CRESST REPORT 838

IDENTIFYING COMMON MATHEMATICAL MISCONCEPTIONS FROM ACTIONS IN EDUCATIONAL VIDEO GAMES

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Abstract

Educational video games provide an opportunity for students to interact with and explore complex representations of academic content and allow for the examination of problem-solving strategies and mistakes that can be difficult to capture in more traditional environments. However, data from such games are notoriously difficult to analyze. This study used a three-step process to examine mistakes students make while playing an educational video game about the identification of fractions. First, cluster analysis was used to identify common misconceptions in the game. Second, a survey was given to determine if the identified in-game misconceptions represented real-world misconceptions. Third, a second educational video game was analyzed to determine whether the same misconceptions would be identified in both games. Results indicate that the in-game misconceptions identified in this study represented real-world misconceptions and demonstrate that similar misconceptions can be found in different representations.

Introduction

Educational video games provide an opportunity to present students with authentic and interesting educational tasks (Edelson, Gordin, & Pea, 1999) in an environment where they can interact with and explore complex representations of serious academic content (Fisch, 2005; National Research Council, 2011). Additionally, educational video games record the exact learning behavior of students, not just the answers given (Romero & Ventura, 2007). This allows for the examination of problem-solving strategies and mistakes that can be impossible to capture on a paper-and-pencil test (Merceron & Yacef, 2004; Quellmalz & Pellegrino, 2009; Rahkila & Karjalainen, 1999) or through students' verbal explanations (Bejar, 1984).

The resulting information can be used to provide detailed measures of the extent to which players have mastered specific learning goals (National Science and Technology Council, 2011) or to support diagnostic claims about students' learning processes (Leighton & Gierl, 2007). Educational games and simulations also have the potential to be used to identify the strengths and weaknesses of individual students (Mehrens, 1992), provide individualized feedback (Brown, Hinze, & Pellegrino, 2008), guide instruction that is optimal for each student (Bejar, 1984; Clark, Nelson, Sengupta, & D'Angelo, 2009; Radatz, 1979), or allow students to track their own progress (Rahkila & Karjalainen, 1999). Additionally, they could be used to improve classroom instruction (Merceron & Yacef, 2004) by allowing for the identification of common errors or determining the relative effectiveness of different pedagogical strategies for different types of students (Romero & Ventura, 2007).

These possibilities have led the government to call for the research and development of educational video games and simulations as platforms to assess the complex skills identified in state and national standards (U.S. Department of Education, 2010) and determine the effectiveness of different instructional practices (U.S. Department of Education, 2012).

However, the interpretation of the rich stream of complex data that results from the tracking of in-game actions is one of the most serious bottlenecks facing researchers examining educational video games and simulations today (Mislevy, Almond, & Lukas, 2004). The task is so difficult that a government task force recently determined that the single biggest challenge to embedding assessment in educational games and simulations is determining methods of drawing inferences from log data (National Research Council, 2011). The process of identifying evidence of student performance in educational video games and simulations is incredibly complex due to the sheer number of observable actions and the variety of potential relationships each action could have with student performance (Frezzo, Behrens, Mislevy, West, & DiCerbo, 2009). Extracting relevant features from the noise in the data is crucial in such environments to make analysis computationally tractable (Masip, Minguillon, & Mor, 2011).

In educational video games or simulations, relevant features of student performance must be extracted from the log files that are automatically generated by the game or simulation as students play (Kim et al., 2008). Though log data are more comprehensive and more detailed than most other forms of assessment data, analyzing log data presents a number of challenging problems (Garcia, Romero, Ventura, de Castro, & Calders, 2011; Mostow et al., 2011) inherent when examining exact learning behaviors in highly unstructured environments (Amershi & Conati, 2011). These environments typically include thousands of pieces of information for each student (Romero, Gonzalez, Ventura, del Jesus, & Herrera, 2009) with no known theory to help identify which information is salient (National Research Council, 2011). In addition to the size of the data, the specific information stored in the log files is not always easy to interpret (Romero & Ventura, 2007), as the responses of individual students are highly context dependent (Rupp, Gushta, Mislevy, & Shaffer, 2010), and it can be very difficult to picture how student knowledge, learning, or misconceptions manifest themselves at the level of a specific action taken by the student in the course of the game.

Due to these difficulties, there is currently no systematic approach to extracting relevant data from log files (Muehlenbrock, 2005) and the field is still in its infancy (Romero, Ventura, Pechenizkiy, & Baker, 2011; Spector & Ross, 2008).

Related Work

Some researchers have resorted to hand coding log files to extract the relevant data. Trained human raters have been used to extract purposeful sets of actions from game logs (Avouris, Komis, Fiotakis, Margaritis, & Voyiatzaki, 2005) and logs of eye-tracking data (Conati & Merten, 2007) and to identify student errors in log files from an introductory programming environment (Vee, Meyer, & Mannock, 2006). Additionally, Amershi and Conati (2011) had raters examine behavior patterns in log files from an exploratory learning environment and categorize students as high learners, thoughtful low learners, or unthoughtful low learners.

A number of other studies used basic aggregate information from the log data from online learning environments, without examining the specific actions taken by students. The aggregate information extracted from the log data were the number of activities completed by the student and the amount of time spent in the activity. The number of activities completed in the online learning environments *Moodle* (Romero et al., 2009) and *ActiveMath* (Scheuer, Mühlenbrock, & Melis, 2007) have been used to predict student grades. The time spent in each activity in an online learning environment has been used to detect unusual learning behavior (Ueno & Nagaoka, 2002). Combinations of the total time spent in the online environment and the number of activities successfully completed have been used to predict student success (Muehlenbrock, 2005) and report student progress (Rahkila & Karjalainen, 1999).

