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EXAMINATIONS IN MATHEMATICS OTHER THAN THOSE SET BY THE TEACHER FOR HIS OWN CLASSES

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THE AMERICAN REPORT

COMMITTEE No. VII

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THE AMERICAN REPORT.

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CONTENTS.

EXAMINATIONS IN MATHEMATICS, OTHER THAN THOSE SET BY THE TEACHER FOR HIS OWN CLASSES.

GENERAL REPORT.[1]

The chairman of this committee, in submitting the reports of the several subcommittees, wishes to call attention to certain general considerations.

The proportion of school children and other students in the United States who are required to take examinations other than those set by the teacher for his own classes is exceedingly small. Except in New York State, where a considerable number of children in the elementary and secondary schools are accustomed to take examinations set by the State education department, almost the only examinations of the kind are those set by a small number of colleges and by the college entrance examination board at the point of transition from the secondary school to the college.

According to the report of the United States Bureau of Education there was in 1908 a total enrollment in educational institutions of the United States of about 19,000,000. About 1,000,000 were attending public high schools and other secondary schools, about 200,000 were attending colleges and universities (excluding professional departments and schools), and about 100,000 were attending law schools, medical schools, and other similar professional schools.

According to the report of the Commissioner of Education there were in 1908 nearly 700 so-called colleges and universities. The Carnegie Foundation for the Advancement of Teaching had placed only 67 of these on its accepted list.[2] Only 7 of the 67 required that all candidates for admission should take entrance examinations. Of the institutions not on the accepted list of the Carnegie Foundation only 4, namely, Bryn Mawr College, Haverford College, the United States Military Academy, and the United States Naval Academy, had a similar requirement. In 1908, according to the fourth annual report of the Foundation, between 4,000 and 5,000 students were enrolled in the entering classes of these 11 institutions.

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[1] The exact topic assigned to the committee preparing this report was "Examinations in mathematics other than those set by the teacher for his own classes."

[2] Several are excluded because of connection with religious denominations, although of high standing educationally.

In estimating the number of candidates examined annually for admission to college it must be remembered that a very large number spread their examinations over several years, taking, for example, an examination in algebra one year and an examination in geometry another year, etc. Then, too, the colleges that admit students on the basis of secondary school certificates frequently require candidates who can not obtain such certificates to take admission examinations. In view of these considerations, it seems probable that the total number of candidates taking admission examinations in 1908 was about 10,000. We may say, therefore, that in the year 1908, of 900,000 children attending secondary schools outside of New York State, less than 1 per cent were required to take examinations other than those set by the teacher for his own classes.

In New York State, on the other hand, examinations set by the education department were used very extensively for the purpose of testing candidates for admission to the public high schools and for promotion in and graduation from such schools. In 1908 the number of candidates so examined was about 100,000.

Notwithstanding the fact that only a very small proportion of the school children in the United States are examined for admission to college, college admission examinations constitute one of the most powerful influences bearing upon the work of the secondary schools. Teachers in the secondary schools in every part of the United States recognize in the examinations set for admission to college a concrete illustration of generally accepted standards; and institutions like the college entrance examination board are in constant receipt of communications, asking that, by means of the examinations, an attempt be made to influence the methods of teaching various subjects.

It has been urged by eminent educators in the United States that the requirements for graduation from the secondary school and the requirements for admission to the college should be made identical. If this be a true educational principle, its application certainly can not mean that the colleges should be compelled to make their requirements for admission conform to standards determined wholly by the secondary schools. And it seems that it would be quite as unreasonable that the secondary schools should be compelled to make their standards of work conform to requirements established by the colleges, without consideration of the conditions actually existing in the secondary schools.

While examinations for admission to college exercise a powerful many institutions had an effect which was only injurious to the best educational interests. The situation was one in which the secondary schools were unable to make their influence felt in any appreciable degree. Each college felt free to act independently, not only of other colleges, but also of the secondary schools as a whole.

