Department of Education
& Early Development

ALASKA MATHEMATICS STANDARDS

Adopted June 2012

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Alaska English/Language Arts and Mathematics Content Standards

High academic standards are an important first step in ensuring that all Alaska's students have the tools they need for success. These standards reflect the collaborative work of Alaskan educators and national experts from the nonprofit National Center for the Improvement of Educational Assessment. Further, they are informed by public comments. Alaskan teachers have played a key role in this effort, ensuring that the standards reflect the realities of the classroom. Since work began in spring 2010, the standards have undergone a thoughtful and rigorous drafting and refining process.

A nationwide movement among the states and employers has called for America's schools to prepare students to be ready for postsecondary education and careers. Standards in English/language arts and mathematics build a foundation for college and career readiness. Students proficient in the standards read widely and deeply in a range of subjects, communicate clearly in written and spoken English, have the capacity to build knowledge on a subject, and understand and use mathematics.

Industry leaders were part of Alaska's standards review. Repeatedly these leaders placed the greatest weight on critical thinking and adaptability as essential skills in the workplace. Industry leaders believe that strengthening our K-12 system will help ensure that Alaskans are prepared for high-demand, good-wage jobs. Instructional expectations that include employability standards will help students prepare for a career.

Additionally, institutions of higher education were engaged in refining Alaska's standards. These educators focused on whether the standards would culminate in student preparedness. Students proficient in Alaska's standards will be prepared for credit-bearing courses in their first year of postsecondary education. It is critical that students can enter institutions of higher education ready to apply their knowledge, extend their learning, and gain technical and job-related skills.

These standards do not tell teachers how to teach, nor do they attempt to override the unique qualities of each student and classroom. They simply establish a strong foundation of knowledge and skills all students need for success after graduation. It is up to schools and teachers to decide how to put the standards into practice and incorporate other state and local standards, including cultural standards. In sum, students must be provided opportunities to gain skills and learn to apply them to real-world life and work situations.

Introduction to Mathematics Standards

The mathematics standards prepare Alaska students to be competitive on the national and world stage. These standards are a set of specific, rigorous expectations that build students' conceptual understanding, mathematical language, and application of processes and procedures coherently from one grade to the next so all students will be prepared for post-secondary experiences. The focus areas for each grade level and each conceptual category narrative establish a depth of knowledge as opposed to a breadth of knowledge across multiple standards in each grade level or content area.

The standards for mathematics stress both conceptual understanding and procedural skills to ensure students learn and can apply the critical information needed to succeed at each level.

- In kindergarten, the standards follow successful international models and recommendations by focusing kindergarten work on the number core: learning how numbers correspond to quantities, and learning how to put numbers together and take them apart (the beginnings of addition and subtraction).

- The K-5 standards provide students with a solid foundation in whole numbers, addition, subtraction, multiplication, division, fractions and decimals--which help young students build the foundation to successfully apply more demanding math concepts and procedures and move into applications.

- Having built a strong foundation in K-5, students can do hands-on learning in geometry, algebra and probability and statistics. Students who have completed 7th grade and mastered the content and skills through the 7th grade will be well-prepared for algebra in grade 8. The middle school standards are robust and provide a coherent and rich preparation for high school mathematics.

- The high school standards set a rigorous definition of readiness by helping students develop a depth of understanding and ability to apply mathematics to novel situations, as college students and employees regularly do.

Organization of Mathematics Standards

The Alaska Mathematics Standards define what students should understand and be able to do in their study of mathematics. Teachers ensure students achieve standards by using a variety of instructional strategies based on their students' needs.

The standards are divided into two areas of equal importance:

1. **The Standards for Mathematical Practice** are embedded at every grade level to establish habits of mind that will empower students to become mathematically literate. Instructional approaches that promote students' development of the Practices are critical to procedural fluency in mathematics.

2. **The Standards for Mathematical Content** are grade-level specific in kindergarten through grade 8. The high school content is organized by conceptual category. Taken together, the K-12 standards provide a scaffold that allows students to become increasingly more proficient in understanding and using mathematics. There is a gradual, steady progression leading to college and career readiness by the time students graduate from high school.

Each grade-level is supported with the inclusion of an Instructional Focus section. The Instructional Focus guides teachers toward the critical areas of emphasis. Each high school Conceptual Category includes a narrative that also guides teachers' instruction.

The Standards for Mathematical Practice

These eight standards bring the complexities of the world into focus and give schema for grappling with authentic and meaningful problems. The practice standards define experiences that build understanding of mathematics and ways of thinking through which students develop, apply, and assess their knowledge.

Algorithmic knowledge is no longer sufficient when preparing our students to become globally competitive. The knowledge of good practitioners goes beyond algorithmic learning and allows them to picture the problem and the many roads that may lead to a solution. They realize that mathematics is applicable outside of the classroom and are confident in their ability to apply mathematical concepts to all aspects of life. The Standards of Mathematical Practice allow students to deepen their understandings of mathematical concepts and cultivates their autonomy as mathematically literate and informed citizens. Employing mathematics as a means of synthesizing complex concepts and making informed decisions is paramount to success in all post-secondary endeavors.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

Kindergarten	1	2	3	4	5	6	7	8	High School

Instruction around the Standards for Mathematical Practices is delivered across all grades K-12. For each Standard for Mathematical Practice, there are grade-span descriptors that are meant to help students, parents and educators determine how these might be demonstrated by students. Implementing the practices to meet the descriptors will involve strengthening current teaching practices.

The Standards for Mathematical Content

Each grade level in the K-8 standards is prefaced with an explanation of instructional focus areas for that grade level. Each conceptual category in the high school standards is prefaced with an explanation of the implication of that category to a student's mastery of mathematics. Specific modeling standards appear throughout the high school standards as indicated by an asterisk (*).

Additional mathematic standards that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics are indicated by a plus symbol (+). The plus symbol indicates that the standard is not required for all students.

K-8 Mathematical Domains:	**High School Conceptual Categories:**
1. Counting and Cardinality – CC	1. Number and Quantity – N
2. Operations and Algebraic Thinking – OA	2. Algebra – A
3. Number and Operations in Base Ten – NBT	3. Functions – F
4. Measurement and Data – MD	4. Modeling – M
5. Number and Operations – Fractions – NF	5. Geometry – G
6. Geometry – G	6. Statistics and Probability – P
7. Ratios and Proportional Relationships – RP	
8. The Number System – NS	
9. Expressions and Equations – EE	
10. Functions – F	
11. Statistics and Probability – SP	

The standards for mathematics stress both conceptual understanding and procedural skills to ensure students learn and can apply the critical information needed to succeed at each level. This creates a learning progression where the mathematics learned in elementary school provides the foundation for the study of statistics, probability, ratio and proportion, geometry, and algebra in middle school. This is, in turn, the base upon which the knowledge needed for success in colleges and careers can be developed in high school.

The standards organization is not intended to convey the order of instruction nor the length of time to devote to the topics. In the standards, the clusters have been arranged in the grade span to show the continuum between grades. The following table outlines the progression of the content from kindergarten through high school.

<table>
<tr><th colspan="10">Standards for Mathematical Content</th></tr>
<tr><th>Kindergarten</th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th><th>7</th><th>8</th><th>High School</th></tr>
<tr><td colspan="2">Counting and Cardinality</td><td colspan="7"></td><td rowspan="3">Number & Quantity</td></tr>
<tr><td colspan="6">Number and Operations in Base Ten</td><td colspan="2">Ratios and Proportional Relationships</td><td></td></tr>
<tr><td colspan="3"></td><td colspan="3">Number and Operations - Fractions</td><td colspan="3">Number System</td></tr>
<tr><td colspan="6" rowspan="2">Operations and Algebraic Thinking</td><td colspan="3">Expressions and Equations</td><td>Algebra</td></tr>
<tr><td colspan="2"></td><td>Functions</td><td>Functions</td></tr>
<tr><td colspan="9">Geometry</td><td>Geometry</td></tr>
<tr><td colspan="6">Measurement and Data</td><td colspan="3">Statistics and Probability</td><td>Statistics and Probability</td></tr>
</table>

Domains

Conceptual Categories

Modeling

Domains are large groups of related standards. Each shaded row shows how domains at the earlier grades progress and lead to conceptual categories at the high school levels. The right side of the chart lists the five **conceptual categories** for high school. Selecting one conceptual category and moving left along the row shows the domains at the middle and elementary school levels from which this concept builds. Modeling, the sixth conceptual category, is incorporated throughout the other five high school categories.

Overall, the progressions of the standards begin and end in different grades, avoiding the re-teaching of concepts that should have been mastered. This allows for higher rigor overall, which is key to laying the foundation for high school mathematics standards and college/career preparedness.

For each of the grade-spans (K-2, 3-5, 6-8, and 9-12) an overview of the topics to be covered follows.

Overview of Mathematical Content Standards

Kindergarten	Grade 1	Grade 2
Counting and Cardinality • Know number names and the count sequence. • Count to tell the number of objects. • Compare numbers. **Operations and Algebraic Thinking** • Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from. • Identify and continue patterns. **Number and Operations in Base Ten** • Work with numbers 11–19 to gain foundations for place value. **Measurement and Data** • Describe and compare measurable attributes. • Classify objects and count the number of objects in categories. • Work with time and money. **Geometry** • Identify and describe shapes. • Analyze, compare, create, and compose shapes.	**Counting and Cardinality** • Know ordinal names and counting flexibility. • Count to tell the number of objects. • Compare numbers. **Operations and Algebraic Thinking** • Represent and solve problems involving addition and subtraction. • Understand and apply properties of operations and the relationship between addition and subtraction. • Add and subtract up to 20. • Work with addition and subtraction equations. • Identify and continue patterns. **Number and Operations in Base Ten** • Extend the counting sequence. • Understand place value. • Use place value understanding and properties of operations to add and subtract. **Measurement and Data** • Measure lengths indirectly and by iterating length units. • Work with time and money. • Represent and interpret data. **Geometry** • Reason with shapes and their attributes.	**Operations and Algebraic Thinking** • Represent and solve problems involving addition and subtraction. • Add and subtract up to 20. • Work with equal groups of objects to gain foundations for multiplication. • Identify and continue patterns. **Number and Operations in Base Ten** • Understand place value. • Use place value understanding and properties of operations to add and subtract. **Measurement and Data** • Measure and estimate lengths in standard units. • Relate addition and subtraction to length. • Work with time and money. • Represent and interpret data. **Geometry** • Reason with shapes and their attributes.

Grade 3	Grade 4	Grade 5
Operations and Algebraic Thinking • Represent and solve problems involving multiplication and division. • Understand properties of multiplication and the relationship between multiplication and division. • Multiply and divide up to 100. • Solve problems involving the four operations, and identify and explain patterns in arithmetic. **Number and Operations in Base Ten** • Use place value understanding and properties of operations to perform multi-digit arithmetic. **Number and Operations—Fractions** • Develop understanding of fractions as numbers. **Measurement and Data** • Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects. • Represent and interpret data. • Geometric measurement: understand concepts of area and relate area to multiplication and to addition. • Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures. **Geometry** • Reason with shapes and their attributes.	**Operations and Algebraic Thinking** • Use the four operations with whole numbers to solve problems. • Gain familiarity with factors and multiples. • Generate and analyze patterns. **Number and Operations in Base Ten** • Generalize place value understanding for multi-digit whole numbers. • Use place value understanding and properties of operations to perform multi-digit arithmetic. **Number and Operations—Fractions** • Extend understanding of fraction equivalence and ordering. • Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers. • Understand decimal notation for fractions, and compare decimal fractions. **Measurement and Data** • Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit and involving time. • Represent and interpret data. • Geometric measurement: understand concepts of angle and measure angles. **Geometry** • Draw and identify lines and angles, and classify shapes by properties of their lines and angles.	**Operations and Algebraic Thinking** • Write and interpret numerical expressions. • Analyze patterns and relationships. **Number and Operations in Base Ten** • Understand the place value system. • Perform operations with multi-digit whole numbers and with decimals to hundredths. **Number and Operations—Fractions** • Use equivalent fractions as a strategy to add and subtract fractions. • Apply and extend previous understandings of multiplication and division to multiply and divide fractions. **Measurement and Data** • Convert like measurement units within a given measurement system and solve problems involving time. • Represent and interpret data. • Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition. **Geometry** • Graph points on the coordinate plane to solve real-world and mathematical problems. • Classify two-dimensional figures into categories based on their properties.

Grade 6	Grade 7	Grade 8
Ratios and Proportional Relationships • Understand ratio concepts and use ratio reasoning to solve problems. **The Number System** • Apply and extend previous understandings of multiplication and division to divide fractions by fractions. • Compute fluently with multi-digit numbers and find common factors and multiples. • Apply and extend previous understandings of numbers to the system of rational numbers. **Expressions and Equations** • Apply and extend previous understandings of arithmetic to algebraic expressions. • Reason about and solve one-variable equations and inequalities. • Represent and analyze quantitative relationships between dependent and independent variables. **Geometry** • Solve real-world and mathematical problems involving area, surface area, and volume. **Statistics and Probability** • Develop understanding of statistical variability. • Summarize and describe distributions.	**Ratios and Proportional Relationships** • Analyze proportional relationships and use them to solve real-world and mathematical problems. **The Number System** • Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers. **Expressions and Equations** • Use properties of operations to generate equivalent expressions. • Solve real-life and mathematical problems using numerical and algebraic expressions and equations. **Geometry** • Draw, construct and describe geometrical figures and describe the relationships between them. • Solve real-life and mathematical problems involving angle measure, area, surface area, and volume. **Statistics and Probability** • Use random sampling to draw inferences about a population. • Draw informal comparative inferences about two populations. • Investigate chance processes and develop, use, and evaluate probability models.	**The Number System** • Know that there are numbers that are not rational, and approximate them by rational numbers. **Expressions and Equations** • Work with radicals and integer exponents. • Understand the connections between proportional relationships, lines, and linear equations. • Analyze and solve linear equations and pairs of simultaneous linear equations. **Geometry** • Understand congruence and similarity using physical models, transparencies, or geometry software. • Understand and apply the Pythagorean Theorem. • Solve real-world and mathematical problems involving volume of cylinders, cones and spheres. **Statistics and Probability** • Investigate patterns of association in bivariate data. **Functions** • Define, evaluate, and compare functions. • Use functions to model relationships between quantities.

Overview of High School Content Standards

Modeling	Number and Quantity	Algebra
Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Specific modeling standards appear throughout the high school standards indicated by an asterisk (*). If the asterisk appears on the heading for a group of standards, it should be understood to apply to all standards in that group. There are other individual standards under clusters, domains and conceptual categories that have connections to modeling. Additionally, model with mathematics is a Standard for Mathematical Practice. This practice will be started in kindergarten.	**The Real Number System** • Extend the properties of exponents to rational exponents. • Use properties of rational and irrational numbers. **Quantities*** • Reason quantitatively and use units to solve problems. **The Complex Number System** • Perform arithmetic operations with complex numbers. • Represent complex numbers and their operations on the complex plane. + • Use complex numbers in polynomial identities and equations. **Vector and Matrix Quantities** • Represent and model with vector quantities. + • Perform operations on vectors. + • Perform operations on matrices and use matrices in applications. +	**Seeing Structure in Expressions** • Interpret the structure of expressions. • Write expressions in equivalent forms to solve problems.* **Arithmetic with Polynomials and Rational Expressions** • Perform arithmetic operations on polynomials. • Understand the relationship between zeros and factors of polynomials. • Use polynomial identities to solve problems. • Rewrite rational expressions. **Creating Equations and Inequalities*** • Create equations and inequalities that describe numbers or relationships. **Reasoning with Equations and Inequalities** • Understand solving equations as a process of reasoning and explain the reasoning. • Solve equations and inequalities in one variable. • Solve systems of equations. • Represent and solve equations and inequalities graphically.

