A Study of the Alignment Between the NAEP Mathematics Framework and the Common Core State Standards for Mathematics (CCSS-M)

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This document is one of four reports by the NAEP Validity Studies Panel that explore the relationship between NAEP and the Common Core State Standards (CCSS) and consider how NAEP can work synergistically with the CCSS assessments to provide the nation with useful information about educational progress. The complete volume with all four reports can be found at www.air.org/common_core_NAEP.

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Executive Summary

Introduction

For decades, prior to the inception of the Common Core State Standards (CCSS), the National Assessment of Educational Progress (NAEP) was the only vehicle through which states could assess the progress of their students using a common metric. Now, 45 states, 4 U.S. territories, and the District of Columbia have adopted the CCSS to provide a clear and consistent curriculum framework to prepare students for college and the workplace. But because NAEP is a critical monitor for comparing results of student achievement across states, it is imperative that the newer CCSS standards and the NAEP frameworks be examined to determine the degree of alignment. The results will allow policymakers to make decisions about what changes, if any, should be made to the NAEP frameworks.

Methodology

This alignment study focuses primarily on the conceptual match between the subtopics and objectives in the NAEP Mathematics Framework and the content standards in the Common Core State Standards for Mathematics (CCSS-M) in Grades K–8. While an item-to-framework study is also critical when inquiring about alignment, items from the CCSS assessment consortia were not available at the time of this study.

Two criteria were used to describe the degree of alignment between the CCSS-M and the NAEP Mathematics Framework: the extent of content coverage and the grade at which the content was covered. To obtain the necessary data, two mappings were conducted: (a) CCSS-M to NAEP Mathematics Framework; and (b) NAEP Mathematics Framework to CCSS-M.

Findings

The study's findings relied on the judgment of four panels of experts who identified the specific CCSS-M content that was not covered well in the NAEP mathematics subtopics and objectives for Grade 4 and Grade 8 and the specific NAEP mathematics content that was not covered well in the CCSS-M at or before the grade level of the NAEP assessment.

The study did not find wide areas of content in the NAEP Mathematics Framework that were not covered in the CCSS-M. Similarly, the study did not find wide areas of content in the CCSS-M that were not covered by the NAEP Mathematics Framework. Nevertheless, there were differences in specificity and conceptual understandings between the CCSS-M and the NAEP Mathematics Framework that are important to note: (1) the CCSS-M have more rigorous content in eighth-grade algebra and geometry; (2) the CCSS-M infuse and distribute the development of mathematical expertise, such as the ability to estimate accurately, throughout the standards for mathematical content, whereas the NAEP Mathematics Framework assesses estimation as a skill in isolation from the vast majority of the content; (3) the CCSS-M attend to developing conceptual understandings of a greater number of mathematical topics (such as unit fractions, patterns, and functions) than does the

NAEP Mathematics Framework; and (4) the CCSS-M introduce some mathematics content, such as probability, at higher grades than does the NAEP Mathematics Framework.

Conclusions, Recommendations, and Next Steps

Certainly, there are differences between the NAEP Mathematics Framework and the CCSS-M. For example, the NAEP Mathematics Framework is an *assessment* framework that prescribes what should be tested on NAEP. The CCSS-M, on the other hand, provide a *curriculum* framework that prescribes what should be taught in classrooms. In those few areas where content is covered by the NAEP Mathematics Framework, but not included in the CCSS-M, and vice versa, studies should be conducted to determine how estimates of students' achievement status and growth are affected by the degree of alignment between what is taught and what is tested.

Historically, the NAEP frameworks have aspired to represent the union of all the various state curricula while reaching beyond these curricula to lead as well as reflect. As a result, NAEP often has pushed on the leading edge of what the nation's children know and should able to do. The introduction of the CCSS-M provides both new opportunities and challenges for NAEP. As the nation moves toward widespread implementation of instruction and assessment based on the CCSS-M, NAEP must balance the goals of comparability over time (i.e., maintaining trend) with current relevance.

CONTENTS

Background

Since its founding in 1963, the National Assessment of Educational Progress (NAEP) has made a unique contribution to American education. Since 1990, when state NAEP was authorized by Congress, NAEP—also referred to as "the Nation's Report Card"—has been the only vehicle through which states can compare the progress of their students against a common standard. Originally, only some states participated in state NAEP, but with the passage of No Child Left Behind, every state receiving Title I funds was required to take state NAEP in reading and mathematics. In 2010, however, the Common Core State Standards (CCSS) for English language arts and mathematics were released, and soon thereafter adopted by 45 states, 4 U.S. territories, and the District of Columbia.

The CCSS Initiative is a state-led effort coordinated by the National Governors Association Center for Best Practices and the Council of Chief State School Officers. The initiative, which includes the development of educational standards, is a collaboration among teachers, school administrators, and experts that was formed to provide a clear and consistent framework of what is needed to prepare American children for college and the workforce. Specifically, the initiative defines the knowledge and skills students should gain during their K–12 education so that they graduate from high school ready to succeed in entry-level, credit-bearing academic college courses or in meaningful workforce training programs. As of this writing, two federally funded state consortia are developing assessments aligned with the CCSS for general education students in Grades 3–8 and high school: the Partnership for Assessment of Readiness for College and Careers (PARCC) and the Smarter Balanced Assessment Consortium (Smarter Balanced). In addition, two other state consortia are developing English language arts and mathematics assessments linked to the CCSS for students with severe cognitive disabilities: the Dynamic Learning Maps Alternate Assessment System Consortium and the National Center and State Collaborative consortium. Finally, the World-Class Instructional Design and Assessment consortium, as well as a second consortium led by WestEd, are developing English language proficiency assessments for English learners.

The Charge

In spring 2011, the National Center for Education Statistics (NCES) asked the NAEP Validity Studies Panel (NVS Panel) to undertake a study of the validity and utility of NAEP in the context of the CCSS. NCES asked that the study address the following questions:

1. What is the conceptual match between NAEP and the CCSS?
2. How should the content in the assessment frameworks and the standards be compared?
3. What could be learned from this comparison?

Two interrelated studies were commissioned: one in reading and writing and the other in mathematics. The purposes of these studies were twofold: (1) to compare the content of the current NAEP reading and mathematics frameworks in grades assessed by NAEP with the content standards of the CCSS in English language arts and mathematics; and (2) to make recommendations to NCES regarding broad issues related to the content comparison of NAEP subtopics and objectives and the CCSS, including the extent of alignment that is appropriate to support NAEP's continuing role as an independent monitor. In the current study, only mathematics is addressed.

NAEP and the Common Core State Standards for Mathematics (CCSS-M): Different Types of Mathematics Frameworks

NAEP began assessing mathematics in 1973, and the long-term trend component of NAEP, which reports on achievement among 9-, 13-, and 17-year-olds, has continued unbroken since that time. A second mathematics trend line, known as "main NAEP," began in 1990 using a new assessment instrument.

The main NAEP mathematics assessment is administered at the national and state levels and in selected urban districts. Results are reported on student achievement at Grades 4, 8, and 12 at the national level and at Grades 4 and 8 at the state level and in large urban districts that volunteer to participate. The main NAEP assessment is based on a framework that is updated periodically, but it has nevertheless been possible to continue the main NAEP trend lines from 1990 through the 2013 assessment for all grade levels. (The greatest changes were introduced in the Grade 12 content objectives in 2009, but special analyses were conducted and confirmed that the Grade 12 trend line could be maintained.)

The NAEP Mathematics Framework is an assessment framework, not a curriculum framework. Because it must fairly assess students from across the country, it spans the full range of mathematics that *could be* taught in America's classrooms. What *is* taught and learned in American classrooms depends on individual state or district mathematics curricula coupled with the educational preparation and instructional practices of teachers and the attentiveness and engagement of students.

The absence of an "official" national curriculum allows for a certain level of flexibility and freedom of choice as to the breadth of content and the depth of coverage in classrooms. This has led to the criticism that the U.S. mathematics curriculum is "a mile wide and an inch deep." The challenge for the CCSS Initiative, then, was to be able to answer the question: What essential mathematical knowledge and skills do students in Grades 4, 8, and 12 need to possess to be equipped to take full advantage of two important postsecondary opportunities—college and careers?

To address this challenge for grades K–12, the CCSS Initiative solicited input, advice, and guidance from professional educators, subject-area experts, policy groups, and the public on how to frame the standards. After reviewing the comments received, the initiative developed the CCSS standards, which were announced in 2009, were released in 2010, and will be assessed across all states in the respective consortia in 2015 when the PARCC and Smarter Balanced assessments are available. The standards were designed to be robust and relevant and based on a careful study of what (1) *is being taught* in countries with whom the United States has to compete and (2) *needs to be taught* to adequately prepare America's young people for successful postsecondary experiences and opportunities. Specifically, for the latter component, the standards' objective was to define a more focused, coherent curriculum framework. In the area of mathematics, the CCSS Initiative developed the CCSS for Mathematics (CCSS-M) content standards to delineate what mathematical content should be taught and learned and what mathematical expertise students should develop.

For more information on the NAEP Mathematics Framework and the CCSS-M, see Appendix A.

Comparing Standards to Standards

Comparing standards to standards can present many challenges and may result in many errors. The purpose of these comparisons is to determine what is substantially the same and what is different about the two sets of standards. One must remember that what is being compared is text. The text is written in a genre that is highly structured, almost in outline form. The authors of the text have choices to make about their structure: what should be superordinate, what should be subordinate, how precisely each topic should be described, and so on. A major goal in comparing standards to standards is to minimize the occurrence of interpretive errors such as pseudo-discrepancies, pseudo-matches, and pseudo-precision. The current study sought to minimize these types of errors.

A *pseudo-discrepancy* can occur when the same material is distributed differently by the compared standards in their respective organizational structures. For example, "estimation" is treated in the NAEP Mathematics Framework as a specific subtopic and also as an "estimation" objective in the content area of Number Properties and Operations, whereas the CCSS-M distribute "estimation" across multiple standards. As such, if the study methodology relied on a literal comparison of words, there would be a finding of discrepancy. The expert panels that participated in the current study were instructed to conduct a more deliberate evaluation of the topic "estimation" that transcended the organizational location of the topic in the text. It was expected that this type of evaluation would reduce the occurrence of pseudo-discrepancy errors.

Similarly, the same term might occur in both standards, leading to a finding of a match based on the literal occurrence of a word or topic. However, the meaning of the word, or the topic, in each context might be quite different, causing a *pseudo-match*. To decrease the occurrence of this type of interpretive error, the panels in this study were asked to evaluate and compare what was said about the topics for each standard and not just rely on the words used.

In addition to errors of pseudo-discrepancy and pseudo-match, there can be errors analogous to drawing inferences beyond the precision of the measurements being made. These are errors of *pseudo-precision*. When the meaning of one text is being compared with the meaning of another text, and the texts, although on the same broad topic, are not organized in parallel, care must be taken in how fine-grained the analysis is. Analyses that are too fine-grained could lead to results that are misleading. Therefore, panelists were asked to make broader judgments, at higher levels of analysis—for example, at the NAEP subtopic or CCSS-M cluster levels—where the differences in organizational structures are less likely to lead to pseudo-precision.

NAEP and the CCSS-M: Risks and Benefits

The large number of states and territories that have adopted the CCSS-M as their state standards has significantly reduced the variation in standards among the states. It is the hope that this will lead to a corresponding reduction in variation among states in curriculum: what instructional materials are used, what gets taught, what gets tested, and what gets learned. NAEP, by its mission, is independent of any particular curriculum. Given this curricular agnostic perspective, we asked the question: What changes, if any, should the National Assessment Governing Board (the Governing Board) consider for NAEP in response to the adoption of the CCSS-M across so many states?

Table 1 lists some possible findings from the comparison of NAEP and CCSS-M, and the risks and benefits associated with each. Each of these "if . . . then" propositions poses consequences for NAEP. As shown in the table, the seriousness of the consequences ranges from medium to high. This study is designed to provide data on the types of findings listed in the first three scenarios.

Table 1. Alignment of NAEP, CCSS-M, and Non-CCSS-M Content and the Consequences for NAEP

IF	THEN	Seriousness of Consequence for NAEP
1. If content is included in the CCSS-M at the grade level assessed by NAEP, but NAEP does not assess it …	Then growth in that content could go undetected by NAEP and NAEP will underestimate growth.	High
2. If content is included in NAEP, but not in the CCSS-M …	Then NAEP growth estimates could be diluted by inclusion of untaught content and NAEP will underestimate current growth; however, NAEP could continue to provide estimates of students' performance in areas of interest for long-term trends.	Medium
3. If there is a large degree of overlap between NAEP assessment objectives and the CCSS-M content standards, but there are states that adopt non-CCSS-M content …	Then growth in non-CCSS-M content will go undetected and NAEP will underestimate growth.	Medium
4. If NAEP item samples have grade level by content-strand interactions (e.g., items sample third-grade place value, fifth-grade graphing, and fourth-grade fractions) …	Then anchor items on the scale and perhaps standard setting may be off.	Medium
5. If NAEP item samples have content by complexity interactions different from the CCSS-M (e.g., higher complexity items with fractions, lower complexity items with operations) …	Then complexity will be confounded with content and the scale could be distorted.	Medium

Note: This list is not intended to be exhaustive.

Conducting Content Alignment Studies: A Review of the Literature

The CCSS-M have been adopted by an overwhelming majority of states; therefore, it is imperative that they be examined to determine whether there is alignment between the standards and the NAEP Mathematics Framework, given that NAEP results are used to make comparisons of student achievement across the states, U.S. territories, and the District of Columbia. Conducting an alignment study between a newly implemented set of standards and a previously used set of standards or assessments allows researchers to determine whether the newer set addresses the same or similar attributes (such as focus, coherence, or rigor) as the older set. The results of an alignment analysis comparing the NAEP Mathematics Framework and the CCSS-M also allow policymakers to make wise decisions about what changes, if any, should be made in the NAEP Mathematics Framework.

Content alignment refers to the degree to which content coverage is the same in two or more frameworks. According to the National Assessment Governing Board (n.d.), it is important to note that regardless of whether the focus of the alignment study is on a framework's attributes or content coverage, alignment refers more to the relationship between the two frameworks (or documents) and less to particular characteristics of either of the documents.

Different methodologies have been used in the various alignment studies that have been conducted over the past decade. Early approaches to the study of alignment were developed by Webb (2002, 2005), Porter (2002, 2006), and Achieve, Inc. (2002). All three approaches use panels made up of individuals with expertise in the content area under study. In each approach, panelists, individually or collectively, rate the degree of alignment using specific criteria. A consensus can be reached by the panel members or there may be interest in reporting the variability that exists among them. The three approaches differ, however, in the types of judgments made by the panelists and in the information that is produced in the alignment study. A detailed discussion of the three approaches and the design to guide implementation of content alignment studies for 12th-grade NAEP assessments in mathematics and reading (as well as other assessments that are used to provide indicators for reporting the preparedness of 12th graders on NAEP in these subjects) can be found at www.nagb.org/publications/design-document-final.pdf.

In fact, there are several different alignment study designs that can be employed: (a) standards to standards; (b) standards to assessment items; (c) assessment items to assessment items; (d) assessment items to assessment frameworks (Daro, Stancavage, Ortega, DeStefano, & Linn, 2007; Everson, Kim, & Butvin, 2009); and (e) assessment frameworks to assessment frameworks. The current study employs a hybrid standards-to-assessment framework design.

Curricular Alignment and the CCSS-M

Interest in the relationships, and particularly the alignment, among standards, assessments, and U.S. students' performance on international as well as national assessments emerged in the late 1990s with the release of the original Third International Mathematics and Science Study (TIMSS) data (Schmidt, McKnight, Valverde, Houang, & Wiley, 1997). The results revealed a downward trend in the performance of U.S. students in Grades 4 through 12 relative to the performance of students in other countries. More than two decades later, the message has not changed. Results from international studies such as TIMSS and the Program for International Student Assessment (PISA), as well as national assessment results from NAEP, echo the mediocre performance of U.S. students, especially in mathematics.

Astute observers of these trends recognize that there are several factors related to low performance (Kilpatrick, Swafford, & Findell, 2001; Schmidt et al., 2001). Some of these factors are embodied in the nature of the curricula (Stancavage et al., 2008). These curricula include not only the written or intended curriculum, but the implemented curriculum (what and how it is taught), the learned curriculum (how and how much of it is learned), and ultimately, the assessed curriculum (how it is assessed). Researchers who study the alignment of intended and assessed curricula and the effects of that alignment on the learned curriculum often operationalize the intended curriculum as curricular content standards, the assessment curriculum as assessment frameworks, and the learned curriculum as student performance or achievement (Porter, 2002; Schmidt & Maier, 2009).

Prior to beginning an alignment study, it is common to identify the criteria that will be used to make judgments about alignment. Quite often, the criteria for excellence or important characteristics of that which is to be examined or compared are identified. In the release of the 1997 TIMSS results, the criterion for excellence that was used to make comparisons among countries consisted of the curriculum standards of all countries whose eighth-grade students performed at the top of the international distribution. These countries were referred to as the A+ countries, and three characteristics of their curriculum standards were identified as important: focus, coherence, and rigor (Schmidt, Wang, & McKnight, 2005). In the 1997 TIMSS study release, a measure of *focus* was defined as "the number of topics covered at each grade that was also aggregated over the first eight grades, by counting the total number of topic-by-grade combinations covered in elementary and middle school" (Schmidt & Houang, 2012, p. 235). Essentially, a set of standards possesses the characteristic of focus to the extent that it has a relatively small number of topics. In addition, Schmidt and Houang (2007) defined a topic-grade combination as coverage of a topic at a particular grade.

Schmidt et al. (2005) considered coherence as the most important characteristic of a set of curriculum standards. They defined *coherence* as a sequence of topics and performances, articulated over time, that is logical and reflects, where appropriate, the sequential and hierarchical nature of the disciplinary content from which the subject matter derives. Thus, coherence refers not only to the coverage of topics within the standards, but more importantly, to whether the sequence in which the topics are covered is consistent with the logical structure of the subject matter from which it is derived. Based on this definition, an international model of coherence, referred to as the A+ model, was derived by an examination and vetting of the coherence found in the national standards of the top-achieving TIMSS countries by a group of mathematicians. Schmidt and Houang (2007) also identified quantitative indicators for both focus and coherence, calculated measures for each of the countries in the A+ group, and related focus and coherence to student achievement. The results of that study suggested that focus is an integral part of the concept of coherence, and their joint influence is positively related to performance on the TIMSS mathematics test.

Schmidt and Houang (2012) also undertook a multicomponent, comprehensive study of the CCSS-M. First, the CCSS-M were compared with the A+ model for congruence. Next, the CCSS-M were compared with state standards to determine the level of congruence. Using data from the Teacher Education and Development Study in Mathematics and *Mathematics Teaching in the 21st Century*, state standards for 50 states were compared with the CCSS-M. The cognitive demand of the CCSS-M and the state standards by grade level was also evaluated using four levels: (1) knowledge—memorizing definitions; (2) performing routine procedures; (3) solving routine problems; and (4) mathematics reasoning, including nonroutine problem solving. Schmidt and Houang (2012) considered cognitive demand to be an indication of a topic's depth, related to the third characteristic of the A+ model—rigor. Last, the authors examined the relationship between the CCSS-M and student achievement, as measured by NAEP, through a simple linear regression. The regression analysis tested the hypothesis that states with standards more congruent to the CCSS-M had higher scores on NAEP in 2009.

A two-dimensional approach was used that consisted of a topic/content specification dimension as well as a performance expectation, or cognitive demand, dimension. To assess the congruence between topic/content and cognitive demand, a matrix was formed with topics in the rows and grades across the columns. Congruence was measured by a combination of focus and coherence. The model of congruence in the Schmidt and Houang (2012) study was the CCSS-M. There were five indicators of congruence that were combined to form one overall measure:

1. A dichotomous (0 or -1) indicator that assessed whether a topic was introduced at an earlier grade level than in the CCSS-M. For every topic for which this was the case, a negative one was added to the indicator; however, a zero was assigned when the topic was introduced on the same grade level as in the CCSS-M.
2. An indicator of *focus* that was calculated by adding a negative one each time a topic was covered at a grade level for which it was not intended in the CCSS-M. These occurrences were then summed over all topics.
3. An indicator of the number of times a topic was not covered at a grade level for which it was intended in the CCSS-M. Every time this occurred, a negative one was added to the topic indicator and summed over all topics.
4. An indicator of whether a topic was covered later than the CCSS-M intended (e.g., decimals were covered in Grade 5 when the CCSS-M had indicated that decimals should not have been covered after Grade 3). Each time this occurred, a negative one was added to the topic indicator and summed across all topics.