When the college entrance examination board was organized in 1900 the principle of cooperation was made fundamental. The board represents, not only a cooperative effort on the part of a large number of colleges and universities, but also a cooperation between the colleges on the one hand and the secondary schools on the other hand, in respect to a matter of vital importance to both.

The college entrance examination board does not claim the right to formulate definitions of the requirements for admission to college. It accepts definitions that have been drawn up and recommended by representative bodies of experts, recognized as having authority to speak for the subjects concerned. The requirements adopted by the board possess the two important characteristics of uniformity and stability. They can be altered only when the desirability of a change has been approved by a properly representative body of teachers.

Judged merely from the point of view of the colleges, the most important result of the board's operations has been the economy of time, money, and energy in the administration of college-admission requirements. Formerly examinations for admission to college were held only in the larger cities, and it was necessary for each of the more important colleges to maintain its own separate system of examinations.

The most important immediate benefit derived by the secondary schools from the operations of the college entrance examination board is the possibility of unity in class work. Not many years ago it was customary at some of the larger preparatory schools to divide the members of the highest classes into as many different classes as there were colleges to which the students were seeking admission.

But perhaps the greatest and most far-reaching effect of the examinations of the board will ultimately be the raising of the standards of work in the secondary schools. The standards set by the board are, undoubtedly, higher than those set by most individual schools. The board's examinations call for a thorough knowledge of the subject in which it holds examinations and, as the influence of the board spreads, the effect on the schools must be more and more marked.

SUBCOMMITTEE 1. NATURE OF PROMOTION IN ELEMENTARY SCHOOLS AND ADMISSION TO SECONDARY SCHOOLS.

Any consideration of the nature of promotions in mathematics at once involves the question of examinations, for in practically no schools is a pupil's fitness to pursue more advanced work determined without some written examination, either formal or informal, given either by the class teacher or by some higher authority. Numerous questions arise, therefore, regarding the nature and extent of these examinations, such as, by whom are they given, to what extent are they a factor in determining promotion, how valuable are they as a basis for promotion, are teachers more particular about promotion in mathematics than in other school studies.

These and related questions seemed to constitute proper lines of inquiry for this subcommittee. In order to secure evidence on these points and to get expressions of opinion from teachers and principals of various sections of this country, a questionnaire was prepared and sent to the cities of over 5,000 inhabitants and to typical normal and private schools. The response to this questionnaire was very general, and replies to the number of 427 were received from all parts of the country. The answers disclosed facts as to current practice and gave indications of the trend of professional opinion. Below are given the questions asked in this questionnaire, with a summary of the replies grouped together as accurately as possible under a few general headings:

1. In what grades are mathematics examinations or tests given?

Second	102	Sixth	423
Third	256	Seventh	420
Fourth	394	Eighth	416
Fifth	409	Above eighth	129

2. By whom are the questions made out?

Grade or critic teacher	203
Superintendent	106
Principal	48
Teachers and superintendent, or superintendent	220
State or county authorities	22

3. How often are examinations given?

Monthly or oftener	142
Bimonthly or oftener	55
Quarterly	48
Twice a year	180
Annually	19
Irregularly	124

4. What are the main purposes of examinations?

Aid in determining promotion or giving marks	99
To compel review	46
To help teacher judge efficiency of her teaching	181
To test knowledge or power of pupils	390

Other reasons given are briefly as follows:

(1) Encourages preparation, stimulates the worker, and prods the lazy.
(2) To cultivate good habits of expression.
(3) As a whip to make pupils work and to satisfy the teacher's desire to see objective results.
(4) To keep teachers on course of study and their pupils up to grade, and to spur children to better appreciation.
(5) To test efficiency of grade and to determine individual needs.
(6) To insure quickness and accuracy in thinking and expression.
(7) To test power to apply knowledge.
(8) For the enlightenment of parents.
(9) To give superintendent a uniform comparison of the work in the whole system.
(10) To accustom pupils to written work.
(11) Urge pupil to make critical preparation, and reach self-reliance.
(12) To act as check upon recitation grades.
(13) Protection of teacher.

