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TRAINING OF TEACHERS OF ELEMENTARY AND SECONDARY MATHEMATICS

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THE AMERICAN REPORT.

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¹ As originally announced the chairman of the committee undertaking this investigation was Prof. George W. Myers, of the University of Chicago. On account of his absence in Europe for the year, his place was taken by Prof. Clifford B. Upton, of Teachers College, Columbia University, New York, N. Y. A part of the report as prepared by him and the rest of the committee appeared in its first form in the Educational Review, April, 1911.

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TRAINING OF TEACHERS OF ELEMENTARY AND SECONDARY MATHEMATICS

SUBCOMMITTEE 1.¹ THE TRAINING OF TEACHERS OF MATHEMATICS IN PROFESSIONAL SCHOOLS OF COL- LEGIATE GRADE, SEPARATE FROM OR CONNECTED WITH COLLEGES OR UNIVERSITIES.

Twenty years ago no professional training of university grade existed in this country to prepare teachers of mathematics for secondary schools. At that time the young teacher's sole preparation for his work was the taking of as many academic courses in mathematics as possible, plus, in some instances, a course on the history of education or some lectures on general pedagogy. On graduation he had had no observation of skillful teaching in secondary mathematics, no practice work, no survey of secondary mathematics from a higher and pedagogic standpoint, and no knowledge of the best literature dealing with the teaching of algebra and geometry in this country or abroad. He gained all his experience in the classroom, and, if progressive, he also acquired a certain pedagogic equipment from private reading and from teachers' associations; if not, his point of view on questions of teaching remained narrow, as is always the case when one fails to come into contact with what others are doing in his field of work. A very large number of candidates for teaching begin their work to-day with this modicum of preparation.

About 15 years ago we find conditions throughout the country beginning to change in this respect. At least five different educational institutions² had by this time established courses on the teaching of algebra and geometry, which, together with a course on general pedagogy, formed a certain professional training for high-school teaching in mathematics. Up to 1900 only four other colleges³ are

¹ No report of the committee as a whole has been prepared, the ground being covered sufficiently well by the reports of the subcommittees that are presented herewith.

² University of Michigan (1898), Michigan State Normal College (1898), Teachers College, Columbia University (1894), Syracuse University (1895), and University of Chicago (1895). The dates indicate the year in which the course was first given.

³ University of Pennsylvania (1897), University of Indiana (1898), Albany Normal College (1898), and University of Illinois (1900).

TRAINING OF TEACHERS OF MATHEMATICS.

known to this committee to have added courses on the pedagogy of secondary mathematics to their programs.

The past 10 years have shown far greater interest in pedagogical matters and a much more rapid growth in courses of this kind. At present about 25 other colleges,¹ in addition to those above mentioned, have developed such courses as "The history and pedagogy of secondary mathematics," "The teaching of mathematics in secondary schools," and "Teachers' course in algebra and geometry."

For the other high-school studies, such as Latin, English, German, physics, and chemistry, there has been a corresponding development of professional courses on the teaching of these subjects.

This general interest in the professional preparation of teachers for secondary education, which is thus being shown in many of our American colleges and universities, has led to a feeling on the part of a number of these institutions that teaching must be recognized as a profession, and that training for it must be as formal and definite as for law, medicine, or engineering. This has resulted in the establishment at a number of our universities of schools of education, each with a formal organization and a separate faculty, these being considered as professional schools and as such ranking, in general, with the schools of law, medicine, and engineering. In these schools attention has been given particularly to preparation for secondary teaching, as the various State normal schools have, in general, solved the problem of preparing teachers for the elementary schools.

Of the various schools of education now existing in the United States the oldest are Teachers College of Columbia University and the School of Pedagogy of New York University. Teachers College was founded in 1888 as a privately endowed institution under the presidency of Nicholas Murray Butler, and was then known as the New York College for the Training of Teachers. It was a school of university grade and enjoyed reciprocal relations with Columbia University. In 1898 it became one of the professional schools of Columbia University, ranking with the schools of law, medicine, and engineering. Teachers College was the first professional college of education of university grade separate from a university, though now a part of Columbia University.

The School of Pedagogy of New York University, established in 1890, takes rank with the other professional schools of that institution, and may be said to be the first professional school of university grade directly connected with a university.

¹ Eleven of the more prominent are: New York University (1902), University of Texas (1903), University of Missouri (1903), University of Minnesota (1905), University of Cincinnati (1906), University of Nebraska (1907), Cornell University (1907), University of Wisconsin (1907), University of Iowa (1908), Drake University (1908), and Ohio State University (1909).

TRAINING IN SCHOOLS OF COLLEGIATE GRADE.

In 1901 a school of education with a distinct faculty and ranking as a professional school was opened at the University of Chicago, and has since become one of the prominent institutions of this kind. Within the last five years, at least 11 other universities¹ have organized similar schools or colleges where the professional training of teachers is carried on. In all but two cases² these schools rank with the other professional schools of the university.