Other studies focused on summarizations or averages of pre-coded events. One such study counted the number of hints students requested and the number of failures they experienced to categorize students as hint-driven learners or failure-driven learners (Yudelson et al., 2006). A second study counted the number of deaths in each area of the game to determine which areas needed improvement (Kim et al., 2008). A third study counted the amount of money earned in a management simulation to determine effective or ineffective players (Ramnarayan, Strohschneider, & Schaub, 1997). A fourth study counted the number of errors, the average economy of motion, and the time it took students to finish a laparoscopic surgery simulation to determine performance (Gallagher, Lederman, McGlade, Satava, & Smith, 2004).

Study Design

In this report, a three-step process is used to examine mistakes students make while playing an educational video game. After the misconceptions were identified, the second step of the study was to give a later sample of students a survey about the identified misconceptions. The purpose of the survey was to determine whether the identified in-game misconceptions reflected real-world misconceptions of fractions and to confirm that the interpretation of each misconception was accurate. Finally, log data from a similar game were examined. The purpose of examining log data from a second educational video game focusing on the same topic was to determine whether the same misconceptions would be identified in both contexts and to ascertain whether individual students were identified as holding the same misconceptions in both games.

Identification of Misconceptions

Study Design

In the initial step of the study, a small sample of students played a fractions game called *Save Patch*. Each action the students took in the game was logged automatically and analyzed using data mining techniques to answer the following research question:

1. Can mathematical misconceptions be identified solely from actions students take in an educational video game?

Methods

Save Patch was designed to address four main fractions concepts: the meaning of the unit, the meaning of addition as applied to fractions, the meaning of the denominator, and the meaning of the numerator (Vendlinski, Delacruz, Buschang, Chung, & Baker, 2010). In order to address these concepts, the game area was represented as a line in one-dimensional levels and a grid in two-dimensional levels to reinforce the idea that a unit can be represented as one whole interval on a number line. Units were represented as gray posts connected by dirt paths, with small red posts indicating the fractional pieces the unit was broken into (see Figure 1). Students were given ropes in the resource bin labeled Path Options and had to break the ropes they were given into the fractional pieces indicated in the level and place the correct number of unit fractions (fractions with a numerator of one) on each sign to guide their character safely to the cage to unlock the prize.

Figure 1. Screen shot from *Save Patch*.

Successful game play required students to determine the unit size for a given grid as well as the size of the fractional pieces making up each unit. The distance the character moved was a function of the number and size of ropes placed on each sign, where one whole rope represented a whole unit and each whole rope could be easily broken into fractional pieces of the desired size by clicking on the arrows next to the rope. Therefore, a successful solution to a given level should indicate a solid understanding of fractions.

To correctly solve the level in Figure 1, the steps of a successful solution would be to (a) convert one of the whole unit coils in the Path Options into fourths by clicking on the down arrow next to the rope, (b) drag a 1/4 piece of rope onto the beige sign to the right of the game character in the upper left corner of the screen, (c) drag another 1/4 piece of rope to the second beige sign, (d) drag another 1/4 piece of rope to the third beige sign, (e) drag a whole unit rope piece to the gold sign, (f) change the direction of the last sign to down by clicking on the arrow beneath the sign, and (g) click on the Go button to submit the answer. Other valid solutions to the level include: (1) placing three 1/4 pieces of rope on the first beige sign, placing nothing on the other beige signs since having 3/4 on the first sign would make the game character walk directly to the gold sign, and placing a whole unit rope on the gold sign, or (2) breaking an additional whole unit rope into fourths in the Path Options and placing four 1/4 pieces of rope on the gold sign rather than one whole unit piece of rope.

This design allowed students to demonstrate knowledge of the meaning of the denominator of a fraction (by breaking up the whole units into the correct denominator) and the numerator of a fraction (by placing the correct number of unit fractions on each sign). Additionally, a number of levels in the game were designed to represent more than one unit, allowing students to demonstrate knowledge of the meaning and importance of the whole unit.

To allow students to demonstrate knowledge of addition as applied to fractions, game play was constrained so that it was not possible to add two fractions with different denominators, rather than allowing the students to make the addition and have the game calculate the resulting distance. This means that the game did not allow students to add 1/2 to 1/3, but instead forced students to convert the 1/2 rope to 3/6 and the 1/3 rope to 2/6 before allowing the fractions to be added together. For the same reason, the game did not allow the creation of mixed numbers (e.g., 1 1/2), forcing players to convert the whole number portion of the mixed number into the appropriate fractional representation (e.g., 2/2) before adding the fractional portion to the whole number portion.

To scaffold students' understanding of fractions and provide a logical progression through the game, *Save Patch* was broken into six stages. The first stage was designed to introduce students to the game mechanics in a mathematical setting they were comfortable with, and therefore included only whole numbers. The second stage introduced fractions via unit fractions, requiring students to identify the denominator while restricting the numerator to one. The third stage combined concepts from the first two stages, with at least one distance in each level representing a unit fraction and at least one other distance representing a distance equivalent to a whole unit. The fourth stage was similar to the third stage, except that the distance representing a whole unit did not start and end on unit markers. Instead, the whole unit distance spanned a unit marker (e.g., extending from 1/3 to 4/3). The fifth stage dealt with proper fractions (where the numerator was greater than one but smaller than the denominator) and was when students were first asked to identify the numerator as well as the denominator of a fraction. The sixth stage completed the identification of fractions concepts by asking students to identify improper fractions (where the numerator was larger than the denominator).

The sample consisted of 155 students from an urban school district in southern California in sixth-grade math, Algebra Readiness, or Algebra 1 courses. These students played *Save Patch* for approximately 40 minutes in their regular math class. The resulting game log data were analyzed using fuzzy feature cluster analysis to group in-game actions that commonly co-occurred, resulting in the identification of two common mathematical misconceptions in the game (details of the process can be found in Kerr & Chung, 2012).

