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MATHEMATICS AT WEST POINT AND ANNAPOLIS

INTERNATIONAL COMMISSION ON THE TEACHING OF MATHEMATICS

THE AMERICAN REPORT

COMMITTEE No. XI

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INTERNATIONAL COMMISSION ON THE TEACHING OF MATHEMATICS

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THE AMERICAN REPORT

Committee No. XI. Mathematics at West Point and Annapolis

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CONTENTS.

MATHEMATICS AT WEST POINT AND ANNAPOLIS

GENERAL REPORT

Of all the technical schools in the United States, probably none exists whose aim is so clearly defined as that of our two great Government schools for the training of officers for the United States Army and Navy. The purpose of these schools is strictly utilitarian, viz, to give to a selected number of young men of this country the best possible technical training for the positions of responsibility in the Army and Navy. That these schools do their work well is amply demonstrated by the efficiency of our military and naval forces.

It is always of interest to see how a school fulfills its aim, how it adjusts its subject matter to meet the particular task in hand, and how it eliminates that which detracts from the main purpose. To teachers of mathematics such an investigation will be of particular interest, especially in view of the tendency of a number of modern educators to demand a justification of every topic in the curriculum. It is the purpose of the two reports submitted herewith to give such an exposition of the teaching of mathematics in these two great schools, the United States Military Academy at West Point and the United States Naval Academy at Annapolis, these being the only schools of the kind in the United States.

These reports will repay a careful reading; they are suggestive to all teachers not only in details of class organization, but in the general handling of subject matter to serve a definite purpose. They will also show why mathematics has so long held and still retains its prominent place in the training of military and naval officers.

To those who are familiar with similar schools abroad, it will be of great interest to compare the aims and results of instruction in mathematics in a famous Government school like the École Polytechnique[1] of Paris with those found in West Point and Annapolis. In the French school the greatest prominence is given to purely theoretical instruction in higher mathematics, with a very limited amount of practical application; in our two schools few topics are taught beyond the essentials of the calculus, and the practical problem is the basis and end of all the work.

[1] The work of the École Polytechnique is described in the report of the French Committee (published by Hachette, Paris) of the International Commission on the Teaching of Mathematics.

REPORT OF SUBCOMMITTEE 1

MATHEMATICS IN THE TRAINING OF ARMY OFFICERS, INCLUDING SCHOOLS FOR GRADUATES OF WEST POINT

The training of Army officers is carried on in the following schools: (1) The Undergraduate school at West Point; (2) the Engineer School at Washington Barracks; (3) the Ordnance School at Sandy Hook, N. J.; and (4) the Coast Artillery School at Fort Monroe.

I. THE UNITED STATES MILITARY ACADEMY

HISTORICAL SKETCH

The United States Military Academy, established in 1802, is situated at West Point, N. Y. It is a school for the practical and theoretical training of cadets for the military service. The students of the academy are known as "cadets." Upon completing its course satisfactorily cadets are eligible for promotion and commission as second lieutenants in any branch of the military service of the United States. The Corps of Cadets as now constituted consists of 1 from each congressional district, 1 from each Territory, 1 from the District of Columbia, 1 from Porto Rico, 2 from each State at large, and 40 from the United States at large, all to be appointed by the President. Those cadets appointed from States or Territories must be actual residents of the congressional or Territorial districts, or of the District of Columbia, or of the States, respectively, from which they are appointed. Four Filipinos, one for each class, are authorized to receive instruction as cadets, to be eligible on graduation to commissions only in the Philippine Scouts. Under the apportionment of Members of Congress, according to the Twelfth Census, the maximum number of cadets is 533.

The total number of graduates from 1802 to 1909, inclusive, is 4,852.

ENTRANCE REQUIREMENTS

The United States Military Academy is the only school in the country that has for its sole object the furnishing of commissioned officers to the United States Army. From the method of selection of appointees referred to above, it is reasonable to believe that the Military Academy receives a more varied class of students and one more broadly representative of all the States than any other educational institution in the country, except the similarly constituted Naval Academy at Annapolis.

Entrance to the academy is by examination. The scope of the entrance requirements in mathematics is as follows:

Algebra.—Candidates will be required to pass a satisfactory examination in that portion of algebra which includes the following range of subjects: Definitions and notation; the fundamental laws; the fundamental operations; factoring; highest common factor; lowest common multiple; fractions, simple and complex; simple or linear equations with one unknown quantity; simultaneous simple or linear equations with two or more unknown quantities; involution, including the formation of the squares and cubes of polynomials; binomial theorem with positive integral exponents; evolution, including the extraction of the square and cube roots of polynomials and of numbers; theory of exponents; radicals, including reduction and fundamental operations, rationalization, and equations involving radicals; operations with imaginary numbers; quadratic equations; equations of quadratic form; simultaneous quadratic equations; ratio and proportion; arithmetic and geometric progressions. Candidates will be required to solve problems involving any of the principles or methods contained in the foregoing subjects.

Plane geometry.—Candidates will be required to give accurate definitions of the terms used in plane geometry, to demonstrate any proposition of plane geometry as given in the ordinary textbooks, and to solve simple geometric problems either by a construction or by an application of algebra.

