Middle School Learners' Ontological 'Trying-on' of Dimensions: A Phenomenological Investigation

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Abstract: This paper shares findings from a post-intentional phenomenological study aimed at understanding learners' experience investigating space and dimension concepts in a fifth and sixth grade mathematics class. Findings indicate experiences in this study manifest as an ontological 'trying-on' of geometric dimensions (e.g., through sight, perception, and motion), leading learners to conjecture about dimensional relationships. The paper discusses implications informing the second iteration of the design-based research project.

Introduction

Becoming is an ontological notion explicated in the phenomenological philosophical tradition of Heidegger and Merleau-Ponty. Heidegger questions the epistemological focus (i.e., consciousness of something) of phenomenological investigations, recognizing that experience is already situated in the world of being. Throughout his major work, *Being and Time* (1927/2008), he uses the phrase 'Being-in-the-world' to indicate the ontological phenomenological starting point. According to Heidegger, phenomenology's task is to question what it is to be in the everyday world. Merleau-Ponty (1945/2002) agrees with Heidegger's notion of 'Being-in-the-world' and makes embodiment a central theme in his work – a subject embedded in a certain position and time in space. This is exemplified in his work with the primacy of perception.

This paper shares middle school learners' experiences conjecturing about the relationships between dimensions following the film *Flatland* (Travis & Johnson, 2007). The film offers viewers an opportunity to contemplate analogously within and between dimensions. In addition to the film and subsequent discourse, learners constructed models to demonstrate 3D/4D relationships. For learners, the investigation was taken up ontologically as they 'tried on' the lenses, motion, and perceptions of imagined characters.

Issue Being Addressed

Rarely do learners reflect on the ontological experience of *becoming* in mathematics class. Yet, children form conceptions of space early in life simply by *being* in the world. They engage in everyday activities like "looking, walking, drawing, building, and manipulating objects" (Lehrer, Jenkins, & Osana, 1998, p. 169). They develop intuitions about spatial structure before formalized words are attached to their concepts (Freudenthal, 1983; Gravemeijer, 1998; Piaget & Inhelder, 1948/1967; van Hiele, 1986). It is possible that their pre-formalized concepts formed by *being* in the world could come in conflict with Euclidean geometric concepts formally taught in school.

These conflicts are shown to relate to the materials and processes used in schools (Lehrer et al., 1998). For example, most textbooks and teachers present images of triangles to students with a horizontal base, although this is not a defining characteristic. This results in students primarily recognizing prototypical examples of triangles to the exclusion of non-prototypical examples. These limited conceptions can inhibit students from reaching the intended goals of instruction and will most likely resurface in later geometry learning. Spiro, Coulson, Feltovich, and Anderson (1988) describe reciprocal misconception compounding as a problem resulting from simplifying complex concepts. Many misconceptions have been identified in geometry and their origins might lie in this simplification process as well as instruction that fail to incorporate learners' prior knowledge (e.g., Carroll, 1998; Monaghan, 2000).

The current educational climate rarely provides opportunities for learners to problematize space in geometry, rather it appears taken-for-granted that everyone shares the same concept – a Euclidean mathematical space represented by x, y, and z coordinates. Even more problematic is that school explorations of space start in an abstract manner, building on idealizations of the point (0 space), the line (1 space), and the plane (2 space) – spaces that are actually impossible to *find* in the world. In this study, we created a learning environment where learners could build on prior 3D experiences and the experiences of characters from the film *Flatland*.

Context

Forty-three 5th and 6th grade mathematics students were shown *Flatland* as a way to follow up on a recent unit investigating properties of polygons. After watching the movie, students questioned what it would be like to live in a four-dimensional world – to eat, to move, to *see.* There were no simplistic answers to their questions, so we conjectured as a class and eventually created three-dimensional shapes out of straws and pipe cleaners. These were dunked in bubbles, which allowed students to make physical models of hyper-shapes (i.e., representations of 4D shapes). A year later, we conducted a post-intentional phenomenological study with students to explore their lived experiences. In particular, we explored if and how children take on the perspective of another dimension not their own (0D, 1D, 2D, 4D).