In addition to the above-mentioned institutions connected with universities there are at least three normal colleges,³ separate from any university or college, where work in the training of teachers is of collegiate grade and leads to a recognized bachelor's degree. These three institutions offer both academic and professional courses in preparation for secondary teaching.

The work in all of the above schools or colleges of education differs from that offered by a university chair or department of education in that professional courses of study are offered in far more generous measure. In these schools one finds courses in general and educational psychology, the history and philosophy of education, school administration, foreign school systems, secondary education, etc., in addition to courses on the teaching of each of the secondary subjects, often accompanied by systematic observation and practice teaching of the subject under expert supervision in some high school. The academic preparation of the prospective teacher is usually carried on for two years in the regular college department of the university before the pupil enters the school of education, and is continued in the college department along with the professional courses in the school of education. In general, academic work is not done in the school of education, the courses there offered being strictly of a professional nature. The principal exception to this is found in those professional schools which are not connected with a university. In all of these schools the professional courses are credited toward the bachelor's degree, and in a few of these institutions graduate courses leading to the master's and doctor's degrees in education are also offered.

¹ University of Cincinnati, College for Teachers (1905); University of Texas, Department of Education (1906); Syracuse University, Teachers' College (1906); University of Missouri, School of Education (1906); University of Minnesota, College of Education (1906); University of Indiana, School of Education (1907); Ohio State University, College of Education (1907); University of Iowa, School of Education (1907); Drake University, College of Education (1907); University of Illinois, School of Education (1908); University of Nebraska, Teachers' College (1909).

(The dates indicate the year of establishment of the school of education. By a comparison of the above with the footnotes on pages 5 and 6 it will be seen that in some universities courses on the teaching of secondary mathematics existed before the organization of a formal school of education, while in others such courses came after the establishment of the professional school.)

² Universities of Illinois and Iowa.

³ State Normal College, Ypsilanti, Mich.; State Normal College, Albany, N. Y., and Normal College of the City of New York.

It is the purpose of this report to deal only with the training of teachers of mathematics as carried on in the above-mentioned schools or college of education.

THE AIM OF SCHOOLS AND COLLEGES OF EDUCATION.

So far as mathematics is concerned, it is the aim of all of these colleges of education to prepare teachers and supervisors of mathematics for public and private high schools. Some of these institutions also have facilities for equipping teachers of mathematics for departmental work in elementary schools and instructors in methods in mathematics for normal schools. At a few schools of education opportunity is offered to principals and teachers to fit themselves for the study of special problems in the teaching of mathematics, such as constructing courses of study for elementary and high schools, and especially courses in mathematics which fill the particular needs of manual training, commercial, industrial, trade, and other specialized forms of schools. The courses found in several of these professional schools are very helpful to those preparing to teach academic courses in mathematics in colleges and universities, while extensive and complete preparation is possible for those who wish to become professors of the pedagogy of mathematics. Facilities are also offered to prepare for research and teaching in the history of mathematics.

THE PRESENT UNDERGRADUATE PROFESSIONAL PREPARATION FOR TEACHING IN SECONDARY SCHOOLS.

In each of the 17 schools of education considered in this report the minimum training of a professional nature for secondary work is a definitely organized course on the teaching of secondary mathematics. This course varies in length from 17 to 20 hours of class work, the average for all of the institutions being 48 hours. The method of handling the course is, in general, by lectures, recitations or assigned readings, discussions, written reports, and quizzes. The subject-matter of this course varies somewhat in each institution, though certain topics are common to practically all of the courses. That some notion may be formed of what is being done in such work the following composite syllabus, prepared from the syllabi sent in by the leading institutions, has been arranged, and may, therefore, be regarded as a sort of maximum course. The course treats of such topics as the reasons for teaching algebra and geometry, the historical development of these subjects and of the methods of presenting them, the world's best literature concerning the teaching of mathematics, the mathematical curriculum in American and foreign secondary schools, the adaptation of a curriculum to the needs of

various types of schools, a study of typical American and foreign texts for secondary schools, and the various movements to reform the teaching of secondary mathematics at home and abroad. Some attention is often given to the methods of conducting a recitation in mathematics with discussions of the heuristic method, the laboratory method, the no-text method, etc. The course also contains a review of typical parts of algebra and geometry offering scientific or pedagogic difficulties, such as the number system of algebra, equivalent equations, factoring, theory of exponents, simultaneous quadratic equations, graphic solutions, the foundations of geometry, the sequence of propositions, the treatment of parallels, loci, limits, incommensurable cases, methods of attacking exercises in geometry, and the nature of the problems and exercises of algebra and geometry. The reading of current periodical literature on the teaching of mathematics and the discussion of such topics as teaching algebra and geometry together, the fusion of plane and solid geometry, etc., are often included.