These entrance examination papers are prepared at the academy. They are furnished to examining boards of Army officers convened annually at such places as the War Department may direct. The examinations are thus held at points widely distributed over the United States and its dependencies. All papers are sent to the Military Academy for correction.

DISTRIBUTION OF TIME

A cadet when admitted to the Military Academy must be over 17 and under 22 years of age. He pursues a course of study lasting 4 years and 3 months.

The instruction in pure mathematics extends from entrance on March 1 to March 1 two years later.

This time is subdivided as follows:

	Periods.
Review of plane geometry	6
Solid geometry	31
Algebra	85
Trigonometry, plane and spherical	31
Analytic geometry, plane and solid	80
Descriptive geometry	62
Differential and integral calculus and theory of errors	95

Each period is 1 hour and 20 minutes long.

The instruction in applied mathematics is distributed through the last three years of the undergraduate curriculum, except that surveying follows immediately upon the completion of trigonometry.

The time assigned to each subject is as follows:

	Periods.	Hours in each period.
Theoretical surveying	20	1
Practical surveying (in field)	13	4
Analytical mechanics	104	
In laboratory	21	2
In recitation room	83	1½
Sound, light, and astronomy	108	1½
Mechanics of engineering	91	1½
Ordnance and gunnery	94	1½

EXTENT OF COURSE IN PURE MATHEMATICS

The course in pure mathematics as laid down in general terms in the regulations of the academy is:

Geometry. Problems in plane geometry. Geometry in space: planes, lines, polyhedrons, cylinders, cones, and the sphere.

Algebra. Detached coefficients, factoring, linear and quadratic equations, graphic representation, the binomial theorem, inequalities, theory of exponents, complex numbers, ratio and proportion, variation, progressions, series, undetermined coefficients, logarithms, the slide rule, interest, combinations, probabilities, determinants, and the theory of equations.

Trigonometry. The measurement of angles, the simple trigonometric functions, the functions of several angles, trigonometric equations, and the solution of plane and spherical triangles.

Analytic geometry. In the plane: Systems of coordinates, change of axes, anharmonic ratios, the straight line, the circle, the parabola, the ellipse, the hyperbola, the polar equation of the conic, the general equation of the second degree, systems of conics, and envelopes. In space: Systems of coordinates, transformations of coordinates, the plane, the straight line, surfaces of the second degree, and plane sections.

Descriptive geometry. Orthographic projections: Points, right lines, and planes, problems on their relative positions, the revolution of objects, curves in space (especially the circle and cylindrical helix), surfaces (including ruled surfaces and surfaces of revolution), tangent planes, intersection of surfaces, trihedrals. Spherical projections. Shades and shadows. Central projections or perspective. Isometric projections.

Differential and integral calculus. The theory of limits, differentiation of the standard elementary forms, simple applications of the derivative to velocity and rates, maxima and minima, curvature, evaluation of indeterminate forms, envelopes, expansion of functions, curve tracing, tangent planes, and normal lines to surface. The fundamental principles of integration, the integration of the standard forms, the measurement of arcs, areas, and volumes, mean values, approximate integration, ordinary differential equations.

Method of least squares. Errors to which observations are liable, the correction of observations, the probability curve and its equation, measures of precision, the deduction and the application of the formulas for probable and mean error, weights of observations, and the formation of equations of condition and normal equations.

The course of mathematics as outlined above is compulsory for all students and the time devoted to the subject is the same for all. A marked difference, however, is made in the amount and the difficulty of the matter required of students differing in ability.

This is accomplished in the following way: Upon admission the students are arranged in sections of 10 to 12 in order of ability as shown by their marks on their entrance examination in mathematics. This is a tentative grading of no great value, and in consequence all sections have identical courses until the examination which takes place at the end of the first three months. At that time a well-defined grading according to mathematical ability is possible. Each class is then separated into three subdivisions, known as upper, middle, and lower thirds.

A standard course in mathematics has been set for the lower third that is believed to contain the minimum of knowledge, a proficiency in which will insure that the student can intelligently pursue the applied courses that follow. A more extended course is provided for the middle third. A still more comprehensive course is provided for the upper third, one that is expected to give a thorough foundation in pure mathematics for all subsequent undergraduate and graduate work.

The difference in the courses for the groups of a class appears in the use of a more comprehensive or difficult textbook for the abler student, or, if the same text be used, in an omission of the more difficult subjects for the less able; also in the grading of the illustrative problems and exercises assigned to the different groups; also in a further advance into the theory of the subject by the abler student, even to the inclusion of an additional branch of mathematics.

Transfers of students are freely made from time to time between the sections in each group and less frequently from one group to another. These transfers are based upon the quality of the work performed by the student as proved by his daily tasks, oral and written. Each group is thus induced to strive for the full development of its mathematical powers.

A daily alternation throughout the course, where possible, of two subjects—usually one geometric and the other analytic—and toward the end of the mathematical course a similar alternation of mathematics with the course in mechanics enable the various branches to supplement and assist each other and also give the student more time for the digestion of each subject or for concentration upon the one that demands of him the greater labor.