Theoretical Perspective

Our perspective acknowledges that humans' experience of phenomena in the world is complex. Phenomena emerge over time, are context dependent, and exhibit complex dynamics such as emergence, ambiguity, and adaptation (e.g., Davis & Sumara, 2006; Jacobson & Wilensky, 2006). Not only is the concept of space and dimension taken up as complex and ill-structured, but designing learning environments is viewed as a complex, iterative process informing and developing theory as well as practice (Barab & Squire, 2004; Gravemeijer & Cobb, 2006). At the early stages in design, our aim is to explicate phenomenological understandings concerning the ways shifts in perspective manifest in complex, emergent environments integrating innovative practices (e.g., modeling, mathematical discourse) and emerging forms of technology (e.g., animations, video cases). In particular, we used conceptual change strategies from science education including a discrepant event and bridging analogies as described by Clement (1993, 2008). These manifestations of shifts in perspective were then used to inform refinements in the design and theoretical conjectures.

There is little insight concerning middle school learners investigating the fourth dimension or even how this may play out using multimedia animations. Banchoff (1990) is one who passes along geometric insights to teachers of young children. He writes "[t]he invitation to examine coordinates from a dimensional standpoint is available at all times: We only have to make students aware of what they are seeing" (p. 33). We agree that the invitation is available but that it also encompasses more than sight. Freudenthal (1973), attributed with developing Realistic Mathematics Education, describes geometry as embodied:

> since it is about the education of children, [geometry] is grasping that space in which the child lives, breathes and moves. The space the child must learn to know, explore, conquer, in order to live, breath and move better in it. (p. 403)

Clements (1998) describes children's investigation of shape through various actions (e.g., touching, drawing, discussing, moving); yet, one of his descriptions stands out – "development of perspective taking" (p. 3). He describes the "three mountains" task, where a doll is placed in several positions around a mountain scene. Children are asked to describe the scene from the perspective of the doll as it is moved, but always describe their own viewpoint instead. He writes, "it is not just familiarity or experience, but *connecting different* viewpoints, that develops perspective-taking ability" (p. 3). How does one connect different viewpoints? Taking on an external perspective is nuanced because children would also have to make a transition from two to three-space. It may help to think about the context of maps to understand viewpoint. According to Presson (1987 as cited in Clements, 1998) children:

> must grow in their ability to treat the spatial relations as separate from their immediate environment. These secondary meanings require people to take the perspective of an abstract frame of reference ("as if you were there") that conflicts with the primary meaning. (p. 13)

Methodological Approach

Vagle's (2010) post-intentional phenomenological (PIP) research method was used in this study to capture the lived-experiences of middle school students investigating space and dimension. PIP brings post-structural thinking in conversation with phenomenology philosophers, such as Husserl (1901/1970) Heidegger (1927/2008), Merleau-Ponty (1945/2002), Gadamer (1975/1994), and methodologists such as van Manen (1990), and Dahlberg, Dahlberg, and Nyström (2008). Phenomena, and humans experience living with them, are seen as *tentative manifestations* – dynamic and continuously changing. Experiences are not only interpreted, but are lived in tentative, partial, and fleeting ways. Vagle (2010) has designed a five-component process for conducting PIP research: (1) Identify a phenomenon in its multiple, partial, and varied contexts, (2) devise a clear, yet flexible process for collecting data appropriate for the phenomenon under investigation, (3) make a bridling plan, (4) read and write your way through your data in a systematic, responsive manner, and (5) craft a text that captures tentative manifestations of the phenomenon it its multiple, partial, and varied contexts.

Phenomenology is the study of experience, or as Dahlberg et al. (2008) write, "the science of the world and its inhabitants, the "things of experience" understood as the world of experience" (p. 33). One cannot investigate phenomena without due attention to the central tenet of intentionality. Intentionality is best understood as an embodied relationship to things and beings in the world. Although similar to the word "intention," or action (i.e., I intend to go to bed early), intentionality is different. We choose to draw on Vagle's (2009) notion of intentionality as related to PIP research, thus situating intentionality within this study. Vagle writes (2010) "intentionality is shifting and forever partial and thus can be read through post-structural frames"

(p. 2). This presents a point of departure from other forms of phenomenology, such as transcendental phenomenology, which seeks to describe essences. Rather, we ascribe to Vagle's approach that identifies phenomena as *tentative manifestations*.

