Analysis of Research IN THE Teaching of Mathematics 1955 and 1956

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Contents

Foreword

THE IMPROVEMENT of teaching in mathematics, as in other subjects, depends to a large degree on the extent to which research conclusions find their way into the classroom. The purpose of this study is to help report and disseminate the findings in the research on mathematics teaching completed during the 2 years 1955 and 1956.

All concerned with the present study hope that it will prove helpful to both researcher and classroom teacher. The Office of Education is grateful to the deans of graduate schools and to research workers in mathematics education who supplied the data on which this study is based. Without their willing cooperation it could not have been prepared.

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Preface

IN THE YEARS 1952, 1953, and 1954 the National Council of Teachers of Mathematics and the U. S. Office of Education cooperated to summarize the research completed in mathematics education during each of these years. The summaries received many favorable comments and suggestions from leaders in mathematics education.

As a result of the suggestions, the office and the council cooperated further, this time summarizing research completed in mathematics education during the 2-year period 1955–56, and also analyzing the material. The present bulletin presents this work.

Cooperation in working with the 1955–56 material, as with the earlier material, was effected through the Research Committee of the National Council of Teachers of Mathematics. The members of that committee were the following: John J. Kinsella, School of Education, New York University; L. Clark Lay, Pasadena City College; Nathan Lazar, College of Education, The Ohio State University; and Kenneth E. Brown, Office of Education, U. S. Department of Health, Education, and Welfare.

ANALYSIS OF RESEARCH IN THE TEACHING OF MATHEMATICS

Introduction

WHAT RESEARCH in the teaching of mathematics was carried on during calendar years 1955 and 1956? What does this research say about effective ways of teaching? In what areas of mathematics education was considerable research carried on? In what areas was there very little?

To help teachers get answers to these and similar questions, the U. S. Office of Education, aided by the Research Committee of the National Council of Teachers of Mathematics, sent an inquiry to 400 colleges that offered graduate work in mathematics and/or whose staffs had made previous contributions in this area. The committee received answers to the inquiry and data on 123 studies from approximately 350 colleges. These studies included 20 studies by college faculty members, 54 master's theses, and 49 doctor's dissertations.

About half of the 123 studies were devoted to methods and approximately half to content. Twenty were concerned with prognosis and evaluation; the others, with the history of mathematics, aids to teaching, teacher education, differences in mathematical ability between the sexes, status of mathematics teachers, and miscellaneous topics. If an attempt were to be made to classify the studies according to major emphasis, the categories would not be mutually exclusive. Several studies, for example, were concerned with both content and methods.

When the studies are classified by the three grade levels—college, high school, elementary school—overlapping again appears. Thus, the college level would contain 40 studies; the high school level, 60; and the elementary school level, 40. Some studies dealing mainly with college-level topics also deal with the training of high school teachers. Likewise, a few studies concern both college and high school and some others concern both high school and elementary school.

Any thought of classifying the studies, then, as a preliminary step to analyzing them for this bulletin, was abandoned. Instead, important questions were posed pertinent to mathematics education on the three levels identified above. The three groups of questions and accompanying analyses follow this introduction. Within each analysis, numbers appearing in parentheses denote corresponding numbers used in the appendix for arranging summaries of all 123 studies alphabetically by author.

Research in the Teaching of College Mathematics

ON THE COLLEGE LEVEL, research in mathematics teaching emphasized the content of freshmen mathematics courses, the preparation needed by mathematics teachers, and the mathematics needed for success in various college subjects. Also included were studies on remedial mathematics programs and programs for college freshmen poorly prepared in mathematics. Questions and commentary on these and other areas follow below.

1. *How poorly prepared in mathematics are some college freshmen?*

The excellent college records of many freshmen indicate a good foundation in mathematics but the large college classes of remedial mathematics remind one that some pupils have neglected the study of high school mathematics.

One study (46) discloses that the remedial students in mathematics were below average in I. Q. and weak in all subjects, and had scored at the first percentile on national norms for a widely used high school achievement test.

2. *Are remedial mathematics programs in college solving the problem of the student poorly prepared in mathematics?*

The remedial programs seem to be helpful to some pupils but no study shows that the college freshman receives, through a short remedial mathematics course, a desirable foundation in high school mathematics. In fact, one study (27) of 141 freshmen shows that the students in the remedial course gained little more than those who omitted it and took, instead, the regular college mathematics course. Attempts (8) are still being made to develop more desirable textbooks for remedial students. It is true that when a student reaches college he may have greater motivation to learn mathematics and more maturity than when he was in high school; yet it is very difficult, if not impossible, for anyone to get a thorough knowledge of high school mathematics from a 3-hour semester course in college.

3. Do students whose mathematics training has been interrupted by service in the armed forces (or for any other reason) make lower grades than their classmates?

The answer seems to be no. According to one study (123) students with a longer time lapse since studying a preceding mathematics course made higher grades than those with a shorter time lapse.

Of 805 students in the study, 172 had taken mathematics with only the summer intervening, and the remaining students had a mean lapse of 17 months. The close-proximity group made higher scores on the pretest, but the long-proximity group made higher scores on the final test. Many superior pupils in the long-proximity group made lower scores on the pretest than some of the less capable pupils in the close-proximity groups. Of course, this raises the question of the validity of a pretest in classifying students for experimental studies if the time lapse since the student's previous study of mathematics is not taken into account.

4. What are the best predictors of college success in mathematics?

A combination of high school marks in mathematics and the score on a mathematics achievement test seems to be a good predictor of success in college mathematics. One university (70) making a study of the placement of 1,832 freshmen found that the combination of scores on a placement test compiled by the university staff, the scores on the Iowa High School Content Examination, the rating on the Ohio State Psychological Examination, and the previous marks in mathematics was a very good predictor, but little better than the combination of high school marks and achievement test score.

5. Does success in high school mathematics lead to success in general college courses?

The students with high ability in high school mathematics usually made higher scores in general college courses than those with low ability in high school mathematics. One study (79) showed that students who failed to complete their college program were those who had made the lowest achievement scores in high school mathematics. Although most college programs seem to require competency in mathematics for a student to be successful, one study (113) shows an exception. Success in high school mathematics apparently had no relationship to the college success of a group of prospective elementary school teachers at one State university. Perhaps this is an indictment, in this scientific age, of a college program devoid of quantitative relationships.

6. Are the poorly prepared students usually from small high schools?

A study (79) of 600 college freshmen in one State indicated no difference in mathematics ability between students graduating from large high schools and those from small ones.

7. Is college general mathematics on its way out?

A recent study (38) based on the offerings of 200 colleges indicates that between 1948 and 1955 the number of colleges offering general mathematics increased. In general, the cultural type of general mathematics course is receiving the attention of research workers. The majority of these courses are 2 semesters with 6 hours credit. Although there is much experimentation with the content (65), (102), (38), (98), (18), usually at least half of it consists of topics from arithmetic, algebra, geometry, and trigonometry. The remainder is topics from analytical geometry, statistics, higher mathematics (65) and recreational mathematics.

Different methods of presenting the material have been used. One experiment (102) showed the geometric approach as slightly superior to the algebraic. A survey of 87 institutions (18) indicated an attempt to integrate topics. The respondents to a questionnaire suggested that a general mathematics course should contain material from the following 9 areas and that the amount in each area should decrease in the order given: Algebra, arithmetic, geometry, trigonometry, analytic geometry, statistics, mathematics of finance, the calculus and logic.

8. What mathematics is needed to understand first-year college subjects?

Several studies concerned with this question were reported. Most of them were based on an analysis of textbooks or other literature in the field. Elementary algebra and arithmetic occurred more frequently than other mathematics. The data reveal the mathematics in the textbook but not necessarily the mathematics that would be desirable to have in the textbook if the student could understand it. In general, the studies show that a knowledge of high school mathematics is needed to understand many first-year college subjects.

A recent study (118) of textbooks in engineering physics revealed that although the majority of the mathematics was algebra and plane geometry, 11 percent, nevertheless, was solid geometry, trigonometry, analytic geometry, and the calculus. This amount is an increase over the amount revealed by previous studies. Another study (45) shows that general physics textbooks used a large amount of elementary algebra but very little other mathematics.

General geology textbooks (107) used little mathematics. The mathematical skills necessary to understand these textbooks could be obtained from traditional high school mathematics courses. The majority of the problems involved algebra and simple ratios. Only a few needed trigonometry or exponential functions for their solution. Two questions were raised by the person making the textbook analysis: Would more effective learning take place under a mathematical, rather than a descriptive, approach? Should additional important topics in geology needing a mathematical approach be included in the freshmen course? The study revealed that many mathematical problems occur in advanced geology and that these problems would enrich general geology if the students had the prerequisite mathematical background.

In agriculture, a questionnaire from 75 colleges (68) revealed that they usually require elementary algebra, but recommended a more general course.

According to one study (45) the skills necessary for success in the physical science course at one State college may be obtained from high school algebra if the course emphasizes direct and inverse variation, functions, graphs, interpretation of data, and the ability to generalize.

The study of textbooks in various fields shows that elementary algebra—solution of equations, translations of verbal statements into diagrams or equations, use of graphs and variation—appears frequently. Advanced algebra—radicals, operations with polynomials, or factoring of quadratics—seldom occurs. The other mathematics concepts appearing are those found in the traditional high school mathematics courses.

9. *Should there be a special mathematics course for agriculture students?*

Of 75 colleges in 40 States (68), half recommended a general mathematics course and half the regular algebra course. The person conducting the survey concluded that there is a growing interest in requiring a special mathematics course for agriculture students.

10. *Are the mathematics offerings in pre-engineering programs of junior college fewer than those recommended by State university officials?*

An analysis (49) of 101 junior college catalogs showed that the offerings included all the topics recommended by the deans of engineering schools in State universities. In fact, the offerings usually exceeded the deans' recommendations.

11. *Should the meaning of mathematics concepts and operation be emphasized at the college level?*

Although fewer experiments on methods were reported at the college level than at any other level, at least two experiments indicated that the meaning of the concepts should receive greater emphasis.

In one experiment (18) on teaching the calculus the emphasis with the experimental group was on understanding concepts. The quality of learning of this group was superior to that of the control group. In another experiment (119), which emphasized understanding operations (in accord with the principles of modern psychology), the achievement of the experimental group exceeded that of the control group. The experimental group rated the course higher than the control group. Building an elementary mathematics course solely upon the wishes of the students is not recommended, but perhaps a favorable student attitude is conducive to effective learning.

12. *Do prospective teachers need more mathematics?*

A survey (10) of the graduates of one large university who are teaching revealed that they feel they should have had more mathematics.

Another survey (2) found that one-fifth of the general science and biology teachers had taken no mathematics in college and one-fourth of the chemistry teachers had taken one course or none at all. Also reported were attempts to teach these and other quantitative subjects to prospective teachers without using any mathematics.

An experiment (59) was reported in teaching the linear regression theory in elementary statistics to students who had not taken the calculus. Elementary school teacher programs (113) having no quantitative considerations in any course are an example of an attempt to educate teachers without introducing any mathematics.

13. *What should be the preparation of mathematics teachers?*

Studies (86), (28), (29) reported in 1955 and 1956 suggest that the undergraduate program should include 26 to 32 hours of mathematics. Non-Euclidean geometry, statistics, and history of mathematics are recommended in addition to the traditional college program through the calculus. The preparation is usually suggested in terms of semester hours of credit rather than specific competency. For example, the respondents to one survey (29) of 151 teachers, 65 administrators, and 35 college professors recommended 70 hours in general education, 24 in mathematics, 24 in education, and 1 year of student teaching.

Questionnaires (101) answered by 951 teachers suggested that the

5th year of teacher education should be composed of 50 percent mathematics courses and not more than 25 percent professional education courses. In these broad categories they recommended that few specific courses be required.

A group of New England teachers replying to a survey (10) recommended statistics in the mathematics teacher education program.

Integrated mathematics courses have been suggested. One research worker (114) concludes that the content should be mathematical and the purpose professional.

Research in the Teaching of High School Mathematics

ON THE HIGH SCHOOL LEVEL, research in mathematics teaching gave considerable emphasis to the need for students to *understand*. Also investigated was the extent to which physical aids and methods are able to contribute to attaining that goal. Experiments with introducing modern mathematics concepts to high school students reflect the desire of frontier thinkers in mathematics education to restudy the entire mathematics program. Discussion of this and other problems follows below.

1. *To what extent has the content of high school mathematics been under investigation and experimentation?*

Experimentation with the content of high school mathematics has taken place in several parts of the country and with different size groups. In one State (61) a group of 36 teachers developed a sequence of mathematics topics which were published as a series of high school textbooks. These topics were from algebra, plane geometry, and trigonometry, with many references to social problems.

In another State (Illinois) an extensive experiment is underway to develop not only new content but also a different emphasis on some of the traditional topics. Some concepts normally reserved for college students are incorporated in the course, some traditional definitions have been altered, and forced correlation is not emphasized nor are compartmental lines adhered to. Leadership stems from the University of Illinois, but the schools taking part in the experiment are not confined to that State.

In contrast to the State University experiment involving several hundreds pupils is an experiment with one elementary algebra class taught by various staff members of a large city college (53). Approximately 40 percent of the elementary algebra course consists of topics from traditional mathematics and applied mathematics.

Some content changes have been small. For example, one research worker (5) changed the content of elementary algebra only to the extent of adding drill lists of important words in algebra.

Perhaps nonmathematical words could have been included in the list, since according to one study (58), the vocabulary of many algebra textbooks is above the vocabulary level of some of the pupils.

2. Are the written problems in high school mathematics practical ones?

A study (120) of a sample of high school mathematics textbooks showed that, on a 4-point scale, more than one-third of the written problems fall in the top category of practicability. Although application problems are sometimes more difficult than the mathematical principle being illustrated, the study indicated that the written problems in general were on a lower vocabulary level than the explanatory sections.

3. Have there been radical changes in plane geometry content?

Even though the content of high school mathematics has been investigated for some time, an analysis (117) of 97 plane geometry textbooks revealed that the sequences for the past 15 years are in 3 general patterns: (1) A modern version of Euclid's sequence, (2) Legendre's sequence, and (3) postulation of the parallel lines-transversal theorem.

Another study (104) shows that the idea of continuity has received a change in emphasis. In fact, the continuity of a function is appearing more frequently in algebra textbooks and less in geometry. The incommensurable case in geometry is receiving more informal, and less rigorous, treatment than previously.

4. What is the content of high school general mathematics?

The content of general mathematics textbooks varies considerably. However, they usually include topics from elementary algebra, informal geometry, and a strong emphasis on arithmetic and its applications to the solution of concrete problems. Since many pupils who take general mathematics do not pursue college studies, some educators look at the mathematics used by semiskilled and unskilled workers as a key to the content of general mathematics. In fact, one investigator after studying the mathematics used by these workers, concludes that a 7th-grade mathematics course would fill their needs.

There are at least two fallacies to the status-quo curriculum approach. First, industry might require a greater use of mathematics if the workers had greater ability in the subject. Second, when the present pupils are adults, a greater knowledge of mathematics may be demanded of semiskilled workers than is now the case.

5. What methods of teaching geometry have been the subject of recent experimentation and study?

Methods of teaching geometry have been the subject of several investigations. Although some interesting studies have been completed, no special technique has been developed that has promise of changing greatly the present practice. Research studies report an emphasis on teaching for meaning through discovery and broader generalizations. One study (87) is devoted to super-generalizations in geometry with an emphasis on step-by-step discovery. In another (83), 2 groups of 21 pupils were matched as to mathematics achievement and I. Q. One group was taught plane geometry by the traditional method; the other was given specially prepared material leading to the discovery of relationships and encouraging generalizations. The author concluded that the two methods were equally effective. After a study of the mathematical ideas in law, a researcher (48) recommends that greater emphasis should be placed on understanding in high school mathematics, especially reasoning in geometry for those who may study law.

6. Does the study of solid geometry improve one's ability to determine space relationships?

Perhaps more research is needed to fully answer this question. One extensive study indicates that little, if any, improvement in space conceptions result from the study of solid geometry. In another experiment (51) with 41 pupils, the experimental group made significant gains when the analytic approach was used.

7. What methods of teaching elementary algebra have been the subject of recent experimentation and study?

As in geometry, there was no research that showed one best method of teaching algebra. A study (40) of elementary algebra textbooks, for example, gave 9 different methods for teaching signed numbers. In an experiment (15) with 55 pupils the emphasis in a unit on direct numbers was on the discovery method. The reported achievement of the control group and the experimental group was about the same. In another experiment (108) a "guess-and-check" method was used as an aid to better understanding of verbal algebra problems. Although no conclusive results were reported, the teacher of the class believed that it aided in understanding the concepts. Much has been written on the need for exploration and discovery in teaching mathematics. One experiment (69) involving 10 algebra classes used drill with one group and exploration and discovery with the other.

The pupils were tested by a special test developed to meet the objectives of the experimental group and also by the Douglass Survey Test. (The experimental group had greater gains in achievement than the control group, when measured by the special test but not significantly different when measured by the Douglass Survey Test.) The individual results on both the special test and the Douglass Survey Test indicate that a large percentage of students showed little achievement.

8. *Is instruction more effective if the pupils are taught in homogeneous groups?*

The answer has not been fully answered by research but it seems to depend upon the area of expected achievement. In one experiment (78) the pupils who were grouped homogeneously achieved more in fundamental skills in mathematics but less in problem solving than those not so grouped. In a survey (92) of a sample of schools in one State, the percentage of schools using homogeneous grouping in mathematics was 67 and in science, 40. These figures may reflect the success of the method in some schools but they do not imply that the method is desirable for all schools. Where homogeneous grouping has been received favorably, usually a combination of criteria is used in determining the grouping. A combination of I. Q., mathematics achievement test scores, previous marks in mathematics, and student interest rating has been used.

9. *Is greater achievement secured by small-group work within a class rather than by instruction to the entire class?*

The experiment (57) reported in this survey shows no significant difference in achievement of high school pupils taught in small groups within a class as compared with those taught in one group in a class. However, there was a most significant difference in pupil achievement under the various teachers in the experiment.

10. *What provisions are made for the superior pupil?*

Much has been written on this topic. A survey (92) in one State seems to be typical of reports from other geographic sections. Nearly half of the schools reported they were providing for the superior pupils through special programs, and two-thirds of the respondents stated they had special sections for the superior. Examples of special sections are 2-track programs in algebra and advanced algebra for college-bound pupils. The special programs were usually in schools with enrollments above 1,000. There seemed to be little or no difference in the methods used between the average pupils and the superior pupils. In the survey, 95 percent of the schools said that enrichment, even in special cases, was the chief means of providing for superior pupils.

11. *Is the core program an effective method of teaching mathematics?*

As with other methods of instruction, the answer to this question seems to depend to a large extent on the teacher. One of the outstanding strengths of the core approach is motivation through a social setting. The lack of teachers with necessary depth and breadth of training for core instruction is a weakness of the program. The fact that the sequential nature of mathematics does not lend itself readily to such an approach has caused many teachers to receive it with disfavor. This unpopularity is reflected in a survey (6) which showed that 12 percent of the schools with the core program included mathematics. In the secondary school, mathematics in the core seemed most successful at the 7th-grade level.

12. *What teaching method do teachers favor?*

A random sample of 131 teachers from one State (56) indicated that they preferred the traditional lecture and recitation method. In fact, 91 percent of these teachers considered it always or frequently valuable. Nearly one-half of them stated that field trips in mathematics are seldom worth the time, and the same percentage indicated student committee work is never or seldom valuable in teaching mathematics.

13. *Are there differences in mathematics ability between the sexes?*

Research has not presented conclusive evidence on this question. Some studies (74), (115) indicate that the number factors are dissimilar for boys and girls. Certain geometric concept patterns seem to be related to sex, which results in some geometry tests being loaded in favor of the boys. One experiment (100) indicates that boys can estimate answers better than girls.