Such books as Smith's *The Teaching of Elementary Mathematics* and *The Teaching of Geometry*, and Young's *The Teaching of Mathematics* are frequently used as outlines or texts for such a course. Fine's *College Algebra and Number System of Algebra*, Chrystal's *Algebra*, and the histories of mathematics by Ball, Fink, Cajori, Gow, and Allman are frequently reported as reference books.

The prerequisites in mathematics for the above course are one year of college mathematics in about half of the cases and through the calculus in others. Even where the former requirement exists, it usually happens that most of the students have had the calculus and such courses as the theory of equations and projective geometry as a minimum preparation before entering the pedagogical course. This pedagogical course is normally taken in the senior year.

In connection with the above course on the teaching of secondary mathematics 12 different colleges of education offer systematic work in observation and practice teaching in secondary classes, which receives academic credit. This work is carried on in most cases in a special high school connected with or under the direction of the school of education. In a few cases arrangements are made for such practical work in public city high schools. The amount of observation and practice teaching required varies from 80 to 60 hours of work.

In some of these colleges other professional courses in preparation for the teaching of mathematics are given in addition to the teachers' course above mentioned. A distinct and separate course on the history of mathematics is offered in seven of these institutions, while in several others such historical work is a definite part of the course on the teaching of mathematics. This historical course aims partic-

ly to show the evolution of the science of mathematics and to trace the causes which have led to its development or stagnation in various epochs. Work in surveying, designed especially for prospective teachers, is also offered at several schools. At Teachers College of Columbia University and at the School of Education of Chicago University several additional interesting professional courses are found, such as "Encyclopedia of elementary mathematics for teachers," "Applied mathematics for teachers in secondary schools," "Psychology of number," "Teaching of secondary mathematics in Europe," and "Critical review of secondary mathematics."

In a few schools special certificates or diplomas, certifying special preparation and fitness for teaching secondary mathematics, are given in addition to the regular degree to students who fulfill certain requirements. In some instances these diplomas are granted by the State board of education to students who have had at the school of education a certain preparation for the teaching of mathematics. In these cases the diploma is a State license to teach for life or for a given number of years. In other instances the diploma is conferred only by the college of education independent of any board of education. The requirements for these diplomas are usually as follows: Courses on educational psychology, the history of education, and the teaching of secondary mathematics, with observation and practice teaching, together with academic work in college mathematics of from 15 to 24 points (a point equaling one hour of work for one semester).

THE PRESENT UNDERGRADUATE PROFESSIONAL PREPARATION FOR TEACHING IN ELEMENTARY SCHOOLS.

Only a few¹ of the schools of education offer preparation for teaching mathematics in elementary schools. One reason for this is that the State normal schools offer, as a rule, sufficient preparation for this field of work. When, however, one wishes to prepare for supervision or departmental work in mathematics in the grades, and hopes to obtain the better positions, it is necessary to have further scholarship in mathematics and education, and a college degree, a type of training which can be found only in the colleges for teachers since they begin, in general, where the normal schools leave off. Students who have done two years of work beyond the high school, in a normal school, are usually able to complete a course and obtain a degree in one of the schools of education by two additional years of study. A part of the preparation for work in the elementary schools is a course on the teaching of arithmetic corresponding to the one described above on the teaching of algebra and geometry. The syllabus

¹ Teachers College of Columbia University, School of Education of the University of Chicago, College for Teachers of the University of Cincinnati, Albany Normal College, and the Indiana State Normal College.

is of the same general type as that for secondary mathematics, except that arithmetic and such parts of algebra and concrete geometry as are taught in the elementary school make up the subject matter for discussion. Students taking such a course have often had a year's work in college mathematics, while the most serious students have usually had more. In Teachers College at Columbia University students also take a thorough course on higher arithmetic, in which special attention is given to modern business practices. In the School of Education of the University of Chicago a course on the psychology of number is usually taken. In addition, students generally elect very freely related courses in the departments of psychology and elementary education. Opportunity for practice teaching and observation in elementary mathematics is offered in connection with this work. It should be added that students who are studying seriously the problems of teaching mathematics in elementary schools usually take also the full preparation for teaching in secondary schools, in order that they may better appreciate the nature of the secondary school work to which the elementary school courses are leading.

GRADUATE COURSES IN THE TEACHING AND HISTORY OF MATHEMATICS.

Graduate courses in the teaching and history of mathematics, leading to the master's and doctor's degrees, are found only at Teachers College of Columbia University and at the School of Education of the University of Chicago. In these two institutions research courses are conducted on the seminar plan, in which the students work on individual topics relating to the history and pedagogy of mathematics. The prerequisite for these courses is a bachelor's degree, which should include a course on the teaching of algebra and geometry in addition to a generous amount of college mathematics. Students who have not had courses on the history and pedagogy of secondary mathematics usually take these courses parallel with the graduate work. A large number of the students pursuing these courses are those who have already had considerable experience in teaching. The work is adapted to the needs of the various classes of students. It is taken by those who wish further preparation for secondary teaching as well as by those who aim to become heads of departments of mathematics in high schools. One also finds a number who are fitting themselves to teach methods in mathematics in normal and training schools.

