenicima osnovne škole. U jednom drugom radu koji je obuhvaćao 8 skupina s po dvije osobe u dobi od 10 ili 11 godina koje su računalno provodile eksperimente s materijalom temeljenim na krugu, novčiću i kocki, Pratt (2000) je otkrio sustavnu predrasudu o vjerojatnosti u svakodnevnom životu učenika. Budući da su njegovi ispitanici bili izloženi brojnim eksperimentima uz primjenu navedenoga materijala i raznih pitanja, omogućio im je da ju prevladaju. Watson i Moritz (2002) su proveli istraživanje o tome kako učenici napreduju kada odgovaraju na pitanja o vjerojatnosti jednog događaja, složenih događaja i uvjetovanih događaja. S tim je ciljem ukupno 2615 učenika (5-11 godina) odgovaralo na 4 pitanja, a njihovi su odgovori vrednovani u kontekstu sljedećih čimbenika: *a) tipovi odgovora, b) usporedbe po dijelovima, c) longitudinalna promjena, d) usporedbe po pitanjima te e) usporedbe po drugim slučajnim pitanjima*. Kada se po skupinama usporedio omjer točnih odgovora na pitanja o uvjetovanoj vjerojatnosti, otkriveno je da se postotak točnih odgovora povećava s obrazovnom razinom. Međutim, nije pronađena nikakva korelacija između obrazovne razine i omjera točnih odgovora na pitanja o vjerojatnosti složenih događaja. Fischbein i Gazit (1984) su proveli istraživanje da bi utvrdili kako pouka o vjerojatnosti utječe na intuiciju o vjerojatnosti. Izraelska djeca u petom, šestom i sedmom razredu (10-13 godina) imala su 12 nastavnih sati posvećenih određenim konceptima (npr. izvjesno, moguće, itd.) i njihovim vjerojatnostima. Dok je većina ideja bila preteška učenicima petih razreda, smatralo se da će oko 60-70% učenika šestih razreda, odnosno 80-90% učenika sedmih razreda moći razumjeti i primijeniti većinu koncepata. Batanero i Serrano (1999) proveli su istraživanje na uzorku od 277 učenika u dobi između 14 i 17 godina u Španjolskoj da bi utvrdili kako se mijenja značenje što ga djeca daju konceptu „slučajnosti" ovisno o njihovoj dobi. U tom je istraživanju korišten Greenov test s 8 pitanja (četiri su se odnosila na slučajne nizove, a četiri na slučajne dvodimenzionalne distribucije). Rezultat istraživanja doveo je do zaključka da dob nije važna pri razumijevanju koncepta „slučajnosti"; to je teško shvatljiv koncept, a da bi se dobro usvojio, ključno je razumjeti mnoge druge koncepte vjerojatnosti kao što su prostor elementarnih događaja, vjerojatnost nekog događaja, usporedbe vjerojatnosti, itd. Weir (1962) je proveo istraživanje da bi otkrio kako tri različita tipa poduke uz primjenu istoga materijala utječu na učenje o vjerojatnosti u skupinama djece u dobi od 5-7 i 9-13 godine. S tim je ciljem 96 učenika različite dobi bilo podijeljeno u 6 skupina po 16 učenika i te su skupine bile izložene različitim intervencijama. Na njih su primijenjene tri nastavne prakse kao što su: *„bez intervencije prema članovima skupine"*, *„reći članovima skupine da postoji rješenje pitanja"* i *„reći članovima skupine da ne postoji rješenje pitanja"*. Polazeći od analize, utvrđeno je da: a) mlađi učenici više vole pomoć i poticaj nego stariji učenici, b) stariji učenici više mijenjaju početne odgovore nakon što dobiju pomoć nego mlađi učenici, c) različite nastavne prakse nemaju nikakvog učinka na učenikov odabir situacija za koje su dobili pomoć, d) stariji učenici teže prevladavaju predrasude. Ukratko, zaključeno je da učenikove predrasude ili prethodno usvojeno znanje imaju važnu ulogu kada odlučuje o konceptima slučajnosti ili vjerojatnosti.

Pregledana literatura pokazuje da se učenikove koncepcije, pogrješke i pogrješne predodžbe mijenjaju ovisno o dobi, obrazovnoj razini, jeziku i kulturi. Nadalje, istraživanja o pogrješnim predodžbama što ih turski učenici imaju o vjerojatnosti pokazala su da njihovo razumijevanje navedenog koncepta jedva ovisi o dobi i obrazovnoj razini. U tom se kontekstu smatra da je njihovo određivanje koncepcija, pogrješaka i pogrješnih predodžbi o vjerojatnosti važno. Ovo istraživanje stoga ima cilj usporediti i vrjednovati pogrješne predodžbe što ih imaju učenici različitih obrazovnih razina (5, 6, 7 i 8 razred) i dobi (11-14) o nekim konceptima vjerojatnosti (*složeni događaji, vjerojatnost događaja, usporedbe vjerojatnosti*).

Istraživanje

Plan

Da bi se odredile predodžbe što ih učenici imaju u odnosu na njihov razred i razumijevanje, obično se koriste međugeneracijska i longitudinalna istraživanja. Unatoč činjenici da međugeneracijsko istraživanje obuhvaća različite kohorte učenika, prihvatljivije je od longitudinalnog istraživanja kada je vrijeme ograničeno (Abraham, Williamson i Westbrook, 1994). Ta se istraživanja mogu provoditi u relativno kraćem vremenskom razdoblju, a cilj im je obično što bolje upoznati promatrane skupine. Međugeneracijska istraživanja, također, pružaju mogućnost da se prate pomaci u konceptualnom razvoju slijedom učenikove zrelosti, boljeg intelektualnog razvoja i daljnjeg učenja.

Uzorak

Istraživanje je provedeno školske 2009./2010. godine na uzorku od ukupno 540 učenika (11-14 godina) jedne osnovne škole na jugoistoku Turske. Sudionici istraživanja uglavnom pripadaju nižim ili srednjim društveno-ekonomskim slojevima. Njihova se škola nalazi u središnjem dijelu regije i ima dobru fizičku infrastrukturu. Tablica 1 prikazuje razrede, dob i veličinu razreda unutar istraživane skupine učenika.

Tablica 1

Obrazovni kontekst

U Turskoj je obrazovanje organizirano na četirima razinama: predškolsko obrazovanje (3–6 godina), osnovno obvezno obrazovanje (6–14 godina), sekundarno obrazovanje (14–17 godina), te visoko obrazovanje (koledži i sveučilišta). Sve škole u zemlji moraju primjenjivati isti kurikul, što ga izrađuje i provodi Ministarstvo obrazovanja Republike Turske. Osim toga, nastava se matematike brzo mijenja posljednjih nekoliko godina i više je usmjerena na samog učenika, što je rezultat konstruktivističkoga pristupa.