Phenomenological philosophers and methodologists have discussed in length and with great detail the phenomenological attitude, sometimes referred to as a scientific or open attitude. In choosing Vagle's approach, we chose a phenomenological attitude as one of openness, not of phenomenological reduction described by researchers such as Giorgi (1997), who uses transcendental notions of bracketing and Epoché to capture the essence of a phenomena. An "open attitude" as described by Dahlberg et al. (2008) means, "having the capacity to be surprised and sensitive to the unpredicted and unexpected" (p. 98). Bridling, described below, is one way both Dahlberg and Vagle suggest remaining open to the phenomena under investigation.

Bridling

Bridling is a way to negotiate complexity with data rather than being rigid and lockstep. The practice better encompasses the way towards an increased understanding of the phenomenon, and to us, the validity of the investigation. Bridling includes three main activities on the part of the researcher: (1) questioning pre-understandings (including assumptions), (2) remaining open, and (3) joining in an ongoing dialogue about the phenomenon. Therefore, bridling does not remove, set aside, or render the researcher non-influential but "animates and illuminates the researcher more fully" (Vagle, 2009, p. 592). The bridling entries in this study were made in three different formats: (1) as comments inserted alongside/in the margins of transcripts, (2) as individual, dated Word document files after analysis sessions, and (3) as notes inserted in books and articles.

Statement of Phenomena of Interest and Questions

The phenomena of interest in this study aims to show the reader "the lived quality and significance" of middle school students experience investigating dimensional relationships in a deep and meaningful way. The primary research question asks, "What is the lived-experience investigating dimensional relationships, including the fourth dimension?" Secondary questions ask: (1) What role does the video *Flatland* play in the experience for students? (2) Are there indications that persistent, or long-term learning occurred?

Data sources by Primary Research Question

All 43 students (21 fifth grade and 22 sixth grade at the time of instruction) from a K-12 independent school in the Northeastern United States were asked to participate in the study. Three fifth graders (1 female and 2 male) and seven sixth graders (6 female and 1 male) agreed to generate lived-experience descriptions one year after instruction. These were used to guide follow-up interviews where all 10 students articulated their experiences more fully and/or clarified their experiences. Second-round interviews were conducted as needed to clarify and expand upon key aspects of the experience. Although the 6^{th} grade interviewees were mostly female, this was representative of the student body (18 females and 4 males).

Because we interviewed former students, we used a "conversational approach" to help alleviate possible power structures (Denzin, 1989). According to Denzin, interviewing "should *not* be a relationship where one party does all the talking and the other only asks questions. When interviews turn into this form, they become asymmetric, authoritarian social relations in which the power of social science determines the information given" (p. 43). To achieve this, we focused on having students describe their experience with as much detail and ability to capture multi-faceted aspects of the phenomenon.

During the instruction, student participation and work was captured through video and photographs. Additional data sources include lesson artifacts and bridling entries. These were used to guide interview questions, support/challenge interview data, and generate hunches as described by Glesne (2011). These additional data sources were also used to generate a multifaceted description of the phenomenon and answer secondary research questions.

Analysis: Whole-Part-Whole

We draw on both Vagle (2010) and Dahlberg et al.'s (2008) suggestions for data analysis, using a whole-part-whole approach. Following is a summary, describing with way data is analyzed both within and across sources:

- *Whole.* The first reading focuses on the whole data collection event, where all data pieces are brought together and read (not analyzed) as a whole.
- *Part.* This is followed by multiple line-by-line readings with researcher notes and follow-up interview questions for each participant. Subsequent line-by-line readings articulate meanings and consider notes, markings, follow-ups, and bridling entries. After saving documents for each participant, the last line-by-line reading articulates analytic thoughts for each part for each participant.
- *Whole.* Subsequent readings start to identify tentative manifestations across the data.

Findings

For this paper focused on becoming, we will explicate a particular manifestation of the learners' experiences *trying on* various dimensions (0D, 1D, 2D, 3D, and 4D). The phenomenon was evidenced through learners' conjectures about what it might be like to see, perceive, and move in the dimensions. Notable in the descriptions from learners was their attention to relationships within the dimensions. For example, when describing sight, perception, and motion in 3D space, they conjectured what it might be like to be a point, a line, a square, a cube, and even a hypercube in 3D space.

Learners described *Flatland* as the point when they started considering dimensional relationships. They related to the 2D Flatlanders' struggle to visualize three-dimensional objects. Students had an analogous struggle visualizing the fourth dimension. Students talked most about the last scene of *Flatland*. This is the point where they saw the fourth dimension animated for the first time and where they watched Spherius, the main 3D character in the movie, deny the existence of a fourth dimension. This denial prompted a lengthy discourse among students about close-mindedness. Students expressed being upset with Spherius for denying a higher dimension in a similar way that 2D characters in the movie denied a third dimension.

