# STEREO ORTHOGONAL AXONOMETRIC PERSPECTIVE FOR THE TEACHING OF DESCRIPTIVE GEOMETRY

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**ABSTRACT**

The representation of figures in mongean projection (double system planned orthographic projection used in the studies of Descriptive Geometry), specially when placed in a particular situation in relation to the projection plans, possesses the quality that, through them, the actual dimensions of represented spatial objects can be found directly and without additional geometric constructions. However, these representations using orthogonal views present a disadvantage that had been observed in which the representation is not always compared to the sight of the actual object. In some complex problems, even with great effort of concentration and imagination, the idea of the represented spatial figure cannot be reached.

For current students and future engineers and architects, spatial visualization in perspective and depth is of most importance. The binocular or stereo perspective is necessary for the improvement of students' spatial reasoning. For this, stereo visualization programs used in computer graphics can be used. They differ from the method of the cylindrical orthogonal axonometric perspective, often taught in the courses of Descriptive Geometry. Thus, it is proposed in this work, the use of computational method anaglyphic, in which the binocular axonometric perspective is obtained from the mongean projections of the object. The developed program is indicated for the teaching/learning Descriptive Geometry.

**KEYWORDS**

Computer graphics; descriptive geometry; axonometric stereoscopic visualization; geogebra software

## 1. INTRODUCTION

The great evolution of computing and electronics allowed the use of new technological resources for the teaching/learning of disciplines on graphics area, such as Descriptive Geometry, Technical Drawing, Space Geometry, etc. This way, we can find some studies in the literature that have been developed in the search for computational solutions to enhance the teaching of such subjects.[18] proposes an intelligent computing environment accessed via Internet, which operates according to the profile of each student. The system presents a database of exercises that must be registered by teachers. The environment has not been tested yet. Also, we found studies that indicate the potential for stereoscopic computer graphic systems for the development of spatial visualization skills [13] and virtual reality techniques to be employed. It is suggested to use a stereoscopic display system using polarized projections, in which students would attend classes using passive polarized glasses. However, the system was not implemented. Another work of the area has a graphical tutorial system, available on the Internet for teaching Descriptive Geometry. The system consists of written content, accompanied by illustrative animations. The system, however, is not intended to be used in classroom, and does not explore stereoscopic capabilities or real-time interactivity [8]. In a more recent study, the author has been making use of augmented reality techniques [2], [4] for visualizing and learning of conical surfaces [9]. [16] implemented the Hipercal3D software, which utilizes virtual reality through language VRML, for the viewing and manipulation of objects generated with interactivity using three-dimensional visualization perspective and solid modeling for descriptive geometry, but it does not use features of stereoscopy. [7] implemented the VirtualGD software, also using the VRML language [19], [20] to create a lively and interactive virtual environment, with manipulation of objects in real time, showing the key elements and processes used in Descriptive Geometry. In another subsequent work [6], the authors present the use of stereoscopy techniques implemented to VirtualGD. The Descriptive Geometry software [17], developed at the University of Lisbon, can be freely downloaded from the website. The program is interactive and has offices in both epure and space representation. The system interface is somewhat overwhelmed by too much information windows and it is not very suitable for beginning students. Since the program AEIOU Descriptive Geometry [1] has many graphics and a clear and friendly interface, it is a commercial software. Both programs do not offer resources of stereoscopy. The use of stereoscopic techniques has great potential for the improvement of spatial visualization ability, particularly for individuals with greater difficulties, because they allow the understanding of spatial (three-dimensional) situations presented in exercises, making its development possible. Stereoscopy is related to the ability of depth perception. A long time ago, it began to be used in order to facilitate the visualization of complex spatial situations and, particularly, in the study of Descriptive Geometry [11]. The first experiments used simple techniques like drawings anaglyphics [12]. Nowadays, with the technological development, more sophisticated devices are available, which opened new possibilities for the use of these techniques. For students, the spatial visualization in perspective and depth is of the most importance and for both the perspective using stereoscopy view is necessary.