In the fifth class algebra alternates with plane and solid geometry. In the fourth class algebra alternates with plane and spherical trigonometry. Later, analytic geometry alternates with descriptive geom-

etry. In the third-class the course in the calculus and least squares alternates during its latter portion with the subject of mechanics.

PURPOSE OF TEACHING MATHEMATICS

The extent of the mathematical curriculum is determined primarily by the requirements of the succeeding scientific courses in this academy and at the graduate schools for engineers, ordnance and artillery officers.

Remarks on the purpose of the study of mathematics have been variously given in the following extracts:

(From report of Congressional Committee on Military Affairs, 1834.)

Mathematics is the study which forms the foundation of the course. This is necessary, both to impart to the mind that combined strength and versatility, the peculiar vigor and rapidity of comparison necessary for military action, and to pave the way for progress in the higher military sciences. All experience shows that the mind, in order that it may act with efficiency, must be accustomed to exertion. It should be taught gradually to develop its own powers, and as it slowly learns their capacity and the manner of employing them, the increasing lights which are thrown upon its course will enable it to go on for an unlimited extent in the path of improvement.

(From a report of the Academic Board of the Academy, 1843.)

The academic board believe that one of the most important objects of the academy is to subject each cadet, previous to his promotion to a higher grade in the Army, to a thorough course of mental as well as military discipline, to teach him to reason accurately, and readily to apply right principles to cases of daily occurrence in the life of the soldier. They are satisfied that a strict course of mathematical and philosophical study, with applications to the various branches of military science, is by far the best calculated to bring about this end, and that the present scientific course at the academy, the result of the experience of many years, is in its main features such a course. They are aware that many of the cadets, as is the case with most of those who pursue a scientific course at other institutions, will have little occasion to make practical applications of the many mathematical formulæ with which they meet, and that they may have passed over particular problems without thoroughly understanding their meaning in all their important points; still, if the course has been carefully taught, the reasoning faculties will have been strongly exercised and disciplined, and a system and habit of thought acquired which are invaluable in the pursuit of any profession, and as desirable for the infantry or dragoon officer as for any other officer in service.

(From a report of Prof. Church, 1860.)

I consider the course of mathematics as now taught to all who pass their final examination sufficient to enable the cadet to acquire a thorough knowledge of all the courses which follow it, and not more than sufficient to enable him to study with advantage the courses of natural and experimental philosophy, engineering, and ordnance and science of gunnery. Moreover, I regard the mental training of the pupil as one of the great objects of the study of mathematics, a training particularly required by the officer of every corps of the Army, and to which many of them owe their distinguished success; and I believe that the scientific reputation of the academy depends in a great degree

upon the thoroughness and extent to which the mathematics and its applications to other sciences are taught, and to diminish them would seriously affect this reputation and the success of the institution.

(From a report of Prof. Bass, 1896.)

The object of the mathematical instruction in this academy is primarily to prepare the pupil for the study of mechanics, wave motion, astronomy, electricity, ordnance and gunnery, and engineering. In addition the study of mathematics develops the reasoning faculties and establishes a mental foundation upon which any branch of knowledge may safely and rapidly be constructed.

(From a report of Prof. Echols, 1907.)

It has been thought best in the past to educate all of our officers in the same institution with practically the same curriculum, thus giving to all branches of the service a nucleus of men well trained along scientific lines and qualified to take up intelligently any graduate work that may be demanded of them * * *.

If we choose to draw a conclusion from the example of other countries, it is that less mathematics is taught on the average to officers of infantry and cavalry. On the other hand, the strides in science in recent years and the increased application of mathematical investigation in electricity, chemistry, ordnance, and engineering demand a thorough foundation in scientific training for officers of engineers, ordnance, and artillery. This foundation must be laid upon a good and sufficient training in pure mathematics. I find that in range the course in mathematics pursued at our Military Academy by the upper sections compares favorably with theirs, constituting about an average. The French and Italian cadet schools cover more ground, the English, Austrian, and German slightly less. In England and Germany there are, however, courses of pure and applied mathematics for specially selected officers at graduate schools.

The conclusion seems to be that the range covered by the upper course in mathematics at West Point is not too great for the proper training of officers of engineers, ordnance, and artillery; that its value may be enhanced by a more thorough training in the subject of differential equations at the expense, if need be, of some other subject.

Since this Government is disposed to take due pains with the education of its officers, irrespective of branch, and since cadets at our institution do not before admission cover with certainty as much ground as at the preparatory schools of other countries, a *general course* in mathematics should be required of all officers for its practical value, but no less for its educational value in training the mind to logical forms of thought, in developing the sense of absolute truthfulness, together with a confidence in the accomplishment of definite results by definite means. A *special course* in mathematics is, on the other hand, to be regarded as the foundation stone in the training of officers for the scientific corps.

ADMINISTRATION

For the purpose of control and instruction, the personnel of the department of mathematics consists of a professor, an associate professor, an assistant professor, and a number of instructors, varying with the size of the classes under instruction. This staff is composed entirely of officers of the United States Army. The professor is permanently in charge as the executive head of the department. He

has military control over the instructors and students while they are on duty under him; he assigns the instructors to their work, makes out the courses of study, superintends the instruction, and is responsible that approved methods are followed.