5. In your opinion, is the use of mathematics examinations increasing or decreasing at present?

Increasing	110
Decreasing	256
No change	93
Don't know	57
Nature of examination is changing	19

6. Promotion may be dependent solely upon examinations or partly upon examinations and partly upon class work, or solely upon class work. What seems to be the general practice of typical schools in your vicinity?

Solely upon examinations	12
Examinations and class work	327
Mostly or solely upon class work	102
Teachers' impressions	12
About one-third on examinations and two-thirds class work	107

7. How efficacious are written examinations as a means of determining the fitness of a class to do the work of the next grade?

Very helpful	147
Slightly helpful	181
Not relied upon alone	176
Teacher's opinion more valuable	51
Depends upon character of examinations	28

Other opinions are as follows:

(1) They are usually a fair test, except in cases of nervous children who rarely do themselves justice under the strain of an examination.
(2) It depends upon the individual pupil, his temperament, his environment, etc.
(3) It is a poor teacher who can't find out without tests.
(4) A child's work may be good, and yet examinations unsatisfactory.

(6) Incentives to application, study, and punctuality.
(7) Not entirely faultless, but as good as any other single scheme.
(8) Useless.
(9) In some instances it appears perfectly satisfactory; in others it is obviously a failure.
(10) Valuable, but chiefly in a supplementary way.
(11) Unfair as means of determining fitness for promotion.

8. Does mathematics in the elementary school differ from other subjects of the curriculum in that a good knowledge of the processes and a reasonable degree of accuracy in manipulaion are indispensable for promotion? Is more care taken to determine fitness for promotion in mathematics than in other subjects, and, if so, why?

Yes	103
No	210
More care than other subjects	155
No more care than other subjects	210
Mathematics more definite and easy to test	59

Other opinions are as follows:

(1) The notion still prevails that mathematics is of preeminent worth.
(2) We do not mean to let mathematics dominate, but the peculiar dependence of each successive step on the last step compels us either to make it too decisive or to reduce the work to a low pressure. In fact we probably do a little of both.

9. To what extent are pupils denied promotion to next grade because of unsatisfactory work in mathematics alone?

Very rarely or never	225
Yes	85
On same basis as other fundamentals	167
Promotion based upon average knowledge and power	35

Other opinions are as follows:

(1) Probably fails more pupils than any other.
(2) Pupils whose work in mathematics alone is unsatisfactory are promoted conditionally.

10. Do you believe that there is an increasing desire for "standardized" tests, i. e., tests in the essentials of mathematics that have been given to enough pupils in representative systems to enable anyone to find out where his pupils stand in the essentials as compared with the pupils of some of the best systems in the United States?

Yes	280
No	92
Not desirable	22
Don't know	55
Desirable	73

GENERAL CONCLUSIONS.

The inevitable vagueness of some of these questions, due to their general nature, makes it impossible to draw conclusions that have statistical accuracy. However, we may venture the following:

(1) Written examinations of one kind or another are given in about all of the elementary schools of the United States. In a considerable number of schools such examinations are given as early as the second grade, but the

current practice seems to be for them to begin in the fourth grade and continue through the eighth.

(2) There seems to be no regular time for giving examinations. In States such as New York, where there is a strong centralized State system, the annual examination set by the State authorities is the controlling factor in determining the fitness of the pupils to enter the secondary school.

(3) There is no regular custom that determines who shall make the questions of the examinations, although in a large number of schools the examinations are framed by the supervising principal and the superintendent in cooperation with the classroom teacher. In nearly as large a number they are made by the grade or critic teacher alone.

(4) A study of the answers to question 4 of the questionnaire shows that a wide variation of opinion exists among teachers as to the purpose of giving examinations. A majority say that they are "to test the knowledge and power of the pupils," but this is a vague phrase and may express a wide range of meanings. Back of most of the answers given there seems to be the idea that it is very desirable to review carefully the instruction of the classroom and to give the pupils an opportunity to show independently what grasp of the subject they have obtained.