In this work, we developed a program, using the GeoGebra educational software [5], in which the binocular axonometric perspective of a given object is created from the mongean projections. We observed that the most articles published in technical and scientific events in the area are still at the level of academic research and are not therefore available for use or do not explore stereoscopic vision. On the other hand, we find some commercial products available, but not always perfectly suited to our educational needs.

We first present the usual matrix method used by computer graphics programs, [3], [15], [21] and that is implemented in C language using the OpenGL graphics library, [10], [14] and later we will introduce the proposed method, which uses orthogonal axonometric perspective and the GeoGebra software [5].

## 2. STEREO ANAGLYPHIC PROJECTION

Stereoscopic vision consists of depth scene perception by the human visual system. For this purpose, it is necessary to capture the image of the object by the two eyes so the brain can estimate the distance between them, it means, between the eyes and the object. We will be using the anaglyphic method, in which the image of the left eye is colored with cyan and the right eye is colored with red.

## 2.1 The Traditional Matrix Method

The following section presents the method commonly used in computer graphics stereoscopic vision and implemented in C language, with the purpose of comparing the figures generated from those obtained by the proposed method, presented in section 2.2.

### 2.1.1 Visualization in Perspective using the Matrix Method in Homogeneous Coordinates

Consider the case in which the object is described by points $P = (x,y,z)$ having its coordinates given in relation to the orthonormal frame in the world and we want to design it (Figure 1) in a projection plan pp located between P and the eye of the beholder $e=(e_1,e_2,e_3)$. It is necessary to provide a point $a=(a_1,a_2,a_3)$ to set the direction to where one should look and as the observer's head orientation vector $\vec{u}_p = (u_{p1}, u_{p2}, u_{p3})$.

Thus the intersection of a point $P=(x,y,z)$ of space with the projection plan pp in relation to the observer's eye ***e*** is calculated by two coordinate transformations:

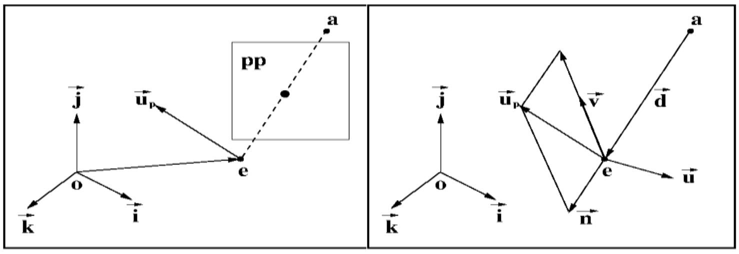


Figure 1. The projection plan (pp) and the referential of the observer's eye.

(I) **Frame in the observer's eye**

Consider the new orthonormal frame $(\vec{n}, \vec{v}, \vec{u})$, obtained from the world orthonormal frame by translation and rotations, with its center in the observer's eye and given by

$$\vec{n} = \frac{e-a}{|e-a|} \qquad \vec{v} = \frac{\vec{u}_p - (\vec{u}_p.\vec{n})\vec{n}}{|\vec{u}_p - (\vec{u}_p.\vec{n})\vec{n}|} \qquad \vec{u} = \vec{v} \times \vec{n}$$

So by a given point $P=(x,y,z)$ to pass to the coordinates of the eye $(X,Y,Z)$ using homogeneous coordinates just consider the transformation

$$(X,Y,Z) = \left(\frac{x'}{w}, \frac{y'}{w}, \frac{z'}{w}\right) \text{ where } \begin{pmatrix} x' \\ y' \\ z' \\ w \end{pmatrix} = \begin{pmatrix} u_1 & u_2 & u_3 & (-e).\vec{u} \\ v_1 & v_2 & v_3 & (-e).\vec{v} \\ n_1 & n_2 & n_3 & (-e).\vec{n} \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Using the OpenGL library [14] this transformation is given by the command ,

$$gluLookAt\,(e_1, e_2, e_3, a_1, a_2, a_3, u_{p1}, u_{p2}, u_{p3});$$

(II) **Normalizing the visualization region and leading the observer to infinity**

