US-China Education Review A, ISSN 2161-623X
June 2013, Vol. 3, No. 6, 365-373

Assessing Translation Misconceptions Inside the Classroom: A Presentation of an Instrument and Its Results*

Pauline Anne Therese M. Mangulabnan
De La Salle University, Manila, Philippines

This is a descriptive research on the difficulties of Filipino high school students in translating algebraic word problems into mathematical equations. This research is composed of three parts: (1) development of an 11-page "Filipinized" questionnaire; (2) analysis of the mathematical thinking processes of the respondents based on the answers to the questionnaire; and (3) identification of the alternative conceptions or errors of the students in translating word problems which lead to incorrect or misconceived answers. Through the instrument and students' answers analysis, this research categorizes the assimilation errors made by high school respondents as LBE (language-based errors), OIE (operational-influenced errors), ATE (algebraic translation errors), and RSE (relational-symbol errors). This research recommends a review of the construction of word problems in terms of its realness and logic to match students' mathematical background.

Keywords: mathematics, classroom assessment, algebraic translation, students' misconceptions

Introduction

Mathematics, no matter how relevant, is usually feared by learners. Mathematics education researchers all over the globe point to the students' classroom bad experience, lack of internal and/or external motivation, natural mathematical ability, social status, and unpleasant encounters with teachers as some of the causes of such anxiety. Because of this anxiety, there are quite a number of college students who would choose as their major the one involving the least amount of mathematics. But no matter how students would want to avoid mathematics, it is an undeniably part of their everyday existence.

A simple form of early mathematics is when a child thinks of his new age on the day of his birthday. From this simple application, mathematics is encountered as one tries to label and explain the complexities of the world. Galileo had expressed this perfectly when he said that mathematics is the language of the universe. All its riddles and mysteries are modeled and resolved by mathematics and problem-solving.

Problem-solving, which is the universal remedy for all mathematical problems, (and mathematics) has been an integral way of life and component of mathematics (Schoenfeld, 2004). Its importance is supported by the fact that it involves high order thinking skills, such as application, synthesis, critical thinking, and

* **Acknowledgements:** The author's deepest gratitude to her college thesis partners, Ms. Cristine Ann Paderes and Ms. Nathalie Joyce Lim, for starting a very valuable research for Filipino math learners and for trusting her to further improve the study they once started, to her dear math education mentor, Dr. Auxencia Limjap, for unconditionally supporting the author with all her researches and idiosyncrasies, to her family and friends who never get tired of dealing with her and her loony work schedule, and to her beloved Philippines for always giving her a reason to continue empowering our educational system through research.

Pauline Anne Therese M. Mangulabnan, Lilyrose School, De La Salle University.

evaluation. “A mathematical problem as a task as defined by Charles and Lester (1982) for which the problem solver wants or needs to find a solution, has no readily available procedure for finding the solution, must make an attempt to find a solution” (Pothier, 2000). Polya (1949) even stressed that to solve a math problem is to find a way where no way is known as off-hand, to find a way out of difficulty, to find a way around an obstacle, and to attain a desired end, which is not immediately attainable by appropriate means. As crucial as it sounds for the students’ mathematical journey, problem-solving is a greater challenge for mathematics teachers. The latter would have to ensure that such ambitious goals are met (i.e., students are able to understand math word problems and translate them into mathematical equations to solve).

Indeed, translating word problems into mathematical equations posits problems which are affected by several factors. This does not only include mathematical skills, but also linguistic skills and verbal skills—particularly in the English language, since both private and public schools in Philippines utilize English as the medium of discourse (Mangulabnan, Paderes, & Lim, 2007). Another obstacle in problem-solving is that a pupil is expected to have good foundations in mathematics (Cathcart, Pothier, Vance, & Bezuk, 2000). Furthermore, in any type of word problem, a problem-solver uses his/her prior knowledge in obtaining an accurate solution and answer. Truly, understanding a word problem in problem-solving is a complex method that involves several determining factors, which either affect or influence the process in which solvers find the correct answer.