Matematika je obvezni nastavni predmet u Turskoj i zauzima mnogo prostora u programima osnovnih i srednjih škola. Nastavu matematike izvode razredni učitelji sve do druge faze obveznog osnovnog obrazovanja (6-8 razred). U tom se smislu može naslutiti da je matematička kompetencija učenika u prvoj fazi (1-5 razred) obveznog obrazovanja povezana s matematičkom kompetencijom razrednog učitelja. Tu činjenicu dakle treba uzeti u obzir pri uspoređivanju skupina. U prošlosti se vjerojatnost poučavala jedino u osmom i desetom razredu, no 2005. godine zaživio je novi kurikul pa se ta tema rasporedila na četvrti, peti, šesti, sedmi i osmi razred osnovne škole skupa s desetim razredom srednje škole. To je podrazumijevalo da će problematika vjerojatnosti biti zastupljena u svakom razredu počevši od prve faze osnovnog obrazovanja (MEB, 2005). Svaka je učenička skupina, obuhvaćena istraživanjem, bila izložena formalnoj pouci o vjerojatnosti. No, njihova je izloženost bila različita. Kada je analiziran nastavni program matematike u sklopu primarnog obrazovanja, koji se počeo provoditi 2005. godine, utvrđeno je da on obuhvaća 3 nastavna sata (3*40 minuta) u petom i 5 nastavnih sati (5*40 minuta) u šestom, sedmom i osmom razredu. Prema tom kurikulu, ciljevi su nastave u petom razredu da učenik: „*procjenjuje vjerojatnost događaja*", „*provodi eksperimente o vjerojatnosti elementarnog događaja i tumači rezultat*" te „*tumači je li neki događaj fer ili nije*". Ciljevi su nastave u šestom razredu da učenik: „*objašnjava pojmove, eksperiment, ishod, prostor elementarnih događaja, događaj, slučajni odabir i jednaku vjerojatnost s obzirom na neki slučaj*", „*objašnjava neki elementarni događaj i njegovu vjerojatnost*", „*rješava probleme povezane s vjerojatnošću nekog događaja i tumači rezultat*", „*objašnjava mogući raspon vjerojatnosti nekoga događaja*" te „*objašnjava sigurne i nemoguće događaje*". U sedmom razredu ciljevi nastave su da učenik: „*odredi eksperiment, prostor elementarnih* *događaja te vjerojatnost diskretnih i indiskretnih događaja*", „*tumači diskretne i indiskretne događaje*", „*procjenjuje vjerojatnost diskretnih i indiskretnih događaja*" te „*procjenjuje vjerojatnost nekog događaja koristeći se znanjem iz geometrije*". Nastavni ciljevi u osmom razredu su da učenik: „*objašnjava ovisne i neovisne događaje*", „*procjenjuje vjerojatnost ovisnih i neovisnih događaja*" te „*objašnjava eksperimentalne, teorijske i subjektivne vjerojatnosti*".

Autori su kao promatrači prisustvovali satima matematike u razredu gdje se interveniralo i gdje se primjenjivao test jedan ili dva sata (1*40 ili 2*40). Unatoč blagim razlikama među skupinama, nastava se uglavnom provodila tako da bude usmjerena na učitelja i orijentirana na provjeru znanja, što se dalo primijetiti. Učenicima se sadržaj tumačio prema udžbeniku, a učitelj je na ploči pisao potrebna objašnjenja. Štoviše, neki problemi kao što je „pretpostavite da bacamo kocku..." ili „pretpostavite da u košari imamo 4 crvene..." bili su posebno postavljeni, a zatim riješeni. Ukratko, provedeni su maštoviti eksperimenti i pronađeni rezultati. U tom su procesu učitelji odgovarali učenicima na pitanja o tom predmetu. Oko 70%-75% nastavnoga sata sastojalo se samo od učiteljeva govora. Poslije predavanja, učitelj je zamolio učenike da odgovore na pitanja na kraju lekcije.

Instrumenti

Instrument se sastojao od 6 pitanja i bio je pripremljen za potrebe ovoga istraživanja uz pomoć literature (Baker i Chick, 2007; Fischbein i sur., 1991; Jones i sur., 1997; Polaki, 2002). Sva su se pitanja u testu odnosila na koncept prostora elementarnih događaja koji čini osnovu za predmet vjerojatnosti. Nadalje, prva se tri pitanja (Q1, Q2, Q3) odnose na koncepte vjerojatnosti nekog događaja i usporedbe vjerojatnosti dok se preostala tri pitanja (Q4, Q5, Q6) tiču koncepata složenih događaja I i II. Tri su pitanja otvorenoga tipa, ali je jedno (tri) dvodimenzionalno pitanje koje se sastoji od višestrukoga izbora i otvorenog odgovora. U svim su pitanjima ispitanici zamoljeni da napišu svoje prosudbe kako bi se dalje istraživali čimbenici koji utječu na njihovo znanje o vjerojatnosti. Skupina matematičkih metodičara i matematičara provjerila je valjanost i pouzdanost testa, a zatim potvrdila sadržajnu valjanost instrumenta. Provedeno je također pilot istraživanje na uzorku od 40 učenika različite obrazovne razine. Trajalo je oko 25 minuta te otkrilo da su pitanja o vjerojatnosti svima razumljiva i jasna. Dva su metodičara neovisno ocijenila pitanja korištena u istraživanju da bi se omogućila pouzdanost rezultata za svako pitanje. Izračunati su Cronbach alpha koeficijenti za svako pitanje, a kretali su se u rasponu od 0.78 do 0.93.

Postupak

Svaka je skupina poučavana o vjerojatnosti, a zatim je primijenjen instrument vrjednovanja. Skupine, međutim, nisu bile vremenski ograničene. Pitanja su poredana nasumično da bi se postigla ravnoteža u učinku redoslijeda.

Analiza podataka

Pri analizi podataka bili smo pod utjecajem istraživanja što su ga proveli Fischbein i sur. (1991). Prvo, pregledani su učenički odgovori, zatim kategorizirani kao „bez odgovora", „točan odgovor" i „netočan odgovor" (zbroj tipova pogrešne predodžbe) i konačno ocijenjeni. Pri izračunu postotka pogrešnih predodžbi u odgovorima uzet je u obzir njihov omjer u odnosu na netočne odgovore. Točni su odgovori kategorizirani kao što prikazuje Tablica 2. U ovom je radu korišten sljedeći simbol za označavanje razreda kojemu navedeni ispitanici pripadaju: G5 znači *Razred 5* a slijede ga 6, 7 ili 8 ukazujući na određeni razred. Postoci točnih odgovora koji pripadaju tim kategorijama izračunati su u odnosu na sve točne odgovore. Zatim su predstavljeni podaci za svako pitanje u tablici postotaka i frekvencijske distribucije prema razredima. Varijable su uspoređivane sa hi-kvadrat testom.

Umjesto ponavljanja pogrešnih predodžbi iz literature ovaj se rad bavi samo pogrješnim predodžbama vezanim za svako pitanje u ovom istraživanju te ih analizira u sklopu dvaju glavnih pogrješnih predodžbi u odnosu na svako pitanje. Takav je pristup omogućio istraživačima da analiziraju učenikove odgovore bez predrasuda. Štoviše, budući da je cilj ovog istraživanja spoznati kako učenici mijenjaju pogrješne predodžbe o vjerojatnosti, a ne potvrditi ili opovrgnuti prethodne rezultate o pogrješnim predodžbama, takav je pristup smatran prihvatljivijim.

Ponekad se u tekstu koriste pojmovi „točan" i „netočan". Svjesni smo da takvo razlikovanje nije apsolutno. Učenici raspolažu reprezentacijama i tumačenjima koja se moraju poštovati sama po sebi, za što postoje razlozi, a mogu odgovarati određenim situacijama. „Točno" jednostavno označava ono što je obično prihvaćeno o standardnoj vjerojatnosti u udžbenicima. S druge strane, mora se uzeti u obzir da djeca ponekad daju po svemu sudeći „točne" odgovore iz pogrješnih razloga. U ovo su istraživanje uključeni neki primjeri odgovora na svako pitanje nekih učenika. Rad stoga predstavlja glavne vrste učenikovih prosudbi. Osim toga, ako učenikovi odgovori ne ulaze u navedene kategorije, ali su zanimljivi, također su uvršteni da bi se povećala čitljivost rada.