Results

The most common misconception held by students involved a misunderstanding of how fractions were partitioned. Students who had partitioning misconceptions could not correctly determine the denominator of the fraction. Rather than counting the number of pieces each unit was broken into to determine the denominator, these students counted the number of dividing marks between pieces. This misconception caused students to consistently identify the denominator of a fraction incorrectly. In the example in Figure 2, a student who knew how to partition a fraction correctly would count the number of spaces in the first unit, which would lead them to identify the denominator as 3. This student would then place three 1/3 pieces on the first sign, one 1/3 piece on the second sign, and one 1/3 piece on the third sign, resulting in a successful attempt to reach the prize.

However, students who held misconceptions about partitioning would count the number of small red posts in the first whole unit, rather than the number of spaces. In the example in Figure 2, this would lead them to identify the denominator as 2. This student would then place three 1/2 pieces on the first sign, one 1/2 piece on the second sign, and one 1/2 piece on the third sign, resulting in an unsuccessful attempt to reach the prize.

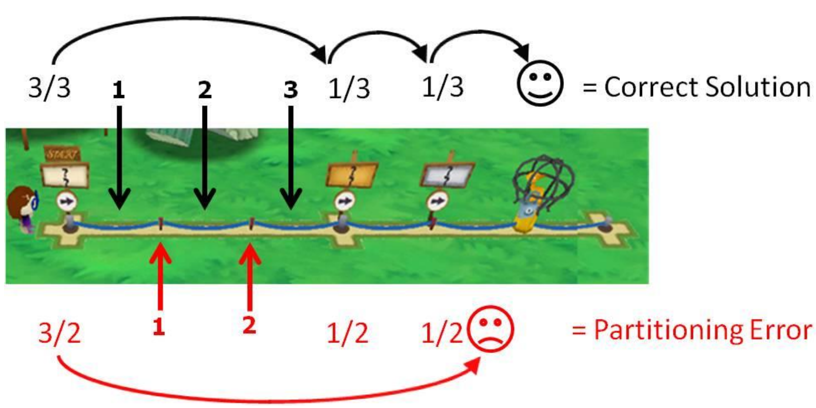

Figure 2. Partitioning errors in *Save Patch*.

This misconception results in a successful solution whenever the fractional representation is a circle. In circular representations of fractions, such as slices of pizza or pie, the number of pieces and the number of lines between the pieces are generally the same. You cut a line into three pieces by making two cuts, but you cut a circle into three pieces by making three cuts. *Save Patch* did not reinforce this misconception because there were no circular representations of fractions in the game.

The other common misconception held by students involved a misunderstanding about the unit. Students who made this error could not correctly determine the number of units being represented. Rather than using the labeling and/or visual cues provided in the representation (such as large dividing marks) to determine the number of units, these students always assumed that the entire representation was one unit across. In the example in Figure 3, a student who knew how to identify the unit correctly would determine that there were two units represented and count the number of spaces in only the first unit to determine the denominator, which would lead them to identify the denominator as 3. This student would then place three 1/3 pieces on the first sign, one 1/3 piece on the second sign, and one 1/3 piece on the third sign, resulting in a successful attempt to reach the prize.

However, students who held misconceptions about unitizing would assume that there was only one unit represented and would, therefore, count the number of spaces in the entire representation (rather than only the number of spaces in the first unit). In the example in Figure 3, this would lead them to identify the denominator as 6. This student would then place three 1/6 pieces on the first sign, one 1/6 piece on the second sign, and one 1/6 piece on the third sign, resulting in an unsuccessful attempt to reach the prize.

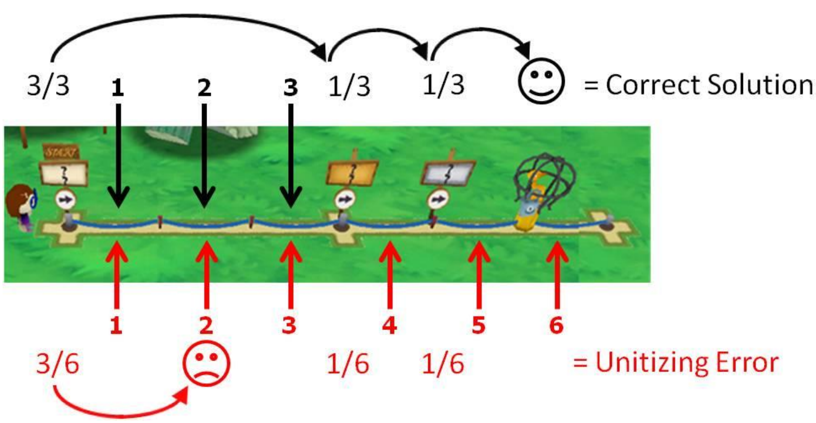

Figure 3. Unitizing errors in *Save Patch*.

This misconception would result in a successful solution whenever the representation consisted of only one unit. In such cases, their assumption that the representation was always one unit across would be correct. *Save Patch* may have reinforced this misconception because approximately a third of the game levels represented a single unit.

The cluster analysis also identified two game-related errors specific to the number line representation in *Save Patch*. The most common game-related error was for students to avoid math by using all the resources in the order in which they were provided, rather than attempting to calculate the denominator represented in the level. For example, if students were given one half, one third, and one fourth in the Path Options, students who made this error would place the one half rope on the first sign on the grid, the one third rope on the second sign on the grid, and the one fourth rope on the third sign on the grid. The other game-related error was a directional error involving misuse of the arrows beneath each sign. Students who made directional errors failed to change the direction of the arrow before submitting their answer in levels where a directional change was required, or changed the direction so that the puppet walked the wrong way on the grid and failed to reach the goal (despite having the correct values on each sign).

Gathering Evidence to Support Inferences

Study Design

In the second step of the study, a subsequent sample of students were given a short survey after game play to answer the following research questions:

1. Do the identified mathematical misconceptions reflect real-world misconceptions of fractions, rather than simply being due to oddities in the game medium?
2. Are the interpretations of how these misconceptions occur accurate?