Theoretical Background

Lesh (1987; as cited as Nickson, 2000) described three significant translations in solving algebraic word problems. Students translate a word problem formulated as a sentence in English to an algebraic sentence (equation), then from the algebraic sentence to an arithmetic sentence, and finally from an arithmetic sentence back to the original problem situation. Therefore, in order to obtain the right answer, it is significant for solvers to first analyze, comprehend, and translate thoroughly the English problem structure into a mathematical equation. However, most students do not recognize the similarities and differences between arithmetic and algebra which impose a problem in translation. Van Ameron (2002) stated that arithmetic deals with the calculations of known numbers, while algebra requires reasoning of unknown variables, making it hard for the students to learn/shift to algebra. Algebraic expressions include letters or symbols which represent unknown numbers, while the symbols written in an arithmetic equation are usually abbreviations or units. When students encounter such difficulty, they resort to other conceptions which they believe will help them solve the math problem regardless whether the method is sensible or not.

In one of his works, Clement (1982) focused on a deeper understanding of a particular type of misconception in word problem translation known as the reversal error. He posited that students’ mistakes on misplaced numerical coefficients are not simple errors due to carelessness but rather are deep-seated difficulties brought about by a thought process—reversal error. Reversal error is an offshoot of student word problem translators to treat numerical variables, as if they stood for objects rather than numbers (Clement, 1982). For example, when a student is asked to comprehend a statement like “for every two students (S), five books (B) are given”, he/she will think of the variables as either students or books but not as numerical quantities of an unknown. Hence, such assimilation may lead to a reversal error (e.g., $5S = 2B$ or $2S = 5B$) in representation by either syntactic method or semantic method. Clement (1982) described the two conceptual sources of reversal errors (which he also observed even in translations that involve pictures to equations, data tables to equations, and equations to sentences) as follows:

(1) Syntactic word order matching process is when the student simply assumes that the order of key words in the problem statement will map directly into the order of symbols appearing in the equation.

In the above given example on students and books, a student who assimilated the problem through syntactic method will translate it into $2S = 5B$ because two is near the word "students" in the same manner that five is near "books". For many problem-solvers, this method is convenient and handy, because it gives them a "sound" equation without much thinking. Also, such method works for some word problems;

(2) Semantic static comparison process is achieved when a student relates real life experiences in reasoning out for the translation that has occurred. There is some semblance of reason in this approach as an intuitive symbolization strategy, but the approach is very literal attempt to compare the sizes of the two groups.

In the above given example on students and books, a student who assimilated the problem through the semantic method will answer and will reason that there should be more books than students. Thus, five is multiplied to S to show a greater quantity.

Using the abovementioned thinking processes, many problem-solvers jump into hasty generalizations and write equations which are not based on any mathematical reasoning. Even more alarming is that Clement (1982) conducted his study to a group of engineering students who, more than students in other disciplines, are tasked to do more difficult mathematical problems.

In this research, there are three research aims: (1) to develop an instrument that classroom teachers and other education stakeholders can use to assess various misconceptions and thinking processes in translation in algebra students; (2) to verify whether the same thought processes are present among Filipino students before they enter college level; and (3) to provide other thinking processes among Filipino students which lead to an alternative conception in translating algebraic word problem into a mathematical equation. This is also crucial for Filipinos, because unlike Clement's respondents, Filipinos learn mathematics in their second language—English. Latu (2006) claimed that if a student's aim is to achieve success in mathematics, but he/her is continually hampered by reading or language problems, frustration and lessening of self-expectation are likely to occur. Hence, problems arising due to language barrier will also be discussed.

Instrument

The core of this phenomenological qualitative research lies in the 11-page questionnaire developed in 2007 by Mangulabnan, Paderes, and Lim. The instrument consists of six word problems with underlying questions in each problem designed to be answered by high school students who had accomplished the Secondary Algebra Curriculum as prescribed by the Philippine Department of Education. Each word problem was constructed in Filipino context. Hence, the word problems involve objects, situations, and nouns that are Filipino in nature or at least familiar to the majority of Filipino students. However, questions (see Table 1) were written in English which is the medium of instruction for high school mathematics in the country and a second language for most of the respondents.