Consider now the dimensions of the window W (Figure 2) providing horizontal coordinates x=l (left), x=r (right), and the vertical coordinates y=b (botton) and y=t ( top) and the region of space where the point lies fixing the previous plan by the depth (near) z=-n and depth (far) z=-f. To calculate final coordinates $(X,Y)$, of the perspective projection of **P** on **pp**, one must now perform the following operations (Figure 2, right):

(a) shear H to centralize windows W about the depth given by the oz axis.

(b) adjust the LCD viewing pyramid for a change of scale S for side plans with 45 $^0$ in relation to the coordinated plans xz and yz

(c) finally use a transformation in perspective N taking the eye of the observer to infinity.

So **P'=(NSH)P** is obtained in homogeneous coordinates using the matrix

$$NSH = \begin{pmatrix} \frac{2n}{r-l} & 0 & \frac{r+l}{r-l} & 0 \\ 0 & \frac{2n}{t-b} & \frac{t+b}{t-b} & 0 \\ 0 & 0 & \frac{-(f+n)}{f-n} & \frac{-2nf}{f-n} \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

Using the OpenGL library, [14], this final transformation is presented in the command,

$$glFrustum(l, r, b, t, n, f);$$

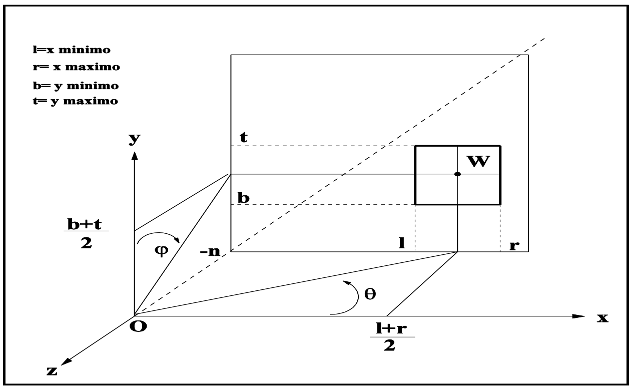


Figure 2. Parameters of the Window W

### 2.1.2 Stereo Anaglyphic viewing with the Matrix Method

By designing a point P on a projection plan (pp), according to the observer, we find the Pl projection on the left eye (le) and the corresponding Pr to right eye (re), which must be on the same horizontal line. The distance between the points designed Pl and Pr is called Parallax and three situations may occur, depending on the relative position of the point P in relation to the projection screen (pp) and the eyes of the observer:

(i) after the screen where we have positive parallax (Figure 3, left).

(ii) on the screen where the parallax is null (Figure 3, center).

(iii) before the screen where we have negative parallax (Figure 3, right).

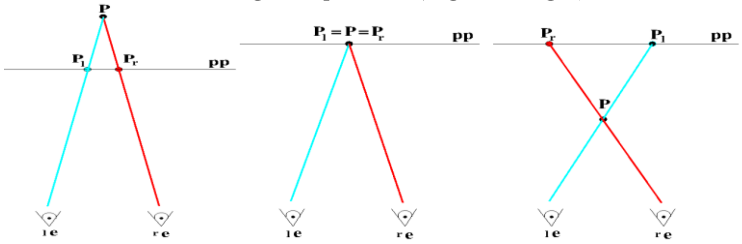


Figure 3. Projections in stereo with positive, null and negative parallax

In order to perceive the image at depth, it is necessary that each eye sees only its respective image. This is done in anaglyphic method as follows using an image editor:

(a) Name the left eye image of e-image and right eye of r-image.

(b) Decompose the left eye image in components R (red), G (green) and B (blue) and in order to keep the cyan color, the component R should be deleted.

(c) Decompose the image of the right eye in the R, G and B components and in order to keep the red color, it is necessary to delete the G and B components.