Methods

This survey was given to 484 sixth-grade students in 22 urban and suburban schools who had previously played *Save Patch* for four days in their regular math class. The survey presented students with two different representations of 4/3 (shown in Figure 4). One representation showed a level from the game where the prize was located at 4/3, and the other representation showed a number line with a question mark located at 4/3. Students were asked to (a) identify the location of the prize and explain how they got their answer, (b) identify the location of the question mark and explain how they got their answer, and (c) state whether they thought the two representations were the same or different and explain why they thought so.

Figure 4. Game and number line representations of the same problem (identifying 4/3).

Results

As can be seen in Figure 5, a majority of the students (76%) recognized that the two questions represented the same problem. Additionally, most of the students who recognized that the questions were the same gave a conceptual explanation of the similarity (e.g., “because they are two different pictures of the same number line” or “because the cage is in the same place as the question mark") while far fewer students based their explanation on the similarity of their answers. In fact, 35% of students who said that the questions were the same actually provided different answers to Question A and Question B. These results demonstrate that most students view the game medium as equivalent to a real-world number line. Additionally, 57% of students who made errors reflecting one of the identified misconceptions on the game question made the same error on the number line question, providing some evidence that the identified misconceptions reflect real-world misconceptions of fractions.

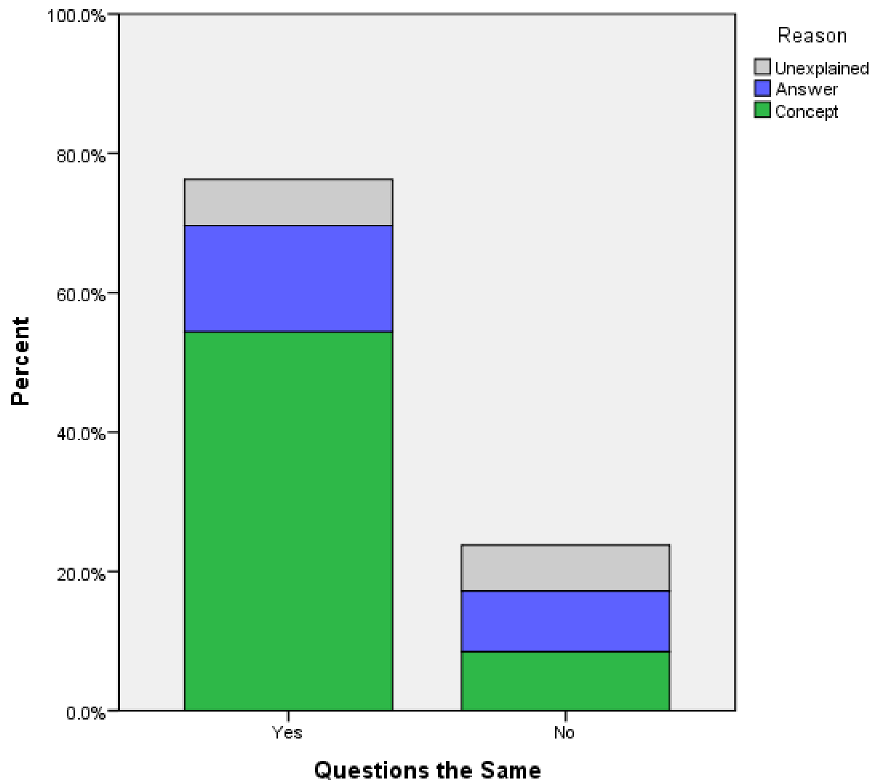

Figure 5. Responses to whether the questions were the same broken out by reason given.

More than half of the students (59%) made errors on either Question A or Question B. While many of the errors involved neither unitizing nor partitioning misconceptions, 38% of errors were partitioning errors and 19% were unitizing errors. In total, 93 of the 484 students in the study made partitioning errors, 38 made unitizing errors, and 16 made errors involving both unitizing and partitioning. Figure 6 shows the percentage of students making each error who provided explanations for their answers that was consistent with the interpretation of the misconception.

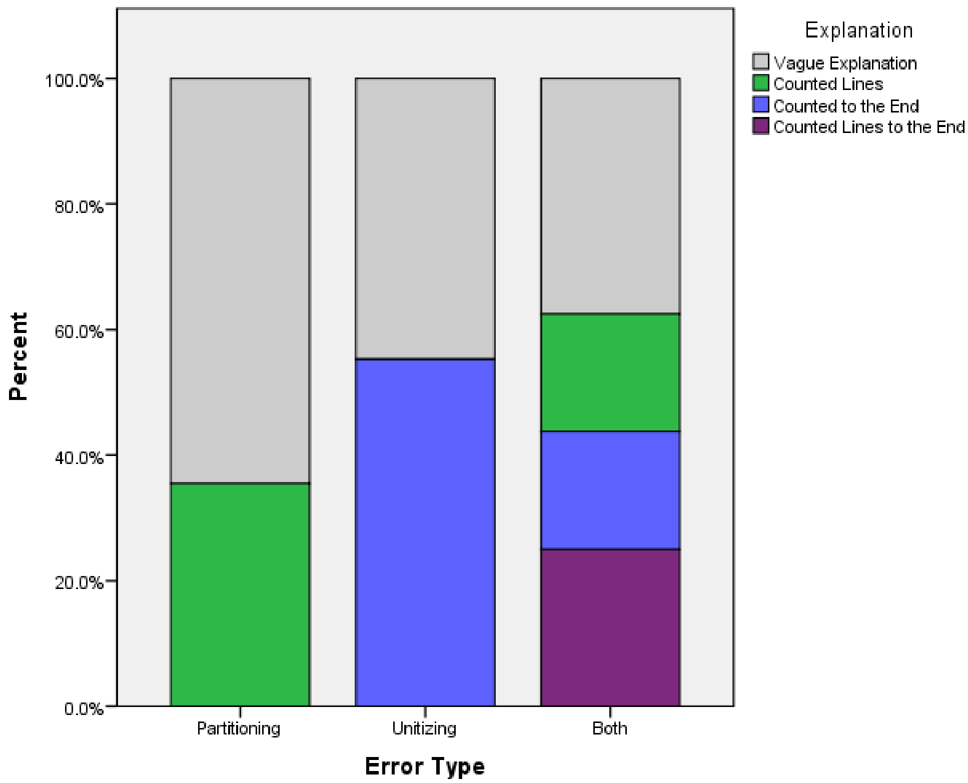

Figure 6. Percentage of each error type where explanation matched expectations.