14. *What is the best predictor of success in elementary algebra?*

Studies (116), (64), (76) seem to indicate that the best predictor of success in high school elementary algebra is the pupil's previous marks in mathematics, score on an aptitude test, score on achievement test, and degree of interest. Additional scores, such as I. Q. or reading test scores, increase the reliability of prediction very little. For success in geometry (16) a combination of the scores on an aptitude test and previous mark in algebra seems to be a valid predictor.

15. *Who has the greatest influence on high school pupils in their decision to pursue the study of mathematics?*

A survey (112) of 327 high school students who were mathematics contest winners during the years 1952–55 indicated that the high school teacher was the greatest motivating factor in their study of the subject. The pupils rated their parents as second in influence.

16. *Are physical devices of value in teaching high school mathematics?*

In this period of competent teacher shortage, many devices have been suggested for teaching mathematics. Mechanical devices for geometry instruction, aids in algebra, models, historical material, and field instruments have been advocated.

The various physical devices for teaching mathematics seem to make only a small contribution. More important than the device itself is the skill of the teacher in using it. After a survey of a sample of mathematics teachers, one research worker (97) concluded that most of the teachers who had concrete devices to aid in teaching mathematics did not know how to use them.

17. *Are mathematic contests for high school pupils desirable?*

A survey of contest winners in one State indicates that contests influenced many of them to pursue the study of mathematics. A national committee (71), however, after considering the disadvantage of contests, was reluctant to recommend them. A questionnaire (71) sent to 200 leaders in mathematics education indicated they were about equally divided as to whether mathematics contests are advisable for high school pupils.

18. *What does research say about the status of mathematics teachers?*

Although much has appeared in the press about the shortage of qualified mathematics teachers, only a few studies were reported on their qualifications or status. A study (109) of the mathematics teachers in one State found that the average mathematics teacher had a bachelor's degree, 18 years' teaching experience, and classes composed of 26 pupils; and received \$3,698 annually. According to another study, more men than women are entering this profession but women stay in it longer. Since other studies in this area were reported to be in progress, perhaps more data will be available in the near future.

19. *Are teachers doing a good job in the teaching of mathematics?*

More high school pupils know more mathematics than at any previous time in our Nation's history, yet the most competent teachers state that a better job ought to be done in teaching mathematics. Research workers (11) in mathematics education find that high school pupils' competency increases according to the number of years of mathematics taken, yet in their zeal for better mathematics teaching the teachers are unsatisfied with the pupils' achievement. The conclusion, from an experimental study (54) where emphasis was on critical thinking, was that little was accomplished.

This is a difficult skill to develop and a most difficult one to test (3). Perhaps too much is expected in too short a time. Another experiment on teaching elementary algebra, with emphasis on meaning of concepts, resulted in the research worker's questioning the value to the pupil of time spent in class. In contrast, we have the large group of scientists trained in our high school mathematics classes who are silently testifying to the excellencies of that training as they push back even further the frontiers of science and mathematics.

Research in the Teaching of Elementary School Mathematics

ON THE ELEMENTARY SCHOOL LEVEL, research in mathematics teaching ranged from a study of physical devices that might be helpful in this teaching area to patterns of thinking done by pupils in problem solving. The type of research varied from examination of textbooks to experimentation that involved teachers in a score of schools. The type and range of research on the elementary school level of mathematics teaching are indicated by the questions and commentary following below.

1. *Is arithmetic textbook vocabulary too difficult for the pupils?*

Studies (105), (43) show that the vocabulary of arithmetic textbooks is usually below the grade level for which the texts are used. For example, the 6th-grade textbook material examined varied from 4.7 to 6.9 with an average of 5.5. The vocabulary of problem solving material rated higher than explanatory sections. One researcher (43) pointed out that a textbook may rate within or below the grade level and yet be very difficult in certain sections. Also the lack of progression from easy to difficult verbal material throughout the textbook was not reflected in an average grade level. The technical vocabulary was about on the same grade level as the general vocabulary.

2. *Does emphasis on meaning of concept result in more effective teaching?*

In an experiment (66) involving 20 pairs of teachers from 20 schools, the rules of arithmetic were developed inductively through pupil participation with the experimental groups and traditional methods with the control groups. Emphasis was placed on the meaning of the operations and the discovery of a variety of ways to solve a given problem. The results of the study were in favor of the experimental group at the 5-percent level of confidence.

In another experiment (91) the meaning of the fraction concept was emphasized. The multiplication concept rather than the division concept of fractions was stressed. Fractions were considered not as broken parts of things but as submultiples or unit fractions. For example, three-eighths is three times one-eighth. The experimenter reported that the experimental group gained a better understanding of fractions than the control group.

3. *Is drill more effective if based on relationships?*

Two groups (89) of pupils were taught second-decade addition and subtraction facts by carefully developing the concept. In the experimental group, however, the drill was based on relationships and in the control group on simple repetitive practice. No difference was found in the achievement of the two groups. This was true of the ability to transfer knowledge to higher decades, to retain facts, and to compute mechanically.

4. *Which method of subtraction is most effective in helping students solve word problems?*

The take-away method, a combination of the additive and take-away methods, and the action method were used in teaching three groups of children. The experiment (22) involved three third-grade classes in each of five schools. In the action method the pupil was required to give the action needed to solve the problem. For example, John has 5¢ and he needs 15¢. How much more does he need? Here the pupil must determine the action and operation for the solution of the problem. It was found that no one method excels to any extent in helping pupils solve word problems in subtraction.

5. *Can computation be taught without teaching meaning?*

The general principle that objectives are best achieved when procedures are carefully planned seems to be true in the case of teaching for meaning (50), (17), (75). When computation was taught with special emphasis on meaning, gains and achievement in meaning resulted. When computation was taught without emphasis on meaning, little or no gain in meaning was the outcome. In teaching a unit on graphs, steady gains in meaning of graphs occurred only when the unit specially emphasized meaning. It would seem from the studies that careful planning is required if computation is to be taught with meaning.

6. *What procedures should be used to teach verbal problems?*

One study indicates that the following procedure is helpful in teaching verbal problems (7): First, the problem should be solved by the child with physical objects such as diagrams, pictures, or drawings, depending on the child's mathematical maturity; second, the pupil should describe the operation in words. That is, he should write what he did and why; third, he should be led to a compact algorism.

7. *Shauld division be taught as successive subtraction?*

An experiment (111) with 12 4th-grade classes, in which the experimental group was taught division as successive subtraction and the control group was taught it by the traditional method, showed no difference between the two groups on routine subtraction. However, the experimental group generalized more readily than the control group. A similar result (84) was reported from an experiment in which the emphasis was on the developmental method. There was no difference between the groups on immediate recall tests, but the developmental group was superior on delayed recall and generalizations.

8. *Are physical devices useful in teaching arithmetic?*

Many studies were reported on the use of physical aids in teaching arithmetic. These varied from cardboard strips to commercially built computing machines. The cardboard strips were used in one experiment (75) to implement discovery methods and in this experiment they seemed helpful. The calculating machine was used by several teachers in a control experiment (33). The experimental group in this study showed greater gains in computation and reasoning. However, the differences between the two groups were not large enough to be statistically significant.

Some experiments indicate that physical devices are helpful in teaching arithmetic, while others show no special value in these aids (55), (34). For example, one researcher (72) suggests that the abacus is a cultural curiosity, not a teaching device. Other leaders in mathematics education suggest that it can be a valuable teaching aid. A questionnaire survey (47) of a sample of elementary teachers revealed that 91.3 percent of them rated the physical devices as having value in teaching arithmetic. However, 85 percent stated that they needed training in using such devices. They also indicated that the infrequent use of physical devices was due to insufficient money to buy them.

A study (21) of 166 commercial aids showed that 109 of them could easily be made by the teachers. With overcrowded classrooms and heavy teaching schedules, however, teachers do not find time to construct many physical devices. Studies were made on the advisability of using games, songs, and poems in teaching arithmetic (14), (12), (121). A limited degree of success was reported.

An indication of the aids actually being used in the classroom is reported by a survey of a sample of teachers in one State (77). These teachers said that they used business forms, charts, pictures, workbooks, chalk boards, and bulletin boards as teaching aids. Most of the other aids were used very little. Perhaps one research worker (30) expresses the findings of many others when he concludes that physical devices, if properly used, may aid the children in learning, just as the chalk board may be an aid, but that the aids seldom teach.

9. In what areas of arithmetic are pupils weakest?

The research report indicated that pupils are weak in understanding arithmetical processes and basic treatments, ability to estimate reasonable answers, and application of processes to verbal problems (35), (81), (60), (24), (19).

Summary

COLLEGE LEVEL

ANALYSIS of the research for 1955–56 shows considerable emphasis on the content of college freshmen mathematics courses, both for students who go into science and mathematics and for students in other fields. Little experimentation was reported, though, on methods that might be more effective in teaching the content of these courses. Likewise, little research was reported on experimentation with different methods of teaching large college classes or on the use of aids, such as study aids, physical devices, or television. Studies concerning desirable preparation of mathematics teachers indicated that they should have a general cultural education in addition to 24–30 hours of mathematics. The courses recommended in mathematics usually include the calculus, non-Euclidean geometry, history of mathematics, and statistics. Considering the present teacher shortage it seems unlikely that additional hurdles in the form of more mathematics courses will be required of prospective teachers.

Are there not important concepts from modern algebra, statistics, and foundations of geometry that could and should be included in the present curriculum without introducing entire courses on these topics? Is there obsolete material in both education courses and mathematics courses that could be replaced with modern concepts? Research reported in 1955–56 does not answer these questions. Nor does it throw much light on the problem of which specific concepts should be included in teacher education courses. It sheds no light at all on the most desirable ways of teaching these concepts. In short, it seems from the 1955–56 research that the important specific understandings in mathematics that should be included in the teacher education program have not been determined.

Reports were received on the preparation of textbooks, some of which included modern topics in mathematics, but these reports contained very little evaluation of the material. Evaluations, if any, were usually of scores on achievement tests which evaluated traditional material rather than modern topics. In general, the evaluations were of students taught by very competent teachers who were especially enthusiastic about presenting modern concepts. No answer was given to the question: How successful would the material be, if taught by an average teacher, in increasing mathematical achievement in the special areas for which it was desired?

For the period 1955–56, studies were received on the mathematics needed in various areas of learning such as geology, physics, and agriculture. Most of these studies consisted of an analysis of textbooks in the specific fields. Although such studies are valuable they point out only the mathematics used in a particular group of textbooks. They do not show what mathematics would be helpful in understanding concepts in geology, physics, agriculture, and other fields. For example, in a freshman geology textbook the authors explain elementary quantitative concepts in highly verbose language. The authors successfully avoid mathematical symbols. However, advanced books and periodicals in geology frequently use mathematical symbols and operations. From an examination of college freshmen textbooks one might erroneously conclude that no mathematics is needed to understand this field. Would not more effective learning take place if: (1) The college freshman had an understanding of basis mathematics, at least through the elementary concepts of calculus? (2) The textbooks used mathematics to describe quantity and quality relationships?

The emphasis to develop science material without mathematics may be detrimental to the capable student. The college of education at one university developed an elementary school teacher program with so little mathematics that high school mathematics experiences had no effect upon the student's success in this program. In other words, prospective elementary school teachers were being taught quantitative considerations of economics, sociology, government, and science through verbose discussion. Later, as teachers, they would have to teach elementary school pupils the value of our precise number system and the value of its notation when they themselves did not have a background in these values.

HIGH SCHOOL LEVEL

Several of the studies on teaching high school mathematics placed emphasis on understanding concepts. The means of securing understandings varied from the use of cardboard strips to pupil discovery and experimentation. The teaching methods, no matter whether they involved physical aids or a grouping within a class, seemed to be beneficial to some extent when used by the advocate. Like the chalk board, the device itself seldom teaches, but when used by skilled teachers it helps instruction. Surveys seem to indicate that the chalk board is the most frequently used aid in mathematics teaching and the lecture-recitation the most popular method.

The research evidenced considerable activity in the study of the content of high school mathematics courses. The research varied from a listing of some practical problems for a course to a control experiment involving teachers in several States. No doubt the research on the frontiers of mathematics education is most valuable; but for implementation to take place, research will need to determine what specific changes can be put into effect with the present teachers. For example: How much modern mathematics and which modern mathematics can present teachers incorporate in their classes? How much change can be made in the content of high school courses and still give security not only to the teachers but to the parents? These and similar questions remain unanswered.

Research has shown that a combination of scores on an aptitude test and an achievement test, and previous mathematics marks is a good predictor of success in high school mathematics; but research has not shown what motivates pupils to pursue the study of mathematics. Research indicates that high school mathematics teachers are a dominant force in causing high school pupils to pursue the study of mathematics. Research has not shown, however, why these teachers are outstanding in their ability to influence pupils.

ELEMENTARY SCHOOL LEVEL

Several studies were reported on the use of physical devices in the teaching of elementary school mathematics. Although some devices seemed to be helpful when used by the experimenter, none of the devices made outstanding contributions.

Teachers indicated that some of the physical devices are desirable but that they lack the money to buy, or the time to make, them. Also that for effective teaching they needed instruction in using the aids.

Many of the studies were concerned with ways to teach arithmetic so that the concepts in operation will have more meaning to the pupil. These studies varied from teaching division as successive subtraction to teaching arithmetic with the aid of a computing machine. But in each case the purpose was to emphasize the meaning of the operation. Investigations in this area usually involve patterns of thinking done by pupils in problem solving. Although the research indicates that meaning should be emphasized in arithmetical instruction, it has not determined how much drill should be merely repetitive or how much it should include repeating the operation in a meaningful social situation.

Recommendations for Future Research.

The research in mathematics education reported for the years 1955–56 indicates that many persons have attacked small problem areas. Some of these areas of investigation are of minor importance. A casual reading of the summaries of research in the appendix reveals student studies of inferior quality with very little contribution to the teaching of mathematics. Little of the research was that of a team approach to the solution of a critical problem in this area. Even the studies of individual faculty members differed materially in their quality and contribution to mathematics education. Some of the investigations resulted in the compilation of teaching material or detailed experiments which were never published or were published in very abbreviated form. Such research has little impact on mathematics education.

This report on research in mathematics education lists certain unanswered questions and problems in mathematics education where there has been little research. However, it should not be assumed that these are the most critical issues in the teaching of mathematics.

Research reflects three important needs in mathematics education. *First, the identification of the crucial problems.* (These problems might be identified by State groups of teachers or by national committees. In any case, they should be identified to give direction to research.) *Second, greater coordination of effort in attacking the identified problems.* (Many of the problems are too large to be solved by a single individual; team work will be necessary.) *Third, publication and wide distribution of research.* (Unpublished research has little impact on classroom practice.)

Future advancement in the teaching of mathematics will depend upon the extent to which we identify the crucial problems, coordinate our efforts to solve them, and make the results known to the classroom teacher.

Unanswered Questions in the Teaching of Mathematics

THE QUESTIONNAIRE for gathering the research completed in 1955–56 asked the research workers to point out one or two questions still unanswered in the teaching of mathematics. These questions, or problems, cover a wide variety, both as to significance and as to type. The solution to many of them would require cooperative research by many teachers for several years.

The following 71 questions were all posed by the research workers. (Some of these questions, in one form or another, were asked by more than one researcher, while others appeared only once.):

1. What is the functional competency in mathematics at each of the following levels: Elementary school, junior high school, senior high school, junior college?
2. How well founded is the assertion that, in general, teachers of elementary school arithmetic do not have sufficient preparation and training?
3. Is the competency of students in college algebra and geometry less than it was 10 years ago?
4. Are the courses offered to high school students in consumer mathematics, general mathematics, etc., adequate to meet the needs of a person who expects to become an elementary school teacher?
5. What modern mathematics should be included in the education of prospective school teachers?
6. Which concepts in modern mathematics can be developed meaningfully with high school pupils?
7. What specific concepts in mathematics should be taught the slow pupil, the average pupil, and the fast pupil in the secondary school?
8. What difference in pupil achievement does the teacher's knowledge of arithmetic really make?
9. What place should modern mathematics have in a high school mathematics program?
10. How can poorly prepared teachers receive additional mathematics education?
11. What provisions can one make for gifted students when enrollments do not allow for homogeneous sectioning of classes?

12. Are high school teachers satisfied with the content of algebra and geometry courses?
13. How can the school show the community the importance of mathematics in every-day living?
14. What are some of the ways by which a teachers' college may finance studies in the teaching of arithmetic?
15. How can the teaching of arithmetic be made more challenging to elementary school children?
16. How can more of the capable students be encouraged to pursue advanced mathematics?
17. In what ways can capable students be encouraged to go to college?
18. What mathematics should we require of high school pupils?
19. To what extent do mathematics teachers move from one school to another, or leave the profession?
20. Is the traditional mathematics approach as effective in teaching mathematics as the general or integrated approach?
21. How successfully can freshman college mathematics be taught to high school pupils?
22. Is a uniform content for high school general mathematics desirable? If so, what should it be and in what year(s) should it be taught to obtain the maximum long-range benefit?
23. What kinds of 2-track programs can a small high school use?
24. What mathematics should there be in the general education program in the secondary school?
25. What is the mathematics deadwood that should be removed from the high school?
26. What in-service education program is of greatest value to mathematics teachers?
27. What are the characteristics of teachers who motivate students to pursue the study of mathematics?
28. What new demands in mathematics is industry placing upon its employees?
29. Should mathematics given to the slow pupil differ in kind or amount from that given to the average pupil?
30. Is the Carnegie unit the best method of recording growth in mathematics?
31. Is instruction in mathematics for 4 days a week for 2 years better than 2 days a week for 4 years?
32. Are mathematics clubs worthwhile?
33. Is color in textbooks better than black and white for promoting learning in mathematics?
34. To what extent can children transfer learning from one situation in mathematics to another?
35. What mathematics, if any, should there be in a college general education program?
36. Is mathematical aptitude a single identity or a pattern of aptitudes?
37. How effective is television in teaching mathematical concepts in the high school?

38. How desirable or effective is ability grouping within the class for teaching secondary school mathematics?
39. In what ways may instruction be improved for large classes?
40. What is the optimum class size in mathematics in the secondary schools?
41. What are the current instruction practices in college freshmen and sophomore mathematics classes?
42. Which is more desirable for elementary and secondary school teachers: Professionalized mathematics or professionalized methods courses?
43. At which grade level can pupils be expected to acquire an understanding of ratios?
44. To what extent do beginning college teachers receive supervision?
45. What projects seem desirable in the secondary school for emphasizing specific important principles in mathematics?
46. What patterns of thinking do children use in solving various kinds of verbal problems in mathematics?
47. What factors determine the mathematical ability of high school pupils?
48. To what extent are mathematics clubs helpful in motivating the study of mathematics?
49. Should the various mathematics algorisms be rationalized? If so, how far should each rationalization be extended and at what grade level?
50. How does the knowledge of the rationalization of algorisms assist transfer from one type to another and simplify subsequent mathematical thinking?
51. To what extent should emphases be placed on meaning in the teaching of mathematics?
52. How should college mathematics be presented to give the greatest contribution to general education?
53. What methods seem to be successful in rating mathematics instructors?
54. How does the achievement of students who have taken integrated courses in mathematics compare with the achievement of those who have taken compartmentalized courses covering the same subject matter?
55. What are the relative merits, from a functional viewpoint, of teaching elementary and secondary mathematics with a limited amount of drill, as compared with teaching it formally with maximum drill and a minimum amount of application to everyday problems?
56. Which operations in the present course of study in geometry, trigonometry, and algebra are little used in further study of mathematics or science?
57. What quantitative experiences do teacher training institutions usually give to prospective elementary school teachers?
58. What practical techniques are most promising for differentiating instruction to rapid and slow learners in heterogenous classes?
59. What mathematical competencies should elementary teachers possess?

