The Transformation App Redux: The Notion of Linearity

Anthony Domenick

Mathematics Department

Bloomfield College, Bloomfield, New Jersey

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Abstract

The notion of linearity is perhaps the most fundamental idea in algebraic thinking. It sets the transition to functions and culminates with the instantaneous rate of change in calculus. Despite its simplicity, this concept poses complexities to a considerable number of first semester college algebra students. The purpose of this observational study was to explore how the TI-84 Transformation Graphing Application, a highly visual and interactive tool, could cultivate the teaching and learning of the elementary slope-intercept form. The study was conducted with 40 subjects during three 70 minute class sessions. Three activities were modelled and subsequently investigated on the effects that changes in slope and y intercept values have on linear equations as well as an application of given data to extend cognitive thinking on patterns and relationships between two variables. It was discovered that the direct exploratory instruction of the essential elements of this application can enable students to visualize and conclude mathematical generalizations about the foundation properties of the function concept.

Introduction

No mathematics educator can rebuff the sophisticated intensity that the TI Nspire can render with respect to elucidating content. However, there is a parallel vigor that is sustained with the interactive milieu created by the Transformation Graphing Apps Feature of the TI-84, the Nspire's predecessor.

Prevalent studies related to the 1974 National Council of Teachers of Mathematics (NCTM) technology decree conclude the irrevocable

advantages calculators have had on students' ability to distill mathematical abstractions. Algebraic problem solving can no longer be an esoteric detachment. That is, the multifarious concepts of function, table, and graphical representation can prove to be an intrinsic value to the student's acquisition of concrete mathematical knowledge.

Donald J. Dessart, Mathematics Researcher from the University of Tennessee, is a staunch advocate of calculator intervention in the primary and secondary classrooms. Unfortunately, he has found that in the past, a multitude of the instructors at this level have thwarted its integration: "Not only has the calculator failed to redirect the curriculum, it has failed to enter most mathematics classrooms....With a cost that was rapidly shrinking, the device appeared too powerful and potentially omnipresent to ignore."

Although that was back in the 1980's, there is still a reluctance today from a considerable faction of pre-college instruction to subscribe to technological influences. Perhaps this residual effect explains the absence of the mastery of fundamental algebraic principles among first year college students nationally. The vast majority of students who enter College Algebra classes at a private liberal arts college in Bloomfield, New Jersey have difficulty articulating the real-world significance of a linear function, translating the interrelationship between standard, slope-intercept and point -slope form, as well as interpreting the graph's linear behavior.

Methodology

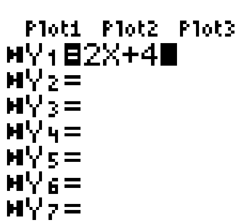
An observational study incorporating the use of the Transformation Graphing Application was conducted to determine whether this feature can augment student's knowledge of the effect m and b have on the graph of a line. This comprehension is pivotal to the construct and synthesis of examining successive function families including quadratic, polynomial, piecewise, exponential and logarithmic- all

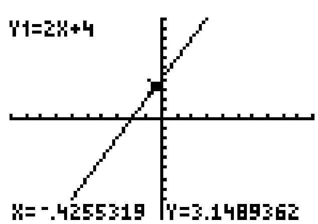
analyzed in the first semester course. Three class sessions of 70 minutes in duration with 40 subjects from a private college in Bloomfield, New Jersey were the subjects of study. Each student used a TI-83 or 84 Plus with the loaded application, and worked in groups of three. Screens shots from the TI Emulator were projected for the students to model.

Three activities were explored:

1. **Acknowledging the linear function in slope-intercept form.**
2. **Exploring the effects of variable changes where A represents the rate of change, and B denotes where the linear system begins.**
3. **Applying the George Polya Guess and Check Problem Solving Strategy by manipulating the variable values leading to a regression model based upon given data.**

In the first activity, students were asked to enter the equation 2x+4 in the Function Editor (y=). Select Zoom 6, Standard Window x[-10,10], y[-10.10] ,and then as a group make a list of 5 characteristics about the equation. Students also took note of the Transformation icon that precedes the sequentially listed equations as a function of y in terms of x.