Tablica 2

Rezultati i diskusija

Rezultati povezani s konceptima vjerojatnosti nekog događaja i usporedbama vjerojatnosti

Tri se pitanja (Q1, Q2, Q3) u instrumentu odnose na koncepte vjerojatnosti nekog događaja i usporedbe vjerojatnosti. Kao što se vidi u Tablici 3, nema značajne razlike među skupinama u smislu omjera pogrješnih predodžbi u pitanju Q1 [χ^2 (3) = 4.871, p = .181]. Može se tako tvrditi da se omjer pogrešnih predodžbi kada je riječ o pitanju broj 1 (Q1) nije mijenjao u odnosu na obrazovnu razinu skupine (36.6%, 32.7%, 29.1% i 41.4%). Kada su se uspoređivale vrste pogrješnih predodžbi što su ih skupine pokazale na ovome pitanju, moglo se vidjeti da je tip II češći od tipa I. Tablica 3 pokazuje da, osim u slučaju osmog razreda, s povećanjem obrazovne razine dolazi do blagoga povećanja broja točnih odgovora. No, nema značajne razlike među skupinama u pogledu točnih odgovora na Q1 [χ^2 (3) = 4.33, *p* = .227]. Prema Tablici 3, CACJ (točan odgovor, točna prosudba) omjer po skupinama (28.3%, 40.7%, 56.4% i 54.2%) pokazao je da se obrazovna razina povećava, pri čemu se CANJ (točan odgovor, nema prosudbe) omjer (67.2%, 39.5%, 32.1% i 30.5%) smanjuje u odnosu na obrazovnu razinu. CAIJ (točan odgovor, netočna prosudba) omjer po skupinama (4.5%, 19.8%, 11.5% i 15.3%) pokazuje povećanje obrazovne razine.

Grafikon 1

Tablica 3

U ovom je istraživanju utvrđeno da su učenici nastojali riješiti problem u prvom pitanju (Q1) koristeći se intuicijom i strategijama neformalne kalkulacije. Potrebno je primijetiti da su se iste strategije koristile u nekoliko sličnih istraživanja (Batanero i Serrano; 1999; Fischbein, Gal-Ezer i Zur, 2004; Gürbüz, 2007; Nello i Marino, 1991; Stavy i Tirosh, 1996a; 1996b). S druge strane, pri rješenju Q1 mnogi su učenici shvatili važnost veličine, ali su se usredotočili na jedan aspekt veličine, pa su pogrješno odgovorili zbog prosudbi u *Tipu I* ili *Tipu II* (Tablica 2). Na primjer, neki su učenici odgovorili, *„B jer su se krugovi okretali istovremeno*" (G5; G6; G7), *,B jer crvena boja pokriva polovicu kruga B*" (G5; G7), *„B jer su crveni, plavi i zeleni dio spojeni* " (G6; G7), *„ A jer je sve crveno kod kruga A*" (G5; G7). S druge strane, jedan je učenik odgovorio na Q1 *„ne možemo ništa reći dok ne znamo tko okreće krugove*" (G5; G8), *„moja omiljena boja je crvena, dakle crveno*" (G5; G6) " *„„bilo bi bolje da podijelimo ove krugove na dva ili četiri jednaka dijela* " (G7; G8), *„budući da ne možemo znati koliko će se sekundi ili minuta okretati krugovi, ne možemo znati gdje će se zaustaviti*" (G6; G7), *„B jer je plava boja u donjem dijelu, a crvena u gornjem dijelu*" (G8).

Kada se pogleda Tablica 3, može se vidjeti da omjer netočnih odgovora na Q1 ne pokazuje značajnu razliku u odnosu na obrazovnu razinu i dob. Prema tom rezultatu, nije moguće komentirati pozitivne ili negativne učinke dobi ili obrazovne razine na otklanjanje pogrešnih predodžbi kod učenika. No, sukladno rezultatima istraživanja što su ga proveli Watson i Moritz (2002), može se tvrditi da učenici donose potpunije prosudbe na odgovore kako im se dob ili obrazovna razina povećavaju. To se može povezati s jezičnim razvojem. Doista, Ford i Kuhs, (1991), Gibbs i Orton, (1994), Kazıma, (2006) te Tatsis i sur. (2008) ističu važnost jezičnoga razvoja u razumijevanju koncepata vjerojatnosti.

Grafikon 2

Tablica 4

Kao što pokazuje Tablica 4, nema značajne razlike među skupinama po omjeru pogrješnih predodžbi u pitanju Q2 [χ^2 (3) = 8.053, *p* = .045]. Prema tom rezultatu, učenici šestih i osmih razreda imaju više pogrješnih predodžbi o pitanju Q2 nego učenici petih i sedmih razreda. Kada se skupine usporede u odnosu na tipove pogrešnih predodžbi, može se vidjeti da je *Tip II* češći od *Tipa I*. Točni odgovori skupina (17.6%, 18.4%, 26.9% i 19.5%) pokazuju porast obrazovne razine kao što je to slučaj s rezultatima istraživanja što su ga proveli Fischbein i Gazit (1984). Štoviše, kada analiziramo Tablicu 4, omjer točnih odgovora i točnih prosudbi pokazuje porast obrazovne razine (26.1%, 29.6%, 33.3% i 44.0%), za razliku od omjera točnih odgovora s izostankom prosudbi (34.8%, 33.3%, 36.1% i 20.0%) te omjera točnih odgovora s netočnim prosudbama (39.1%, 37.1%, 30.6% i 36.0%). S druge strane, omjeri se praznih odgovora na pitanje Q2 značajno smanjuju (38.1%, 25.1%, 29.1%, 24.2%).

Otkriveno je da su učenici koji su imali pogrješne predodžbe o pitanju Q2 imali različite prosudbe za svoje netočne odgovore. Davali su netočne odgovore ili zato što nisu razumjeli problem ili su ga pogrešno razumjeli (mislili su da trebaju odabrati ili novčić ili kocku) kao što je to pokazalo Fastovo istraživanje (1997). Na primjer, „*Odabrao bih novčić jer se baca*" (G5), „*Odabrao bih novčić jer je vjerojatnost da se pokaže glava veća kod novčića*" (G5; G7), „*Odabrao bih kocku jer ima broj na svakoj strani* " (G5), „*Odabrao bih kocku jer pruža 3 mogućnosti dok s novčićem postoji samo jedna*" (G6; G7; G8), „*Odabrao bih kocku jer je broj parnih brojeva kod nje veći nego broj glava kod novčića*" (G6; G8), "*Odabrao bih kocku jer će poštena kocka uvijek pobijediti*" (G8). Štoviše, budući da su neki učenici povezali pitanje Q2 sa sposobnošću ili su imali nedostatke u razumijevanju koncepta prostora elementarnih događaja, odgovorili su na temelju tih pogrješnih predodžbi. Odgovarali su tako da su pokazivali nerazumijevanje poput „*Odabrao bih kocku jer mogu dobiti glavu u kraćem vremenskom roku*" (G5; G8), „*Odabrao bih novčić jer ima 2 strane, a vjerojatnost dobivanja glave je 1/2. Međutim, kocka ima 6 strana, a vjerojatnost dobivanja glave je 1/6*" (G6; G8), „*Odabrao bih novčić jer je vjerojatnost dobivanja glave 50% dok je vjerojatnost dobivanja parnog broja na kocki skoro 33%* (G6; G7; G8)", „*Odabrao bih novčić jer bih mogao varati s njim i uvijek dobiti glavu, ali ne bih mogao to isto s kockom*" (G6; G8), „*Odabrao bih kocku jer vjerojatnost dobivanja parnog broja iznosi 80%* (G8)", „*To je kao kocka*, *ali ja bih pokušao da budem siguran. Odabrao bih ono što se najčešće događa*" (G8).

Pokazalo se da su točni odgovori na pitanje Q3 s netočnim prosudbama uglavnom pripisivani slučajnosti i prijašnjim iskustvima. Stanoviti broj autora koji su koristili isti tip pitanja (Fischbein i sur., 1991; Lecoutre, 1992, Pratt, 1998; Amir i Williams, 1999; Batanero i Serrano, 1999; Baker i Chick, 2007; Nilsson, 2007) izvijestili su o sličnim zaključcima u svojim istraživanjima.