5. An indicator of whether a topic was covered across consecutive grades, but was covered in only certain grades in the CCSS-M (e.g., in Grades 5 through 8 versus Grades 5, 6, and 8). These occurrences were coded as in indicator 2 above.

According to Schmidt and Houang (2012), the five indicators were summed across all topics to produce a negative value, which indicated the degree of lack of congruence between the standards and the CCSS-M and, more specifically, the degree of deviation from the CCSS-M. To facilitate interpretation of the results, the overall scale for measuring congruence was converted from a negative scale to a positive one, ranging from 0 to 1,000—with 1,000 indicating perfect agreement with the set of standards that represented the model of congruence

The results showed that the CCSS-M are coherent and focused when compared with the A+ model, even though the CCSS-M contain three additional topics (and the topics in the CCSS-M are not ordered in the same way as in the A+ model). Only three topics in the A+ model were introduced at earlier grades than in the CCSS-M, but several topics were introduced earlier in the CCSS-M than in the A+ model. Overall, there were no significant differences between the CCSS-M and the A+ model (i.e., they are congruent), as the two had a degree of consistency of 85 percent.

The results also revealed that from a maximum of 1,000 points on the measure of congruence with the CCSS-M, states ranged in scores from 662 to 826, with a mean of 762 (SD = 33.5). The 50 states were placed into five categories ranging from *most like CCSS mathematics* to *least like CCSS mathematics* based on their congruence with the CCSS-M. The most congruent states were California, Florida, Georgia, Indiana, Alabama, Minnesota, Oklahoma, Michigan, Mississippi, and Washington; the least congruent states were Arizona, Nevada, Iowa, Kansas, Louisiana, New Jersey, Wisconsin, Rhode Island, and Kentucky.

With regard to the *focus* component of the congruence measure, the CCSS-M required slightly fewer topics than the state standards at Grades 1 through 5, but there was little difference between the CCSS-M and the state standards at Grades 6 through 8. Furthermore, when examining cognitive demand, only 3 percent of the state standards reached the highest level—level 4: “mathematics reasoning, including non-routine problem solving.” By way of contrast, 61 percent of the state standards were at the lowest level—level 1: “knowledge—memorizing definitions.”

The results of a simple linear regression, which included all 50 states, revealed a weak relationship between CCSS-M congruence (that is, congruence between the CCSS-M and state standards) and performance on state NAEP. The states were then divided into two groups: Group 1 consisted of states with standards that varied in their level of congruence with the CCSS-M and the NAEP scores; Group 2 had a high level of congruence with the CCSS-M, but lower NAEP scores. Correlations between the level of CCSS-M-state congruence and performance on state NAEP were then calculated for each group. These analyses revealed there was a positive relationship between congruence and NAEP scores in Group 1, but there was no significant relationship between congruence and NAEP scores in Group 2. After controlling for this group difference, the results showed that states with standards that are more congruent to the CCSS-M generally had higher NAEP mathematics scores.

The current study assesses the alignment of the NAEP Mathematics Framework and the CCSS-M. Unlike Schmidt and Houang (2012), who examined focus and coherence in the CCSS-M, the current study does not specifically examine the extent to which there is focus and coherence in the NAEP framework. Nevertheless, the results of the study could very well lead to the following question: What does the extent of alignment between the NAEP Mathematics Framework and the CCSS-M tell us about the focus and coherence of the NAEP Mathematics Framework, and what effect will that have on NAEP's role as a monitor of student performance in the context of the CCSS-M?

Methodology

In the absence of CCSS-M assessments (which were under development at the time of the study), this alignment study focuses primarily on the conceptual match between the subtopics and objectives in the NAEP Mathematics Framework and the CCSS-M content standards. The Governing Board oversees the development of the NAEP Mathematics Framework, which describes the specific knowledge and skills to be assessed at Grades 4, 8, and 12. The *2011 NAEP Mathematics Assessment Framework* was used in comparisons with the CCSS-M. A subsequent study that compares items from the CCSS-M assessments with the NAEP items will answer other important questions.

The NAEP Mathematics Framework is organized into five broad areas of mathematics content:

- **Number Properties and Operations (NPO),** including computation and understanding of number concepts
- **Measurement (M),** including use of instruments, application of processes, and concepts of area and volume
- **Geometry (G),** including spatial reasoning and applying geometric properties
- **Data Analysis, Statistics, and Probability (DASP),** including graphical displays and statistics
- **Algebra (A),** including representations and relationships

Each content area is divided into subtopics, and each subtopic consists of one or more objectives. These divisions are not intended to separate mathematics into discrete, nonoverlapping elements. Rather, they are intended to provide a helpful classification scheme that describes the universe of mathematical content assessed by NAEP.

The CCSS-M consist of two components: the Standards for Mathematical Content and the Standards for Mathematical Practice. The two components operate in concert to provide school mathematics experiences that, according to the authors, are “substantially more focused and coherent in order to improve mathematics achievement …” in the United States. The CCSS-M set grade-specific content standards for Grades K–8 and subject-specific standards for high school. The grade-level standards are organized into standards, clusters, and content domains. Each content domain consists of clusters of related standards. Standards define what students should understand and be able to do. (See Appendix A for a detailed discussion of how the NAEP Mathematics Framework and the CCSS-M are organized.)

For the current study, two mappings were conducted: (a) CCSS-M content standards to NAEP Mathematics Framework subtopics and objectives; and (b) NAEP Mathematics Framework subtopics and objectives to CCSS-M content standards.

Mappings

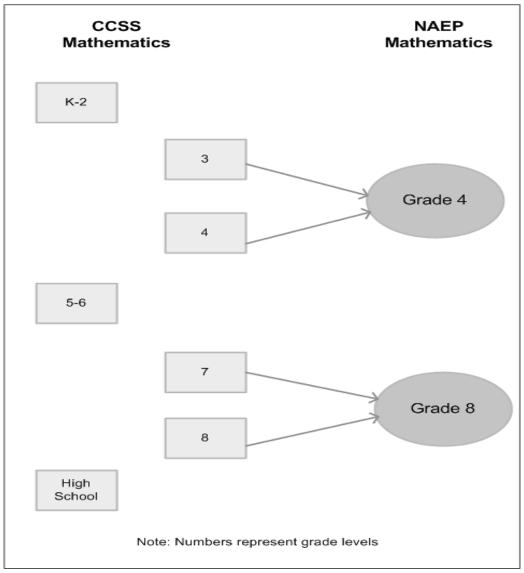
Mapping 1: CCSS-M Standards to NAEP Mathematics Framework (CCSS-M → NAEP)

The mapping from the CCSS-M to the NAEP Mathematics Framework subtopics and objectives was expected to provide answers to the following question, which became **Research Question 1**:

> Which CCSS-M clusters and standards in Grades 3 and 4 or Grades 7and 8 are not represented at all or are not explicitly addressed among the subtopics and objectives for Grade 4 or Grade 8, respectively, in the current NAEP Mathematics Framework? Where there is good representation, in what ways are the CCSS-M clusters/standards and NAEP subtopics/objectives different (i.e., in concept meaning or perspective, specificity of coverage, coverage by grade level, or cognitive demand or complexity)?

Although the CCSS-M span Grades K–8 and high school, Figure 1 shows the specific grade-level mappings referenced in Research Question 1—clusters and standards from Grades 3 and 4 in the CCSS-M to subtopics and objectives for Grade 4 in the NAEP Mathematics Framework, and clusters and standards from Grades 7 and 8 in the CCSS-M to subtopics and objectives for Grade 8 in the NAEP Mathematics Framework. For each mapping, we used the CCSS-M grade that is the same as the grade assessed by NAEP and the CCSS-M grade that is one grade below the grade assessed by NAEP. The absence of arrows means that there was no direct comparison of those grade-level clusters and standards with the subtopics and objectives of the grades assessed by NAEP; however, where important differences or similarities occurred, they were noted.

Figure 1. Mapping From the CCSS-M to the NAEP Mathematics Framework



Note: Numbers represent grade levels

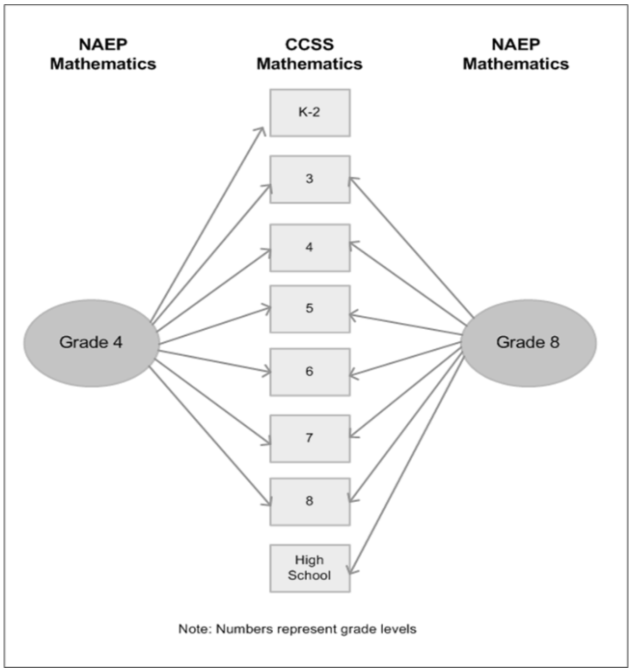
Mapping 2: NAEP Mathematics Framework to CCSS-M Standards (NAEP → CCSS-M)

Any specific Grade 4 or Grade 8 NAEP mathematics objective may suggest a broader breadth of content than any of the CCSS-M specific grade-level standards. Thus, the mapping from NAEP subtopics and objectives to the CCSS-M standards was expected to provide an answer to the following question, which became **Research Question 2**:

> Which NAEP subtopics/objectives for Grade 4 and Grade 8 are not addressed on grade level or have been deemphasized in the CCSS-M?

Figure 2 illustrates how the comparisons for Research Question 2 were operationalized. Each objective in the NAEP Mathematics Framework for Grade 4 and Grade 8 was matched to one or more standards in the CCSS-M. The standards in the CCSS-M that were matched to the objectives in the NAEP Mathematics Framework could be on the grade level of the grade assessed by NAEP or below or above the grade level. The arrows in Figure 2 that extend from Grade 4 and Grade 8 indicate that a "match" could occur across a wide band of clusters and standards in CCSS-M grades. The absence of an arrow from Grade 4 or Grade 8 to a particular CCSS-M grade indicates that a match is not likely to occur among objectives for the grade assessed by NAEP and the standards for that grade in the CCSS-M.

Figure 2. Mappings From the NAEP Mathematics Framework to the CCSS-M



Note: Numbers represent grade levels

Design of the Alignment Study

Specification of Criteria to Determine the Degree of Alignment Between the Two Frameworks

Two criteria were used to describe the degree of alignment between the CCSS-M and the NAEP Mathematics Framework subtopics and objectives: the extent of content coverage and the grade at which the content was covered. The extent of content coverage was rated using four descriptive levels:

- Covered with few differences
- Covered with differences related to specificity
- Covered with differences related to conceptual understanding
- Not covered.

The study also sought to determine the match between the K–8 grades in which the CCSS-M content is supposed to be taught and the grades at which matched objectives appear in the NAEP Mathematics Framework. A mismatch in content by grade could result in an underestimation of students' achievement. For example, if content appears in the NAEP Grade 4 assessment, but that content does not appear in the CCSS-M until later grades, then students who take the NAEP Grade 4 assessment would not have had an opportunity to learn the content. Similarly, if the content appears in the NAEP Grade 4 assessment, but the content is introduced in earlier grades at a level that is less mature than that assessed at Grade 4, then students may not be able to handle the cognitive demand or complexity of the content on the NAEP Grade 4 assessment.

In both cases, students may be underprepared to respond successfully to items or tasks in the NAEP Grade 4 assessment; hence, their mathematics achievement is likely to be underestimated by NAEP. On the other hand, if content that appears in the NAEP Grade 4 assessment is taught in earlier grades in ways that become increasingly more cognitively demanding, then students who take that assessment are better prepared to respond successfully to items or tasks on the NAEP Grade 4 assessment.

Panelists' Procedures for Conducting the Alignment Analysis

Use of Expert Panels: Fourteen experts were divided into four mathematics content panels—two panels each for Grade 4 and Grade 8. At each grade level, one panel addressed the research questions using the NAEP mathematics content areas of Number Properties and Operations and Algebra, while the other panel addressed the research questions using the NAEP mathematics content areas of Measurement; Geometry; and Data Analysis, Statistics, and Probability. Also, two panels examined the alignment of CCSS-M clusters and standards in each of the K–8 grades with the NAEP Grade 4 subtopics and objectives, and two panels examined the alignment of CCSS-M clusters and standards in each of Grades 3 through 8 as well as high school with the NAEP Grade 8 subtopics and objectives.

Composition of Panels: Each panel for Grades 4 and 8 consisted of three or four experts. Experts were drawn from the following four groups: elementary and secondary school teachers and/or school-based mathematics specialists; mathematics educators; mathematicians; and mathematics consultants. Panels were formed based on participants' school-level teaching experience, gender, race/ethnicity, and knowledge of the NAEP Mathematics Framework and the CCSS-M.

Panel Procedures: Panelists reviewed information prior to attending a panel meeting in person. At the panel meeting, panelists discussed their independent judgments about the answers to Research Questions 1 and 2. Then, as a panel, they were asked to reach a consensus about answers to the research questions and to write a panel summary.

To facilitate the panel's review and comparison of the NAEP subtopics and objectives and the CCSS-M clusters and standards and to assist the panelists in answering the research questions, two preliminary mappings—CCSS-M → NAEP and NAEP → CCSS-M—were conducted by Deborah Holtzman, one of the authors of this paper. The results, which were referred to as "Deb's Analysis," were recorded on spreadsheets and sent to the respective panelists.[2]

"Deb's Analysis" was done to reduce the voluminous amount of information about the alignment of CCSS-M clusters and standards with NAEP subtopics and objectives into a manageable quantity. It would not have been possible to ask the panelists to have done this work given the large number of hours these preliminary analyses required. For Grade 4 and Grade 8, the analysis consisted of examining each set of standards organized under a CCSS-M cluster and writing a statement about the extent to which each cluster was covered in a set of NAEP objectives organized by subtopic and grade. "Deb's Analysis" made the examination of the information more manageable, and it also provided a perspective, as a starting point, for panelists to express different levels of agreement with the judgments made about the alignment of the CCSS-M and NAEP Mathematics Framework.

For the CCSS-M → NAEP mapping, "Deb's Analysis" matched groups of standards within each cluster and content domain in the CCSS-M for Grades 3 and 4 with the appropriate objectives, subtopics, and content areas for Grade 4 in the NAEP Mathematics Framework. For example, the CCSS-M Grade 3 standards 8 and 9 in the content domain "Operations and Algebraic Thinking," cluster A "Solve problems involving the four operations and identify and explain patterns in arithmetic" (notated as 3.OA.A.8 and 3.OA.A.9), were matched with the NAEP Grade 4 content area "Number Properties and Operations," subtopic 3 "Number Operations," objective f "Solve application problems involving numbers and operations" (notated as 4NPO3f), and subtopic 5 "Properties of Number Operations," objective e "Apply basic properties of operations" (notated as 4NPO5e). In addition, 3.OA.A.9 was matched to a NAEP Grade 4 objective in the algebra content area, subtopic 1 (4A1a).

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[2] Deborah Holtzman is a Ph.D.-level analyst with expertise in mathematics education at American Institutes for Research.

Similar comparisons were conducted between the CCSS-M for Grades 7 and 8 and the Grade 8 objectives, subtopics, and content areas in the NAEP Mathematics Framework.

For the NAEP → CCSS-M mapping, subtopics and objectives for Grades 4 and Grade 8 in the NAEP Mathematics Framework were compared with standards and clusters in the CCSS-M for Grades K–8. The goal of this mapping was to identify in what grades and to what degree content objectives in the NAEP framework were aligned with standards in the CCSS-M. Thus, for each objective in the NAEP framework for Grade 4 and Grade 8, "Deb's Analysis" identified the grade(s) in which a similarly stated standard was found in the CCSS-M. Furthermore, a judgment statement was recorded about the extent of the content alignment with the NAEP objectives for Grades 4 and 8.

Panelists were asked to review "Deb's Analysis" and indicate whether they agreed or disagreed with each of the mappings. The panelists were also asked to write comments at the standards level for the CCSS-M → NAEP mapping and at the objective level for the NAEP → CCSS-M mapping, in cases where they did not mark "Agree." The purpose of the comments was to note any perceived misinterpretations or additional information needed in "Deb's Analysis." Finally the panelists were asked to review their ratings of agreement and comments across all standards and objectives and to write summaries of their conclusions. The summaries were to be written at the cluster level for the alignment of the CCSS-M to NAEP and at the subtopic level for the alignment of NAEP to the CCSS-M.

The panelists completed these assignments prior to attending a two-day meeting in person. The results of their preliminary work were used to frame panel discussions and to create panel summaries for each CCSS-M cluster and NAEP subtopic comparison.

A leader for each panel was selected from among its members. The panel leader was charged with facilitating the panel's discussions and submitting the panel's cluster and subtopic summaries.

Analysis and Reporting of Findings

Panelists made two types of judgments—one at a more micro level and one at a more macro level—about the alignment of the NAEP Mathematics Framework and the CCSS-M. The micro-level judgments were related to their individual levels of agreement with the results from "Deb's Analysis." The macro-level judgments were related to their collective level of agreement, in the form of panel summaries, about each CCSS-M cluster and NAEP subtopic.

The findings of the study are represented both qualitatively and quantitatively. The *qualitative* findings are represented by identifying the specific NAEP content that is not covered well in the CCSS-M and the specific CCSS-M content that is not covered well in the NAEP framework, based on the panelists' judgments. Content that is covered well and matched by grade level in the two documents carries no major negative consequences for NAEP. Content that is not aligned well may result in negative consequences for NAEP—some of which were noted in Table 1. The *quantitative* aspects of the study are related, in part, to the "spread" of the content alignment across CCSS-M grades. This spread, or the number of grades in which NAEP objectives are addressed in the CCSS-M, speaks to the extent of coverage between the CCSS-M and NAEP frameworks. Both types of findings are captured in the Results and Discussion section below, separately by cluster in the CCSS-M (Tables 2 through 5) and by subtopic in the NAEP Mathematics Framework (Tables 6 and 7). The tables present the panel summaries and also use shading to denote differences in the extent of content coverage and the amount of spread across grades.

There was no attempt to represent findings in terms of correlation coefficients or other statistical representations of alignment.

Results and Discussion

This section presents the results and discussion of the two mappings—CCSS-M → NAEP and NAEP→ CCSS-M—in connection with the panelists' considerations of the answers to Research Questions 1 and 2.

The results of the analysis and subsequent discussion could potentially serve at least two purposes: (1) provide valuable information about the level of student preparedness in the CCSS-M for the mathematics knowledge and skills that the NAEP assessment is designed to measure in Grade 4 and Grade 8; and (2) make recommendations to NCES regarding broad issues related to the content comparison of NAEP and the CCSS-M, including the extent of alignment that is appropriate to support NAEP's continuing role as an independent monitor.

Research Question 1: *Which CCSS-M clusters and standards in Grades 3 and 4 or Grades 7 and 8 are not represented at all or are not explicitly addressed among the subtopics and objectives for Grade 4 or Grade 8, respectively, in the current NAEP Mathematics Framework? Where there is good representation, in what ways are the CCSS-M clusters/standards and NAEP subtopics/objectives different (i.e., in concept meaning or perspective, specificity of coverage, coverage by grade level, or cognitive demand or complexity)?*

Results for CCSS-M Grades 3 and 4 → NAEP Grade 4

To answer Research Question 1, four panels examined the CCSS-M → NAEP mapping. Two panels examined specifically the alignment of CCSS-M clusters and standards in Grades 3 and 4 with the NAEP Grade 4 subtopics and objectives, and two panels examined specifically the alignment of CCSS-M clusters and standards in Grades 7 and 8 with the NAEP Grade 8 subtopics and objectives. The rationale for targeting two adjacent grades in the CCSS-M was to determine the nature of the alignment of the CCSS-M clusters/standards with the NAEP Grade 4 and Grade 8 framework objectives *at or immediately below* Grade 4 and Grade 8, respectively.