*Standards with connections to modeling. If asterisk appears on the category, domain, or cluster for a group of standards, it should be understood to apply to all standards in that group. There may be individual standards within clusters with connections to modeling.
+ Standards include additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics.

Functions

Interpreting Functions

- Understand the concept of a function and use function notation.
- Interpret functions that arise in applications in terms of the context.
- Analyze functions using different representations.

Building Functions

- Build a function that models a relationship between two quantities.
- Build new functions from existing functions.

Linear, Quadratic, and Exponential Models*

- Construct and compare linear, quadratic, and exponential models and solve problems.
- Interpret expressions for functions in terms of the situation they model.

Trigonometric Functions

- Extend the domain of trigonometric functions using the unit circle.
- Model periodic phenomena with trigonometric functions.
- Prove and apply trigonometric identities.

Geometry

Congruence

- Experiment with transformations in the plane.
- Understand congruence in terms of rigid motions.
- Prove geometric theorems.
- Make geometric constructions.

Similarity, Right Triangles, and Trigonometry

- Understand similarity in terms of similarity transformations.
- Prove theorems involving similarity.
- Define trigonometric ratios and solve problems involving right triangles.
- Apply trigonometry to general triangles. +

Circles

- Understand and apply theorems about circles.
- Find arc lengths and areas of sectors of circles.

Expressing Geometric Properties with Equations

- Translate between the geometric description and the equation for a conic section.
- Use coordinates to prove simple geometric theorems algebraically.

Geometric Measurement and Dimension

- Explain volume formulas and use them to solve problems.
- Visualize relationships between two-dimensional and three-dimensional objects.

Modeling with Geometry

- Apply geometric concepts in modeling situations.*

Statistics and Probability*

Interpreting Categorical and Quantitative Data

- Summarize, represent, and interpret data on a single count or measurement variable.
- Summarize, represent, and interpret data on two categorical and quantitative variables.
- Interpret linear models.

Making Inferences and Justifying Conclusions

- Understand and evaluate random processes underlying statistical experiments.
- Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

Conditional Probability and the Rules of Probability

- Understand independence and conditional probability and use them to interpret data.
- Use the rules of probability to compute probabilities of compound events in a uniform probability model.

Using Probability to Make Decisions

- Calculate expected values and use them to solve problems. +
- Use probability to evaluate outcomes of decisions. +

*Standards with connections to modeling. If the asterisk appears on the category, domain, or cluster for a group of standards, it should be understood to apply to all standards in that group. There may be individual standards within clusters with connections to modeling.

+ Standards include additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics.

Guide to Reading the Mathematical Content Standards

There are eleven domains within the K-8 Standards. Students advancing through the grades are expected to meet each year's grade-specific standards, and retain or further develop skills and understandings mastered in preceding grades. An instructional focus is included before each grade to support the implementation of the content.

K-8 Mathematical Domains:

1. Counting and Cardinality - CC
2. Operations and Algebraic Thinking - OA
3. Number and Operations in Base Ten - NBT
4. Measurement and Data - MD
5. Number and Operations - Fractions - NF
6. Geometry - G
7. Ratios and Proportional Relationships - RP
8. The Number System - NS
9. Expressions and Equations - EE
10. Functions - F
11. Statistics and Probability - SP

Domains are intended to convey coherent groupings of content. All domains are underlined.
Clusters are groups of related standards. Cluster headings are bolded.
Standards define what students should understand and be able to do. Standards are numbered. Any standard followed by an (L) indicates the standard is to be locally assessed.

K- 8 Grade Level →

Grade 4

Domain → **<u>Number and Operations in Base Ten 4.NBT</u>**

Cluster

Generalize place value understanding for multi-digit whole numbers.

4.NBT.1. Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.*

4.NBT.2. Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on the value of the digits in each place, using >, =, and < symbols to record the results of comparisons.

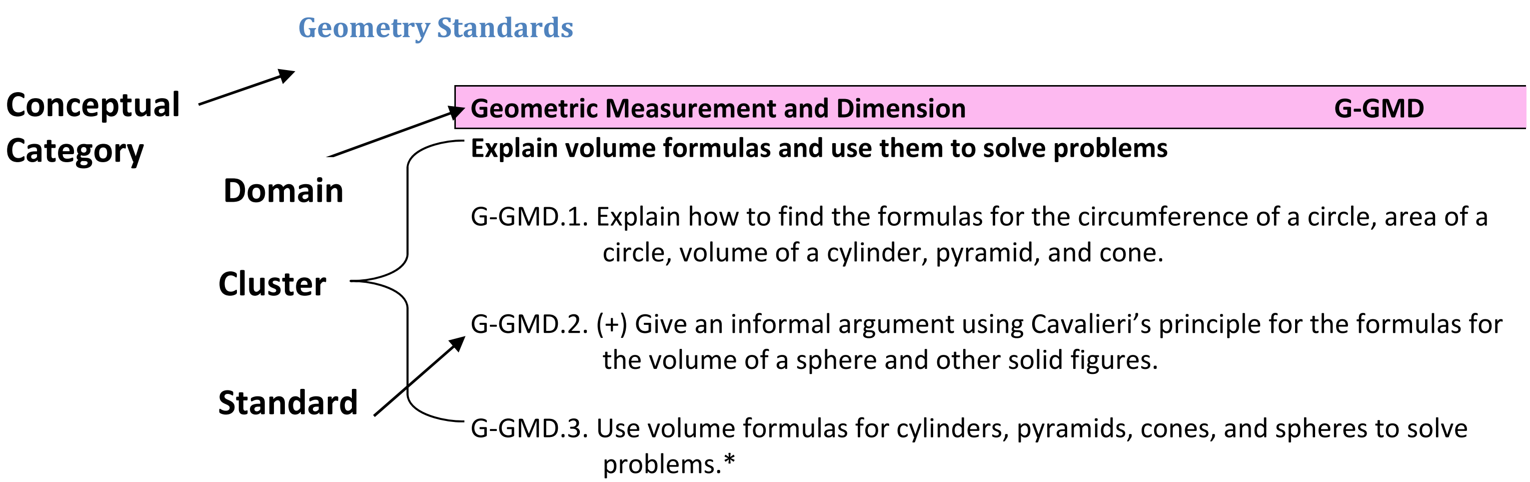
Standard → 4.NBT.3. Use place value understanding to round multi-digit whole numbers to any place using a variety of estimation methods; be able to describe, compare, and contrast solutions.

The high school standards specify the mathematics that all students should study in order to be career and college ready. They are organized into conceptual categories, which are intended to portray a coherent view of high school mathematics. A student's work with any set of standards crosses a number of traditional course boundaries. For example, the Functions Standards would apply to many courses such as Algebra I or Algebra II. It is a district decision how to design course offerings covering the mathematics standards. Districts can use the traditional approach of Algebra I, Geometry, and Algebra II or implement an integrated approach. There are various high school math pathways to be considered.

There are six conceptual categories for high school. Each conceptual category in the high school standards is prefaced with a narrative and an explanation of the implication of that category to a student's mastery of mathematics.

High School Mathematical Conceptual Categories:

1. Number and Quantity - N
2. Algebra - A
3. Functions - F
4. Modeling - M
5. Geometry - G
6. Statistics and Probability - P



(+) Additional standards for advanced courses
(*) Standards with connection to modeling

Standards for Mathematical Practice

Alaska Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council’s report Adding It Up: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy).

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

Each Standard for Mathematical Practice listed below is followed by a set of grade-span descriptors. These descriptors of the Standards of Mathematical Practice are meant to help students, parents and educators to picture how these practices might be demonstrated by students. Within the grade span, students should apply the practices using specific grade-level content. Additionally, students at higher grade spans may revisit earlier grade-span proficiencies as the rigor of the content increases.

Connecting the Standards for Mathematical Practice and Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction. The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word “understand” are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices. In this respect, those content standards which set an expectation of understanding are potential “points of intersection” between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

In grades K-2 mathematically proficient students will:

- focus on the problem and check for alternate methods
- check if the solution makes sense

In grades 3-5 mathematically proficient students will:

- explain correspondences between equations, verbal descriptions, tables, and graphs
- draw diagrams of important features and relationships, graph data, and search for regularity or trends
- use concrete objects or pictures to help conceptualize and solve a problem
- understand the approaches of others to solving complex problems
- identify correspondences between different approaches
- check if the solution makes sense

In grades 6-8 mathematically proficient students will:

- explain correspondences between a new problem and previous problems
- represent algebraic expressions numerically, graphically, concretely/with manipulatives, verbally/written
- explain connections between the multiple representations
- determine the question that needs to be answered
- analyze a problem and make a plan for solving it
- choose a reasonable strategy
- identify the knowns and unknowns in a problem
- use previous knowledge and skills to simplify and solve problems
- break a problem into manageable parts or simpler problems
- solve a problem in more than one way

In grades 9-12 mathematically proficient students will:

- make connections between a new problem and previous problems
- determine the question that needs to be answered
- choose a reasonable strategy
- identify the knowns and unknowns in a problem
- use previous knowledge and skills to simplify and solve problems
- break a problem into manageable parts or simpler problems
- represent algebraic expressions numerically, graphically, concretely/with manipulatives, verbally/written
- explain connections between the multiple representations
- solve a problem in more than one way
- explain the meaning of a problem and look for an entry point
- analyze a problem and make a plan for solving it
- explain correspondence between differing approaches to identify regularity and trends
- check answer using a different method
- identify correspondence between different approaches
- monitor and evaluate progress and change course if necessary
- check the answers to problems using a different method and continually ask, “Does this make sense?”

2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

In grades K-2 mathematically proficient students will:

- represent a situation symbolically and/or with manipulatives
- create a coherent representation of the problem
- use units of measurement consistently

In grades 3-5 mathematically proficient students will:

- represent a situation symbolically
- create a coherent representation of the problem
- have the ability to show how problem has a realistic meaning
- reflect during the manipulation process in order to probe into the meanings for the symbols involved
- use units consistently

In grades 6-8 mathematically proficient students will:

- represent a situation symbolically and carry out its operations
- create a coherent representation of the problem
- translate an algebraic problem to a real-world context
- explain the relationship between the symbolic abstraction and the context of the problem
- compute using different properties
- consider the quantitative values, including units, for the numbers in a problem

In grades 9-12 mathematically proficient students will:

- decontextualize to abstract a given situation and represent it symbolically and manipulate the representing symbols.
- reflect during the manipulation process in order to probe into the meanings for the symbols involved
- create a coherent representation of the problem
- make sense of quantities and their relationships in problem situations
- attend to the meanings of quantities
- use flexibility with different properties of operations and objects
- translate an algebraic problem to a real-world context
- explain the relationship between the symbolic abstraction and the context of the problem
- compute using different properties
- consider the quantitative values, including units, for the numbers in a problem

3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

In grades K-2 mathematically proficient students will:

- construct arguments using concrete referents such as objects, drawings, diagrams, and actions
- justify conclusions, communicate conclusions
- listen to arguments and decide whether the arguments make sense

In grades 3-5 mathematically proficient students will:

- construct arguments using concrete referents such as objects, drawings, diagrams, and actions
- justify conclusions, communicate conclusions, listen and respond to arguments, decide whether the argument makes sense, and ask questions to clarify the argument
- reason inductively about data, making plausible arguments that take into account the context from which the data arose

In grades 6-8 mathematically proficient students will:

- construct arguments using both concrete and abstract explanations
- justify conclusions, communicate conclusions, and respond to the arguments
- listen to arguments, critique their viability, and ask questions to clarify the argument
- compare effectiveness of two arguments by identifying and explaining both logical and/or flawed reasoning
- recognize general mathematical truths and use statements to justify the conjectures
- identify special cases or counter-examples that don’t follow the mathematical rules
- infer meaning from data and make arguments using its context

In grades 9-12 mathematically proficient students will:

- construct arguments using both concrete and abstract explanations
- justify conclusions in a variety of ways, communicate the methodology, and respond to the arguments
- reason inductively about data and make plausible arguments that take into account the context from which the data arose
- understand and use stated assumptions, definitions, and previously established results in constructing arguments
- make conjectures and build a logical progression of statements to explore the truth of the conjectures
- analyze situations by breaking them into cases and recognize and use counter-examples
- recognize general mathematical truths and statements to justify the conjectures
- identify special cases or counter-examples that don’t follow the mathematical rules
- infer meaning from data and make arguments using its context
- compare effectiveness of two arguments by identifying and explaining both logical and/or flawed reasoning

4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

In grades K-2 mathematically proficient students will:

- apply mathematics to solve problems in everyday life
- identify important quantities in a practical situation and model the situation with manipulatives or pictures
- interpret mathematical results in the context of the situation and reflect on whether the results make sense

In grades 3-5 mathematically proficient students will:

- apply mathematics to solve problems arising in everyday life
- identify important quantities in a practical situation and model the situation using such tools as manipulatives, diagrams, two-way tables, graphs or pictures
- interpret mathematical results in the context of the situation and reflect on whether the results make sense
- apply mathematical knowledge, make assumptions and approximations to simplify a complicated situation

In grades 6-8 mathematically proficient students will:

- apply mathematics to solve problems arising in everyday life and society
- identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, and formulas
- interpret their mathematical results in the context of the situation and reflect on whether the results make sense
- make assumptions and approximations to simplify a situation, realizing the final solution will need to be revised
- analyze quantitative relationships to draw conclusions
- reflect on whether their results make sense
- improve the model if it has not served its purpose

In grades 9-12 mathematically proficient students will:

- apply mathematics to solve problems in everyday life, society, and workplace
- identify important quantities in a practical situation and map the relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas
- consistently interpret mathematical results in the context of the situation and reflect on whether the results make sense
- apply knowledge, making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later
- make assumptions and approximations to simplify a situation, realizing the final solution will need to be revised
- identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, and formulas
- analyze quantitative relationships to draw conclusions
- improve the model if it has not served its purpose

5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

In grades K-2 mathematically proficient students will:

- select the available tools (such as pencil and paper, manipulatives, rulers, and available technology) when solving a mathematical problem
- be familiar with tools appropriate for the grade level to make sound decisions about when each of these tools might be helpful
- identify relevant external mathematical resources and use them to pose or solve problems
- use technological tools to explore and deepen their understanding of concepts

In grades 3-5 mathematically proficient students will:

- select the available tools (such as pencil and paper, manipulatives, rulers, calculators, a spreadsheet, and available technology) when solving a mathematical problem
- be familiar with tools appropriate for their grade level to make sound decisions about when each of these tools might be helpful
- identify relevant external mathematical resources and use them to pose or solve problems
- use technological tools to explore and deepen their understanding of concepts
- detect possible errors by strategically using estimation and other mathematical knowledge
- know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data

In grades 6-8 mathematically proficient students will:

- select and use tools appropriate to the task: pencil and paper, protractor, visual and physical fraction models, algebra tiles, geometric models, calculator, spreadsheet, and interactive geometry software.
- use estimation and other mathematical knowledge to confirm the accuracy of their problem solving
- identify relevant external and digital mathematical resources and use them to pose or solve problems
- represent and compare possibilities visually with technology when solving a problem
- explore and deepen their understanding of concepts through the use of technological tools

In grades 9-12 mathematically proficient students will:

- select and accurately use appropriate, available tools (such as pencil and paper, concrete or virtual manipulatives such as geoboards and algebra tiles, graphing and simpler calculators, a spreadsheet, and available technology) when solving a mathematical problem
- identify relevant external and digital mathematical resources and use the resources to pose or solve problems
- detect possible errors by strategically using estimation and other mathematical knowledge
- use technology to visualize the results of varying assumptions, exploring consequences, comparing predictions with data, and deepening understanding of concepts