Kao što pokazuje Tablica 4, CACJ (točan odgovor, točna prosudba), omjer u odnosu na Q2 povećao se ovisno o obrazovnoj razini. Taj je rezultat u skladu s rezultatima istraživanja što ga je proveo Polaki (2002). Ovdje možemo reći da su učenici u različitim uvjetima učenja davali slične odgovore jer su imali iskustvo sličnih mentalnih procesa. U ovom je istraživanju utvrđeno da odgovori na Q2 sadrže određene pogrješne predodžbe jer neki učenici nisu mogli razumjeti pitanje ili su ga pogrešno razumjeli. Može se tvrditi da je pogrješno razumijevanje pitanja povezano s jezičnim razvojem. Doista, Ford i Kuhs (1991), Gibbs i Orton (1994), Kazıma (2006) te Tatsis i sur. (2008) istaknuli su važnost jezičnog razvoja u razumijevanju koncepata vjerojatnosti. Neki su odgovori na ovo pitanje čak sadržavali pogrješne predodžbe zato što učenici nisu mogli točno odrediti prostor elementarnih događaja ili su imali pogrješne predodžbe o razlomcima. Navedeni rezultat odgovara onima o kojima su izvijestili Carpenter i sur. (1981), Jones i sur. (1997), Ritson (1998), Polaki (2002) i Chernoff (2009). Nadalje, učenici su se općenito radije koristili postotcima (%) kada su rješavali probleme ove vrste, vjerojatno zato što se rado koriste postotcima i u svakodnevnim sličnim situacijama.

Grafikon 3

Tablica 5

Polazeći od Tablice 5, može se shvatiti da omjeri pogrješnih predodžaba o Q3 po skupinama (54.2%, 73.7%, 70.1%, i 73.4%) pokazuju značajne razlike u obrazovnoj razini [χ^2 (3) = 15.502, p = .001]. Kada se usporede te pogrješne predodžbe, može se primijetiti da je *Tip II* češće zastupljen nego *Tip I*. Omjeri su točnih odgovora po skupinama (4.8%, 4.8%, 6.0%, i 7.8%), pokazalo se, sasvim niski i otkrivaju blagi porast u smislu obrazovne razine. S druge strane, CANJ (točan odgovor, nema prosudbe) omjeri (50%, 42.8%, 37.5%, i 30%) se smanjuju u odnosu na obrazovnu razinu. CAIJ (točan odgovor, netočna prosudba) omjer (16.7%, 28.6%, 25%, i 30%) pokazuje porast na razini obrazovanja, a omjer praznih odgovora (41.2%, 21.8%, 23.9%, i 18.8%) smanjenje na toj istoj razini.

Pokazalo se da su učenici s pogrješnim predodžbama u pitanju Q3 navodili različite prosudbe u svojim netočnim odgovorima. Imali su pogrješne predodžbe jer su odgovore povezivali s općim čimbenikom slučajnosti i konceptom omiljene boje ili su se usredotočili na položaj, odnosno broj lopti u košari. Na primjer, „*ne možemo reći ništa a da ne znamo koja je omiljena boja osobe koja bira lopte*" (G5), „*crvena i plava jer ženske osobe više vole crvenu, a muške plavu boju*" (G6; G7), „*zelena jer će zelene lopte na dnu izbiti na površinu kada se uzmu crvene i plave*" (G5; G6; G8), „*plave i zelene jer su na dnu košare*" (G7), „*crvene jer kada stavimo ruku u košaru odmah dotaknemo crvenu loptu*" (G6; G8), „*samo crvena I plava jer su te lopte na vrhu*" (G5; G7), „*zelena jer će zelena lopta ostati u košari najduže*"(G5), „*crvena i plava jer su se smanjile i zamijenile mjesta*" (G6; G8), „*plava jer je najmanje plavih u košari*" (G7). Navedeni pristupi učenika koji su dali netočne odgovore na pitanje Q3 u skladu su s pristupima učenika iz istraživanja što su ih proveli Gürbüz (2010), Gürbüz, Çatlıoğlu, Birgin i Erdem (2010) te Jones i sur. (1997). Nadalje, nekolicina je učenika koji su pogrješno povezali situacije s pitanjem netočno odgovorila pokazujući pogrešne predodžbe. Na primjer, *„zelena jer su to zavisni događaji"* (G8) i *„crvena i plava jer ovdje imamo jedan zavisni događaj"* (G7). Osim toga, nekolicina je učenika shvatila da trebaju uspostaviti vezu s prostorom elementarnih događaja. Budući da međutim nisu uspostavili takve poveznice, njihovi su odgovori netočni i sadrže pogrješne predodžbe. Na primjer, *„crvena i plava jer odabrane lopte nisu vraćene u košaru pa se prostor elementarnih događaja smanjio"* (G7), *„zelena jer se vjerojatnost odabira zelene lopte povećava zbog smanjenja prostora elementarnih događaja"* (G7).

Nekolicina učenika dala je netočne odgovore u kojima su sadržane pogrješne predodžbe zato što nisu razumjeli pitanje Q3 ili zato što su ga pogrješno razumjeli, a to je slično rezultatima Fastova istraživanja (1997). Ti su učenici uglavnom otkrili vjerojatnost odabira svake lopte umjesto onoga što se doista pitalo. Naprimjer, "*P(b)=3/9, P(g)=4/9 i P(r)=2/9*" (G5; G7), *"P(g)=4/9 P(r)=1/7 i P(b)=2/7"* (G6; G8), *"P(g)=4/9, P(r)=1/9 i P(b)=2/9"* (G6; G7). Ovdje su donosili prosudbe paralelne s rezultatima istraživanja koje su proveli Gürbüz (2006), Gürbüz i sur. (2010) te Jones i sur. (1997); svoje su odgovore povezivali s omiljenom bojom ili položajem lopti u košari.

Pitanje Q3 izgleda lagano, ali učenicima je bilo teško riješiti ga kada se usporedi s drugim pitanjima. Pokazalo se da su oni koji su u svojim odgovorima imali pogrješne predodžbe kalkulirali zanemarujući važnost prostora elementarnih događaja. Navedeni rezultat odgovara onima o kojima su izvijestili Carpenter i sur. (1981), Jones i sur. (1997), Ritson (1998), Polaki (2002) i Chernoff (2009). Nadalje, odgovori na ovo pitanje pokazuju da učenici nisu dobro razumjeli pitanje i da su se usredotočili na položaj lopti, što odgovara rezultatima Jonesa i sur. (1997) i Gürbüza (2007).

Rezultati i diskusija povezani s konceptima složenog događaja I i II

Q4 (Pitanje 4) i Q5 (Pitanje 5) u ovom dijelu odnose se na koncepte „složeni događaj I", a Q6 (Pitanje 6) na koncept „složeni događaj II".

Grafikon 4

Tablica 6

Prema Tablici 6, omjeri pogrješnih predodžbi po skupinama (32.8%, 30.5%, 24.6% i 31.3%) ne pokazuju značajnu razliku u odnosu na obrazovnu razinu [χ^2 (3) = 2.461, p = .482]. Kada se pogrješne predodžbe, vezane za ovo pitanje, usporede po skupinama, može se vidjeti da je *Tip II* češći od *Tipa I*. Točni odgovori po skupinama (24.4%, 32.7%, 35.8% i 41.4%) pokazuju porast u odnosu na obrazovnu razinu, kao rezultati Fischbeina i Gazita (1984). Slično tome, CACJ (točan odgovor, točna prosudba) omjer po skupinama (6.2%, 6.3%, 16.6% i 15.1%) blago je povećan u odnosu na tu razinu. Štoviše, CANJ (točan odgovor, nema prosudbe) omjer (56.3%, 52.1%, 43.8% i 35.9%) pokazuje smanjenje u odnosu na razinu obrazovanja, a CAIJ (točan odgovor, netočna prosudba) omjer (37.5%, 41.6%, 39.6% i 49.0%) porast u odnosu na tu istu razinu.