Taking on Sight and Perception from the Various Dimensions

Almost every student talked about what it might be like to *be* in a dimension. For some, this took the form of articulating what it actually means to be in 3D space and how they are only seeing parts of solids, but not all sides at once. They called this *seeing in 2D*. They talked about perceiving depth and *thinking* that we see in 3D. Somehow it was easy for them to then imagine by analogy what it must be like to see and perceive in the fourth dimension – they called this *seeing in 3D*.

Students engaged in their learning experience by trying on different dimensional lenses (or contemplating being other dimensions) and then visiting other dimensions, even the fourth. Alan is one student who attempted to generalize sight and perception for all dimensions and used Euclidean mathematical notions of the coordinate system in his discussion. He demonstrated an ability to perceive as "creatures" from different dimensions, related to the movie *Flatland*:

> Pointland is a land of the zero dimension, where all that exists is a point. No axis. Lineland is only a line, the first dimension with the x-axis (or y). Flatland is the second dimension, with the x and y-axis. The creatures in the movie could only see a line wherever they looked. Spaceland is the third dimension, where we exist. With the x, y, and z-axis, it is most likely the largest 'land.' The creatures of this land can see a 2D image that can be perceived in 3D.

Interesting here was his distinction between sight and perception in 2D Flatland and 3D Spaceland. His ability to connect dimensions to the x, y, and z-axis demonstrated what Freudenthal termed *mathematization* (1973). He continued the process as he connected the fourth dimension into this same x, y, and z coordinate framework.

> If someone asked me to describe the fourth dimension, I would say something along the lines of...basically a dimension more complex than ours as another axis or existence…starts with none in the zero, x in the first, y in the second, z in the third. We'll have to make a new letter in the alphabet for the fourth dimension. Or 'a'…it would probably work, yeah a different sort of direction. The fourth axis is probably where the idea of how I perceived hypercube came from. Cause it's a cube connected by another axis…like a cube in the center connected by the line coming from the outer corner. And I was thinking of those lines as being a fourth axis.

Alan was the only student who talked about the x, y, z, and "a" axis in relation to the dimensions, attributing *Flatland* as his starting point. He said previously he had only learned about the second dimension, but not the zero, first, third, or fourth dimensions. He talked about his experience as a whole:

> Investigating the fourth dimension is to learn what the world is like mathematically and also to teach us more about the dimension we exist in. The fourth dimension was to explain why things happen, why the world is like it is. Well, before that, I didn't really understand. I had a vague understanding of the second dimension, but not much of the first, zero, third, or fourth. The biggest thing I learned is that humans exist in the third dimension but we can only actually see the second dimension – we just perceive the third. As we evolved to – not evolved – as we exist, only able to see the dimension below it and not the actual dimension it's in.

Dimensions Lower than Three (0-2)

This section incorporates students' discussion of the zero, first, and second dimensions. Students described these in various ways, taking the perspectives of the different types of dimensional creatures (0D – Point; 1D –

Linelander; 2D – Flatlander; 3D - Spacelander). For example, taking on a Linelander's perspective of its own dimension, or a Linelander's perspective of a Flatlander. In addition, they described the dimensions either by appearance, motion, perception, and even the sight available to a particular dimension.

Students talked least about the zero dimension, but almost all students recalled the "me" song that the Point character from *Flatland* sang. In the following description, Annie talked about the perspective of a point:

> I had never really thought about it before, like a line can only see side to side of where it is or a point only notices itself because it can't - it's not really anything else, I thought that was really cool because I'd never really taken that in perspective before.

Students' discussion of the first dimension highlights their attention to limitations of sight and motion available to a line. For example, Susie expressed, "their [Linelanders] only way of traveling was right and left and only it couldn't have any sort of up and down. It was only just in the two directions." Edward, Sabrina, and Daria also described Lineland, but they talked more about the Linelanders' point of view when other dimensional beings came into their line. For example, Edward talked about one Linelander's point of view towards Arthur Square, a Flatlander visiting the line in the movie:

> In Lineland it could only see down it's line so it like it couldn't see up or down so it wouldn't be able to see him…it would only be able to see him if his eye or if his - if he was right on it's line, so I thought that was kind of cool how it had no other power to see anywhere else.