There are also a few students who are preparing to teach mathematics in colleges and universities. This seems to be a good omen, for it indicates that students are beginning to realize that there are important pedagogical problems to be solved, especially in connection with the teaching of the first year of college mathematics, where

in general, more poor instruction is found than in any other place in our educational system, a situation which is easily explained, since as a rule the teachers in the first year of the college courses are young men, fresh from the study of higher mathematics, who have never had experience in teaching and who have never so much as discussed the most elementary topic in relation to such work. The fact that the men who are preparing in the schools of education for this college work usually follow at the same time the courses offered on the teaching of secondary mathematics shows that they realize that the problems of teaching in the first years of college are almost identical with those in the last two years of the high school. France and Germany long ago fully appreciated this when they included in the courses of study for their secondary schools the mathematics which we usually teach in the first two college years. The further fact that these young men also pursue as many advanced courses as possible in pure and applied mathematics indicates that they feel strongly the necessity of combining mathematical scholarship with this pedagogic training. In these schools of education there is no dogmatic instruction in reference to college teaching; such work would be fatal. In the seminars these students usually work out some problem connected with freshman or sophomore college work and, at the same time, get fully into touch with all of the movements and problems in secondary teaching which bear so directly upon their field of work. They thus obtain a broad basis for the pedagogy of freshman college mathematics; they get perspective if nothing else.

In these graduate courses men are also prepared for college teaching and research in the history of mathematics. The facilities for this work are exceptional at Teachers College of Columbia University, where very complete libraries² of old mathematical books and manuscripts are available for the use of graduate students. In the seminars one also finds a number of principals and supervisors who are interested in the new types of industrial, trade, and commercial schools, which are so rapidly springing up in this country, and who are working on special studies relative to the courses of study in mathematics adapted to the needs of these schools.

PRESENT TENDENCIES.

A number of the schools of education report plans for enlarging the work for the professional training of teachers of mathematics. In several of these colleges where short courses on the teaching of secondary mathematics are now offered it is proposed to make within a year or so a considerable increase in the length of the course. In a few institutions where no practice teaching or observation is now

² The large private libraries of Prof. David Eugene Smith and George A. Plimpton, Esq.

given, it is hoped to mature arrangements for such work in the near future. In a number of other schools serious efforts are being made to increase the efficiency of this work, especially on the side of practice teaching.¹ Several professors have also announced their intention of offering courses on the history of mathematics; others are planning graduate courses in the teaching of mathematics. At several of our colleges of education there seems to be a strong feeling that secondary teachers should have a synoptic course in mathematics somewhat along the lines of Klein's "Elementarmathematik vom höheren Standpunkte aus." There is no question as to the advisability of such a course; the main reason why such work has not been more generally given in America is probably the lack of a suitable textbook. As a number of American professors are now jointly preparing a work of this kind, courses of this nature will soon be found, no doubt, in a number of our teachers' colleges.

CONCLUDING REMARKS.

The ideal preparation for teaching in secondary schools and the first two years of college should be something more complete than that now generally found. On the side of scholarship it should approach the completeness of the preparation in France and Germany, where one must have a State license, obtained only by passing a most rigid State examination, before he can hope to do anything worth while in secondary teaching. On the other hand, it would probably be detrimental to the life and elasticity of our system if a similar license should be demanded in this country. Yet with us, where the public is now beginning to recognize that teaching is a profession, a feeling which will certainly increase as the years go by, the time will undoubtedly come when secondary teaching will be sufficiently attractive financially to enable us to demand from the prospective teacher some such preparation as the following: On the side of pure mathematics we may expect the calculus, differential equations, solid analytic geometry, projective geometry, theory of equations, theory of functions, theory of curves and surfaces, theory of numbers, and some group theory. On the applied side we should demand a strong course in mechanics, theoretical and practical astronomy, descriptive geometry, and some mathematical physics with a thorough course in experimental physics. To this should be added special courses on surveying and general applications of mathematics that the student may see to what all of the above work is leading. As pedagogical training there should be included a strong course on

¹An interesting article on "Practice work in university departments of education," by F. E. Farrington, has recently appeared in the Bulletin of the University of Texas, No. 134, Nov. 1, 1909. It may also be found in the Publications of the National Society of College Teachers of Education for 1909.

the teaching of secondary mathematics with observation and practice teaching under expert supervision, a course on the history of mathematics, at least one graduate course on the history and teaching of mathematics, and a course of an encyclopedic nature dealing critically with the field of elementary mathematics from the higher standpoint. A foundation in psychology and the history of education is also necessary. Such a preparation may at first seem excessive, but it is the ideal, and, with the exception of about half of the pedagogic training outlined, it is no more severe than the requirements in France to-day for the secondary teaching license known as the "agrégation." We can demand this if the public will give teaching the recognition it deserves.

