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INVITED ARTICLE

Fostering Middle School Students' Relational Thinking of the Equal Sign Using GeoGebra

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Current reforms in mathematics education have called for a stronger emphasis on the teaching and learning of algebra for all students at all grade levels. Succeeding in algebra can prepare students to learn and understand more advanced mathematics in the future. One topic in algebra—the equal sign—has received considerable attention in middle school mathematics because it is a fundamental concept to understand algebra. In particular, middle school is a vital transition for students to develop from arithmetical reasoning of elementary school to algebraic reasoning of secondary school. Although middle school students' difficulties with the equal sign are well documented, to date, little is known about how to use GeoGebra—a dynamic and an interactive tool—to develop middle school students' understanding of the equal sign. In this paper, we explore one GeoGebra dynamic worksheet focusing on comparing two functions of the equal sign with graphical and algebraic representations. We also describe how this GeoGebra dynamic worksheet is used to promote middle school students' relational thinking of the equality.

Introduction

The equal sign is fundamental to learning and understanding mathematics in general, particularly in algebra. Consequently, the equal sign has garnered great emphasis in school mathematics. The wealth of research investigating students' conceptions of the equal sign, however, shows that many students at all grade levels possess inadequate understanding of the equal sign (Falkner, Levi, & Carpenter, 1999; Knuth, Stephens, McNeil, & Alibali, 2006; McNeil & Alibali, 2005). Many elementary and middle school students view the equal sign as an arithmetic operation rather than a mathematical equivalence (Knuth et al., 2006; McNeil & Alibali, 2005). These misconceptions might persist among high school and college students in understanding the meaning of the equal sign (Carpenter, Franke, & Levi, 2003; Knuth et al., 2006). Indeed, students' inadequate understanding of the equal sign should not come as a

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surprise because the equal sign has typically been introduced to students in early elementary school mathematics and little instructional time has spent on this symbol in the later grades (Knuth et al., 2006).

Research has suggested that treating two given quantities on both sides of the equal sign as functions, and representing them in the Cartesian coordinate system, is one way to foster students' appropriate interpretations and uses of the equality (Carraher, Schliemann, & Schwartz 2008). Further, a growing recognition of using multiple representations with technology is crucial to support students' algebraic learning and thinking (Erbas, Ledford, Orrill, & Polly, 2005; NCTM, 2000). To date, there is limited research on how to use GeoGebra—a free and dynamic software—to enhance middle school students' relational view of the equal sign. Middle school is a critical grade level to study given that it represents a vital transition from the concrete, arithmetical reasoning of elementary school to the abstract, algebraic reasoning of secondary school. To contribute to this research area, the purpose of this study is to explore a GeoGebra example for middle school mathematics teachers to support their students' learning of the equal sign meaningfully.

The equal sign serves as a vehicle for middle school students to support their algebraic reasoning and thinking. However, students' difficulties with the interpretation of the equal sign are well documented (Knuth et al., 2006; Knuth, Alibali, Hattikudur, McNeil, & Stephens, 2008). Particularly, the equal sign is usually introduced as an operational view rather than with a relational view of the equality in middle school curricula (McNeil & Alibali, 2005). The relational view of the equal sign is defined as "looking at expressions and equations in their entirety, noticing number relations among and within these expressions and equations" (Jacobs, Megan, Carpenter, Levi, & Battey, 2007, p. 260). Carpenter et al (2005) pointed out that many students tend to compute both sides of $8+4=\square+5$ to find the correct answer for the box. On the contrary, students who think relationally are able to notice that 5 is one more than 4 and that the number in the box should be one less than 8. Engaging students in relational thinking of the equal sign can help them use properties of algebraic expressions and number operations rather than merely carry out arithmetic procedures (Carpenter et al. 2003, 2005; Stephens, 2008).