Kada je riječ o Q4, najveći broj točnih odgovora bilježimo kod učenika petih (42.8%) i sedmih razreda (39.6%), a najmanji kod učenika osmih razreda (27.3%).

Kada se pogledaju odgovori, uočava se da učenici koji u pitanju Q4 pokazuju pogrješne predodžbe imaju različite prosudbe za svoje netočne odgovore. Neki su učenici pod utjecajem općeg čimbenika slučajnosti, a neki percepcije da je vjerojatnost nekog događaja jednaka. Na primjer, „*Ne mogu to komentirati jer ovisi o slučajnosti*" (G5; G6), „*Vjerojatnosti su jednake jer su brojevi jednom napisani i to su isti brojevi*" (G5; G6), „*vjerojatnosti su jednake jer su krugovi isti*" (G5), „*Budući da su svi brojevi isti, vjerojatnost dobivanja istih brojeva je veća. Jer su se mjesta brojeva na krugovima promijenila*" (G5; G6; G7; G8), „*vjerojatnost dobivanja istih brojeva je veća* jer se *krugovi istovremeno okreću*" (G6; G7), „*vjerojatnosti su jednake jer je isto u (1,1), (2,2) i (3,3), a različito u (1,2), (1,3) i (2,3)*" (G7). Kao što je spomenuto, učenici misle da vjerojatnost nekog događaja prirodno ovisi o slučajnosti pa tako događaji imaju jednake vjerojatnosti. Fischbein i sur. (1991), Lecoutre (1992), Pratt (1998), Amir i Williams (1999), Batanero i Serrano (1999), Baker i Chick (2007), te Nilsson (2007) navode slične zaključke u svojim istraživanjima.

Paralelno s rezultatima Fastova istraživanja (1997), neki su učenici na primjeru pitanja Q4 pokazali pogrješne predodžbe zato što nisu dobro razumjeli pitanje ili su se usredotočili na vještine osoba koje okreću krugove. Primjerice, „Budući da ga istovremeno okreću i zaustavljaju, vjerojatnost je događaja *(3, 1) veća*" (G7), „*budući da ima više neparnih brojeva na krugovima, vjerojatnost neparnog broja je veća*" (G5), „*nemoguće je dobiti iste brojeve jer su brojevi pomiješani*" (G5)", „*Veća je vjerojatnost dobivanja broja 2 jer se položaj broja 2 nije promijenio*" (G6; G8), „*ne mogu ništa reći prije nego što vidim osobu koja okreće krugove*" (G5), „ako djeca okreću *krugove, to ovisi o slučajnosti. Ali ako ga okreću odrasli, ovisi na koji broj ciljaju*" (G6; G8). Ovdje se može vidjeti da neki učenici misle kako su ishodi pod nadzorom onih koji okreću krugove. Amir i Williams (1999), Greer (2001) i Sharma (2006) izvješćuju o sličnim zaključcima svojih istraživanja.

U ovom istraživanju većina je učenika pogrješno odgovorila na pitanje Q4 jer su se više usredotočili na položaj brojeva na krugovima nego što su povezali pitanje s prostorom elementarnih događaja, a to odgovara onome o čemu su izvijestili Jones i sur. (1997), Gürbüz (2007) te Gürbüz i sur. (2010). Nekolicina učenika koja nije bila svjesna odnosa između vjerojatnosti i prostora elementarnih događaja nije mogla razviti sustavnu tehniku proizvodnje svih mogućih ishoda, pa nisu ni mogli pronaći točan rezultat. Na primjer, neki su učenici mislili da su (1, 2) i (2, 1) istovjetni te su proizveli netočna rješenja. Baker i Chick (2007) kao i Gürbüz (2010) došli su do sličnih rezultata s paralelnim pitanjem. Većina učenika koji su točno odgovorili na to pitanje nisu mogli prosuditi vlastite razloge ili su im prosudbe, kao što ćemo vidjeti iz njihovih odgovora, pogrešne. Možda nisu mogli navesti točne razloge zbog nedostatka znanja o prostoru elementarnih događaja i o razlomcima. Takav rezultat potvrđuje ono o čemu pišu Carpenter i sur. (1981), Jones i sur. (1997), Ritson (1998), Polaki (2002) te Chernoff (2009).

Grafikon 5

Tablica 7

Prema Tablici 7, omjeri pogrješnih predodžbi po skupinama (75.6%, 73.5%, 69.4% i 68.8%) pokazuju djelomično smanjenje u odnosu na obrazovnu razinu, ali ta promjena nije značajna [χ^2 (3) = 2.461, p = 0.553]. Slično tome, Dooren i sur. (2003) u svome istraživanju nisu pronašli povezanost između obrazovne razine i pogrješnih predodžbi. Kada se pogrješne predodžbe usporede po skupinama, može se primijetiti da je Tip II češći od Tipa I. Omjeri točnih odgovora po skupinama (16.0%, 16.3%, 20.1% i 20.3%) sasvim su niski i pokazuju blagi porast u odnosu na obrazovnu razinu. Međutim, nije otkrivena nikakva povezanost između CACJ (točan odgovor, točna prosudba) (9.5%, 20.8%, 7.4% i 19.2%), CANJ (točan odgovor, nema prosudbe) (66.6%, 33.3%, 70.4% i 65.3%) i CAIJ (točan odgovor, netočna prosudba) (23.8%, 45.9%, 22.2% i 15.3%) omjera po skupinama i obrazovne razine.

Potrebno je primijetiti da točni odgovori po skupinama pokazuju djelomični porast kada je riječ o obrazovnoj razini, ali ti omjeri ostaju sasvim niski. Osim toga, većina učenika na svim obrazovnim razinama pokazuje pogrješne predodžbe. Najčešća pogrješna predodžba, koja se javlja u svim skupinama, je ona drugoga tipa. Nekoliko je istraživanja problematiziralo slične pogrješne predodžbe (Amir i Williams, 1999; Çelik i Güneş, 2007; Fischbein i sur., 1971; 1991; Fischbein i Gazit, 1984; Lecoutre i Durand, 1988). Na primjer, Lecoutre i Durand (1988) su otkrili da je većina sudionika odgovorila netočno, da postoji jednaka vjerojatnost za dva događaja te da je ta predrasuda vrlo otporna (Vidi u Fischbein i sur., 1991). Lecoutre i Durand (1988) su identificirali nekoliko „modela" kojima se ispitanici koriste da bi svoje odgovore potkrijepili prosudbama. U slučaju odgovora „ dvije su vjerojatnosti različite" pronašli su prosudbu kao što je ova: „Rjeđe se dva puta dobiva isti rezultat". Fischbein i sur. (1991) te Çelik i Güneş (2003) dobili su slične rezultate kao Lecoutre i Durand (1988). Ovdje se može tvrditi da pouka o vjerojatnosti u različitim uvjetima učenja s različitim učiteljima, starija dob kao i daljnji mentalni razvoj nisu učinkoviti kada je riječ o otklanjanju tih pogrješnih predodžbi.