So, by using the anaglyphic glasses with red lens in the left and cyan lens in the right, we will be able to visualize the image at depth, as shown in Figure 4 the right image.

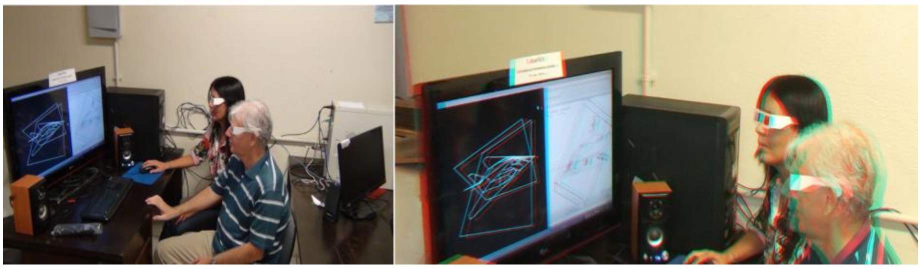

Figure 4. Monocular image (left) and anaglyphic binocular image (right), (LaborMA)

For the creation of a virtual stereoscopic image, the object must be drawn into a virtual screen considering:

(i) right eye:

glFrustum(xleft1, xright1, top, bottom, near, far);
gluLookAt( re, ra, $\overrightarrow{u_p}$);
and to extract the red component, we must consider the command,
glColorMask(GL_TRUE, GL_FALSE,GL_FALSE,GL_TRUE);

(ii) left eye

glFrustum( xleft2, xright2, top, bottom, near, far);
gluLookAt(le, la, $\overrightarrow{u_p}$);
and to extract the green (G) and blue (B) components to have the cyan color, we must consider the command, glColorMask(GL_FALSE, GL_TRUE, GL_TRUE,GL_TRUE);

After doing that, one just should superpose the images of each eye and the result will be an anaglyphic stereo image. Observe the hexahedron and the F117 airplane projections as anaglyphic stereo images in Figure 5.

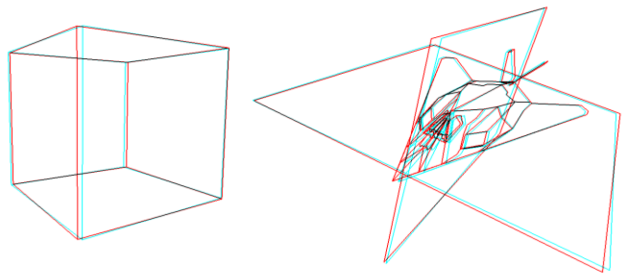

Figure 5. Perspectives in anaglyphic stereo using OpenGL of a hexahedron (on the left) and a F117 airplane (right)

## 2.2 The Proposed Method

This paper proposes the use of computational anaglyphic method, in which the binocular axonometric perspective is obtained from the mongean projections of the object. All these concepts are studied in Descriptive Geometry course which emphasizes the educational role of the software. It not only provides the appropriate spatial visualization, but also demonstrates and explores the concepts of mongean and axonometric perspective projection studied in the course. Its innovative nature comes from the implementation of stereoscopic vision from traditional methods used in Descriptive Geometry. The developed software uses the free educational program GeoGebra, [5].

### 2.2.1 Descriptive Geometry Basic Concepts and the Spatial Visualization Problem

Descriptive Geometry is an early discipline, specially of engineering and architecture courses. It addresses to the representation and solution of problems involving spatial mathematical entities such as points, lines, planes, polyhedrons, etc., through its cylindrical orthogonal projections in two planes perpendicular to each other, in which one is rotated over the other, with the aid of plane geometric properties, as shown in Figure 6. The figure obtained after this revolution is called Epure or mongean representation [10].

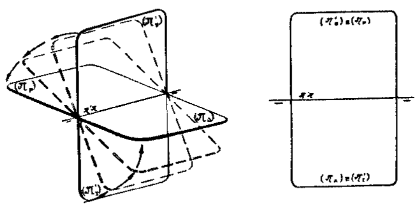

Figure 6. Revolution of the vertical plane on the horizontal plane (left). Epure or mongean representation (right)

The distance from one point to the horizontal plane is called quota. The quota coordinate is conventionally represented in Epure by the distance from the vertical point projection to the intersection planes line.