60. To what extent is it desirable for pupils to collect mathematical data through experiments or surveys for their mathematical classes?
61. Is there an arithmetic readiness? If so, how can the teacher help pupils reach this state?
62. If persons in high school achieve the 29 competencies listed by the National Council of Teachers of Mathematics Post War Commission, will they be prepared for college mathematics?
63. What mathematical concepts are usually known or not known by pupils at various grade levels, 1–12?
64. Should mathematics be a laboratory science?
65. What reading skills are necessary for success in statement problems in algebra?
66. To what extent can a teacher use visual aids to develop concepts in higher mathematics?
67. What change in the attitude of students toward plane geometry would result if construction work were reduced or eliminated?
68. What competencies in geometry are needed by teachers in the elementary school?
69. What type of home work should high school students be required to do in mathematics?
70. To what extent should solid geometry be incorporated in the plane geometry course?
71. How does a school-wide testing program in mathematics affect the teaching of mathematics?

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Appendix: Summary of Research Studies

1. ADKINS, JULIA ELIZABETH. Historical and Analytical Study of the Tally, the Knotted Cord, the Fingers, and the Abacus. (Ph. D., 1956, The Ohio State University, Columbus.)

Major Faculty Adviser.—Nathan Lazar.

Problem.—To investigate man's use, through the ages, of physical devices to record and communicate numbers and survey school practices from *ca.* 600 B. C. to the present in teaching arithmetic with objective materials.

Procedures.—A wide sampling of research studies, books, and periodicals pertinent to the subject.

Major Findings and Conclusions.—Physical devices commonly used by adults became the basis for teaching arithmetic to children. It has been taught more years without Hindu-Arabic numerals than with them. Until the 16th century A. D. school arithmetic consisted almost entirely of learning to use some physical device. Only then did facility in using numerals become a goal in teaching arithmetic. This study suggests a sequence of activities for teaching it, providing a gradual transition from tally objects to structured abacus to Hindu-Arabic numerals.

2. ANDERSON, JAMES ANDREW. The Mathematical Background of Science Teachers in Southern Minnesota. (M. S., State Teachers College, Mankato, Minn.)

Major Faculty Adviser.—G. M. Wisnik.

Problem.—To determine the mathematics preparation of southern Minnesota high school science teachers and the adequacy of their preparation for teaching science.

Procedures.—A study was made of available material on the mathematical background of science teachers in the United States and of the answers to a questionnaire sent to southern Minnesota high school science teachers.

Major Findings and Conclusions.—Approximately one-fifth of the southern Minnesota high school general science teachers had never taken a course in college mathematics. Approximately one-fourth of the southern Minnesota high school chemistry teachers had taken one course in college mathematics or none at all.

3. BADGER, BLANCHE CRISP. An Analysis of the Evolving Evaluation Program in Elementary Geometry. (Ph. D., 1956, George Peabody College for Teachers, Nashville, Tenn.)

Major Faculty Adviser.—F. Lynwood Wren.

Problem.—To trace the development of the present evaluation program in elementary geometry and to present its educational implications.

Procedures. — Published literature was analyzed to determine current objectives for teaching elementary geometry. Each question of every selected test was considered in the light of its objectives. A check list of objectives was used to record data and summary tables were made.

Major Findings and Conclusions.—Widely used tests in elementary geometry seem to test knowledge of the subject adequately. The use of nongeometric materials is an emerging practice in teaching geometry but not in constructing tests. These rarely include questions on the history of geometry nor do they measure information on geometries other than the traditional Euclidean pattern. "The habit of clear thinking and precise expression," rated as the No. 1 objective by many authorities in the field, seems neglected as an objective in most of these tests. Although tests have been devised to measure understanding of the nature of proof and critical thinking, they have not been used extensively in the evaluation program of elementary geometry.

4. Bastram, Chester Enoch. An Analysis and Synthesis of Research Relating to Selected Areas in the Teaching of Arithmetic. (Ph. D., 1956, The Ohio State University, Columbus.)

Major Faculty Adviser.—Lowry W. Harding.

Problem.—To analyze and synthesize significant research in selected areas of arithmetic teaching as a basis for critically examining teaching practices.

Procedures.—Approximately 600 studies screened, 400 discarded as failing to meet the established criteria; the remaining 200 carefully analyzed and reported.

Major Findings and Conclusions.—(1) Number readiness is a matter of degree and is operative at all grade levels and with all number learning experiences. Several interrelated factors (experience, intelligence, maturation, degree of understanding, and intrinsic purpose) influence readiness to learn arithmetic. (2) Arithmetic problem solving is learned better, and computational skills are learned as well, through an activity program as through one more rigidly organized. In addition, activity programs contribute more to the child's social and emotional adjustment. (3) Regardless of the curriculum pattern, arithmetic instruction should be definitely planned. (4) Inductive methods are superior to deductive methods in the teaching of processes and generalizations. (5) Computational skills should be so presented that the sequential nature of the number system and its relationships are not violated. (6) Children profit from meaningful practice. This should be motivated by a social need and should follow understanding. (7) Methods allowing children to use crude procedures, concrete materials, and crutches are superior to methods which short-cut all these. (8) Cue words should not be used to teach problem solving. Instead, teachers should stress the meaning of the separate process as well as the interrelationships so that children can think rationally in selecting a process. (9) The activity method combined with classroom grouping appears to be the most satisfactory means to individual needs. (10) Several techniques and devices (standardized tests, teacher-made tests, interviews, observations, and anecdotal records) have a key place in arithmetic evaluation.

5. Bates, James Watt. A Remedial Program Based On Word Concepts to Aid the Slow Learner in First-Year Algebra. (A. M., 1954, Boston University, Boston, Mass.)

Major Faculty Adviser.—Henry W. Syer.

Problem.—To develop a remedial program in first-year algebra. To make this program a supplement to regular classroom procedure.

Procedures.—A list of the important words in Algebra was compiled and divided into 10 parts with drill exercises for each. Three methods of administering the drill exercises are given.

Major Findings and Conclusions.—Drill sections may be done without the teacher's help.

6. BAUCOM, THOMAS VICTOR. The Place of Mathematics in a Junior High School Core Curriculum. (Ed. D., 1956, Teachers College, Columbia University, New York, N. Y.)

Major Faculty Adviser.—Howard F. Fehr.

Problem.—To study the relationship of mathematics to the core program in junior high schools.

Procedures.—Questionnaires and personal letters were sent to schools and individuals in many sections of the United States and to every county in Maryland. An attempt was made to contact every school experimenting to combine mathematics in a core program.

Major Findings and Conclusions.—Mathematics and the core are better suited to correlation than to integration. The core class can provide social background and motivation. Combining mathematics with the core program has been more successful in the 7th grade than in other grades.

7. BELL, CARMEN. Making the Concepts and Operations of Arithmetic Meaningful Through the Use of Expanded Algorisms. (M. A., 1955, The Ohio State University, Columbus.)

Major Faculty Adviser.—Nathan Lazar.

Problem.—To investigate the intermediate steps between (a) use of devices to give meaning to arithmetical operations and (b) the use of the compact algorisms adopted by society to record its operations.

Procedures.—The intermediate steps consisted of algorisms expressed in greater detail ("expanded algorisms"). The sequence: (1) Verbal and numerical problems are solved by performing the operations with physical objects, diagrams, pictures, etc. (2) Each concept and operation is described in words by the child himself. (3) Various kinds of expanded algorisms were used to typify the operations performed with various devices and to lead to the usual compact algorism.

Major Findings and Conclusions.—The teacher should incorporate the three steps into teaching of arithmetical operations.

8. BEZUSKA, STANLEY J. Semitutorial methods in mathematics (Freshmen and Seniors in the College of Arts and Sciences). (1955, Boston College, Boston, Mass.)

Problem.—To prepare a college mathematics textbook containing a review of high school mathematics for poorly prepared college freshmen.

Procedures.—A textbook was written which consisted essentially of two sections: (1) College freshman mathematics content; (2) a series of supplementary lessons written in simplified and detailed manner dealing with the essentials of algebra, trigonometry, and analytic geometry. The purpose of section 2 was to provide necessary background and familiarity with techniques for understanding section 1.

Major Findings and Conclusions.—Poorly prepared students can succeed in college mathematics under this semitutorial scheme.

9. BROTHEN, FRANK J., *and* THERRIEN, PAUL J. Secondary School Mathematics Requirements of Industrial Firms of Greater Boston. (Ed. M., 1954, Boston University, Boston, Mass.)

Major Faculty Adviser.—Henry W. Syer.

Problem.—To discover the amount of mathematics a skilled worker found essential in his job.

Procedures.—Questionnaire sent to personnel of 50 industrial plants in Boston and of 10 trade unions; personal visits made to industrial organizations.

Major Findings and Conclusions.—For most people, no further mathematics education is needed beyond the 9th-grade course. This course should include the significant principles of arithmetic, algebra, geometry, drafting, and the elementary notions of other mathematics subjects.

10. BROWN, RICHARD GILBERT. A Followup Study of the Mathematics Graduates of Boston University, School of Education. (M. Ed., 1955, Boston University, Boston, Mass.)

Major Faculty Adviser.—Henry W. Syer.

Problem.—To determine how satisfactory is the training in the teaching of mathematics offered by the School of Education, Boston University, as judged by the graduates.

Procedures.—Questionnaire sent to a randomly selected sample of the university's graduates.

Major Findings and Conclusions.—Most graduates considered their university training necessary and sufficient. They judged the following areas less than sufficient: Methods for teaching arithmetic, general mathematics, algebra I, and plane geometry. They recommended more required work in mathematics and a course in elementary statistics. They also recommended that the State Board should establish (1) a special State mathematics coordinator or supervisor—an appointive position with money and authority to carry out a program—and (2) in-service training for mathematics teachers.

11. BROWN, ROBERT CARL. Functional Competence in Mathematics of Louisiana High School Seniors. (Ph. D., 1956, Peabody College, Nashville, Tenn.)

Major Faculty Adviser.—J. Houston Banks.

Problem.—To determine: (1) The degree of functional mathematics competence attained by high school seniors in Louisiana; (2) the types of errors students make in applying their mathematics; (3) the significant differences in mathematics competence according to the training of the pupils; and (4) recommendations for improvements based upon the findings of the study, the literature in the field, and the experiences of the writer.

Procedure.—The Davis Test of Functional competence in Mathematics was administered to the seniors of 28 selected public high schools in the State. ("Functional competence" is defined by the 29 items of the check list in the guidance pamphlet prepared by the Commission on Postwar Plans. A group is considered functionally competent on an item if 67 percent of them get the correct answer.) The traditional programs for 2, 3, and 4 years are still the best for attaining functional competence. The more years of mathematics a student has had, the higher is his functional competence.

Major Findings and Conclusions.—Basic weaknesses in functional mathematics competence appeared in the following areas: (1) Nature of the decimal system, both place value and base; (2) estimating amounts and answers; (3) nature of approximate numbers and measurement; (4) statistics; (5) reading tables, interpolation, interpretation; (6) how and when to use the Pythagorean Theorem; (7) conversion of units; (8) formulas and problems involving formulas; (9) exponents; (10) trigonometry—tangent and sine relationships; and (11) consumer problems.

12. BURK, BILLIE ROSSON. Sources, Descriptions, and Suggested Uses of Stories, Poems, and Games Involving Numbers in Grade One. (M. A., 1956, University of Texas, Austin.)

Major Faculty Adviser.—Frances Flournoy.

Problems.—To compile enrichment materials for first-grade arithmetic, and to suggest procedures for using them.

Procedures.—A survey was made of story books, songs, poems, and games to determine those judged suitable for the first grade. (Judgment was based on the writer's experience as a first-grade teacher.) The mathematical understandings to be developed were determined by a study of materials published as a guide to the content of first-grade arithmetic.

Major Findings and Conclusions.—A wide variety of materials is available for first-grade use (included was reference to 30 books, 50 songs, 40 jingles, 40 games). Teachers should acquaint parents with materials of these kinds to give their children simple number experiences as a preparation for first-grade arithmetic.

13. BURKHARD, SARAH. A Study of Concept Learning in Differential Calculus. (Ph. D., 1956, Teachers College, Columbia University, New York, N. Y.)

Major Faculty Adviser.—Howard F. Fehr.

Problem.—To determine methods and materials to increase understanding of concepts in the calculus.

Procedures.—Mathematical literature was searched to determine the nature and important aspects of the concepts of differential calculus. The experimental portion of this study used by 9 regular classes in differential calculus representing 235 students. Two classes were taught in the conventional manner with emphasis on skills and problem-solving; the others, in the experimental manner with increased emphasis on understanding concepts of the calculus.

Major Findings and Conclusions.—The differences observed between the two groups of students were primarily those of quality of concept.

14. BRUNSKILL, BENTON CLARK. A Critical Compilation of Arithmetic Games for the Elementary School. (M. A., 1956, The Ohio State University, Columbus.)

Major Faculty Adviser.—Lowry W. Harding.

Problem.—The problem is 3-fold: (1) Is there a place for drill in teaching arithmetic? (2) If drill is necessary, are arithmetic games a desirable means to secure recurrent experiences? (3) Where can suitable games be found that will provide necessary drill?

Procedures.—Books, periodicals, and yearbooks were consulted to find out what specialists in the field of arithmetic thought about the need for drill and the value of arithmetic games. Most of the games were described in periodicals.

Major Findings and Conclusions.—Drill is needed in arithmetic teaching. Understanding must come first. Arithmetic games, carefully chosen as to certain criteria and number concepts to be developed in each grade, afford practice. A teacher would have to look through many magazines and manuals. The greater part of this study is devoted to a compilation of games, organized according to the concepts to be developed.

15. BURT, PAULINE LUCILLE. Experimental Introductory Unit With Directed Numerals. (M. A., 1956, Illinois State Normal University, Normal.)

Major Faculty Adviser.—Francis R. Brown.

Problems.—To create interest in algebra through an introductory unit based on the study of number development and of directed numerals. To encourage students to make discoveries on their own.

Procedures.—The early history of numbers and the study of directed numerals were taught to an experimental group of 17. A control group of 45 (not matched with the experimental group) was taught directed numerals following the textbooks. Evaluation was made by comparing the two groups' gains on the directed numerals test, by using the student interest inventory, and by a daily log of the students' behavior in the classroom.

Major Findings and Conclusions.—(1) The experimental group indicated a definite interest in the use of the discovery method. (2) The discovery method seemed useful at all learning levels. (3) The gains of both groups were about the same.

16. CAMMACK, BOYD VOLORIS. Prediction of Achievement in Plane Geometry. (M. A., 1956, Iowa State College, Ames.)

Major Faculty Adviser.—Frank Wellman.

Problem.—To develop a table of probability as an aid to predict achievement in plane geometry.

Procedures.—Quadriserial correlations and loss tests were computed as the variables were dropped one by one. When Otis gamma I. Q. and English grade-point average were dropped individually, the adjusted quadriserial correlations between the remaining variables and the criterion were 0.6113 and 0.6138, respectively. When both variables, Otis gamma I. Q. and English grade-point average, were dropped, the other two variables produced a quadriserial correlation of 0.607 with the criterion.

Major Findings and Conclusions.—The algebra grade-point average and plane geometry aptitude test score were the best variables of the four investigated to develop a discriminant equation for predicting achievement in plane geometry.

17. CHABOUDY, FRANK REXFORD. The Value of Planned Instruction in the Reading of Graphs at the Sixth-Grade Level. (M. A., 1956, San Diego State College, San Diego, Calif.)

Major Faculty Adviser.—Francis A. Ballantine.

Problem.—To determine the effect of planned and systematic instruction upon the graph-reading ability of 6th-grade pupils.

Procedures.—A controlled experiment consisting of 4 6th-grade classes from 3 elementary schools. No attempt was made to equate the classes as to number, sex, or scholastic ability. The 2 control classes (50 pupils) were evaluated as one group against the 2 experimental classes (64 pupils) as the other group. A teacher-prepared pretest and posttest were administered consisting of 15 problems on circle graphs, 15 on line graphs, and 15 on bar graphs. Each class was taught by the regular classroom teacher.

Major Findings and Conclusions.—A significant difference between the groups at the beginning of the experiment increased to such magnitude by the end that obviously the experimental groups learned considerably more during the instructional period than had control group during the school year. Sixth-grade pupils find circle graphs the easiest to comprehend, bar graphs second, and line graphs the most difficult.

18. CHERKAUER, RUDOLPH J. Mathematics in the General Education of Four-Year College Students. (D. Ed., 1955, University of Buffalo, Buffalo, N. Y.)

Major Faculty Adviser.—Richard M. Drake.

Problem.—To ascertain the role of mathematics in the general education of 4-year college students.

Procedures.—A questionnaire was developed and mailed to the chairman of the mathematics departments at United States colleges and universities having general education programs. From a possible 103, 87, or 85 percent, were returned. Available literature and a second questionnaire were used to ascertain educators' views on the subject.

Major Findings and Conclusions.—Seventy-two of the institutions, or 83 percent, include mathematics as part of the general education program. The major purposes given for mathematics in general education, along with the percent of institutions listing them, were: (1) To develop and use essential concepts, understandings, relationships, and generalizations—85 percent; (2) to gain cultural value and understanding of the method of mathematics and its potentialities—72 percent; (3) to gain basic mathematics skills—67 percent; (4) to see, understand, and use applications of mathematics—63 percent. Areas in their order of frequency were algebra, arithmetic, geometry, trigonometry, analytical geometry, statistics, finance, the calculus, and logic.

19. COLLIER, CALHOUN C. The Development of a Noncomputational Mathematics Test for Grades 5 and 6. (Ph. D., 1956, The Ohio State University, Columbus.)

Major Faculty Adviser.—Ruth Streitz.

Problem.—To develop a specific reliable and valid instrument to measure the objectives now commonly neglected in evaluation—arithmetical understanding and reasoning ability.

Procedures.—The procedures included: (1) Examination, for test content, of methods books, 5 courses of study, and 7 series of textbooks; (2) administration of tryout tests to 660 pupils in grades 5 and 6 in selected elementary schools in Ohio, representing four different socioeconomic groups; (3) administration of the final form of the test to 624 pupils in grades 5 and 6, forming a representative sample; (4) application of the split-halves (odds-evens) method and the Spearman-Brown Prophecy formula to determine the reliability of Test Form D; (5) determination of the validity of Test Form D by obtaining Pearsonian correlation coefficients between scores on the test and scores on six external criteria.

Major Findings and Conclusions.—The following conclusions were drawn: (1) Improvement in evaluation instruments and procedures has not kept pace with the changing emphasis in instructional objectives in arithmetic; (2) the test items, in general, are closely related to real-life situations of 5th- and 6th-grade pupils; (3) pupils in both grades show serious weaknesses in ability to form sound judgments on the basis of quantitative data; (4) pupils from a relatively high socioeconomic environment appear to have developed a higher degree of arithmetical understanding and reasoning ability than pupils from one relatively low.

20. CONNOR, RONNOC HILL II. High School Geometry in Transition. (M. A., 1956, Illinois State Normal University, Normal.)

Major Faculty Adviser.—Douglas R. Bey.

Problem.—To determine the purpose, content, method of presentation, and type of student who should take high school geometry.

Procedures.—A jury of 10 men prominent in the fields of pure mathematics and mathematics education and 14 in-service-teachers cooperated by filling out a questionnaire on the high school geometry course. The data thus obtained formed the basis of the study.

Major Findings and Conclusions.—The jury decided that the major purposes of high school geometry are to develop a maturity in mathematics and to teach the scientific approach to problem solving. The course should include such topics as non-Euclidean geometry, projective geometry, and the basic concepts of geometry. Development of critical thinking is no longer considered the major reason for teaching high school geometry.

21. CRAVEN, PATRICIA GILLETTE. A Survey of Certain Instructional Materials for Developing Concepts and Skills in Primary Arithmetic. (M. A., 1956, University of Texas.)

Major Faculty Adviser.—Thomas A. Horn.

Problem.—To survey, describe, give sources, and point out the skills and concepts that might be better developed through certain materials falling into any one of four categories: (1) Commercial materials, (2) teacher-made materials, (3) films and filmstrips, (4) children's literature. The materials selected from each of these four types were only those which would concern the development of arithmetic skills and concepts.

Procedures.—First, second, and third grade materials on description and sources were organized as to teaching (1) the number system, (2) the four fundamental processes, (3) fractions, measurement, and money, (4) number vocabulary. A specific value in concept or skill was pointed out for each item in each category.