Most explanations were too vague to be interpreted (e.g., "I counted" or "Because"). However, 39% of students who made a partitioning error explicitly stated that they counted lines to determine the denominator and 52% of students who made a unitizing error explicitly stated that they counted "all the way to the end" to determine the denominator. Of the students who made both unitizing and partitioning errors, 25% stated that they counted lines all the way to the end, an additional 19% stated that they counted lines (but did not mention counting all the way to the end) and 19% stated that they counted all the way to the end (but did not mention counting lines). Most importantly, not a single student stated that they counted something besides lines to determine the denominator or stated that they stopped somewhere besides the end, indicating that the interpretations of how these misconceptions occur are largely accurate.

Identifying Misconceptions in Multiple Contexts

Study Design

In the third step of the study, students played a different game about identifying fractions before playing *Save Patch* and took a paper-and-pencil test before and after game play. This study investigated the following research questions:

1. Can the same mathematical misconceptions be identified in other games that address the same topic?
2. Are individual students identified as holding the same misconceptions in both games?

Methods

The sample consisted of 854 sixth-grade students from 20 urban and suburban schools. These students participated in the study for 12 non-consecutive days in their regular math classes, for approximately 40 minutes each day. On the first day, students took a pretest measuring their prior knowledge of fractions. On Days 2 through 3, students played *Wiki Jones*, an educational game designed to remediate the identification of fractions issues identified in *Save Patch*. On Days 4 through 7, students played *Save Patch*. On Days 8 through 11, students played other educational games not related to the identification of fractions, and on Day 12 students took a posttest measuring their understanding of fractions.

Wiki Jones was designed to address three of the four main fractions concepts addressed in *Save Patch*: the meaning of the unit, the meaning of the denominator, and the meaning of the numerator. In order to address these concepts, the game area was represented as a number line superimposed over images of objects in the background (see Figure 7). In this game, students are detectives on the trail of bacon thieves. Depending on the prompt given, students would have to identify each whole unit correctly, divide each unit into the correct number of pieces, or locate the correct position on the number line to advance the story.

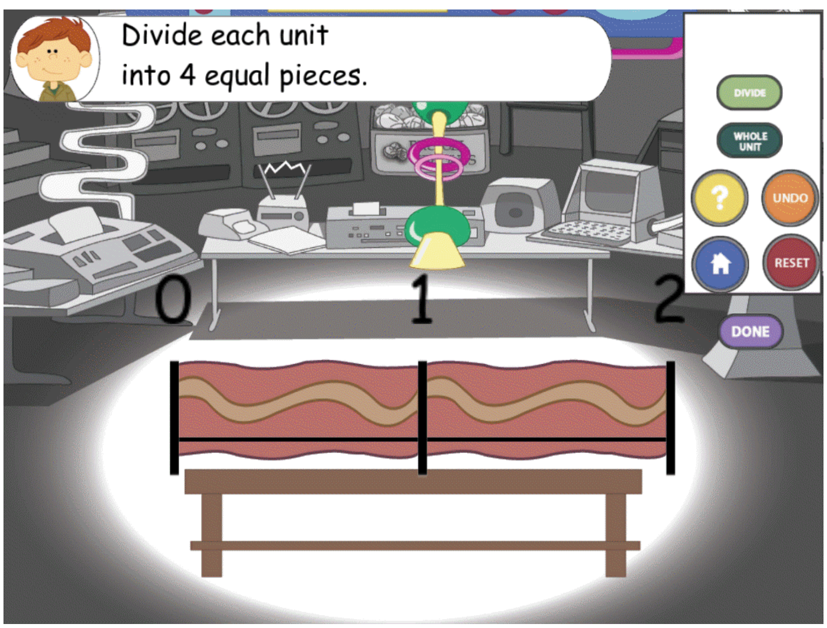

Figure 7. Screen shot from *Wiki Jones.*

Students could choose which action to take by clicking on the corresponding oval button in the upper right hand corner of the screen. The Whole Unit button activated a bagging machine that students could use to identify individual units on the number line. In the level in Figure 7, students could select the Whole Unit machine and then drag the bag from 0 to 1 and then from 1 to 2 to identify the two units shown on the number line. The Divide machine activated a laser that would cut the image into smaller pieces. In the level in Figure 7, students are being asked to select the Divide machine and draw three equally spaced lines across the bacon between the 0 and the 1 and three additional equally spaced lines across the bacon between the 1 and the 2. The Locate button (not shown in this level) would activate a claw machine that would grab whatever was located at the position indicated. If asked to locate an item at 6/4, students would select the Locate machine and then click on the 6/4 position on the number line.

This design allowed students to demonstrate knowledge of the meaning and importance of the whole unit (by breaking up a number line into whole units), the meaning of the denominator of a fraction (by breaking up the whole units into the correct denominator) and the numerator of a fraction (by identifying the location on the number line of the value indicated). Because successful game play required students to determine the unit size for a given number line, the size of the fractional pieces making up each unit, and the location on the number line of a specified value, a successful solution to a given level should indicate a solid understanding of fractions.

To scaffold students' understanding of fractions and provide a logical progression through the game, *Wiki Jones* was broken into five stages. The first stage addressed only the meaning of the whole unit by requiring students to identify the number of whole units being represented. The second stage built on the first stage by introducing the meaning of the denominator, requiring students to break each unit into the desired number of pieces. The third stage built on the previous stages by introducing the meaning of the numerator, requiring students to identify the location of a given fraction on a number line already divided into the appropriate number of pieces. The fourth stage combined skills from the previous stages by requiring students to first break each unit into the desired number of pieces before identifying the location of a given fraction. The final stage reversed the process by asking students to write down the fractional value of the indicated location on the number line.