SUBCOMMITTEE 2. STATE NORMAL SCHOOLS.

The State normal schools exist for the purpose of training teachers for the public schools. They are generally supported by appropriations by the State legislatures. A small number of these schools are supported by a specified State tax, supplemented by appropriations by the State legislatures, and in a few cases they receive some income from the sale or rental of public land grants. A few State normal schools have small permanent endowments. Tuition is free, except for slight incidental fees, to persons declaring an intention to teach in the State in which the school is located. With the exception of a few schools in the South the State normal schools are coeducational.

This report is prepared from data obtained from the catalogues of State normal schools, and from the answers to questionnaires. A questionnaire was sent to each of 185 State normal schools and to 8 city training schools. Answers were received from 65 State normal schools and from 8 city training schools. The number of city training schools from which data were received being small, these schools have been classed with the State normal schools.

THE RELATION OF NORMAL SCHOOLS TO OTHER SCHOOLS.

The requirements for entrance to normal schools vary in different parts of the country. Of 64 schools reporting on this topic, 23 per cent require high-school graduation. The majority of these schools are in New England and New York, and offer a two-year course of study, which is composed largely of professional work. The remaining schools require for admission only the completion of the work of the eighth grade. Such schools usually offer two courses of study, a four-year course for the students who have completed only the work of the eighth grade and a two-year course for high-school graduates. A few schools, chiefly in the Middle West, offer two

ERRATUM.

The sentence beginning on the seventh line from the bottom of page 15 should read as follows: Of the schools reporting, 89 per cent give courses in methods of teaching arithmetic and 9 per cent give courses in methods of teaching algebra and geometry.

years of work in addition to the above courses, for the completion of which a bachelor's degree is granted.

The work of the normal schools now receives better recognition from the colleges than formerly. Of the schools reporting to this committee 56 per cent receive full credit in the colleges for work done in mathematics, and 20 per cent receive partial credit. Frequently the colleges allow junior-class standing to those graduates of the normal schools who have done two years of work beyond the high school. It is estimated that about 13 per cent of the graduates of the normal schools later pursue advanced work in college. The larger estimates come from the West, where the normal schools are used as preparatory schools by a considerable number of students from the rural districts, because the high schools are not so well developed as those in the East. Since a considerable number of their graduates later go to college, the normal schools of this section have come to offer considerable elective work. It thus becomes possible for the student to do in the normal schools the mathematics usually done in the first two years in college.

Practically all normal-school graduates teach at some time in the public schools. It is estimated that 73 per cent of the graduates teach five years or more. The large majority of them teach in the grades. Sixty-four per cent of the schools report that some of their graduates teach in the high schools in the smaller cities and towns.

THE AIM OF THE INSTRUCTION IN MATHEMATICS.

Each school was asked to give the aim of the instruction in mathematics. Of the schools reporting on this question, 51 per cent claim to pay equal attention to mathematics as a science (the so-called culture value) and to mathematics as an art (the so-called utilitarian value). About 28 per cent claim to emphasize more the culture aspect (except in arithmetic), and 21 per cent put greater stress upon the utilities.

Since a large majority of the students are preparing to teach in the elementary schools, arithmetic is the most important subject in the mathematical curriculum of the normal school. The mathematical instruction has as its chief aim the preparation of efficient teachers of arithmetic. One means used to accomplish this end is to give formal instruction in methods of teaching this subject. Of the schools reporting, 89 per cent also give courses in methods of teaching algebra and geometry. The general opinion is that such courses are distinctly valuable when preceded by a thorough grounding in the subject matter. In 66 per cent of the schools offering these courses they are given wholly, and in 8 per cent in part, by the same teachers that give the academic courses in mathematics.

It is the opinion in 23 per cent of the schools that there should be no difference, except in courses in methods, in the instruction in mathematics offered to students preparing to teach and to other students. The remaining schools take the point of view that in courses for prospective teachers more attention should be paid to the aims in teaching mathematics, to methods of explanation and orderly presentation, and to the historical development and economic importance of the subject.

METHODS OF INSTRUCTION AND BRANCHES OF STUDY.