Major Findings and Conclusions.—At least 166 commercial materials are available to develop primary-grade arithmetic skills and concepts; and, in addition, 109 materials that teachers could use for the same purpose. Twelve films and 41 children's books were found that use concepts usually taught in primary-grade arithmetic.

22. CRUMLEY, RICHARD D. A Comparison of Different Methods of Teaching Subtraction in the Third Grade. (Ph. D., 1956, University of Chicago.)

Major Faculty Adviser.—Maurice L. Hartung.

Problem.—To determine what effects different methods of teaching the concept and skills of subtraction have upon the development of skills to solve problems involving numerical quantities.

Procedures.—Three 3d-grade classes at each of five schools were taught by the subtractive-subtraction method, the combination method, and the action method, respectively. The experimenter observed the instruction and conferred with the teachers once a week. He prepared a test to measure achievement in solving one-step problems involving addition or subtraction, and immediately after the experimental period, interviewed five pupils from each class, asking each to demonstrate the meaning of subtraction.

Major Findings and Conclusions.—None of the three methods proved to be superior to a great extent. Many factors were not controlled, including the pupils' tendency to decide upon a process on the basis of the generalization: Add to get a larger number, subtract to get a smaller one.

23. CURTIS, ETHEL LOUISE. A Study of the Mathematical Backgrounds of students Who Are Preparing To Be Elementary Teachers and Who Are Enrolled in Certain Colleges in Minnesota. (Ph. D., 1955, University of Minnesota, Minneapolis.)

Major Faculty Advisers.—P. R. Grimm and L. J. Brueckner.

Problem.—To ascertain the achievement in elementary arithmetic and mathematical relationships attained by prospective elementary teachers.

Procedures.—(1) A test was constructed to measure the understanding of mathematical concepts in elementary arithmetic. (2) A battery of tests was administered and the resulting data were analyzed to ascertain the status of these students in elementary arithmetic.

Major Findings and Conclusions.—A wide range of achievement in each group was observed on each test. Students were most successful in problem solving and computation in that order. The test of quantitative relationships proved to be the most difficult. All tests were correlated positively. The highest coefficient was +0.785 between problem solving and computation. The lowest was +0.450 between computation and quantitative relationships.

24. DAVEY, MELVIN LEROY. Survey of Arithmetic Skills in Grades 6, 7, and 8 at Plentywood, Mont. (M. E., 1956, Montana State University, Missoula.)

Major Faculty Adviser.—John F. Staehle.

Problem.—To measure the arithmetic abilities of the students in grades 6, 7, and 8 at Plentywood, Mont., and to identify skills with which each class as a whole had difficulty.

Procedures.—The Iowa Every-Pupil Test of Basic Skills, Test D, Form O, was administered to 52 6th-grade students, 49 7th-grade students, and 45 8th-grade students during the 3d week of September 1955. The 73 items on the test were divided into 22 skill areas.

Major Findings and Conclusions.—The 3 grades scored the following mean percentages for the entire test: 6th grade, 34 percent; 7th grade, 55 percent; 8th grade, 69 percent. The skills on which the three grades scored below their means for the entire test were: (1) Knowledge of common geometric figures and terms, (2) ability to estimate or describe quantitatively, (3) ability to compare the size of numbers and fractions, (4) knowledge of the common processes, and (5) division of decimals.

25. DAVIS, SHULA FIERCE. General Mathematics in Selected Accredited Negro Secondary Schools in Virginia. (M. A., 1956, Virginia State College, Petersburg.)

Major Faculty Adviser.—A. G. Macklin.

Problem.—To ascertain the status of general mathematics in a selected group of Negro accredited secondary schools of Virginia.

Procedures.—A list of the Negro accredited secondary schools of Virginia was obtained from the State Board of Education. To each of these 98 schools questionnaires were mailed concerning instruction in general mathematics. Fifty-seven of the 98 schools returned usable data.

Major Findings and Conclusions.—(1) The objectives of general mathematics emphasizing practical and transfer values are considered most important. (2) The basic texts most commonly used are *Refresher Arithmetic* by Stein, *Living Arithmetic* by Boswell and others, and *Mathematics in Life* by Schorling and Clark. (3) The content of general mathematics is basically arithmetic, algebra, and geometry. (4) Pupil reactions toward pursuit of courses in general mathematics are largely indifferent in both required and elective courses. (5) Fifty-nine percent of the respondents to the questionnaire were mathematics majors in undergraduate school, and 56 percent had studied beyond the bachelor's degree. (6) The handicaps most frequently encountered by teachers of general mathematics in both required and elective courses are inadequate background and a low level of pupil interest.

26. DICKEY, DARRELL ALBERT. Mathematical Recreations for Secondary Schools. (M. A., 1956, Illinois State Normal University, Normal.)

Major Faculty Adviser.—T. E. Rine.

Problem.—To collect a usable list of mathematical recreations that may be used in secondary school mathematics classes.

Procedures.—Examination of literature and its analysis to determine usability in algebra, geometry, and general mathematics.

Results.—(1) A list of recreational materials was prepared. (2) Considerable material on fallacies in reasoning was compiled.

27. DICKEY, JOHN W. Arithmetic Abilities of College Freshman. (1955, State Teachers College, Newark, N. J.)

Problem.—To develop a remedial program for college freshmen.

Procedures.—Form A of the Basic Skills in Arithmetic Test, published by the Science Research Associates, was given to 141 freshmen. Twenty-four of them placed in the bottom quarter in terms of 8th-grade norms and they became the remedial group. Each student was asked to purchase an 8th-grade workbook and systematically work through the material during the first semester. These students, divided into 6 groups, met by themselves several times for self-help. The department faculty on several occasions met with those groups who were a part of their regular classes in mathematics 104.

Form B of the same standard test was administered to the entire group of 141, including the remedial group of 24.

Major Findings and Conclusions.—Although the remedial group of 24 made significant gains with remedial help, all 141 students made significant gains without this help. The data point both to the ineffectiveness of the remedial program, and to by-product values of the course in mathematics 104 (arithmetic for colleges) in terms of substantial growth in arithmetic skills.

28. DI PIETRO, ALPHONSO JOSEPH. A Program in Mathematics Education for West Virginia Teachers of Secondary Mathematics. (Ph. D., 1956, George Peabody College for Teachers, Nashville, Tenn.)

Major Faculty Adviser.—F. Lynwood Wren.

Problem.—To investigate and develop a functional program of mathematics education for prospective teachers of secondary mathematics in West Virginia.

Procedures.—A tentative program for prospective teachers was formulated. It was evaluated by a selected jury of 109 mathematics education specialists. The jury's comments were used to write a description of courses that should go into a mathematics education program.

Major Findings and Conclusions.—The prospective teacher of secondary mathematics should take no fewer than 32 semester hours in certain courses under supervision of the mathematics department. The first year of the present mathematics education program in West Virginia should include a thorough treatment of the number concept, the nature of proof, the concepts of function and measurement, and algebra, which would incorporate some of the elementary aspects of modern algebra and trigonometry.

In the second year, analytic geometry should be integrated with the calculus. The mathematics education program must provide for considerable work beyond the calculus and must include at least some training in modern algebra, modern geometry, statistics, history of mathematics, and applications and methods in teaching secondary mathematics.

Modern mathematics should be emphasized in the teacher training program.

29. DONOVAN, SISTER MARY MATHEW. A Study of Selected Data Relative to the Education of Texas Teachers of Secondary School Mathematics in Order To Suggest a Program for Their Future Education. (Ed. D., 1956, University of Houston, Houston, Tex.)

Major Faculty Adviser.—William J. Yost.

Problem.—(1) To investigate the opinions and attitudes of secondary school mathematics teachers and administrators and college mathematics professors as to what constitutes an adequate preparation for secondary school mathematics teachers in Texas. (2) To study and evaluate the data in order to draw up a program for ways and means of giving prospective teachers of secondary mathematics the preparation they need.

Procedures.—Questionnaires were sent to 300 teachers of secondary mathematics and 200 secondary school administrators and college professors. Responses came from 151 teachers, 65 administrators, and 35 college professors. Reports, programs, and research papers pertaining to the education of mathematics teachers for secondary schools also provided data for the study.

Major Findings and Conclusions.—(1) Emphasis should be placed on a broad general education, including courses in art, music, religion, and philosophy. (2) Courses in algebra, geometry, the calculus, trigonometry, and history of mathematics have value. (3) A minimum of 70 hours in general education, 24 in mathematics, and 24 in professional courses was suggested. (4) Methods courses should be taught by a member of the mathematics department who has had experience in teaching secondary mathematics, or by professor responsible to both the department of mathematics and the department of education. (5) Student teaching should cover 1 year. (6) College mathematics teachers have a responsibility to create in the students an enthusiasm for mathematics and a desire to teach it.

30. EIDSON, WILLIAM PATTON. The Role of Instructional Aids in Arithmetic Education. (Ph. D., 1956, The Ohio State University, Columbus.)

Major Faculty Adviser.—Lowry W. Harding.

Problem.—To define the role of instructional aids in the elementary school arithmetic program.

Procedures.—A comprehensive analysis of pertinent literature was made. Criteria for arithmetic instructional aids were developed by applying commonly accepted principles of education.

Major Findings and Conclusions.—(1) The most effective results are obtained by placing major emphasis on arithmetic instructional aids as sources of data rather than as supplementary devices. (2) Instructional aids themselves seldom teach arithmetic; the teacher's role is paramount.

31. EUSTRATIADES, DEMETRIUS JOHN. Historical Aspects of Solid Geometry. (M. A., 1956, The Ohio State University, Columbus.)

Major Faculty Adviser.—Nathan Lazar.

Problem.—To trace the origin and development of solid geometry and collect and present historical material related to its concepts and facts.

Procedures.—A review and analysis of the literature in the field.

Results.—The theorems in Euclid pertaining to solid geometry were compared with those found in solid geometry textbooks. The contribution to the study of the regular polyhedra, pyramids, cylinders, cones, and spheres made by the Babylonians, Egyptians, and Greeks, are discussed in detail. Many ancient mathematicians, especially the Greeks, are mentioned and quoted and the story is told of the famous duplication problems. A list of the theorems of the geometry of the sphere appears in the conclusions.

32. REDMAN, JEAN CAMERON. The Remedial Teaching of Fractions in the Secondary School. (M. A., 1956, The Ohio State University, Columbus.)

Major Faculty Adviser.—Nathan Lazar.

Problem.—To study some approaches to remedial teaching of the meaning of fractions in the secondary school.

Procedures.—A review was made of the literature in this field.

Major Findings and Conclusions.—This study suggests physical manipulations to show the student the relative sizes of different fractions, as these fractions apply to objects and groups of objects of the same size, and also to objects and groups of objects of different sizes. The concept of equivalent fractions and the comparison of fractions by changing to a common denominator are other fundamental aspects of meaning developed in the study, especially through the use of "quasi-three-dimensional" cardboard strips, which seem particularly well suited for this purpose.

33. FEHR, HOWARD F. (Teachers College, Columbia University), MCMEEN, GEORGE, Newark, N. J.), and SOBEL, MAX (South Side High School, Newark, N. J.). The Use of Hand-operated Calculating Machines in Learning Arithmetic. (1956, Memorial School, Cedar Grove, N. J.)

Problem.—To determine the degree of improvement in computation and reasoning ability made by 5th-grade pupils using calculating machines in the study of arithmetic.

Procedures.—Four 5th-grade classes were selected as the exprimental group. Two classrooms were equipped with Monroe educator model hand-operated calculating machines, one to each pupil. The regular textbook and syllabus were followed without any deviation. The control group was scattered in four communities where the same textbook and/or syllabus was used. The control and experimental groups were matched according to socio-economic backgrounds, I. Q., and mathematics achievement.

Major Findings and Conclusions.—The experimental group made significant gains in both computation and reasoning. Although the gains were significant, the difference in achievement was not sufficiently large to be statistically significant. That is, there was not a significant difference in achievement between the two groups at the end of the experiment. All indications were, however, that the machine-taught group would have excelled if the experiment could have run for 1 full year.

34. FLORY, HERBERT SPENCER. The Use of Physical Devices in the Teaching of 6th-Grade Arithmetic. M. A., 1955, The Ohio State University, Columbus.)

Major Faculty Adviser.—Ruth Streitz.

Problem.—To determine whether physical devices in teaching arithmetic or the textbook-drill method of instruction gave children a better understanding of arithmetical concepts.

Procedures.—An experimental group used physical devices, participated in individual and group activities that required the use of arithmetic skills, and used the textbook as supplemental material. Three control groups were taught by the textbook-drill method. Forms AA and DD of the California Arithmetic Test, Grade

4–6, were used to compare the experimental group with the control groups.

Major Findings and Conclusions.—The children of the experimental group who were in the low average and high average intelligence quotients range had a higher mean gain in years and months than did the children of any of the control groups. In the test of number concepts the experimental groups had a higher mean gain than any of the control groups.

35. FORRESTER, HAZEL B. A Study of 329 4th-Grade Pupils' Understanding of a Selected Arithmetical Vocabulary. (M. Ed., 1955, University of Texas, Austin.)

Major Faculty Adviser.—Thomas D. Horn.

Problem.—(1) To determine the knowledge that 4th-grade children in the Austin public schools have of a list of arithmeical terms considered essential to normal progress at their grade level. (2) To compare the type of arithmetical terms most known with those terms least known by the pupils. (3) To evaluate how well the aim of helping 4th grade pupils acquire a technical vocabulary in arithmetic is being realized.

Procedures.—A vocabulary test, consisting of 150 arithmetical terms, was developed and administered to 329 4th-grade pupils in the Austin, Tex., public schools. The pupils' understanding of the arithmetical vocabulary, as measured by the test instrument, was determined.

Major Findings and Conclusions.—A substantial number of the terms were apparently not known by the pupils; at least as measured by the vocabulary test. Of the 329 testees, 70 percent knew at least 107, or 71 percent, of the terms tested. Twenty-one, or 14 percent of the terms tested, were known by less than 59 percent of the testees. Of the 149 terms tested, only 20, or 15 percent, were known by 90 to 94 percent of the testees. No basic term involved in the 4 fundamental processes of addition, subtraction, multiplication, and division was known by 60 percent or less of the pupils. Eight of the terms were known by 60 percent or less of the pupils.

36. FULLERTON, CRAIG KERR. A Comparison of the Effectiveness of Two Prescribed Methods of Teaching Multiplication of Whole Numbers. (Ph. D., 1955, State University of Iowa, Iowa City.)

Major Faculty Adviser.—Herbert F. Spitzer.

Problem.—To compare the effectiveness of two methods of teaching the "easy" multiplication facts in the 3d grade. The prescribed developmental method is an inductive method characterized by pupil development of multiplication facts from word problems by such means as counting, making marks, drawing pictures and diagrams, adding, and using the number line. The other method, the prescribed conventional method, introduces multiplication process and facts through addition examples and pictures printed in the lesson material. The pupil takes no active part in developing the facts.

Procedures.—The data for the investigation were contributed by 30 3d-grade classes in Iowa.

Major Findings and Conclusions.—The prescribed developmental method is superior to the conventional method.

37. GAGER, WILLIAM A. Functional Mathematics in the Secondary Schools. (University of Florida, Gainesville.)

Problem.—To study the teaching methods, outcome, organizations, and content of mathematics courses currently taught in secondary schools in order to find out how to improve and enrich the secondary mathematics curriculum.

Procedures.—Thirty-six secondary mathematics teachers from all types of high schools and all levels of instruction, with a large number of specialists who served as consultants, studied school practices and the pertinent literature. They made a list of concepts and then tried to determine the fundamental principles of mathematics through which the concepts would have to function. Finally, they prepared textbooks for each grade, 7 through 12, of the secondary school.

Major Findings and Conclusions.—(1) The separate traditional courses, as now taught in most high schools, overemphasize the learning of abstract ideas by drill procedure. (2) Meaning must always take precedence over drill and must precede it in all mathematical training. (3) The secondary mathematics curriculum should completely integrate all traditional and general mathematics courses in such a way as to present the essential concepts and principles in their structural form.

38. GALLION, ZACHARY TAYLOR. A Determination and Appraisal of the Content of Freshman General Mathematics Courses in Selected Colleges and Universities. (Ph. D., 1955, Louisiana State University, Baton Rouge.)

Major Faculty Adviser.—W. A. Lawrence.

Problem.—To determine the content of general mathematics courses in the freshman programs of selected colleges and universities; to get an evaluation of this content from leaders in the field of general mathematics; and to formulate a syllabus for use in freshman general mathematics courses.

Procedures.—Two hundred replies from 273 inquiries were analyzed to show trends and content of general mathematics courses. The 17 textbooks listed by 2 or more institutions were analyzed according to the percentage of pages devoted to the various mathematics topics. The heads of the mathematics departments, by means of the original questionnaire, selected 85 leaders in the fields of general mathematics to evaluate the content. The results of the appraisal were used to formulate a syllabus for a general mathematics course. The syllabus consisted of the 39 topics judged by the appraisal group to be of most value for the content of a freshman general mathematics course.

Major Findings and Conclusions.—(1) The general student is dissatisfied with the traditional courses of algebra and trigonometry. (2) Since 1948 a marked trend has appeared toward organizing general mathematics courses. (3) The typical pattern is to offer a 2-semester course, granting 3 hours for each course. (4) Of the total pages 54.3 were devoted to traditional subjects.

39. GAME, RICHARD I. Two Lesson Plans for Basic Technical Mathematics. (M. A., 1956, Adelphi College, Garden City, N. Y.)

Major Faculty Adviser.—Agnes Snyder.

Problem.—To develop two special lesson plans for teaching simple equations and trigonometry to students in the technical division of the State University of New York's Agricultural and Technical Institute at Farmingdale.

Procedures.—A survey of technical mathematics books and other pertinent literature.

Major Findings and Conclusions.—Two lesson plans were prepared, representing approximately 6 weeks of the basic technical mathematics curriculum offered at the State University of New York's Agricultural and Technical Institute at Farmingdale. Only basic elementary mathematics was included in the plans and no attempt was made to include the study of any topics classified as "higher mathematics."

40. GERHART, FAMA EILEEN. The Methods of Developing the Concept of Signed Numbers and the Rules of Addition and Subtraction. (M. M.; 1955, The Ohio State University, Columbus.)

Major Faculty Adviser.—Nathan Lazar.

Problem.—To organize in one place the different methods of developing the concept of signed numbers and the rules for addition and subtraction.

Procedures.—The material was collected from the following sources: 7th- and 8th-grade mathematics textbooks, 9th-grade algebra textbooks, books on methods for teaching mathematics, miscellaneous books and periodicals concerned with the topic of signed numbers, and lectures of Dr. Nathan Lazar at The Ohio State University during the autumn quarter, 1954.

Major Findings and Conclusions.—Nine methods are reported for developing signed numbers.

41. GLABE, GORDON RICHARD. Indirect Proof In College Mathematics. (Ph. D., 1955, The Ohio State University, Columbus.)

Major Faculty Adviser.—Nathan Lazar.

Problem.—To formulate a precise definition for indirect proof and discover a method or methods as applicable to indirect proofs at the elementary levels of mathematics as at the more advanced levels.

Procedures.—Beginning with elementary plane geometry and extending through some of the more advanced courses of college mathematics, textbooks were examined for definitions of indirect proof as well as for discussions and explanations of the method. Definitions of indirect proof were also traced through books on methods of teaching, reports of committees, books on logic, and articles in various periodicals.

Major Findings and Conclusions.—The following definitions of direct and indirect proofs were proposed:

> *A Direct Method of Proof* is any method from which, by reasoning from the hypothesis and axioms, postulates, definitions, and previously proved theorems alone, the conclusion is deduced.
>
> *An Indirect Method of Proof* is any method from which, instead of proceeding as in the direct method of proof, (1) an equivalent proposition is proved, or (2) the contradictory of the given proposition is disproved, or (3) a proposition equivalent to the contradictory of the given proposition is disproved.

42. GLAVAS, CHRISTOS BASIL. Plane Coordinate Systems in Mathematics Study. (Ed. D., Teachers College, Columbia University, New York, N. Y.)