(5) The use of examinations as the chief basis for promotion is apparently decreasing, and the present tendency seems to place more emphasis upon class work than upon the formal examination. Teachers find examinations more or less helpful in determining a pupil's fitness for promotion, but at the same time realize that they are often misleading.

(6) The definite and exact nature of mathematics makes it more easy than in other subjects to measure the results obtained from instruction; consequently many teachers tend to be more careful regarding fitness for promotion in mathematics, but in only a few cases are pupils denied promotion on account of failure in mathematics alone.

(7) There seems to be a pronounced desire throughout the country for standardized tests in mathematics, that is, tests which will enable teachers to measure fairly accurately the efficiency of their instruction and to know whether their pupils are as proficient as those in other localities.

This subcommittee believes that the tendencies in the country as indicated above are along sound lines. Examinations in mathematics as a means of determining fitness for promotion are invaluable, but should not be a controlling factor to the extent of not giving due value to the excellence of daily class work. Such examinations should measure not only the extent to which pupils have mastered the regular work of the course of study, but also their ability to apply their knowledge to new situations. The great need of the future is to develop standards by which schools may measure fairly readily the results of their work. This can be done only after much experimentation and careful study. We need to know more clearly what are the essentials of a course of study, and to devise ways and means of determining whether pupils have mastered those essentials sufficiently well. We should try to reach some consensus of opinion as to what is a proper mastery of the subject matter of the curriculum. A standard may be set so high that much educational waste will an adequate return in practical efficiency. We have at present no reasonable standards that are accepted by a majority of teachers. Many educators feel that this should be the next step forward.

SUBCOMMITTEE 2. ENTRANCE TO COLLEGE BY COLLEGE EXAMINATIONS.

THE RISE OF THE AMERICAN COLLEGE ENTRANCE EXAMINATIONS IN MATHEMATICS.

The practice of examining candidates for entrance into college in mathematics seems to have arisen about the beginning of the nineteenth century, although 150 years before that time candidates for admission to Harvard were required to show proficiency in Latin. It was not until 1802, on the contrary, that mathematics was required of all. At that time the standard for admission was raised, and a knowledge of arithmetic, including the rule of three, was demanded at Harvard. Bowdoin at the same time required the fundamentals of arithmetic. The University of North Carolina called for arithmetic as early as 1795, and Princeton in 1819. By 1825 entrance requirements had become somewhat more systematized. A little algebra was added to the Harvard program in 1819, and in 1825 the applicant was required to know algebra through simple equations including arithmetical and geometrical progression. The standard in arithmetic was quite high, involving much of which the youth of the present day are entirely ignorant: Vulgar and decimal fractions; proportion, compound and simple; single and double fellowship; alligation, medial and alternate, etc. This requirement seems to have been somewhat in advance of what prevailed elsewhere. Yale and the University of Vermont called for arithmetic only. Arithmetic constituted the sole mathematical demand of the University of Pennsylvania in 1833. In 1826 the visitors to the United States Military Academy at West Point reported to Congress that the requirements for admission were low. The University of Virginia has never had entrance examinations.

By 1850 considerable progress had been made in leveling up the standard for admission. Harvard still led in being the only college to require geometry, although at Western Reserve it was stated that a knowledge of algebra and geometry was advisable, though not indispensable. On the other hand, Harvard had fallen into the highly objectionable habit of examining not in subjects, but in particular books. The requirement was Davies's and Hill's arithmetics, Euler's or Davies's algebra to roots, Hill's "Introduction to Geombegan promptly at 6 a. m. Yale required, besides arithmetic, algebra to quadratics; Columbia called for arithmetic through cube root and algebra through simple equations, substantially the same as was demanded by Princeton and the universities of Pennsylvania, North Carolina, and South Carolina. The University of Vermont more modestly asked for a knowledge of common arithmetic and the elements of algebra, while Oberlin held no examinations, but demanded a six-months' period of probationary residence. It is interesting that the only instance found of a college demanding a knowledge of quadratic equations at this date was the University of Waterville, Me., which required a knowledge of the doctrine of powers and roots sufficient for the solution of equations. It would be interesting to know if these were quadratic equations pure or affected.