The resulting game log data were analyzed using fuzzy feature cluster analysis in the same process as applied to *Save Patch* in Kerr and Chung (2012). While the individual actions recorded in the log data from *Wiki Jones* differ significantly from the individual actions recorded in the log data from *Save Patch*, the cluster analysis should identify the same misconceptions in both games, given that both games address the same mathematical content.

To determine whether there was a relationship between the errors made in the two games, correlations between the number of partitioning errors on the pretest, the number of partitioning errors made in *Wiki Jones*, the number of partitioning errors made in *Save Patch*, and the number of partitioning errors made on the posttest were calculated. Similar correlations were calculated for unitizing errors.

Results

As in *Save Patch*, the most common misconception held by students in *Wiki Jones* involved a misunderstanding of how fractions were partitioned. Students who had partitioning misconceptions could not correctly divide a unit into the required number of pieces. Rather than drawing one line less than the number of pieces required (e.g., drawing three lines to make four pieces), these students drew the same number of lines as the number of pieces required (e.g., drawing four lines in an attempt to make four pieces). The misconception caused students to consistently divide units into the wrong fractional amounts. In the example in Figure 8, a student who knew how to partition correctly would divide each unit into four pieces by drawing three evenly spaced lines inside each unit. Students who held misconceptions about partitioning would attempt to divide each unit into four pieces by drawing four evenly spaced lines inside each unit, resulting in five pieces rather than four.

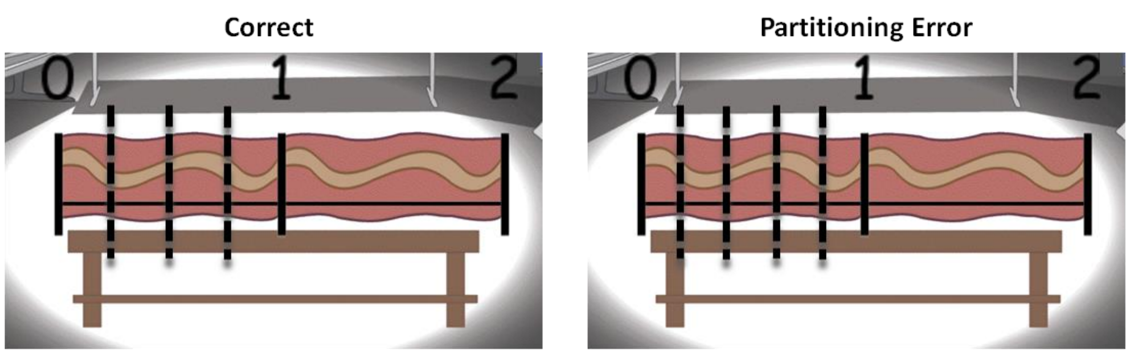

Figure 8. Partitioning errors in *Wiki Jones.*

As in *Save Patch*, the other common mathematical misconception held by students in *Wiki Jones* involved a misunderstanding about the unit. Students who had unitizing misconceptions could not correctly determine the number of units being represented. Rather than using the labeling and/or visual clues provided in the representation to determine the number of units, these students always assumed that the entire representation was one unit across. In the example in Figure 9, a student who knew how to identify the unit correctly would determine that there were two units represented and draw three lines in the first unit to break it into four pieces. Students who held misconceptions about unitizing would assume that there was only one unit represented (despite the fact that both units are clearly labeled in the representation) and would, therefore, draw three lines evenly spaced between the beginning and the end of the represented number line. In the example in Figure 9, this would result in the student dividing each unit into two pieces rather than the intended four pieces.

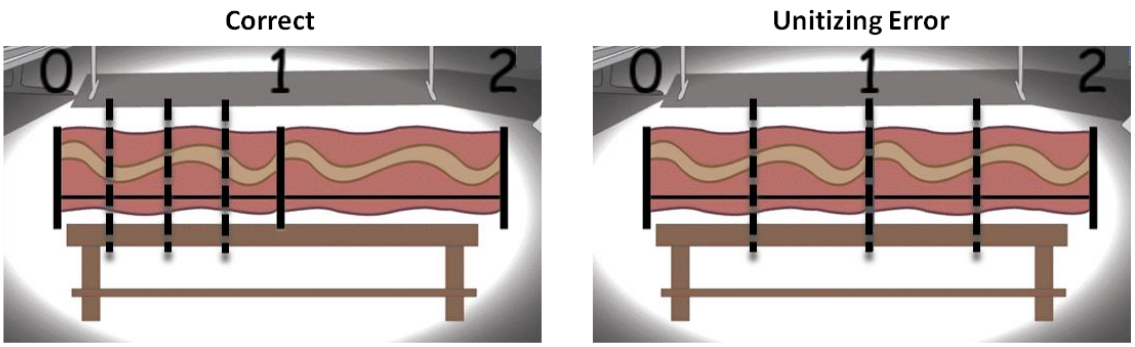

Figure 9. Unitizing errors in *Wiki Jones.*

While the cluster analysis process resulted in the identification of the same mathematical errors in both games, entirely different game-related errors were identified. Because *Wiki Jones* did not include directional indicators in any of its representations, it was not possible for students to make directional errors in this game. Nor was it possible for students to avoid math by using all of the resources in the order in which they were given, because *Wiki Jones* does provide students with a list of available resources. Instead, students in *Wiki Jones* made other game-related errors specific to the game's representation of fractions.

The game-related errors in *Wiki Jones* involved accidentally dropping two in-game tools: the wrapping tool and the cutting tool. If students dropped the wrapping tool by accidentally letting go of the cursor, this error resulted in the identification of a unit that began and ended at the same point (and was invisible to students, so it could not be corrected, but would still result in an error when students submitted their answer). If a student dropped the cutting tool before the line went all the way through the bacon, the game would not make the desired cut. Instead, the line would disappear and the student would have to draw it again.