Major Faculty Adviser.—Howard F. Fehr.

Problem.—The development, value, and use of plane coordinate systems in the study of mathematics.

Procedures.—The history of coordinate systems was investigated.

Major Findings and Conclusions.—Up to the present time the study of analytic geometry has been based almost exclusively on a single coordinate system. Other coordinate systems, however, should be used. The greatest educational value comes from using the coordinate system best suited to the locus under study. Various coordinate systems were suggested for specific loci in analytic geometry.

43. GLOTT, RALPH. An Investigation of the Verbal Matter in Recently Published Arithmetic Textbooks and Workbooks for the Intermediate Grades. (Ed. D., 1955, University of Pittsburgh, Pittsburgh, Pa.)

Major Faculty Adviser.—G. A. Yoakam.

Problem.—To determine the readability of verbal matter in arithmetic books and workbooks for the intermediate grades.

Procedures.—An analysis was made of arithmetic textbooks and workbooks for grades 4, 5, and 6, published from 1950 through 1954.

Major Findings and Conclusions.—Arithmetic textbooks and workbooks are placed within or below the intended grade range. They are above grade range in 11 percent of the areas sampled, within it in 44 percent, and below it in 48 percent. On the average, 30 percent of the words rate above the 10th thousand on the Thorndike list. In statement problems, the verbal matter consistently rates higher than that of the textbooks and workbooks, as a whole, and also of the developmental and explanatory material. The lack of progression from easy to difficult verbal matter is without doubt an important factor in textbook readability. A textbook may be entirely too difficult in sections even though it rates within or below grade range. Greater care in distributing the vocabulary load would result in textbooks more nearly fitted to children's needs.

44. GUNDLACH, BERNARD H. The Teaching of Mathematics as Mathematics: A Critique and Proposal.

Problem.—To develop a critique and proposal for teaching mathematics to elementary school teachers.

Procedures.—Undertaken originally under the auspices of the Ford Foundation's Arkansas experiment in teacher education, 1954–56, in the College of Arts and Sciences of the University of Arkansas, this study, somewhat modified, is now being used in a cooperative experiment of the departments of mathematics and education, Bowling Green State University, Ohio. This experiment showed how to present certain areas even to young students from the viewpoint of invariance under transformations. Basic principles of association theory and Gestalt psychology were integrated into a dynamic approach capable of bringing about desirable changes in student and teacher attitudes.

Major Findings and Conclusions.—The study featured a double approach: (1) *Long-range.* Preparation of a new generation of elementary teachers better equipped to handle pressing demands to (*a*) encourage gifted students in further mathematics and science study; (*b*) prepare future citizens to cope more effectively with modern society's increasing complexities. (2) *Short-range.* Remedial services for current teachers, including school visits and classroom demonstrations.

Various practicable schemes for both approaches are discussed with corresponding cost estimates. A detailed study outline is given for a new teacher-training course in elementary mathematics. Main emphasis here is shifted away from problem *solving* to problem *construction*, since ability in the latter generally implies ability in the former. Problem construction ability can free the teacher from dependence upon texts, guides, and workbooks and give scope to his initiative.

45. HANNON, HERBERT HAROLD. An Analysis of the Mathematical Concepts Necessary for a College Physical Science Course. (Western Michigan College, Kalamazoo.) (Paper presented to NARST at Atlantic City, 1957.)

Problem.—To analyze the physical science course at Western Michigan

College to determine the mathematical concepts with which a student must be familiar in order to understand the physical concepts involved in the course material.

Procedures.—The outline workbook prepared by the physical science staff at the college was used for the material studied. This was supplemented by adequate references to three well-known physical science textbooks.

Major Findings and Conclusions.—The frequency of occurrence of mathematical implications is great. By actual count approximately 60 percent of the exercises listed involve computation and/or require making generalizations that involve mathematical concepts. The number of different mathematical skills needed is not large. A student may learn these skills from a course in traditional high school algebra if additional emphasis is placed upon direct and inverse variation, functions and their graphs, interpretation of data, and the ability to generalize.

46. HARVEY, ROBERT HENRY. A Course in Remedial Mathematics for Freshmen at Knoxville College. (Ed. D., 1956, Teachers College, Columbia University, New York, N. Y.)

Major Faculty Adviser.—Howard F. Fehr.

Problem.—To determine the specific mathematical deficiencies of freshmen at Knoxville College and devise a more adequate course in remedial mathematics than the present course.

Procedures.—Remedial students' scores on the Knoxville 1950–54 placement tests were analyzed. Student responses to each item of the arithmetic section were also analyzed.

Major Findings and Conclusions.—The remedial student is extremely weak in mathematics. His score on the arithmetic section of the placement test generally falls below the first percentile of the national norms. He ranks below the first quartile of his class on the arithmetic test in virtually all cases. He tends to be weak in English and below average in intelligence as measured by the tests administered to freshmen at Knoxville College. A course in mathematics was developed which included units on number and number systems, the development of the real number system, the equation, percentage, and reasoning.

47. HERTZ, PAULINE. A Study of the Use of Manipulative Devices in the Teaching of Arithmetic in the Primary and Elementary Grades. (M. A., 1956, College of Idaho, Caldwell.)

Major Faculty Adviser.—Donald J. Mammen.

Problem.—To determine: (1) What manipulative devices for teaching arithmetic were available to the teacher, commercially produced or teacher-pupil made; (2) which of these devices the teachers and pupils actually used; (3) why some teachers were not using them.

Procedures.—A list of known devices, commercial or teacher made, was compiled from information received from State departments, educational associations, teachers colleges, arithmetic authorities, and commercial companies from a cross section of the United States and Canada. This list was included in a questionnaire sent to a cross section of mathematics teachers in California, Idaho, and Oregon to determine which of the devices were actually used in the classroom.

Major Findings and Conclusions.—Of 222 questionnaires sent to teachers, 153 were returned. All except 8 of the teachers who replied use manipulative devices to some extent in teaching arithmetic, and 91.3 percent believe these devices are of value. Of the 150 teachers who gave their opinions as to the need for more training in the use of manipulative devices, 85.4 percent believe that teachers need more of it and 36.6 percent that it should be received in workshops. Over 40 percent (41.8) believe that workshop training should be combined with training elsewhere. The main reasons for not using manipulative devices are lack of time and lack of money.

48. HESCH, ELIZABETH B. The Nature of Mathematical Evidence and Its Significance for The Teaching of Secondary School Mathematics. (Ph. D., 1955, Columbia University, New York, N. Y.)

Major Faculty Adviser.—Howard F. Fehr.

Problem.—To investigate the nature of mathematical evidence as it functions in human affairs and consider its relationship to secondary school mathematics for the preprofessional student.

(The term "evidence" is used to designate any fact, proposition, principle, relationship, or combination of these, as grounds for forming judgments; making decisions; establishing proof, belief, knowledge, or law; settling questions; drawing inferences; making predictions; convincing others; acting intelligently—or for establishing relationships among any of the foregoing.)

Procedures.—The literature on occurrence of evidence was surveyed and analyzed.

Major Findings and Conclusions.—The greatest degree of precision in establishing rules to evaluate evidence has been attained in law. Scientific research is second, followed by historical research and judgments of individuals in the course of daily life. Mathematical evidence occurs in the form of data or in the form of a model.

A new approach is needed for teaching secondary school mathematics to the preprofessional student—an approach which emphasizes the functions of the subject. This would require conscious effort to reach for understanding of basic principles and ideas, presently not emphasized. Thus a student would develop an attitude of approaching problems on the basis of available evidence and in the light of the given framework.

49. HILL, WILLIAM FLOYD. An Analysis and Evaluation of the Mathematics Content in Six Selected Curricula of a Technical and Commercial Nature in Public Junior Colleges of California, Iowa, and Texas. (Ph. D., 1955, Baylor University, Waco, Tex.)

Major Faculty Adviser.—C. G. Strickland.

Problem.—To determine what mathematics experiences are being provided by the public junior colleges of California, Iowa, and Texas in preengineering, junior engineering, shopwork, prebusiness, general business, and secretarial training. To determine what mathematics experiences should be required for each of these curricula as indicated by the opinions of deans or directors of the corresponding schools in State universities or colleges.

Procedures.—Junior college bulletins describing the nature of the mathematics offerings were analyzed, and topics for each curriculum were drawn from 101 junior college catalogues. A check list of these topics was sent to the junior colleges, asking them which ones they had in their curriculum. A check list was also sent to State universities, asking them to indicate which topics they considered essential to the various curricula.

Major Findings and Conclusions.—For preengineering, junior engineering, and shopwork, the percent of the junior colleges which included the mathematics topics was, with every few exceptions, equal to or greater than the percent of State universities which rated the corresponding topics essential. More emphasis was placed on trigonometry, algebra, and business mathematics in the commercial curricula than considered essential.

50. HINTON, LOREN NELSON. How Students' Ability To Use Arithmetical Skills Compares With Understanding of the Reasons for Their Use. (M. Ed., 1956, The Ohio State University, Columbus.)

Major Faculty Adviser.—Lowry W. Harding.

Problem.—To find how a student's ability to compute compares with his ability to give reasons for the computation.

Procedures.—The writer devised two tests of similar construction. One was a computation test, and the other asked for the reason why a certain mathematical process or procedure was used. The tests were given to the same experimental group at the beginning and the end of the 7A semester and at the end of the 8A semester.

Major Findings and Conclusions.—If an effort is made to teach arithmetical meanings, the degree of understanding can be measured, and the students' scores on the reasoning test compare favorably with their scores on the computational test.

51. HOLCOMB, JOHN DEWITT. Solid Geometry in the 10th Grade. (M. A., 1956, Illinois State Normal University, Normal.)

Major Faculty Adviser.—Douglas R. Bey.

Problem.—To develop and teach 2 units in solid geometry in the 10th grade.

Procedures.—Two different experimental units in solid geometry were developed and taught to 41 plane geometry students. The "Euclidean unit," designed for an average class of sophomores in plane geometry, was taught to 2 classes for 2 weeks. The "analytic unit," designed for an above-average class of sophomores in plane geometry, was taught to a third class for the same length of time. The units were evaluated in four ways—by a final test on content, a record of student and critic teacher opinion, the space relations test of the differential aptitude series, administered before and after the units were taught, and a comparison of scores on this test.

Major Findings and Conclusions.—The gain in the content of the units was satisfactory in all three classes. A significant gain in space relations ability was found for 2 of the 3 classes. The greatest gain in space relations ability was made by the class to which the "analytic unit" was taught.

52. HOLLWEDEL, HENRY CHARLES, JR. The Problem of the Slow Child in Junior High School Mathematics. Ed. M., 1955. (University of Buffalo, Buffalo, N. Y.)

Major Faculty Adviser.—L. O. Cummings.

Problem.—(1) To find the requirements industry places on the mathematics education of unskilled and semiskilled workers. (2) To find where present workers are failing in terms of these requirements. (3) To plan a curriculum that will satisfy the current needs of the slower students during the last 2 years of required mathematics courses (7th and 8th grades).

Procedures. — One hundred questionnaires were sent to industrial organizations in Jamestown, N. Y.; 17, representing 4,159 men, were returned. The resulting data were used as the basis for developing a 7th-grade mathematics workbook.

Major Findings and Conclusions.—The 4,159 men used the fundamental operations of arithmetic in their work. Approximately 18 percent were rated unsatisfactory in addition, 35 percent unsatisfactory in multiplication, and 68 percent unsatisfactory in division. A workbook was compiled which emphasized the fundamentals of arithmetic.

53. HORTON, ROBERT EUGENE. Report on the Experimental Classes in the First Course in Alegbra at Los Angeles City College Covering Two Semesters—the Spring and the Fall of 1954. (1956, Los Angeles City College, Los Angeles, Calif.)

Problem.—To design a course that will fulfill the needs of students who do not plan to take any mathematics beyond elementary algebra and geometry.

Procedures.—Roughly speaking, the content was grouped under three headings: (1) Conventional Algebra, (2) the unconventional topics, and (3) applied mathematics. The experiment is being continued at Los Angeles City College, where various instructors take turns teaching successive classes for two semesters. They draw upon the experience of their predecessors and add to it from their own background and educational philosophy.

Major Findings and Conclusions.—Certainly there was no loss to any student. Those who did not get much out of the course did not get much out of conventional algebra, either. To others the meaning of mathematics became clearer.

54. HUFFORD, ROGER ALDEN. An Analysis of Critical Thinking in Secondary Schools. (M. A., 1956, Illinois State Normal University, Normal.)

Major Faculty Adviser.—Douglas B. Bey.

Problem.—To investigate the effect of present-day methods on critical thinking and determine which classes are contributing most to their students' concept of critical thinking.

Procedures.—Equivalent forms of the Watson-Glaser critical thinking appraisal were given at the beginning and end of the fall semester to sophomores at University High School, Illinois State Normal University, Normal. Eighty-four sophomores took both the initial and the final test.

Major Findings and Conclusions.—(1) The sophomore class showed no significant change in critical thinking ability after a semester of high school work. (2) With the variable of original test score held even relatively constant, no class appeared to have any advantage from the teaching of critical thinking. (3) If the pattern observed in University High School is typical, the study has implications for mathematics teachers. If other subjects can teach critical thinking in life situations as effectively as mathematics, then mathematics teachers should be relieved of the responsibility and given the opportunity to teach more of the advanced mathematics on which our technological culture is based.

55. IMMERZEEL, GEORGE E. A Comparative Investigation of the Use and Nonuse of Manipulative Devices in Teaching 7th-Grade Mathematics. (M. A., 1956, Iowa State Teachers College, Cedar Falls.)

Major Faculty Adviser.—H. C. Trimble.

Problem.—Is the study of the 7th-grade mathematics more meaningful when taught using manipulative devices than when taught without their use?

Procedures.—The author used two classes at Horace Mann Junior High School in Burlington, Iowa. Classes were equated in initial tests and taught by the author. Comparisons on basis of growth were measured by tests, attitude scales, anecdotal records, and records of voluntary activities, etc.

Major Findings and Conclusions.—The study aroused community and school interest and led to purchase and construction of teaching materials. The experimental group made greater progress than the control group as measured by standardized tests, teacher-made tests, questionnaires, and daily class records.

56. JAFFE, MITCHELL. A Study of Methods and Techniques of Teaching Preferred by Secondary Mathematics Teachers. (Ed. M., 1953, Boston University, Boston, Mass.)

Major Faculty Adviser.—Henry W. Syer.

Problem.—To investigate secondary teachers' preferences in certain methods and techniques of teaching mathematics.

Procedures.—Two hundred-eighteen mathematics teachers in 70 Massachusetts secondary schools were sent check-list type questionnaires designed to bring out their teaching preferences. Responses were received from 131 teachers in 58 schools.

Major Findings and Conclusions.—The traditional lecture and recitation remained high on the preferred list. Discussion was recognized as a valuable teaching method. Supervised study and laboratory methods were favored more strongly by teachers with more than 15 years' experience than by the less experienced. Teachers with less experience preferred the recitation, a teacher-pupil centered activity, more strongly than the others. Homework assignments appeared rather traditional, with little variety and with little allowance for individual differences or for motivation of the superior pupil.

57. JOHNSON, DONOVAN A. A Study of the Relative Effectiveness of Group Instruction. (1955, University of Minnesota, Minneapolis.)

Problem.—To compare the effectiveness of one type of small group instruction with regular class instruction.

Procedures.—In one situation the class was divided into small groups, each taught separately in the regular classroom. In the control situation the entire class was taught by the regular teacher in the traditional manner.

Advanced algebra classes in the University High School at the University of Minnesota were used for this experiment. Two experimental classes and two control classes were selected by stratified random sampling of 48 students. They were divided into upper and lower strata according to achievement in previous mathematics topics. An experimental and a control section of equal size were randomly selected from each stratum using a table of random numbers. As a result, an experimental and a control class were formed, each with 13 students; also an experimental and a control class, each with 11 students. Then each experimental class was again divided into two subgroups for group instruction. Although these classes were small, they approached the size of the advanced algebra class of a typical high school.

The highly significant teacher factor was controlled by having each instructor teach one experimental and one control class.

Major Findings and Conclusions.—There was no difference between the achievement of the class taught in small groups and that of the class taught in the traditional manner.

58. JOHNSON, DONOVAN A. The Readability of Mathematics Books. (1955, University of Minnesota High School, Minneapolis.)

Problem.—To describe one way of testing the readability of mathematical material and report the results of the technique applied to a variety of mathematics books.

Procedures.—This study used the modified Flesch formula to measure the readability of a variety of mathematics books. This method involves finding the average number of words per sentence and the average number of 1-syllable words and then converting these figures into a readability score as follows: (1) Select a random sample of written material. Starting with a sentence, count off 100 words. (2) Count the number of sentences in the sample. (3) Count the number of 1-syllable words in the sample. (4) Compute the reading ease (R. E.) of the sample of substituting in the formula:

$$R.\ E. = 1.6\ s - 1.0\ w - 31.5$$

s=number of 1-syllable words

w=average number of words per sentence

Major Findings and Conclusions.—The data obtained are summarized in the table below.

Readability of Mathematics Books

Books and number	R. E. score	Grade level
7th grade textbooks:		
1	72	8.0
2	76	7.5
3	65	9.0
8th grade textbooks:		
1	71	8.0
2	63	9.0
3	65	9.0
Algebra textbooks:		
1	61	9.5
2	62	9.0
Geometry textbooks:		
1	63	9.0
2	63	9.0
3	61	9.5
Supplementary books:		
1	67	8.0
2	53	11.5
3	68	8.0
4	71	8.0
5	40	15.0
6	48	13.0
7	54	11.5

59. KARST, OTTO JACOB. Linear Regression Using Least Absolute Values. (Ph. D., 1956, New York University, New York.)

Major Faculty Adviser.—John J. Kinsella.

Problem.—To develop the statistical and educational implications of a theory of linear regression based on least absolute values (L. A. V.).

(The linear regression theory, as usually presented, is based on the least squares property of the regression line. This requires, for adequate understanding, a knowledge of differential calculus which many students do not have. A theory of regression can be built on the properties of a regression line so that the sum of the absolute values of the deviations of the data from it is a minimum. Furthermore, the requisite mathematical background for understanding this theory does not go beyond elementary algebra and plotting straight lines.)

Procedures.—The following statistical aspects of the study were developed: (1) A general method, with examples, for obtaining the L. A. V. regression line from ungrouped, grouped, and weighted data. (2) The principles for extending this method to 3-dimensional data. (3) A general method, with examples for finding a "best" L. A. V. line for a given set of data. (4) The L. A. V. properties of the median of a set of numbers, with implications for the wider use of the median as a measure of central tendency of the arithmetic mean. (5) The comparative study by means of T, F, and X^2 tests of the relative abilities of the L. A. V. and L. S. lines to estimate the slope of the regression line of a theoretical model population.

Major Findings and Conclusions.—Using only the mathematical prerequisites of elementary algebra and plotting of straight lines, it is possible to develop a L. A. V. theory of linear regression which can be taught to students in elementary statistics courses.

60. KLIEBHAN, SISTER MARY CAMILLE. An Experimental Study of Arithmetic Problem-Solving Ability of 6th-Grade Boys. (1955, Ph.D., The Catholic University of America, Washington, D. C.)

Major Faculty Adviser.—Francis J. Houlahan.

Problem.—To ascertain, by the "t" test of the significance of differences between means, how groups of 6th-grade boys designated as "high achievers" and "low achievers" in problem solving in arithmetic differ in regard to the certain variables, selected from among the findings of previous research as being representative of the more important components of problem-solving ability.

Procedures.—A battery, consisting of 8 standardized tests and 9 tests constructed for the present study, was administered to 479 boys from 27 classes in 22 parochial schools in the Archdiocese of Milwaukee. On the bases of mental-age scores obtained on the Kuhlmann-Anderson test and performance on the Stanford arithmetic reasoning test, matched groups were made of high-achieving and low-achieving boys.

Major Findings and Conclusions.—Ability to estimate answers to problems appears to be the most significant factor in differentiating these groups of high and low achievers.