The process of raising the mathematical standard for admission continued through the years 1850–1875. In the last-named year Harvard mentions the metric system as a part of required arithmetic, and, what is of much more significance, announces that tables will be supplied for those problems which involve logarithms. The algebra requirement had been extended to include quadratics, and in geometry the first 13 chapters of Peirce's book—i. e., all plane geometry but maxima and minima—are assigned. The geometry requirement at Yale was somewhat less, being Euclid, Books I and II, or Legendre I, III, and IV. Arithmetic was also called for, and Loomis's Algebra to logarithms. Substantially the same demands were made by Columbia, the University of Pennsylvania, and Amherst, while Oberlin had discarded the probationary half year, and called for Olney's Algebra and plane geometry. The academic department of Princeton made a lesser requirement in geometry, only Euclid, Book I, or an equivalent, but in the school of science plane and solid geometry were needed. Cornell and the University of Vermont required arithmetic and algebra through quadratics only.

The changes in the preceding 25-year periods were quite eclipsed by those which took place between 1875 and 1900. The requirement of arithmetic was abolished in most places before the end of the century. At Harvard, in 1900, algebra through quadratics and plane geometry were required of all candidates; advanced algebra, solid geometry, and trigonometry were optional advanced subjects, and an option in analytic geometry had been but recently discontinued for lack of pupils properly prepared. At Yale elementary algebra fell into two parts, the line of division being before quadratic equations. Geometry also was bisected, the second part including problems in mensuration, metric system, and logarithms. For the Sheffield Scientific School, trigonometry, solid geometry, and advanced algebra were requisite to admission. Amherst and Cornell had about latter trigonometry included the right spherical triangle. The same examinations were required at Columbia, the University of Pennsylvania, and Princeton. Columbia, however, had no options, while the latter two omitted the option in advanced algebra. The Princeton school of science continued to demand arithmetic, but made weight at the other end by demanding both plane and solid geometry. While there were many minor differences in the requirements of the various colleges during this period, there was a marked desire to unify the requirements; and that some progress was made to this end is indicated by the fact that a portion of the statement of the entrance requirement in mathematics in the catalogues of Yale and Princeton was the same verbatim for a few years before 1900.

To sum up, the period from 1800–1825 was characterized by the general introduction of mathematical, i. e., arithmetical, examinations for entrance. Between 1825 and 1850 algebra was brought forward, and a faint rumor of geometry might be noticed. Between 1850 and 1875 algebra was moved ahead through quadratic equations, and geometry firmly established. The years 1875–1900 saw the abolition of the requirement of arithmetic and the establishment of optional examinations in subjects of a more advanced grade. It is on the whole remarkable that, in a century when the interchange of views upon academic subjects was much more difficult than at present, the mathematical requirements for admission never differed very greatly; no one college having an unchallenged lead, but all progressing steadily together in the direction of improvement and progress.

PRESENT CONDITIONS.

In recent years an elaborate system of entrance to college by school certificates has developed, and there are at present three well differentiated entrance methods used in the United States. The one requiring an examination in all subjects for admission either by the college or by the college entrance examination board; the second admitting almost entirely by school certificates; and the third a combination of the two other methods, some students being examined by the college and others admitted on school certificate. Examples of colleges for men adhering to the first method are Harvard, Yale, Princeton, Columbia, West Point, Annapolis, Haverford, and the Massachusetts Institute of Technology. Most of the State universities of the West use the second method, that of admitting almost entirely by school certificate. The heads of two of these universities recently authorized the statement that they belived that that method is perfectly satisfactory to all who have had to do with the matter, and that they know of no serious movement for a change.