All panelists had access to the results of "Deb's Analysis" as a starting point for making individual judgments about content coverage by grade level between the subtopics and objectives in the NAEP Mathematics Framework and the clusters and standards in the CCSS-M. When the panelists convened for a two-day meeting, the individual panelists' judgments were used to form, by consensus, panel summaries for each CCSS-M cluster and NAEP subtopic.

Table 2 presents the panel summaries that describe the alignment between the CCSS-M standards for Grade 3 and the NAEP subtopics and objectives for Grade 4. Each CCSS-M cluster for Grade 3 is listed in the left-hand column of the table. In addition, there is a visual representation of the nature of the content coverage (or alignment) between the CCSS-M and NAEP, as judged by the panelists, in the right-hand column. Table 3 is set up identically to Table 2, but compares the CCSS-M clusters for Grade 4 and the subtopics and objectives in the NAEP framework for Grade 4.

The panel summaries reveal that the content coverage between the CCSS-M and NAEP could be described in essentially four ways: (1) covered with few differences; (2) covered with differences related to specificity; (3) covered with differences related to conceptual understanding; and (4) not covered. For the purposes of this report, (3) and (4) are combined and illustrated together.

An example of *(1) covered with few differences* is found in Table 3 for CCSS-M Grade 4 content domain "Number and Operations: Fractions," cluster C "Understand decimal notation for fractions, and compare decimal fractions." (notated 4.NF.C) The panel summary for this cluster stated the following: "This cluster is closely aligned with NAEP objectives 4NPO1b, 4NPO1e, and 4NPO1i in subtopic Number Sense and 4NPO3a in subtopic Number Operations.*"* Furthermore, there were no statements from the panel about major differences between the CCSS-M standards and the NAEP objectives. This type of content coverage is denoted by a pattern.

Examples of alignment results that illustrate *(2) covered with differences related to specificity* are numerous. For example, in Table 2, the panel summary for the CCSS-M Grade 3, content domain "Measurement and Data," cluster A "Solve problems involving measurement and estimation" (3.MD.A), stated the following: "The CCSS-M are more detailed in their requirements and also more specific in connecting problem solving and measurement data." In another example in Table 3, the panel summary for the CCSS-M Grade 4 content domain "Operations and Algebraic Thinking," cluster A "Use the four operations with whole numbers to solve problems" (4.OA.A), indicated that: "Computational objectives for CCSS-M and NAEP are aligned in NAEP objectives 4NPO3e and 4NPO3f; however, the representation of multiplication as a comparative operation in CCSS-M is not included (or specified) in NAEP." This type of content coverage is denoted by dark gray.

The two types of alignment that could potentially have more negative consequences for NAEP in its role as a monitor—because they could result in NAEP underestimating student performance—are related to *(3) covered with differences related to conceptual understanding* and *(4) not covered*. An example of alignment (3) can be found in Table 2 for the CCSS-M Grade 3 content domain "Operations and Algebraic Thinking," cluster A "Represent and solve problems involving multiplication and division" (3.OA.A). Here the panel summary noted: "CCSS-M goes beyond the NAEP objectives, which concentrate primarily on procedural skill… It is unclear whether both sets of expectations hold the same conceptual understanding." Differences in cognitive demand could also be related to differences in conceptual understanding. For example, within the same content domain, the panel summary for cluster D, "Solve problems involving four operations, and identify and explain patterns in arithmetic" (3.OA.D), stated: "[S]tandards in this cluster require students to '*explain* patterns in arithmetic,' whereas in NAEP objective 4A1a, the expectation is to '*extend* numerical patterns'." These types of content coverage are denoted by light gray.

Table 2. Coverage of the CCSS-M Grade 3 Clusters in the NAEP Grade 4 Mathematics Framework[1,2]

CCSS-M Grade 3 Clusters	Panel Summaries on Alignment of CCSS-M Grade 3 With NAEP Grade 4	Coverage in the NAEP Grade 4 Mathematics Framework
3.OA: Operations and Algebraic Thinking		
Cluster A: Represent and solve problems involving multiplication and division.	Both the CCSS-M and the NAEP framework expect students to solve problems involving multiplication and division. The CCSS-M in this cluster are mapped to the NAEP Grade 4 subtopic Number Operations, objectives 4NPO3b, 4NPO3c, 4NPO3e, and 4NPO3f. Panelists note that the CCSS-M go beyond the NAEP Grade 4 objectives, which concentrate primarily on procedural skill. It is unclear whether both sets of expectations hold the same conceptual understanding.	░
Cluster B: Understand properties of multiplication and the relationship between multiplication and division.	Although conceptually aligned, the CCSS-M in this cluster clearly set the groundwork for algebraic expressions, which are not covered in the NAEP Grade 4 framework. Some content is covered in the NAEP subtopic Properties of Number and Operations, Grade 4 objective 4NPO5e.	▓
Cluster C: Multiply and divide within 100.	Topical coverage is aligned; however, the CCSS-M expectation includes both fluency and from memory whereas the NAEP Grade 4 objectives, 4NPO3b and 4NPO3c, include the use of a calculator.	▓
Cluster D: Solve problems involving the four operations, and identity and explain patterns in arithmetic.	The CCSS-M expect students to solve two-step word problems with equations. It is unclear whether the expectation of application problems found in NAEP Grade 4 objectives 4NPO3f or 4NPO5e includes two-step problems. Also, standards in this cluster require students to "explain patterns in arithmetic," whereas in the NAEP Grade 4 objective 4A1a, the expectation is to "extend numerical patterns."	░
3.NBT: Number and Operations in Base Ten		
Cluster A: Use place value understanding and properties of operations to perform multidigit arithmetic.	There is an explicit expectation that understanding of place value is used to round whole numbers, and fluency is used to add and subtract and to multiply one-digit numbers by multiples of 10. The explicit expectation of rounding is not included in the NAEP Grade 4 objectives. Rather, rounding is mentioned, parenthetically, in the NAEP Grade 4 objective 4NPO2b, which states: "Makes estimates appropriate to ... whole numbers ... by … selecting the appropriate method of estimation (e.g., rounding)." Additionally, the CCSS-M expect fluency, whereas the NAEP Grade 4 framework allows calculators and provides guidelines for what computations will be assessed with and without the use of calculators. Some content coverage of the standards in this CCSS-M cluster can also be found in objectives in three NAEP Grade 4 subtopics: Number Sense—4NPO1a; Number Operations—4NPO3a, b, and e; and Properties of Number and Operations—4NPO5e.	░

3.NF: Number and Operations: Fractions		
Cluster A: Develop understanding of fractions as numbers.	Conceptual understanding of fractions as numbers, especially using a number line, is an expectation in the CCSS-M; however, this expectation is absent in the NAEP Grade 4 framework. The framework suggests models as representations of fractions in the NAEP Grade 4 objective 4NPO1e. Both the CCSS-M standard 3.NF.A.3d and the NAEP Grade 4 objective 4NPO1i address “comparing fractions,” but the CCSS-M make explicit the validity of comparisons in the context of the same whole. Furthermore, reasoning about the size of fractions in the CCSS-M expects a lot more than simply comparing numbers as indicated in the NAEP Grade 4 objectives.	░
3.MD: Measurement and Data		
Cluster A: Solve problems involving measurement and estimation.	The CCSS-M are more detailed in their requirements and also more specific in connecting problem solving and measurement data. The panelists thought that the NAEP Grade 4 objectives 4NPO3f, 4M1c, and 4M1e aligned well. The CCSS-M focus on time, volume, and weight only—not on temperature, as does the NAEP Grade 4 objective 4M1b, which specifically mentions temperature.	■
Cluster B: Represent and interpret data.	The standards in this cluster on solving problems related to a data set do not appear to be as tightly focused as the NAEP Grade 4 objectives. Similarly, the standards’ focus on measuring and plotting the measurements in a line plot does not seem to be fully captured by the NAEP Grade 4 objectives.	■
Cluster C: Geometric measurement: understand concepts of area and relate area to multiplication and to addition.	The standards in this cluster—3.MD.C.5 through 3.MD.C.7d—make up a much more specific, prescriptive, and detailed treatment of student learning outcomes than the NAEP Grade 4 objective, 4M1g, which simply states “solve problems involving area of squares and rectangles.” The CCSS-M describe the process of measuring area in much greater detail. The CCSS-M are also very specific about representing the distributive property using areas of rectangles. This treatment continues in the NAEP Grade 4 objectives, but is not nearly as specific.	■
Cluster D: Geometric measurement: recognize perimeter.	Both the CCSS-M and NAEP Grade 4 framework address solving problems involving perimeter; however, the CCSS-M are more specific and focused than the NAEP Grade 4 objectives. For example, problems in the CCSS-M might involve rectangles with the same perimeter and different areas or with the same area and different perimeters. The relevant NAEP Grade 4 objective 4M1f simply states, “solve problems involving perimeter of plane figures.”	■
3.G: Geometry		
Cluster A: Reason with shapes and their attributes.	This cluster is another example of the CCSS-M being more targeted than what would be found in the NAEP Grade 4 framework, especially at the standard level. The NAEP Grade 4 framework and the CCSS-M also use slightly different language around definition, classification, categories, and so on. The standard 3.G.A.2 in this cluster is more about fractions than about geometry. Also, the NAEP Grade 4 objective 4NPO1e, in the subtopic Number Sense, seems a better fit for the CCSS-M standard 3.G.A.2 than any of the NAEP Grade 4 objectives for Geometry.	░

▨	Covered with few differences
■	Covered with differences related to specificity
░	Covered with differences related to conceptual understanding

[1] *Notation for the CCSS-M:* Grade level, content domain, cluster, standard number within domain. For example, 3.OA.D.8 is read as Grade 3, Operations and Algebraic Thinking, Cluster D, Standard 8.

[2] *Notation for NAEP objectives:* Grade level, content area, subtopic, objective. For example, 4NPO1i is read as Grade 4, Number Properties and Operations, Subtopic 1, Objective i.

Table 3. Coverage of the CCSS-M Grade 4 Clusters in the NAEP Grade 4 Mathematics Framework[1,2]

CCSS-M Grade 4 Clusters	Panel Summaries on Alignment of CCSS-M Grade 4 With NAEP Grade 4	Coverage in the NAEP Grade 4 Mathematics Framework
4.OA: Operations and Algebraic Thinking		
Cluster A: Use the four operations with whole numbers to solve problems.	Computational standards in the CCSS-M are aligned with NAEP Grade 4 objectives 4NPO3e and 4NPO3f; however, the representation of multiplication as a comparative operation found in the CCSS-M is not among the NAEP objectives. The standard 4.OA.A.3 in this cluster also includes estimation strategies (e.g., rounding) to determine the reasonableness of an answer. A similar expectation can be found in the NAEP Grade 4 objectives 4NPO2b and 4NPO2c under the subtopic Estimation.	■
Cluster B: Gain familiarity with factors and multiples.	In the CCSS-M, whole numbers in the range of 1 to 100 are classified as prime or composite. Although factor pairs for 1 to 100 are determined, the CCSS-M do not specify prime or composite factorizations. In fact, in the CCSS-M, there is no mention of prime factorization per se. In NAEP Grade 4 objective 4NPO5b, however, there is an expectation to "recognize, find, or use factors, multiples, or prime factorization."	■
Cluster C: Generate and analyze patterns.	Similar topical coverage of patterns can be found across NAEP Grade 4 objectives 4A1a, 4A1b, 4A1c, and 4A1d. Although the generation of patterns using a rule is a common expectation in the CCSS-M and the NAEP Grade 4 framework, this CCSS-M cluster also expects students to be able to analyze patterns and explain attributes of the elements of the pattern. This "analysis of patterns" expectation is not found among the NAEP Grade 4 objectives.	■
4.NBT: Number and Operations in Base Ten		
Cluster A: Generalize place value understanding for multidigit whole numbers.	The connection between place value and comparing and ordering whole numbers is not specifically made in the NAEP Grade 4 objectives. Rounding is a strategy explicitly called for in the CCSS-M, but is offered as an example of an estimation strategy in the NAEP Grade 4 framework. (See objective 4NPO2b under the NAEP Grade 4 subtopic Estimation.)	░
Cluster B: Use place value understanding and properties of operations to perform multidigit arithmetic.	Illustrations and explanations of computational results by using equations, rectangular arrays, and/or area models are included in the CCSS-M in this cluster, but are not included in the NAEP Grade 4 framework. Fluency in adding and subtracting multidigit whole numbers is an expectation unique to the CCSS-M.	░

4.NF: Number and Operations: Fractions		
Cluster A: Extend understanding of fractions equivalence and ordering.	The NAEP Grade 4 framework does not sufficiently address fractions as a quantity. The CCSS-M include the statement “Recognize that comparisons are valid only when the two fractions refer to the same whole.” This is an important concept on which the NAEP Grade 4 framework is silent. The NAEP framework does not include the symbols <, >, or =, nor does it connect estimation to comparing numbers. Topics related to comparing fractions can be found in the NAEP Grade 4 subtopics Number Sense (objective 4NPO1i) and Estimation (objective 4NPO2a), but there is no reference to using visual fraction models to compare fractions as in the CCSS-M.	░
Cluster B: Build fractions from unit fractions.	Both the CCSS-M in this cluster and NAEP Grade 4 objective 4NPO3a address addition and subtraction of fractions; however, the CCSS-M approach to building fractions with the use of unit fractions and operations on whole numbers is unique to the CCSS-M. Further, NAEP Grade 4 objectives do not include multiplication of fractions.	░
Cluster C: Understand decimal notation for fractions, and compare decimal fractions.	This cluster is closely aligned with NAEP Grade 4 objectives 4NPO1b, 4NPO1e, and 4NPO1i in the subtopic Number Sense and NAEP Grade 4 objective 4NPO3a in the subtopic Number Operations. These objectives in the NAEP framework cover “representing numbers using models” (as in the case of decimal fractions), comparing decimal fractions, and operations on fractions and decimals.	▒
4.MD: Measurement and Data		
Cluster A: Solve problems involving measurement and conversion of measurements.	Alignment is good; however, some differences are related to specificity. For example, the span of the NAEP Grade 4 subtopics and objectives that map to this CCSS-M cluster reflects the tendency of the CCSS-M to draw together topics from multiple NAEP Grade 4 objectives, including Number Properties and Operations—4NPO3f; Measurement—4M1b, 4M1f, and 4M1g; and Algebra—4A1e. Standard 4.MD.A.2 in this cluster covers solving problems involving simple fractions and decimals. Solving problems involving multiplication or division with fractions or decimals is not represented in any Grade 4 objective in the NAEP framework; rather, this expectation is covered in the NAEP Grade 8 objective 8NPO3f.	▓
Cluster B: Represent and interpret data.	Alignment is good, with exceptions worth noting. The CCSS-M include expectations that students will be given multiple opportunities to analyze and interpret data that they have collected or been given. The CCSS-M require that students know and use multiple ways of representing data and be able to communicate and justify their thinking. The CCSS-M place more emphasis on line plots than do the objectives in the NAEP Grade 4 framework. The NAEP Grade 4 objectives that map to this cluster are in Number Properties and Operations—4NPO3f; and Data Analysis, Statistics, and Probability—4DASP1a and 4DASPb.	▓
Cluster C: Geometric measurement: understand concepts of angle and measure angles.	The CCSS-M’s approach to measurement appears to be a better balance of building a conceptual basis for later procedural skills, whereas the approach in the NAEP Grade 4 framework seems to be more procedural. This CCSS-M cluster is mapped to the NAEP Grade 4 Geometry objective 4G1c, which states: “Identify or draw angles and other geometric figures in the plane.” Measuring angles and drawing angles of a specific measure are emphasized in the CCSS-M, but are less specific in the NAEP Grade 4 objective 4G1c.	░

4.G: Geometry		
Cluster A: Draw and identify lines and angles, and classify shapes by properties of their lines and angles.	Alignment between the CCSS-M and the NAEP Grade 4 objectives 4G1c and 4G1d is good for this CCSS-M cluster.	▨

▨	Covered with few differences	■	Covered with differences related to specificity	□	Covered with differences related to conceptual understanding

[1] *Notation for the CCSS-M:* Grade level, content domain, cluster, standard number within domain. For example, 3.OA.D.8 is read as Grade 3, Operations and Algebraic Thinking, Cluster D, Standard 8.

[2] *Notation for NAEP objectives:* Grade level, content area, subtopic, objective. For example, 4NPO1i is read as Grade 4, Number Properties and Operations, Subtopic 1, Objective i.

Discussion of the Extent of Alignment Between CCSS-M Grades 3 and 4 and NAEP Grade 4

The computational requirements in the CCSS-M at Grades 3 and 4 are matched by the requirements in the objectives in the NAEP framework at Grade 4. One exception to matching computational demands is that the CCSS-M include multiplication of fractions by whole numbers, but the Grade 4 NAEP objectives do not. The CCSS-M also emphasize representing quantitative relationships in a real-world problem by an expression or equation as well as in two-step problems, whereas Grade 4 NAEP objectives do not. An item-to-item comparison in subsequent assessment alignment studies will reveal if these differences are of concern when it comes to NAEP's ability to continue to assess states' educational progress and thereby provide valid information.

Some of the specific understandings in the CCSS-M number domains that are not included in the NAEP framework at Grade 4 are (1) understanding that place value in base 10 implies that each place is worth 10 times as much as the place to its right, (2) illustrating and explaining a multiplication calculation by using equations, (3) using rectangular arrays and/or area models in problem solving, (4) understanding fractions as numbers, and (5) generating fraction equivalence (except by "comparison").

The CCSS-M Grade 3–4 measurement domain is more detailed and specific than are the NAEP Grade 4 objectives. This situation could eventually lead to differences in emphases between Grade 4 measurement items used for NAEP and the CCSS-M. For example, although both the CCSS-M and the NAEP framework include area of rectangles at Grade 4, only the CCSS-M ask for understanding of the connection between area and multiplication and the additivity of areas. In addition, the CCSS-M specify fractional and decimal lengths for measurement problems, while the NAEP framework is not as specific.

A close examination should be undertaken by the Governing Board of each cluster in the CCSS-M for which there is coverage in the NAEP framework at Grade 4, but "with differences related to specificity" or "with differences related to conceptual understanding," or where there is "no coverage in the NAEP framework" at Grade 4.

Importantly, an item-to-item comparison in subsequent studies will reveal if these differences between the CCSS-M and the Grade 4 NAEP subtopics and objectives are associated with variances in the tests.

Results for CCSS-M Grades 7 and 8 → NAEP Grade 8

Table 4 provides panel summaries that describe the alignment between the CCSS-M standards for Grade 7 and the subtopics and objectives in the NAEP Mathematics Framework for Grade 8. Each CCSS-M cluster for Grade 7 is listed in the left-hand column of the table. There is also a visual representation of the nature of the content coverage (or alignment) between the CCSS-M and the NAEP Grade 8 framework, as judged by the panelists. Table 5 is set up identically to Table 4, but compares the CCSS-M clusters for Grade 8 and the NAEP subtopics and objectives at Grade 8.

The panel summaries for the CCSS-M for Grade 7 and Grade 8 reveal that the content coverage between the CCSS-M and the NAEP subtopics and objectives for Grade 8 framework could be described in the same way as the analyses reported for Grades 3 and 4 above: (1) covered with few differences, (2) covered with differences related to specificity, (3) covered with differences related to conceptual understanding, and (4) not covered. For the purposes of this report, (3) and (4) are combined and illustrated together.

An example of *(1) covered with few differences* is found in Table 4 for cluster A "Use properties of operations to generate equivalent expressions" in the CCSS-M Grade 7 content domain "Expressions and Equations" (7.EE.A). The panel summary for this cluster stated: "The CCSS-M specify rational coefficients, while the NAEP Grade 8 framework does not." This type of difference is not major. This was the only cluster in this mapping where coverage between the CCSS-M at Grade 7 and the NAEP subtopics and objectives at Grade 8 were judged to have few differences.