The courses of study in 64 schools were obtained either from answers to questionnaires or from the catalogues of the schools. The following table shows the different courses given, the number of schools offering each course, and the average number of hours given to each course:

Subject.	Number of schools.	Average number of hours.
Arithmetic.....	All.	100
Elementary algebra.....	All.	171
Plane geometry.....	All.	143
Advanced algebra.....	9	108
Solid geometry.....	18	72
Methods of teaching.....	32	63
Mathematics.....		
Plane trigonometry.....	40	77
Spherical trigonometry.....	4	80
Analytics.....	25	98
Calculus.....	23	87
Theory of equations.....	3	52
Differential equations.....	1	48
Surveying.....	11	57
Analytical mechanics.....	1	48
History of mathematics.....	8	70

High-school graduates taking a two-year course in a normal school are required to review arithmetic, and, in some schools, algebra and geometry. Students who enter the normal school from the eighth grade are required to take courses in arithmetic, algebra, and geometry. If more advanced courses in mathematics are offered they are as a rule elective. The number of students electing mathematics is small in most schools, as the only students interested in the higher courses are those who desire to secure advanced standing in college.

or to teach the subject in a high school. No elective work is offered in 32 per cent of the schools reporting on this question.

Recitations by the students, supplemented by an occasional lecture, is the practically universal method of conducting classes. In nearly all cases a text is used as the basis of a course, the text being supplemented at the pleasure of the instructor.

One of the most important parts of the professional training of a student in a normal school is the practice teaching under supervision. In the schools reporting, 80 is the average number of class periods of practice teaching in mathematics which it is possible for a student to take who is making a specialty of this subject. In 76 per cent of the schools all students are required to do some practice teaching in mathematics, and the average number of class periods required is 54. This teaching is always under the direct supervision of a critic teacher. In 51 per cent of the schools reporting, the head of the department of mathematics is said to exercise some supervision over the practice teaching, but in some of these schools the supervision is very slight. Such supervision seems to the committee to be very desirable in order to unify the work in mathematics throughout the school, to make the work of the training school more efficient, and to keep the teachers of mathematics in the normal school in touch with elementary school work.

Correlation was also made a subject of inquiry. Of the schools reporting, 12 per cent say that no attempt is made to correlate the different mathematical subjects. Thirty-eight per cent are attempting to make some systematic correlation. The remainder attempt some correlation, but nothing systematic. Twelve per cent of the schools reporting on the question think it advisable to eliminate entirely the traditional boundaries between arithmetic, algebra, and geometry; 19 per cent think these boundaries should be eliminated to a large extent; 7 per cent not at all. Other schools suggest methods by which related topics in the different subjects may be correlated.

Sixty-nine per cent of the schools report some attempts at correlating mathematics with other subjects, usually with the sciences, but close correlation is made, probably, in but few schools. The replies to the questionnaire indicate that it is the general opinion that in arithmetic the larger part of the problems in the lower grades and some of the problems in the upper grades should be taken from the other subjects that the pupils are pursuing and from the pupil's environment.

Much more is now being done than ever before in the normal schools in the way of using algebra and geometry, as well as arithmetic, to solve problems taken from the other school subjects and from the experience of the pupils. Some of these problems are taken

from texts, which now furnish more problems of this kind than formerly, some are obtained from published lists and from periodicals, and many more are made by the teachers themselves. Some of the teachers in normal schools have been active in arousing interest in genuine applied problems. The great difficulty, as has been often expressed in the replies received by the committee, is to find applied problems that are of real interest to a large number of students.

The question was asked, What causes operate to produce failures in mathematic in normal schools that do not operate in other subjects? About one-fourth of the answers say none. Other answers say that too much time is devoted to other subjects; that we have poor methods of instruction and poor elementary training; that too much is attempted; that the subject, as it has been taught, lacks interest.

The libraries of the normal schools are generally well supplied with works on elementary mathematics, and on the history and teaching of the subject. The departments are also supplied with apparatus necessary for illustrative work in teaching weights and measures, and geometric models for use in teaching mensuration and solid geometry.

EXAMINATIONS.

Thirty-seven per cent of the schools reporting say that examinations are held at the option of the instructor. In such cases the examinations are given at intervals of from 2 to 6 weeks at the completion of important topics. The rule in the remaining schools is to hold examinations at the end of each term or semester. In one school no examinations are given. The examinations are written in 83 per cent of the schools; in the remainder, both written and oral. In 41 per cent of the schools the examination periods are from 40 to 50 minutes; in 46 per cent from 1 to 2 hours; in 13 per cent from 3 to 4 hours.

The principal objects of the examinations are to test the knowledge of the students, to furnish a test of the teaching, to give unity to the subject, and to secure a review. In 5 per cent of the schools the grading of pupils depends entirely upon the examinations, and in 2 per cent not at all. In 20 per cent the examinations count for one-half in determining grades; in 44 per cent for one-third; and in 29 per cent for less than one-third. In 44 per cent of the schools there is a tendency to give less consideration to written examinations given at stated intervals, and in 45 per cent there is a tendency toward abolishing them.

In three States the governing board of the normal schools or the State department of education conducts annual examinations, which in some cases determine promotion and graduation.