All six major algebraic word problems are open-ended questions, so that the students will be able to reason out their answers. The first two questions lifted from John Clement's paper. However, the original problem from Clement's paper used strudel cake which was changed to banana cake to adapt to Philippine setting. Clement's problems in translation were used to verify whether the semantic and syntactic errors under the reversal error of the Western students also apply to Filipinos.

The next four questions were constructed after long and rigid researching of different word problems seen in different books—Algebra-related and those that are not. The remaining four questions were finalized with the help of experienced high school algebra teachers and in accordance to the limitation of the instrument which is on solutions of linear equations for high school students. The underlying questions helped in pinpointing the thinking processes or alternative conceptions which are emerging in the course of the word problem translation and solving for the final answer. The most important component of each underlying question is the restatement of the problem. This is an initial filter for identifying whether the difficulty of answering the problem is due to language comprehension or mathematical in nature. Also, the underlying questions for every item were based on the hypothesis of various math educators towards the possible difficulties of student respondents. These underlying questions are product of experts' suggestions and pilot-testing results.

Table 1

Instrument Word Problems With Underlying Questions and Percentage of Wrong Answers During Pilot Testing

Questions	Total respondent	Percentage of wrong final answers (%)
A1. At Mindy's restaurant, for every four people who ordered cheesecake, there were five people who ordered banana cake. (Use *C* and *B* as variables) 1. What are the given? 2. How will you represent the number of cheesecake and the number of banana cake? Explain your representation fully.	29	93.10
A2. There are six times as many students as professors in this university. (Use *S* and *P* as variables) 1. What is the given? 2. How will you represent the number of students and the number of professors? Explain your representation fully.	21	76.19
B. Two persons are to run a race. Jose can run 10 meters per second while Cristine can run six meters per second. The race track is 600 meters. If Cristine had a 50 meter head start, who will win the race? 1. What is the given? 2. How did you understand the phrase "If Cristine has a 50 meter head start"? Are you going to use it to solve the problem? Why or why not? 3. Do you know the distance formula? Did you consider using it? 4. Who won the race? Why did you say so?	37	35.14
C. A farmer has cows and chickens in his farm. He had counted 13 heads and 36 feet. How many cows and how many chickens does he have all in all? 1. What is the given? 2. What is being asked in the problem? 3. What does 13 heads and 36 feet mean?	30	30.00
D. The weight of the tub and a monkey inside it is eight kilos. The weight of the same tub and a bear in it is 802 kilos. If the combined weight of the monkey and the bear is 800 kilos, what is the weight of the tub? 1. What is the given? 2. What is being asked in the problem? 3. How will you explain the problem in your own words?	17	88.24
E. When Jane's friend visited her, Jane had just finished eating one-fourth of the eight ChocNut candies. Jane and her friend each ate one-half of the remaining ChocNut. How many ChocNut did Jane's friend eat? 1. What is the given? 2. What is being asked in the problem? 3. What does the sentence "Jane had just finished eating one-fourth of the eight ChocNut candies" mean? 4. What does the sentence "Jane and her friend each ate one-half of the remaining ChocNut" mean?	21	28.57

Results and Analysis

The instrument was answered by 120 fourth-year students from four different schools in Philippines. Before the start of the test, the respondents were given the complete instructions. The students should use a ball pen; the students must show their complete solutions; erasures are okay; students are not allowed to have any other scratch papers, everything must be written in the questionnaire; the students may explain the answers in Filipino, if they wish to; and students are encouraged to take their time as they answer the instrument. It was stressed that the goal of the study is to understand how the respondents think individually and that is the reason why they were not allowed to talk to their seatmates. All the questions raised by the participants were noted down. The respondents were also instructed to write down their reactions towards the level of difficulty of each question and/or did they understand the problem or not. After the test, random interviews were done to further probe on the answers of the students.

Figure 1 shows the percentages of the respondents who got the answers correctly. Figure 2 shows the comparison of respondents' wrong final answer between the pilot-testing and the actual implementation of the instrument.

Question	Percentage of Correct Answers by		Percentage of Wrong Answers
	Algebra Solution	Trial & Error	
A1	0.00%	0.00%	100.00%
A2	7.41%	0.00%	92.59%
B	25.00%	0.00%	75.00%
C	0.00%	34.26%	65.74%
D	10.19%	7.41%	82.41%
E	39.81%	0.00%	60.19%

Figure 1. Results of the actual implementation of the instrument to 120 respondents.