The distance from one point to the vertical plane is called afastament. This coordinate is represented in Epure by the distance from the horizontal point projection to the intersection planes line, according to Figure 7.

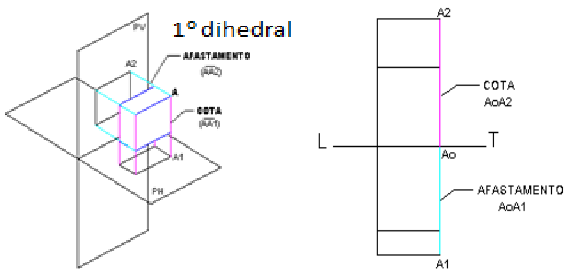


Figure 7. Afastament and quota’s first dihedral point. Spatial representation (left) and mongean representation (right)

It has been observed that a large number of students have serious difficulties in relation to spatial visualization ability, and this is one of the main factors for failure in learning this discipline. The reduced workload and the lack of a prior contact with the basic concepts of the discipline are some of the factors that contribute to worsen this situation. The non-development of spatial reasoning during the school results in students, newcomers to undergraduate courses with difficulties to understand the concepts dealt with in disciplines like Descriptive Geometry. In addition to this factor, there is a lack of alternative learning that could minimize these difficulties and support the mental abstraction processes necessary for the development of spatial visualization skills. The real models are used to facilitate three dimensional understanding. Thus, we see the need to seek the use of modern techniques that could minimize this problem.

### 2.2.2 Anaglyphic Stereo viewing with the Proposed Method

The perspective is a graphical representation that shows the objects as they appear to our sight, with its three dimensions. The axonometric perspective presented in this work is an orthographic cylindrical projection on an oblique plan in relation to the three dimensions of the object being represented. This perspective is essentially technical. It is an issue of particular interest on the part of the students of engineering and architecture, since they need to develop their capacity for perception and creation of three-dimensional objects, in addition to being able of reproducing them in their orthogonal or mongean views, studied in Descriptive Geometry. With teaching/learning purposes, more often than not, it is necessary to draw an axonometry in order to clarify details that had not been fully understood in the mongean views. The axonometric perspective, which construction is based on the inscription of the solid in a trihedron tri-rectangle (three right angles), in which only the edges, called axes (axon=shaft, metria=measure), are considered. The projection of this trihedron in a plan can lead to the following types of axonometric perspective:-Conic: projection from an objective point.

-Cylindrical projection from a point of infinity

- Orthogonal: projection lines, orthogonal to the plane of projection.
- Oblique: projection lines, oblique to the plane of projection.

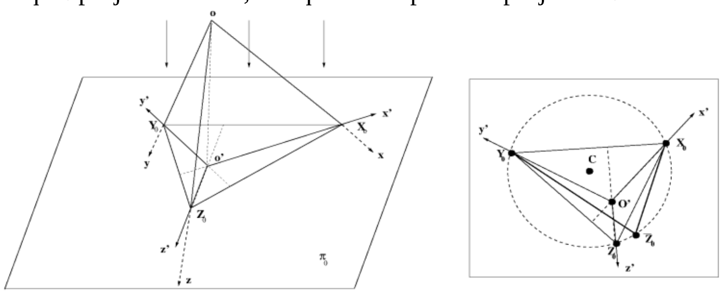

Figure 8. Orthogonal axonometric projection elements (left) and the second axonometric triangle (right)

Figure 8 on the left presents the method of cylindrical orthogonal axonometric perspective, with particular distinction to the elements used in its construction, where:

- **O** – The vertex of the trihedron goal. For being the closest point of the observer (found at infinity) is always visible on projection.
- **x**, **y** and **z** – axonometric axes
- **x’**, **y’** and **z’** projections of axonometric axes
- **Triangle** $\mathbf{X_oY_oZ_o}$ – axonometric or fundamental triangle
- **O’** – orthographic projection of the vertex **O**. There is a basic Geometry theorem which demonstrates that **O’** always falls on the orthocenter (date of heights) of the triangle $\mathbf{X_oY_oZ_o}$.