His juniors are all younger officers detached by the War Department from the several branches of the service and sent for periods of four years or more to serve on the teaching force of the academy. They are nominated to the War Department by the professor. Their selection is dictated by the fact that they have shown special ability, aptitude, and predilection for mathematics by their undergraduate record; also by the fact that they are believed to possess those other important qualities of the instructor that are essential for the molding of character in the student.

The junior instructors, whenever it is possible, are trained upon the course they are assigned to teach by a series of conferences conducted by the professor or one of the more experienced instructors.

Under normal conditions only one-fourth of the body of instructors is changed annually. This insures continuity in the teaching staff.

An instructor has charge of two sections of about 10 cadets each which recite to him daily (6 days per week).

METHODS, REVIEWS, AND EXAMINATIONS

A certain portion of a standard textbook is assigned in advance for each day's recitation. Upon this theory and upon the exercises applying it the student is expected to spend from three to four hours in preparation. When the section reports to the instructor for recitation, the latter devotes as much of the beginning of the period as he judges best in answering the questions of the student upon the different points of the lesson for the day, solving illustrative problems and volunteering such explanations and elucidations of the subject matter as he may think advisable. An endeavor is made to select textbooks that present the subject matter in sufficient detail to enable an earnest student of normal intelligence by thorough application to comprehend clearly the theory enunciated. It is the purpose to require this thoughtful application which develops the spirit of mental independence and consciousness of power. A lack of proper effort to accomplish his daily task, therefore, is regarded as a military delinquency on the part of the student and is punished as such.

It is the custom, though not an invariable one, to take each subject of study three times.

First, an advance is made, during which the lessons are short and explanations and demonstrations are numerous. The student is expected to ask and to receive assistance from his instructor upon any

difficult point. He is questioned and induced by hints from the instructor to originate questions upon matters that may have failed to suggest themselves to him. On the advance all work of instructor and student is performed upon the blackboard to the benefit of the entire section. It is believed that much profit is derived from seeing the errors of a fellow-student's work corrected and also from observing its excellencies. The sections are composed of pupils of approximately equal ability. Each member is thus encouraged to take an active part in any discussions and receives his full share of the instructor's time and attention.

At regular intervals a review of the advance work is made. The lessons are longer, fewer explanations are necessary, and more applications of theory to the solutions of problems are required of the student. Occasionally an entire period on this partial review is devoted to the solution in writing of the same set of problems by all cadets of the same group.

A second or general review takes place at the close of each subject. This covers the entire course, in long lessons continued over several days. Each day a thorough written test is held upon the portion of the subject assigned for that day. This includes both demonstrations and practical applications. After the completion of the work by the student, solutions are given and explained in full at the blackboard by an instructor and questions relating thereto are answered by him. The student who has qualified upon these written tests is considered proficient on the work of the term without further examination. The one who has not so qualified is given a regular written examination of four to eight hours covering the entire course.

COURSE IN MECHANICS AND ASTRONOMY

The courses in the calculus and in mechanics begin at the same time; the calculus is taught daily from September 1 to November 1, and alternates daily with mechanics from November 1 to March 1; mechanics is taught on alternate days from September 1 to June 1.

The lessons in mechanics in September and October are two-hour periods in the section room, without a preparatory study hour. The principles developed in this part of the course relate to so much of the subject of statics as does not involve the use of the calculus. Simple illustrative experiments are made by the pupils, and the first part of the text is followed in such a way as to bring out the essential points with the least possible loss by misdirected study. The purpose of this part of the course is not only to master the fundamental principles of statics, but also to direct the study so that the pupil shall always use his mathematical knowledge and the language of the text merely as tools with which to shape accurate physical con-

ceptions of the subject. During this preliminary part of the course only such simple definitions as are necessary to the work in hand, are required of the students, and the experience gained here is an effective preparation for mastering the more difficult definitions and distinctions in the subject of kinetics.

By November 1 the class has covered enough of the calculus to take up the study of mechanics with mutual advantage to the two courses, and the recitations in mechanics from that date have a study period for preparation, alternating with the calculus.

It is the aim of the course to cover the subject as thoroughly as practicable in a little more than 100 lessons; or, stated in the proportion of the student's entire work for an academic year, about one-fourth.

The subject of statics is carried far enough to include the mechanical powers and a short treatment of the simple principles of graphic statics. The course in rigid kinetics is carried up to as general an explanation of simple gyroscopic motion as may be made by differential equations, without the time integrals; and the year's work is concluded by a short study of fluid equilibrium and flow. Astronomy is carried far enough to secure some facility in handling the sextant, and in solving time, latitude, and longitude problems. This work occupies about 54 days.

The course in ordnance and gunnery requiring previous mathematical training is, in general terms, as follows: Interior ballistics; calculations of the effects of explosion; calculations of the strength of guns of both built-up and wire-wound construction; calculations relating to the construction of the rifling curves of guns; recoil and recoil brakes; exterior ballistics; calculations of the strength of projectiles and the arming resistance of fuses; calculations of the forces brought on the parts of gun carriages by the discharge of the gun; calculations of the stresses in gun carriage parts; calculations relating to gears and gearing; and calculations relating to recoil springs.