MODERN TENDENCIES IN THE TEACHING OF MATHEMATICS.

The committee asked for opinions concerning some recent movements to improve the teaching of elementary mathematics. While it is probable that not all of these recommendations are carried out in the schools from which they come, they are valuable, at least, as indicating present tendencies.

One question asked for recommendations for improving the initial instruction in arithmetic, algebra, and geometry; and a number of recommendations were received. About 18 per cent of the schools recommend that the study of arithmetic be begun later. There should be more objective work, especially in the lower grades, and the work should be less mechanical. There should be drill on the fundamental operations until the pupils have a much higher degree of speed and accuracy than at present.

In teaching algebra, the recommendation comes from almost all schools that this subject be more closely connected with arithmetic, so that the transition from arithmetic to algebra may be simpler and more natural. To accomplish this it is suggested that the literal notation be introduced not later than the seventh grade and used freely in the eighth grade. Furthermore, to make algebra more interesting and useful, let it be used in solving real problems.

In respect to geometry, the answers show a general tendency to make the subject less formal in the beginning. By the use of simple illustrations attempts are made to base the elementary concepts more completely upon the experiences of the pupils. Numerical and other applied problems are used to give the subject more meaning.

Fifty-one schools sent replies to the question "What traditional topics should be omitted from the course of study in arithmetic, algebra, and geometry?" In the case of arithmetic, the general recommendations are to omit all obsolete and purely technical topics and unduly complicated problems. The number of topics which one or more schools recommend for omission is large. Thirteen schools recommend the omission of cube root, except by factoring; 15, the omission of true discount; 9, highest common factor by the Euclidean method; 6, partnership and compound proportion; 5, partial payments and progressions; 4, compound and annual interest, much of percentage, and complicated problems in fractions; 3, bank discount and longitude and time. Other topics are listed for omission by one or more schools. Although a very small per cent of the schools would omit all of the topics just named, the list possesses some interest in that it shows how many of the traditional topics are being removed from the course. There are further suggestions that indicate a somewhat general tendency to abridge the above topics when they are not omitted.

It is generally recommended that the work in algebra be less formal; that we omit or postpone until late in the course complicated multiplications and divisions of polynomials, complicated problems in factoring, fractions, and radicals, and highest common factor by division. The notion of function and of variation should be introduced early. A large number of applied problems should be given throughout the course.

A number of schools recommend that limits, variables, and incommensurables be omitted from elementary geometry. Other schools think these topics should be retained, but that in discussing them only illustrations should be used, and that no proofs should be attempted. In some schools it is thought that with students of the grade of maturity of those in normal schools, it is profitable to discuss the formal proofs of some of the elementary theorems in the above topics. There is general agreement that the proofs of some of the more difficult theorems in plane geometry may be omitted.

Twenty-six per cent of the schools report that some laboratory work is done in mensuration; 15 per cent do field work in trigonometry or surveying; 33 per cent more make some application of the mathematics either in laboratory work or in applied problems. The results from laboratory work are generally thought to be good, but not in all cases.

About 14 per cent of all schools reporting to this committee have made some experiments in teaching mathematics. These were in the main experiments to determine better methods of teaching arithmetic. One of these, for example, was an experiment to determine the efficiency of daily drills in the fundamental operations and to determine the effects of such drills upon the reasoning power. It is encouraging to know that a number of teachers of mathematics in normal schools are undertaking to obtain in a scientific way data concerning improvements in teaching mathematics.

The schools were asked to state what dangers they see, if any, in recent movements to make secondary mathematics more intuitive, useful, and attractive. The answers show a wide range of opinion, as might be expected. Forty-four per cent of the answers say that no danger is seen. Others say that the movements in question have caused inefficient teaching; that drill is neglected; that the disciplinary value of the subject is lost; and that there results superficial and unrelated knowledge.

SUBCOMMITTEE A. PRIVATE NORMAL SCHOOLS.

The great majority of the private normal schools conduct their own preparatory departments, and hence have no specific entrance requirements. They generally receive students of all grades and fit them for entrance to such courses as they offer. Applicants for advanced standing are usually allowed credit on high-school diplomas or on recommendation from former instructors, though students coming from high schools are very often found deficient in the most elementary parts of mathematics. It is recommended that in the lower schools persistent drills be given in the fundamental operations of arithmetic, so that students may perform these operations not only with facility, but with absolute certainty as to the correctness of their results and may be able to prove the accuracy of their answers.

There seems to be no uniformity whatever in the credits given by the higher institutions of learning for mathematics covered in private normal schools. Each one of these schools stands on its own merits and has its own reputation. The State universities usually give full credit for all the work done in mathematics in the private normal schools of their own State. The comparison of the widely differing reports would seem to indicate that about 30 per cent of the graduates of the private normal schools enter universities and technical schools for the purpose of doing more advanced work, while about 50 per cent of the graduates teach for five years or more, usually in the public schools.