To examine the relationship between misconceptions in *Wiki Jones* and misconceptions in *Save Patch*, correlations were run between the number of unitizing errors made in each game and the number of unitizing errors made on the pretest and posttest (see Table 1). All correlations were significant at $p < .001$, indicating that there is a significant relationship between unitizing errors made in all four environments. However, the strength of the relationships varied. The relationship between unitizing errors in *Wiki Jones* and in *Save Patch* was not as strong as the relationship between pretest unitizing errors and posttest unitizing errors ($p < .001$).

Table 1

Correlations Between In-Game Unitizing Errors and Pretest and Posttest Unitizing Errors

	Unitizing errors			
Unitizing errors	Pretest	*Wiki Jones*	*Save Patch*	Posttest
Pretest	—			
Wiki Jones	.163	—		
Save Patch	.229	.137	—	
Posttest	.422	.120	.262	—

Note. All correlations significant at $p < .001$.

Similar correlations were run for partitioning errors (see Table 2). All correlations were significant at $p < .001$, indicating that there is also a significant relationship between partitioning errors made in all four environments. All relationships for partitioning errors were stronger than their equivalent relationships for unitizing errors. However, as with unitizing errors, the strength of the relationships varied. The relationship between partitioning errors in *Wiki Jones* and in *Save Patch* was not as strong as the relationship between pretest partitioning errors and posttest partitioning errors ($p < .001$).

Table 2

Correlations Between In-Game Partitioning Errors and Pretest and Posttest Partitioning Errors

	Partitioning errors			
Partitioning errors	Pretest	*Wiki Jones*	*Save Patch*	Posttest
Pretest	—			
Wiki Jones	.258	—		
Save Patch	.312	.293	—	
Posttest	.494	.229	.333	—

Note. All correlations significant at $p < .001$.

Discussion

This study used a three-step process to examine mistakes students made while playing an educational video game. First, in-game mathematical misconceptions were identified using cluster analysis. Then, a survey was given to determine whether the identified in-game misconceptions reflected real-world misconception of fractions and to confirm that the interpretation of each misconception was accurate. Finally, a similar game was analyzed to determine whether the same misconceptions would be identified in both contexts and to ascertain whether individual students were identified as holding the same misconceptions in both games.

In the first part of the study, the cluster analysis successfully identified two in-game mathematical misconceptions in *Save Patch*: misconceptions involving partitioning a unit into the correct number of fractional pieces and misconceptions involving identifying the unit. The cluster analysis also identified two game-related errors specific to the in-game representation of fractions. The findings of this part of the study indicate that cluster analysis is a promising method of identifying both misconceptions about the content being taught and specific game-related errors by examining the actions students take while trying to solve problems in an educational video game.

In the second part of the study, most students stated that the in-game representation and the standard number line representation were the same question, and a majority of students who made an error involving either identifying or partitioning a unit on the in-game representation made the same error on the number line representation. Additionally, student explanations of how they arrived at their answers corresponded to the identified misconceptions about identifying and partitioning a unit. The results of this part of the study indicate that the cluster analysis process identified real-world mathematical misconceptions that accurately represented student thought processes, providing initial evidence that cluster analysis is a valid method of identifying misconceptions from in-game actions.

In the third part of the study, the cluster analysis successfully identified the same mathematical misconceptions in a second game, called *Wiki Jones*. While the mathematical misconceptions were the same in both games, the cluster analysis identified different game-related errors in *Wiki Jones* than in *Save Patch*, indicating that the cluster analysis process can be used to differentiate between representation-specific errors and underlying mathematical misconceptions. Correlations between occurrences of each specific mathematical misconception in each game indicate that there is a significant relationship between errors across environments, but the relationship between in-game errors was not as strong as the relationship between pretest and posttest errors. This effect may arise because the games provide feedback when errors are made (both explicitly in text form and implicitly when the desired effect does not occur), which may influence students to change their behavior either temporarily or for the long term.

In sum, the results of this study indicate that the analysis of student actions in educational video games can provide valuable information about underlying misconceptions that reflect students' real-world beliefs which can be prohibitively difficult to capture through standard assessments. Extracting this information from in-game actions is the first step in addressing the serious challenges inherent in embedding assessment in educational games, and demonstrates that salient information about student performance can be extracted from game log files.

Additionally, this study provides information that could be valuable in the classroom. A short paper-and-pencil diagnostic test to identify the presence of misconceptions involving identifying or partitioning a unit has been developed from the information provided by this study (see Appendix A). The scoring rubric for use with the test is provided in Appendix B.

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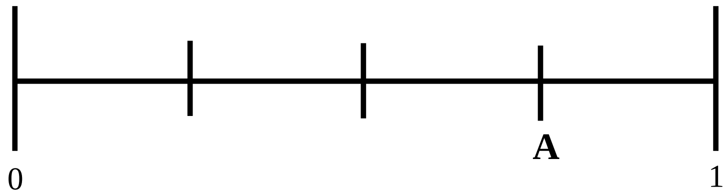
Appendix A
Diagnostic Fractions Assessment

Name: ______________________________________

Diagnostic Fractions Assessment

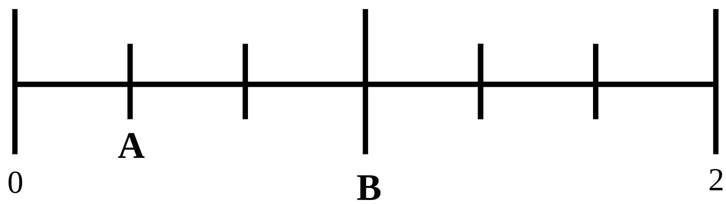
1. Examine the number line below.



   a. **Where** is **A** located on the number line above? Write your answer as a fraction.