-$\boldsymbol{\pi_o}$ – projection plan, always considered in horizontal position. Keeping fixed the trihedron, the lifting or lowering of $\boldsymbol{\pi_o}$ raises similar axonometric triangles, which results in the same perspective, because this depends only on the shape of the axonometric triangle and not on the size of their sides.

The trihedron formed by orthogonal coordinate systems is projected on the projection plan, in such a way that the scale along the axes x, y and z are duly rearranged on the projected axes x ', y ', z ' according to the orthographic projection. This procedure is called a ranking of axes designed. Graduating the axes is to score on them a specific given unit. In fact, what is sought in order to draw the object perspective is the result of this degree in the projections of the axes (perspective unity).

As the angle formed by two shafts at the apex is 90°, the triangle is always an acute triangle axonometric (presents all three internal angles smaller than 90°). According to the size of the sides of the triangle, the cylindrical orthogonal axonometric perspective receives the following names:

Trimetry or anisometry: If the axonometric triangle is a scalene one, there will be three different perspectives of distinct units for the same unit, one for each axe designed.

Dimetry: If the triangle is isosceles, two perspectives will be alike and one will be different.

Isometry: it is the most widely used case, if the triangle is equilateral, all three perspectives will be the same.

In order to graduate the projections, we must first determine the real magnitude of the axes, to mark on them the given unit and to accomplish that, we must refute a pair of axes on the plane of the painting. After that, it is possible to obtain the degree from projections of axes, as presented in Figure 9.

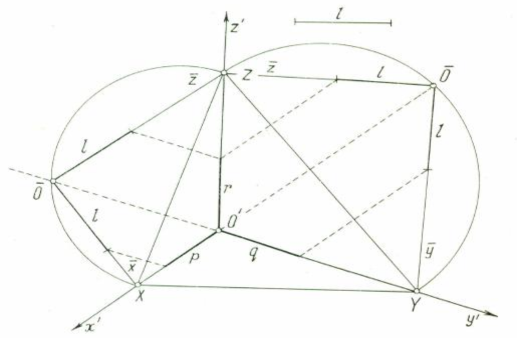

Figure 9. Graduate axonometric axes

Using GeoGebra educational software two axonometric projections with two different axonometric triangles are built. The triangles have two common vertices and the third one is shifted in space as shown in Figure 8 on the right. Using the anaglyphic method, with the figure in red to the right eye and the cyan one to the left eye, the projections of the hexahedron and the F117 airplane are obtained, as shown in Figure 10.

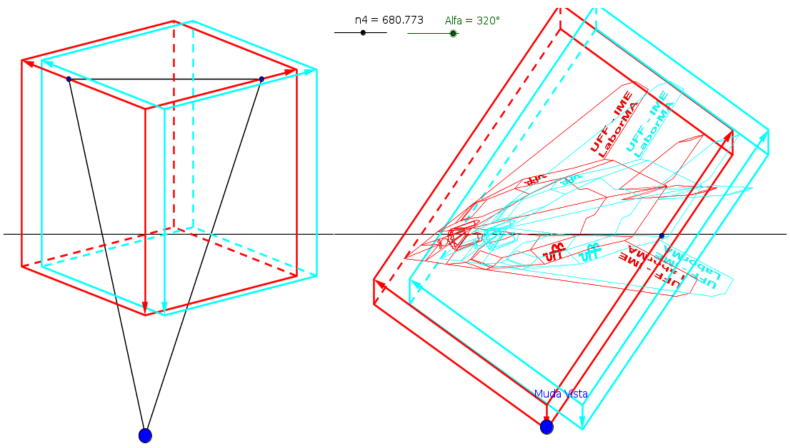


Figure 10. Anaglyphic stereo visualization using orthogonal axonometry of the hexahedron and the F117

Figure 11 shows the mongean projection and the stereo anaglyphic axonometric projections of the F117, as it appears on the screen of GeoGebra.