6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

In grades K-2 mathematically proficient students will:

- give thoughtful explanations to each other
- use clear definitions and reasoning in discussion with others
- state the meaning of symbols they choose, including using the equal sign consistently and appropriately

In grades 3-5 mathematically proficient students will:

- give carefully formulated explanations to each other
- use clear definitions and reasoning in discussion with others
- state the meaning of symbols, including using the equal sign consistently and appropriately
- specify units of measure, and label axes to clarify the correspondence with quantities in a problem
- calculate accurately and efficiently
- express numerical answers with a degree of precision appropriate for the problem context

In grades 6-8 mathematically proficient students will:

- use clear definitions in explanations
- understand and use specific symbols accurately and consistently: equality, inequality, ratios, parenthesis for multiplication and division, absolute value, square root
- specify units of measure, and label axes to clarify the correspondence with quantities in a problem
- calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context

In grades 9-12 mathematically proficient students will:

- communicate precisely to others
- use clear definitions in explanations
- use symbols consistently and appropriately

- specify units of measure, and label axes to clarify the correspondence with quantities in a problem
- calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context
- examine claims and make explicit use of definitions

7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

In all grade levels mathematically proficient students will:

- discern a pattern or structure
- understand complex structures as single objects or as being composed of several objects
- check if the answer is reasonable

8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

In all grade levels mathematically proficient students will:

- identify if calculations or processes are repeated
- use alternative and traditional methods to solve problems
- evaluate the reasonableness of their intermediate results, while attending to the details

K-8 Mathematical Content Standards

Instructional Focus: Kindergarten through Second Grade

Kindergarten	Grade 1	Grade 2
In Kindergarten, instructional time should focus on two critical areas: (1) representing, relating, and operating on whole numbers, initially with sets of objects; (2) describing shapes and space. More learning time in Kindergarten should be devoted to number than to other topics. (1) Students use numbers, including written numerals, to represent quantities and to solve quantitative problems, such as counting objects in a set; counting out a given number of objects; comparing sets or numerals; and modeling simple joining and separating situations with sets of objects, or eventually with equations such as 5 + 2 = 7 and 7 – 2 = 5. (Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged, but it is not required.) Students choose, combine, and apply effective strategies for answering quantitative questions, including quickly recognizing the cardinalities of small sets of objects, counting and producing sets of given sizes, counting the number of objects in combined sets, or counting the number of objects that remain in a set after some are taken away.	In Grade 1, instructional time should focus on four critical areas: (1) developing understanding of addition, subtraction, and strategies for addition and subtraction within 20; (2) developing understanding of whole number relationships and place value, including grouping in tens and ones; (3) developing understanding of linear measurement and measuring lengths as iterating length units; and (4) reasoning about attributes of, and composing and decomposing geometric shapes. (1) Students develop strategies for adding and subtracting whole numbers based on their prior work with small numbers. They use a variety of models, including discrete objects and length-based models (e.g., cubes connected to form lengths), to model add-to, take-from, put-together, take-apart, and compare situations to develop meaning for the operations of addition and subtraction, and to develop strategies to solve arithmetic problems with these operations. Students understand connections between counting and addition and subtraction (e.g., adding two is the same as counting on two). They use properties of addition to add whole numbers and to create and use increasingly sophisticated strategies based on these properties (e.g., "making tens") to solve addition and subtraction problems within 20. By comparing a variety of solution strategies, children build their understanding of the relationship between addition and subtraction.	In Grade 2, instructional time should focus on four critical areas: (1) extending understanding of base-ten notation; (2) building fluency with addition and subtraction; (3) using standard units of measure; and (4) describing and analyzing shapes. (1) Students extend their understanding of the base-ten system. This includes ideas of counting in fives, tens, and multiples of hundreds, tens, and ones, as well as number relationships involving these units, including comparing. Students understand multi-digit numbers (up to 1000) written in base-ten notation, recognizing that the digits in each place represent amounts of thousands, hundreds, tens, or ones (e.g., 853 is 8 hundreds + 5 tens + 3 ones). (2) Students use their understanding of addition to develop fluency with addition and subtraction within 100. They solve problems within 1000 by applying their understanding of models for addition and subtraction, and they develop, discuss, and use efficient, accurate, and generalizable methods to compute sums and differences of whole numbers in base-ten notation, using their understanding of place value and the properties of operations. They select and accurately apply methods that are appropriate for the context and the numbers involved to mentally calculate sums and differences for numbers with only tens or only hundreds.

(2) Students describe their physical world using geometric ideas (e.g., shape, orientation, spatial relations) and vocabulary. They identify, name, and describe basic two-dimensional shapes, such as squares, triangles, circles, rectangles, and hexagons, presented in a variety of ways (e.g., with different sizes and orientations), as well as three-dimensional shapes such as cubes, cones, cylinders, and spheres. They use basic shapes and spatial reasoning to model objects in their environment and to construct more complex shapes.	(2) Students develop, discuss, and use efficient, accurate, and generalizable methods to add within 100 and subtract multiples of 10. They compare whole numbers (at least to 100) to develop understanding of and solve problems involving their relative sizes. They think of whole numbers between 10 and 100 in terms of tens and ones (especially recognizing the numbers 11 to 19 as composed of a ten and some ones). Through activities that build number sense, they understand the order of the counting numbers and their relative magnitudes. (3) Students develop an understanding of the meaning and processes of measurement, including underlying concepts such as iterating (the mental activity of building up the length of an object with equal-sized units) and the transitivity principle for indirect measurement.[1] (4) Students compose and decompose plane or solid figures (e.g., put two triangles together to make a quadrilateral) and build understanding of part-whole relationships as well as the properties of the original and composite shapes. As they combine shapes, they recognize them from different perspectives and orientations, describe their geometric attributes, and determine how they are alike and different, to develop the background for measurement and for initial understandings of properties such as congruence and symmetry [1]Students should apply the principle of transitivity of measurement to make indirect comparisons, but they need not use this technical term.	(3) Students recognize the need for standard units of measure (centimeter and inch) and they use rulers and other measurement tools with the understanding that linear measure involves an iteration of units. They recognize that the smaller the unit, the more iterations they need to cover a given length. (4) Students describe and analyze shapes by examining their sides and angles. Students investigate, describe, and reason about decomposing and combining shapes to make other shapes. Through building, drawing, and analyzing two- and three-dimensional shapes, students develop a foundation for understanding area, volume, congruence, similarity, and symmetry in later grades.

Alaska Mathematics Standards Grades K-2

Grade K	Grade 1	Grade 2
Counting and Cardinality K.CC **Know number names and the count sequence.** K.CC.1. Count to 100 by ones and by tens. K.CC.2. Count forward beginning from a given number within the known sequence. K.CC.3. Write numbers from 0 to 20. Represent a number of objects with a written numeral 0 - 20 (with 0 representing a count of no objects). **Count to tell the number of objects.** K.CC.4. Understand the relationship between numbers and quantities; connect counting to cardinality. a. When counting objects, say the number names in standard order, pairing each object with one and only one number name and each number name with one and only one object. b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. c. Understand that each successive number name refers to a quantity that is one larger.	Counting and Cardinality, 1.CC **Know ordinal names and counting flexibility.** 1.CC.1. Skip count by 2s and 5s. 1.CC.2. Use ordinal numbers correctly when identifying object position (e.g., first, second, third, etc.). 1.CC.3. Order numbers from 1 - 100. Demonstrate ability in counting forward and backward. **Count to tell the number of objects.** 1.CC.4. Count a large quantity of objects by grouping into 10s and counting by 10s and 1s to find the quantity.	

Grade K	Grade 1	Grade 2
K.CC.5. Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array or a circle, or as many as 10 things in a scattered configuration; given a number from 1-20, count out that many objects. **Compare numbers.** K.CC.6. Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group (e.g., by using matching, counting, or estimating strategies). K.CC.7. Compare and order two numbers between 1 and 10 presented as written numerals.	**Compare numbers.** 1.CC.5. Use the symbols for greater than, less than or equal to when comparing two numbers or groups of objects. 1.CC.6. Estimate how many and how much in a given set to 20 and then verify estimate by counting.	
<u>Operations and Algebraic Thinking K.OA</u> **Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.** K.OA.1. Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps) acting out situations, verbal explanations, expressions, or equations.	<u>Operations and Algebraic Thinking 1.OA</u> **Represent and solve problems involving addition and subtraction.** 1.OA.1. Use addition and subtraction strategies to solve word problems (using numbers up to 20), involving situations of adding to, taking from, putting together, taking apart and comparing, with unknowns in all positions, using a number line (e.g., by using objects, drawings and equations). Record and explain using equation symbols and a symbol for the unknown number to represent the problem.	<u>Operations and Algebraic Thinking 2.OA</u> **Represent and solve problems involving addition and subtraction.** 2.OA.1. Use addition and subtraction strategies to estimate, then solve one- and two-step word problems (using numbers up to 100) involving situations of adding to, taking from, putting together, taking apart and comparing, with unknowns in all positions (e.g., by using objects, drawings and equations). Record and explain using equation symbols and a symbol for the unknown number to represent the problem.

Grade K	Grade 1	Grade 2
K.OA.2. Add or subtract whole numbers to 10 (e.g., by using objects or drawings to solve word problems). K.OA.3. Decompose numbers less than or equal to 10 into pairs in more than one way (e.g., by using objects or drawings, and record each decomposition by a drawing or equation). *For example, 5 = 2 + 3 and 5 = 4 + 1.* K.OA.4. For any number from 1- 4, find the number that makes 5 when added to the given number and, for any number from 1- 9, find the number that makes 10 when added to the given number (e.g., by using objects, drawings or 10 frames) and record the answer with a drawing or equation. K.OA.5. Fluently add and subtract numbers up to 5.	1.OA.2. Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20 (e.g., by using objects, drawings and equations). Record and explain using equation symbols and a symbol for the unknown number to represent the problem. **Understand and apply properties of operations and the relationship between addition and subtraction.** 1.OA.3. Apply properties of operations as strategies to add and subtract. (Students need not know the name of the property.) *For example: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known (Commutative property of addition). To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12 (Associative property of addition). Demonstrate that when adding zero to any number, the quantity does not change (Identity property of addition).* 1.OA.4. Understand subtraction as an unknown-addend problem. *For example, subtract 10 - 8 by finding the number that makes 10 when added to 8.*	

Grade K	Grade 1	Grade 2
	Add and subtract using numbers up to 20. 1.OA.5. Relate counting to addition and subtraction (e.g., by counting on 2 to add 2). 1.OA.6. Add and subtract using numbers up to 20, demonstrating fluency for addition and subtraction up to 10. Use strategies such as • counting on • making ten (8 + 6 = 8 + 2 + 4 = 10 + 4 = 14) • decomposing a number leading to a ten (13 - 4 = 13 - 3 - 1 = 10 - 1 = 9) • using the relationship between addition and subtraction, such as fact families, (8 + 4 = 12 and 12 - 8 = 4) • creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).	**Add and subtract using numbers up to 20.** 2.OA.2. Fluently add and subtract using numbers up to 20 using mental strategies. Know from memory all sums of two one-digit numbers.
	Work with addition and subtraction equations. 1.OA.7. Understand the meaning of the equal sign (e.g., read equal sign as "same as") and determine if equations involving addition and subtraction are true or false. *For example, which of the following equations are true and which are false?6 = 6, 7 = 8 - 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2).* 1.OA.8. Determine the unknown whole number in an addition or subtraction equation. *For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 6 + 6 = ?, 5 = ? - 3.*	**Work with equal groups of objects to gain foundations for multiplication.** 2.OA.3. Determine whether a group of objects (up to 20) is odd or even (e.g., by pairing objects and comparing, counting by 2s). Model an even number as two equal groups of objects and then write an equation as a sum of two equal addends. 2.OA.4. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns. Write an equation to express the total as repeated addition (e.g., array of 4 by 5 would be 5 + 5 + 5 + 5 = 20).

Grade K	Grade 1	Grade 2
Identify and continue patterns. K.OA.6. Recognize, identify and continue simple patterns of color, shape, and size. <u>Number and Operations in Base Ten K.NBT</u> **Work with numbers 11-19 to gain foundations for place value.** K.NBT.1. Compose and decompose numbers from 11 to 19 into ten ones and some further ones (e.g., by using objects or drawings) and record each composition and decomposition by a drawing or equation (e.g., 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight or nine ones.	**Identify and continue patterns.** 1.OA.9. Identify, continue and label patterns (e.g., aabb, abab). Create patterns using number, shape, size, rhythm or color. <u>Number and Operations in Base Ten 1.NBT</u> **Extend the counting sequence.** 1.NBT.1. Count to 120. In this range, read, write and order numerals and represent a number of objects with a written numeral. **Understand place value.** 1.NBT.2. Model and identify place value positions of two digit numbers. Include: a. 10 can be thought of as a bundle of ten ones, called a "ten". b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight or nine ones. c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90, refer to one, two, three, four, five, six, seven, eight or nine tens (and 0 ones). 1.NBT.3. Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, <.	**Identify and continue patterns.** 2.OA.5. Identify, continue and label number patterns (e.g., aabb, abab). Describe a rule that determines and continues a sequence or pattern. <u>Number and Operations in Base Ten 2.NBT</u> **Understand place value.** 2.NBT.1. Model and identify place value positions of three digit numbers. Include: a. 100 can be thought of as a bundle of ten tens --called a "hundred". b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones). 2.NBT.2. Count up to 1000, skip-count by 5s, 10s and 100s. 2.NBT.3. Read, write, order up to 1000 using base-ten numerals, number names and expanded form. 2.NBT.4. Compare two three-digit numbers based on the meanings of the hundreds, tens and ones digits, using >, =, < symbols to record the results.

Grade K	Grade 1	Grade 2
	Use place value understanding and properties of operations to add and subtract. 1.NBT.4. Add using numbers up to 100 including adding a two-digit number and a one-digit number and adding a two-digit number and a multiple of 10. Use: • concrete models or drawings and strategies based on place value • properties of operations • and/or relationship between addition and subtraction. Relate the strategy to a written method and explain the reasoning used. Demonstrate in adding two-digit numbers, tens and tens are added, ones and ones are added and sometimes it is necessary to compose a ten from ten ones. 1.NBT.5. Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used. 1.NBT.6. Subtract multiples of 10 up to 100. Use: • concrete models or drawings • strategies based on place value • properties of operations • and/or the relationship between addition and subtraction. Relate the strategy to a written method and explain the reasoning used.	**Use place value understanding and properties of operations to add and subtract.** 2.NBT.5. Fluently add and subtract using numbers up to 100. Use: • strategies based on place value • properties of operations • and/or the relationship between addition and subtraction. 2.NBT.6. Add up to four two-digit numbers using strategies based on place value and properties of operations. 2.NBT.7. Add and subtract using numbers up to 1000. Use: • concrete models or drawings and strategies based on place value • properties of operations • and/or relationship between addition and subtraction. Relate the strategy to a written method and explain the reasoning used. Demonstrate in adding or subtracting three-digit numbers, hundreds and hundreds are added or subtracted, tens and tens are added or subtracted, ones and ones are added or subtracted and sometimes it is necessary to compose a ten from ten ones or a hundred from ten tens. 2.NBT.8. Mentally add 10 or 100 to a given number 100-900 and mentally subtract 10 or 100 from a given number.