Analizirajući prirodu pogrješnih predodžbi, može se uočiti da gotovo svi odgovori potvrđuju kako su dva ishoda jednako vjerojatna. Zamisao da je dvostruko vjerojatnije dobiti par 5-6 nego par 6-6- može proisteći samo iz reprezentacije o odgovarajućem prostoru elementarnih događaja. Osnovna prosudba koju djeca navode u oba slučaja odnosi se na slučajne događaje. Navodimo sljedeće odgovore: *„ista je vjerojatnost jer se može dobiti 6-6 i 6-5 ili nijedno od toga"* (G5; G6; G7), *„po mome mišljenju vjerojatnost je ista jer dobiveni broj predstavlja iznenađenje "* (G5; G6; G8), *„vjerojatnost je ista jer su isti brojevi 6 i 5 na objema kockama"* (G5; G6; G7), *„ista je vjerojatnost jer se ne može odrediti nešto što ovisi samo o kretanju jednog malog predmeta kao što je kocka, koju svaka osoba baca različito"* (G6; G7; G8), *„jer su kocke neovisne P(a) = 1/6.1/6=1/36, P(b) = 1/6.1/6 = 1/36"* (G7; G8), *„kocke su neovisne P(6)=1/6, P(5)=1/6, pa je odgovor c"*

(G7; G8), *„budući da kocka teži dobiti veće brojeve, odgovor je b"* (G5; G7), *„ishod se ne može predvidjeti jer kocke udaraju jedna o drugu kada se bacaju"* (G7; G8). Ovi učenici su uglavnom otkrili vjerojatnost odabira svake lopte umjesto onoga što se pitalo. Naprimjer, "*P(b)=3/9, P(g)=4/9 i P(r)=2/9*" (G5; G7), *"P(g)=4/9 P(r)=1/7 i P(b)=2/7"* (G6; G8), *"P(g)=4/9, P(r)=1/9 i P(b)=2/9"* (G6; G7).

Kada se pregledaju netočni odgovori na pitanje Q5, može se vidjeti da je njihov razlog uglavnom u nedostatku znanja o konceptu prostora elementarnih događaja i tipovima događaja. Dvije glavne ideje se tada osobito koriste da bi se prosudilo o jednakoj vjerojatnosti dobitka 6-6 i 5-6: (a) primitivnija ideja da su oba događaja rezultat slučajnosti pa nema razloga očekivati jedan događaj više od drugoga; i (b) sofisticiranija ideja da su 5 i 6 jednako vjerojatni pa tako svaki događaj, predstavljajući binarnu kombinaciju, ima jednaku vjerojatnost. Primjećeno je štoviše da su učenici bili svjesni vjerojatnosti nekog događaja, ali zbog pogrješnih pristupa nisu mogli ponuditi točna rješenja pri određenju prostora elementarnih događaja i događaja. Ovi rezultati uglavnom potvrđuju rezultate što su ih dobili Lecoutre i Durand (1988), Fischbein i sur. (1991) te Çelik i Güneş (2007).

Grafikon 6

Tablica 8

Kada se pogleda Tablica 8, može se vidjeti da povećani omjeri pogrješnih predodžbi po grupama (54.9%, 62.6%, 56.0%, i 57.8%) prate povećanje obrazovne razine. No, kao što su utvrdili Dooren i sur. (2003), to povećanje ne pokazuje značajnu razliku [χ^2 (3) = 1.993, p = .574]. Kada se ovdje usporede pogrješne predodžbe po grupama, uočava se da je Tip II češći od Tipa I. Omjeri točnih odgovora po grupama (38.2%, 30.6%, 35.1% i 29.7%) pokazuju povećanje u odnosu na obrazovnu razinu. Međutim, CACJ (točan odgovor, točna prosudba) (12.0%, 17.8%, 21.3% i 23.7%) i CAIJ (točan odgovor, netočna prosudba) omjeri po grupama se povećavaju (24%, 60%, 34% i 47.7%) s obrazovnom razinom. Njihovi CANJ (točan odgovor, nema prosudbe) omjeri (64%, 22.2%, 44.7% i 28.9%) pokazuju smanjenje u odnosu na obrazovnu razinu.

Utvrđeno je da učenici koji su imali pogrješne predodžbe o pitanju Q6 imaju različite prosudbe za svoje pogrješne odgovore. Na primjer, neki su se učenici koristili općim čimbenikom slučajnosti, drugi su imali percepciju da će vjerojatnosti biti jednake, a mali broj njih je dao nelogične prosudbe. *„Obitelji Muse i Meryem trebalo bi istražiti. Onaj tko je rođen u sretnijoj obitelji bit će pobjednik."* (G7; G8), *„Onaj koji ima svoj sretan dan je favorit jer ovisi o slučajnosti."* (G7; G8), *„Budući da je 3 pola od 6, oboje će imati priliku pobijediti."* (G5), *„Musa i Meryem imaju istu priliku zato što je na kocki jedan broj 3 i jedan broj 6."* (G6; G7; G8), "*3+6=9:2=4,5 njihove šanse za pobjedu su jednake."* (G6), *„Musa i Meryem imaju istu priliku. Zato što su 3 neparna i 3 parna broja."* (G6; G8), *„Meryem i Musa imaju istu priliku jer se ne zna koji će broj dobiti."* (G5; G7; G8), *„Meryem i Musa imaju istu priliku jer je 3 premalo, a 6 previše."*

(G5; G6), *„Može se shvatiti na kraju igre. U ovom trenutku ne možemo ništa reći."* (G8), *„Musa je moj favorit. Zato što je vjerojatnost za male ishode veća kod kocke."* (G6; G7). Ovdje se može primijetiti da učenici, što su stariji i na višem obrazovnom stupnju, nastoje odgovore na ovo pitanje povezati s čimbenikom slučajnosti. Drugim riječima, smatrali su da svaki događaj prirodno ovisi o slučajnosti te da stoga ima jednaku vjerojatnost. Fischbein i sur. (1991), Lecoutre (1992), Pratt (1998), Amir i Williams (1999), Batanero i Serrano (1999), Baker i Chick (2007), te Nilsson (2007) izvijestili su o sličnim rezultatima u svojim istraživanjima.

Neki su učenici s pogrješnim predodžbama odgovarali koristeći se vjerojatnim ishodima, kao što navode Amir i Williams (1999). Na primjer, *„Musa je moj favorit. Zato što kada su ishodi (1,2) i (2,1), njihov bi broj bio 3. Međutim, da bi dobili 6, samo je jedan ishod (3,3)"* (G7), *„Musa i Meryem imaju istu priliku. Jer za 3, (1,2),(2,1) i za 6, (1,5),(5,1)"* (G7; G8). Za odgovore koji sadrže ovakve pogrešne predodžbe moglo bi se reći da im nedostaje potrebno znanje o konceptu prostora elementarnih događaja. Paralelno s tim, Keren (1984), Fischbein i sur. (1991), Polaki (2002), Baker i Chick (2007), Gürbüz (2007; 2010), Nilsson (2007) te Chernoff (2009) pokazali su u svojim istraživanjima da znanje učenika o tom konceptu igra važnu ulogu u njihovim odgovorima na pitanja o vjerojatnosti. Nekolicina je učenika odgovorila s pogrješnim predodžbama razmatrajući čimbenik roda, kao što navode Amir i Williams (1999). Na primjer, *„Musa je favorit. Zato što muškarci imaju više sreće u ovoj vrsti igara na sreću."* (G5; G7; G8), *„Meryem pobjeđuje jer su djevojčice sretne ruke."* (G6; G7). Prema Tablici 8, može se vidjeti da pouka o vjerojatnosti, dob I mentalni razvoj nisu učinkoviti kada je riječ o otklanjanju pogrješnih predodžbi povezanih s ovim pitanjem.