There are several examples of alignment *(2) covered with differences related to specificity*. For example, in Table 4, for cluster B, "Solve real-life and mathematical problems using numerical and algebraic expressions and equations" in CCSS-M Grade 7 content domain "Expressions and Equations" (7.EE.B), the panel summary stated: "…the CCSS-M standard 7.EE.B.3 in this cluster includes 'assess the reasonableness of answers using mental computation and estimation strategies,' which is not explicitly emphasized in NAEP." Another example is in Table 5 for cluster A, "Know that there are numbers that are not rational, and approximate them by rational numbers," in the CCSS-M Grade 8 content domain "Number System" (i.e., 8.NS), where the panel summary stated: "…the CCSS-M for Grade 8 address irrational numbers more explicitly than the NAEP Grade 8 framework. The NAEP Grade 8 framework addresses irrational numbers in two subtopics—Number Sense (objective 8NPO1e) and Estimation (objective 8NPO2a)—where "common irrational numbers such as e and π are applied in contexts."

An example of alignment (*3) covered with differences related to conceptual understanding* can be found in cluster A, "Analyze proportional relationships and use them to solve real-world and mathematical problems," in the CCSS-M for the Grade 7 content domain "Ratios and Proportional Relationships" (7.RP.A). Panelists noted: "Even though there is somewhat of a match between the NAEP Grade 8 objectives … and the CCSS-M cluster, the NAEP Grade 8 objectives do not require the depth of conceptual understanding called for in the CCSS-M." An example of a cluster in which panelists observed that there was "no coverage" can be found in cluster A, "Define, evaluate and compare functions," in the CCSS-M Grade 8 content domain "Functions" (8.F.A). Panelists noted: "…the CCSS-M standard 8.F.A.1 is not addressed in the NAEP Grade 8 framework."

Table 4. Coverage of the CCSS-M Grade 7 Clusters in the NAEP Grade 8 Mathematics Framework[1,2]

CCSS-M Grade 7 Clusters	**Panel Summaries on Alignment of CCSS-M Grade 7 With NAEP Grade 8**	**Coverage in the NAEP Grade 8 Mathematics Framework**
7.RP: Ratios and Proportional Relationships		
Cluster A: Analyze proportional relationships and use them to solve real-world and mathematical problems.	Even though there are similarities between the NAEP Grade 8 objectives 8M1i (i.e., solving problems involving ratios) and 8NPO4b, 8NPO4c, and 8NPO4d (i.e., using fractions to represent ratios and proportions) and the standards in this CCSS-M cluster, the NAEP Grade 8 objectives do not require the depth of conceptual understanding called for in the CCSS-M. Items in the NAEP Grade 8 assessment generated from these NAEP objectives could be solved by setting up proportions without understanding the underlying concepts related to proportionality.	░
7.NS: The Number System		
Cluster A: Apply and extend previous understandings of operations with fractions.	The computational aspect of CCSS-M standard 7.NS.A.1 in this cluster is addressed in the NAEP Grade 8 objective 8NPO3a (i.e., perform computations with rational numbers); however, there is no mention of number line representations of fractions in the NAEP Grade 8 objective, as there is in standard 7.NS.A.1. CCSS-M standard 7.NS.A.2d refers to terminating or repeating decimal forms. Division of rational numbers is inferred in NAEP Grade 8 objective 8NPO3a, but explicit knowledge of terminating or repeating decimals is not. Other standards in this cluster map onto NAEP Grade 8 objectives 8NPO3d and 8NPO3e.	░
7.EE: Expressions and Equations		
Cluster A: Use properties of operations to generate equivalent expressions.	Standard 7.EE.A.1 in this cluster is addressed in NAEP Grade 8 objectives across two content areas: Number Properties and Operations—8NPO5e; and Algebra—8A3c. The CCSS-M specify rational coefficients, while the NAEP Grade 8 objectives do not. This latter difference in expectation is not major.	▒
Cluster B: Solve real-life and mathematical problems using numerical and algebraic expressions and equations.	Standard 7.EE.B.3 in this cluster includes "assess the reasonableness of answers using mental computation and estimation strategies," which is not emphasized in the NAEP Grade 8 framework. Also, 7.EE.B.3 emphasizes "apply properties of operations to calculate with numbers in any form," whereas the NAEP Grade 8 objective only references the calculations. 7.EE.B.3 also includes performing operations with tools to solve numeric problems, not just linear algebraic expressions. In addition, 7.EE.B.4a includes "compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach," which is not included in the NAEP Grade 8 framework.	▓

7.G: Geometry		
Cluster A: Draw, construct, and describe geometrical figures and describe the relationships between them.	The objectives in the NAEP Grade 8 framework do not focus as much on work with triangles as do the CCSS-M. By focusing on triangles, the CCSS-M pave the way to formal high school work with triangle congruence criteria.	░
Cluster B: Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.	Standard 7.G.B.4 in this cluster calls for an informal derivation of the relationship between the circumference and area of a circle, which does not appear in any of the objectives in the NAEP Grade 8 framework. The CCSS-M also have a greater focus on solving for unknown angle measures in preparation for standard 8.G.A.5 and the standards in the Geometry domain for high school. In the CCSS-M, there are more obvious opportunities for employing the Standards for Mathematical Practice.	■
7.SP: Statistics and Probability		
Cluster A: Use random sampling to draw inferences about a population.	Variability among sample means is not addressed in the NAEP Grade 8 objectives. The standards in this cluster focus on variation, generating a sample, and randomness as a tool for making samples representative. The standards are mapped to the NAEP Grade 8 objectives 8DASP3a and 8DASP3b under the subtopic Experiments and Samples.	■
Cluster B: Draw informal comparative inferences about two populations.	The standards in this cluster focus on making informal comparisons between two populations using measures of variability and central tendency. The NAEP Grade 8 objective 8DASP2d, which states: "Using appropriate statistical measures, compare ... two different populations…," infers the use of measures of variability and central tendency for making comparisons, but is not as specific as the standards in this cluster.	■
Cluster C: Investigate chance processes and develop, use, and evaluate probability models.	The CCSS-M are more specific about expectations and results from greater versus fewer numbers of trials. The CCSS-M elaborate more fully the idea of sample space. The relevant NAEP Grade 8 objectives under the subtopic Probability include 8DASP4a, b, c, d, e, f, g, and j.	■

▨ Covered with few differences
■ Covered with differences related to specificity
░ Covered with differences related to conceptual understanding

[1] *Notation for the CCSS-M:* Grade level, content domain, cluster, standard number within domain. For example, 3.OA.D.8 is read as Grade 3, Operations and Algebraic Thinking, Cluster D, Standard 8.

[2] *Notation for NAEP objectives:* Grade level, content area, subtopic, objective. For example, 4NPO1i is read as Grade 4, Number Properties and Operations, Subtopic 1, Objective i.

Table 5. Coverage of the CCSS-M Grade 8 Clusters in the Grade 8 NAEP Mathematics Framework[1,2]

CCSS-M Grade 8 Clusters	**Panel Summaries on Alignment of CCSS-M Grade 8 With NAEP Grade 8**	**Coverage in the NAEP Grade 8 Mathematics Framework**
8.NS: The Number System		
Cluster A: Know that there are numbers that are not rational, and approximate them by rational numbers.	The Grade 8 CCSS-M cover irrational numbers more broadly than the NAEP Grade 8 framework. The NAEP Grade 8 framework addresses rational numbers in objectives in two subtopics—Number Sense (objective 8NPO1e) and Estimation (objective 8NPO2a)—and focuses on “common irrational numbers,” such as e and π in applied contexts.	■
8.EE: Expressions and Equations		
Cluster A: Work with radical and integer exponents.	The standards in this cluster address exponents more specifically and radicals/roots more conceptually than the objectives in the NAEP Grade 8 framework. For example, neither standard 8.EE.A.1 (“laws of integer exponents”) nor standard 8.EE.A.2 (“represent solutions to equations of the form x^2= p and x^3 = p, where p is a positive rational number” and “know that $\sqrt{2}$ is irrational”) is covered in the NAEP Grade 8 framework. The CCSS-M expect students to perform operations with numbers expressed in scientific notation, including multiplicative comparisons. The NAEP Grade 8 objectives 8NPO1f, 8NPO2d, and 8A3c cover scientific notation, estimating square and cube roots, and performing basic operations on roots, respectively.	▒
Cluster B: Understand the connections between proportional relationships, lines, and linear equations.	The intent of this cluster is to address the connections between proportional relationships, lines, and equations. This is not a focus in the NAEP Grade 8 framework. Standard 8.EE.B.5 in this cluster is mapped to the following NAEP Grade 8 objectives: 8NPO4c, 8A1f, 8A2a, 8A2b, and 8A4d. These NAEP objectives appear in the subtopics Ratios and Proportional Reasoning; Patterns, Relations, and Functions; Algebraic Representations; and Equations and Inequalities. Standard 8.EE.B.6 is not covered in the NAEP Grade 8 framework.	■
Cluster C: Analyze and solve linear equations and pairs of simultaneous linear equations.	The standards in this cluster are not covered well in the NAEP Grade 8 framework. Standard 8.EE.C.8, “Analyze and solve simultaneous systems of linear equations,” is not covered at all in the NAEP Grade 8 framework. Standard 8.EE.C.7a, “Give examples of linear equations with different number of solutions,” also is not covered. In addition, the NAEP Grade 8 framework does not address linear equations with the distributive property, as called for in this cluster. What is addressed is found in the NAEP Grade 8 objective 8A4a, “Solve linear equations or inequalities,” and Grade 8 objective 8A4c, “Analyze situations or solve problems using linear equations and inequalities with rational coefficients symbolically or graphically.”	▒

8F: Functions		
Cluster A: Define, evaluate, and compare functions.	Standard 8.F.A.1 is not addressed in the NAEP Grade 8 framework; however, components of the remaining standards, 8.F.A.2 and 8.F.A.3, are mapped to the following NAEP Grade 8 objectives: 8A1c, 8A1e, and 8A1f, which address patterns, relations, and functions; 8A2a, 8A2b, and 8A2f, which address algebraic representations; and 8A4d, which focuses on interpretations of relationships between symbolic linear expressions and their graphical representations.	░
Cluster B: Use functions to model relationships between quantities.	The standards in this cluster are mapped to the following NAEP Grade 8 objectives: 8A1c and 8A1e; 8A2a, 8A2b, and 8A2f; and 8A5a. These NAEP Grade 8 objectives are subsumed under the subtopics Patterns, Relations, and Functions; Algebraic Representations; and Mathematical Reasoning in Algebra, respectively.	■
8G: Geometry		
Cluster A: Understand congruence and similarity using physical models, transparencies, or geometry software.	The NAEP Grade 8 framework does not have the same explicit focus on triangles and angles that appears in standard 8.G.A.5. In the NAEP Grade 8 framework, transformations are not explicitly connected to congruence and similarity. In the CCSS-M, transformations provide the undergirding for an understanding of these ideas. The CCSS-M also provide for the use of technology as a tool for work in this cluster. The properties of transformations are made explicit in the CCSS-M, but not in the NAEP Grade 8 framework. Standards in this cluster are mapped to the following NAEP Grade 8 objectives: 8G2c,8G2e, 8G2f, 8G3f, and 8G4d.	░
Cluster B: Understand and apply the Pythagorean Theorem.	The CCSS-M go further than the NAEP Grade 8 framework in expectations of fluency with the Pythagorean Theorem. Standard 8.G.B.6 specifies “explain a proof” and standard 8.G.B.7 covers work in two and three dimensions. Some of these concepts are mapped to the NAEP Grade 8 objective 8G3d.	■
Cluster C: Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.	Standard 8.G.C.9 requires work with volume of a sphere, cone, or cylinder. This standard can be mapped to the NAEP Grade 8 Measurement objective 8M1h, which focuses on solving problems involving the volume or surface area of rectangular solids, cylinders, prisms, or composite shapes.	■
8SP: Statistics and Probability		
Cluster A: Investigate patterns of association in bivariate data.	The CCSS-M and the NAEP Grade 8 framework include work with scatterplots, but the CCSS-M go beyond finding a line of best fit and interpreting slope to having students make scatterplots and interpret various patterns of distribution. The CCSS-M also cover modeling relationships between quantities using scatterplots. Bivariate categorical data are missing from the NAEP Grade 8 framework. The standards in this cluster are mapped to the NAEP Grade 8 objectives from Data Analysis, Statistics, and Probability (8DASP2e) and Algebra (8A1f).	■

▨	Covered with few differences	■	Covered with differences related to specificity	░	Covered with differences related to conceptual understanding

[1] *Notation for the CCSS-M:* Grade level, content domain, cluster, standard number within domain. For example, 3.OA.D.8 is read as Grade 3, Operations and Algebraic Thinking, Cluster D, Standard 8.

[2] *Notation for NAEP objectives:* Grade level, content area, subtopic, objective. For example, 4NPO1i is read as Grade 4, Number Properties and Operations, Subtopic 1, Objective i.

Discussion of the Extent of Alignment Between CCSS-M Grades 7 and 8 and NAEP Grade 8

There are some differences between the CCSS-M at Grades 7–8 and the NAEP framework at Grade 8 that are related to conceptual understanding; these may lead to differences in learning and in the development of the respective assessments. The emphasis in the CCSS-M's "ratio and proportionality" on unit rate (constant of proportionality) is not matched in the NAEP framework at Grade 8; however, the NAEP framework covers the CCSS-M topics in ratio and proportionality. The CCSS-M make explicit the use of number lines in specifying understanding of number systems, whereas the NAEP framework at Grade 8 does not.

Expressions and Equations (algebra) is one content domain in the CCSS-M for which students may be learning mathematics that goes untested and undetected by NAEP at Grade 8. This is perhaps the most dangerous risk to the NAEP mission, given the national priority on algebra for all. It is fundamental to NAEP's mission that its assessments be able to detect progress in this high-priority domain. By not testing what the CCSS-M recommend should be taught, NAEP risks underestimating progress. Increases in student enrollment in Algebra I in eighth grade have already exposed NAEP to this risk, even prior to the development of the CCSS-M.

Whereas Expressions and Equations in the CCSS-M begins the study of topics traditionally taught in Algebra I in the United States, the NAEP framework's treatment of expressions and equations at Grade 8 is more typical of prealgebra. The CCSS-M reflect the migration of Algebra I content to lower grades in the United States over the last two decades. At the time the NAEP Mathematics Framework was originally written, few American eighth graders took Algebra I. The number of eighth graders enrolled in Algebra I has increased substantially—from approximately 15 to 20 percent in the late 1980s and early 1990s to approximately 30 percent in 2009 (Stein, Kaufman, Sherman, & Hillen, 2011). Many of the same topics appear in prealgebra and in Algebra I, but with a real difference in depth, rigor, and technical demand. It appears that something like this difference exists between the NAEP Mathematics Framework and the CCSS-M in Expressions and Equations. As an example, the CCSS-M, but not the NAEP Mathematics Framework, require the use of properties of operations to generate equivalent expressions, laws of exponents, the correspondences between proportional relationships, lines and equations, and analyze and solve linear equations and pairs of simultaneous linear equations. (However, the CCSS-M do not complete the study of Algebra I topics in Grade 8, only going as far as systems of linear equations. Polynomials and quadratic formulas, for example, are in the CCSS-M for high school, not Grade 8.)

Geometry may be another area where the CCSS-M at Grades 7 and 8 go further than the NAEP Mathematics Framework and expose NAEP to underestimating progress. The CCSS-M are more explicit about the mathematical understandings associated with a given topic than is the NAEP framework in geometry at Grade 8. The topics are mostly aligned, but differ in their specificity. In the NAEP framework, for example, students apply the Pythagorean Theorem to solve problems, but understanding and proof are not explicit objectives, as they are in the CCSS-M. Although both the NAEP framework and the CCSS-M have a transformational approach to geometry, the properties of transformations are made explicit in the CCSS-M, but not in the NAEP framework (nor does the NAEP framework have the same explicit focus on triangles and angles that appears in the CCSS-M).

In statistics, the CCSS-M explicitly call for a comparison that involves the use of both a measure of central tendency and a measure of variability. The NAEP framework at Grade 8 does not explicitly call for the use of both measures; rather, it calls for the "use of appropriate statistical measures." The CCSS-M also include bivariate categorical data, whereas the NAEP framework does not. Otherwise, the alignment is adequate.

In both Grade 4 and Grade 8, the NAEP Mathematics Framework's approach to broader mathematical expertise is spotty compared with the CCSS-M's approach. (For instance, the NAEP framework does not have anything comparable to the CCSS-M's Standards for Mathematical Practice.) Finally, the NAEP framework has incorporated mathematical reasoning in several places, but lacks the explicitness of the CCSS-M.

Research Question 2: *Which NAEP subtopics/objectives for Grades 4 and Grade 8 are not addressed on grade level or have been deemphasized in the CCSS-M?*

Four panels examined the NAEP → CCSS-M mapping to answer Research Question 2. For this mapping, the panels were organized by grade levels assessed by NAEP and by the content areas in the NAEP Mathematics Framework: Grade 4 and Grade 8, Number Properties and Operations and Algebra; and Grade 4 and Grade 8, Measurement; Geometry; and Data Analysis, Statistics, and Probability.

Results for NAEP Grade 4 → CCSS-M

Table 6 presents a graphical representation of the alignment between the subtopics and objectives in the NAEP Mathematics Framework for Grade 4 and the CCSS-M for Grades 1–8. The graphical representation was produced by shading all grade levels where the CCSS-M were matched with objectives in the NAEP Mathematics Framework under a subtopic. For example, the subtopic Number Sense has six objectives. The CCSS-M standards that were matched with the six objectives in the NAEP Mathematics Framework for Grade 4, Number Sense, included the following: 2.NBT.A.1, 2.MD.B.6, 2.NBT.A.3, 2.G.A.2, 2.G.A.3, 3.NF.A.2, 4.NBT.A.2, and 5.NBT.A.3a. These standards represent an alignment spread across Grades 2–5. The different kinds of shading in Table 6 represent different levels of alignment or coverage.

Table 6 reveals that all but one subtopic in the NAEP Grade 4 framework under the content area Number Properties and Operations is covered to some extent in the CCSS-M during or prior to Grade 4. The only exception is the NAEP subtopic Ratios and Proportional Reasoning, which is initially introduced in the CCSS-M at Grade 5. Under the content area Algebra, three of the six subtopics are covered in the Grade 4 CCSS-M: patterns, relations, and functions; algebraic representations; and mathematical reasoning with algebra. The depth of coverage for two algebra subtopics—namely, Variables, Expressions, and Operations; and Equations and Inequalities—is minimal in the CCSS-M, with gaps at Grade 4 and Grade 5.

The two subtopics under the NAEP content area of Measurement are covered in Grade 2 through Grade 5 in the CCSS-M. All objectives in the NAEP framework in geometry at Grade 4 have some coverage in the Grade 4 CCSS-M. There is some concern, however, that there is a difference in specificity between the NAEP objectives in the subtopic Position, Direction, and Coordinate Geometry at Grade 4 and the CCSS-M. The NAEP subtopic Dimension and Shape is covered in the CCSS-M in Grade 2 through Grade 4. Furthermore, objectives in the subtopic Mathematical Reasoning in Geometry are inferred in the CCSS-M across Grade 2 through Grade 6, in part because mathematical reasoning is part of the Standards for Mathematical Practice and is therefore infused throughout the CCSS-M. Finally, there are quite a few gaps in the coverage of objectives in the NAEP content area of Data Analysis, Statistics, and Probability.

More details about the CCSS-M coverage for each Grade 4 NAEP subtopic are provided below and in Appendix B.

Table 6. Coverage of NAEP Grade 4 Mathematics Subtopics in the CCSS-M Grades 1–8

NAEP Subtopic	Where Taught in the CCSS-M?							
Number Properties and Operations (by Grade)								
	1	2	3	4	5	6	7	8
Number Sense		▨	▨	▨	▨			
Estimation		■		■		■		
Number Operations		▨	▨	▨	▨	▨		
Ratios and Proportional Reasoning					░	░		
Properties of Number and Operations		■	■	■				
Mathematical Reasoning Using Number		■	■	■	■			
Algebra (by Grade)								
	1	2	3	4	5	6	7	8
Patterns, Relations, and Functions			▨	░	░		▨	
Algebraic Representations				░	▨			
Variables, Expressions, and Operations			▨			▨		
Equations and Inequalities			▨					
Mathematical Reasoning in Algebra		■	■	■	■			

Measurement (by Grade)								
	1	2	3	4	5	6	7	8
Measuring Physical Attributes		■	■	■	■			
Systems of Measurement		■		■	■			
Geometry (by Grade)								
	1	2	3	4	5	6	7	8
Dimension and Shape		░	░	░				
Transformation and Shapes and Preservation of Properties				░				■
Relationships Between Geometric Figures	■	■		■				
Position, Direction, and Coordinate Geometry				■	■	▨		■
Mathematical Reasoning in Geometry		■	■	■	■	■		
Data Analysis, Statistics, and Probability (by Grade)								
	1	2	3	4	5	6	7	8
Data Representation					▨	▨		
Characteristics of Data Set						░		
Probability							░	

▨ Covered with few differences
■ Covered with differences related to specificity
░ Covered with differences related to conceptual understanding

Number Properties and Operations (NPO): Grade 4

The six subtopics under the Number Properties and Operations (NPO) content area in the NAEP framework are number sense, estimation, number operations, ratios and proportional reasoning, properties of numbers and operations, and mathematical reasoning using numbers. The following descriptions identify the primary areas where there is not a match between the subtopics and objectives in the NAEP framework at Grade 4 and the CCSS-M.