II. ENGINEER SCHOOL OF APPLICATION

WASHINGTON BARRACKS, D. C.

OBJECT OF SCHOOL

This school aims to give to the junior officers of the Corps of Engineers a theoretical and practical knowledge in the various departments of engineering required of them in their professional duties as engineers in charge of fortifications, rivers and harbors, lighthouses and aids to navigation, and military engineering in time of peace and war.

ORGANIZATION

The organization of the school is changed from year to year. In 1912 the department of military engineering will be transferred to Fort Leavenworth. The instructors are detailed from the officers of the Corps of Engineers supposed to be specially fitted by their experience in the charge of works pertaining to the department to which they are detailed.

STUDENTS

The student officers are all graduates from West Point and have the benefit of the technical training at that academy; after graduation they are ordered to special designated stations of river and harbor or fortification work, where they are required to observe the operations of the work of construction and submit these as the results of their observations. These being satisfactory, at the end of one year they are ordered to the Engineer School to take up the required course. No examinations are required for entrance, it being assumed that the preparation as outlined above is sufficient.

COURSES OF STUDY.

The courses of study are not definitely fixed, as changes are constantly necessary in order to keep up with the knowledge gained by recent works of construction and operation.

In general the courses are about as follows:

Civil engineering, about 12 weeks; military engineering, about 8 weeks; and electricity, mechanics, and power, about 32 weeks.

STUDY AND APPLICATION OF MATHEMATICS

Pure mathematics is not studied at the Engineer School. The application of the principles of mathematics is required in all departments. A detailed statement of the requirements is shown below.

Roads and pavements.—An elementary knowledge is required of algebra, geometry, trigonometry, analytic geometry; an extensive knowledge is required of descriptive geometry.

Foundations, roofs, and bridges.—These require an extensive knowledge of algebra, geometry, trigonometry, descriptive geometry, analytic geometry, and calculus.

Building construction, heating, and ventilation.—The course in these subjects requires a fair knowledge of geometry and an extensive knowledge of descriptive or practical geometry.

Water supply, sewage disposal, etc.—These courses cover principally the practical application to hydraulics of such mathematical subjects as algebra, geometry, trigonometry, descriptive geometry, analytic geometry, and calculus, of which a fair knowledge is necessary. The practical and descriptive parts of the above courses require no mathematics.

Surveying and practical astronomy.—These are practically studies in mathematics and, particularly in the case of astronomy, require an extensive knowledge of the application of the principles of algebra, geometry, trigonometry, descriptive geometry, analytic geometry, calculus, and least squares.

Harbor improvement, wave action, etc.—The course in these studies is principally a course of reading as to the practical application of the effects of tides, ocean currents, river currents, jetties, etc., in the formation of harbors. In the reading as outlined above frequent mathematical deductions are made, requiring a fair knowledge of algebra, geometry, trigonometry, descriptive geometry, analytic geometry, calculus, and least squares.

Ordnance and gunnery.—The course is principally one of reading, covering the construction of battleships and principles of design; these require an elementary knowledge of algebra, geometry, trigonometry, descriptive geometry, analytic geometry, calculus, and, at times, least squares.

Design of coast fortifications.—This course is supplemented by visits to seacoast fortifications and mine fields at New York, Fort Monroe, or other suitable seacoast defenses. Students are required to design plans for defense of certain portions of the seacoast, showing plans of defense complete and in detail. This course requires an extensive knowledge of geometry, trigonometry, descriptive geometry, and analytic geometry.

Electricity, practical.—This course is very complete, including experiments of all kinds with electricity in its application to motors, dynamos, lights, etc., for use in seacoast defenses, mines, etc. An extensive knowledge is required of algebra, geometry, descriptive geometry, and analytic geometry.

Mechanics.—This is a further study of mechanics and civil engineering as studied at the United States Military Academy. The course is short, but in portions requires a fair knowledge of algebra, geometry, descriptive geometry, trigonometry, analytic geometry, and calculus.

Power, engines, plants, etc.—This course covers only an incomplete knowledge of the principles of operation of the larger classes of engines, but requires a very good knowledge of the smaller ones. Tests are made of the smaller engines; their efficiencies are obtained; the advantages and disadvantages of their application to the works of construction and operation of rivers and harbors and seacoast defense are systematically studied. These studies require an extensive knowledge of the principles of algebra, geometry, trigonometry, descriptive geometry, and analytic geometry.

METHODS OF INSTRUCTION

Instruction is both practical and theoretical. The course for the year is discussed by the instructors in the school at a board meeting at which the commandant of the school presides. The time allotted to each department is here decided upon, and a schedule is submitted by each department to the commandant. Upon his approval, the course of this department is adopted. Certain books are selected for reading by the students, and portions of these books are required; other important books on the subjects are listed, the reading of which is optional.