The specific-reading skills measured by tests of reading to note irrelevant detail and reading to note numerical detail are more effective in distinguishing high and low achievers than general reading ability.

No significant difference was found to exist between high- and low-achieving boys, a fact which demonstrates that, in this population, 6th-grade boys who are doing poorly in arithmetic problem solving also tend to be weaker in other arithmetical abilities and skills. Although the findings are not submitted as proof of this, they suggest that boys are poor in arithmetic problem solving because they lack competence in these other skills.

61. KOKOMOOR, FRANKLIN WESLEY, *and* GAGER, WILLIAM A. Functional Mathematics in the Secondary Schools. (1956, University of Florida, Gainesville.)

Problem.—A study of the teaching methods, outcomes, organization, and content of mathematics courses currently taught in secondary schools to determine ways of improving and enriching the seconding mathematics curriculum.

Procedures.—Thirty-six secondary mathematics teachers from all types of high schools and all levels of instruction, with a large number of specialists serving as consultants, carried out this study. The group studied school practices and also literature that had a bearing on the problem. They then listed all mathematics concepts basic to a sound mathematical training in secondary schools and determined the fundamental principles of mathematics through which the concepts would have to function. Using materials exemplifying these principles and concepts, they prepared a textbook for each grade, 7 through 12.

Major Findings and Conclusions.—(1) The separate traditional courses, as now taught in most high schools, overemphasize the learning of abstract ideas by drill procedure. (2) The traditional courses, as now taught in most high schools, are suitable for only a very small percent of mathematics pupils. (3) The traditional high school mathematics courses do not prepare the majority of their pupils for either college or life. (4) Meaning must always have preference over drill and must precede it in all mathematical training. (5) To assure a higher degree of meaning, the secondary mathematics curriculum should completely integrate all traditional and general mathematics courses so as to present essential concepts and principles in their proper structural form.

63. KUECHLE, RICHARD THEODORE. The Locus Concept in Geometry. (M. A., 1956, The Ohio State University, Columbus.)

Major Faculty Adviser.—Nathan Lazar.

Problem.—To examine books on methods of teaching mathematics, articles from periodicals, and textbooks of geometry to ascertain how authors treat the topic of locus.

Procedures.—Textbooks and other pertinent material were analyzed as to the presentation of locus and terms used in the presentation.

Major Findings and Conclusions.—Writers disagree on definitions of terms and presentation. The study inclundes models and practical uses of loci.

64. KUEHN, HENRY JOHN. An Evaluation of a Test of Primary Mental Abilities, Two Tests of Arithmetic Achievement, and a Test of Reading Ability as Predictors of Success on a Test in Elementary Algebra. (Ed. M., 1955, University of Washington, Seattle.)

Major Faculty Adviser.—August Dvorak.

Problem.—To evaluate four tests: The California reading, The Stanford achievement test (computation), The Seattle arithmetic test, and the Science Research Associates test of primary mental abilities as predictors of success on the coperative test of elementary algebra.

Procedures.—Data for the study were obtained from the personal record folders of 489 pupils in 3 junior high schools of Seattle. The evaluation was made in terms of 9 Pearson product-moment coefficients of correlation and coefficients of multiple correlation for 4 different combinations of variables obtained by the Horst iterative reduction method.

Major Findings and Conclusions.—The Seattle arithmetic test, used alone, was almost as good a predictor of success on the algebra test as any combination of the variables. The available evidence indicated that the SRA part scores did not measure such abilities as reasoning, number, word fluency and verbal meaning. Only the SRA total score should be used to decide whether or not a pupil should take algebra in the 9th grade.

65. LAFFERTY, WILLIAM ANDREW. The Selection of Subject Matter in Mathematics for General Education. (Ed. D., 1956, Teachers College, Columbia University, New York, N. Y.)

Major Faculty Adviser.—Howard F. Fehr.

Problem.—To improve the course in mathematics for general education at Northwest Missouri State College.

Procedures.—Five criteria for the selection of subject matter are set up and validated by an appeal to pertinent authority.

Major Findings and Conclusions.—The following criteria resulted from the survey and were used as the basis for developing mathematics for general education: (1) The course should be geared to the top level of student ability. (2) It should require no previous mathematics beyond that provided by 1 year of general mathematics or algebra in the usual 4-year high school. (3) The course should be not merely *about* mathematics, but should also require the performance of mathematics, leading at all times to development of meaningful learning. (4) It should reveal the spirit and nature of mathematics, particularly modern mathematics, and should explain what it is that mathematicians do.

66. LANKFORD, FRANCIS G. JR. and PATTISHALL, EVAN G. JR. An Experimental Project in The Teaching of Arithmetic.

Problem.—To improve methods of teaching addition and subtraction of fractions in the elementary grades.

Procedures.—Twenty pairs of teachers from twenty schools took part in the experiment. Each pupil was given an individual copy of each piece of material and each teacher was visited at least twice—at the beginning and end of the project.

Major Findings and Conclusions.—(1) The initial tests (California arithmetic test and California test of mental maturity) were slightly in favor of the control groups, but these differences did not prove to be statistically significant. (2) Test I on addition and subtraction of fractions showed a significant difference, at the 5-percent level, in mean number of correct answers favoring the experimental groups. (3) In computing the answers for test I, the control groups had to use their pencils twice as much as the experimental groups. (4) The pupils in the experimental groups more frequently expressed a liking for the opportunity to compute answers mentally. (5) The experimental groups chose freedom to work and solve fraction problems in their own way rather than have the teacher method.

67. LANKFORD, FRANCIS G., JR. and PATTISHALL, EVAN G., JR. Experimental Study in the Teaching of Arithmetic. (1955, University of Virginia, Charlottesville.)

Problem.—To study methods of teaching arithmetic through pupil participation.

Procedures.—Experimental and control conditions were randomly assigned to each of 18 pairs of nonhomogeneous 5th-grade classes. Addition and subtraction of fractions was taught emphasizing two important features: (1) Ideas and rules of arithmetic were developed inductively through pupil participation rather than by the more usual method of teacher-explained rule followed by practice. (2) Pupils were encouraged to learn arithmetic thoughtfully and independently, using mental arithmetic with emphasis on varied approaches. (The best teaching procedures, as normally employed by designated control teachers, were used in the control situations.)

Major Findings and Conclusions.—The experimental groups were (1) significantly superior in the mean number of correct answers on final achievement tests and (2) significantly superior in expressing arithmetical concepts through mental computation. Another significant result was an improved attitude toward arithmetic.

68. LAYTON, WILLIAM ISAAC. College Mathematical Training for Students Specializing in Agriculture. (1955, Stephen F. Austin State College, Nacogdoches, Tex.)

Problem.—To survey the mathematical requirements of a representative group of colleges offering programs in agriculture.

Procedures.—Seventy-five colleges of varying size and type in 40 States participated in the survey by returning a questionnaire. The three principal sections of the questionnaire dealt with college entrance requirements in mathematics, present requirements in college mathematics, and recommendations concerning college mathematics for students of agriculture.

Major Findings and Conclusions.—Seventy-five colleges had mathematics entrance requirements for agricultural students ranging from 0 to 3 high school units. Mathematics *required* most frequently for Smith-Hughes programs in agriculture was college algebra. Agricultural mathematics, ranking second, was required by half as many colleges as those requiring college algebra. The mean of all mathematics required was 3.91 semester hours.

Under the heading of mathematics *recommended* for those specializing under Smith-Hughes programs, agricultural mathematics was first, followed by college algebra and trigonometry.

A question was posed on the questionnaire as to the desirability of 73 topics for agriculture mathematics. Forty six were rated highly desirable by 50 percent or more of the 40 respondents who advocated agricultural mathematics instead of college algebra and/or trigonometry. These 46 topics were in arithmetic, algebra, and their special applications to agricultural problems.

Interest seems to be growing to require agricultural mathematics instead of college algebra and/or trigonometry for college students specializing in agriculture.

69. LEWIS, EDWIN HIRSCHI. A Concept Approach to the Teaching of Algebra. (Ed. D., University of Utah, Salt Lake City.)

Major Faculty Adviser.—Leo G. Provost.

Problem.—To investigate the effectiveness of two distinct methods of teaching first-year algebra, each method employing unique materials and procedures adapted to its particular philosophy. One of the two methods is called the traditional method throughout this study while the other is termed the concept method.

Procedures.—An effort was made to distinguish between the two methods, not only by using different textbooks, but also by specifically identifying the characteristics of the teaching-learning situation in each case. Both methods were designed for a full school year of elementary algebra and included the topics and treatment necessary as a foundation course for students planning to pursue additional formal mathematics.

Students were placed in sections on the basis of their own selections with no grouping as to interest or ability. The 10 algebra sections participated. The control groups, using the regular State-adopted text, were taught in the usual way with a rather formal and teacher-structured learning situation. The experimental groups used the specially prepared textbook, designed with emphasis upon a unique sequence, fewer but broader concepts, less drill, and student exploration and discovery. Careful nurturing of concepts was the guide for teaching the experimental groups.

Major Findings and Conclusions.—The experimental method used in this study was definitely superior to the control method when the results were measured by the special test. No significant advantage showed up for either method when results were measured by the Douglass survey test. Apparently neither method was a handicap to the teacher's effectiveness. The experimental method was superior for above-median students but showed no advantage for below-median students.

The individual results on both the special test and the Douglass survey test indicated that a large percentage of students showed so little achievement in the course of a year's instruction that the time consumed could not be justified. Since the amount of drill in the experimental test was so drastically curtailed and yet the results as shown on the special test were superior, it appears that much of the drill in traditional algebra is unproductive of understanding and insight.

70. LINSCHEID, HAROLD WILBERT. A Study of the Freshman Mathematics Placement Program at the University of Oklahoma. (Ph. D., 1955, The University of Oklahoma, Norman.)

Major Faculty Advisers.—J. C. Brixey and F. A. Balyeat.

Problem.—To determine criteria that may be used to supplement the mathematics placement test given at the University of Oklahoma. To determine the relative value of the 40 items of the placement test in separating students into ability groups.

Procedures.—The mathematics department of the University of Oklahoma gives a mathematics placement test to all entering freshmen. The results are used to place students in mathematics R (remedial mathematics), mathematics 2 (intermediate algebra), or mathematics 21 (a first course in mathematical analysis). An analysis of the data on 1,832 freshmen who entered the University of Oklahoma in September 1952 covered performance on the placement test, the Iowa High School content examination (OSPE), the grade earned in the first mathematics course taken in college, and grade-point averages in mathematics as well as general grade-point averages in both high school and college.

Major Findings and Conclusions.—The probable error of prediction of college grade points in mathematics was approximately two-thirds of a grade point, based upon regression equations in which the independent variables were X_1 (placement test raw score), X_2 (decile rank on mathematics section of the IHSC), X_3 (decile rank on OSPE total score), and X_4 (high school mathematics grade points). The probable error of prediction increased slightly when only X_1 and X_2 were used as independent variables. From a practical standpoint, the single function X_1+X_2 could be used to place the students in the 3 groups.

The placement test served better as an instrument for separating students into mathematics 2 and mathematics 21 than for separating them into remedial mathematics and mathematics 2. Based upon responses by 1,211 students, 8 items of the placement test had more than a 50 percent correct response, while 13 items had less than a 20-percent correct response.

71. LLOYD, DANIEL B. The National Status of Mathematics Contests in Secondary Schools. (1955, District of Columbia Teachers College, Washington.)

Problem.—To collect and disseminate information for other groups wishing to launch contests or improve existing contests and (2) to investigate the desirability of conducting a national contest by asking the opinions of those now working with contests.

Procedures.—A questionnaire was distributed to each of the State representatives of the National Council of Teachers of Mathematics and 80 other leaders in mathematics education.

Major Findings and Conclusions.—Thirty-six contests, involving 3,900 schools were reported. There were no reports from 18 States. Six States reported that no contests were held. Considerable sentiment for and against national scholarships was expressed. A national contest was not conducted.

72. LUTZ, MARTHA LUCILLE. A Comparative Study of the Chinese and Japanese Abaci. (M. A., 1956, The Ohio State University, Columbus.)

Major Faculty Adviser.—Nathan Lazar.

Problem.—A study of the Chinese and Japanese abaci.

Procedures.—The literature on history of the abacus was reviewed.

Major Findings and Conclusions.—Modern forms of the Chinese and Japanese abaci are structurally similar. There is very little basic difference in the operating methods used with the different forms of abaci. The Chinese and Japanese abaci are important mainly as an item of cultural curiosity, not as a teaching device nor as a remedial teaching device in arithmetic.

73. McALISTER, RUTH C. How Mentally Retarded Children Learn Basic Number Concepts. (M. Ed., 1953, University of Texas, Austin, Tex.)

Major Faculty Adviser.—William G. Wolfe.

Problem.—How are mentally retarded children taught basic number concepts?

Procedures.—A review of literature of the field. A review of methods and materials suggested by 35 authorities in the areas of arithmetic and special education.

Major Findings and Conclusions.—A unit of study was developed for teaching these basic skills to the mentally retarded.

74. McCALL, JOHN R. Sex Differences in Intelligence: A Comparative Factor Study. (1955, Ph. D., The Catholic University of America, Washington, D. C.)

Major Faculty Adviser.—James Vanderveldt.

Problem.—In a study of the factorial patterns involved in 31 subtests from several "intelligence tests" involving 451 boys and girls in the 8th grade, Dr. James T. Curtin had found 5 different factors. Two of these were numerical in nature. The suggestion was made that possibly sex differences in numerical ability accounted for the appearance of the two different but similar numerical factors. To determine these sex differences this study was begun.

Procedures.—The data of the Curtin study were broken up into 2 sets—1 for the 206 boys; 1 for the 245 girls. Matrices of coefficients of correlation were set up for each sex. These were factored by Thurstone's centroid method and the axes rotated orthogonally toward simple structure for psychological interpretation of the results.

Major Findings and Conclusions.—Three factors appeared for each sex. A certain amount of similarity was found between the verbal factors with somewhat less similarity between the space factors. The number factors, however, were quite dissimilar. The girls' number factor seemed to be more restricted and circumscribed, being limited to simple arithmetic operations.

75. McKAY, LOUISE DI PRAMPERO. A Physical Approach to Teaching Addition and Subtraction of Fractions. (M. E., 1956, The Ohio State University, Columbus.)

Major Faculty Adviser.—Nathan Lazar.

Problem.—To study the teaching of addition and subtraction of fractions by rectangular cardboard strips.

Procedures.—Books and articles on the teaching of arithmetic and textbooks of elementary arithmetic were reviewed as to teaching addition and subtraction of fractions. Concrete and semiconcrete devices and techniques suggested by the authors were evaluated.

Major Findings and Conclusions.—The main advantage of the cardboard strips over the other devices for teaching fraction concepts and processes is that they enable students to discover *by themselves* the solutions to all types of problems involving addition and subtraction of fractions.

76. McMillan, LaVerne Ruth. A Comparative Study of the Prediction of Achievement in 9th-Grade Algebra. (M. A., 1956, University of Texas.)

Major Faculty Adviser.—Royal B. Embree.

Problem—To investigate the relation of certain measurable factors to success in 9th-grade algebra as an aid to more effective guidance and instruction.

Procedures.—The factors considered in this study were 8th-grade mathematics marks, the Lee test of algebra ability, the California arithmetic test, and the Detroit alpha intelligence test. Success in algebra was measured by teachers' marks and algebra achievement tests. The subjects for the study were the 286 pupils enrolled in algebra classes during 1953-54 in the Woodrow Wilson Junior High, Port Arthur, Tex., and the 296 who were enrolled in algebra classes in this same school during 1954-55.

Major Findings and Conclusions.—Marks in 8th-grade arithmetic seemed to be a good predictor of success in elementary algebra. The test of algebra ability was helpful.

77. Marske, Herold. A Study of the Use of 6th-Grade Arithmetic Aids in Rural Schools of Southeastern Minnesota. (M. S., 1955, Winona State Teachers College, Winona, Minn.)

Major Faculty Adviser.—Glen Fishbaugher.

Problem.—To ascertain (1) which of 15 selected aids are most widely used in teaching 6th-grade arithmetic in rural schools of southeastern Minnesota, (2) the value of these aids in the opinion of the rural teachers, and (3) why some aids are not being used.

Procedures.—A questionnaire was designed to obtain the reactions of teachers using particular aids and to discover which aids are not being used and why. Two hundred copies of the questionnaires were sent to teachers in Winona, Wabasha, Olmsted, Houston, and Fillmore counties of southeastern Minnesota. Eighty-one teachers completed and returned the questionnaire.

Major Findings and Conclusions.—The 11 aids used by more than half the teachers in order of their use were business forms, charts and diagrams, pictures, workbooks, bulletin boards, stories, tricks, puzzles, and field trips. The four least used aids were films, slidefilms, the abacus, and dramatizations.

The teachers ranked the aids in the following order: Charts and diagrams, business forms, construction activities, table materials, games, slidefilms, workbooks, films, pictures, newspapers, catalogs, magazines, bulletin board, abacus, stories, tricks, and puzzles, dramatizations, and field trips. Films and slide-films were ranked high but used little because not available.

78. Mastbaum, Sol. A Comparison of the Relative Effectiveness of Homogeneously Grouped Classes and Heterogenously Grouped Classes in Junior High School Mathematics. (M. A., 1956, University of Minnesota, Minneapolis.)

Major Faculty Adviser.—Donovan A. Johnson.

Problem.—To determine whether homogeneously grouped classed differed from heterogeneously grouped classes in achievement.

Procedures.—Pairs of pupils were matched in sex, I. Q., grade level, and scores on test D, Iowa every-pupil tests of basic skills.

Major Findings and Conclusions.—The homogeneous groups made significantly greater gains in mathematics skills.

79. MILLER, MYRON S. An Analysis of Clinical Cases and Nonclinical Cases as Determined by an Arithmetic Proficiency Test. (D. Ed., 1956, Michigan State University of Agriculture and Applied Science, East Lansing.)

Major Faculty Advisor.—C. V. Millard.

Problem.—(1) Do low levels of performance on an arithmetic proficiency test occur in any particular patterns within the new-student population at Michigan State University? (2) Does the level of performance on the arithmetic proficiency test indicate the probable student attrition at Michigan State University? (3) Does the level of performance on the arithmetic proficiency test indicate probable student achievement at Michigan State University?

Procedures.—For the first question, eight categories within the new-student population were analyzed. Scores on a reading, a writing, and a psychological test were also utilized. For the 2d and 3d questions, a follow-up study over a 4-year period was completed on some 600 freshmen with the highest scores, the middle scores, and the lowest scores on the arithmetic test given in the fall of 1951.

Major Findings and Conclusions.—The percentage of females with low scores on the arithmetic test was significantly higher than the percentage of males. There was no difference as between students from large high schools and those from small high schools. Most students with low scores on the arithmetic test received relatively low scores on the quantitative section of the psychological test. Students with low scores on the arithmetic test showed a significantly lower rate of return than students with middle scores. Students with high scores were higher in achievement than the other students.

80. MYERS, SHELDON STEPHEN. The Nature of Definition in High-School Geometry: A Critique of Current Practices. (Ph. D., 1955, The Ohio State University, Columbus.)

Major Faculty Adviser.—Nathan Lazar.

Problem.—To study the differences, if any, between definitions in contemporary plane geometry textbooks and those recommended by modern mathematics and logic books.

Procedures.—This study surveyed the extent to which the following six defining practices originated in the works of Aristotle and the extent to which they occur in contemporary geometry textbooks: (1) Definitions should be in the form of "genus at differentia." (2) Definitions should not use obscure language but should use that are prior and more intelligible. (3) Definitions should not contain redundant or superfluous information. (4) Definitions must be reversible. (5) There is only one definition of a thing. (6) Definitions state what a thing is, but not the fact that the thing exists.

Major Findings and Conclusions.—Widespread and serious discrepancies were found between textbook practices and the treatment of definition recommended in mathematical and logical literature. Much of the Aristotelian theory of definition was shown to be no longer valid.

In geometry, besides the Aristotelian class-difference method there are other methods to formulate definitions—such as the genetic, the operational, and the descriptive.