For *Number Sense*, NAEP objective 4NPO1b, which refers to using a two-dimensional model for representing numbers, and objective 4NPO1d, which refers to writing or renaming whole numbers, have been deemphasized in the CCSS-M. Otherwise, the panelists agreed that there is good alignment between the objectives in the NAEP subtopic Number Sense and the CCSS-M.

For *Estimation*, several CCSS-M standards refer to estimation or cover estimation in the context of solving a word problem. The panelists noted that the NAEP subtopic Estimation is not covered so much in the CCSS-M content standards as in the CCSS-M's Standards for Mathematical Practice. Furthermore, the CCSS-M address estimation much more in the context of measurement than in the context of number properties and operations.

For *Number Operations*, references to the operations of addition, subtraction, multiplication, and division of whole numbers; fractions with like denominators; and decimals to the hundredths place are covered in the CCSS-M, especially in domains 2.NBT to 5.NBT, 2.OA to 4.OA, 3.NF, and 3.MD to 4.MD. The explicit reference to the use of the calculator as a method for dealing with multiplication of large whole numbers in the NAEP framework could not be found in the CCSS-M.

The *Ratios and Proportional Reasoning* subtopic contains one NAEP Grade 4 objective: use of "simple ratios to describe problem situations." Information in this subtopic does not appear in the CCSS-M until Grade 5 (in standard 5.NF.B.3) and Grade 6 (in standard 6.RP.A.1).

For *Properties of Numbers and Operations*, the NAEP Grade 4 objective "identifying odd and even numbers" (4NPO5a) is covered in the CCSS-M in standard 2.OA.C.3—two grades below the grade at which it is assessed by NAEP. Beyond Grade 2, even and odd numbers are not the subject of any standard in the CCSS-M. Otherwise, this subtopic receives good coverage in Grades 2 through 5 in the CCSS-M.

The only objective in the subtopic *Mathematical Reasoning Using Numbers* focuses on explaining or justifying "a mathematical concept or relationship." For example, one might be asked to explain why 15 is odd or why 7 minus 3 does not equal 3 minus 7. It is instructive to note that "mathematical reasoning" appears in other subtopics in the NAEP Mathematics Framework: "mathematical reasoning in algebra" and "mathematical reasoning in geometry." Expectations for explanation and justification are evident throughout the CCSS-M, in part because in the CCSS-M, mathematical reasoning is linked to the mathematical practices and not necessarily to any particular content standard. In different ways, the NAEP framework and the CCSS-M treat "reasoning" in a distributed way. Both provide evidence that "reasoning" is pervasive in mathematics.

Measurement: Grade 4

There are two subtopics under the NAEP content area of Measurement at Grade 4: measuring physical attributes and systems of measurement.

Measuring Physical Attributes is covered in several CCSS-M standards before Grade 4. The coverage starts as early as Grade 2 (e.g., 2.MD.A.1–4) and extends to Grade 5 (5.NF.B.4b), where students are specifically expected to solve area problems in which figures have fractional sides. Panelists noted that some of the less explicit attention to estimation in the CCSS-M standards might be compensated for by the emphasis on the mathematical practice "precision." It was noted by the panelists that in many measurement situations, an exact measurement is not called for; thus, "appropriate precision" can be read as an endorsement of estimation when it is needed. Panelists noted that measurement of temperature was completely absent from the CCSS-M.

For *Systems of Measurement*, the panelists observed that all but one of the NAEP Grade 4 objectives in this subtopic are covered in the CCSS-M's Standards for Mathematical Practice. These objectives focus on selecting or using an appropriate type and size of unit for the attribute being measured and determining situations in which highly accurate measurement is important. The one exception is the NAEP Grade 4 objective 4M2b, which focuses on solving problems involving conversions and is covered in the Grade 4 and Grade 5 CCSS-M standards 4.MD.A.1 and 5.MD.A.1.

Geometry: Grade 4

The NAEP content area of Geometry consists of five subtopics: dimension and shape; transformation of shapes and preservation of properties; relationships between geometric figures; position, direction, and coordinate geometry; and mathematical reasoning in geometry.

For *Dimension and Shape*, two of the NAEP Grade 4 objectives are covered in the CCSS-M standards much earlier than at Grade 4—specifically, in kindergarten and Grades 2 and 3 (K.G.A.1, K.G.A.3, 2.G.A.1, and 3.G.A.1). The panelists agreed that, in general, the content coverage for the treatment of solid figures in the CCSS-M is almost nonexistent after kindergarten.

Three of the four objectives under the subtopic *Transformation of Shapes and Preservation of Properties* are covered either in the Grade 4 or Grade 8 CCSS-M. Symmetrical figures, lines of symmetry, and attributes of area are covered in the CCSS-M in 4.G.A.2 and 4.G.A.3, and the identification of images that result from flips (reflections), slides (translations), and turns (rotations) is covered in the CCSS-M in 8.G.A.3 and 8.G.A.4. The NAEP Grade 4 objective 4G2e, which focuses on matching or drawing congruent figures in a given collection, is not explicitly covered in the CCSS-M.

The conceptual match between the NAEP Grade 4 subtopic *Relationships Between Geometric Figures* and the CCSS-M was mixed. The match at the objective level ranged from "covered with few differences" for the description and comparison of properties of simple and compound figures composed of triangles, squares, and rectangles to "covered with differences related to specificity" for the objective that focuses on recognizing two-dimensional faces or three-dimensional shapes. The deemphasis on two- and three-dimensional shapes in the CCSS-M also was mentioned in the discussion on the subtopic of Dimensions and Shape. An objective involving patterns, which is under the subtopic Patterns, Relations, and Functions in the NAEP Algebra content area, appears in the CCSS-M as the context of "patterns of geometric figures." The panelists noted that there is also somewhat of a match between the NAEP Grade 4 objective 4G3a involving geometric patterns and the CCSS-M geometry standard 5.G.A.2.

For the subtopic *Position, Direction, and Coordinate Geometry*, the panel judged that "the subtopic is covered for the most part in a nice progression." Parallelism and perpendicularity are covered in the Geometry domain at Grades 4 and 8 in the

CCSS-M, and the concept of representing geometric figures using rectangular coordinates is covered in standards 5.G.B.3 and 6.G.A.3.

Data Analysis, Statistics, and Probability: Grade 4

At Grade 4, the NAEP content area of Data Analysis, Statistics, and Probability includes three of the four subtopics that also appear at Grade 8 and Grade 12. The subtopics are data representation, characteristics of data sets, and probability. The subtopic excluded from Grade 4 is experiments and samples.

It is fair to say that the only subtopic in this NAEP Grade 4 content area that has adequate coverage in the CCSS-M is *Data Representation*. Even with that analysis, the panelists noted different emphases. For example, data representation in the NAEP Grade 4 framework can take the form of pictographs, bar graphs, circle graphs, line graphs, line plots, and tallies, whereas in the CCSS-M, line graphs are not addressed nor is there any mention of tallies. Tables are mentioned in the Grade 4 CCSS-M, but in the context of a very specific mandated activity (e.g., record measurement equivalents in a two-column table).

The subtopics *Characteristics of Data Sets* and *Probability* could not be matched to any of the CCSS-M standards in Grades 3 through 5. All of the objectives in these two subtopics are introduced in the CCSS-M at Grade 6 or 7, where they appear as standards 6.SP.A.2, 6.SP.B.5c, and 7.SP.C.7.

Algebra: Grade 4

The NAEP content area of Algebra consists of five subtopics: patterns, relations, and functions; algebraic representations; variables, expressions, and operations; equations and inequalities; and mathematical reasoning in algebra.

According to the panelists, the *Patterns, Relations, and Functions* subtopic of the NAEP Grade 4 framework exhibits more dissonance with the CCSS-M than any of the other subtopics in the Algebra content area. The panelists suggested that the concept of "pattern" in the CCSS-M conveys something slightly different from anything found in the NAEP framework at Grade 4. For example, the NAEP Grade 4 "pattern" objectives ask one to (a) recognize, describe, or extend a pattern, or (b) given a pattern or sequence, construct a rule that can generate the terms of the pattern or sequence. The CCSS-M standards 3.OA.D.9, 4.OA.C.5, and 5.OA.B.3 emphasize generating a pattern from a rule and analyzing and explaining patterns.

For *Algebraic Representations*, the emphasis in NAEP on translating between the different forms of representations (symbolic, numerical, verbal, or pictorial) of whole number relations is not explicitly referenced in the CCSS-M. In the CCSS-M, the emphasis is on using different types of representation; hence, translation is implied rather than explicit. The NAEP Grade 4 objective on graphing or interpreting points with whole numbers or letters on a grid is covered in the Grade 5 CCSS-M standards 5.G.A.1 and 5.G.A.2.

The remaining three subtopics—*Variables, Expressions, and Operations; Equations and Inequalities;* and *Mathematical Reasoning in Algebra*—are all covered in the CCSS-M. It was noted by the panelists that two of these subtopics (variables, expressions, and operations; and equations and inequalities) are covered in the CCSS-M as part of solving word problems in Grade 3 and Grade 4 and that the third subtopic (mathematical reasoning in algebra) is mentioned as part of understanding the operations and computations in base 10.

Discussion of the Extent of Alignment Between NAEP Grade 4 and the CCSS-M

At Grade 4, most of the content in the NAEP framework is also included in the CCSS-M. For example, alignment between the NAEP framework and the CCSS-M was quite good for the content domain Number Properties and Operations, with only one subtopic misaligned by grade level—ratio and proportional reasoning. The objectives of ratio and proportional reasoning are introduced at a later grade level in the CCSS-M than in NAEP. In the Algebra content area, the match is good, with two exceptions: (a) the treatment of patterns has a different perspective in the CCSS-M than in NAEP; and (b) the CCSS-M emphasize generating patterns from rules while the NAEP framework emphasizes inferring the next step in a pattern or inferring a rule from a pattern. Whether these differences in perspective will lead to different kinds of test items can only be determined in a future comparative item-to-item study. Even if the items differ in some systematic way, it remains an empirical question how this difference will affect performance. The Measurement and Geometry content areas in the NAEP Grade 4 framework and the CCSS-M do not show major differences.

The clearest difference between the NAEP Grade 4 framework and the CCSS-M is in Data Analysis, Statistics, and Probability. The NAEP framework has substantially more emphasis on data and probability by Grade 4 than do the CCSS-M. It is worth noting, however, that this difference disappears by Grade 8. The CCSS-M concentrate data and probability in fewer and later grades (particularly in Grade 7) than does the NAEP framework. This may lead to a scenario in which students taught under the CCSS-M but tested by NAEP will encounter data and probability constructs they have not been taught, a circumstance which could depress overall NAEP scores. It would be possible, and worthwhile, to study the correlation between the CCSS-M implementation and performance on the Data Analysis, Statistics, and Probability subscale of NAEP over time.

Results for NAEP Grade 8 → CCSS-M

Table 7 illustrates the alignment of Grade 8 NAEP subtopics and objectives with the CCSS-M for Grade 1 to Grade 8. Overall, the NAEP Grade 8 objectives in the content areas Number Properties and Operations, Algebra, and Geometry have very good coverage in the CCSS-M in Grade 6 through Grade 8. Gaps in coverage in the CCSS-M for NAEP Grade 8 objectives appear in the content areas Measurement and Data Analysis, Statistics, and Probability.

The NAEP Grade 8 subtopics appear to have fewer gaps (or greater coherence across grade bands) than the NAEP Grade 4 subtopics. NAEP Grade 8 objectives in the subtopic Number Operations are mapped to standards in the CCSS-M across seven continuous grades—from Grade 2 to Grade 8. Four of the six NAEP Grade 8 subtopics under the content area Number Properties and Operations have continuous coverage across at least six grades. Similarly, the subtopics in the content area Geometry—including Dimension and Shape, Relationships Between Geometric Figures, and Mathematical Reasoning in Geometry—are covered to various degrees across six continuous grades.

More details about the CCSS-M coverage for each Grade 8 NAEP subtopic are provided below and in Appendix C.

Table 7. Coverage of Grade 8 NAEP Mathematics Subtopics in the CCSS-M Grades 1–8

NAEP Subtopic	**Where Taught in the CCSS-M?**							
	Number Properties and Operations (by Grade)							
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Number Sense			▨	▨	▨	▨	▨	▨
Estimation			░	░	░	░	░	░
Number Operations		▨	▨	▨	▨	▨	■	▨
Ratios and Proportional Reasoning					▨	▨	▨	▨
Properties of Number and Operations				▨	▨	▨	▨	
Mathematical Reasoning			▨	▨	▨	▨	▨	▨
Algebra (by Grade)								
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Patterns, Relations, and Functions				▨	▨			▨
Algebraic Representations						▨	▨	▨
Variables, Expressions, and Operations						▨	▨	▨
Equations, and Inequalities						▨	▨	▨
Mathematical Reasoning in Algebra						▨	▨	▨
Measurement (by Grade)								
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Measuring Physical Attributes		▨	▨	▨	▨	▨	▨	
Systems of Measurement				░				
Measurement in Triangles							▨	▨

Geometry (by Grade)								
	1	2	3	4	5	6	7	8
Dimension and Shape			■	■	░	░	░	░
Transformation and Shapes and Preservation of Properties				▨		■	■	▨
Relationships Between Geometric Figures			▨	▨	▨	▨	▨	▨
Position, Direction, and Coordinate Geometry				▨		▨	▨	▨
Mathematical Reasoning in Geometry			░	░	░	░	░	░

Data Analysis, Statistics, and Probability (by Grade)								
	1	2	3	4	5	6	7	8
Data Representation						░	░	░
Characteristics of Data Set						■	■	■
Experiments and Samples							░	
Probability							▨	

▨ Covered with few differences
■ Covered with differences related to specificity
░ Covered with differences related to conceptual understanding

Number Properties and Operations: Grade 8

Most of the NAEP Grade 8 objectives under the subtopic *Number Sense* are covered in the CCSS-M prior to Grade 8. For example, NAEP objective 8NPO1a, in which place value is used to model and describe integers and decimals, is not mentioned in the CCSS-M beyond Grade 5. NAEP Grade 8 objectives 8NPO1b, 8NPO1g, and 8NPO1h—which include modeling rational numbers, modeling and applying absolute value, and comparing rational numbers using various representations (e.g., fractions, decimals, percentages or integers)—are covered in the CCSS-M in Grade 6 and Grade 7. (The specific CCSS-M standards that are matched to this subtopic are 6.RP.3b and 7.ND.2d.) Expressing or interpreting numbers using scientific notation from real-life contexts (8NPO1f) is the only NAEP Grade 8 objective in the Number Sense subtopic that appears to be introduced for the first time in eighth grade in the CCSS-M.

The NAEP Grade 8 objectives in the *Estimation* subtopic focus primarily on the accuracy and appropriateness of estimation in a particular context or situation. For example, the NAEP Grade 8 objective 8NPO2d, which covers estimation of square roots or cube roots of numbers less than 1,000, is very similar to the CCSS-M Grade 8 standard 8.NS.A.2, which focuses on the use of rational approximations of irrational numbers for comparing the size of irrational numbers. There is one important difference between the expectation in the NAEP Grade 8 objective and the expectation in CCSS-M standard 8.NS.A.2: namely, 8.NS.A.2 involves a two-step process (first, the estimation of the irrational number by a rational approximation;

second, a comparison of the rational approximations). Furthermore, standard 8.NS.A.2 does not identify an upper limit (e.g., 1,000) for selecting examples nor does it explicitly mention the use of calculators or computers to verify results, as is the case for NAEP Grade 8 objective 8NPO2c. There are CCSS-M practice standards that refer to estimation; however, there are no CCSS-M content standards at Grade 8 that are specifically about estimation.

The NAEP Grade 8 subtopic *Number Operations* covers performing operations on rational numbers, interpreting the results of number operations, and solving application problems involving rational numbers and operations. The objectives under this subtopic allow for the use of exact answers or estimates “as appropriate” for problem solving, whereas CCSS-M standard 7.EE.B.3 calls for assessing the “reasonableness of answers using mental computation and estimation strategies.”

The NAEP Grade 8 subtopic *Ratios and Proportional Reasoning* includes the use of fractions to represent and express ratios and proportions in problem situations, particularly solving problems involving percent increase and decrease, interest rates, and part/whole relationships. All of these objectives are covered in the CCSS-M with standards 6.RP.A and 7.RP.A. The NAEP Grade 8 objective 8NPO4c, “using proportional reasoning to model and solve problems,” is also covered in the CCSS-M.

The NAEP objectives in the subtopic *Properties of Numbers and Operations* are mapped to standards that are introduced and taught prior to Grade 8. Prime and composite numbers are covered in the CCSS-M by standard 4.OA.B.4. Greatest common factors and least common multiples are mentioned in the CCSS-M standard 6.NS.B.4, and the application of basic properties of operations is covered in the CCSS-M at Grade 6 and Grade 7. Operations with odd and even numbers and rules of divisibility, however, are not specifically mentioned in the CCSS-M.

The NAEP subtopic *Mathematical Reasoning Using Number,* which includes “explaining operations with two or more fractions,” is represented in a standard in the CCSS-M for Grade 5 that involves multiplication of fractions as well as in a standard for Grade 6 that involves division of a fraction by a fraction. The panelists also noted that even though an objective in this subtopic calls for explanations and justifications of mathematical concepts or relationships, “justifications” are seldom asked for in the CCSS-M for Grade 6 through Grade 8. They are, however, mentioned in the CCSS-M Standards for Mathematical Practice, which apply to all grades.

Measurement: Grade 8

The subtopic *Measuring Physical Attributes* has six objectives, all of which are covered in the CCSS-M, but at various grade levels from Grade 2 to Grade 7. Three NAEP Grade 8 objectives—8M1b, which focuses on comparing objects with respect to some measurement attribute; 8M1c, which asks individuals to estimate the size of an object with respect to a measurement attribute; and 8M1e, which requires individuals to use an appropriate measurement instrument or create a given unit of measure—are mapped to standards in grades much lower than Grade 8 (e.g., Grade 2 and Grade 3) in the CCSS-M. The remaining three objectives under this subtopic—8M1f, 8M1h, and 8M1i—all involve solving problems related to perimeter, area, volume, rates, and population density. These latter three objectives are covered in Grade 5 to Grade 7; however, concepts of density, including population density, do not appear in the CCSS-M until high school.

Two of the NAEP Grade 8 objectives in S*ystems of Measurement*—focus on estimation and determining the appropriate size of a unit of measurement—are not matched with any of the CCSS-M standards. Instead, both NAEP Grade 8 objectives (and the closely parallel and Grade 4 objectives) are more aligned with several of the CCSS-M Standards for Mathematical Practice, particularly SMP5 and SMP6. (See Mathematical Practice Standards in Appendix A.)

The NAEP Grade 8 objectives under the subtopic *Measurement in Triangles* focus on solving problems involving indirect measurement. These objectives are covered in the CCSS-M by 7.G.A.1, 7.G.B.6, 8.G.A.4, and 8.EE.B.6.

Geometry: Grade 8

Under the subtopic *Dimension and Shape,* NAEP objective 8G1a, which refers to drawing or describing a path of shortest length between points to solve problems in context, was judged by the panelists to be "not covered" in the CCSS-M. However, upon close examination of the CCSS-M Grade 8 standards, it appears that 8.G.B.8, which refers to applying the Pythagorean Theorem to find the distance between two points in a coordinate system, could be a conceptual match for NAEP objective 8G1a. All other objectives under this subtopic are covered in the CCSS-M for Grade 6 and Grade 7, with the exception of objective 8G1b, which asks individuals to identify a geometric object given a written description of its properties. This latter objective is covered in the CCSS-M at Grades 3, 4, and 5 (standards 3.G.A.1, 4.G.A.2, and 5.G.B.3).