To support middle school students to develop relational views of the equal sign, we propose that a focus on comparing graphical representations associated with algebraic representations of two given quantities on both sides may facilitate their mathematical thinking and reasoning. In order to accomplish this goal, it is worth designing GeoGebra examples for teachers to teach the equal sign at the middle school level. In this paper, we explore one GeoGebra task for teachers that are intended to provide middle school students with meaningful experiences with respect to the equal sign, and argue a possible contribution to visual learning as well as explorative learning.

A GeoGebra Example of the Equal Sign

The *Principles and Standards for School Mathematics* recommends that technology is an "essential tool for teaching, learning, and doing mathematics" (NCTM, 2000, p. 24). In particular, GeoGebra, dynamic and interactive mathematics learning environments (DIMLE) (Karadag, Martinovic, & Freiman, 2011; Martinovic & Karadag, 2012), provide individuals with dynamic and interactive representations of mathematical concepts. One advantage of working in DIMLE is that students can learn mathematics through explorations, and solve problems in multiple ways. In the remainder of the paper, we analyze a GeoGebra example of the equal sign and demonstrate how a DIMLE can be used to engage students in explorative learning and to promote their visualization.

Description of the GeoGebra Task

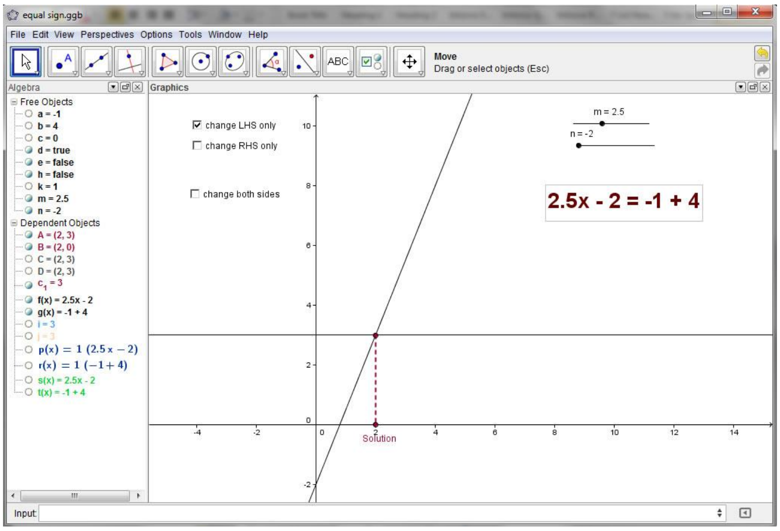

Figure 1: A GeoGebra Example of the Equal Sign

Figure 1 illustrates the graphical and algebraic representations of two functions—$f(x)=mx+n$ and $g(x)=a+b$—and an algebraic expression equating these two functions. $f(x)=mx+n$ is the function illustrating the left-hand side (LHS) of the equal sign, while $g(x)=a+b$ defines the right-hand side (RHS). Three check boxes listed on the second quadrant of the geometric view provide students with opportunities to control only changing left-hand side, right-hand side, or both sides of the equation. Since *change LHS only* check box is active in Figure 1, only the sliders controlling the variables *m* and *n* are visible, and therefore, only manipulation of the variables m and n is allowed. In contrast, the change of the parameters *a* and *b* are restricted. In sum, students are able to manipulate the parameters *m* and *n*, to visually explore the change in the graphs, and to conjecture about the effect of each manipulation displayed below the sliders.