Students were not accustomed to inquiry based instruction. Initially there were long pauses before students volunteered to comment. Students were then prompted with, "What is the line doing? That is, how would you describe the behavior of this line?

Comment 1 (Student A) ***It looks like it is increasing from left to right.***

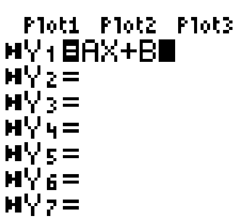
Comment 2 (Student B) ***The line starts at 4.***

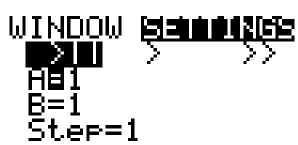
Comment 3 (Student C) ***Two is the slope and 4 is the y intercept.***

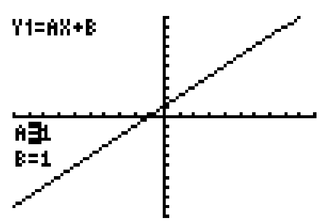
"How can you prove that two is the slope from the y intercept of 4?"

The question, at first triggered silence, but then induced discussion among the groups regarding where the graph crosses the y axis, which other equations slant in the same or different directions, and the generic significance of slope. Students were instructed to use the right arrow from lower Quadrant 3 to trace, which then incrementally displays the increasing values of the x and y variables of the function.

The second task directed students to use the slope-intercept generic form of AX+B, relating the coefficient of A as synonymous with M, while B is maintained as the y-intercept since the Transformation App uses only coefficients A,B,C and D. Also, it was essential to place this generic form in Play-Pause Mode by selecting Settings from the Window feature. This would enable students to change the values of A and B with respect to a chosen increment value, and view the simulated changes in the curves.



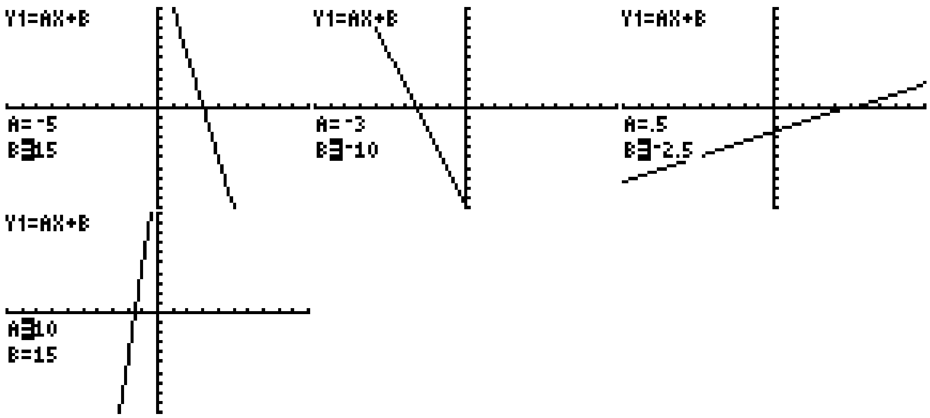




Students were asked to experiment with the values of A and B by toggling the up and down arrows for each coefficient, and the left or right arrows to decrease or increase the values within a Standard Window. After experimentation with the A and B values, the task was to generalize the effect of positive and negative changes with

respect to both A and B, and the behavior of the graph when such changes are performed.

Volunteers from the groups displayed sample graphs which explored both discrete, integer and continuous values.



Comment 4 (Student C) ***When the A is negative, the graph shows a decrease.***

Comment 5 (Student D) ***The decimals made our graph almost flat compared to the other graphs.***

Comment 6 (Student E) ***Our line looks the steepest of all because of the A value of 10.***

The third exercise required students to apply data provided by the Bureau of Transportation Statistics on population (in millions) and the number of licensed drivers (in millions) from selected states in the year 2003. This involved investigating additional features of the TI-84, and their significance to a linear relationship.