Grade K	Grade 1	Grade 2
		2.NBT.9. Explain or illustrate the processes of addition or subtraction and their relationship using place value and the properties of operations.
<u>Measurement and Data K.MD</u> **Describe and compare measurable attributes.** K.MD.1. Describe measurable attributes of objects (e.g., length or weight). Match measuring tools to attribute (e.g., ruler to length). Describe several measurable attributes of a single object. K.MD.2. Make comparisons between two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter.* **Classify objects and count the number of objects in each category.** K.MD.3. Classify objects into given categories (attributes). Count the number of objects in each category (limit category counts to be less than or equal to 10).	<u>Measurement and Data 1.MD</u> **Measure lengths indirectly and by iterating length units.** 1.MD.1. Measure and compare three objects using standard or non-standard units. 1.MD.2. Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps.	<u>Measurement and Data 2.MD</u> **Measure and estimate lengths in standard units.** 2.MD.1. Measure the length of an object by selecting and using standard tools such as rulers, yardsticks, meter sticks, and measuring tapes. 2.MD.2. Measure the length of an object twice using different length units for the two measurements. Describe how the two measurements relate to the size of the unit chosen. 2.MD.3. Estimate, measure and draw lengths using whole units of inches, feet, yards, centimeters and meters. 2.MD.4. Measure to compare lengths of two objects, expressing the difference in terms of a standard length unit. **Relate addition and subtraction to length.** 2.MD.5. Solve addition and subtraction word problems using numbers up to 100 involving length that are given in the same units (e.g., by using drawings of rulers). Write an equation with a symbol for the unknown to represent the problem. 2.MD.6. Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1,2, …, and represent whole-number sums and differences within 100 on a number line diagram.

Grade K	Grade 1	Grade 2
Work with time and money. K.MD.4. Name in sequence the days of the week. K.MD.5. Tell time to the hour using both analog and digital clocks. K.MD.6. Identify coins by name.	**Work with time and money.** 1.MD.3. Tell and write time in half hours using both analog and digital clocks. 1.MD.4. Read a calendar distinguishing yesterday, today and tomorrow. Read and write a date. 1.MD.5. Recognize and read money symbols including $ and ¢. 1.MD.6. Identify values of coins (e.g., nickel = 5 cents, quarter = 25 cents). Identify equivalent values of coins up to $1 (e.g., 5 pennies = 1 nickel, 5 nickels = 1 quarter). **Represent and interpret data.** 1.MD.7. Organize, represent and interpret data with up to three categories. Ask and answer comparison and quantity questions about the data.	**Work with time and money.** 2.MD.7. Tell and write time to the nearest five minutes using a.m. and p.m. from analog and digital clocks. 2.MD.8. Solve word problems involving dollar bills and coins using the $ and ¢ symbols appropriately. **Represent and interpret data.** 2.MD.9. Collect, record, interpret, represent, and describe data in a table, graph or line plot. 2.MD.10. Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart and compare problems using information presented in a bar graph.

<table>
<tr><th>Grade K</th><th>Grade 1</th><th>Grade 2</th></tr>
<tr>
<td><u>Geometry K.G</u>
(shapes include squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres)
Identify and describe shapes.
K.G.1. Describe objects in the environment using names of shapes and describe their relative positions (e.g., above, below, beside, in front of, behind, next to).

K.G.2. Name shapes regardless of their orientation or overall size.

K.G.3. Identify shapes as two-dimensional (flat) or three-dimensional (solid).

Analyze, compare, create, and compose shapes.
K.G.4. Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices), and other attributes (e.g., having sides of equal lengths).

K.G.5. Build shapes (e.g., using sticks and clay) and draw shapes.

K.G.6. Put together two-dimensional shapes to form larger shapes (e.g., join two triangles with full sides touching to make a rectangle).</td>
<td><u>Geometry 1.G</u>
Reason with shapes and their attributes.
1.G.1. Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes. Identify shapes that have non-defining attributes (e.g., color, orientation, overall size). Build and draw shapes given specified attributes.

1.G.2. Compose (put together) two-dimensional or three-dimensional shapes to create a larger, composite shape, and compose new shapes from the composite shape.

1.G.3. Partition circles and rectangles into two and four equal shares. Describe the shares using the words, halves, fourths, and quarters and phrases half of, fourth of and quarter of. Describe the whole as two of or four of the shares. Understand for these examples that decomposing (break apart) into more equal shares creates smaller shares.</td>
<td><u>Geometry 2.G</u>
Reason with shapes and their attributes.
2.G.1. Identify and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces compared visually, not by measuring. Identify triangles, quadrilaterals, pentagons, hexagons and cubes.

2.G.2. Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.

2.G.3. Partition circles and rectangles into shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.</td>
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Instructional Focus: Third Grade through Fifth Grade

Grade 3	Grade 4	Grade 5
In Grade 3, instructional time should focus on four critical areas: (1) developing understanding of multiplication and division and strategies for multiplication and division within 100; (2) developing understanding of fractions, especially unit fractions (fractions with numerator 1); (3) developing understanding of the structure of rectangular arrays and of area; and (4) describing and analyzing two-dimensional shapes. (1) Students develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models; multiplication is finding an unknown product, and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size. Students use properties of operations to calculate products of whole numbers, using increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors. By comparing a variety of solution strategies, students learn the relationship between multiplication and division. (2) Students develop an understanding of fractions, beginning with unit fractions. Students view fractions in general as being built out of unit fractions, and they use fractions along with visual fraction models to represent parts of a whole. Students understand that the size of a fractional part is relative to the size of the whole. For example, 1/2 of the paint in a small bucket could be less paint than 1/3 of the paint in a larger bucket, but 1/3 of a ribbon is longer than 1/5 of the same ribbon because when the ribbon is	In Grade 4, instructional time should focus on three critical areas: (1) developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends; (2) developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers; (3) understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry. (1) Students generalize their understanding of place value to 1,000,000, understanding the relative sizes of numbers in each place. They apply their understanding of models for multiplication (equal-sized groups, arrays, area models), place value, and properties of operations, in particular the distributive property, as they develop, discuss, and use efficient, accurate, and generalizable methods to compute products of multi-digit whole numbers. Depending on the numbers and the context, they select and accurately apply appropriate methods to estimate or mentally calculate products. They develop fluency with efficient procedures for multiplying whole numbers; understand and explain why the procedures work based on place value and properties of operations; and use them to solve problems. Students apply their understanding of models for division, place value, properties of operations, and the relationship of division to multiplication as they develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multi-digit dividends. They select and accurately apply appropriate	In Grade 5, instructional time should focus on three critical areas: (1) developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions); (2) extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations; and (3) developing understanding of volume. (1) Students apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators. They develop fluency in calculating sums and differences of fractions, and make reasonable estimates of them. Students also use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (Note: this is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.) (2) Students develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations. They finalize fluency with multi-digit addition, subtraction, multiplication, and division. They apply their understandings of models for decimals, decimal notation, and properties of operations to add and subtract decimals to hundredths. They develop fluency in these computations, and make

divided into 3 equal parts, the parts are longer than when the ribbon is divided into 5 equal parts. Students are able to use fractions to represent numbers equal to, less than, and greater than one. They solve problems that involve comparing fractions by using visual fraction models and strategies based on noticing equal numerators or denominators. (3) Students recognize area as an attribute of two-dimensional regions. They measure the area of a shape by finding the total number of same-size units of area required to cover the shape without gaps or overlaps, a square with sides of unit length being the standard unit for measuring area. Students understand that rectangular arrays can be decomposed into identical rows or into identical columns. By decomposing rectangles into rectangular arrays of squares, students connect area to multiplication, and justify using multiplication to determine the area of a rectangle. (4) Students describe, analyze, and compare properties of two-dimensional shapes. They compare and classify shapes by their sides and angles, and connect these with definitions of shapes. Students also relate their fraction work to geometry by expressing the area of part of a shape as a unit fraction of the whole.	methods to estimate and mentally calculate quotients, and interpret remainders based upon the context. (2) Students develop understanding of fraction equivalence and operations with fractions. They recognize that two different fractions can be equal (e.g., 15/9 = 5/3), and they develop methods for generating and recognizing equivalent fractions. Students extend previous understandings about how fractions are built from unit fractions, composing fractions from unit fractions, decomposing fractions into unit fractions, and using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number. (3) Students describe, analyze, compare, and classify two-dimensional shapes. Through building, drawing, and analyzing two-dimensional shapes, students deepen their understanding of properties of two-dimensional objects and the use of them to solve problems involving symmetry.	reasonable estimates of their results. Students use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain why the procedures for multiplying and dividing finite decimals make sense. They compute products and quotients of decimals to hundredths efficiently and accurately. (3) Students recognize volume as an attribute of three-dimensional space. They understand that volume can be measured by finding the total number of same-size units of volume required to fill the space without gaps or overlaps. They understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. They select appropriate units, strategies, and tools for solving problems that involve estimating and measuring volume. They decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of arrays of cubes. They measure necessary attributes of shapes in order to determine volumes to solve real-world and mathematical problems.

Alaska Mathematics Standards Grades 3-5

Grade 3	Grade 4	Grade 5
Operations and Algebraic Thinking 3.OA **Represent and solve problems involving multiplication and division.** 3.OA.1. Interpret products of whole numbers (e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each). *For example, show objects in rectangular arrays or describe a context in which a total number of objects can be expressed as 5 × 7.*	Operations and Algebraic Thinking 4.OA **Use the four operations with whole numbers to solve problems.** 4.OA.1. Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 groups of 7 and 7 groups of 5 (Commutative property). Represent verbal statements of multiplicative comparisons as multiplication equations.	Operations and Algebraic Thinking 5.OA **Write and interpret numerical expressions.** 5.OA.1. Use parentheses to construct numerical expressions, and evaluate numerical expressions with these symbols.
3.OA.2. Interpret whole-number quotients of whole numbers (e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each). *For example, deconstruct rectangular arrays or describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.*	4.OA.2. Multiply or divide to solve word problems involving multiplicative comparison (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem or missing numbers in an array). Distinguish multiplicative comparison from additive comparison.	5.OA.2. Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 x (8 + 7). Recognizing that 3 x (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.*
3.OA.3. Use multiplication and division numbers up to 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem).	4.OA.3. Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.	

Grade 3	Grade 4	Grade 5
3.OA.4. Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 x ? = 48, 5 = ? ÷ 3, 6 x 6 = ?* **Understand properties of multiplication and the relationship between multiplication and division.** 3.OA.5. Make, test, support, draw conclusions and justify conjectures about properties of operations as strategies to multiply and divide. (Students need not use formal terms for these properties.) • Commutative property of multiplication: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. • Associative property of multiplication: 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. • Distributive property: Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. • Inverse property (relationship) of multiplication and division. 3.OA.6. Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.*	**Gain familiarity with factors and multiples.** 4.OA.4. • Find all factor pairs for a whole number in the range 1–100. • Explain the correlation/differences between multiples and factors. • Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. • Determine whether a given whole number in the range 1–100 is prime or composite.	

Grade 3	Grade 4	Grade 5
Multiply and divide up to 100. 3.OA.7. Fluently multiply and divide numbers up to 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 ×5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers. **Solve problems involving the four operations, and identify and explain patterns in arithmetic.** 3.OA.8. Solve and create two-step word problems using any of the four operations. Represent these problems using equations with a symbol (box, circle, question mark) standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. 3.OA.9. Identify arithmetic patterns (including patterns in the addition table or multiplication table) and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*	**Generate and analyze patterns.** 4.OA.5. Generate a number, shape pattern, table, t-chart, or input/output function that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. Be able to express the pattern in algebraic terms. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.* 4.OA.6. Extend patterns that use addition, subtraction, multiplication, division or symbols, up to 10 terms, represented by models (function machines), tables, sequences, or in problem situations. (L)	**Analyze patterns and relationships.** 5.OA.3. Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.*

Grade 3	Grade 4	Grade 5
Number and Operations in Base Ten 3.NBT **Use place value understanding and properties of operations to perform multi-digit arithmetic.** 3.NBT.1. Use place value understanding to round whole numbers to the nearest 10 or 100. 3.NBT.2. Use strategies and/or algorithms to fluently add and subtract with numbers up to 1000, demonstrating understanding of place value, properties of operations, and/or the relationship between addition and subtraction. 3.NBT.3. Multiply one-digit whole numbers by multiples of 10 in the range 10-90 (e.g., 9 x 80, 10 x 60) using strategies based on place value and properties of operations.	Number and Operations in Base Ten 4.NBT **Generalize place value understanding for multi-digit whole numbers.** 4.NBT.1. Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.* 4.NBT.2. Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on the value of the digits in each place, using >, =, and < symbols to record the results of comparisons. 4.NBT.3. Use place value understanding to round multi-digit whole numbers to any place using a variety of estimation methods; be able to describe, compare, and contrast solutions. **Use place value understanding and properties of operations to perform multi-digit arithmetic.** 4.NBT.4. Fluently add and subtract multi-digit whole numbers using any algorithm. Verify the reasonableness of the results.	Number and Operations in Base Ten 5.NBT **Understand the place value system.** 5.NBT.1. Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left. 5.NBT.2. Explain and extend the patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain and extend the patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10. 5.NBT.3. Read, write, and compare decimals to thousandths. a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form [e.g., 347.392 = 3 x 100 + 4 x 10 + 7 x 1 + 3 (1/10) + 9 (1/100) + 2 (1/1000)]. b. Compare two decimals to thousandths place based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.

Grade 3	Grade 4	Grade 5
	4.NBT.5. Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. 4.NBT.6. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	5.NBT.4. Use place values understanding to round decimals to any place. **Perform operations with multi-digit whole numbers and with decimals to hundredths.** 5.NBT.5. Fluently multiply multi-digit whole numbers using a standard algorithm. 5.NBT.6. Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, number lines, real life situations, and/or area models. 5.NBT.7. Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between the operations. Relate the strategy to a written method and explain their reasoning in getting their answers.

Grade 3	Grade 4	Grade 5
<u>Number and Operations—Fractions 3.NF</u> (limited in this grade to fractions with denominators 2, 3, 4, 6, and 8) **Develop understanding of fractions as numbers.** 3.NF.1. Understand a fraction 1/*b* (e.g., 1/4) as the quantity formed by 1 part when a whole is partitioned into *b* (e.g., 4) equal parts; understand a fraction *a*/*b* (e.g., 2/4) as the quantity formed by *a* (e.g., 2) parts of size 1/*b*. (e.g., 1/4) 3.NF.2. Understand a fraction as a number on the number line; represent fractions on a number line diagram. a. Represent a fraction 1/*b* (e.g., 1/4) on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* (e.g., 4) equal parts. Recognize that each part has size 1/*b* (e.g., 1/4) and that the endpoint of the part based at 0 locates the number 1/*b* (e.g., 1/4) on the number line. b. Represent a fraction *a*/*b* (e.g., 2/8) on a number line diagram or ruler by marking off *a* lengths 1/*b* (e.g., 1/8) from 0. Recognize that the resulting interval has size *a*/*b* (e.g., 2/8) and that its endpoint locates the number *a*/*b* (e.g., 2/8) on the number line.	<u>Number and Operations—Fractions 4.NF</u> (limited in this grade to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100) **Extend understanding of fraction equivalence and ordering.** 4.NF.1. Explain why a fraction *a*/*b* is equivalent to a fraction ($n \times a$)/($n \times b$) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. 4.NF.2. Compare two fractions with different numerators and different denominators (e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2). Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions (e.g., by using a visual fraction model).	<u>Number and Operations—Fractions 5.NF</u> **Use equivalent fractions as a strategy to add and subtract fractions.** 5.NF.1. Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)* 5.NF.2. Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators (e.g., by using visual fraction models or equations to represent the problem). Use benchmark fractions and number sense of fractions to estimate mentally and check the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.*

Grade 3	Grade 4	Grade 5
3.NF.3. Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. a. Understand two fractions as equivalent if they are the same size (modeled) or the same point on a number line. b. Recognize and generate simple equivalent fractions (e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent (e.g., by using a visual fraction model). c. Express and model whole numbers as fractions, and recognize and construct fractions that are equivalent to whole numbers. *For example: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.* d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions (e.g., by using a visual fraction model).	**Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.** 4.NF.3. Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$. a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions (e.g., by using a visual fraction model). *For example: 3/8 = 1/8 + 1/8 + 1/8 ;3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.* c. Add and subtract mixed numbers with like denominators (e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction). d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators (e.g., by using visual fraction models and equations to represent the problem).	**Apply and extend previous understandings of multiplication and division to multiply and divide fractions.** 5.NF.3. Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers (e.g., by using visual fraction models or equations to represent the problem). *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?* 5.NF.4. Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. a. Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)* b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

Grade 3	Grade 4	Grade 5
	4.NF.4. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. a. Understand a fraction *a*/*b* as a multiple of 1/*b*. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).* b. Understand a multiple of *a*/*b* as a multiple of 1/*b*, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)* c. Solve word problems involving multiplication of a fraction by a whole number (e.g., by using visual fraction models and equations to represent the problem). Check for the reasonableness of the answer. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*	5.NF.5. Interpret multiplication as scaling (resizing), by: a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence *a*/*b* = (*n* × *a*)/(*n* × *b*) to the effect of multiplying *a*/*b* by 1. (Division of a fraction by a fraction is not a requirement at this grade.) 5.NF.6. Solve real-world problems involving multiplication of fractions and mixed numbers (e.g., by using visual fraction models or equations to represent the problem).