Students talked even more about the second dimension. Mary felt like the movie gave her a better understanding of the second dimension. "If you had a piece of paper, it helped me understand what it would be like if you had a circle that could, or a sphere that could sink into it." Although this description may seem simplified upon first reading, Mary started articulating a foundational calculus concept of iteratively slicing a sphere. Another student, Susie, described the Flatlander's point of view and motion. Here, she started to insert herself into the second dimension showing how she took on the role of dimensional motion:

> It's kind of crazy for me thinking about actually only living in the flat surface. I sort of remember always thinking about the idea of how do they move? Do they slide along on the floor or what? Or crawl? I remember when we started watching, that was basically one of the first things that popped out to me. I was like, how are they moving? Its kinda' cool to think about how – wasn't he saying that they only have northwards and southwards or something? That they don't have up and down? Yeah cause it's flat.

The second dimension is typically taught as a concrete dimensional plane. The movie allowed her to contemplate motion in this dimension, whereas her prior experiences limited her to only seeing two-dimensional drawings in books. It created another way of conceptualizing the second dimension, an unexpected (from the teacher's point of view) experience of taking on the role of seeing and moving in different dimensions.

The Third Dimension

Students' descriptions of the third dimension reflected a problematizing of their own 3D space. These students had little formal instruction concerning the third dimension at the start of the lesson. Edward gave a prototypical example of problematizing sight from his own third dimension, "We don't see circles, we see spheres. I mean we see circles but…and you can't really get anything completely flat except for like if you're looking at a screen. I guess that like what's inside would be totally flat." In his description, there is tension as he tried to reconcile seeing "flat" but at the same time seeing, or perceiving, space. Although he did not use the word perceive, many students did. For example, Alan talked about seeing and perceiving. "The third dimension is something we can only perceive, and that we can't really understand further than that in a visual way."

The Fourth Dimension

Students conjectured most about the fourth dimension. *Flatland* only gave a glimpse of a rotating tesseract at the end of the movie, but this seemed to be the scene that affected them most. During class, student questions about the existence of a fourth dimension were persistent and eventually led to the hands-on investigation with straw and bubble shapes. This led to several conjectures about the fourth dimension, elaborated below.

Sight in the Fourth Dimension

Mary talked about sight and being in the fourth dimension. It's as if she not only learned about the fourth dimension but also strengthened her understanding of the 0, 1st, 2nd, and 3rd dimensions and their relationship. She talked about what it would be like to see and perceive from each of these dimensions:

> I want to know more about what it would actually be like to be in a fourth dimension. I have an idea. You'd be able to see all the sides of an object at once, but it would be like in Flatland. The square guy could only see the sphere when he was in his field of vision. So it would be weird to think that there would be things that we can't see…you can see all sides of something. I'm continuously getting the vision of your eyes popping out of your head and like curving around an object so you could see the back of it.

Mary's curiosity led her to think about seeing in a fourth dimension. Other students experienced this as well. For example, Edward said:

> To see every side of a cube, your eyes would have to be…probably have to have one like, like coming out of like something that's like this [makes motion of hand making a hook coming from forehead curving out to front of face] I guess so you could see like this way toward…like you could see yourself…and you could see like anywhere I guess. That would be very difficult to see ourselves, like our eyes.

Alan also talked about sight in the fourth dimension, distinguishing between sight and perception:

> Assuming that any creature can perceive in, can perceive a dimension below it – can SEE the dimension below it, but perceive its own. Well, that would imply the fourth dimensional creature would have to be able to see a third dimension. So basically what I imagine a fourth dimensional creature being like is a sphere with eyes on the inside of it, so it could look at something from every angle. And it would have to have some way of getting, getting things in there to look at, but I hadn't gotten past that. Yeah, you'd have to be able to see all sides at once. First my…could imagine seeing all three sides of a cube – all six sides of a cube at once.

Motion in the Fourth Dimension

In addition to conjecturing about the sight of four-dimensional beings, students also discussed motion. For example, Annie inserted the human form into the fourth dimension. "I think it would be pretty insane to live in the fourth dimension, especially if we were 4D humans, cause that'd mean like our guts would be like rotating inside of us." Edward used motion to think about what it must be like to eat a hamburger. "It will be forever going in and out of itself so you'd never get a bite with both [the bun and the meat] in it cause sometimes you'd get just the meat and sometimes you'd get just the bun." When Annie was asked how she would describe the fourth dimension to someone else, she said, "I would tell them that it's kind of like…I'm still not like 100% sure, but it's kinda' like an object…it…attached kind of inside an object, but it's kind of rotating so it's always on the inside or the outside." It's interesting that she was okay admitting not being 100% sure about describing the fourth dimension, yet continued trying. This seemed to happen for all the students interviewed.