The NAEP Grade 8 objectives in the subtopic *Transformation of Shapes and Preservation of Properties* are covered for the most part in CCSS-M standards 8.G.A.2, 8.G.A.3, and 8.G.A.4; however, the foundational understandings of combining, subdividing, and changing shapes of plane figures and solids are in the CCSS-M for Grade 6 and Grade 7 (standards 6.G.A.1, 7.G.A.3, and 7G.B.4). In addition, lower levels of cognitive demand, which ask individuals to "identify" or "recognize" lines of symmetry in plane figures, appear in the CCSS-M for Grade 4 (standard 4.G.A.3).

For the NAEP Grade 8 subtopic *Relationships Between Geometric Figures*, the panelists noted that there is a strong match between the NAEP objectives and the CCSS-M for Grade 3 through Grade 8. The CCSS-M that were matched with the NAEP Grade 8 objectives in this subtopic included standards 3.G.A.1, 4.G.A.1, 4.G.A.2, 5.G.B.3, 5.G.B.4, 6.G.A, 7.G.A, and 8.G.A–C.

The NAEP Grade 8 objectives in the subtopic *Position, Direction, and Coordinate Geometry* cover the grade span from Grade 4 to high school with a gap at Grade 5 in the CCSS-M. NAEP objective 8G4a (which focuses on describing relative positions of points and lines using geometric ideas of midpoint, parallelism, and perpendicularity) is first introduced in CCSS-M standards 4.G.A1, 4.G.A.2, and 4.G.A.3. Furthermore, for standards 8.G.A.1–5, students use congruence, similarity, or geometric software to meet NAEP objective 8G4a.

The NAEP objective under the subtopic *Mathematical Reasoning in Geometry* is not specifically covered in the CCSS-M, but is conceptually aligned with the CCSS-M Standards for Mathematical Practice.

Data Analysis, Statistics, and Probability: Grade 8

The objectives under the NAEP Grade 8 subtopic *Data Representation* that focus on (a) reading, interpreting, interpolating or extrapolating from data; and (b) graphing and solving problems related to data (8DAS1a, 8DASP1b) are covered in the CCSS-M in standards 6.SP.A.2, 7.SP.A.1, 8.SP.A.3, and 8.SP.A.4. The remaining NAEP objectives (8DASPc, 8DASPd, and 8DASPe), which focus on (a) solving problems by estimating; (b) determining whether information in a graph is represented effectively and appropriately; and (c) comparing/contrasting the effectiveness of different representations of the same data, are reflected in the CCSS-M Standards for Mathematical Practice. The specific standards that apply are SMP1—make sense of problems and persevere in solving them; SMP2—reason abstractly and quantitatively; SMP3—construct viable arguments; SMP5—use appropriate tools strategically; and SMP6—attend to precision. Circle graphs, which appear in NAEP Grade 8 objective 8DASP1b, are deemphasized in the CCSS-M.

In the NAEP Grade 8 subtopic *Characteristics of Data Sets*, the mean and median are covered in CCSS-M standards 6.SP.A.3 and 7.SP.B.4 as "measures of center;" however, there is no specific reference to mode in the CCSS-M. Also, the CCSS-M seem to place greater emphasis on understanding and interpreting the measures of center and spread than on calculating them. The NAEP Grade 8 objective 8DASP2c, on outliers, is covered by two CCSS-M standards (6.SP.B.5c and 8.SP.A.1); however, these standards do not specifically address the *effect* of outliers on measures of central tendency and spread as does the NAEP objective.

The NAEP Grade 8 subtopic *Experiments and Samples* is covered somewhat in the CCSS-M, primarily in Grade 7. The NAEP Grade 8 objectives focus broadly on issues related to sampling design, whereas the CCSS-M focus only on the need for a sample to be random. The NAEP objective 8DASP3d, "evaluate the design of an experiment," is covered in the CCSS-M high school statistics and probability content domain.

The NAEP Grade 8 subtopic *Probability* is covered in the CCSS-M at Grade 7 in standard 7.SP (all clusters). The panelists noted that while there is a strong match between the NAEP framework and the CCSS-M for this subtopic, the NAEP framework goes further than the CCSS-M in including a focus on independent and dependent events. The CCSS-M address the probability of independent and dependent events in high school statistics.

Algebra: Grade 8

Within the NAEP Grade 8 subtopic *Patterns, Relations, and Functions*, objectives related to numerical or geometric patterns and sequences are covered in the CCSS-M for the elementary grades in 4.OA.C.5 and 5.NBT.A.2, but are not found in any of the standards for the middle grades (Grade 6 through Grade 8). However, objectives related to linear functions—that is, how to calculate their slopes and intercepts and how they contrast with nonlinear functions—are covered in the CCSS-M by Grade 8 (in standards 8.SP.A.1 and 8.SP.A.2).

The NAEP Grade 8 subtopic *Algebraic Representations* has objectives that are covered throughout the CCSS-M for Grade 6 through Grade 8. For example, NAEP objective 8A2c (graphing or interpreting points on a rectangular coordinate system) and objective 8A2d (solving problems involving coordinate pairs) are covered in standard 6.NS.C.8. Objective 8A2f (identifying or representing functional relationships in meaningful contexts) is covered in standards 8.EE.B.5 and 8.F.B.5. Analyzing or interpreting linear relationships—objective 8A2b—is covered in standard 8.F.A.3.

The NAEP Grade 8 subtopic *Variables, Expressions, and Operations* has two objectives. NAEP objective 8A3b, which deals with writing algebraic expressions, equations, or inequalities, is covered in CCSS-M standards 6.EE.B.6, 6.EE.B.7, 6.EE.B.8, and 7.EE.A.2. NAEP objective 8A3c, which focuses on performing basic operations and using appropriate tools on linear expressions, is addressed broadly in the CCSS-M content domain Expressions and Equation at Grades 5–8. Objective 8A3c also is covered by SMP5 in the CCSS-M Standards for Mathematical Practice (see Appendix A).

Solving equations and inequalities and interpreting the meaning of the equal sign are covered in the NAEP Grade 8 subtopic *Equations and Inequalities*. There also is a focus on demonstrating how to use and evaluate common formulas. These areas are covered in the CCSS-M for Grade 6 through Grade 8, primarily in the content domain Expressions and Equation.

The NAEP Grade 8 subtopic *Mathematical Reasoning in Algebra* asks that individuals make, validate, and justify conclusions and generalizations about linear relationships. This topic is covered in the CCSS-M content domain Expressions and Equation at Grades 6 and 8 and in the Standards for Mathematical Practice.

Discussion of the Extent of Alignment Between NAEP Grade 8 and the CCSS-M

Every content area in the NAEP Grade 8 framework has been covered in the CCSS-M by Grade 8 and, in most cases, is initially presented at an earlier grade. Under ideal conditions, it is not likely that students taking the NAEP Grade 8 assessment would encounter topics that they have not been taught. Thus, the risk of underestimating growth by diluting scores with untaught material is small for the NAEP Grade 8 assessment.

There are some differences in specificity and conceptual understandings between the NAEP Mathematics Framework and the CCSS-M, and these differences might matter when assessing students. In Number Properties and Operations, the NAEP framework treats "estimation" as a content area whereas the CCSS-M distribute estimation among other content domains and the Standards for Mathematical Practice. Some topics in measurement are covered in much lower grades in the CCSS-M than in the NAEP framework. This could lead to "less mature" versus "more mature" differences in the conceptualization of these topics. To be certain of this will require item-to-item comparisons in subsequent studies. In Data Analysis, Statistics, and Probability, experimental design and conditional probability are not taught until high school in the CCSS-M, but get some attention in the NAEP Grade 8 framework.

Conclusions and Recommendations

When Should NAEP Change the Yardstick?

When a set of common standards on which common assessments would be based, and which nearly all states would adopt, became a reality in 2010 with the introduction of the CCSS, it became necessary for NAEP to attend to shifting definitions and emphases of subject matter competence and to determine how these might affect claims about progress or lack thereof on a national, state, or district level (National Center for Education Statistics, 2012).

Historically, the NAEP frameworks have aspired to represent the union of all the various state curricula while reaching beyond these curricula to lead as well as reflect. As a result, NAEP often has pushed on the leading edge of what the nation's children know and should able to do. The introduction of the CCSS-M provides both new opportunities and challenges for NAEP. As the nation moves toward widespread implementation of instruction and assessment based on the CCSS-M, NAEP must balance the goals of comparability over time (i.e., maintaining trend) with keeping itself relevant.

NAEP in the Context of the CCSS-M

This study found the preponderance of content in the CCSS-M also is found in the NAEP Mathematics Framework, but with some differences. The differences are potentially important and should receive attention in the normal revision of the framework and the assessments. Four types of discrepancies were observed. Compared with the NAEP framework, the CCSS-M have

1. More rigorous content in eighth-grade algebra and geometry
2. More extensive and systematic treatment of mathematical expertise (found in the Standards for Mathematical Practice)
3. A more conceptual perspective on many mathematical topics, explicitly stating the mathematics to be understood rather than the type of problem to be solved
4. Some content taught at higher grades than is assessed in the fourth-grade NAEP assessment. For example, the study of proportional relationships is concentrated in Grades 6 and 7, and data sets and probability are taught in Grades 6 and 7, respectively.

These are important differences and these areas should be considered a priority in the normal revision of the NAEP Mathematics Framework.

The study also found that the CCSS-M include the preponderance of content included in the NAEP framework by the grade level assessed, with several important exceptions noted in the results reported above. Subsequently, where content is assessed by NAEP, but not included in the CCSS-M, analyses should be conducted to estimate the effect that dropping this content from the curriculum that align with the CCSS-M might have on overall NAEP scores. This should be done to avoid misinterpreting this effect as a general decline in mathematics achievement, when it may be due to a specific decline in a subdomain that has been intentionally deemphasized in the CCSS-M.

Recommendations and Next Steps

Based on the results of our research, we offer the following recommendations with respect to the NAEP Mathematics Framework and the NAEP mathematics assessments in the context of the CCSS-M:

- NAEP should continue to maintain its role as an independent monitor of the academic achievement of the nation's students.
- NAEP should not aim to be a replica of the assessments that are based on the CCSS-M, but should make use of advances in item development technology associated with the CCSS-M assessments, particularly those related to assessing mathematical practices—an area that has not been a strong point for NAEP, especially when designing items of high complexity.
- NAEP should review its mathematics framework to ensure that objectives remain current and reflect the coursetaking patterns of the nation's students (e.g., algebra I enrollment in eighth grade versus ninth grade and the placement of content assessed on the fourth-grade NAEP, such as proportionality and probability, in higher grades in the CCSS-M curriculum).
- NAEP should continue to lead improvements in item design and should pay particular attention to avoiding items biased toward a characterization of mathematics as merely a domain of problems organized as topics. The items should also assess conceptual understanding of the mathematics, explanations of solutions, reasoning and content, and manipulation of expressions or equations for a purpose.
- NAEP should consider improving its strategy for assessing mathematical expertise, perhaps expanding and adding a broader set of objectives to the assessment frameworks that cut across content areas and focus on what in the CCSS-M are called "mathematical practices." A move in this direction can already be seen for "mathematical reasoning." In 2005, mathematical reasoning appeared only in the Geometry content area in NAEP; however, by 2013, mathematical reasoning appeared as a subtopic in Number Properties and Operations, Algebra, and Geometry in Grades 4 and 8. NAEP has extensive experience in assessing skills in reading and writing and should draw on this expertise to do something similar in mathematics.
- NAEP should continue to serve as a leader, especially in the areas of scoring, interpreting, and reporting assessment data and information from different sources (e.g., providing linkages among district, state, national, and international assessments).
- When the CCSS-M items are available, NAEP should carry out a study comparing how well NAEP items reflect the CCSS-M standards and how well the CCSS-M items fit into the NAEP Mathematics Framework.

A major trend to which NAEP must respond if it is to remain relevant in the future is outlined in the report titled *NAEP: Looking Ahead—Leading Assessment Into the*

Future (National Center for Education Statistics, 2012). That trend is NAEP's capacity to assess a broader set of learning outcomes. NAEP needs to remain responsive in a changing and dynamic curriculum and assessment milieu. Whether the issues are related to high-stakes versus low-stakes, status versus growth, or assessment *of* learning versus assessment *for* learning, NAEP's role must be clear and unambiguous. If change is coming to NAEP, and particularly the NAEP frameworks, it must be deliberate and not reactionary, thoughtful and not superfluous. NAEP has undergone notable changes to meet expanded new demands in the past. NAEP also can meet new demands successfully—not only now, but also in the future in the context of the Common Core State Standards.

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Appendix A. Features of the NAEP Mathematics Framework and the CCSS-M

NAEP Mathematics Framework: An Assessment Framework

The National Assessment Governing Board oversees the development of the NAEP Mathematics Framework—a framework that describes the specific knowledge and skills to be assessed in each NAEP content area and grade level. The various stakeholders to whom NAEP results are made available are the same stakeholders who provide input on the framework: content experts, school personnel, teachers, parents, policymakers, and others.

The mathematics knowledge and skills that are assessed in NAEP must necessarily take into account the constraints of a large-scale assessment, with its limitations on time, space, and resources. In practical terms, this means that the frameworks are developed with the understanding that some concepts, skills, and activities in mathematics as taught are not suitable to be assessed by NAEP even though they may be very important components of a school curriculum.

The Grade 4 and Grade 8 objectives in the *2011 Mathematics Framework* were used as the basis for making comparisons with the CCSS-M in the current study and have served as the basis for the NAEP assessment at these grade levels since 2005. These are the same Grade 4 and Grade 8 objectives that are in the *2013 Mathematics Framework*. Therefore, the results of the analyses about the alignment of the 2011 mathematics framework with the CCSS-M are applicable to the 2013 mathematics framework as well.

The NAEP Mathematics Framework is organized into five broad areas of mathematics content:

- **Number Properties and Operations (NPO),** including computation and understanding of number concepts
- **Measurement (M),** including use of instruments, application of processes, and concepts of area and volume
- **Geometry (G),** including spatial reasoning and applying geometric properties
- **Data Analysis, Statistics, and Probability (DASP)**, including graphical displays and statistics
- **Algebra (A),** including representations and relationships

Each content area is divided into subtopics, and each subtopic consists of one or more objectives. These divisions are not intended to separate mathematics into discrete, nonoverlapping elements. Rather, they are intended to provide a helpful classification scheme that describes the universe of mathematical content assessed by NAEP.

Number Properties and Operations measures students' understanding of ways to represent, calculate, and estimate with numbers. It consists of the following subtopics: number sense, estimation, number operations, ratios and proportional reasoning, properties of number operations, and mathematical reasoning using numbers. At Grade 4, objectives cover number properties and operations and focus on computation with or understanding of whole numbers and common fractions and decimals. At Grade 8, students are expected to compute with rational and common irrational numbers as well as solve problems using proportional reasoning and apply properties of select number systems.

Measurement assesses students' knowledge of units of measurement for such attributes as capacity, length, area, volume, time, angles, and rates. It consists of the following subtopics: measuring physical attributes, systems of measurement, and measurement in triangles. At Grade 4, objectives focus on customary units, such as inch, quart, pound, and hour, and common metric units, such as centimeter, liter, and gram. Length as a geometric attribute also is addressed. At Grade 8, students are expected to know how to use rates and square units for measuring area and surface area, cubic units for measuring volume, and degrees for measuring angles.

Geometry measures students' knowledge and understanding of shapes in two and three dimensions and relationships between shapes, such as symmetry and transformations. It consists of the following subtopics: dimension and shape; transformation of shapes and preservation of properties; relationships between geometric figures; position, direction, and coordinate geometry; and mathematical reasoning in geometry. At Grade 4, objectives focus on simple figures such as cubes and spheres. At Grade 8, the focus is on properties of plane figures, especially parallel and perpendicular lines, angle relationships in polygons, cross-sections of solids, and the Pythagorean Theorem.

Data Analysis, Statistics, and Probability consists of the following subtopics: data representation, characteristics of data sets, experiments and samples, and probability. At Grade 4, objectives focus on how data are collected and organized, how to read and interpret various representations of data, and basic concepts of probability. At Grade 8, the student is expected to know how to organize and summarize data in various formats, such as tables, charts, and graphs; analyze statistical claims; and solve problems involving probability.

Algebra measures students' understanding of patterns, using variables, algebraic representation, and functions. At Grade 4, objectives focus on students' understanding of algebraic representation, patterns, and rules; graphing points on a line or a grid; and using symbols to represent unknown quantities. At Grade 8, the focus is on understanding patterns and functions; algebraic expressions, equations, and inequalities; and algebraic representations, including graphs.

Levels of Complexity in the Framework

In addition to the content dimension of the objectives of the NAEP framework, there is a complexity dimension that classifies items into three levels of complexity: (1) low, (2) moderate, and (3) high.

The objectives that generate *low-complexity* items usually are statements of recall and recognition of previously learned concepts and principles. The following statements are typical of the demands of objectives that might lead to low-complexity items:

- Recall or recognize a fact, term, or property
- Recognize an example of a concept
- Compute a sum, difference, product, or quotient
- Evaluate an expression in an equation
- Solve a one-step problem
- Draw or measure a simple geometric figure

The objectives that generate *moderate-complexity* items involve more flexibility of thinking as well as informal methods of reasoning and problem solving. These objectives bring together skills and knowledge from various content areas. The following statements are typical of what may lead to moderate-complexity items:

- Solve a word problem using multiple steps
- Provide justification for steps in a solution process
- Extend a pattern
- Retrieve information from a graph, table, or figure and use it to solve a problem

High-complexity items are generated from statements that require more abstract thinking, planning, analysis, and creative thought. The following are examples of statements of objectives that may generate high-complexity items:

- Perform a procedure with multiple decision points
- Generate a pattern
- Formulate an original problem, given a scenario
- Describe, compare, and contrast solution methods
- Analyze the assumptions of a mathematical model
- Solve a novel problem

The final form of the assessment depends on the assessment blueprint or test specifications. These define how the content and the levels of complexity of the items are to be distributed.

CCSS-M: A Curriculum Framework

The CCSS-M framework consists of two components: the Standards for Mathematical Content and the Standards for Mathematical Practice. The two components operate in concert to provide school mathematics experiences that, according to the authors, are "substantially more focused and coherent in order to improve mathematics achievement …" in the United States. The CCSS-M set grade-specific content standards for Grades K–8 and subject-specific standards for high school. The grade-level standards are organized into clusters and content domains. The standards define what students should understand and be able to do, clusters are groups of related standards, and content domains are larger groups of related clusters. The following is an example showing the organization of one cluster in the Geometry content domain for Grade 4.

> **Geometry: 4.G** ***(CCSS-M—Content Domain)***
>
> **A. Draw and identify lines and angles, and classify shapes by properties of their lines and angles.** *(Cluster)*
>
> 4.G.A.1. Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures. *(Standard)*
>
> 4.G.A.2. Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles. *(Standard)*
>
> 4.G.A.3. Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry. *(Standard)*

The organization of the CCSS-M hierarchy is very similar to the organization of the NAEP Mathematics Framework. For NAEP Grades 4, 8, and 12, groups of objectives form subtopics, and groups of subtopics are subsumed under content areas. For example,

> **Geometry: Grade 4** ***(NAEP—Content Area)***
>
> **Dimension and shape** *(Subtopic)*
>
> a. Explore properties of paths between points. *(Objective)*
>
> b. Identify or describe (informally) real-world objects using simple plane figures (e.g., triangles, rectangles, squares, and circles) and simple solid figures (e.g., cubes, spheres, and cylinders.) *(Objective)*
>
> c. Identify or draw angles and other geometric figures in the plane. *(Objective)*

Items for the NAEP assessments are constructed from the statements of the objectives. Items for the CCSS-M assessments will be constructed from statements at the content standards level in concert with the appropriate Standards for Mathematical Practice.