THE ARGUMENT FOR THE RETENTION OF THE COLLEGE ENTRANCE EXAMINATIONS AS PRESENTED BY AN EXPERIENCED SCHOOL PRINCIPAL.

In judging of the value of any method of admission to college two aspects must be examined: First, how does it fulfill its primary purpose, namely, to pick out from the mass of applicants those fit to pursue college courses; and, secondly, what are its effects upon the methods of the secondary schools.

It will be admitted that all methods have their unavoidable limitations and fulfill their purpose imperfectly. This is proved by the large number of unfit students dropped by colleges of all types at the end of the first quarter. Against the examination system it is urged that a single paper gives the examiner too slender a foundation for judging of a student's knowledge and capacities, even if it reflected them truly; that this latter is often not the case, because students work under emotionally highly disturbing conditions which handicap the habitually careful but nervous student and favor the one who has been in the habit of taking chances; that there are cram methods of preparing for examinations, by which a superficial gloss may be imparted which will outlast an admission examination but not the test of actual work. Nevertheless every teacher of considerable experience will be ready to testify that undeserved failures are very rare, less so than undeserved success, which is as it ought to be, and that as a whole, the examinations represent the deserts of the students very fairly.

The second aspect of the subject, the effect upon the methods of the secondary schools, is in the opinion of many the more important. The consciousness that at the end of the year the student's knowledge and his ability to apply it will be tested by some one who will judge by results only, stimulates in the highest degree the teacher's thoughtfulness. The student must not only have known his subject some time, but he must have ready command of the whole of it. He must also be trained to clear, neat, and concise presentation, and he must have had frequent opportunities to deal with problems that lie outside his daily prepared work and call for the exercise of independent judgment. Neglect of any of these points will surely lead to disappointing results in the admission examination.

It is quite true that these are the aims of all good teaching, everywhere; but under no system are the rewards for good work so prompt and the penalties for perfunctory teaching so swift as under the examination system. The high degree of stimulation this system gives to thoroughness is its principal recommendation, and in this respect it is thought to be much superior to the certificate system. And it appeals to the stronger students. A teacher in a prominent school states that the weaker students gravitate toward the certificate colleges, which bear, rightly or wrongly, among students very generally the reputation of being easy. As far as the public high schools are concerned there is every reason why it should be so. Wherever there are State universities, public opinion considers graduation from the high school as equivalent to an entrance certificate for the university, and while the grade required for graduation must of necessity be such that a majority of the students can reach it without serious difficulty, every teacher knows that as a matter of fact only a fraction of that majority is fit for more advanced work. Though some schools have attempted to set up one grade for graduation and another grade for a college certificate, this distinction has proved impracticable; and so, why should a student choose greater inconvenience, if the less will apparently land him in the same place?

For all these reasons it seems to those who favor the examination system adapted to furnish the college a constituency of reasonably homogeneous preparation and that therefore its preservation is desirable.

The introduction of the certificate system means the abolishing of the examinations. The former is the only system in existence west of the Alleghenies. The examination system is in use only in about half a dozen of the eastern colleges, although some of these are the oldest and among the strongest universities in the country. Like the system of admission by examinations the other one has its characteristic strong points, and its equally characteristic weaknesses. Let us examine both.

The certificate system, as President Angell, who was virtually its creator, has said, is a conscious adaptation to American conditions of the German system. In Germany a student who has graduated from the gymnasium passes ipso facto into the university. But in Germany the gymnasium is a State institution; character, time, and extent of its course of study are minutely and carefully elaborated and prescribed, and the carrying out of this plan is supervised by the same authority that controls the university. The articulation is complete, and yet the certificate of maturity is given only after the passing of a severe examination. This examination, it is true, is given by the teachers themselves; but it has previously to be laid before the supervising authorities, who may order any changes they see fit to make, or may reject the paper altogether, and set one of their own.