Lectures are given during the year on the subjects of study. The lecturers are generally officers of the Corps of Engineers who have had experience in the subject under discussion; other engineers of special experience in similar work are secured as lecturers whenever possible. The library of the school is a very good scientific library.

Enough copies of each book are kept to provide each student with such reading matter as is necessary for his use at the time the particular subject is being studied.

EXAMINATIONS

Written examinations are required in certain subjects, principally those covered by courses in reading. Experiments, if successful, are accepted in lieu of examination in certain parts of the experimental courses.

III. ORDNANCE SCHOOL OF APPLICATION

SANDY HOOK PROVING GROUND, N. J.

The object of this school is to impart to those officers of the Army who are detailed to duty in the ordnance department a theoretical and practical knowledge of the science of designing and using ordnance.

METHOD OF OBTAINING STUDENTS

Officers of the Army at large desiring service in the ordnance department are permitted to take, upon application, a competitive examination held annually. The officers selected are detailed to duty in the department and, as a rule, are sent to the school during their first year's service. The majority of the students are graduates of the United States Military Academy and the remainder of other colleges and technical schools throughout the country.

The entrance examination is intended to demonstrate that each successful candidate has a practical working knowledge of arithmetic, algebra, geometry, trigonometry, analytic geometry, descriptive geometry, differential and integral calculus, and mechanics.

COURSE OF STUDY

1. Pure mathematics. Differential equations. Four weeks. Limited courses in elliptic functions, hyperbolic functions, vectors, and determinants are now being prepared.
2. Ordnance engineering, 21 weeks.
3. Chemistry of explosives, gunpowders, oils, and fuels, 8 weeks.
4. Practical and theoretical electricity, 16 weeks.

Nearly all branches of mathematics are required in the solution of problems connected with the design of guns, gun carriages, projectiles, fuses, and other ordnance material; with the determination of the action of powder in guns and of high explosive bursting charges in projectiles, and with the course in alternating current electricity. The solution of the problems involved in the designing of gun carriages requires a more complete mastery of the different

branches of mathematics than any other class of ordnance and perhaps any other class of engineering work.

The course in differential equations embraces: Equations of the first order and first degree, equations of the first order but not of the first degree, singular solutions, linear equations with constant coefficients, linear equations with variable coefficients, ordinary differential equations with more than two variables, and partial differential equations.

METHOD OF INSTRUCTION

Instruction is given by lectures, on which the students take notes. The instruction is limited to the deduction of fundamental principles, generally expressed in equations or formulæ, and to explanations of the practical applications and conditions covered by each problem.

Recitations are not held. The results of each exercise and test are carefully scrutinized and checked, and each principle of which there is an indication of any lack of full understanding is fully explained in a subsequent lecture. During lectures full and free discussion of the principles involved is invited. The discussion among students of their work is encouraged, but the results must be the production of the individual. Stress is laid on the making of calculations correctly. The instructors and students live together in a mess and the latter are under the constant observation of and in contact with the former.

IV. THE COAST ARTILLERY SCHOOL

This school, situated at Fort Monroe, Va., is a school of application, the object of which is to prepare officers and enlisted men of the Coast Artillery Corps for the active duties of their arm of the service; to make research in such branches of science as relate to practical gunnery, submarine mining, and torpedoes in coast defense; to make experiments and to disseminate such knowledge as may be desirable in the interests of the Coast Artillery service.

COURSES OF STUDY

The regular and advanced courses for officers are embraced in two departments, as follows:

I. Department of artillery and gun defense:
   Regular course—
      1. Artillery proper.
      2. Artillery defense.
      3. Explosives.
   Advanced course—
      1. Ballistics.
      2. Artillery defense, advanced.
      3. Explosives, advanced.

II. Department of electricity and mine defense:
Regular course—
1. Electricity and mine defense.
2. Power.
Advanced course—
1. Electricity and mine defense, advanced.
2. Power, advanced.

The object of the advanced course is to amplify for specially selected officers the instruction and work of the regular course, with a view to improving their qualifications as instructors, fitting them for board and technical work, instructing them in the duties of the general staff of an army, and preparing them for duty at the Army War College.

The courses of instruction comprise practical exercises, problems, research, partial examinations, conferences, and lectures.

The course in exterior and interior ballistics requires advanced mathematical training and is, for the greater part, a graduate course in mathematics. The following topics are taken up in this course:

Exterior ballistics. The principal and secondary problems. Accuracy and theory of errors. The calculation of constants, including the coefficient of form and the drift constant. Classification of trajectories. Deductions of empirical formulas. Perforations. Practical work in setting up, adjusting, and using ballistic machines.

Interior ballistics. Relation of maximum pressure to charges. Mode of combustion of powder and its relation to pressures. Initial pressure on the rifling. Characteristics of a powder. Variations. Recoil.

REPORT OF SUBCOMMITTEE 2

MATHEMATICS IN SCHOOLS FOR THE TRAINING OF NAVAL OFFICERS, INCLUDING SCHOOLS FOR GRADUATES OF ANNAPOLIS

The United States Naval Academy, at Annapolis, Md., is under the supervision of the United States Navy Department and has for its purpose the training of officers of the Navy. The students of the Naval Academy are called "midshipmen." According to the present law two midshipmen are allowed for each Senator, Representative, and Delegate in Congress, and two for the District of Columbia; in addition to these, the President appoints five each year from the United States at large.