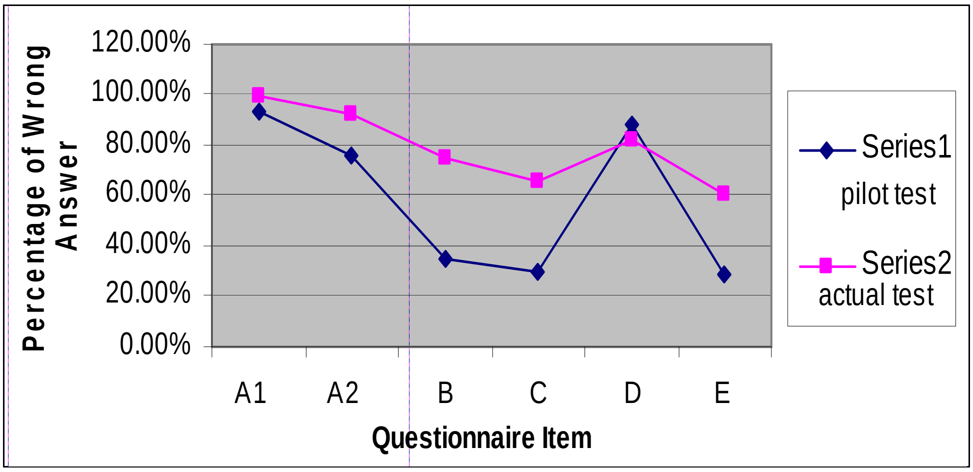

Figure 2. Comparison of the results of pilot-testing and actual implementation.

It was interesting to note that at a larger scale, the errors ballooned. Such number for the actual implementation was chosen to ensure that the answers of the respondents were already exhausted, and repetition of reasoning was attained for the reliability of the categories (on errors and alternative conceptions) formed.

After the analysis, four alternative conceptions or errors on the students' thinking processes on translating word problems into mathematical equations were categorized: (1) LE (language error); (2) OIE (operation-influenced error); (3) ATE (algebraic-translation error); and (4) RSE (relational-symbol error).

Among the four types, LE were the most prevalent one among the respondents. This is an error committed when students cannot understand words, phrases, or even sentences in the given word problem. Most of the students commented that the problems look easy, but they were hard to translate as a whole, so some refused to answer the items they cannot already restate. Words, like bear, candy, gave, etc., are ordinary words which are difficult to translate when taken collectively. The phrase "for every" was a difficult for the majority of the students, so they just deleted that phrase in the restatement of the problem. Because they were not able to understand the phrase, students just added the numbers and equated it to zero (e.g., $4C + 5B = 0$, $6S + P = 0$). The phrase "head start" was another difficult phrase for the students; some said that it was just a nuisance in the problem; others added the "head start" to the length of the race track or to the speed of Christine (e.g., Christine runs 56 m/s, race track is 650 m). The phrase, "The farmer counted 13 heads and 36 feet", was another difficult phrase for many. Some argued that 13 is the heads of the chicken and 36 is the feet of the cow. Others who were not able to understand said that it is impossible to have 13 heads, because the number of feet is an even. There were others also who said that 13 heads included the farmer.

The misunderstanding that happened in the last problem is mostly with the interpretation of the phrase "Jane and her friend each ate one-half of the remaining ChocNut". The common restatement is that Jane and her friend had shared in three ChocNut candies. They did not understand that from the remaining six candies, it is divided into two three for Jane and the other three for her friend. Mostly, the error in understanding this problem lies on the comprehension of fractions. They do not know whether one fourth will be multiplied, divided, or subtracted from eight. When the respondents were asked to state what do they do when they cannot understand the problem, the majority answers were to guess an equation based on how the numbers are positioned from the variables, to perform arithmetic operations until an answer is achieved, to see whether there is something like that in real life, so they can just relate it and get an answer and think of a formula they have learned before and substitute the numbers to get an answer (many of the respondents used for problem *C*). A majority of the students mentioned that they had a hard time for answering, because they are not good in English. It must also be noted that Clement's reversal errors are present among Filipinos.