Here in the United States conditions are very different and the differences are such as greatly to weaken the value of the system. In at least one State the school law prescribes the modus operandi of the formation of high schools and provides for their support, but beyond this it does not go. There is not a word as to subjects, or methods, or extent of study. Every high school is a law unto itself; everything is left to the school district and to the school directors. High schools may be large or small, weak or strong; the teachers well paid or poorly paid; they may know their business or not. And what thus may be is, and a high-school certificate of graduation may have all degrees of value. Just how much it is worth no one can tell, unless he knows the school that issues it. And what has been said of this one State is largely true of the others.

Inspection of schools by the universities, and lists of approved schools, preparatory departments attached to some of the colleges, are intended to remedy some of the disadvantages resulting from the almost total lack of regulation by the State; but especially in the younger States, but in some of the older ones too, the articulation is very loose, and the colleges are much handicapped by the heterogeneousness of the student material they get.

Of course in the larger cities (and they are rapidly multiplying) the means, the will, and the intelligence are at hand for the building up of high schools of high rank, and it is in the relations of those with the colleges that the certificate system appears at its best. The following advantages are claimed for it:

It allows of a natural arrangement of courses of studies, which the examination system sometimes disturbs, on account of the necessity of having all subjects reasonably fresh at the time of the examinations. It enables a school to distribute the work evenly, without congestion at any point; it finally allows the student to step from one institution into the other without the nerve-racking trial of an examination, purely on his merits, as ascertained and set forth by the man who knows him best—his teacher. These are great advantages, and if they were unqualifiedly true would constitute an almost overwhelming argument in their favor. But, beside what has already been said, there are some serious modifying facts to be considered.

While the examination papers issued by the colleges that admit by that method constitute an authoritative commentary on the real meaning of the definitions of the college standards as set forth in the college requirements, such a criterion is lacking in the case of the certificate colleges; hence there is a certain vagueness on this point.

Inspection of the schools is good as far as it goes. But to be really effective it should be much more frequent and searching than it is. It is true that the performance of the student after entrance is the best criterion of the character of the instruction given at the school, but it takes a number of students and several years to reach a safe conclusion, and by that time the school may, through change in direction and change of teachers, have totally changed.

Finally, and this is probably its weakest point, it puts the burden of responsibility upon the shoulders least able to bear it. The principal of the school may himself have an imperfect or low ideal of what constitutes good work, or he may have the feeling of insecurity of tenure strong enough to make him unwilling to refuse a certificate in certain cases, and quite willing to shift the burden to backs stronger than his own.

Whether the preponderance of advantages or disadvantages lies on the one or the other side, or whether what is true now will continue to be true in the future, it is difficult to decide. The certificate method, if sufficiently safeguarded, would no doubt be the ideal method; but as long as the defects of organization of the high schools are as glaring as they are at present it seems to many that the examination method offers the better guaranties for the fulfillment of the purposes of higher education.

RECOMMENDATIONS AS TO DESIRABLE IMPROVEMENTS IN PREPARING FOR THESE EXAMINATIONS.

As the examination for college admission rests on the methods pursued in the schools, some criticisms and recommendations as to the teaching of mathematics in the schools seem to be in place in this report.

Algebra is usually begun in the first year of the high school without any previous preparation for the use of literal instead of numerical expressions. The opportunity to prepare for this transition in the grammar schools in connection with arithmetic is ordinarily neglected. The massing of definitions in algebra in the beginning of the textbooks is confusing; because of the severely technical language, the definitions are frequently misunderstood and speedily forgotten. This gives a fundamental unsoundness which perseveres. Our textbooks and teachers do not display sufficient skill in grouping the work in each topic; the advance from simple to excessively and needlessly complex examples is too sudden; difficulties introduced merely as difficulties are useless. In the practice of the schools 50 per cent of the pupils forfeit the benefit of this elementary training because in these complex examples a single error vitiates the entire result. The matter of problems is inadequately explained; pupils frequently do not understand that in attacking problems they are called upon to translate conditions from the English language into algebra language. Many more problems should be analyzed through and with the pupils, and a method of analysis should be developed.