      Answer: __________

2. Examine the number line below.



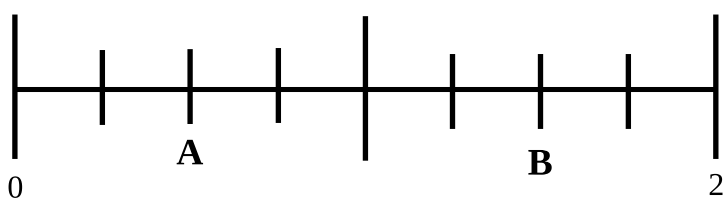
   a. **Where** is **A** located on the number line above? Write your answer as a fraction.

      Answer: __________

   b. **Where** is **B** located on the number line above? Write your answer as a fraction.

      Answer: __________

3. Examine the number line below.



   a. What is the **distance** between **A** and **B** on the number line above? Write your answer as a fraction.

      Answer: __________

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b. **Where** is **B** located on the number line above? Write your answer as a fraction.

Answer: _________

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Appendix B
Scoring Rubric

Scoring Rubric

Use the information in the *Using the Scoring Rubric* section on the following page to determine the category (know, partitioning, or unitizing) for each student's answer to the questions in the *Diagnostic Fractions Assessment*. The labels above each question identify the specific content addressed by each portion of each item. These labels correspond to the stages for *Save Patch*.

Student Name	Proper Fractions (Question 1a)	More Than One Unit (Question 2a)	Identifying a Whole Unit (Question 2b)	Wholes Across a Unit Bar (Question 3a)	Improper Fractions (Question 3b)
	☐ know ☐ partitioning ☐ unitizing	☐ know ☐ partitioning ☐ unitizing	☐ know ☐ partitioning ☐ unitizing	☐ know ☐ partitioning ☐ unitizing	☐ know ☐ partitioning ☐ unitizing
	☐ know ☐ partitioning ☐ unitizing	☐ know ☐ partitioning ☐ unitizing	☐ know ☐ partitioning ☐ unitizing	☐ know ☐ partitioning ☐ unitizing	☐ know ☐ partitioning ☐ unitizing
	☐ know ☐ partitioning ☐ unitizing	☐ know ☐ partitioning ☐ unitizing	☐ know ☐ partitioning ☐ unitizing	☐ know ☐ partitioning ☐ unitizing	☐ know ☐ partitioning ☐ unitizing
	☐ know ☐ partitioning ☐ unitizing	☐ know ☐ partitioning ☐ unitizing	☐ know ☐ partitioning ☐ unitizing	☐ know ☐ partitioning ☐ unitizing	☐ know ☐ partitioning ☐ unitizing
	☐ know ☐ partitioning ☐ unitizing	☐ know ☐ partitioning ☐ unitizing	☐ know ☐ partitioning ☐ unitizing	☐ know ☐ partitioning ☐ unitizing	☐ know ☐ partitioning ☐ unitizing
	☐ know ☐ partitioning ☐ unitizing	☐ know ☐ partitioning ☐ unitizing	☐ know ☐ partitioning ☐ unitizing	☐ know ☐ partitioning ☐ unitizing	☐ know ☐ partitioning ☐ unitizing
	☐ know ☐ partitioning ☐ unitizing	☐ know ☐ partitioning ☐ unitizing	☐ know ☐ partitioning ☐ unitizing	☐ know ☐ partitioning ☐ unitizing	☐ know ☐ partitioning ☐ unitizing
	☐ know ☐ partitioning ☐ unitizing	☐ know ☐ partitioning ☐ unitizing	☐ know ☐ partitioning ☐ unitizing	☐ know ☐ partitioning ☐ unitizing	☐ know ☐ partitioning ☐ unitizing

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Using the Scoring Rubric

Question 1a: If a student answers 3/4, check the "know" box for question 1a.
If a student answers 3/3 or 3/5, check the "partitioning" box for question 1a.

Question 2a: If a student answers 1/3, check the "know" box for question 2a.
If a student answers 1/2 or 1/4, check the "partitioning" box for question 2a.
If a student answers 1/6, check the "unitizing" box for question 2a.

Question 2b: If a student answers 1/1 or 3/3, check the "know" box for question 2b.
If a student answers 3/2 or 3/4, check the "partitioning" box for question 2b.
If a student answers 3/6, check the "unitizing" box for question 2b.

Question 3a: If a student answers 1/1 or 4/4, check the "know" box for question 3a.
If a student answers 4/3 or 4/5, check the "partitioning" box for question 3a.
If a student answers 4/8 or 1/2, check the "unitizing" box for question 3a.

Question 3b: If a student answers 6/4 or 3/2, check the "know" box for question 3b.
If a student answers 6/3, 2/1 or 6/5, check the "partitioning" box for question 3b.
If a student answers 6/8 or 3/4, check the "unitizing" box for question 3b.

Instructional Ramifications

Partitioning: If your students are making a lot of partitioning errors, you may want to use non-circular representations of fractions (such as number lines or brownie trays) so that the number of lines and the number of spaces are not the same, and explicitly explain that the denominator is the number of parts, not the number of spaces.

Unitizing: If your students are making a lot of unitizing errors (or knew question 1a but not question 2a), you may want to provide representations that include more than one unit (e.g., a number line or more than one pizza or tray of brownies) and provide explicit instructions on how to determine the number of units represented.

Question 2b: If most of your students did not know question 2b, you may want to explain that fractions are also representations of division and show that (for example) 4/4 is the same as 1 because 4 divided by 4 is one.

Question 3a: If your students knew question 2b but not question 3a, you may want to have students find distances equivalent to 1 that start and end between whole numbers, and explain it as a subtraction problem (e.g., 6/4 - 2/4 = 4/4 = 1/1 = 1).

Question 3b: If most of your students did not know 3b, you may want to provide some representations of improper fractions and talk about why you would want to keep them as improper rather than converting to mixed numbers (e.g., it is easier to add them to another fraction if they stay in fractional form).