OIE was categorized for a prevailing use of the basic mathematical operations, such as addition, subtraction, multiplication, and division in the solutions of students. Although the basic mathematical operations are really what we use in solving mathematical problems, OIE pertains to an illogical and irrational use of such operations just to get an answer which could have not been thought wisely in the first place. This error includes the wrong assimilation of the students to multiply, add, subtract, and/or divide all the numbers that are in the problem to come up with an exact number. Thus, the assimilation of a problem-solver to find an exact number for an answer falls under this error (many sees non-integers as wrong answers for the lack of exposure to such solution set). One of the best examples is when a student was trying to answer item *D*. The student made use of this equation, where to solve for the missing weight. The student said that he/she had made up this equation, because weight is being asked in the problem. More so, he/she was expecting that from this equation, he/she will be able to come up with the answer, because he/she could not really figure out what is the supposedly correct answer. This particular student was able to correctly restate the problem. However, because of his lack of algebraic understanding, this was answered. Other variations include adding $4c$, $5b$, $c + b$, etc.. For the students, when there are numbers and operations involved, answers come out easily. That is the reason why they always try to substitute a number for the variables like in the equation respondent got $x = 5$, because y is represented by the number zero. This alternative conception falls under OIE, because the respondent wanted to get an exact value for x despite the fact that an algebraic representation is being asked in the problem. In another form, students wrote equations and reasoned that with addition, they will know the total number of what is being asked. In this alternative conception, students see an equation as a representation of an operation. Unfortunately, any arithmetic error that is part of what they did not learn from grade school (e.g., poor mastery of fractions, ratios, percentages, etc.) is carried out together with this misconception.

ATE is the error in writing the variable representation in algebraic equations. ATE is an error in the direct translation of the word problems into mathematical sentences. It also includes the misconception that algebra is all about solving for x and y, and that representation only involves those variables. There are respondents who constructed the equation, $2x + 4x = 36$. The numerical coefficients are correct, but the literal coefficients are wrong. In the equation, x is used to represent for both the feet of chickens and of the cows where they do not understand that the feet of chickens is different from the feet of the cows, such that two different variables must be used. This shows that students did not know how to use variables correctly and could not differentiate one variable representation from the other. In problem *A*2, there were many students who had made a variable representation of: let x be the number of students and $6x$ be the number of professors. These students correctly comprehended what was being asked by the problem, but used a different variable disregarding what was given.

Students also tended to disregard some of the givens in the problem when they could not represent the phrase in a mathematical sense. For example, in problem *C*, there were students who use the information of the 13 heads and disregarded the 36 feet, because they cannot represent the given algebraically. From 13 heads, the respondents tried to guess numbers (although trial and error is an acceptable solution, this is not what is assessed in the test) like six and seven and allot these numbers of chickens and cows respectively. It is clear that there are students who disregard some information in the problem, if they do not know how to represent them correctly. With the same problem, another student represented x as the cow's head and that $\frac{1}{2}y$ for the chicken's head which is wrong. They thought that since chicken's head is smaller than a cow's head, they can represent it as $\frac{1}{2}y$. In problem *D*, a student represented x as the weight of the monkey and $x + 1$ as the weight of the tub. He understood the problem comprehensively but was not able to come up with the correct variable representation. He just assumed that the weight of the tub is one kilo more than the weight of the monkey. He does not understand that the statements in the problem should be used to solve for the problem. In this error, the common mistake is that the students try to represent everything in one variable without considering its relationship to the other variables concerned.

Lastly, a RSE (relational-symbol error) is an alternative conception resulting to the misuse of relation symbols like "= sign", "< sign", and "> sign" in a translation of word problems. An alternative conception is created when a student tries to take into consideration comparison of the two different numbers found in the problem which then affects the generalization of what mathematical relationship symbol will be used. RSE is common in problems *A*1 and *A*2, but is not present in the rest of the items of the instrument.