The Standards for Mathematical Practice describe different types of practices and habits of mind that mathematics educators at all levels should seek to develop in their students. These practices and mindsets are not new to the mathematics education community. They are based on important processes and proficiencies embedded in the core work of the National Council of Teachers of Mathematics (NCTM) and the National Research Council (NRC), respectively. The "processes" are based on the NCTM's process standards of problem solving, reasoning and proof, communication, representation, and connections. The "proficiencies," or habits of mind, are based on the mathematical proficiencies described in the National Research Council's report, *Adding It Up* (Kilpatrick, Swafford, & Findell, 2001): adaptive reasoning, strategic competence, conceptual understanding, procedural fluency, and productive disposition. Unlike the content standards for mathematics, which are grade-specific, the eight Standards for Mathematical Practice are consistent at and apply to all grades (kindergarten through high school). Because they play an important role in decisions about the level of the alignment of the NAEP objectives with the CCSS-M content standards and in the design of assessment items based on the CCSS-M, a brief description of each of the Standards for Mathematical Practice (SMP) is provided below.

1. **Make sense of problems and persevere in solving them (SMP1).** Mathematically proficient students check their answers to problems using different methods, and they continually ask themselves, "Does this make sense?"
2. **Reason abstractly and quantitatively (SMP2).** Mathematically proficient students bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize and the ability to contextualize. The ability to decontextualize allows them to abstract a given situation, represent it symbolically, and manipulate the representing symbols as if they have a life of their own. On the other hand, the ability to contextualize allows students to add meaning to the symbols involved. Quantitative reasoning involves creating a coherent representation of a problem, knowing and flexibly using different and appropriate properties of operations, and computing them accurately.
3. **Construct viable arguments and critique the reasoning of others (SMP3).** Mathematically proficient students make conjectures and build a logical progression of statements to explore the truth of their conjectures. They justify their conclusions, communicate them to others, and respond to the arguments of others. If there is a flaw in an argument, they can explain what it is. They reason inductively about data, making plausible arguments that take into account the context from which the data arose.
4. **Model with mathematics (SMP4).** Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In the early grades, this may involve writing an addition equation to describe a situation. In the middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. In high school, a student might use geometry to solve design problems or use a function to describe how one quantity of interest depends on another. They interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.
5. **Use appropriate tools strategically (SMP5).** Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions

about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations.

6. **Attend to precision (SMP6).** Mathematically proficient students try to communicate precisely to one another. They are careful about specifying units of measure and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, expressing numerical answers with a degree of precision appropriate for the problem context.
7. **Look for and make sure of structure (SMP7).** Mathematically proficient students look closely to discern a pattern or structure. In the early grades, students might notice that 3 + 7 is the same as 7 + 3, or they may sort a collection of shapes according to how many sides the shapes have. Later, they may recognize that 7 × 8 equals 7 × (5 + 3), which is the same as 7 × 5 + 7 × 3.
8. **Look for and express regularity in repeated reasoning (SMP8).** Mathematically proficient students notice if calculations are repeated and look for both general methods and mathematically sound shortcuts. They continually evaluate the reasonableness of their intermediate results.

The authors of the CCSS-M suggest that designers of curricula, professional development, instruction, and assessments should attend to the need to connect mathematical practices to mathematical content. Expectations in content standards that begin with the word "understand" are often good opportunities to connect to the practices of the content. For example, CCSS-M standard 4.NF.B.3b (a Grade 4 standard in the content domain of Number and Operations: Fractions) states that students will:

> "Understand a fraction a/b with a > 1 as the sum of fractions 1/b and decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fractional model."

According to this standard, students are expected to extend previous understandings about how fractions are built, composed, and decomposed from unit fractions. In addition, students are expected to use the meaning of fractions and multiplication to multiply a fraction by a whole number (e.g., 3/8 = 3 × 1/8 = (1/8 + 1/8 + 1/8). Evident in this content standard and the accompanying example are at least three of the Standards for Mathematical Practice: SMP2, SMP4, and SMP7.

Appendix B. Coverage of Grade 4 NAEP Mathematics Objectives in the CCSS-M

Table B-1. Coverage of Grade 4 NAEP Mathematics Objectives in the CCSS-M[1,2]

NAEP content area: Number properties and operations

NAEP subtopic: (1) Number sense			
NAEP objective ID	**NAEP objective**	**Where taught in the CCSS-M?**	**Comments**
4NPO1a	(a) Identify place value and actual value of digits in whole numbers.	2.NBT.A.1, 4.NBT.A.1	
4NPO1b	(b) Represent numbers using models such as base 10 representations, number lines, and two-dimensional models.	2.MD.B.6	Exception: Two-dimensional models for representing numbers are not covered.
4NPO1c	(c) Compose or decompose whole quantities by place value (e.g., write whole numbers in expanded notation using place value: 342 = 300 + 40 + 2).	2.NBT.A.1, 4.NBT.A.2	
4NPO1d	(d) Write or rename whole numbers (e.g., 10 = 5 + 5, 12 – 2, or 2 × 5).	2.NBT.A.3	
4NPO1e	(e) Connect model, number word, or number using various models and representations for whole numbers, fractions, and decimals.	2.NBT.A.3, 2.MD.B.6, 2.G.A.2, 2.G.A.3, 3.NF.A.2, 4.NBT.A.2, 5.NBT.A.3a	
4NPO1i	(i) Order or compare whole numbers, decimals, or fractions.	2.NBT.A.4, 4.NBT.A.2, 5.NBT.A.3b	
NAEP subtopic: (2) Estimation			
NAEP objective ID	**NAEP objective**	**Where taught in the CCSS-M?**	**Comments**
4NPO2a	(a) Use benchmarks (well-known numbers used as meaningful points for comparison) for whole numbers, decimals, or fractions in contexts (e.g., ½ and .5 may be used as benchmarks for fractions and decimals between 0 and 1.00).		
4NPO2b	(b) Make estimates appropriate to a given situation with whole numbers, fractions, or decimals by • Knowing when to estimate, • Selecting the appropriate type of estimate, including overestimate, underestimate, and range of estimate, or • Selecting the appropriate method of estimation (e.g., rounding).	2.MD.A.3, 4.NBT.A.3	

4NPO2c	(c) Verify solutions or determine the reasonableness of results in meaningful contexts.	4.OA.A.3, 6.EE.B.5	Also covered in the Standards for Mathematical Practice.
NAEP subtopic: (3) Number operations			
NAEP objective ID	**NAEP objective**	**Where taught in the CCSS-M?**	**Comments**
4NPO3a	(a) Add and subtract: • Whole numbers, or • Fractions with like denominators, or • Decimals through hundredths.	2.OA.A.1, 2.OA.B.2, 2.NBT.B.6, 2.NBT.B.7, 3.NBT.A.2, 4.NF.B.3c, 5.NBT.B.7	
4NPO3b	(b) Multiply whole numbers: • No larger than two-digit by two-digit with paper-and-pencil computation, or • Larger numbers with use of Calculator	3.NBT.A.3, 5.NBT.B.5	Use of calculators is not mentioned in the CCSS-M.
4NPO3c	(c) Divide whole numbers: • Up to three digits by one digit with paper-and-pencil computation, or • Up to five digits by two digits with use of calculator.	3.OA.C.7, 4.NBT.B.6, 5.NBT.B.6	
4NPO3d	(d) Describe the effect of operations on size (whole numbers).	3.NF.A.2a–2b, 3.NF.A.3, 5.NBT.A.2	The match of CCSS-M standards with this objective is more indirect than direct.
4NPO3e	(e) Interpret whole-number operations and the relationships between them.	3.OA.A.1, 3.OA.A.2, 3.OA.B.6	
4NPO3f	(f) Solve application problems involving numbers and operations.	1.OA.D.8, 2.OA.A.1, 2.MD.C.8, 3.OA.A.3, 3.OA.D.8, 3.MD.C.7b, 3.MD.D8, 4.OA.A.2, 4.OA.A.3, 4.NF.B.4c, 4.MD.A.2, 4.MD.C.7, 5.NF.A.2, 5.NF.B.3, 5.NF.B.6, 5.NF.B.7c	
NAEP subtopic: (4) Ratios and proportional reasoning			
NAEP objective ID	**NAEP objective**	**Where taught in the CCSS-M?**	**Comments**
4NPO4a	(a) Use simple ratios to describe problem situations.	5.NF.B.3, 6.RP.A.1	

NAEP subtopic: (5) Properties of numbers and operations			
NAEP objective ID	**NAEP objective**	**Where taught in the CCSS-M?**	**Comments**
4NPO5a	(a) Identify odd and even numbers.	2.OA.C.3	
4NPO5b	(b) Identify factors of whole numbers.	3.OA.A.4, 4.OA.B.4	
4NPO5e	(e) Apply basic properties of operations.	3.OA.A.4, 4.OA.B.4	
NAEP subtopic: (6) Mathematical reasoning using numbers			
NAEP objective ID	**NAEP objective**	**Where taught in the CCSS-M?**	**Comments**
4NPO6a	(a) Explain or justify a mathematical concept or relationship (e.g., explain why 15 is an odd number or why 7–3 is not the same as 3–7).	2.NBT.B.9, 3.NFA.3, 4.NF.A.1, 5.NBT.A.2, 5.NF.B.5b	Also covered in the Standards for Mathematical Practice.

NAEP content area: Algebra

NAEP subtopic: (1) Patterns, relations, and functions			
NAEP objective ID	**NAEP objective**	**Where taught in the CCSS-M?**	**Comments**
4A1a	(a) Recognize, describe, or extend numerical patterns.	3.OA.D.9	
4A1b	(b) Given a pattern or sequence, construct or explain a rule that can generate the terms of the pattern or sequence.	4.OA.C.5, 5.OA.B.3, 5.NBT.A.2	
4A1c	(c) Given a description, extend or find a missing term in a pattern or sequence.	5.OA.B.3	
4A1d	(d) Create a different representation of a pattern or sequence given a verbal description.		Not found in the CCSS-M
4A1e	(e) Recognize or describe a relationship in which quantities change proportionally.	7.RP.A.2a	
NAEP subtopic: (2) Algebraic representations			
NAEP objective ID	NAEP objective	**Where taught in the CCSS-M?**	**Comments**
4A2a	(a) Translate between the different forms of representations (symbolic, numerical, verbal, or pictorial) of whole-number relationships (such as from a written description to an equation or from a function table to a written description).	8.F.A.2	The content in this standard may be more than what is expected at fourth grade.
4A2c	(c) Graph or interpret points with whole-number or letter coordinates on grids or in the first quadrant of the coordinate plane.	5.G.A.1, 5.G.A.2	
NAEP subtopic: (3) Variables, expressions, and operations			
NAEP objective ID	**NAEP objective**	**Where taught in the CCSS-M?**	**Comments**
4A3a	(a) Use letters and symbols to represent an unknown quantity in a simple mathematical expression.	3.OA.A.3, 6.EE.A.2a, 6.EE.C.9	
4A3b	(b) Express simple mathematical relationships using number sentences.	6.EE.A.2b	

NAEP subtopic: (4) Equations and inequalities			
NAEP objective ID	**NAEP objective**	**Where taught in the CCSS-M?**	**Comments**
4A4a	(a) Find the value of the unknown in a whole-number sentence.	3.OA.A.4	
NAEP subtopic: (5) Mathematical reasoning in algebra			
NAEP objective ID	**NAEP objective**	**Where taught in the CCSS-M?**	**Comments**
4A5a	(a) Verify a conclusion using algebraic properties.	Taught throughout the CCSS-M content standards.	Also covered in the Standards for Mathematical Practice.

NAEP content area: Measurement

NAEP subtopic: (1) Measuring physical attributes			
NAEP objective ID	**NAEP objective**	**Where taught in the CCSS-M?**	**Comments**
4M1a	(a) Identify the attribute that is appropriate to measure in a given situation.		Not found in the CCSS-M.
4M1b	(b) Compare objects with respect to a given attribute, such as length, area, volume, time, or temperature.	2.MD.A.2, 2.MD.A.4	
4M1c	(c) Estimate the size of an object with respect to a given measurement attribute (e.g., length, perimeter, or area using a grid).	2.MD.A.3, 3.MD.A.2	
4M1e	(e) Select or use appropriate measurement instruments, such as a ruler, meter stick, clock, thermometer, or other scaled instruments.	2.MD.A.1	
4M1f	(f) Solve problems involving perimeter of plane figures.	3.MD.D.8, 4.MD.A.3	
4M1g	(g) Solve problems involving area of squares and rectangles.	3.MD.C.5, 3.MD.C.6, 3.MD.C.7, 4.MD.A.3, 5.NF.B.4b	
NAEP subtopic: (2) Systems of measurement			
NAEP objective ID	**NAEP objective**	**Where taught in the CCSS-M?**	**Comments**
4M2a	(a) Select or use an appropriate type of unit for the attribute being measured, such as length, time, or temperature.	2.MD.A.1	
4M2b	(b) Solve problems involving conversions within the same measurement system, such as conversions involving inches and feet or hours and minutes.	4.MD.A.1, 5.MD.A.1	
4M2d	(d) Determine appropriate size of unit of measurement in problem situation involving such attributes as length, time, capacity, or weight.	4.MD.A.1	
4M2e	(e) Determine situations in which a highly accurate measurement is important.		May be covered in the Standards for Mathematical Practice.

NAEP content area: Geometry

NAEP subtopic: (1) Dimension and shape

NAEP objective ID	NAEP objective	Where taught in the CCSS-M?	Comments
4G1a	(a) Explore properties of paths between points.	4.G.A.1	
4G1b	(b) Identify or describe (informally) real-world objects using simple plane figures (e.g., triangles, rectangles, squares, and circles) and simple solid figures (e.g., cubes, spheres, and cylinders).	K.G.A.1, 2.G.A.1,	
4G1c	(c) Identify or draw angles and other geometric figures in the plane.	4.G.A.1	
4G1f	(f) Describe attributes of two- and three-dimensional shapes.	K.G.A.3, K.G.B.4, 3.G.A.1	

NAEP subtopic: (2) Transformation of shapes and preservation of properties

NAEP objective ID	NAEP objective	Where taught in the CCSS-M?	Comments
4G2a	(a) Identify whether a figure is symmetrical or draw lines of symmetry.	4.G.A.3	
4G2c	(c) Identify the images resulting from flips (reflections), slides (translations), or turns (rotations).	8.G.A.3, 8.G.A.4	
4G2d	(d) Recognize which attributes (such as shape and area) change or do not change when plane figures are cut up or rearranged.	4.G.A.3 (introduction)	See High School Geometry, Congruence.A.2, A.3, and B.6
4G2e	(e) Match or draw congruent figures in a given collection.	8.G.A.2	See High School Geometry, Congruence.B.7 and B.8

NAEP subtopic: (3) Relationships between geometric figures

NAEP objective ID	NAEP objective	Where taught in the CCSS-M?	Comments
4G3a	(a) Analyze or describe patterns of geometric figures by increasing number of sides, changing size or orientation (e.g., polygons with more and more sides).	3.G.A.1, 5.G.B.3	
4G3b	(b) Assemble simple plane shapes to construct a given shape.	1.G.A.2	
4G3c	(c) Recognize two-dimensional faces of three-dimensional shapes.	7.G.A.2	
4G3f	(f) Describe and compare properties of simple and compound figures composed of triangles, squares, and rectangles.	1.G.A.2, 2.G.A.1, 4.G.A.2	

NAEP subtopic: (4) Position, direction, and coordinate geometry

NAEP objective ID	NAEP objective	Where taught in the CCSS-M?	Comments
4G4a	(a) Describe relative positions of points and lines using the geometric ideas of parallelism or perpendicularity.	4.G.A.1, 4.G.A.2, 5.G.A.1, 8.G.A.1c	
4G4d	(d) Construct geometric figures with vertices at points on a coordinate grid.	6.G.A.3	

NAEP subtopic: (5) Mathematical reasoning in geometry			
NAEP objective ID	**NAEP objective**	**Where taught in the CCSS-M?**	**Comments**
4G5a	(a) Distinguish which objects in a collection satisfy a given geometric definition and explain choices.	5.G.B.3, 5.G.B.4	

NAEP content area: Data Analysis, Statistics, and Probability

NAEP subtopic: (1) Data representation			
NAEP objective ID	**NAEP objective**	**Where taught in the CCSS-M?**	**Comments**
	Pictographs, bar graphs, circle graphs, line graphs, line plots, tables, and tallies.		
4DASP1a	(a) Read or interpret a single set of data.	6.SP.A.2	
4DASP1b	(b) For a given set of data, complete a graph (limits of time make it difficult to construct graphs completely).	5.MD.B.2, 6.SP.B.4	
4DASP1c	(c) Solve problems by estimating and computing within a single set of data.	6.SP.B.5	
NAEP subtopic: (2) Characteristics of data sets			
NAEP objective ID	**NAEP objective**	**Where taught in the CCSS-M?**	**Comments**
4DASP2b	(b) Given a set of data or a graph, describe the distribution of data using median, range, or mode.	6.SP.A.2, 6.SP.B.5c	
4DASP2d	(d) Compare two sets of related data.	7.SP.B.3, 7.SP.B.4	
NAEP subtopic: (3) Probability			
NAEP objective ID	**NAEP objective**	**Where taught in the CCSS-M?**	**Comments**
4DASP4a	(a) Use informal probabilistic thinking to describe chance events (i.e., likely and unlikely, certain and impossible).	7.SP.C.5	
4DASP4b	(b) Determine a simple probability from a context that includes a picture.	7.SP.C.6, 7.SP.C.7	
4DASP4e	(e) List all possible outcomes of a given situation or event.	7.SP.C.7	
4DASP4g	(g) Represent the probability of a given outcome using a picture or other graphic.	7.SP.C.6	

[1] *Notation for CCSS-M standards*: Grade level, content domain, cluster, standard number within domain. For example, 3.OA.D.8 is read as Grade 3, Operations and Algebraic Thinking, Cluster D, Standard 8.

[2] *Notation for NAEP objectives:* Grade level, content area, subtopic, objective. For example, 4NPO1i is read as Grade 4, Number Properties and Operations, Subtopic 1, Objective i.

Appendix C. Coverage of Grade 8 NAEP Mathematics Objectives in the CCSS-M

Table C-1. Coverage of Grade 8 NAEP Mathematics Objectives in the CCSS-M[1,2]

NAEP content area: Number properties and operations

NAEP subtopic: (1) Number Sense

NAEP objective ID	NAEP objective	Where taught in the CCSS-M?	Comments
8NPO1a	(a) Use place value to model and describe integers and decimals.	5.NBT.A.1-3, 5.NBT.B.6-7	Place value of decimals is covered in Grade 5, but not mentioned beyond Grade 5. Negative integers are introduced in Grade 6.
8NPO1b	(b) Model or describe rational numbers or numerical relationships using number lines and diagrams.	3.NF.A.2, 4.NF.B.4, 5.NF.B.6, 6NS.A.1, 7.RP.A	Modeling is covered in the Standards for Mathematical Practice and the high school mathematics standards.
8NPO1d	(d) Write or rename rational numbers.	3.NF.A.3, 4.NF.A, 4.NF.B, 5.NBT.A.3, 6.NS.C,7.RP.A, 7.NS.A, 8.NS.A	
8NPO1e	(e) Recognize, translate, or apply multiple representations of rational numbers (fractions, decimals, and percents) in meaningful contexts.	6.NS.C, 7.RP, 7.NS.A, 8.NS.A	
8NPO1f	(f) Express or interpret numbers using scientific notation from real-life contexts.	8.EE.A.3, 8.EE.A.4	
8NPO1g	(g) Find or model absolute value or apply to problem situations.	6.NS.C.7, 7.NS.A.1c	
8NPO1h	(h) Order or compare rational numbers (fractions, decimals, percents, or integers) using various models and representations (e.g., number line).	3.NF., 4.NF.A., 4.NF.B, 4.NF.C 5.NBT.A.3b, 5.NF.B.5a, 6.NS.C	
8NPO1i	(i) Order or compare rational numbers including very large and small integers, and decimals and fractions close to zero.		This objective is very similar to 8NPO1h; however, the CCSS-M do not address “very large and small integers” or “decimals and fractions close to zero.”