Grade 3	Grade 4	Grade 5
	Understand decimal notation for fractions, and compare decimal fractions. 4.NF.5. Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.* 4.NF.6. Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.* 4.NF.7. Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions (e.g., by using a visual model).	5.NF.7. Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.* b. Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.* c. Solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions (e.g., by using visual fraction models and equations to represent the problem). *For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?*

Grade 3	Grade 4	Grade 5
<u>Measurement and Data 3.MD</u> **Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.** 3.MD.1. Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes or hours (e.g., by representing the problem on a number line diagram or clock). 3.MD.2. Estimate and measure liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). (Excludes compound units such as cm^3 and finding the geometric volume of a container.) Add, subtract, multiply, or divide to solve and create one-step word problems involving masses or volumes that are given in the same units (e.g., by using drawings, such as a beaker with a measurement scale, to represent the problem). (Excludes multiplicative comparison problems [problems involving notions of "times as much."])	<u>Measurement and Data 4.MD</u> **Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit, and involving time.** 4.MD.1. Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4-ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36).* 4.MD.2. Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.	<u>Measurement and Data 5.MD</u> **Convert like measurement units within a given measurement system and solve problems involving time.** 5.MD.1. Identify, estimate measure, and convert equivalent measures within systems English length (inches, feet, yards, miles) weight (ounces, pounds, tons) volume (fluid ounces, cups, pints, quarts, gallons) temperature (Fahrenheit) Metric length (millimeters, centimeters, meters, kilometers) volume (milliliters, liters), temperature (Celsius), (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real-world problems using appropriate tools. 5.MD.2. Solve real-world problems involving elapsed time between world time zones. (L)

Grade 3	Grade 4	Grade 5
3.MD.3. Select an appropriate unit of English, metric, or non-standard measurement to estimate the length, time, weight, or temperature (L)	4.MD.3. Apply the area and perimeter formulas for rectangles in real-world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.* 4.MD.4. Solve real-world problems involving elapsed time between U.S. time zones (including Alaska Standard time). (L)	
Represent and interpret data. 3.MD.4. Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.*	**Represent and interpret data.** 4.MD.5. Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.*	**Represent and interpret data.** 5.MD.3. Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.*
3.MD.5. Measure and record lengths using rulers marked with halves and fourths of an inch. Make a line plot with the data, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters. 3.MD.6. Explain the classification of data from real-world problems shown in graphical representations. Use the terms minimum and maximum. (L)	4.MD.6. Explain the classification of data from real-world problems shown in graphical representations including the use of terms range and mode with a given set of data. (L)	5.MD.4. Explain the classification of data from real-world problems shown in graphical representations including the use of terms mean and median with a given set of data. (L)

Grade 3	Grade 4	Grade 5
Geometric measurement: understand concepts of area and relate area to multiplication and to addition. 3.MD.7. Recognize area as an attribute of plane figures and understand concepts of area measurement. a. A square with side length 1 unit is said to have "one square unit" and can be used to measure area. b. Demonstrate that a plane figure which can be covered without gaps or overlaps by *n* (e.g., 6) unit squares is said to have an area of *n* (e.g., 6) square units. 3.MD.8. Measure areas by tiling with unit squares (square centimeters, square meters, square inches, square feet, and improvised units).	**Geometric measurement: understand concepts of angle and measure angles.** 4.MD.7. Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand the following concepts of angle measurement: a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles. b. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees. 4.MD.8. Measure and draw angles in whole-number degrees using a protractor. Estimate and sketch angles of specified measure. 4.MD.9. Recognize angle measure as additive. When an angle is divided into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems (e.g., by using an equation with a symbol for the unknown angle measure).	**Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition**. 5.MD.5. Recognize volume as an attribute of solid figures and understand concepts of volume measurement. a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume. b. A solid figure that can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units. 5.MD.6. Estimate and measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and non-standard units.

Grade 3	Grade 4	Grade 5
3.MD.9. Relate area to the operations of multiplication and addition. a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. *For example, after tiling rectangles, develop a rule for finding the area of any rectangle.* b. Multiply side lengths to find areas of rectangles with whole number side lengths in the context of solving real-world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning. c. Use area models (rectangular arrays) to represent the distributive property in mathematical reasoning. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real-world problems. *For example, the area of a 7 by 8 rectangle can be determined by decomposing it into a 7 by 3 rectangle and a 7 by 5 rectangle.*		5.MD.7. Relate volume to the operations of multiplication and addition and solve real-world and mathematical problems involving volume. a. Estimate and find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Demonstrate the associative property of multiplication by using the product of three whole numbers to find volumes (length x width x height). b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole number edge lengths in the context of solving real-world and mathematical problems. c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real-world problems.

Grade 3	Grade 4	Grade 5
Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures. 3.MD.10. Solve real-world and mathematical problems involving perimeters of polygons, including: • finding the perimeter given the side lengths, • finding an unknown side length, • exhibiting rectangles with the same perimeter and different areas, • exhibiting rectangles with the same area and different perimeters.	.	
<u>Geometry 3.G</u> **Reason with shapes and their attributes**. 3.G.1. Categorize shapes by different attribute classifications and recognize that shared attributes can define a larger category. Generalize to create examples or non-examples. 3.G.2. Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.*	<u>Geometry 4.G</u> **Draw and identify lines and angles, and classify shapes by properties of their lines and angles.** 4.G.1. Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular, parallel, and intersecting line segments. Identify these in two-dimensional (plane) figures. 4.G.2. Classify two-dimensional (plane) figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles. 4.G.3. Recognize a line of symmetry for a two-dimensional (plane) figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.	<u>Geometry 5.G</u> **Graph points on the coordinate plane to solve real-world and mathematical problems.** 5.G.1. Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate).

Grade 3	Grade 4	Grade 5
		5.G.2. Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation. **Classify two-dimensional (plane) figures into categories based on their properties.** 5.G.3. Understand that attributes belonging to a category of two-dimensional (plane) figures also belong to all subcategories of that category. *For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.* 5.G.4. Classify two-dimensional (plane) figures in a hierarchy based on attributes and properties.

Instructional Focus: Sixth Grade through Eighth Grade

Grade 6	Grade 7	Grade 8
In Grade 6, instructional time should focus on four critical areas: (1) connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems; (2) completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers; (3) writing, interpreting, and using expressions and equations; and (4) developing understanding of statistical thinking. (1) Students use reasoning about multiplication and division to solve ratio and rate problems about quantities. By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative size of quantities, students connect their understanding of multiplication and division with ratios and rates. Thus students expand the scope of problems for which they can use multiplication and division to solve problems, and they connect ratios and fractions. Students solve a wide variety of problems involving ratios and rates. (2) Students use the meaning of fractions, the meanings of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense. Students use these operations to solve problems. Students extend their previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, and in particular negative integers. They reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plane.	In Grade 7, instructional time should focus on four critical areas: (1) developing understanding of and applying proportional relationships; (2) developing understanding of operations with rational numbers and working with expressions and linear equations; (3) solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume; and (4) drawing inferences about populations based on samples. (1) Students extend their understanding of ratios and develop understanding of proportionality to solve single- and multi-step problems. Students use their understanding of ratios and proportionality to solve a wide variety of percent problems, including those involving discounts, interest, taxes, tips, and percent increase or decrease. Students solve problems about scale drawings by relating corresponding lengths between the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. Students graph proportional relationships and understand the unit rate informally as a measure of the steepness of the related line, called the slope. They distinguish proportional relationships from other relationships. (2) Students develop a unified understanding of number, recognizing fractions, decimals (that have a finite or a repeating decimal representation), and percents as different representations of rational numbers. Students extend addition, subtraction, multiplication, and division to all rational numbers, maintaining the properties of operations and the relationships between addition and subtraction,	In Grade 8, instructional time should focus on three critical areas: (1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; (2) grasping the concept of a function and using functions to describe quantitative relationships; (3) analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem. (1) Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions ($y/x = m$ or $y = mx$) as special linear equations ($y = mx + b$), understanding that the constant of proportionality (m) is the slope, and the graphs are lines through the origin. They understand that the slope (m) of a line is a constant rate of change, so that if the input or x-coordinate changes by an amount A, the output or y-coordinate changes by the amount $m \cdot A$. Students also use a linear equation to describe the association between two quantities in bivariate data (such as arm span vs. height for students in a classroom). At this grade, fitting the model and assessing its fit to the data are done informally. Interpreting the model in the context of the data requires students to express a relationship between the two quantities in question and to interpret components of the relationship (such as slope and y-intercept) in terms of the situation. Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of

(3) Students understand the use of variables in mathematical expressions. They write expressions and equations that correspond to given situations, evaluate expressions, and use expressions and formulas to solve problems. Students understand that expressions in different forms can be equivalent, and they use the properties of operations to rewrite expressions in equivalent forms. Students know that the solutions of an equation are the values of the variables that make the equation true. Students use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. Students construct and analyze tables, such as tables of quantities that are in equivalent ratios, and they use equations (such as $3x = y$) to describe relationships between quantities. (4) Building on and reinforcing their understanding of number, students begin to develop their ability to think statistically. Students recognize that a data distribution may not have a definite center and that different ways to measure center yield different values. The median measures center in the sense that it is roughly the middle value. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. Students recognize that a measure of variability (interquartile range or mean absolute deviation) can also be useful for summarizing data because two very different sets of data can have the same mean and median yet be distinguished by their variability. Students learn to describe and summarize numerical data sets, identifying clusters, peaks, gaps, and symmetry, considering the context in which the data were collected.	and multiplication and division. By applying these properties, and by viewing negative numbers in terms of everyday contexts (e.g., amounts owed or temperatures below zero), students explain and interpret the rules for adding, subtracting, multiplying, and dividing with negative numbers. They use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems. (3) Students continue their work with area from Grade 6, solving problems involving the area and circumference of a circle and surface area of three-dimensional objects. In preparation for work on congruence and similarity in Grade 8 they reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions, and they gain familiarity with the relationships between angles formed by intersecting lines. Students work with three-dimensional figures, relating them to two-dimensional figures by examining cross-sections. They solve real-world and mathematical problems involving area, surface area, and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes and right prisms. (4) Students build on their previous work with single data distributions to compare two data distributions and address questions about differences between populations. They begin informal work with random sampling to generate data sets and learn about the importance of representative samples for drawing inferences.	logical equivalence, they maintain the solutions of the original equation. Students solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane; these intersect, are parallel, or are the same line. Students use linear equations, systems of linear equations, linear functions, and their understanding of slope of a line to analyze situations and solve problems. (2) Students grasp the concept of a function as a rule that assigns to each input exactly one output. They understand that functions describe situations where one quantity determines another. They can translate among representations and partial representations of functions (noting that tabular and graphical representations may be partial representations), and they describe how aspects of the function are reflected in the different representations. (3) Students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students show that the sum of the angles in a triangle is the angle formed by a straight line, and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. Students understand the statement of the Pythagorean Theorem and its converse, and can explain why the Pythagorean Theorem holds, for example, by decomposing a square in two different ways. They apply the Pythagorean Theorem to find distances between points on the coordinate plane, to find lengths, and to analyze polygons. Students complete their work on volume by solving problems involving cones, cylinders, and spheres.

Students in Grade 6 also build on their work with area in elementary school by reasoning about relationships among shapes to determine area, surface area, and volume. They find areas of right triangles, other triangles, and special quadrilaterals by decomposing these shapes, rearranging or removing pieces, and relating the shapes to rectangles. Using these methods, students discuss, develop, and justify formulas for areas of triangles and parallelograms. Students find areas of polygons and surface areas of prisms and pyramids by decomposing them into pieces whose area they can determine. They reason about right rectangular prisms with fractional side lengths to extend formulas for the volume of a right rectangular prism to fractional side lengths. They prepare for work on scale drawings and constructions in Grade 7 by drawing polygons in the coordinate plane.		

Grade 6	Grade 7	Grade 8
Ratios and Proportional Relationships 6.RP **Understand ratio concepts and use ratio reasoning to solve problems.** 6.RP.1. Write and describe the relationship in real life context between two quantities using ratio language. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."* 6.RP.2. Understand the concept of a unit rate (a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship) and apply it to solve real-world problems (e.g., unit pricing, constant speed). *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."*	Ratios and Proportional Relationships 7.RP **Analyze proportional relationships and use them to solve real-world and mathematical problems.** 7.RP.1. Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction 1/2/1/4 miles per hour, equivalently 2 miles per hour or apply a given scale factor to find missing dimensions of similar figures.* 7.RP.2. Recognize and represent proportional relationships between quantities. Make basic inferences or logical predictions from proportional relationships. a. Decide whether two quantities are in a proportional relationship (e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin). b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships in real-world situations.	

Grade 6	Grade 7	Grade 8
6.RP.3. Use ratio and rate reasoning to solve real-world and mathematical problems (e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations). a. Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios, and understand equivalencies. b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?* c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. d. Use ratio reasoning to convert measurement units between given measurement systems (e.g., convert kilometers to miles); manipulate and transform units appropriately when multiplying or dividing quantities.	c. Represent proportional relationships by equations and multiple representations such as tables, graphs, diagrams, sequences, and contextual situations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as* $t = pn$. d. Understand the concept of unit rate and show it on a coordinate plane. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and $(1, r)$ where r is the unit rate. 7.RP.3. Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*	

Grade 6	Grade 7	Grade 8
The Number System 6.NS **Apply and extend previous understandings of multiplication and division to divide fractions by fractions.** 6.NS.1. Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions (e.g., by using visual fraction models and equations to represent the problem). *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?*	The Number System 7.NS **Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.** 7.NS.1. Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. a. Show that a number and its opposite have a sum of 0 (additive inverses). Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.* b. Understand addition of rational numbers ($p + q$ as the number located a distance $\|q\|$ from p, in the positive or negative direction depending on whether q is positive or negative). Interpret sums of rational numbers by describing real-world contexts. c. Understand subtraction of rational numbers as adding the additive inverse, $p – q = p + (–q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. d. Apply properties of operations as strategies to add and subtract rational numbers.	The Number System 8.NS **Know that there are numbers that are not rational, and approximate them by rational numbers.** 8.NS.1. Classify real numbers as either rational (the ratio of two integers, a terminating decimal number, or a repeating decimal number) or irrational. 8.NS.2. Order real numbers, using approximations of irrational numbers, locating them on a number line. *For example, show that √2 is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.* 8.NS.3. Identify or write the prime factorization of a number using exponents. (L)

Grade 6	Grade 7	Grade 8
Compute fluently with multi-digit numbers and find common factors and multiples. 6.NS.2. Fluently multiply and divide multi-digit whole numbers using the standard algorithm. Express the remainder as a whole number, decimal, or simplified fraction; explain or justify your choice based on the context of the problem. 6.NS.3. Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation. Express the remainder as a terminating decimal, or a repeating decimal, or rounded to a designated place value. 6.NS.4. Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4 (9 + 2).*	7.NS.2. Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers and use equivalent representations. a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(–1)(–1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $–(p/q) = (–p)/q = p/(–q)$. Interpret quotients of rational numbers by describing real-world contexts. c. Apply and name properties of operations used as strategies to multiply and divide rational numbers. d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats. e. Convert between equivalent fractions, decimals, or percents.	

