THE ANSWERING PROCESS FOR MULTIPLE-CHOICE QUESTIONS IN COLLABORATIVE LEARNING: A MATHEMATICAL LEARNING MODEL ANALYSIS

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ABSTRACT

In this paper, we introduce a mathematical model for collaborative learning and the answering process for multiple-choice questions. The collaborative learning model is inspired by the Ising spin model and the model for answering multiple-choice questions is based on their difficulty level. An intensive simulation study predicts the possibility of modification of the Nitta model, which describes the transition of the number of students answering multiple-choice questions correctly following discussions among students, using a master equation.

KEYWORDS

Collaborative learning, Mathematical learning model

1. INTRODUCTION

In recent years, the theoretical study of teaching-learning processes has attracted the attention of an increasing number of scientists. Early studies on those processes have been conducted by psychologists and sociologists (Piaget, 1929; Vygotsky, 1978). Although this topic is so complex that there remain many open questions, the study of cognitive processes has developed into an active area of multidisciplinary investigation as the number of physicists interested in research areas such as economics, social science, biology, et cetera, increases. Since Hake (1998) reported that student performance can be enhanced using a teaching approach involving collaborative group work, in contrast to traditional non-interactive lectures, the processes of learning and understanding physics and mathematics have become the focus of cognitive research. In order to further the research in this area, a mathematical model of the teaching-learning process is proposed and studied in this paper.

The aim of using a mathematical model to study the teaching-learning process is to investigate the influence of the structure of group work on student achievement and to design effective curricula. We classify existing mathematical models of the teaching-learning process into three categories: Ising spin modeling, Differential equation modeling, and Stochastic process modeling.

In relation to mathematical modeling of the teaching-learning process, various models have been proposed. For example, Bordogna and Albano (2001) proposed a mathematical model of the teaching-learning process in a classroom using the constructive approach. In this model, interactions between students and teachers are described with a set of equations similar to those that describe magnetism in materials. Pritchard et al. (2008) proposed models of the teaching-learning process using a differential equation. These models are based on various theories of learning: tabula rasa, constructivist, and tutoring. They predict an improvement in the post-test as a function of the pre-test score, depending on the type of instruction given. Although the model itself is quite simple, and an exact solution can be obtained analytically, it fits existing data by sharply determining a parameter. However, these models do not describe collaborative learning among students.

In contrast to the models above that describe long-term learning gain, Nitta's (2010) model describes a short-time learning process. He developed a phenomenological theory of peer instruction and modeled the transition of the number of students answering correctly for multiple-choice questions (MCQs) following discussions among students, using a master equation. The master equation was simplified analytically and he demonstrated that the number of correct answers after peer discussion is approximately given by a simple function of the number of correct answers before discussion. The theoretical curve agrees with data obtained from lectures implementing the peer instruction. However, it is impossible to ignore the assumption involved in the process of simplifying the original master equation. In this paper, we check this reasonability using a mathematical learning model, and in particular a model of the answering process for MCQs.

This paper is organized as follows. First, we briefly review the Nitta model and point out issues related to this model. In section 3, we introduce the mathematical learning model used to analyze the Nitta model. In the final section, we summarize this paper.

2. BRIEF REVIEW OF THE NITTA MODEL

Nitta modeled the transition of the number of students answering correctly for MCQs after discussions among students, using the following master equation

$$\rho_2(c) = \rho_1(c) + \sum_{d \neq c} T_{cd}\, \rho_1(d) - \sum_{d \neq c} T_{dc}\, \rho_1(c), \tag{1}$$

where $\rho_1(c)$ is the ratio of students who select the correct answer and $\rho_1(d)$ is distractors before discussion. $\rho_2(c)$ is the ratio of students who select the correct answer after discussion. Note that $\rho_{1,2}(c) = 1 - \sum_{d \neq c} \rho_{1,2}(d)$. $T_{cd}(0 \leq T_{cd} \leq 1)$ is the transition probability of selecting distractors before discussion to selecting the correct answer after discussion. With the assumption of $T_{cd} \simeq \bar{T}$ and $T_{dc} \simeq 0$, equation (1) can be rewritten as

$$\rho_2(c) = \rho_1(c) + \bar{T}\big(1 - \rho_1(c)\big). \tag{2}$$

Furthermore, $\bar{T}$ is assumed to expand into the power series of $\rho_1(c)$ as

$$\bar{T} = k_0 + k_1\rho_1(c) + k_2\big(\rho_1(c)\big)^2 + \cdots + k_n\big(\rho_1(c)\big)^n \cdots. \tag{3}$$

When $\rho_1(c)$ is small enough, $k_0 \simeq 0$ is derived because the transition probability from selecting distractors to selecting the correct answer after discussions is negligible. Then equation (3) becomes

$$\bar{T} \simeq k_1\rho_1(c) \tag{4}$$

under the linear assumption. Simple analysis leads to the relation

$$\rho_2 = \rho_1 + \rho_1(1 - \rho_1) \tag{5}$$

with $k_1 = 1$ from equations (2) and (4). Here, we rewrite $\rho_{1,2}(c)$ as $\rho_{1,2}$ for simplicity.

Equation (5) is the main result of the Nitta model but we have questions regarding equation (4). When $\rho_1(c)$ is not very small, in other words, in the case of an easy question, equation (3) cannot be approximated as equation (4). Therefore, the main result (5) of the Nitta model does not include the effect of question difficulty.

In the next section, we introduce a mathematical model for collaborative learning and answering MCQs that includes difficulty information.