Neki su točni odgovori na pitanje Q6 zasnovani na pogrješnim razlozima, kao što pokazuje Tablica 2. Neki su učenici ovdje opisali sve moguće parove, ali nisu shvatili da se svaki par različitih brojeva mora dva puta razmotriti, odnosno redoslijed treba također uzeti u obzir. Na sličan su se način neki učenici usredotočili samo na magnitudu. Na primjer, *„Meryem ima prednost jer on ima veći broj"* (G7), *"Meryem ima veću prednost jer je s tim vjerojatnije dobiti veći broj"* (G8), *„Meryem je favorit jer je 6 veće od 3"* (G7; G8), *„Meryem ima prednost jer s dvjema kockama skoro uvijek dobivaš brojeve veće od 3"* (G6). Čini se da postoji samo jedno objašnjenje: neki su učenici birali veći broj kao favorita bez obzira na prostor elementarnih događaja. U literaturi su slični učenički pristupi sličnim pitanjima vrjednovani kao racionalno razmišljanje (Fischbein & Schnarch, 1997; Greer, 2001; Hope & Kelly, 1983; Kahneman & Tversky, 1972; Polaki, 2002; Shaughnessy, 1993). U pitanju Q6 doista je vjerojatnije da će veći broj 6 pobijediti pa je njihov odabir naoko točan (iako se u biti temelji na pogrešnom razlogu). Dva su glavna tipa prosudbi u prilog većem broju: a) neki su učenici jednostavno odabrali veći broj jer „veći je"; b) drugi su nastojali odrediti parove koji bi postigli dobitak 6 i 3, ali su ozbiljno pogriješili birajući parove čiji zbroj postiže 3 ili 6. Kao u slučaju ostalih pitanja, smatra se da su prosudbe učenika u pitanju Q6 pod utjecajem njihova individualnog učenja, iskustava, kultura i uvjerenja. Amir i

Williams (1999), Fischbein i sur. (1991), Shaughnessy (1993), Fischbein (1991; 1995) te Sharma (2006) navode slične rezultate u svojim istraživanjima.

Učenici su općenito intuitivno svjesni odnosa između vjerojatnosti nekog događaja i prostora elementarnih događaja. No, utvrđeno je da nedostatno raspolažu znanjem o prostoru elementarnih događaja kada je riječ o problemima vjerojatnosti. Nekim učenicima nedostaje sustavna tehnika izvođenja svih mogućih ishoda povezanih s nekim događajem. Veličina prostora elementarnih događaja uglavnom utječe na vrjednovanje učenika pri rješavanju problema vjerojatnosti (Greer, 2001; Gürbüz, 2007; 2010; Gürbüz, Çatlıoğlu, Birgin i Toprak, 2009; Stavy i Tirosh, 1994). Čini se da učenici bolje procjenjuju vjerojatnost nekog događaja čiji je prostor elementarnih događaja malen. Postoci točnih odgovora na pitanja Q1 i Q5 mogu se, na primjer,usporediti da bi se potvrdila navedena poveznica. Koncept prostora elementarnih događaja nije koncept koji se može razviti sam po sebi. Da bi se razvio, potrebni su različiti eksperimenti, dok prostor elementarnih događaja u tim eksperimentima treba biti istaknut, kao što to određuju Vidakovic i sur. (1998), Speiser i Walter (1998) te Gürbüz i sur. (2010).

Opća diskusija

Cilj ovoga rada jest usporediti i vrjednovati pogrješne predodžbe što ih učenici različitog stupnja obrazovanja imaju o konceptima vjerojatnosti. Test koji je sadržavao 3 pitanja o konceptima složenog događaja I i II te 3 pitanja o konceptima vjerojatnosti nekog događaja i usporedbama vjerojatnosti (ukupno 6 pitanja) bio je posebno pripremljen i primijenjen na skupinama. Pri usporedbi praznih odgovora na pitanja Q1, Q5, Q6, odnosno Q2, Q3, Q4 primijećeno je da su omjeri praznih odgovora na pitanja Q1, Q5 i Q6 značajno veći od onih u slučaju pitanja Q2, Q3, Q4. To se može objasniti činjenicom da su pitanja Q1, Q5 i Q6 prikladnija za format višestrukog izbora. Gotovo svi glavni ispiti u Turskoj imaju upravo taj format. Učenici različitog obrazovnog stupnja donekle su slično razumjeli iste koncepte vjerojatnosti. Paralelno s rezultatima istraživanja Doorena i sur. (2003), nije pronađena značajna razlika među pogrješnim predodžbama učenika na različitim obrazovnim razinama u pogledu nekih koncepata vjerojatnosti. To je prilično iznenađujuće jer bismo očekivali da se razumijevanje učenika povećava s višom razinom obrazovanja. Još više iznenađuje na primjeru pitanja Q3 to da porast pogrješnih predodžbi prati razinu obrazovanja (Tablica 5). Opći je razlog tomu činjenica da učenici uče o konceptima vjerojatnosti površno ili vrlo teorijski. Oni zapravo usvajaju te koncepte od četvrtoga razreda. Pojedincima je teško promijeniti znanje koje su stekli u svakodnevnom životu i formalnom obrazovanju, čak i kada su na višem stupnju obrazovanja. Mnoga su istraživanja doista pokazala da pojedinci odolijevaju promjeni postojećega znanja te da su teško vjeruju u promjenu (Fast, 1997, 2001; Garfield i delMas, 1991; Green, 1982; Hope & Kelly, 1983; Kahneman i Tversky, 1972; Konold, 1988, 1989; Shaugnessy, 1981). Osim toga, nastavna se metoda ne mijenja čak ni onda kada se povećava stupanj obrazovanja.

Činjenica da učitelj ima vrlo dominatnu ulogu u nastavi u Turskoj te da na satu uveliko ovisi o udžbenicima prije potiče učenje napamet nego konceptualno učenje. Učenici su tako primorani memorirati gradivo i davati odgovore kada im se postavljaju pitanja na ispitima. Na taj način ne mogu razvijati znanstveno shvaćanje ključnih koncepata, kao što su koncepti vjerojatnosti, ili pokazati vlastito razumijevanje kada im je ono potrebno u svakodnevnom životu ili znanstvenim situacijama. Unatoč činjenici da su glavni klasifikacijski ispiti usredotočeni više na ponavljanje činjenica nego na razumijevanje koncepata, nije to jedino što sprječava razumijevanje koncepata koje ovdje istražujemo. Ostali čimbenici kao što su nastavni proces, postojeće znanje učenika, učitelji i nastavni programi matematike utječu na njihovo razumijevanje. Učitelji u Turskoj ističu zastupljenost gradiva i poučavaju o jednostvnim pristupima ili trikovima za rješenje ispitnih problema. Iako je Ministarstvo obrazovanja strukturiralo nove nastavne programe iz matematike na temelju konstruktivizma i učenja s učenikom u središtu zanimanja, trebat će vremena da se postojeća situacija promijeni i da se učitelji uvjere u njihovu učinkovitost. Dakle, nadamo se da će s vremenom to novo nastojanje donijeti uspjeh.

Važno je koristiti se u nastavi tipom pitanja koja ovdje spominjemo. Iz naših rezultata proizlazi zaključak da mnoga djeca ne otkrivaju identičnu matematičku strukturu u praktično različitim situacijama. Ono što ne razumiju je da se situacije moraju promatrati kao sama utjelovljenja idealnih eksperimenata u kojima se matematika bavi idealnim, apstraktnim operacijama i entitetima. Učiti dijete da je vjerojatnost grana matematike i da se, samim time, povezuje s apstraktnim idealom i formalno definiranim predmetima poput geometrije ili aritmetike jedan je od glavnih nastavnih zadataka. Uspoređivati različita utjelovljenja identičnih matematičkih struktura učinkovit je postupak za postizanje toga cilja, ne samo formalno nego i intuitivno. Matematika ima povijesni karakter, a matematički su koncepti zapravo psihološki uvjetovani. Mijenjaju se definicije i tumačenja. Ipak, danas je općeprihvaćeno da se svaka matematička teorija zasniva na sustavu aksioma koji su eksplicitno formulirani. Aksiomi se mogu zamijeniti pod uvjetom da ne će dovesti do kontradiktornosti (Fischbein i sur., 1991). Matematika se stoga treba poučavati na temelju metoda koje bi predstavile navedeni pristup i koristile se sličnim pitanjima.