Fourth Dimension Appearance

In addition to talking about sight and motion in the fourth dimension, students made conjectures about the appearance of fourth dimensional shapes. In these descriptions students used terms like "hyper-human." For example, Susie drew upon her science class to conjecture about "a human inside of a human" as a hyper-shape. Susie's remarks indicated that she was trying to picture humans as they might exist in these conjectured spaces.

An Open Mind

Students did more than describe the fourth dimension. They also talked about being open-minded and close-minded in relation to dimensions other than our own. For example, Mary said:

> Before [learning about the 4th dimension] I always thought the 3rd dimension is as far as you can go in the dimensions or like, I don't know, not really the best, but we're the highest you can go or whatever. But now it's like…there might be something above us…not above us, but something different, or more, or whatever.

Mary talked about thinking that the third dimension was the highest. It's interesting that she chose the phrase, "not really the best, but we're the highest you can go or whatever." Edward did a similar thing:

> He's [Spherius] like but there could never be a fourth dimension. So I think they can believe like numbers lower than them but they don't think there's anything higher than them because

they all want to be the best, I guess. Well, I thought that it would be hard to believe there was a fourth dimension.

This idea seems provocative and indicative of most human thinking, similar to people believing they are smarter than animals. Several students discussed Flatland characters' denial of higher dimensions, arguing about whether you could perceive other dimensions if you could never actually experience them. These discussions indicated a tension regarding whether students' perception of the world was closed or open.

Discussion and Conclusions

We chose these particular manifestations of the phenomenon to show how the experience led them to think about the various dimensions in provocative ways (motion, sight, perception, etc.). The findings indicate that *Flatland* served as what Zeck et al. (1998) refer to as a video anchor, a place to start conjecturing about dimensions. The fact that their conjectures persisted one year after their experience suggests that experiencing dimensional relationships ontologically may be a way to sustain student learning over time. Students were able to articulate specific scenes from the movie, as well as use them to continue conjecturing during the interviews. It may be that the video anchor, which provided a powerful analogy, was able to promote thinking and allow learners to *try on* a being outside their own dimension.

Although we asked non-phenomenological questions, such as conjecturing questions, we feel like their responses give researchers and teachers insight into the capability of middle school students' ability to reason deeply about space and dimension. We admit not attempting to formalize their learning by connecting the investigation to abstract symbols, although we did look at patterns between the dimensions (e.g., number of vertices, edges, faces). The main focus was encouraging them to engage in a conjecturing activity related to their interests following the film. Allowing the space for this to happen gives students an opportunity to engage in mathematical processes, much like a scientist problematizing their world. This seems in line with many of the Realistic Mathematics Education (RME) principles (e.g., Cobb, Zhao, & Visnovska, 2008; Gravemeijer, 1998; van den Heuvel-Panhuizen & Wijers, 2005).

Future Directions

We have already designed and are currently analyzing a second iteration of an instructional program that builds on this study. The revised program incorporates 70 cases of space and perspective in a hypermedia format and collaborative forms of reflective discourse to support the development of robust, flexible concepts among learners (Jacobson, 2008; Kolodner, 2006; Spiro et al., 1988). As part of our research, we have developed a framework grounded in the constructivist perspectives of Cognitive Flexibility Theory (e.g., Spiro et al., 1988) and RME.

At this point, we have developed a media rich set of cases for learners to examine multiple perspectives of space (e.g., noticing surprising similarities and surprising differences between variational instances of a concept) as a way to understand their inherent complexity. One investigation asks students to consider the various ways we capture and represent space. Cases include emergent video techniques that capture sporting events using slow motion or gyro cameras. Concerning invisible spaces, students are asked to consider fractal and hypercube animations representing dimension and spatial relationships we were unable to visualize until recently. At this still emergent stage of design, we are most interested in the ways students experience their engagement with the cases and the learning environment as a whole. Guided by the primary question, what is it for students to find themselves perceiving space as mediated in a variety of ways through technology, we are using post-intentional phenomenological methods to continue our investigation.

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