ENTRANCE REQUIREMENTS

Candidates are required to pass mental and physical examinations to qualify for entrance. The first mental examination is held on the third Tuesday in April and is conducted by the Civil Service Commission at certain authorized places in each State; the second examination is held only at Annapolis, Md., and is under the direction of the Superintendent of the Naval Academy.[1] Mental examinations are given in the following subjects, and applicants may be rejected if found deficient in any one of them: Punctuation, spelling, English, grammar, geography, general history, United States history, arithmetic, algebra through quadratic equations, and plane geometry.

Candidates passing the mental examinations are examined physically at the Naval Academy by a board composed of three medical officers of the Navy. A candidate must be of good moral character, physically sound, well formed, and of robust constitution; any one of 20 stated physical defects is sufficient to cause his rejection. He must be between 16 and 20 years of age. Candidates passing both the mental and the physical examination are admitted as students to the academy. On admission, each midshipman must sign articles binding him to service in the United States Navy for eight years, unless sooner discharged.

[1] Beginning with the year 1912, both examinations will be conducted by the Civil Service Commission.

COURSE OF INSTRUCTION

The instruction given in the academy extends over a period of four years and includes work in mathematics, mechanics, descriptive geometry, drawing, physics, chemistry, engineering, navigation, ordnance and gunnery, history, English, and modern foreign languages. The following outline indicates only the ground covered in theoretical and applied mathematics during the course:

FIRST YEAR.

	Periods per week.
First term:	
Mechanical drawing	4
Algebra and geometry	6
Second term:	
Mechanical drawing and descriptive geometry	3
Trigonometry	6

SECOND YEAR.

First term:	
Mechanical drawing	3
Analytic geometry and the calculus	5
Second term:	
Integral calculus, spherical trigonometry, and stereographic projection	5

THIRD YEAR.

First term, theoretical mechanics	4
Second term:	
Astronomy and navigation	2
Applied mechanics	3

FOURTH YEAR.

First term, theory and practice of navigation	5
Second term:	
Theory of compass deviations, surveying, and practical navigation	4
Engineering mechanics	3

DISTRIBUTION OF TIME

The academic year begins October 1 and closes the first week in June. The members of the three undergraduate classes are sent on a cruise each summer for a period of three months. On these cruises the midshipmen are required to perform all the general duties of a navigator, with particular reference to the adjustment and use of the navigational instruments on board, to practical pilotage, and to the determination of the ship's position by observation.

During the academic year each class in mathematics requires a period of two hours for recitation; classes recite in sections of 12 students each.

Midshipmen complete the courses in pure mathematics during the first two years in the academy and the courses in applied mathematics in various other departments during the last two years.

PURPOSE OF TEACHING MATHEMATICS

In the teaching of mathematics at the Naval Academy, as in any technical school, the aim is purely practical, the course consisting essentially of the solution of problems which a midshipman must understand in order to become a naval officer.

All of the textbooks used in the department of mathematics have been written expressly for the midshipmen by men in the department. No more theory is presented in these books than is absolutely necessary. The recitation work, which is strictly competitive, consists of solving numerous problems on the blackboard; and perhaps 90 per cent of all monthly, semiannual, and annual examinations are composed of practical problems.

ADMINISTRATION

Instruction in the Naval Academy is carried on through the following departments: Mathematics and mechanics, navigation, seamanship, physics and chemistry, marine engineering and naval construction, electricity, English and modern languages. Each department is supervised by a naval officer. Teaching is done in some departments by naval officers detailed for such duty and in others by both naval officers and civilian instructors. In the department of mathematics there are 13 instructors, of whom 7 are naval officers, and 6 civilian instructors, having been appointed from civil life.

METHODS, REVIEWS, AND EXAMINATIONS

Accurate marking of work done by the midshipmen during their four years' course at the academy is of the utmost importance, for their marks or credits determine the order of their promotion after graduation; consequently all of their work is strictly competitive, and all possible care is taken, therefore, in assigning lessons, in organizing new sections with different instructors each month, and in marking daily recitations and examination papers, so that any advantage or disadvantage in the methods of instruction may be equally shared by all.

The lessons for each day of the month are assigned by the head of the department at the beginning of the month; these assignments are noted by the instructors and given to the midshipmen at each recitation, so that all midshipmen in the same class study and recite precisely the same work at the same time.

A further effort to give all of the midshipmen an equal opportunity in the recitation room is made by a new arrangement of sections and instructors each month. The 12 midshipmen receiving the highest marks in the monthly examination constitute the first section for the following month; the 12 receiving the next highest,

the second section, and so on.[1] The instructors are assigned different sections from month to month, so that all midshipmen are placed on an equal footing so far as advantages or disadvantages of different sections and different instructors are concerned.