Grade 6	Grade 7	Grade 8
Apply and extend previous understandings of numbers to the system of rational numbers. 6.NS.5 Understand that positive and negative numbers describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explain the meaning of 0 in each situation. 6.NS.6. Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; Recognize that the opposite of the opposite of a number is the number itself [e.g., –(–3) = 3] and that 0 is its own opposite. b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.	7.NS.3. Solve real-world and mathematical problems involving the four operations with rational numbers. (Computations with rational numbers extend the rules for manipulating fractions to complex fractions.) *For example, use models, explanations, number lines, real life situations, describing or illustrating the effect of arithmetic operations on rational numbers (fractions, decimals).*	

Grade 6	Grade 7	Grade 8
6.NS.7. Understand ordering and absolute value of rational numbers. a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right.* b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write –3 ^{o}C > –7 ^{o}C to express the fact that –3 ^{o}C is warmer than –7 ^{o}C.* c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of –30 dollars, write |–30| = 30 to describe the size of the debt in dollars.* d. Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than -30 dollars represents a debt greater than 30 dollars.* 6.NS.8. Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.		

Grade 6	Grade 7	Grade 8
Expressions and Equations 6.EE **Apply and extend previous understandings of arithmetic to algebraic expressions.** 6.EE.1. Write and evaluate numerical expressions involving whole-number exponents. *For example, multiply by powers of 10 and products of numbers using exponents.* ($7^3 = 7•7•7$) 6.EE.2. Write, read, and evaluate expressions in which letters stand for numbers. a. Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5 – y.* b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.* c. Evaluate expressions and formulas. Include formulas used in real-world problems. Perform arithmetic operations, including those involving whole number exponents, in the conventional order with or without parentheses. (Order of Operations) 6.EE.3. Apply the properties of operations to generate equivalent expressions. Model (e.g., manipulatives, graph paper) and apply the distributive, commutative, identity, and inverse properties with integers and variables by writing equivalent expressions. *For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x.*	Expressions and Equations 7.EE **Use properties of operations to generate equivalent expressions.** 7.EE.1. Apply properties of operations as strategies to add, subtract, factor, expand and simplify linear expressions with rational coefficients. 7.EE.2. Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."* **Solve real-life and mathematical problems using numerical and algebraic expressions and equations.** 7.EE.3. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*	Expressions and Equations 8.EE **Work with radicals and integer exponents.** 8.EE.1. Apply the properties (product, quotient, power, zero, negative exponents, and rational exponents) of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$. 8.EE.2. Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational. 8.EE.3. Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as* 3×10^8 *and the population of the world as* 7×10^9, *and determine that the world population is more than 20 times larger.* 8.EE.4. Perform operations with numbers expressed in scientific notation, including problems where both standard notation and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities. Interpret scientific notation that has been generated by technology.

Grade 6	Grade 7	Grade 8
6.EE.4. Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions $y + y + y$ and $3y$ are equivalent because they name the same number regardless of which number y stands for.* **Reason about and solve one-variable equations and inequalities.** 6.EE.5. Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true. *For example: does 5 make $3x > 7$ true?* 6.EE.6. Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set. 6.EE.7. Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.	7.EE.4. Use variables to represent quantities in a real-world or mathematical problem, and construct multi-step equations and inequalities to solve problems by reasoning about the quantities. a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?* b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.*	**Understand the connections between proportional relationships, lines, and linear equations.** 8.EE.5. Graph linear equations such as $y = mx + b$, interpreting m as the slope or rate of change of the graph and b as the y-intercept or starting value. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.* 8.EE.6. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b. **Analyze and solve linear equations and pairs of simultaneous linear equations.** 8.EE.7. Solve linear equations in one variable. a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). b. Solve linear equations with rational coefficients, including equations whose solutions require expanding expressions using the distributive property and combining like terms.

Grade 6	Grade 7	Grade 8
6.EE.8. Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams. **Represent and analyze quantitative relationships between dependent and independent variables.** 6.EE.9. Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation* $d = 65t$ *to represent the relationship between distance and time.*		8.EE.8. Analyze and solve systems of linear equations. a. Show that the solution to a system of two linear equations in two variables is the intersection of the graphs of those equations because points of intersection satisfy both equations simultaneously. b. Solve systems of two linear equations in two variables and estimate solutions by graphing the equations. Simple cases may be done by inspection. *For example,* $3x + 2y = 5$ *and* $3x + 2y = 6$ *have no solution because* $3x + 2y$ *cannot simultaneously be 5 and 6.* c. Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

Grade 6	Grade 7	Grade 8
Geometry 6.G **Solve real-world and mathematical problems involving area, surface area, and volume.** 6.G.1. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing or decomposing into other polygons (e.g., rectangles and triangles). Apply these techniques in the context of solving real-world and mathematical problems. 6.G.2. Apply the standard formulas to find volumes of prisms. Use the attributes and properties (including shapes of bases) of prisms to identify, compare or describe three-dimensional figures including prisms and cylinders. 6.G.3. Draw polygons in the coordinate plane given coordinates for the vertices; determine the length of a side joining the coordinates of vertices with the same first or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems. 6.G.4. Represent three-dimensional figures (e.g., prisms) using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems. 6.G.5. Identify, compare or describe attributes and properties of circles (radius, and diameter). (L)	Geometry 7.G **Draw, construct, and describe geometrical figures and describe the relationships between them.** 7.G.1. Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale. 7.G.2. Draw (freehand, with ruler and protractor, and with technology) geometric shapes including polygons and circles with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle. 7.G.3. Describe the two-dimensional figures, i.e., cross-section, that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids. **Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.** 7.G.4. Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle. 7.G.5. Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.	Geometry 8.G **Understand congruence and similarity using physical models, transparencies, or geometry software.** 8.G.1. Through experimentation, verify the properties of rotations, reflections, and translations (transformations) to figures on a coordinate plane). a. Lines are taken to lines, and line segments to line segments of the same length. b. Angles are taken to angles of the same measure. c. Parallel lines are taken to parallel lines. 8.G.2. Demonstrate understanding of congruence by applying a sequence of translations, reflections, and rotations on two-dimensional figures. Given two congruent figures, describe a sequence that exhibits the congruence between them. 8.G.3. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates. 8.G.4. Demonstrate understanding of similarity, by applying a sequence of translations, reflections, rotations, and dilations on two-dimensional figures. Describe a sequence that exhibits the similarity between them.

Grade 6	Grade 7	Grade 8
	7.G.6. Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.	8.G.5. Justify using informal arguments to establish facts about • the angle sum of triangles (sum of the interior angles of a triangle is 180º), • measures of exterior angles of triangles, • angles created when parallel lines are cut be a transversal (e.g., alternate interior angles), and • angle-angle criterion for similarity of triangles. **Understand and apply the Pythagorean Theorem.** 8.G.6. Explain the Pythagorean Theorem and its converse. 8.G.7. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. 8.G.8. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system. **Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.** 8.G.9. Identify and apply the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

Grade 6	**Grade 7**	**Grade 8**
Statistics and Probability 6.SP **Develop understanding of statistical variability.** 6.SP.1. Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.* 6.SP.2. Understand that a set of data has a distribution that can be described by its center (mean, median, or mode), spread (range), and overall shape and can be used to answer a statistical question. 6.SP.3. Recognize that a measure of center (mean, median, or mode) for a numerical data set summarizes all of its values with a single number, while a measure of variation (range) describes how its values vary with a single number. **Summarize and describe distributions.** 6.SP.4. Display numerical data in plots on a number line, including dot or line plots, histograms and box (box and whisker) plots.	Statistics and Probability 7.SP **Use random sampling to draw inferences about a population.** 7.SP.1. Understand that statistics can be used to gain information about a population by examining a reasonably sized sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences. 7.SP.2. Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.* **Draw informal comparative inferences about two populations.** 7.SP.3. Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.*	Statistics and Probability 8.SP **Investigate patterns of association in bivariate data.** 8.SP.1. Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association. 8.SP.2. Explain why straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line. 8.SP.3. Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and y-intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.* 8.SP.4. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects and use relative frequencies to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*

Grade 6	Grade 7	Grade 8
6.SP.5. Summarize numerical data sets in relation to their context, such as by: a. Reporting the number of observations (occurrences). b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement. c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range), as well as describing any overall pattern and any outliers with reference to the context in which the data were gathered. d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered. 6.SP.6. Analyze whether a game is mathematically fair or unfair by explaining the probability of all possible outcomes. (L) 6.SP.7. Solve or identify solutions to problems involving possible combinations (e.g., if ice cream sundaes come in 3 flavors with 2 possible toppings, how many different sundaes can be made using only one flavor of ice cream with one topping?) (L)	7.SP.4. Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.* **Investigate chance processes and develop, use, and evaluate probability models.** 7.SP.5. Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event. 7.SP.6. Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.*	

Grade 6	Grade 7	Grade 8
	7.SP.7. Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. a. Design a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.* b. Design a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?* 7.SP.8. Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event.	

Grade 6	Grade 7	Grade 8
	c. Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?*	<u>Functions 8.F</u> **Define, evaluate, and compare functions.** 8.F.1. Understand that a function is a rule that assigns to each input (the domain) exactly one output (the range). The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. For example, use the vertical line test to determine functions and non-functions. 8.F.2. Compare properties of two functions, each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.* 8.F.3. Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*

Grade 6	Grade 7	Grade 8
		Use functions to model relationships between quantities. 8.F.4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. 8.F.5. Given a verbal description between two quantities, sketch a graph. Conversely, given a graph, describe a possible real-world example. *For example, graph the position of an accelerating car or tossing a ball in the air.*

High School Mathematical Content Standards

Courses and Transitions

The high school standards specify the mathematics that all students should study in order to be career and college ready. They are organized into conceptual categories, which are intended to portray a coherent view of high school mathematics. A student's work with any set of standards crosses a number of traditional course boundaries. For example, the Functions Standards would apply to different courses such as Algebra I or Algebra II.

These standards do not mandate the sequence of high school courses. However, the organization of high school courses is a critical component to implementation of the standards. It is a district decision how to design course offerings covering the mathematics standards. Districts can use the traditional approach of Algebra I, Geometry, and Algebra II or implement an integrated approach. There are various high school math pathways to be considered and likely additional model pathways based on these standards will become available as well.

The standards themselves do not dictate curriculum, pedagogy, or delivery of content. In particular, districts may handle the transition to high school in different ways. For example, many students in the U.S. today take Algebra I in the 8th grade, and in some districts and states this is a requirement. By completing grade 7 standards successfully, students have met the prerequisites and are prepared for Algebra I by 8th grade. The standards are designed to permit districts and states to continue existing policies concerning Algebra I in 8th grade.

Another major transition is the transition from high school to post-secondary education for college and careers. The evidence concerning college and career readiness shows clearly that the knowledge, skills, and practices important for readiness include a great deal of mathematics prior to the boundary defined by (+) symbols in these standards. Indeed, some of the highest priority content for college and career readiness comes from grades 6-8. This body of material includes powerfully useful proficiencies such as applying ratio reasoning in real-world and mathematical problems, computing fluently with positive and negative fractions and decimals, and solving real-world and mathematical problems involving angle measure, area, surface area, and volume. Because important standards for college and career readiness are distributed across grades and courses, systems for evaluating college and career readiness should reach as far back in the standards as grades 6-8.

Narrative of Standards – Modeling

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.

Some examples of such situations might include:

- Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.
- Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player.
- Designing the layout of the stalls in a school fair so as to raise as much money as possible.
- Analyzing stopping distance for a car.
- Modeling savings account balance, bacterial colony growth, or investment growth.
- Engaging in critical path analysis, e.g., applied to turnaround of an aircraft at an airport.
- Analyzing risk in situations such as extreme sports, pandemics, and terrorism.
- Relating population statistics to individual predictions.

In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations.

One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations. Models can also shed light on the mathematical structures themselves, for example, as when a model of bacterial growth makes more vivid the explosive growth of the exponential function.

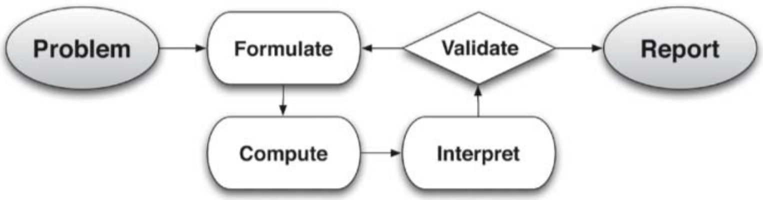
The basic modeling cycle is summarized in the diagram below. It involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.

In descriptive modeling, a model simply describes the phenomena or summarizes them in a compact form. Graphs of observations are a familiar descriptive model— for example, graphs of global temperature and atmospheric CO_2 over time.

Analytic modeling seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters that are empirically based; for example, exponential growth of bacterial colonies (until cut-off mechanisms such as pollution or starvation intervene) follows from a constant reproduction rate. Functions are an important tool for analyzing such problems.

Graphing utilities, spreadsheets, computer algebra systems, and dynamic geometry software are powerful tools that can be used to model purely mathematical phenomena (e.g., the behavior of polynomials) as well as physical phenomena.

Modeling Standards. Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by an asterisk (*).



Narrative of Standards - Number and Quantity

Numbers and Number Systems. During the years from kindergarten to 8th grade, students must repeatedly extend their conception of number. At first, "number" means "counting number": 1, 2, 3... . Soon after that, 0 is used to represent "none" and the whole numbers are formed by the counting numbers together with zero. The next extension is fractions. At first, fractions are barely numbers and tied strongly to pictorial representations. Yet by the time students understand division of fractions, they have a strong concept of fractions as numbers and have connected them, via their decimal representations, with the base-ten system used to represent the whole numbers. During middle school, fractions are augmented by negative fractions to form the rational numbers. In Grade 8, students extend this system once more, augmenting the rational numbers with the irrational numbers to form the real numbers. In high school, students will be exposed to yet another extension of number, when the real numbers are augmented by the imaginary numbers to form the complex numbers.

With each extension of number, the meanings of addition, subtraction, multiplication, and division are extended. In each new number system—integers, rational numbers, real numbers, and complex numbers—the four operations stay the same in two important ways: They have the commutative, associative, and distributive properties and their new meanings are consistent with their previous meanings.

Extending the properties of whole-number exponents leads to new and productive notation. For example, properties of whole-number exponents suggest that $(5^{1/3})^3$ should be $5^{(1/3)3} = 5^1 = 5$ and that $5^{1/3}$ should be the cube root of 5.

Calculators, spreadsheets, and computer algebra systems can provide ways for students to become better acquainted with these new number systems and their notation. They can be used to generate data for numerical experiments, to help understand the workings of matrix, vector, and complex number algebra, and to experiment with non-integer exponents.