Contrary to the usually accepted opinion, it is probably true that it is not the chief object of the private normal schools to prepare students for teaching. The majority of students later go into business, and a considerable number into technical work. Consequently the utilitarian side of the study of mathematics is probably the phase that is chiefly emphasized. Special courses for students bearing upon the history and teaching of mathematics are rarely, if ever, offered.

Classes are always sustained in arithmetic, the time required of the student depending on his proficiency upon entering the school. From 40 to 60 weeks are usually given to algebra, and very satisfactory courses are generally offered. These often include college algebra. Usually about 36 weeks are given to plane geometry. Many of the schools which offer four-year courses require about six months' work in analytic geometry and perhaps the same time in the study of the calculus, though, not infrequently, the course offered in the latter subject extends throughout an entire year. Very little of the instruction is given in the form of lectures to the class, recitations on the part of the pupils being the well-known universal rule.

The recitation periods are very generally one hour in length, five recitations per week. The school year contains from 46 to 50 weeks, without intermission. There appears to be a growing tendency to give frequent tests or examinations. These are usually written and required at various intervals. Some schools give tests as often as once a fortnight, others once a month, and still others not oftener than once in 6 or 10 weeks. Final grades do not usually depend wholly upon result of these tests. Perhaps a weight of about 50 per cent is given to them and about 50 per cent to the record made in class.

SUBCOMMITTEE 4. TEACHERS FOR NORMAL SCHOOLS.

There are about twice as many men as women teaching mathematics in the normal schools of the United States. In the eastern part of the country, however, the women far outnumber the men.

The training of the teachers varies widely. About 60 per cent of the teachers have had high-school training, 55 per cent have had normal-school training, 30 per cent have had college training, and 25 per cent have done graduate work at the universities. The average training for the men is considerably higher than that of the women. There is a marked tendency toward a requirement of graduate work as a prerequisite for teaching mathematics in normal schools. There is also a wide variation in the degrees held by the teachers; about one-third of them have no degree, 42 per cent have the bachelor's degree, 18 per cent the master's degree, and 6 per cent the doctor's degree. The men who hold the higher degrees far outnumber the women.

The term of service in the present teaching position averages about seven and one-half years and is practically the same for men as for women. Twenty-three per cent of all the teachers have served in their present positions for 10 or more years, and 50 per cent have served 5 years or more.

The teachers in the normal schools have not contributed many books or articles to the literature of their field. (This may be due, in part, to the fact that most of the teachers meet a large number of classes each week and consequently have neither the time nor the energy necessary for authorship.) There is a tendency to lighten the work of the teacher in the normal schools, and this may result in a more productive scholarship from this source. Practically all of the contributions made up to the present time have been by the men.

None of the schools make a practice of employing their undergraduates as instructors, and only a few of the schools do employ their graduates without further training.

Most of the teachers of mathematics would prescribe work through the calculus as the minimum academic preparation for teachers of mathematics in normal schools. About 18 per cent of the teachers would include more advanced courses as the minimum preparation.

A large per cent of the teachers express the opinion that in addition to the academic training the prospective teacher of mathematics should also be required to take a professional course, including psychology, history of education, school management, and special methods in the courses that he is to teach.

About 30 per cent of the teachers express the belief that an experience of from one to five years in elementary and secondary schools should be a prerequisite for normal-school teaching.

Most of the normal schools of the United States were established primarily to prepare teachers for the work of the elementary schools, and the various subjects of the curriculum were determined largely by this ideal. Graduate work was not a prerequisite for the normal-school teacher, for his margin of knowledge was usually sufficient if he had completed the undergraduate work at a reputable college. In recent years, however, the normal schools are sending an increasingly large number of their graduates into positions in the secondary schools. Some schools in the Central West send almost half of their graduates into such positions.

This extension of the functions of the normal school has necessitated an extension of the curriculum into the more advanced subjects. A few years ago no mathematics was offered in the normal schools beyond solid geometry and trigonometry. To-day analytics and calculus are offered in several of the best schools, and there is a marked tendency to offer these advanced subjects in most of the schools. These subjects are now offered in order that the prospective teacher of mathematics may have a broader view of the subject and a proper perspective for teaching mathematics in the secondary schools.

The extension of the curriculum into the more advanced subjects means that the teachers of mathematics in the normal schools must be selected from those who have done graduate work in mathematics. Forty-five per cent of the men and 18 per cent of the women now teaching mathematics in the normal schools have completed at least one year of graduate work in the subject, and there is a marked tendency in this country to require at least a year of graduate work from all prospective teachers. A year of graduate work will be required, not because of the degree that may thereby be secured, but because of the amount of work that it represents, and because of the better basis that it gives for a rational presentation of the subject to those who are to become teachers.