NAEP subtopic: (2) Estimation

NAEP objective ID	NAEP objective	Where taught in the CCSS-M?	Comments
8NPO2a	(a) Establish or apply benchmarks for rational numbers and common irrational numbers (e.g., π) in contexts.	8.EE.A.2	

8NPO2b	(b) Make estimates appropriate to a given situation by: • Identifying when estimation is appropriate, • Determining the level of accuracy needed, • Selecting the appropriate method of estimation, or • Analyzing the effect of an estimation method on the accuracy of results.		Covered in the Standards for Mathematical Practice.
8NPO2c	(c) Verify solutions or determine the reasonableness of results in a variety of situations, including calculator and computer results.	4.OA.A.3, 6.EE.B.5;	Also covered in the Standards for Mathematical Practice. Use of calculator or computer is not specifically addressed in the CCSS-M.
8NPO2d	(d) Estimate square or cube roots of numbers less than 1,000 between two whole numbers.	8.NS.A.2	
NAEP subtopic: (3) Number Operations			
NAEP objective ID	**NAEP objective**	**Where taught in the CCSS-M?**	**Comments**
8NPO3a	(a) Perform computations with rational numbers.	3.OA, 3.NBT, 3.NF, 4.OA, 4.NBT, 4.NF, 5.NBT.B.5, 5.NBT.B.6, 5.NBT.B.7, 5.NF, 6.RP.A, 6.NS.B.2, 6.NS.B.3, 7.RP.A, 7.NS.A, 7.EE.A, 8.EE.A.4	
8NPO3d	(d) Describe the effect of multiplying and dividing by numbers, including the effect of multiplying or dividing a rational number by: • Zero, or • A number less than zero, or • A number between zero and one, • One, or • A number greater than one.	3.OA.C.7, 3.NBT.A.3, 4.NF.A, 4.NF.B, 5.NF.B.3, 5.NF.B.4, 5.NF.B.5, 5.NF.B.7 6.NS.A.1, 7.NS.A.2b	
8NPO3e	(e) Interpret rational number operations and the relationships between them.	7.NS.A	
8NPO3f	(f) Solve application problems involving rational numbers and operations using exact answers or estimates as appropriate.	7.RP.A, 7.NS.A, 7.EE.B.3	
NAEP subtopic: (4) Ratios and proportional reasoning			
NAEP objective ID	**NAEP objective**	**Where taught in the CCSS-M?**	**Comments**
8NPO4a	(a) Use ratios to describe problem situations.	6.RP.A, 7.RP.A	

8NPO4b	(b) Use fractions to represent and express ratios and proportions.	6.RP.A, 7.RP.A	
8NPO4c	(c) Use proportional reasoning to model and solve problems (including rates and scaling).	7.RP.A.1, 7.RP.A.2, 7.RP.A.3, 8.EE.B.5	
8NPO4d	(d) Solve problems involving percentages (including percent increase and decrease, interest rates, tax, discount, tips, or part/whole relationships).	6.RP.A, 7.RP.A.3, 7.EE.A	
NAEP subtopic: (5) Properties of number and operations			
NAEP objective ID	**NAEP objective**	**Where taught in the CCSS-M?**	**Comments**
8NPO5a	(a) Describe odd and even integers and how they behave under different operations.	Inferred in 4.OA.C.5	
8NPO5b	(b) Recognize, find, or use factors, multiples, or prime factorization.	4.OA.A.1, 4.OA.B.4, 4.NF.B.4, 6.NS.B.4	Factors and multiples are covered, but prime factorization is not.
8NPO5c	(c) Recognize or use prime and composite numbers to solve problems.	4.OA.B.4	No reference to using prime numbers to solve problems
8NPO5d	(d) Use divisibility or remainders in problem settings.	4.OA.A.3	Remainders are mentioned; however, divisibility is not specifically covered in the CCSS-M.
8NPO5e	(e) Apply basic properties of operations.	5.NF.B.4, 5.NF.B.7, 6.NS.A, 6.NS.B, 7.NS.A	
NAEP subtopic: (6) Mathematical reasoning using numbers			
NAEP objective ID	**NAEP objective**	**Where taught in the CCSS-M?**	**Comments**
8NPO6a	(a) Explain or justify a mathematical concept or relationship (e.g., explain why 17 is prime).		Covered in the Standards for Mathematical Practice.
8NPO6b	(b) Provide a mathematical argument to explain operations with two or more fractions.	5.NF.A, 5.NF.B, 6.NS.A.1,	Also covered in the Standards for Mathematical Practice.

NAEP content area: Algebra

NAEP subtopic: (1) Patterns, Relations, and Functions			
NAEP objective ID	**NAEP objective**	**Where taught in the CCSS-M?**	**Comments**
8A1a	(a) Recognize, describe, or extend numerical and geometric patterns using tables, graphs, words, or symbols.	4.OA.C.5, 8.SP.A.4	
8A1b	(b) Generalize a pattern appearing in a numerical sequence, table, or graph using words or symbols.	4.OA.C.5, 8.SP.A.4	

8A1c	(c) Analyze or create patterns, sequences, or linear functions given a rule.	8.F.B.4	
8A1e	(e) Identify functions as linear or nonlinear or contrast distinguishing properties of functions from tables, graphs, or equations.	8.F.A.2	
8A1f	(f) Interpret the meaning of slope or intercepts in linear functions.	8.EE.B.5, 8.F.A.3	Applied in Modeling in high school.
NAEP subtopic: (2) Algebraic representations			
NAEP objective ID	**NAEP objective**	**Where taught in the CCSS-M?**	**Comments**
8A2a	(a) Translate between different representations of linear expressions using symbols, graphs, tables, diagrams, or written descriptions.	8.F.A.2	
8A2b	(b) Analyze or interpret linear relationships expressed in symbols, graphs, tables, diagrams, or written descriptions.	8.F.A.3	
8A2c	(c) Graph or interpret points represented by ordered pairs of numbers on a rectangular coordinate system.	6.NS.C.6b, 6.NS.C.6c, 7.RP.A.2a	
8A2d	(d) Solve problems involving coordinate pairs on the rectangular coordinate system.	6.NS.C.8	Further developed in High School Geometry: HSG.B.7
8A2f	(f) Identify or represent functional relationships in meaningful contexts, including proportional, linear, and common nonlinear (e.g., compound interest, bacterial growth) in tables, graphs, words, or symbols.	8.EE.B.5, 8.F.B.5	Further developed in High School Functions and Modeling.
NAEP subtopic: (3) Variables, expressions, and operations			
NAEP objective ID	**NAEP objective**	**Where taught in the CCSS-M?**	**Comments**
8A3b	(b) Write algebraic expressions, equations, or inequalities to represent a situation.	6.EE.A.2, 6.EE.B.6-8, 7.EE.A.2	
8A3c	(c) Perform basic operations, using appropriate tools, on linear algebraic expressions (including grouping and order of multiple operations involving basic operations, exponents, roots, simplifying, and expanding).	6.EE.A, 7.EE.A, 8.EE.A.2-4	
NAEP subtopic: (4) Equations and inequalities			
NAEP objective ID	**NAEP objective**	**Where taught in the CCSS-M?**	**Comments**
8A4a	(a) Solve linear equations or inequalities (e.g., ax + b = c or ax + b = cx + d or ax + b > c).	6.EE.B, 7.EE.B, 8.EE.C	
8A4b	(b) Interpret “=” as an equivalence between two expressions and use this interpretation to solve problems.	1.OA.D.7, 6.EE.B, 7.EE.B	Notation of equivalence introduced in Grade 1.

8A4c	(c) Analyze situations or solve problems using linear equations and inequalities with rational coefficients symbolically or graphically (e.g., ax + b = c or ax + b = cx + d).	6.EE.B.7, 6.EE.B.8, 6.EEB.9, 6.G.A, 7.EE.B.4, 8.EE.C.7	
8A4d	(d) Interpret relationships between symbolic linear expressions and graphs of lines by identifying and computing slope and intercepts (e.g., know in y = ax + b, that a is the rate of change and b is the vertical intercept of the graph).	8.EE.B	Also covered in the Standards for Mathematical Practice.
8A4e	(e) Use and evaluate common formulas (e.g., relationship between a circle's circumference and diameter [C = πd], distance, and time under constant speed).	5.MD.C.5b, 6.EE.C.9, 6.G.A.2, 7.G.B.4	

NAEP subtopic: (5) Mathematical reasoning in algebra

NAEP objective ID	NAEP objective	Where taught in the CCSS-M?	Comments
8A5a	(a) Make, validate, and justify conclusions and generalizations about linear relationships.	6.EE.B.5, 8.EE.B	Also covered in the Standards for Mathematical Practice.

NAEP content area: Measurement

NAEP subtopic: (1) Measuring physical attributes

NAEP objective ID	NAEP objective	Where taught in the CCSS-M?	Comments
8M1b	(b) Compare objects with respect to length, area, volume, angle measurement, weight, or mass.	2.MD.A.4	
8M1c	(c) Estimate the size of an object with respect to a given measurement attribute (e.g., area).	2.MD.A.2, 3.MD.A.2	
8M1e	(e) Select or use appropriate measurement instrument to determine or create a given length, area, column, angle, weight, or mass.	2.MD.A.1, 3.MD.C.5, 3.MD.C.6	
8M1f	(f) Solve mathematical or real-world problems involving perimeter or area of plane figures, such as triangles, rectangles, circles, or composite figures.	3.MD.D.8, 4.MD.A.3, 6.G.A.1, 7.G.B.4	
8M1h	(h) Solve problems involving volume or surface area of rectangular solids, cylinders, prisms, or composite shapes.	5.MD.C.3, 5MD.C.4, 5.MD.C.5, 6.G.A.2, 7.G.B.6,	
8M1i	(i) Solve problems involving rates such as speed or population density.	6.RP.A.2, 6.RP.A.3b, 7.RP.A.2b	Concepts of density, including population density, do not appear in the CCSS-M until high school.

NAEP subtopic: (2) Systems of measurement

NAEP objective ID	NAEP objective	Where taught in the CCSS-M?	Comments
8M2a	(a) Select or use an appropriate type of unit for the attribute being measured, such as length, area, angle, time, or volume.	4.MD.A.1	
8M2b	(b) Solve problems involving conversions within the same measurement system, such as conversions involving square inches and square feet.	4.MD.A.1, 6.RP.A.3d,	
8M2c	(c) Estimate the measure of an object in one system given the measure of that object in another system and the approximate conversion factor. For example: • Distance conversion: 1 kilometer is approximately 5/8 of a mile. • Money conversion: U.S. dollars to Canadian dollars. • Temperature conversion: Fahrenheit to Celsius.	4.MD.A.1, 6.RP.A.3d	Covered in the Standards for Mathematical Practice.
8M2d	(d) Determine appropriate size of unit of measurement in problem situation involving such attributes as length, area, or volume.		Covered in the Standards for Mathematical Practice.
8M2e	(e) Determine appropriate accuracy of measurement in problem situations (e.g., the accuracy of each of several lengths needed to obtain a specified accuracy of a total length) and find the measure to that degree of accuracy.		Covered in the Standards for Mathematical Practice.

NAEP subtopic: (3) Measurement in triangles

NAEP objective ID	NAEP objective	Where taught in the CCSS-M?	Comments
8M3a	(a) Solve problems involving indirect measurement, such as finding the height of a building by comparing its shadow with the height and shadow of a known object.	7.G.A.1, 7.G.B.6, 8.G.A.4	

NAEP content area: Geometry

NAEP subtopic: (1) Dimension and shape

NAEP objective ID	NAEP objective	Where taught in the CCSS-M?	Comments
8G1a	(a) Draw or describe a path of shortest length between points to solve problems in context.	Context application in 8.G.B.8, Pythagorean Theorem.	
8G1b	(b) Identify a geometric object given a written description of its properties.	3.G.A.1, 4.GA.2, 5.G.B.3	

8G1c	(c) Identify, define, or describe geometric shapes in the plane and in three-dimensional space given a visual representation.	6.G.A.4	
8G1d	(d) Draw or sketch from a written description polygons, circles, or semicircles.	6.G.A.3, 7.G.A.1, 7.G.A.2	
8G1e	(e) Represent or describe a three-dimensional situation in a two-dimensional drawing from different views.	7.G.A.3	
8G1f	(f) Demonstrate an understanding about the two- and three-dimensional shapes in our world through identifying, drawing, modeling, building, or taking apart.	6.G.A.3, 7.G.A.1, 7.G.A.2	
NAEP subtopic: (2) Transformation of shapes and preservation of properties			
NAEP objective ID	**NAEP objective**	**Where taught in the CCSS-M?**	**Comments**
8G2a	(a) Identify lines of symmetry in plane figures or recognize and classify types of symmetries of plane figures.	4.G.A.3	
8G2c	(c) Recognize or informally describe the effect of a transformation on two-dimensional geometric shapes (reflections across lines of symmetry, rotations, translations, magnifications, and contractions).	8.G.A.3	
8G2d	(d) Predict results of combining, subdividing, and changing shapes of plane figures and solids (e.g., paper folding, tiling, cutting up, and rearranging pieces).	6.G.A.1, 7.G.A.3, 7.G.B.4, 7.G.B.6	The foundational understandings are addressed in these standards. “Predicting,” per se, with respect to this objective, is not specifically evident in the CCSS-M.
8G2e	(e) Justify relationships of congruence and similarity, and apply these relationships using scaling and proportional reasoning.	8.G.A.2, 8.G.A.4	
8G2f	(f) For similar figures, identify and use the relationships of conservation of angle and proportionality of side length and perimeter.	8.G.A.4	
NAEP subtopic: (3) Relationships between geometric figures			
NAEP objective ID	**NAEP objective**	**Where taught in the CCSS-M?**	**Comments**
8G3b	(b) Apply geometric properties and relationships in solving simple problems in two and three dimensions.	6.G.A, 7.G.A	
8G3c	(c) Represent problem situations with simple geometric models to solve mathematical or real-world problems.	6.G.A, 7.G.A, 8.G.A	

8G3d	(d) Use the Pythagorean Theorem to solve problems.	8.G.B.7, 8.G.B.8	
8G3f	(f) Describe or analyze simple properties of, or relationships between, triangles, quadrilaterals, and other polygonal plane figures.	3.G.A.1, 5.G.B.3, 5.G.B.4, 6.G.A, 7.G.A, 8.G.A.2-4	
8G3g	(g) Describe or analyze properties and relationships of parallel or intersecting lines.	4.G.A.1, 4.G.A.2, 8.G.A.1c	
NAEP subtopic: (4) Position, direction, and coordinate geometry			
NAEP objective ID	**NAEP objective**	**Where taught in the CCSS-M?**	**Comments**
8G4a	(a) Describe relative positions of points and lines using the geometric ideas of midpoint, points on common line through a common point, parallelism, or perpendicularity.	High School Geometry	
8G4b	(b) Describe the intersection of two or more geometric figures in the plane (e.g., intersection of a circle and a line).	High School Geometry	
8G4c	(c) Visualize or describe the cross-section of a solid.	7.G.A.3	
8G4d	(d) Represent geometric figures using rectangular coordinates on a plane.	6.G.A.3, 8.G.A.3	
NAEP subtopic: (5) Mathematical reasoning in geometry			
NAEP objective ID	**NAEP objective**	**Where taught in the CCSS-M?**	**Comments**
8G5a	(a) Make and test a geometric conjecture about regular polygons.		Covered in the Standards for Mathematical Practice.

NAEP content area: Data Analysis, Statistics, and Probability

NAEP subtopic: (1) Data representation			
NAEP objective ID	**NAEP objective**	**Where taught in the CCSS-M?**	**Comments**
	Histograms, line graphs, scatterplots, box plots, bar graphs, circle graphs, stem and leaf plots, frequency distributions, and tables.		
8DASP1a	(a) Read or interpret data, including interpolating or extrapolating from data.	6.SP.A.2, 7.SP.A.1, 7.SP.B.4	No mention of interpolating or extrapolating from data in the CCSS-M.
8DASP1b	(b) For a given set of data, complete a graph and then solve a problem using the data in the graph (histograms, line graphs, scatterplots, circle graphs, and bar graphs).	6.SP.B.4, 8.SP.A.1-3,	Solving problems from data in graphs is addressed in the elementary grades in Measurement and Data.

8DASP1c	(c) Solve problems by estimating and computing with data from a single set or across sets of data.	7.SP.A.2, 7.SP.B.3-4	Also covered in the Standards for Mathematical Practice.
8DASP1d	(d) Given a graph or a set of data, determine whether information is represented effectively and appropriately (histograms, line graphs, scatterplots, circle graphs, and bar graphs).		Covered in the Standards for Mathematical Practice.
8DASP1e	(e) Compare and contrast the effectiveness of different representations of the same data.	7.SP.B.3	Also covered in the Standards for Mathematical Practice.
NAEP subtopic: (2) Characteristics of data			
NAEP objective ID	**NAEP objective**	**Where taught in the CCSS-M?**	**Comments**
8DASP2a	(a) Calculate, use, or interpret mean, median, mode, or range.	6.SP.A.3, 6.SP.B.5c, 7.SP.A.2, 7.SP.B.3, 7.SP.B.4	The CCSS-M use the term “measure of center” and refer only to the mean and median.
8DASP2b	(b) Describe how mean, median, mode, range, or interquartile ranges relate to distribution shape.	6.SP.B.5d, 7.SP.B.4	
8DASP2c	(c) Identify outliers and determine their effect on mean, median, mode, or range.	6.SP.B.5c, 8.SP.A.1	
8DASP2d	(d) Using appropriate statistical measures, compare two or more data sets describing the same characteristic for two different populations or subsets of the same population.	7.SP.B.4	
8DASP2e	(e) Visually choose the line that best fits given a scatterplot and informally explain the meaning of the line. Use the line to make predictions.	8.SP.A.2	
NAEP subtopic: (3) Experiments and samples			
NAEP objective ID	**NAEP objective**	**Where taught in the CCSS-M?**	**Comments**
8DASP3a	(a) Given a sample, identify possible sources of bias in sampling.	7.SP.A.2	Bias, per se, is not mentioned in the CCSS-M.
8DASP3b	(b) Distinguish between a random and nonrandom sample.	7.SP.A.1	Coverage of nonrandom samples is inferred.
8DASP3d	(d) Evaluate the design of an experiment.		Covered in High School Statistics and Probability
NAEP subtopic: (4) Probability			
NAEP objective ID	**NAEP objective**	**Where taught in the CCSS-M?**	**Comments**
8DASP4a	(a) Analyze a situation that involves probability of an independent event.	7.SP.C.5	

8DASP4b	(b) Determine the theoretical probability of simple and compound events in familiar contexts.	7.SP.C.6	
8DASP4c	(c) Estimate the probability of simple and compound events through experimentation or simulation.	7.SP.C.8b, 7.SP.C.8c	
8DASP4d	(d) Use theoretical probability to evaluate or predict experimental outcomes.	7.SP.C.7	
8DASP4e	(e) Determine the sample space for a given situation.	7.SP.A.2	
8DASP4f	(f) Use a sample space to determine the probability of possible outcomes for an event.	7.SP.C.8a, 7.SP.C.6	
8DASP4g	(g) Represent the probability of a given outcome using fractions, decimals, and percent.	7.SP.C.5	Representation of probability using fractions is explicit in Grade 7 in the CCSS-M; representation using decimals and percent is implicit.
8DASP4h	(h) Determine the probability of independent and dependent events. (Dependent events should be limited to a small sample size.)		Covered in High School Statistics and Probability.
8DASP4j	(j) Interpret probabilities within a given context.	7.SP.C.6	

[1] *Notation for the CCSS-M:* Grade level, content domain, cluster, standard number within domain. For example, 3.OA.D.8 is read as Grade 3, Operations and Algebraic Thinking, Cluster D, Standard 8.

[2] *Notation for NAEP objectives:* Grade level, content area, subtopic, objective. For example, 4NPO1i is read as Grade 4, Number Properties and Operations, Subtopic 1, Objective i.

Appendix D. NAEP and CCSS-M Alignment Study Panel Assignments—July 2012

Elementary (Grade 4): Number Properties and Operations; Algebra

Sandra Alberti, Panel Leader
Sharon Gaines
Roger Howe
Tad Wantanabe

Elementary (Grade 4): Measurement; Geometry; Data Analysis, Statistics, and Probability

William Bush, Panel Leader
Brittany Gaines
Andy Isaacs
Norman Mattox

Secondary (Middle Grade 8): Number Properties and Operations; Algebra

Elaine Abbas
Diane Briars, Panel Leader
Jason McNeil

Secondary (Middle Grade 8): Measurement; Geometry; Data Analysis, Statistics, and Probability

Pamela Beck
Brad Findell
Carole Philip, Panel Leader