In item *A*1, there are students who answered $C < B$. The students considered that since $4 < 5$, respondents believed that the correct representation should be $C < B$ with C representing the number of people who ordered cheesecakes and B representing the number of people who ordered banana cakes. Another example of this appears in item $A2$, there are students who misinterpreted the phrase “six times as many students as professors” as $6S > P$. There are respondents who justified such representation by saying, “I write $6S$, because the students are greater than the professors”. To further justify the representation, there is a respondent who wrote that there are 600 students and 120 professors to show that their representation is correct. The equations $C = B$ and $S = P$ also appears in the respondents’ answers. In these cases, students overlook the concept of equality. They used the equality sign to formulate a mathematical equation, because they have this notion that an equation should include an equal sign. It is clearly stated in the problem that the numbers of people who ordered cheese cakes and banana cakes are different, but they still equated them without considering the other givens in the problem. Another student represented the problem like this: let six be equal to the number of students and > 6 be the number of professors. Instead of using the “= sign” for the representation, they use the “> sign”, because they would want to emphasize the relationship between the number of the students and the number of professors, but forget about other rules in representation in algebra like equality symbol used to show that two quantities are the same.

Conclusions

Even though a mathematical problem may be answered in infinitely many ways, mathematics still offers an elegant solution which is an accumulation of prior mathematical concepts learned. It is difficult for a math learner to learn a topic without mastering the previous one. In the same manner, the errors identified in this research are intertwined.

Upon reading the problem and the student does not understand the mathematical concept presented, he/she may result to the use of semantic or syntactic method to achieve a “sound” equation. From such equation, the student will then manipulate the variables using the operations he/she learned from his/her previous math subjects. Hence, with little understanding of the units involved in the problem, AET or PSE will occur. Thus, a problem-solver gets either a right or a wrong answer which is never based on any mathematical concept. In the same way, the anxiety caused by limited knowledge in arithmetic makes a problem-solver rely on algebraic representation that will allow him/her to do away with fractions, ratios, or other arithmetic concepts and get an answer which is a product of mixed mathematical concepts used inappropriately.

Many respondents admitted in the interview that they were able to get some items correctly or they were at least able to understand the problem through the guide questions. In this case, sadly, this research shows how unprepared Filipino students are for independent problem-solving. It is alarming to know that many fourth-year students are still unequipped even with the most basic mathematical concepts of arithmetic. In this regard, when will they be ready for a more abstract math which is needed for economic development?

Perhaps, future researchers can make a comparative study of the usage of lingua franca in word problem translation as opposed to the usage of English as the medium of instruction. Also, a review of how arithmetic is being bridged to algebra, and how the former’s mastery will aid in a deeper understanding of the latter should be accomplished. Math education stakeholders and classroom teachers should also probe whether the errors in this research are present among their students. From there, the misconceptions may be used as a springboard for classroom learning designs and preparation of content flow. Most importantly, math teachers must reflect whether they are, as educators, contributing to the alternative conceptions or errors of their own students.

References

Cathcart, G., Pothier, Y., Vance, J., & Bezuk, N. (2001). *Learning mathematics in elementary and middle schools* (2nd ed.). Upper Saddle River, New Jersey: Prentice Hall, Inc..

Clement, J. (1982). Algebra word problem solutions: Thought processes underlying a common misconception. *Journal for Research in Mathematics Education, 13*(1), 16-30.

Foster, D. (2007). Making meaning in algebra: Examining students' understandings and misconceptions. *Assessing Mathematical Proficiency, 53*. MSRI Publications.

Latu, V. (2006). *Language factors that affect mathematics teaching and learning of Pasifika students*. University of Auckland. Retrieved from http://www.merga.net.au/documents/RP532005.pdf

Mangulabnan, P., Paderes, C., & Lim, N. (2007). Alternative conceptions on the assimilation of senior high school students in translating algebraic word problems to mathematical equations (Bachlor's thesis, College of Education, De La Salle University).

Nickson, M. (2000). *Teaching and learning mathematics: A teacher's guide to recent research.* Cassell Publishing.

Schoenfeld, A. (2004). The math wars. *Educational Policy*, January and March. Retrieved from http://www.mathismore.net/articles/pdf/Schoenfeld_MathWars.pdf