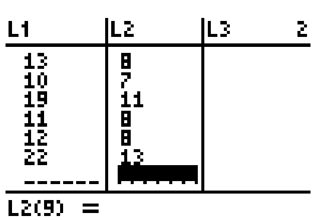
Table 1 Licensed Drivers in 2003

State	Population	Licensed Drivers
California	35	22
Florida	17	13
Illinois	13	8
Michigan	10	7
New York	19	11
Ohio	11	8
Pennsylvania	12	8
Texas	22	13

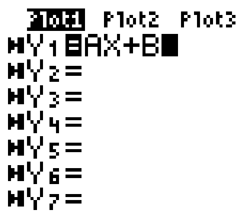
Source: Bureau of Transportation Statistics

- **Entering data in the List feature.**
- **Plotting the Data using the Scatter icon.**
- **Switching to Zoom Stat window versus the previous standard widow.**
- **Creating a scatter plot from the given data by determining dependent and independent variables.**
- **Providing a line of fit for the data via guess and check manipulation of the A and B coefficients. (Analytical statistical formulas and the Stat Graphing regression model feature of the TI were not introduced here.)**
- **Observing patterns in the data with respect to a state's given population and the corresponding number of drivers.**

The state population values were entered in List 1 while the number of licensed drivers were entered in List 2.

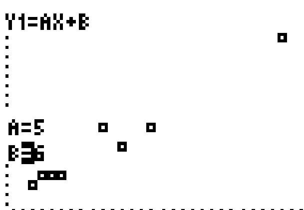


The generic AX+B equation remained in the y= editor.



The Plot feature was applied to visually display the data points.

Zoom Stat will automatically display the ordered pairs.



The groups could see that previous activity values of A and B had no bearing on the graph for the data. So they experimented with estimating values for the slope and y intercept until they concluded a consistent model to represent the data.

Comment 7 (Student F). ***It doesn't look like the whole numbers work.***

Comment 8 (Student G). ***I tried a small decimal for A and did get a line.***

Comment 9 (Student H). ***But it doesn't go through the points.***

Comment 10 (Student I). ***A .5 works really good for the slope, but I can't get the B***

Comment 11 (Student J). ***1 works o.k.; it comes close to the points.***

In summary, an analysis of the verbal responses made by the group members during the classroom tasks revealed some critical outcomes.

1. The Transformation Graphing application enabled students to recognize the visual effect changes in A and B had on the parameters of a line.
2. Students' conceptual understanding of slope was enhanced relating the steepness of the line as the experimentation with the A value displayed either a decrease or an increase in the line's behavior.
3. Students acknowledged the y-intercept in terms of where the line initiates a rise to run relationship.
4. Although students used an effective guess and check method for the licensed driver application values of A and B, there wasn't an effective connection that the unit of measure signified the ratio of drivers to population.
5. Students needed more examples of dependent and independent variables to position on the respective axes.

The TI-84 Transformation App is a multi-purpose instrument for the teaching and learning of Algebra. Mathematical researchers, Helen Doerr and Roxana Zangor, generalize the TI-84 into domains of usage that encompass data collection, computation, visualization and a checking tool. An adaptation of Doerr and Zangor's model is applied here as a summation of the pedagogical findings associated with this Transformation Graphing observational study.

Table 2

TI Usage	Student's Mathematical Behavior
Graphing Tool	Students entered equations in the y=editor in the form of Y=AX+B in a Zoom 6 Standard window X [-10,10] Y [-10,10].
Integrative Tool	Students linked equation, graph, and table as a multiple-representation process.
Verification Tool	Students confirmed their knowledge of graphs created by the Transformation App through prior knowledge of graphing equations on graph paper.
Visualization Tool	Students entered changing values of A and B, and perceived the different effect the slope and y-intercept had on the graph of the line (steepness, shallowness, slants of increase or decrease).

Conclusion

The implications of this observational study maintain the research hypothesis that graphing calculator technology can translate the abstractions of algebra into a concrete framework of mathematical transference. The graphing calculator Apps and all TI Catalog functions transcend traditional pedagogy. The domains cited by Doerr and Zangor as vital features of TI technology prove to be a significant catalyst for stoking student engagement in advanced mathematical ideas that would normally be denied them if only introduced with the mundane paper and pencil approach.

References

Dessart, Donald J. "*The Teacher of Mathematics, Issues for Today and tomorrow,"* Proceedings of A Conference, October 1987.

Doerr, Helen M. and Zangor, Roxana, "*Creating Meaning for and with the Graphing Calculator",* Educational Studies in Mathematics. Vol. 41, No. 2, (February 2000), pp. 143-163.

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