3. MATHEMATICAL LEARNING MODEL

3.1 Collaborative Learning Model

Few mathematical models of collaborative learning have thus far been developed. Among those that have been developed are, for example, Bordogna and Albano's (2001) Ising spin model and related models (Yasutake, 2011; Ogawa et al., 2013). Our model is based on the Ogawa model. When students engage in group discussion, each student has the cognitive impact of the student-student interaction in a group described as

$$C_i^{SS}(t) = \sum_{j=1, j\neq i}^{N} \left[I_{ij}(t)\big(1 - C_i(t)\big) \frac{S_j(t) + S_i(t)}{2} \right], \tag{6}$$

where $C_i^{SS}(t)$ is the cognitive impact on the student i at time t, $I_{ij}(t)$ is the student-student interaction, $C_i(t)$ is the confidence of student i, and $S_i(t)(0 \leq S_i(t) \leq 1)$ is the knowledge level of student i at time t. The student's knowledge level is updated according to the cognitive impact level as

$$S_i(t+1) = \begin{cases} S_i(t) + \Delta S \text{ with the probability } e^{\beta(C_i^{SS}(t)-\alpha)}/Z \\ S_i(t) \text{ with the probability } 1/Z \\ S_i(t) - \Delta S \text{ with the probability } e^{\beta(-C_i^{SS}(t)-\alpha)}/Z \end{cases} \tag{7}$$

where $Z = e^{\beta(C_i^{SS}(t)-\alpha)} + 1 + e^{\beta(-C_i^{SS}(t)-\alpha)}$ and $\Delta S = (1 - 2|S - 1/2|)/10$.

3.2 Mathematical Model for answering MCQs

In this paper, MCQs have four answer choices. When a student i has a knowledge level $S_i(t)$ at the time of answering the MCQ with a difficulty level of $D(0 \leq D \leq 1)$, the probability of the student choosing a correct answer is

$$\text{Prob} = \frac{\tanh\{2(S_i(t) - D)\} + 1}{2}. \tag{8}$$

Equation (8) means that the probability of a student choosing the correct answer is approximately 1/2 when there is no difference between knowledge level and difficulty level and this probability becomes 1 (or 0) when the difficulty level is low (or high).

4. SIMULATION

Students answer MCQs according to the probability (8) before discussion (at time $t = 0$) and collaborative learning occurs in each group according to the knowledge dynamics (6) and (7). After discussion (we set $t = 100$ in this paper), students again answer MCQs with a knowledge level of $S_i(100)$. Using this simulation, the percentage of questions answered correctly before and after discussion can be calculated. We set the initial knowledge level randomly between $S_i(0) = 0.01$ and $S_i(0) = 0.99$ and made a simulation 33,000 times for each difficulty level of MCQ. We then averaged the percentage $\bar{\rho}_1$ and $\bar{\rho}_2$ of a question answered correctly before and after discussion and averaged the transition probability $\bar{T}$. We can calculate k_1 and compare the assumption $k_1 = 1$ set by Nitta.

The simulation result is summarized in Table 1. As we can see from Table 1, the coefficient k_1, which was assumed to be 1, varies according to difficulty level D.

Table 1. Relations between $D, \bar{\rho}_1, \bar{\rho}_2, \bar{T}$, and k_1.

D	$\bar{\rho}_1$	$\bar{\rho}_2$	$\bar{T}$	k_1
0.0	0.827	0.943	0.674	0.815
0.2	0.718	0.888	0.648	0.903
0.4	0.573	0.814	0.595	1.038
0.6	0.427	0.712	0.515	1.206
0.8	0.286	0.565	0.400	1.399
1.0	0.172	0.389	0.268	1.558

5. CONCLUSION

We made a simulation using a mathematical model for collaborative learning and answering MCQs in order to improve the validity of the assumption introduced in the Nitta model. From our simulation results, the coefficient k_1, which was assumed to be 1, varies according to difficulty level *D*. This means that Nitta's formula, equation (5), could be modified by introducing difficulty information for MCQs. The theoretical curve equation (5) seemed to agree with data obtained from lectures implementing peer instruction, but further investigation with a large sample of empirical data could prove our prediction from a simulation study. This is future work.

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REFERENCES

Bordogna, C. M. and Albano, E. V., 2001. Theoretical description of teaching-learning processes: A multidisciplinary approach, *Phys. Rev. Lett.*, Vol. 87, No. 11, pp. 118701-1-4.

Hake, R. R., 1998. Interactive-engagement vs. traditional methods: A six-thousand-student survey of mechanics test data for introductory physics courses, *Am. J. Phys.*, Vol. 66, No. 1, pp. 64-74.

Nitta, H., 2010. Mathematical theory of peer-instruction dynamics, *Phys. Rev. ST Phys. Educ. Res.*, Vol. 6, pp. 020105-1-4.

Ogawa, K. et al, 2013. Mathematical model for collaborative learning: Acquiring hierarchic-structured knowledge, *Proceedings of the 21st International Conference on Computers in Education*, Bali, Indonesia, pp. 247-251.

Piaget, J., 1929. *The Child's Conception of the World*, Harcourt, Brace, New York.

Pritchard, D. H. et al, 2008. Mathematical learning models that depend on prior knowledge and instructional strategies, *Phys. Rev. ST Phys. Educ. Res.*, 4, pp. 010109-1-8.

Vygotsky, L. S. et al, 1978. *Mind in Society: Development of Higher Psychological Processes*, edited and translated by Cole, M. et al, Harvard University Press, Cambridge, Massachusetts.

Yasutake, K. et al, 2011. A simulation analysis on learning effects through interactions in networked communities (*in Japanese*), *Transactions of Japanese Society for Information and Systems in Education*, Vol. 28, No. 1, pp. 50-60.