Quantities. In real-world problems, the answers are usually not numbers but quantities: numbers with units, which involves measurement. In their work in measurement up through grade 8, students primarily measure commonly used attributes such as length, area, and volume. In high school, students encounter a wider variety of units in modeling, e.g., acceleration, currency conversions, derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages. They also encounter novel situations in which they themselves must conceive the attributes of interest. For example, to find a good measure of overall highway safety, they might propose measures such as fatalities per year, fatalities per year per driver, or fatalities per vehicle-mile traveled. Such a conceptual process is sometimes called quantification. Quantification is important for science, as when surface area suddenly "stands out" as an important variable in evaporation. Quantification is also important for companies, which must conceptualize relevant attributes and create or choose suitable measures for them.

Number and Quantity Standards

The Real Number System	N – RN

Extend the properties of exponents to rational exponents.

N-RN.1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.*

N-RN.2. Rewrite expressions involving radicals and rational exponents using the properties of exponents. *For example: Write equivalent representations that utilize both positive and negative exponents.*

Use properties of rational and irrational numbers.

N-RN.3. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Quantities*	N – Q

Reason quantitatively and use units to solve problems.

N-Q.1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

N-Q.2. Define appropriate quantities for the purpose of descriptive modeling.

N-Q.3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

The Complex Number System	N - CN

Perform arithmetic operations with complex numbers.

N-CN.1. Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.

N-CN.2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

N-CN.3. (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

Represent complex numbers and their operations on the complex plane.

N-CN.4. (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.

N-CN.5. (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,* $(1 - \sqrt{3}i)^3 = 8$ *because* $(1 - \sqrt{3}i)$ *has modulus 2 and argument 120°.*

N-CN.6. (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

Use complex numbers in polynomial identities and equations.

N-CN.7. Solve quadratic equations with real coefficients that have complex solutions.

N-CN.8. (+) Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4$ *as* $(x + 2i)(x – 2i)$.

N-CN.9. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Vector and Matrix Quantities — N – VM

Represent and model with vector quantities.

N-VM.1. (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., ***v***, |***v***|, ||***v***||, ***v***).

N-VM.2. (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.

N-VM.3. (+) Solve problems involving velocity and other quantities that can be represented by vectors.

Perform operations on vectors.

N-VM.4. (+) Add and subtract vectors.

a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.

b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.

c. Understand vector subtraction ***v*** – ***w*** as ***v*** + (–***w***), where –***w*** is the additive inverse of ***w***, with the same magnitude as ***w*** and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.

N-VM.5. (+) Multiply a vector by a scalar.

a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$.

b. Compute the magnitude of a scalar multiple $c\mathbf{v}$ using $||c\mathbf{v}|| = |c|\mathbf{v}$. Compute the direction of $c\mathbf{v}$ knowing that when $|c|\mathbf{v} \neq 0$, the direction of $c\mathbf{v}$ is either along $\mathbf{v}$ (for $c > 0$) or against $\mathbf{v}$ (for $c < 0$).

Perform operations on matrices and use matrices in applications.

N-VM.6. (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.

N-VM.7. (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.

N-VM.8. (+) Add, subtract, and multiply matrices of appropriate dimensions.

N-VM.9. (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.

N-VM.10. (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.

N-VM.11. (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.

N-VM.12. (+) Work with 2×2 matrices as a transformations of the plane, and interpret the absolute value of the determinant in terms of area.

Narrative of Standards - Algebra

Expressions. An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Conventions about the use of parentheses and the order of operations assure that each expression is unambiguous. Creating an expression that describes a computation involving a general quantity requires the ability to express the computation in general terms, abstracting from specific instances.

Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning. For example, $p + 0.05p$ can be interpreted as the addition of a 5% tax to a price p. Rewriting $p + 0.05p$ as $1.05p$ shows that adding a tax is the same as multiplying the price by a constant factor.

Algebraic manipulations are governed by the properties of operations and exponents, and the conventions of algebraic notation. At times, an expression is the result of applying operations to simpler expressions. For example, $p + 0.05p$ is the sum of the simpler expressions p and $0.05p$. Viewing an expression as the result of operation on simpler expressions can sometimes clarify its underlying structure.

A spreadsheet or a computer algebra system (CAS) can be used to experiment with algebraic expressions, perform complicated algebraic manipulations, and understand how algebraic manipulations behave.

Equations and Inequalities. An equation is a statement of equality between two expressions, often viewed as a question asking for which values of the variables the expressions on either side are in fact equal. These values are the solutions to the equation. An identity, in contrast, is true for all values of the variables; identities are often developed by rewriting an expression in an equivalent form.

The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables form a set of ordered pairs of numbers, which can be plotted in the coordinate plane. Two or more equations and/or inequalities form a system. A solution for such a system must satisfy every equation and inequality in the system.

An equation can often be solved by successively deducing from it one or more simpler equations. For example, one can add the same constant to both sides without changing the solutions, but squaring both sides might lead to extraneous solutions. Strategic competence in solving includes looking ahead for productive manipulations and anticipating the nature and number of solutions.

Some equations have no solutions in a given number system, but have a solution in a larger system. For example, the solution of $x + 1 = 0$ is an integer, not a whole number; the solution of $2x + 1 = 0$ is a rational number, not an integer; the solutions of $x^2 – 2 = 0$ are real numbers, not rational numbers; and the solutions of $x^2 + 2 = 0$ are complex numbers, not real numbers.

The same solution techniques used to solve equations can be used to rearrange formulas. For example, the formula for the area of a trapezoid, $A = ((b_1+b_2)/2)h$, can be solved for h using the same deductive process.

Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them.

Connections to Functions and Modeling. Expressions can define functions, and equivalent expressions define the same function. Asking when two functions have the same value for the same input leads to an equation; graphing the two functions allows for finding approximate solutions of the equation. Converting a verbal description to an equation, inequality, or system of these is an essential skill in modeling.

Algebra Standards

Seeing Structure in Expressions	**A - SSE**

Interpret the structure of expressions.

A-SSE.1. Interpret expressions that represent a quantity in terms of its context.*

a. Interpret parts of an expression, such as terms, factors, and coefficients.

b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1+r)^n$ *as the product of P and a factor not depending on P.*

A-SSE.2. Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 – y^4$ *as* $(x^2)^2 – (y^2)^2$, *thus recognizing it as a difference of squares that can be factored as* $(x^2 – y^2)(x^2 + y^2)$.

Write expressions in equivalent forms to solve problems.

A-SSE.3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.*

a. Factor a quadratic expression to reveal the zeros of the function it defines. *For example,* $x^2 + 4x +3 = (x +3)(x +1)$.

b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. *For example,* $x^2 + 4x + 3 = (x + 2)^2 - 1$.

c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression* 1.15^t *can be rewritten as* $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*

A-SSE.4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.**

Arithmetic with Polynomials and Rational Expressions	**A - APR**

Perform arithmetic operations on polynomials.

A-APR.1. Add, subtract, and multiply polynomials. Understand that polynomials form a system similar to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication.

Understand the relationship between zeros and factors of polynomials.

A-APR.2. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x – a$ is $p(a)$, so $p(a) = 0$ if and only if $(x – a)$ is a factor of $p(x)$.

A-APR.3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Use polynomial identities to solve problems.

A-APR.4. Prove polynomial identities and use them to describe numerical relationships.
For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 – y^2)^2 + (2xy)^2$ *can be used to generate Pythagorean triples.*

A-APR.5. (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.

Rewrite rational expressions.

A-APR.6. Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.

A-APR.7. (+) Add, subtract, multiply, and divide rational expressions. Understand that rational expressions form a system similar to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression.

Creating Equations and Inequalities* — A – CED

Create equations and inequalities that describe numbers or relationships.

A-CED.1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

A-CED.2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

A-CED.3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing cost constraints in various situations.*

A-CED.4. Rearrange formulas (literal equations) to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law $V = IR$ to highlight resistance R.*

Reasoning with Equations and Inequalities — A – REI

Understand solving equations as a process of reasoning and explain the reasoning.

A-REI.1. Apply properties of mathematics to justify steps in solving equations in one variable.

A-REI.2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

Solve equations and inequalities in one variable.

A-REI.3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

A-REI.4. Solve quadratic equations in one variable.

a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x – p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.

b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers *a* and *b*.

Solve systems of equations.

A-REI.5. Show that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

A-REI.6. Solve systems of linear equations exactly and approximately, e.g., with graphs or algebraically, focusing on pairs of linear equations in two variables.

A-REI.7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line* $y = -3x$ and the circle $x^2 + y^2 = 3$.

A-REI.8. (+) Represent a system of linear equations as a single matrix equation in a vector variable.

A-REI.9. (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3×3 or greater).

Represent and solve equations and inequalities graphically.

A-REI.10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

A-REI.11. Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*

A-REI.12. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Narrative of Standards - Functions

Functions. describe situations where one quantity determines another. For example, the return on $10,000 invested at an annualized percentage rate of 4.25% is a function of the length of time the money is invested. Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models.

In school mathematics, functions usually have numerical inputs and outputs and are often defined by an algebraic expression. For example, the time in hours it takes for a car to drive 100 miles is a function of the car's speed in miles per hour, v; the rule $T(v) = 100/v$ expresses this relationship algebraically and defines a function whose name is T.

The set of inputs to a function is called its domain. We often infer the domain to be all inputs for which the expression defining a function has a value, or for which the function makes sense in a given context.

A function can be described in various ways, such as by a graph (e.g., the trace of a seismograph); by a verbal rule, as in, "I'll give you a state, you give me the capital city;" by an algebraic expression like $f(x) = a + bx$; or by a recursive rule. The graph of a function is often a useful way of visualizing the relationship of the function models, and manipulating a mathematical expression for a function can throw light on the function's properties.

Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships.

A graphing utility or a computer algebra system can be used to experiment with properties of these functions and their graphs and to build computational models of functions, including recursively defined functions.

Connections to Expressions, Equations, Modeling, and Coordinates. Determining an output value for a particular input involves evaluating an expression; finding inputs that yield a given output involves solving an equation. Questions about when two functions have the same value for the same input lead to equations, whose solutions can be visualized from the intersection of their graphs. Because functions describe relationships between quantities, they are frequently used in modeling. Sometimes functions are defined by a recursive process, which can be displayed effectively using a spreadsheet or other technology.

Functions Standards

Interpreting Functions **F - IF**

Understand the concept of a function and use function notation.

F-IF.1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.

F-IF.2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

F-IF.3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1$, $f(n + 1) = f(n) + f(n - 1)$ *for* $n \geq 1$.

Interpret functions that arise in applications in terms of the context.

F-IF.4. For a function that models a relationship between two quantities,

- interpret key features of graphs and tables in terms of the quantities, and
- sketch graphs showing key features given a verbal description of the relationship.

*Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.**

F-IF.5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then negative numbers would be an appropriate domain for the function.**

F-IF.6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*

Analyze functions using different representations.

F-IF.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

c. Graph polynomial functions, identifying zeros (using technology) or algebraic methods when suitable factorizations are available, and showing end behavior.

d. (+) Graph rational functions, identifying zeros and discontinuities (asymptotes/holes) using technology, and algebraic methods when suitable factorizations are available, and showing end behavior.

e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

F-IF.8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, *and classify them as representing exponential growth or decay.*

F-IF.9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically, in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Building Functions F - BF

Build a function that models a relationship between two quantities.

F-BF.1. Write a function that describes a relationship between two quantities.*

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*

c. (+) Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.*

F-BF.2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.*

Build new functions from existing functions.

F-BF.3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. *Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

F-BF.4. Find inverse functions.

a. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. *For example,* $f(x) = 2x^3$ *for* $x > 0$ *or* $f(x) = (x + 1)/(x - 1)$ *for* $x \neq 1$.

b. (+) Verify by composition that one function is the inverse of another.

c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.

d. (+) Produce an invertible function from a non-invertible function by restricting the domain.

F-BF.5. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Linear, Quadratic, and Exponential Models*	**F – LE**

Construct and compare linear, quadratic, and exponential models and solve problems.

F-LE.1. Distinguish between situations that can be modeled with linear functions and with exponential functions.

a. Show that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.

b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

F-LE.2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or input-output table of values.

F-LE.3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

F-LE.4. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where *a*, *c*, and *d* are numbers and the base *b* is 2, 10, or *e*; evaluate the logarithm using technology.

Interpret expressions for functions in terms of the situation they model.

F-LE.5. Interpret the parameters in a linear or exponential function in terms of a context.

Trigonometric Functions — F - TF

Extend the domain of trigonometric functions using the unit circle.

F-TF.1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

F-TF.2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

F-TF.3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosines, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.

F-TF.4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

Model periodic phenomena with trigonometric functions.

F-TF.5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.*

F-TF.6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

F-TF.7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.*

Prove and apply trigonometric identities.

F-TF.8. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to calculate trigonometric ratios.

F-TF.9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Narrative of Standards - Geometry

An understanding of the attributes and relationships of geometric objects can be applied in diverse contexts—interpreting a schematic drawing, estimating the amount of wood needed to frame a sloping roof, rendering computer graphics, or designing a sewing pattern for the most efficient use of material.

Although there are many types of geometry, school mathematics is devoted primarily to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates). Euclidean geometry is characterized most importantly by the Parallel Postulate, that through a point not on a given line there is exactly one parallel line. (Spherical geometry, in contrast, has no parallel lines.)

During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs. Later in college some students develop Euclidean and other geometries carefully from a small set of axioms.

The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamental are the rigid motions: translations, rotations, reflections, and combinations of these, all of which are here assumed to preserve distance and angles (and therefore shapes generally). Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes—as when the reflective symmetry of an isosceles triangle assures that its base angles are congruent.

In the approach taken here, two geometric figures are defined to be congruent if there is a sequence of rigid motions that carries one onto the other. This is the principle of superposition. For triangles, congruence means the equality of all corresponding pairs of sides and all corresponding pairs of angles. During the middle grades, through experiences drawing triangles from given conditions, students notice ways to specify enough measures in a triangle to ensure that all triangles drawn with those measures are congruent. Once these triangle congruence criteria (ASA, SAS, and SSS) are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures.

Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of "same shape" and "scale factor" developed in the middle grades. These transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent.

The definitions of sine, cosine, and tangent for acute angles are founded on right triangles and similarity, and, with the Pythagorean Theorem, are fundamental in many real-world and theoretical situations. The Pythagorean Theorem is generalized to nonright triangles by the Law of Cosines. Together, the Laws of Sines and Cosines embody the triangle congruence criteria for the cases where three pieces of information suffice to completely solve a triangle. Furthermore, these laws yield two possible solutions in the ambiguous case, illustrating that Side-Side-Angle is not a congruence criterion.

Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving. Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions. This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. Geometric transformations of the graphs of equations correspond to algebraic changes in their equations.

Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena in much the same way as computer algebra systems allow them to experiment with algebraic phenomena.

Connections to Equations. The correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof.

Geometry Standards

Congruence	**G - CO**

Experiment with transformations in the plane.

G-CO.1. Demonstrates understanding of key geometrical definitions, including angle, circle, perpendicular line, parallel line, line segment, and transformations in Euclidian geometry. Understand undefined notions of point, line, distance along a line, and distance around a circular arc.

G-CO.2. Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

G-CO.3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

G-CO.4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

G-CO.5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

Understand congruence in terms of rigid motions.

G-CO.6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

G-CO.7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

G-CO.8. Explain how the criteria for triangle congruence (ASA, SAS, SSS, AAS, and HL) follow from the definition of congruence in terms of rigid motions.

Prove geometric theorems.

G-CO.9. Using methods of proof including direct, indirect, and counter examples to prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.

G-CO.10. Using methods of proof including direct, indirect, and counter examples to prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.

G-CO.11. Using methods of proof including direct, indirect, and counter examples to prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.

Make geometric constructions.

G-CO.12. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.