Definitions should be formulated by pupils after they have investigated examples of the term being defined and have developed a group of possible defining properties.

Such figures as quadrilaterials can be arranged and classified in different ways, thus giving rise to different definitions. Contrary to Aristotle, the sequence of definitions in geometry is not absolute.

81. NATTKEMPER, JOHN. Measure of Some Understanding of Arithmetic Processes. (M. A., 1956, University of California, Berkeley.)

Major Faculty Adviser.—John U. Michaelis.

Problem.—To test children's understanding of addition, subtraction, multiplication, and division as taught in the 5th and 6th grades.

Procedures.—A test was devised to measure knowledge of usefulness of each process, appreciation of reasonableness of results, and relationship of one process to the other processes. The results of testing 140 5th-graders and 142 6th-graders were analyzed to determine concept difficulties, grade-level difficulties, and pupil needs.

Major Findings and Conclusions.—The concept of usefulness of each process was grasped fairly well (70 percent of the 5th-graders and 85 percent of the 6th-graders responded correctly to items measuring usefulness). Fewer children responded correctly to items on reasonableness of results (60 percent of the 5th-graders and 68 percent of the 6th-graders). Very few children responded correctly to items of relationships among processes (29 percent of the 5th-graders and 41 percent of the 6th-graders).

82. NELSON, THEODORA S. Results of General Mathematics Tests. (1955, Nebraska State Teachers College, Kearney.)

Problem.—To study problems missed on general mathematics tests in Inter-High School Contests.

Procedures.—The questions most frequently missed were analyzed in each test for the years 1950–55. The data were classified by type of problem.

Major Findings and Conclusions.—Areas showing significant weaknesses were (1) verbal problems, (2) applications of percent, (3) evaluation and use of simple formulas, (4) interpretation of graphs, (5) all phases of simple algebra that were tested.

83. NICHOLS, EUGENE. Comparison of Two Approaches to the Teaching of Selected Topics in Plane Geometry. (Ph. D., 1956, University of Illinois, Urbana.)

Major Faculty Adviser.—K. B. Henderson.

Problem.—To compare the effectiveness of learning certain facts in plane geometry by (1) the dependence approach and (2) the structured search approach. In the former, the teacher gave out the information (statements of assumptions, definitions, and theorems) to the students using a textbook as an aid. The presentation was highly verbal and abstract. In the latter, the teacher organized the students' experience by means of printed instructional materials developed by the investigator so that the students discovered the information for themselves.

Procedures.—Two groups of 21 students each were matched as to their knowledge of plane geometry, I. Q., and age. Certain topics of plane geometry were selected. One group was given the information by the dependence approach. This information was imparted to the other group by the structured search approach. Each group was taught alternately by 3 teachers in order to "randomize" the effect of the teacher.

Major Findings and Conclusions.—The structured-search approach and the dependence approach are equally effective in teaching plane geometry to high school freshmen.

84. NORMAN, MARTHA. Three Methods of Teaching Basic Division Facts. (Ph. D., 1955, State University of Iowa, Iowa City.)

Major Faculty Adviser.—Herbert F. Spitzer.

Problem.—To investigate the effects of 3 methods of teaching certain basic division facts to children in 24 3d-grade classes. The methods were the textbook, the conventional, and the developmental.

Procedures.—Relatively little control was applied to the textbook methods. Teachers were asked to follow their usual procedures, making sure, however, that they covered certain textbook pages each day so that conditions within the textbook group would be comparable. Complete teacher and pupil materials (adapted from a representative 3d-grade textbook) were supplied for the conventional group. The conventional materials introduced division facts by means of story settings and problems, their liberal use providing repetition. Complete teacher and pupil materials were supplied also for the developmental group and instructional aids were used—the number line, counters, pupil drawings, and a number chart, as well as procedures to encourage pupils to generalize.

Major Findings and Conclusions.—There were no significent differences among the three groups in immediate recall. In delayed recall of taught facts both the developmental and conventional methods were superior to the textbook method.

85. O'DAFFER, PHARES GLEN. Teaching of the Congruence Theorems in Plane Geometry. (M. A., 1956, Illinois State Normal University, Normal.)

Major Faculty Adviser.—T. E. Rine.

Problem.—How can we teach congruence of triangles so that it will be more in harmony with the present-day objectives of secondary school mathematics than the method of proof-by-superposition, still widely used today?

Procedures.—An experimental unit was developed.

Major Findings and Conclusions.—On the basis of both objective testing and teacher observations, the unit in congruence as presented was an effective instrument for teaching congruence in a plane geometry course.

86. OSBORNE, EDMUND COLE. A Comparison of Two Curriculums for the Preparation of Teachers of Mathematics in Secondary Schools and of the Students Trained Under Each. (Ed. D., 1956, Boston University, Boston, Mass.)

Major Faculty Adviser.—Henry W. Syer.

Problem.—To compare the two curriculums which the author has administered at the State Teachers College, Worcester, for the preparation of mathematics teachers in secondary schools in order to determine: (1) Is either suitable to the purpose for which it is intended? (2) If both are suitable, which is better? (3) If neither is suitable, what changes should be made?

Procedures.—A statistical study was made of the high school, college, and post-college records of the 320 members of the classes of 1928 through 1953, who, at the time they entered the State Teachers College at Worcester, chose mathematics as their freshman elective, thus indicating that they intended to prepare themselves as secondary school mathematics teachers. This procedure was followed in order to discover the quality of the students attracted to, and retained by, each of the two curriculums. The literature on the preparation of teachers of secondary mathematics was reviewed.

Major Findings and Conclusions.—All prospective teachers of secondary mathematics should have: (1) Broad training in mathematics far beyond any mathematics which the proespective teacher will ever teach; (2) a course built around a core curriculum consisting of college algebra, solid geometry, trigonometry, analytic geometry, differential and integral calculus (total, 18 semester hours); (3) a course in teaching secondary school mathematics; (4) a course in the history of mathematics; (5) an opportunity to teach mathematics under supervision in a junior or senior high school; (6) some work in fields closely related to mathematics, preferably a minor in physics; (7) one or more courses in applied mathematics, (8) several opportunities to observe the work of a master teacher of mathematics in a demonstration junior or senior high school; (9) one or more courses in non-Euclidian geometry; (10) a course in the use of multisensory aids to mathematics teaching; (11) 30 semester hours of content courses in mathematics. Neither of the curriculums met these criteria. The curriculums attracted well-qualified students but failed to retain them as mathematics majors. A new curriculum should be adopted to meet the 11 criteria.

87. PETTYPOOL, CHARLES EVERETT. The Place of Supergeneralizations in Plane Geometry. (M. A., 1956, The Ohio State University, Columbus.)

Major Faculty Adviser.—Nathan Lazar.

Problem.—To study methods of teaching generalizations.

Procedures.—The literature pertinent to the subject was reviewed.

Major Findings and Conclusions.—To discover supergeneralizations, a student must recognize them through a great number of examples in discovery exercises. He may be helped if he learns that a supergeneralization is formed by enlarging the class of figures to which a generalization applies. A well-known supergeneralization is Ceva's theorem, which includes the statement of the concurrency of medians, angle bisectors, and certain altitudes of a triangle.

88. PIERCE, ROBERT FREDERICK. Resources in Europe of Interest to Mathematics Teachers. (Ed. M., 1953, Boston University, Boston, Mass.)

Major Faculty Adviser.—Henry W. Syer.

Problem.—To plan a guided tour in Europe for secondary school teachers with emphasis on present and past mathematical and allied science contributions.

Procedures.—All information for the tour was gathered or confirmed by correspondence (236 letters sent; 174 received).

Major Findings and Conclusions.—An itinerary was made up for an 8-week tour. The students liked the trip and gained much information on the history of mathematics and the use of resources. They also gained a respect for the school systems visited.

89. PINCUS, MORRIS. An Investigation Into the Effectiveness of Two Methods of Instruction in Addition and Subtraction Facts. (Ph. D., 1956, New York University, New York, N. Y.)

Major Faculty Adviser.—John J. Kinsella.

Problem.—To compare the effectiveness of two methods of teaching second-decade addition and subtraction facts to children in Grade 3.

Procedures.—The concepts underlying second-decade and subtraction facts were developed and the children were then divided into 2 groups of 2 classes each and matched directly. The method for the experimental group developed the concepts and then used a drill based on relationships. The method for the control group also developed the concepts, but did not use a drill based on relationships.

Instead, it used one consisting of repetitive practice.

Each of the classes was taught by a different teacher. To minimize the element of teacher ability and personality as factors, the teachers exchanged classes half-way through the experiment.

Major Findings and Conclusions.—(1) Both the experimental and the control groups show considerable gains regardless of the type of drill used. (2) There is no significant statistical difference in effectiveness between the 2 methods of instruction in 2-decade addition and subtraction facts. (3) Neither method shows a significant statistical difference in ability to transfer automatic recall of 2-decade facts to higher decades, to retain the automatic recall of 2-decade facts after a lapse of time, or to retain the ability to transfer the automatic recall to higher decades.

90. RANDLETT, RICHARD ROSS. Analysis of Training, Experience and Salaries of Secondary School Mathematics Teachers in Massachusetts, 1924-1953. (M. Ed., Boston University, Boston.)

Major Faculty Adviser.—Henry W. Syer.

Problem.—A survey of mathematics teachers in the Massachusetts public high schools.

Procedures.—A study was made of the secondary schools of Massachusetts. Schools were divided into four classes according to pupil enrollment as follows: Class I, enrollment of more than 500; Class II, 201-500; Class III, 101-200; Class IV, 100 or less.

Major Findings and Conclusions.—There is a trend among men mathematics teachers, but not among women mathematics teachers, to get a master's degree. There was a decrease in the percent of teachers with no degrees in all classes of schools except Class IV, which had an increase in 1952-53. More men than women with 0 to 4 years of experience, in all classes, indicates that, regardless of salaries, the profession has a great attraction for more young men than women. Apparently the women who entered the profession remained, while the men were more likely to leave for one reason or another.

91. RIESS, ANITA P. A New Rationale for the Teaching of Fractions. (1955, University of Bridgeport, Bridgeport, Conn.)

Problem.—The implications of the history of fractions, especially that of the unit fraction, for teaching in the intermediate grades.

Procedures.—Experimental teaching was conducted during August and September 1954 in a public school at Hamburg, Germany, and in January 1955 in Mill Plain School at Fairfield, Conn. The approach to fractions was from the angle of multiplication rather than division.

Major Findings and Conclusions.—The method seemed to help the pupils gain a better understanding of fractions.

92. ROACH, GEORGE E. A Survey of the Mathematics and Science Programs for the Gifted Students in the Secondary City Schools of Indiana. (M. S., 1956, Indiana State Teachers College, Terre Haute.)

Major Faculty Adviser.—W. O. Shriner.

Problem.—To ascertain the number of secondary city schools of Indiana that have special programs of study designed especially for gifted students in mathematics and science and to investigate the nature or core of these programs.

Procedures.—A total of 180 questionnaires was mailed to every town or city school system in Indiana.

Ninety-one, or 70 percent, came back. These were analyzed. Students with I. Q. 120 or above were classified as gifted students.

Major Findings and Conclusions.—Ninety-five percent of the reporting schools stated that enrichment was their chief method for aiding gifted students in mathematics and science. Sixty-seven percent of these schools used homogeneous grouping in mathematics and 40 percent, homogeneous grouping in science. More and more schools that have studied the gifted-student problem are beginning to realize the advantages of homogeneous grouping.

93. ROSENBERG, HERMAN. The Impact of Modern Mathematics on Trigonometry: A Study of the Significance of Certain Concepts of Higher Mathematics for the Teacher of Trigonometry. (Ph. D., 1955, New York University, New York, N. Y.)

Major Faculty Adviser.—John J. Kinsella.

Problem.—To investigate the significance of certain concepts of higher mathematics for teachers of trigonometry.

Procedures.—Procedures for solving the problem are based upon certain assumptions relating to the nature of (1) the concepts of higher mathematics to be selected for investigation and (2) the *implications* to be determined from the selected concepts. Techniques for *selecting* concepts include examination of available source materials (textbooks, articles, etc.) and preliminary investigation of the potential significance of possible concepts for trigonometry teachers. Techniques for *determining* implications of selected concepts include application of deductive reasoning and fundamental mathematical processes, such as the major operations (addition, differentiation, etc.)

Major Findings and Conclusions.—Selected ideas from modern mathematics enrich trigonometry through: (1) Alternative methodology in proving basic theorems or in solving exercises. (Thus, the law of cosines is derived by utilizing such concepts as polar and exponential forms of complex variables and the dot product of two vectors.) (2) Broad ideas providing insight, suggestions for unification, or possibilities for alternate approaches. (Thus, the existence and classification of various trigonometries are developed by utilizing such concepts as functional notation and equations of conic sections from analytic geometry.) (3) Additional notations and terminology (such as the determinant notation and "constructive" terminology used by the intutionist school of thought on the foundations of mathematics). (4) Additional practice exercises (such as "trigonometric congruences").

94. RUSSELL, HARVEY ROSWELL. Patterns of Cooperation Between Industry and Education. (Ed. D., 1956, Teachers College, Columbia Univserity, New York, N. Y.)

Major Faculty Adviser.—Hubert M. Evans.

Problem.—How do industry and education work together in educational activities?

Procedures.—Literature on the subject was reviewed and the activities of the education committee of the western Connecticut section (Stamford area), American Chemical Society were analyzed. Two hundred and five cases were abstracted and classified as to type of activity.

Major Findings and Conclusions.—The most frequent activities were the following, in the order named: Providing information, assisting high school mathematics and science projects, providing speakers, giving vocational counsel, and arranging plant trips.

95. SABOL, ANNA. Important "Page" From the History of Mathematics. (M. of Ed., 1955, The Ohio State University, Columbus.)

Major Faculty Adviser.—Nathan Lazar.

Problem.—To select, from the great mathematical documents of the past, the important "pages" which might be used in the mathmatics classroom to enrich the student's knowledge of the nature of elementary mathematics so that he will gain a real understanding of the growth of mathematics.

Procedures.—A survey was made of important mathematical works.

Major Findings and Conclusions.—Important "pages" were photographically reproduced.

96. SATHOFF, ROBERTA SPECK. The Use of Historical Material in the Teaching of Trigonometry. (M. of Ed., 1955, The Ohio State University, Columbus.)

Major Faculty Adviser.—Nathan Lazar.

Problem.—To collect historical material about trigonometry that will enrich the subject matter for student.

Procedures.—A survey was made of documents that might contain historical material about trigonometry.

Major Findings and Conclusions.—A brief history of trigonometry is presented in chronological sequence. Following this brief account, extended treatment is given to historical material.

97. SCHILHAB, ELGIN JAMES. A Study of the Attitudes of 1st-Year Algebra Students. (M. A., 1956, University of Texas.)

Major Faculty Adviser.—I. I. Nelson.

Problem.—To discover: (1) The relationship between intellectual achievement and attitudes, (2) the relationship between past achievement in mathematics and attitudes, (3) the influence of reading achievement on student's attitudes, (4) the relationship between intelligence quotients and attitudes, (5) the relationship between social class and attitudes.

Procedures.—Sixty-two boys and 39 girls, 1st-year algebra students in the summer session of the Austin (Tex.) public schools, were used for the study. The majority were making up work in mathematics which they had failed. They gave pertinent personal data and answered questions intended to reveal attitudes. Other data regarding I. Q., achievement in arithmetic, reading vocabulary, and reading comprehension were secured from school records and past grades from teachers' registers.

Major Findings and Conclusions.—Student attitudes were influenced by arithmetic achievement test scores, intelligence quotients, and high school grade averages. The same attitudes were not influenced by reading achievement test scores nor by family social class. A poor foundation in the fundamentals of mathematics and no realization of a practical value for algebra caused negative attitudes.

98. SEBER, ROBERT C. The Development and Organization of Teaching Materials in a Collegiate Mathematics Program for Students of the Nonphysical Science—Part I. (Phi. D., 1956, State University of Iowa, Iowa City.)

Major Faculty Adviser.—H. Vernon Price.

Problem.—To develop a course in mathematics for students of nonphysical science.

Procedures.—A course sequence with a prerequisite of college algebra and trigonometry was proposed. The sequence included topics of algebra, geometry, analysis, and statistics. Emphasis was placed on the style of presentation. Each topic was considered with reference to its descriptive background, formalization as a mathematical system, and application in describing phenomena of nonphysical science.

Major Findings and Conclusions.—The course in mathematics with illustrative material was prepared.

99. SHIRLEY, THOMAS WILLIAM. Survey of Field-Instruments Study and Usage in the Public High Schools of Illinois. (M. A., 1956, Illinois State Normal University, Normal.)

Major Faculty Adviser.—T. E. Rine.

Problem.—To determine the status of field-instrument instruction in mathematics in the medium-size public high schools of Illinois.

Procedures.—The questionnaires returned by 184 Illinois high schools were analyzed.

Major Findings and Conclusions.—(1) "Theoretical" aspects of mathematics are emphasized over the "practical." (2) Most mathematics teachers are not familiar with the operation of field instruments. (3) Mathematics teachers believe that work with field instruments should be done in the secondary school, but do not feel prepared to do this themselves.

100. SISTER MARY DOMINIC ENGELHARD. An Experimental Study of Arithmetic Problem-Solving Ability of 6th-Grade Girls. (1955, Ph. D., The Catholic University of America, Washington, D. C.)

Major Faculty Adviser.—Francis J. Houlahan.

Problem.—To determine, by a comparison of differences between means, the abilities differentiating high- and low-achieving girls in arithmetic problem solving at the 6th-grade level.

Procedure.—A battery of 17 tests—8 standardized, 9 constructed for purposes of this study—was administered to 500 6th-grade girls of the Milwaukee Archdiocese. On the basis of scores attained on the criterion tests—Stanford arithmetic reasoning and Kuhlmann-Anderson — matched groups were constituted.

Major Findings and Conclusions.—All the variables investigated in this study are very significantly associated with problem-solving ability when the entire M. A. range of the population is considered; some factors assume greater or less importance at the high and low M. A. levels. Both high- and low-achieving girls are favorably inclined toward arithmetic.

The girls differ from the boys by (1) a higher M. A. and I. Q., but a lower chronological age; (2) a more favorable attitude toward arithmetic; (3) superior achievement, especially on tests of reading factors.

The boys manifest superior ability to estimate answers to verbal arithmetic problems and also a better understanding of quantitative concepts, principles, and relationships.

101. SMALL, DWAIN E. Opinions of Secondary Mathematics Teachers Concerning the Fifth Year of Teacher Education. (Ed. D., 1955, Indiana University, Bloomington.)

Major Faculty Adviser.—Howard T. Batchelder.

Problem.—To secure the opinions of secondary mathematics teachers concerning the fifth year of teacher education for mathematics teachers.

Procedures.—A random sample of 1,465 teachers who were members of the National Council of Teachers of Mathematics was selected from the council mailing list. A total of 951 teachers representing every State in the United States answered the questionnaire. The opinions and the association of the opinions with factors were analyzed by IBM distributions.

Major Findings and Conclusions.—The fifth year of teacher education for mathematics teachers should include work in professional education, mathematics, research, advanced student teaching, and cultural areas. Approximately 50 percent of the work should be done in mathematics courses including laboratory, and not more than 25 percent in professional education courses.

The program of the fifth year of teacher education for mathematics teachers should be broad and flexible, with few specific course requirements in either mathematics or education. Workshops should be provided to relate the areas of mathematics and science as well as to develop techniques for the laboratory as an integral part of the mathematics classroom. Courses should be given in the history of mathematics, basic number theory, mathematical statistics, modern algebra, modern geometry, theory of equations, and the mathematics of finance. The instructional staff should be composed of persons who have had recent experience with secondary pupils. During the summer session it should utilize some visiting professors.

102. SMITH, ROLAND FREDERICK. An Experimental Comparison of Two Liberal Arts Courses in General Mathematics at Syracuse University. (Ph. D., 1955, Syracuse University, Syracuse, N. Y.)