Rezultati su ovoga istraživanja pokazali da neki učenici ne razumiju slučajne događaje, a razmišljanja su im zasnovana na subjektivnim uvjerenjima. Jedan od najvažnijih čimbenika koji utječe na odgovore učenika jest ta da njihova sposobnost povezivanja slučajnih i uvjetovanih događaja nije potpuno razvijena (Gürbüz, 2007). Štoviše, ti su učenici formirali odgovore polazeći prije od svojeg subjektivnog razmišljanja nego od prave vjerojatnosti nekog eksperimenta. Na primjer, na pitanje Q3 odgovorili su na temelju položaja lopti u košari, dok su za pitanje Q4 odabrali položaj brojeva na krugovima kao polazište za odgovore. To ih je subjektivno zaključivanje udaljilo od znanstvenih odgovora. Ford i Kuhs (1991), Gibbs i Orton (1994), Kazıma (2006) te Tatsis i sur. (2008) utvrdili su u svojim istraživanjima da je jezični razvoj bitan u nastavi matematike na razini primarnog obrazovanja. Međutim, pokazalo se da neki učenici imaju poteškoće pri razumijevanju onoga što čitaju. Rezultati su štoviše pokazali da neki učenici shvaćaju važnost prostora elementarnih događaja kada daju odgovore na pitanja. No njihova svjesnost, utvrđeno je, više potječe od intuitivnih procjena i osobnih vještina pojedinca, a manje od stečenog obrazovanja. Može se reći da se intuitivna sposobnost spontano unapređuje s dobi.

Utvđeno je da neki učenici povezuju vjerojatnost sa slučajnošću ili svojim uvjerenjima i tako traže rješenja zanemarujući određene eksperimentalne uvjete te pokazuju predrasude prema tome predmetu. Ti su učenici, zbog nespremnosti na koncept vjerojatnosti, izjavili da treba uzeti u obzir vještine i sreću igrača kada je riječ o nekoj igri na sreću, a da Božju volju ne treba uplitati. U svojim su prosudbama pomiješali koncept slučajnosti, intuitivnu misao, osobnu vještinu i probabilno razmišljanje s uvjerenjima koja nisu ni znanstvena niti smislena. Navedeni je rezultat u skladu s onim koje navode Amir i Williams (1994), Fischbein (1991), Greer (2001) i Sharma (2006). Amir i Williams (1994) tvrde da se takve ideje često pojavljuju kod manje kompetentnih pojedinaca ili kod onih koji su odgajani u relativno religioznijim društvima.

Zaključci i implikacije

Istraživanje je pokazalo da s porastom obrazovne razine dolazi do povećanja postotka točnih odgovora, dok je pogrješnih predodžbi, povezanih s pitanjima o konceptu složenoga događaja I, manje. Međutim, pokazalo se također da se s porastom obrazovne razine smanjuje postotak točnih odgovora, a pogrješnih je predodžbi o pitanjima povezanim s konceptom složenih događaja II više. Povećanje u slučaju pogrješnih predodžbi ipak nije značajno kada je riječ o navedenom konceptu. Štoviše, pitanja Q2 i Q3, povezana s konceptom vjerojatnosti nekog događaja i uporedbama vjerojatnosti, pokazala su da se omjer točnih odgovora i pogrješnih predodžbi znatno povećao s razinom obrazovanja. Kod pitanja Q1 o konceptima vjerojatnosti nekog događaja i usporedbama vjerojatnosti nema poveznice između obrazovne razine i postotka točnih odgovora ili pogrješnih predodžbi. Može se također reći da se broj prosudbi povećava s višom razinom obrazovanja.

Čini se da su koncepti vjerojatnosti mnogo složeniji u brojnim aspektima nego što se obično smatra. Ako se nastavom namjerava razviti snažna, točna, koherentna, formalna i intuitivna podloga za probabilno razmišljanje, potrebno je voditi računa o nizu nerazumijevanja, pogrješnih predodžbi, predrasuda i emocionalnih sklonosti. Do takvih iskrivljenja može doći zbog jezičnih poteškoća, nedostatka sposobnosti logičnog razmišljanja, problema izdvajanja matematičke strukture iz praktičnog utjelovljenja ili prihvaćanja činjenice da se slučajni događaji mogu analizirati s determinističko-predvidivog stajališta. O sličnim su poteškoćama već pisali Jones i sur. (1997), Gürbüz (2007; 2010) te Gürbüz i sur. (2010).

Nema prirodnog prihvaćanja činjenice da kod prostora elementarnih događaja treba razlikovati i odvojeno promatrati moguće ishode ako je broj njihovih elementarnih komponenti različit. Može se odmah pomisliti: koliko je važno intuitivno znanje kada se određuje vjerojatnost nekoga događaja? Ili, igrali li veličina prostora elementarnih događaja ulogu kada se određuje vjerojatnost nekoga događaja? Fischbein i sur. (1991), Vidakovic i sur. (1998), Speiser i Walter (1998), Polaki (2002) te Gürbüz (2007) u svojim su istraživanjima pronašli da i intuitivno znanje i znanje o prostoru elementarnih događaja imaju ulogu kada se određuje takva vjerojatnost. Rezultati u ovome radu također to potvrđuju.

Prema rezultatima Fischbeinova rada (1975), sposobnost sinteze nužnog i slučajnog u koncept vjerojatnosti predstavljaju dva najvažnija čimbenika koja utječu na neke učenike kada određuju odgovore na pitanja. Zamisao je ta da ishod stohastičkog eksperimenta ovisi samo o slučajnosti - bez obzira na uvjete. Ovo je vrlo važno otkriće te mu učitelji i didaktičari trebaju pažljivo pristupiti.

U tradicionalnim nastavnim sredinama učenike se uglavnom poučava tako da se u središtu nalazi učitelj koji rješava određene primjere i vježbe. Pratt (2000), Gürbüz i sur. (2009; 2010) kao i Gürbüz (2010) ističu u svojim istraživanjima da se pogrješne predodžbe i netočna intuicija u pitanju vjerojatnosti trebaju otklanjati uz pomoć praktičnih vježbi s materijalom jer se učenici prije uvjere u eksperimentalne rezultate nego u teorijske dokaze. Uporaba materijala (pikado, krug, kocka, novčić, karte za igru, itd.) u nastavi o vjerojatnosti, kao i razmatranje mogućih pogrješnih predodžbi, smatraju se važnim za točno razumijevanje ove teme. Dakle, kada poučavamo o vjerojatnosti, trebalo bi više provoditi nastavu uz primjenu pomagala u kojoj su učenici aktivni.

U svjetlu rezultata ovoga istraživanja i odgovarajuće literature može se zaključiti da su pogrješke i pogrješne predodžbe međusobno vrlo slične unatoč promjeni njihovih frekvencija. To pokazuje da različite nastavne metode, teorije učenja i infrastrukturna oprema u nastavnim sredinama nisu dovoljno učinkovite da bi se potpuno otklonile sve pogrješke i pogrješne predodžbe. Osim nastavnih sredina i materijala, potrebna su daljnja istraživanja da bi se odredili čimbenici koji utječu na konceptualno učenje i pogrješne predodžbe.

Konačno, treba spomenuti jedno stajalište na koje se još nismo osvrnuli u ovome radu, iako se nalazi među našim rezultatima: razlike među učiteljima. Otkrili smo da postoje značajne razlike u rezultatima učenika koje poučavaju različiti učitelji. Ne postoje neki važni čimbenici u školama koji mogu poboljšati rad učitelja u Turskoj. U tom se slučaju može tvrditi da je najvažniji čimbenik pri oblikovanju učiteljevih rezultata njegova savjest. S toga se gledišta može zaključiti da bi učitelji trebali biti moralno motivirani da dobro obavljaju svoje zadatke i praćeni konkretnom evaluacijom njihova rada.