Midshipmen are marked on a basis of 2.5 for passing, or satisfactory, and 4 for perfect. They receive daily marks for their work in the recitation, and the average of the daily marks is taken as the weekly mark; the average of four weekly marks is taken as the monthly recitation mark. An examination is given at the close of each month and the examination mark is combined with the monthly recitation mark, the former counting one-third. For the information of the midshipmen, a sheet is posted giving the final monthly mark and class number or standing of each member of the class. This plan is followed each month during the year except the months of January and May, when the monthly examinations are omitted, because these months immediately precede the semiannual and final examinations. The semiannual examination mark is combined with the average of the monthly marks from October to January, inclusive, to determine the final mark for the first half year; the semiannual examination mark counting one-fourth. Similarly for the second half year the final mark is combined with the average of the monthly marks for the final mark of the second half year, the final mark counting one-fourth. Semiannual and annual sheets similar to the monthly sheets are posted, giving the mark and rank of each member of the class. The monthly examinations are two hours in length, while the semiannual and final examinations are each five hours. All examination papers must be marked independently by two instructors, the mean of the two markings being taken as the mark for the examination. In case there is a considerable difference between the two markings, the paper is re-marked by the head of the department, and the different marks are then properly adjusted. Instead of regular recitations on Saturday morning, the classes of the first and the fourth years have written recitations of two hours each. The mark received on this paper is counted as the recitation mark for that day. Such recitations are not so frequent in the other two classes.

When a midshipman is badly deficient in one study and fails to make a satisfactory mark (2.5) in any other subject for either the first or second half year, he is dropped from the academy; the vacancy of course is not filled until the following year.

The final term average of every midshipman in each branch of study is multiplied by the coefficient assigned that branch for the term, and the sum of the products is the aggregate mark for the

[1] In the upper classes there are usually not more than 8 or 10 midshipmen in each section.

year. Coefficients are assigned to the different branches according to the time devoted to these branches. For mathematics these coefficients are as follows: Algebra and geometry, 3; trigonometry, 3; calculus, 5; applied calculus and stereographic projection, 6; and mechanics, 11.

The merit roll of the graduating class is prepared at the completion of the four years' course: the names of the graduates are arranged in the order of merit according to the graduating mark, which is the arithmetical sum of the final marks of the four years of the course.

After graduation midshipmen are sent to sea for two years, after which they are given the final graduating examination, and if found satisfactory are eligible for promotion to the commissioned rank of ensign.

THE NATURE OF THE COURSES IN MATHEMATICS

The Naval Academy is a strictly technical school, and the courses in mathematics are essentially practical problem courses. More condensed courses in mathematics have been necessitated at the academy during the last two years by the increased time devoted to the professional subjects of marine and electrical engineering. All subjects ordinarily treated in mathematical courses, but which are not adapted to the needs of a naval officer, are omitted, while certain other important features are given greater emphasis.

In the algebra, particular emphasis is placed upon the following: Computation by logarithms, the graph of the general polynomial and its use, the tracing of curves of the first and second degree, the theory of equations, identities, undetermined coefficients, and series. The subject of choice and chance is merely touched upon, while the subjects of harmonic progression, interest and annuities, continued fractions, theory of numbers, and determinants are omitted. Special free-hand methods are used for tracing curves of the second degree by means of limiting tangents, diameters, and asymptotes.

The work in trigonometry emphasizes particularly accurate logarithmic work in the solution of plane and spherical triangles—the oblique spherical triangles being solved by means of a perpendicular and Napier's rules. The text includes a course in stereographic projections covering as a special feature the projection and solution of the astronomical triangle.

Except for its subsequent use in the calculus, analytic geometry occupies a very small place in the academy compared with that usually assigned it in colleges and universities. The tracing of curves of the third or higher degrees by use of the analytical triangle may be mentioned as a special feature.

The calculus is a rather thorough practical course, emphasizing particularly the following topics: Length and area of curves, surfaces, volumes, center of gravity, center of pressure, and moments of inertia.

USE OF MATHEMATICS IN OTHER STUDIES

Studies requiring mathematical knowledge for their pursuit and the kind of mathematics required for each are as follows: Ordnance and gunnery—trigonometry, calculus, and mechanics; navigation—trigonometry; marine engineering and naval construction—trigonometry and calculus; and electrical engineering—algebra, trigonometry, and calculus.

OPPORTUNITIES FOR FURTHER STUDY

The more important schools attended by certain graduates of the Naval Academy are the School of Marine Engineering, established at the Naval Academy for young officers pursuing graduate courses in this field of study, and the Massachusetts Institute of Technology, for those who are to become naval constructors. Only graduates of the Naval Academy are permitted to take the entire course in the latter institution. The course at the Massachusetts Institute of Technology is one of three years in length and leads to the degree of master of science. In arranging this course the objects sought are the addition to the training already obtained at the Naval Academy of those subjects which are peculiar to naval architecture, and such an extension and rounding out of that training as will best enable a naval constructor to meet the varied and exacting demands of his official position. The course includes theoretical naval architecture, marine engineering, steam and electrical engineering, steam turbines, warship design, sanitation, metallurgy, and metallography. In mathematics all students are required to take advanced calculus, differential equations, and the method of least squares; this is supplemented by courses in applied mechanics, including the strength of materials, graphic statics, and the theory of elasticity.