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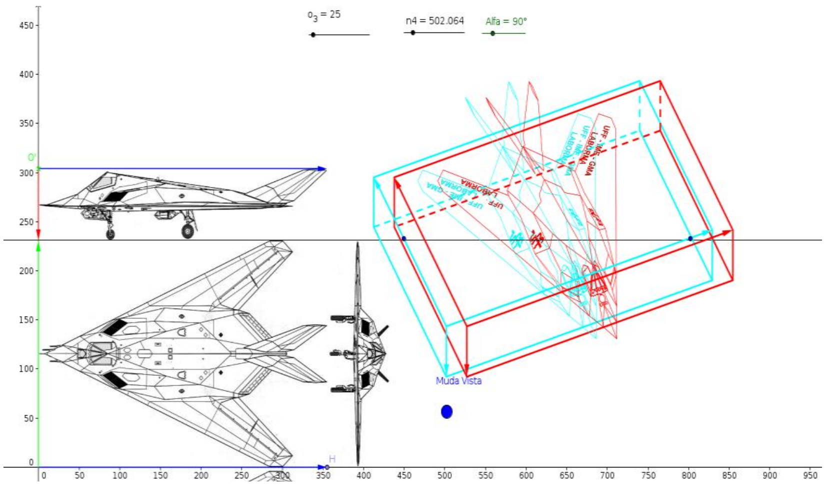


Figure 11. F117 with mongean projections and anaglyphic axonometric orthogonal projection

The mongean and perspective projections side by side and interactive, i.e. changing the mongean projection the stereo perspective changes automatically.

## 3. RESULTS

The tutorials were developed in the Laboratory of Applied Mathematics (LaborMA) for the Descriptive Geometry students of Engineering course, during the year of 2013, as seen in Figure 12. The tutorials were designed to test the proposed method with students who faced great difficult in learning this discipline. Two groups of fifty students each were provided with the tutorials. It became evident that the stereo perspective visualization using the axonometry offered a superior stereo vision at depth in relation to the matrix method used as default by the programs of stereo views in computer graphics. Every student who participated in the tutorials responded that the proposed method generates figures in stereo with better image quality than the traditional method. The use of stereoscopic techniques has great potential for the improvement of spatial visualization ability, particularly for individuals with greater difficulties, because they allow the understanding of spatial (three-dimensional) situations presented in exercises, making its development possible. Another benefit is that the monocular method of axonometry is a tool used in Descriptive Geometry for geometric solids visualization and obtained from the mongean projections of the object, also studied in this discipline. The software improved both: the visualization ability and the Descriptive Geometry basic concepts, which points out its educational role. In addition, we can state that the educational software GeoGebra also helps in the learning process, because it is not only easy to handle but also interactive, allowing the student to immediate change of point of view, increasing the understanding of the solid, from its projections. The discipline Professors observed an improvement in the academic performance of students who attended the tutorials and a study to evaluate the educational benefit of the tool is being prepared.

Figure 12. Students using anaglyphic stereo axonometric orthogonal projections in LaborMA

## 4. CONCLUSION

This work presented a new methodology for spatial visualization: the use of computational binocular axonometric anaglyphic method, in which the axonometric perspective is obtained from mongean projections of the object. These concepts are studied in the course of Descriptive Geometry which emphasizes the educational role of the software. The method was presented to the students during the year of 2013 and provided a greater depth of vision than the method of projective transformations used by default in stereo views in computer graphics programs. Its innovative characteristic comes from the implementation of stereoscopic vision from traditional methods used in Descriptive Geometry. The software was developed using the free educational software GeoGebra. For a future work, we intend to investigate the method using conical axonometric perspective.

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