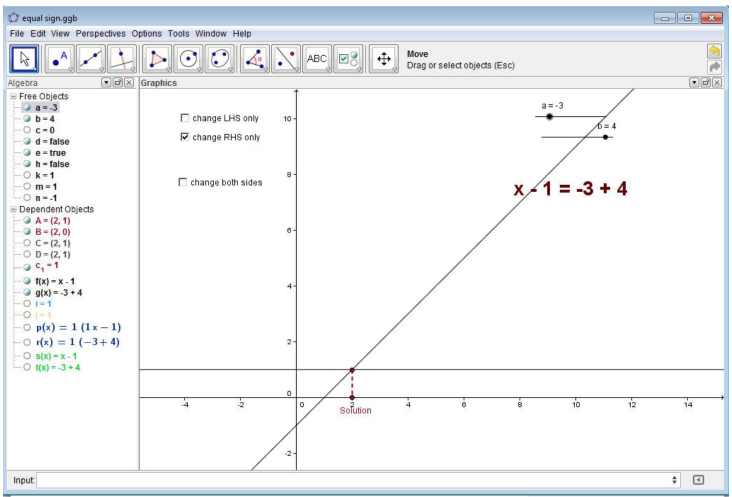

Figure 2: An Extensive GeoGebra Example from Figure 1

In a similar vein, students who manipulate RHS of the equation can change *g*(*x*). As for the second check box, students can only change RHS (see Figure 2) and the sliders control the parameters of *g*(*x*). While the sliders *a* and *b* are displayed, LHS of the equation disappears. Taken together, Figure 2 allows students to change the function *g*(*x*)=*a*+*b*, where *a* and *b* are the parameters. Students are also able to conjecture if the solution of the equation changes while dragging the sliders in this mode.

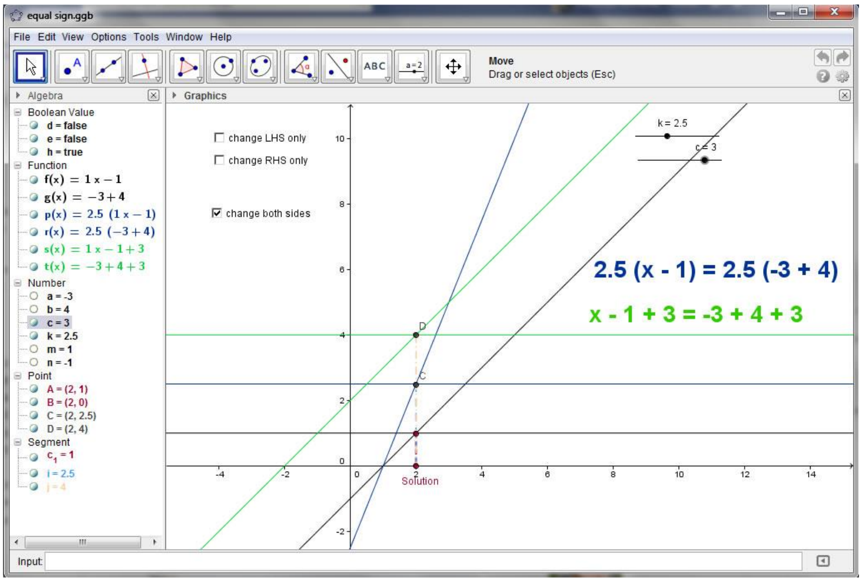

Figure 3: An Extensive GeoGebra Example from Figure 2

In contrast to the one sided effect of the first two options, the third option—check box—allows students to multiply both sides of the equation by using the slider *k*. Students are merely able to add a number to the both sides of the equations, which are represented as two functions in the geometrical view. The green and blue graphical representations of these two manipulations can be monitored while the change of the solution occurs. During the processes of exploring a GeoGebra example shown in Figure 3, students are expected to experiment the change in the solution.

Implications for a GeoGebra Example

The intention for this specific GeoGebra example described in this paper is to demonstrate how middle school students can explore both cognitive and visual aspects of the equal sign in the context of *f*(*x*)=*g*(*x*). Based on our teaching experiences, students may develop correct conjectures about the difference in the visual representations when the change occurs on one side of the equation only, versus it occurring on both sides of the equality. Given the importance of the equal sign that plays in the middle school curriculum, better preparing middle school students to develop the relational view of this symbol for learning algebra is necessary (Knuth et al., 2008). Presenting students with a variety of tasks involving the comparisons of two functions of the equal sign using GeoGebra can help them solve problems in different ways, and foster their understanding of the meaning of the equality.

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