Major Faculty Adviser.—R. M. Exner.

Problem.—To study an algebraic and a geometric method of teaching college general mathematics.

Procedures.—A total of 141 students were divided into 3 groups. The same instructor taught each group, using the algebraic method for the first and the geometric method for the second. The third group served as the control. The statistical analysis was conducted by the Johnson-Neyman method extended to take account of differences among the three groups.

Major Findings and Conclusions.—Although results did not uniformly favor either method, for most students, apparently, the advantages of the geometric method outweighed those of the algebraic.

103. SPECIAL COMMITTEE ON COLLEGE MATHEMATICS FOR NON-SCIENCE STUDENTS, A SUBCOMMITTEE OF THE CALIFORNIA COMMITTEE FOR THE STUDY OF EDUCATION. (1956, Los Angeles, Calif.)

Problem.—To determine current practices in the field of college mathematics for general education.

Procedures.—Questionnaires were sent to about 250 representative colleges throughout United States and Canada.

Major Findings and Conclusions.—Of 110 colleges answering the questionnaires, 67 offered special courses in mathematics for nonscience majors and 16 expressed a desire to give such courses, although at the time they did not do so. The remaining 27 implied that the usual sequence of mathematics courses was adequate for all their students. Thirty-three different textbooks were being used, and in 6 colleges, instructors used their own manuscripts. The books used most frequently were Richardson; Newsom and Eves; Leonhardy; Cooley, Gans, Kline and Wahlert; Trimble, Peck and Bolser; Jones; Griffin. No single course meets the needs of all students and all instructors. The choice of topics is not so important for the success of the course as are the attitude and interests of the teacher, and the attitudes and emotions of the students.

104. STEINBRENNER, ARTHUR HENRY. A Study of the Concept of Continuity for Teachers of Secondary School Mathematics. (Ph. D., 1955, Columbia University, New York, N. Y.)

Major Faculty Adviser.—Howard F. Fehr.

Problem.—To analyze the historical development and existing practices in teaching continuity and related concepts of irrational numbers and limits.

Procedures.—Historical and philosophical methods were used. To bring to light the way in which the concept of continuity enters into secondary school mathematics, an analysis was made of certain elementary textbooks and books on mathematics teaching.

Major Findings and Conclusions.—In plane geometry the question of continuity arises in connection with those theorems involving the incommensurable case. Present-day practice is either to postulate these theorems or to treat them informally. In algebra textbooks, modern practice is to give a broader interpretation to the meaning of number and a more general definition of irrational number than at previous times. Limit of a sequence is important to an understanding of continuity, and arises in connection with some of the material in secondary mathematics. Rigorous definition is out of place in an elemantary discussion, but an informal definition may be given.

105. STILES, EULA S. A Frequency Count of Arithmetical Terms in Texas State-Adopted Arithmetic Textbooks (1955), Grades 3 to 5. (M. Ed., 1955, University of Texas, Austin.)

Major Faculty Adviser.—Thomas D. Horn.

Problem.—To find out what arithmetical vocabulary is used by the State-adopted arithmetic textbooks in Texas.

Procedures.—A basic vocabulary list of 533 terms was compiled from earlier studies and from terms used by the textbooks in Texas. The study showed the number of times each of the 533 terms appeared in each series and each grade, and the grand total for each term. The average frequency was figured for each grade level and the grand total for words used 50 or more times in all series. The ranges for each grade and the grand total were shown for these words. Words not common to all series were also listed.

Major Findings and Conclusions.—Of the 533 words or expressions analyzed in this study, 217, or nearly 41 percent, occurred fewer than 50 times. Ninety-seven terms were found fewer than 10 times and none of these were common to all series. Some of these words were used too few times to warrant their inclusion in the textbooks. More technical words (161) were found than any other kind, but there were almost as many words denoting time, space, quantity, value, or degree (150). Three hundred sixty-eight, or 69 percent, of the terms were introduced by the end of grade 3. The study showed that the authors provided for the repetition of terms.

106. STIPANOWICH, JOSEPH JEAN. The Development and Appraisal of a Course in Basic Mathematics for Prospective Elementary School Teachers. (Ed. D., 1956, Northwestern University, Evanston, Ill.)

Major Faculty Adviser.—E. H. C. Hildebrandt.

Problem.—What specific topics should be included in an initial course in basic mathematics for college students preparing to teach in the elementary school?

Procedures.—(1) A survey was made of the professional literature and textbooks on the subject. (2) The opinion of 68 educators was secured by a questionnaire. (3) an experimental course in basic mathematics for prospective elementary school teachers was taught at Western Illinois State College.

Major Findings and Conclusions.—(1) All prospective elementary school teachers should be required to take mathematics. (2) of the 68 educators, 93 percent recommended the same 26 topics (all arithmetical) for an initial course in basic mathematics. (3) For an experimental-initial course, the following subject-matter units were developed: (a) Mathematics and the needs of prospective elementary school teachers, (b) Ancient and modern number systems, (c) Whole numbers in the Hindu-Arabic decimal number system, (d) Common fractions in the Hindu-Arabic decimal number system, (e) Decimal fractions in the Hindu-Arabic decimal system, (f) Percent, (g) Systems of measurement and mensuration, (h) Introduction to intuitive geometry and elementary algebra.

At the beginning of the experimental course 45 percent of the group disclosed (in writing) that they definitely disliked it. By the end of the course 70 percent of the same group indicated that they liked arithmetic and 10 percent that they still disliked it.

107. STURIKY, ERIC. Mathematics in the College General Geology Course. (Ed. D., 1955, Teachers College, Columbia University, New York, N.Y.)

Major Faculty Adviser.—Howard F. Fehr.

Problem.—To show how high school mathematics might be used in the general course in college geology.

Procedures.—The topics that might be included in a general course in college geology were taken from a pamphlet of the National Science Teachers Association. These topics were checked in 17 general geology books and 10 general science books. All the topics were found in most of the books, and most of the topics were found in all the books. Specialized books used a large amount of mathematics, and general textbooks, little mathematics.

The author attempted to (1) point out areas in the general geology courses where mathematics might be used, (2) show how mathematics might be used in these areas, (3) show where a student's previous mathematical experience might help introduce him to problems in geology.

Major Findings and Conclusions.—Teachers of general geology could use more problem material involving mathematics. This material is not found in general geology and general science textbooks, but is found in specialized textbooks.

Teachers of elementary and high school mathematics could enrich their teaching by using problem material from geology.

108. THOMPSON, ETHEL HART. Application of the Guess-and-Check Method to the Early Use of the Multiple-Equation Method for the Solution of Verbal Problems in Elementary Algebra. (M. E., 1956, The Ohio State University, Columbus.)

Major Faculty Adviser.—Nathan Lazar.

Problem.—To adapt the material in a high school algebra textbook to the "guess-and-check" method.

Procedures.—The author rearranged the material in an algebra textbook and taught an elementary algebra class the "guess-and-check" method of solving verbal problems.

Major Findings and Conclusions.—Guessing can be made a respectable learning technique. This method helps the student acquire a real understanding of verbal problems, since it is based upon his experimental background. The guess-and-check method, applied to the multiple-equation approach to the solution of verbal problems, may be employed by any classroom teacher who is willing to devote a small amount of time to rearranging the material in a traditional algebra textbook.

109. TOMMEY, EILEEN THERESA. Mathematics Teaching in the Public Secondary Schools of Rhode Island (1951-1952). (A. M., 1952, Boston University, Boston, Mass.)

Major Faculty Adviser.—Henry W. Syer.

Problem.—To determine and analyze the status of secondary mathematics in the public senior and junior high schools of Rhode Island during the school year 1951-1952.

Procedures.—A questionnaire was sent to every principal and mathematics teacher in the public secondary schools of the State through the office of the department of education, accompanied by a communication from the commissioner. Returns were received from 95 percent of the schools and 78 percent of the teachers.

Major Findings and Conclusions.—The average secondary mathematics teacher in Rhode Island during the school year 1951-52 had a bachelor's degree, received a salary of $3,648, and had taught 18 years. His average class consisted of 26 pupils, he had an enrollment of 115, and he taught 22 periods a week. In some cases he taught a triple combination of mathematics, English, and social studies, or a dual combination of mathematics and science or mathematics and social studies.

The number of men who teach mathematics in Rhode Island is increasing steadily.

The traditional sequence of courses (elementary algebra, plane geometry, intermediate algebra, trigonometry, and solid geometry) was prevalent. General mathematics courses were confined mostly to the junior high school. A double-track plan of functional courses and traditional courses in the senior high schools was not apparent.

110. TYSON, THOMAS T. Enrichment Material for the Brighter Student of Plane Geometry. (M. E., 1956, The Ohio State University, Columbus.)

Major Faculty Adviser.—Nathan Lazar.

Problem.—To provide enrichment material to supplement the teaching of bright students in plane geometry.

Procedures.—The topics, chosen from advanced geometry textbooks, were simplified for bright students in plane geometry on the secondary level. They were topics closely related to the important material and theorems found in a typical high school geometry course.

Major Findings and Conclusions.—Three topics—Centroids, Ceva's theorem, and homothetic figures—are presented.

111. VAN ENGEN, H. *and* GIBB, GLENADINE. Higher Mental Functions Associated with Division. (Iowa State Teachers College, Cedar Falls.)

Problem.—To see whether there are any educational differences between teaching division by the usual method and teaching it by the method of successive subtraction of multiples of the divisor.

Procedures.—Twelve self-contained 4th-grade classrooms were selected at random from Sioux City and Des Moines, Iowa. Six classes were taught by each method for 1 year and results were reviewed by analysis of convariance.

Major Findings and Conclusions.—No statistical difference between the two methods in so far as the process is concerned. The successive subtraction method was more meaningful. Children taught by this method generalized more readily (statistically significant) than those taught by the usual method.

112. VANNATTA, GLEN D. Background, Choices, and Opinions of Superior Mathematics Students as a Basis for and Attack on the Scientific Manpower Shortage. (Ph. D., 1956, Indiana University, Bloomington.)

Major Faculty Adviser.—Philip Peak.

Problem.—(1) To determine the present activities of students who ranked as winners in the State comprehensive mathematics contest. (2) To determine some of the characteristics of the winners. (3) To determine what caused them to make their particular choices.

Procedures.—Every contest winner was listed from 1952 to 1955. Of these 406 winners, 327 filled out and returned questionnaires.

Major Findings and Conclusions.—To a small degree mathematics contests encourage students with high potential. A very high percentage of these superior students attend college. Most commonly, their reasons for not attending are marriage and insufficient funds. A high percentage receive aid from scholarships but over 50 percent also carry part- or full-time jobs. The high school teacher has the greatest influence on students' decisions to enter the field of mathematics; parents rank second. The facets of scientific work appealing most to the student appear to be challenge of the unknown and mental stimulation.

113. WAGGONER, WILBUR JAMES. The Relationship of High-School Mathematics to Success in College. (Ed. D., 1956, University of Wyoming, Laramie.)

Major Faculty Adviser.—R. I. Hammond.

Problem.—A statistical study of the relationship between the number and type of high school mathematics units completed and success in the college of education, University of Wyoming.

Procedures.—The sample was taken from all students who completed a 4-year program or the 2-year portion of the 5-year plan in elementary education in the college of education during the 4-year period, 1951–55. The sample was divided into 3 groups. The university cumulative grade average was used as the criterion of college success. The 8 independent variables used in this study were the following: (1) High-school grade average, (2) raw score on the history and social studies section of the Iowa high school content examination, (4) whether a student majored or minored in mathematics or physical sciences, (5) number of units in high school general mathematics, (6) number of units in high school algebra, (7) number of units in high school geometry, and (8) number of units in high school trigonometry.

Major Findings and Conclusions.—The number and type of high school mathematics units had no significant relationship with sucess in the college of education. By using the standard error of estimate, the predicted grade average for an individual could vary almost a full grade point from the grade actually earned.

114. WALTERS, ELEANOR BOYD. Concept, Place, and Purpose of Professionalized Subject-Matter in the Education of Teachers of Secondary Mathematics. (Ed. D., 1955, Teachers College, Columbia University, New York, N. Y.)

Major Faculty Adviser.—Howard F. Fehr.

Problem.—To give a satisfactory answer to the question: What is professionalized subject matter in mathematics at the undergraduate level for prospective teachers of secondary school mathematics?

Procedures.—A brief review was made of the history of professionalized subject matter. A definition of professionalized subject matter was then developed in light of the needs of secondary mathematics teachers, of the inadequacy (where such exists) of the traditional mathematics and methods courses to meet those needs, and of the recommendations of responsible committees and individuals concerned with the problem of educating teachers.

Major Findings and Conclusions.—Professionalized subject matter is that which is related to the subject matter that the prospective teacher will teach—selected, organized, and taught in such a way as to meet the teaching needs of prospective teachers. The content is mathematical; the purpose, professional.

The final chapter of the study gives a rather thorough treatment of the concept of proof to indicate the nature of the content of such a professionalized course as proposed.

115. WATTERS, LORAS J. Factors in Achievement in Mathematics. (Ph. D., 1954, The Catholic University of America, Washington, D. C.)

Major Faculty Adviser.—Francis J. Houlahan.

Problem.—To study boys' and girls' factor patterns as reflected on a mathematics test. (This is one of several factor-pattern studies at The Catholic University of America.)

Procedures.—The mathematics section of the Iowa high school contest examination, Form L, was given to nearly 500 juniors in 22 Catholic coeducational schools in Iowa. The centroid method was used to analyze the data.

Major Findings and Conclusions.—A comparison of the 1st- and 2d-order factor structure for the two groups indicated that the factor patterns were similar in many respects. The differences, however, pointed to a greater integration of the boys' abilities. Almost every type of geometry problem in the test proved to have been loaded in favor of the boys. Another notable difference was a mathematical vocabulary factor for the girls.

116. WHITESEL, WILLIAM FRANK. A Study to Predict Success in Algebra of 9th-Grade Students in the Puyallup Junior High School. (M. Ed., 1956, University of Washington, Seattle.)

Major Faculty Adviser.—August Dvorak.

Problem.—To predict the probable succes in algebra of each student entering the 9th grade.

Procedures.—The following variables were studied on 160 cases: (1) Screening test scores, (2) 8th-grade total grade points, (3) I. Q. scores, (4) reading vocabulary scores, (5) reading comprehension scores, and (6) days present.

Major Findings and Conclusions.—The screening test was the best variable for predicting success in algebra.

117. WHITMORE, EDWARD HUGH. A Study of Sequences in Elementary Geometry. (Ph. D., 1956, The Ohio State University, Columbus.)

Major Faculty Adviser.—Nathan Lazar.

Problem.—To study the sequence of definitions, axioms, postulates, and propositions leading to the proof of the proposition on the sum of the interior angles of a triangle.

Procedures. — Ninety-seven plane geometry textbooks, published from 1703 through 1955, were analyzed.

Major Findings and Conclusions.—The 8 most widely used sequences during the last 15 years are the following: (1) A modern version of Euclid's sequence; (2) Legendre's sequence, in which the proposition "parallel lines make equal corresponding angles with a transversal" and its converse were proved directly; (3) the sequence in which the statement is postulated that parallel lines are lines which make equal corresponding angles with a transversal.

118. WILLIAMSON, DOUGLAS B. The Mathematics Essential to the Learning of Engineering Physics. (Ed. D., 1956, Teachers College, Columbia University, New York, N. Y.)

Major Faculty Adviser.—Howard F. Fehr.

Problem.—To determine what mathematics is used in a textbook of college physics for students of engineering, the physical sciences, and mathematics.

Procedures.—An appropriate text and the solutions to 408 of its problems were analyzed.

Major Findings and Conclusions.—Although most of the applications involved elementary algebra or plane geometry, 11 percent were from solid geometry, trigonometry, analytic geometry, and the calculus. Many more applications, especially of these more advanced courses, were found than had been reported by previous investigators.

119. WILLIAMSON, ROBERT GORDON. A Theory of Learning and Its Application to a Class in College Mathematics. (Ed. D., 1956, University of Maryland, College Park.)

Major Faculty Adviser.—H. Gerthon Morgan.

Problem.—To use a philosophical approach to deduce an original method for teaching college subject matter.

Procedures.—Previous learning theories and developments in modern science and mathematics were synthetized into a theory of learning. A procedure was devised applying the theory as a highly individualized method of instruction with emphasis on self-involvment and communication. A class in freshman college mathematics was taught, using the procedure and measuring outcomes in relation to selected factors. The method permitted students to proceed at their own rate under individualized instruction.

Major Findings and Conclusions.—(1) Significant gains in subject matter skill were achieved. (2) Student ratings and evaluation indicated a preference for the method.

120. WILSON, ROSLYN ADELL. A Classified List of Practical Mathematical Problems for Grades 9 Through 12. (Ed. M., 1955, Boston University, Boston, Mass.)

Major Faculty Adviser.—Henry W. Syer.

Problem.—To set up a 4-point scale by which to judge a list of classified practical mathematics problems (grades 9 through 12) found mainly outside the realm of the average classroom textbook.

Procedures.—Textbooks were randomly selected and problems were arbitrarily chosen. The problems were divided into four piles ranging from "practical" to "impractical." The results were tabulated and recorded.

Major Findings and Conclusions.—Out of the 137 problems, 49 were rated "4", 24 were rated "3", 33 were rated "2" and 31 were rated "1". (Rating of "4" is most practical.) Teachers were unanimous on only 6 problems.

121. WISE, MIRIAM MEYERS. An Investigation of the Feasibility of the Use of Teaching Aids, Games, and Devices in the Teaching of Arithmetic. (M. A., 1955, The Ohio State University, Columbus.)

Major Faculty Adviser.—Lowry W. Harding.

Problem.—To explore the feasibility of the use of teaching aids, games, and devices in teaching arithmetic; to determine criteria for use of these aids; to classify and catalog these aids in order to make them available to other interested teachers.

Procedures.—The author investigated games, aids, and devices described in periodicals, commercial games, and games and devices in the toy department of several stores.

Major Findings and Conclusions.—Games and devices can be practical and valuable aids in teaching arithmetic. A list of teaching aids was compiled.

122. WISER, RALPH. The University and College Preparatory Mathematics Curriculum and Enrollment in Selected High Schools of Tehema, Glen and Butte Counties. (M. A., 1955, Chico State College, Chico, Calif.)

Major Faculty Adviser.—Philip M. Iloff.

Problem.—To determine (1) the mathematics courses being offered to university and college preparatory students in some typical Northern California high schools, (2) the proportion of the students enrolled in these courses, and (3) the emphasis placed on mathematics.

Procedures.—Data for this study were obtained by personal interviews with administrators and mathematics instructors in eight secondary schools.

Major Findings and Conclusions.—Sufficient offerings were found in algebra I and plane geometry, but not in the more advanced courses. Offering courses in alternate years not only increased the problem of proper sequence but also the possibility of schedule conflicts. Over one-half the 12th-grade students had taken algebra I and over one-third, plane geometry.

Students graduating from the high schools in this study who have an adequate mathematics background constitute too small a percentage of the total number graduating from these schools.

123. WOLFE, JACK M. Proximity of Prerequisite Learning and Success in Trigonometry in College. (1956, Brooklyn College, Brooklyn, N. Y.)

Problem.—To determine whether there is a significant difference in the final grades in college trigonometry among students who differ as to the time lapse since their last mathematics course.

Procedures.—A total of 280 unselected Brooklyn College students who had registered for trigonometry were pretested in mathematics at the beginning of the fall semester. At the end of this semester, the records of 806 unselected students who had completed the trigonometry course were examined to discover the time lapse between their last previous mathematics course (generally intermediate algebra) and this trigonometry course. Of these 806 students, 172 had completed their last previous course in June—only a summer's vacation before beginning trigonometry. The remaining 633 had a minimum lapse of 8 months and a mean lapse of 17 months.

Major Findings and Conclusions.—The students who had completed a mathematics course the previous June started trigonometry in September with greater proficiency in the prerequisite topics than the students with a longer time lapse. The latter, however, earned significantly higher final grades in trigonometry and had a significantly lower failing rate.

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