G-CO.13. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Similarity, Right Triangles, and Trigonometry — G - SRT

Understand similarity in terms of similarity transformations.

G-SRT.1. Verify experimentally the properties of dilations given by a center and a scale factor:

a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.

b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

G-SRT.2. Given two figures, use the definition of similarity in terms of transformations to explain whether or not they are similar.

G-SRT.3. Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

Prove theorems involving similarity.

G-SRT.4. Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely.*

G-SRT.5. Apply congruence and similarity properties and prove relationships involving triangles and other geometric figures.

Define trigonometric ratios and solve problems involving right triangles.

G-SRT.6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

G-SRT.7. Explain and use the relationship between the sine and cosine of complementary angles.

G-SRT.8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.*

Apply trigonometry to general triangles.

G-SRT.9. (+) Derive the formula $A = 1/2\, ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.

G-SRT.10. (+) Prove the Laws of Sines and Cosines and use them to solve problems.

G-SRT.11. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Circles — G - C

Understand and apply theorems about circles.

G-C.1. Prove that all circles are similar.

G-C.2. Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*

G-C.3. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

G-C.4. (+) Construct a tangent line from a point outside a given circle to the circle.

Find arc lengths and areas of sectors of circles.

G-C.5. Use and apply the concepts of arc length and areas of sectors of circles. Determine or derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

Expressing Geometric Properties with Equations — G - GPE

Translate between the geometric description and the equation for a conic section.

G-GPE.1. Determine or derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

G-GPE.2. Determine or derive the equation of a parabola given a focus and directrix.

G-GPE.3. (+) Derive the equations of ellipses and hyperbolas given foci and directrices.

Use coordinates to prove simple geometric theorems algebraically.

G-GPE.4. Perform simple coordinate proofs. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).

G-GPE.5. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

G-GPE.6. Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

G-GPE.7. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.*

Geometric Measurement and Dimension — G - GMD

Explain volume formulas and use them to solve problems.

G-GMD.1. Explain how to find the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.

G-GMD.2. (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.

G-GMD.3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. *For example: Solve problems requiring determination of a dimension not given.**

Visualize relationships between two-dimensional and three-dimensional objects.

G-GMD.4. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

Modeling with Geometry — G - MG

Apply geometric concepts in modeling situations.

G-MG.1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*

G-MG.2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).*

G-MG.3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).*

Narrative of Standards - Statistics and Probability*

Decisions or predictions are often based on data—numbers in context. These decisions or predictions would be easy if the data always sent a clear message, but the message is often obscured by variability. Statistics provides tools for describing variability in data and for making informed decisions that take it into account.

Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. Quantitative data can be described in terms of key characteristics: measures of shape, center, and spread. The shape of a data distribution might be described as symmetric, skewed, flat, or bell shaped, and it might be summarized by a statistic measuring center (such as mean or median) and a statistic measuring spread (such as standard deviation or interquartile range). Different distributions can be compared numerically using these statistics or compared visually using plots. Knowledge of center and spread are not enough to describe a distribution. Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken.

Randomization has two important uses in drawing statistical conclusions. First, collecting data from a random sample of a population makes it possible to draw valid conclusions about the whole population, taking variability into account. Second, randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments. A statistically significant outcome is one that is unlikely to be due to chance alone, and this can be evaluated only under the condition of randomness. The conditions under which data are collected are important in drawing conclusions from the data; in critically reviewing uses of statistics in public media and other reports, it is important to consider the study design, how the data were gathered, and the analyses employed as well as the data summaries and the conclusions drawn.

Random processes can be described mathematically by using a probability model: a list or description of the possible outcomes (the sample space), each of which is assigned a probability. In situations such as flipping a coin, rolling a number cube, or drawing a card, it might be reasonable to assume various outcomes are equally likely. In a probability model, sample points represent outcomes and combine to make up events; probabilities of events can be computed by applying the Addition and Multiplication Rules. Interpreting these probabilities relies on an understanding of independence and conditional probability, which can be approached through the analysis of two-way tables.

Technology plays an important role in statistics and probability by making it possible to generate plots, regression functions, and correlation coefficients, and to simulate many possible outcomes in a short amount of time.

Connections to Functions and Modeling. Functions may be used to describe data; if the data suggest a linear relationship, the relationship can be modeled with a regression line, and its strength and direction can be expressed through a correlation coefficient.

Statistics and Probability Standards*

Interpreting Categorical and Quantitative Data **S - ID**

Summarize, represent, and interpret data on a single count or measurement variable.

S-ID.1. Represent data with plots on the real number line (dot plots, histograms, and box plots).

S-ID.2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.

S-ID.3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). *For example: Justify why median price of homes or income is used instead of the mean.*

S-ID.4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

Summarize, represent, and interpret data on two categorical and quantitative variables.

S-ID.5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

S-ID.6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.*

b. Informally assess the fit of a function by plotting and analyzing residuals. *For example: Describe solutions to problems that require interpolation and extrapolation.*

c. Fit a linear function for a scatter plot that suggests a linear association.

Interpret linear models.

S-ID.7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

S-ID.8. Compute (using technology) and interpret the correlation coefficient of a linear fit.

S-ID.9. Distinguish between correlation and causation.

Making Inferences and Justifying Conclusions — S - IC

Understand and evaluate random processes underlying statistical experiments.

S-IC.1. Understand statistics as a process for making inferences about population parameters based on a random sample from that population.

S-IC.2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*

Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

S-IC.3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.

S-IC.4. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.

S-IC.5. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.

S-IC.6. Evaluate reports based on data.

Conditional Probability and the Rules of Probability — S - CP

Understand independence and conditional probability and use them to interpret data.

S-CP.1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).

S-CP.2. Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.

S-CP.3. Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.

S-CP.4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in 10th grade. Do the same for other subjects and compare the results.*

S-CP.5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*

Use the rules of probability to compute probabilities of compound events in a uniform probability model.

S-CP.6. Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A,* and interpret the answer in terms of the model.

S-CP.7. Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.

S-CP.8. (+) Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.

S-CP.9. (+) Use permutations and combinations to compute probabilities of compound events and solve problems.

Using Probability to Make Decisions — S - MD

Calculate expected values and use them to solve problems.

S-MD.1. (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.

S-MD.2. (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.

S-MD.3. (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value*. For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.*

S-MD.4. (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?*

Use probability to evaluate outcomes of decisions.

S-MD.5. (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.

a. Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.*

b. Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.*

S-MD.6. (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).

S-MD.7. (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

Glossary for Alaska Mathematics Standards

addition and subtraction within 5, 10, 20, 100, or 1000

Addition or subtraction of two whole numbers with whole number answers, and with sum or minuend in the range 0-5, 0-10, 0-20, or 0-100, respectively. Example: 8 + 2 = 10 is an addition within 10, 14 – 5 = 9 is a subtraction within 20, and 55 – 18 = 37 is a subtraction within 100.

additive inverses

Two numbers whose sum is 0 are additive inverses of one another. Example: 3/4 and – 3/4 are additive inverses of one another because 3/4 + (– 3/4) = (– 3/4) + 3/4 = 0.

associative property of addition

See Table 3 in this Glossary.

associative property of multiplication

See Table 3 in this Glossary.

bivariate data

Pairs of linked numerical observations. Example: a list of heights and weights for each player on a football team. Box plot. A method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.[1]

cardinality

Cardinal numbers, known as the "counting numbers," indicate quantity.

commutative property

See Table 3 in this Glossary.

complex fraction

A fraction *A/B* where *A* and/or *B* are fractions (*B* nonzero).

computation algorithm

A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. *See also:* computation strategy.

computation strategy

Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. *See also:* computation algorithm.

congruent

Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations).

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[1] Adapted from Wisconsin Department of Public Instruction, http://dpi.wi.gov/standards/mathglos.html, accessed Mar 2, 2010.

counting on

A strategy for finding the number of objects in a group without having to count every member of the group. For example, if a stack of books is known to have 8 books and 3 more books are added to the top, it is not necessary to count the stack all over again. One can find the total by counting on—pointing to the top book and saying "eight," following this with "nine, ten, eleven. There are eleven books now."

dot plot

See: line plot

dilation

A transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor.

expanded form

A multi-digit number is expressed in expanded form when it is written as a sum of single-digit multiples of powers of ten. For example, 643 = 600 + 40 + 3.

expected value

For a random variable, the weighted average of its possible values, with weights given by their respective probabilities.

first quartile

For a data set with median *M*, the first quartile is the median of the data values less than *M*. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the first quartile is 6.[2] *See also:* median, third quartile, interquartile range.

fraction

A number expressible in the form *a/b* where *a* is a whole number and *b* is a positive whole number. (The word fraction in these standards always refers to a non-negative number.) *See also:* rational number.

identity property of 0

See Table 3 in this Glossary.

independently combined probability models

Two probability models are said to be combined independently if the probability of each ordered pair in the combined model equals the product of the original probabilities of the two individual outcomes in the ordered pair.

integer

A number expressible in the form *a* or –*a* for some whole number *a*.

interquartile range

A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the interquartile range is 15 – 6 = 9. See also: first quartile, third quartile.

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[2] Many different methods for computing quartiles are in use. The method defined here is sometimes called the More and McCabe method. See Langford, E., "Quartiles in Elementary Statistics," *Journal of Statistics Education*, Volume 14, number 3 (2006).

line plot

A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line. Also known as a dot plot.[3]

mean

A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list.[4] Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21.

mean absolute deviation

A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolute deviation is 20.

median

A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11.

midline

In the graph of a trigonometric function, the horizontal line halfway between its maximum and minimum values. Multiplication and division within 100. Multiplication or division of two whole numbers with whole number answers, and with product or dividend in the range 0-100. Example: 72 ÷ 8 = 9.

multiplicative inverses

Two numbers whose product is 1 are multiplicative inverses of one another. Example: 3/4 and 4/3 are multiplicative inverses of one another because 3/4 × 4/3 = 4/3 × 3/4 = 1.

number line diagram.

A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.

ordinality

Ordinal numbers indicate the order or rank of things in a set (e.g., sixth in line; fourth place).

percent rate of change

A rate of change expressed as a percent. Example: if a population grows from 50 to 55 in a year, it grows by 5/50 = 10% per year.

probability distribution

The set of possible values of a random variable with a probability assigned to each.

probability

A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin, selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition).

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[3] Adapted from Wisconsin Department of Public Instruction, op. cit.

[4] To be more precise, this defines the *arithmetic mean*.

probability model

A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. *See also:* uniform probability model.

properties of equality

See Table 4 in this Glossary.

properties of inequality

See Table 5 in this Glossary.

properties of operation

See Table 3 in this Glossary.

random variable

An assignment of a numerical value to each outcome in a sample space. Rational expression. A quotient of two polynomials with a non-zero denominator.

rational expression

A quotient of two polynomials with a non-zero denominator.

rational number

A number expressible in the form *a/b* or – *a/b* for some fraction *a/b*. The rational numbers include the integers.

rectilinear figure

A polygon all angles of which are right angles.

rigid motion

A transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are here assumed to preserve distances and angle measures.

repeating decimal

The decimal form of a rational number. *See also:* terminating decimal.

sample space

In a probability model for a random process, a list of the individual outcomes that are to be considered.

scatter plot

A graph in the coordinate plane representing a set of bivariate data. For example, the heights and weights of a group of people could be displayed on a scatter plot.[5]

similarity transformation

A rigid motion followed by a dilation.

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[5] Adapted from Wisconsin Department of Public Instruction, op. cit.

tape diagram

A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar model, fraction strip, or length model.

terminating decimal

A decimal is called terminating if its repeating digit is 0.

third quartile

For a data set with median *M*, the third quartile is the median of the data values greater than *M*. *For example*: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15. *See also:* median, first quartile, interquartile range.

transitivity principle for indirect measurement

If the length of object A is greater than the length of object B, and the length of object B is greater than the length of object C, then the length of object A is greater than the length of object C. This principle applies to measurement of other quantities as well.

uniform probability model

A probability model which assigns equal probability to all outcomes. *See also:* probability model.

vector

A quantity with magnitude and direction in the plane or in space, defined by an ordered pair or triple of real numbers.

visual fraction model

A tape diagram, number line diagram, or area model.

whole numbers

The numbers 0, 1, 2, 3,...

Table 1: Common addition and subtraction situations[1]

	Result Unknown	Change Unknown	Start Unknown
Add To	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 – ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? – 2 = 3
	Total Unknown	**Addend Unknown**	**Both Addends Unknown[2]**
Put Together/ Take Apart[3]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 – 3 = ?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare[4]	(“How many more?” version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? (“How many fewer?” version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 – 2 = ?	(Version with “more”): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with “fewer”): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with “more”): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with “fewer”): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

[1] *Adapted from Box 2-4 of the National Research Council (2009, op. cit., pp. 32, 33).*

[2] *These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean makes or results in but always does mean is the same number as.*

[3] *Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.*

[4] *For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.*

Table 2: Common multiplication and division situations[5]

	Unknown Product	Group Size Unknown ("How many in each group?" Division)	Number of Groups Unknown ("How many groups?" Division)
	3 x 6 = ?	3 x ? = 18, and 18 ÷ 3 = ?	? x 6 = 18, and 18 ÷ 6 = ?
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? Measurement example. You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? Measurement example. You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? Measurement example. You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Array[6], Area[7]	There are 3 rows of apples with 6 apples in each row. How many apples are there? Area example. What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? Area example. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? Area example. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? Measurement example. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? Measurement example. A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? Measurement example. A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	*a x b = ?*	*a x ? = p, and p ÷ a = ?*	*? x b = p, and p ÷ b = ?*

[5] *The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.*

[6] *The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.*

[7] *Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.*

Table 3. The properties of operations.

Here a, b and c stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.

Associative property of addition	$(a + b) + c = a + (b + c)$
Commutative property of addition	$a + b = b + a$
Additive identity property of 0	$a + 0 = 0 + a = a$
Existence of additive inverses	For every a there exists $-a$ so that $a + (-a) = (-a) + a = 0$.
Associative property of multiplication	$(a \times b) \times c = a \times (b \times c)$
Commutative property of multiplication	$a \times b = b \times a$
Multiplicative identity property of 1	$a \times 1 = 1 \times a = a$
Existence of multiplicative inverses	For every $a \neq 0$ there exists $1/a$ so that $a \times 1/a = 1/a \times a = 1$.
Distributive property of multiplication over addition	$a \times (b + c) = a \times b + a \times c$

Table 4. The properties of equality.

Here *a*, *b* and *c* stand for arbitrary numbers in the rational, real, or complex number systems.

Reflexive property of equality	$a = a$
Symmetric property of equality	If $a = b$, then $b = a$.
Transitive property of equality	If $a = b$ and $b = c$, then $a = c$.
Addition property of equality	If $a = b$, then $a + c = b + c$.
Subtraction property of equality	If $a = b$, then $a - c = b - c$.
Multiplication property of equality	If $a = b$, then $a \times c = b \times c$.
Division property of equality	If $a = b$, then b may be substituted for a in any expression containing a.
Substitution property of equality	If $a = b$ and $c \neq 0$, then $a \div c = b \div c$.

Table 5. The properties of inequality.

Here *a, b* and *c* stand for arbitrary numbers in the rational or real number systems.

Exactly one of the following is true: $a < b$, $a = b$, $a > b$.
If $a > b$ and $b > c$ then $a > c$.
If $a > b$, then $b < a$.
If $a > b$, then $-a < -b$.
If $a > b$, then $a \pm c > b \pm c$.
If $a > b$ and $c > 0$, then $a \times c > b \times c$.
If $a > b$ and $c < 0$, then $a \times c < b \times c$.
If $a > b$ and $c > 0$, then $a \div c > b \div c$.
If $a > b$ and $c < 0$, then $a \div c < b \div c$.