The subject of factoring is poorly taught. The presentation of factoring under 8 or 10 different cases is misleading to the pupils. They do not study intensively the quantity that is to be factored, but endeavor to guess under what case it is to be rubricated; this is the cause of much trouble. There should be more interpretation of the relations found in an algebraic expression; then the method of factor. g will reveal itself naturally to the pupil.

Makers of textbooks forget the immaturity of the pupils and their previous meager mental outfit, and the language of the textbook is frequently lacking in concreteness. In the attempt to compass the whole of the elementary algebra in one year, as undertaken by most schools, the advance from absolute ignorance of the subject to the intricacies of radicals and equations of the second degree is too rapid to promote genuine understanding of the subject matter at this age; it is as though a beginner in Latin were expected in one year to advance from an elementary study of Latin forms to an intelligent grasp of Cicero. The advance should be more deliberate, and at least two years, with about three recitations per week, be devoted to the subject; the pupil needs time to digest. The failures in algebra at examination and the lack of grasp shown in the first college year are frequently traceable to this cause. There should be far more teaching in the class, where the teacher should develop every new subject fully before assigning any part of it for home work. It is wasteful, and even absurd, to let pupils drift without suggestion from the teacher. Home work in the earlier stages of the high school should confirm and reenforce insight gained in the class. Even the errors perpetrated in class may, if immediately traced to their origin, prove a valuable help to knowledge. Such class teaching is far more stimulating to the entire student body than the present system.

Again, our algebra should contain a greater wealth of examples and of problems of moderate difficulty, and these should more frequently refer to conditions in which they actually present themselves to pupils in practical life; that is, by real problems. All mechanical memorizing of rules should be discouraged. The definitions of the textbook should be emphasized as the best and most serviceable expression of algebraic facts and principles, but the pupils' attempts to formulate definitions should not be discouraged; they may be guided toward accuracy.

Geometry.—The lack of all constructional and mensurational work in the earlier school years is a serious drawback; many geometric relations ought to be recognized in the grammar-school period. If geometry in the high school does not precede algebra, it should at least be carried on pari passu.

The nature of demonstrational geometry as an exercise in logic is not made sufficiently explicit by texts and teachers. The massing of definitions at the beginning of the geometry textbooks, when many of these definitions refer to figures that do not come up for consideration until many months have passed, is bad teaching. The progress in demonstration should at first be very deliberate. Unconventional questioning by the teacher should bring out what the crucial or pivotal point in every demonstration is. There should be variation of the form in which propositions are brought before pupils. Sometimes, instead of the formal language of the theorem, a drawing of the figure and the algebraic expression of relations should be given, together with the crucial point, leaving the pupil to find expression for the hypothesis and the conclusion.

Many believe that plane geometry also should be spread over two years; the system of crowding the plane geometry into one year makes the advance too rapid and discourages the pupils. Three years of five periods per week for the joint study of algebra and plane geometry seem best, the teacher determining according to needs the subject to which two and three hours respectively should be given in each year. Practical applications of theorems in geometry should be at first quite simple. Some of them should follow immediately the propositions which entered into their solution. Another set, not too difficult, should be massed promiscuously at the end of each book of geometry, and at the close of the volume. All possible use should be made of illustrative material, and the pupils encouraged to make occasional models in illustration of principles. Invite students to invent demonstration; even faulty procedure may be turned to good account. Break up the belief that there is only one sequence of propositions; more often even than in algebra does cooperative class work seem the best means to secure intelligent appreciation of topics. The work is slower, but surer. Whilst exceptional pupils may gain satisfaction from unaided pondering over elementary mathematics, the majority of secondary pupils under such a system grope in the dark and lose the valuable stimulus that carefully guided advance furnishes.

RECOMMENDATIONS AS TO IMPROVEMENTS IN SETTING THESE EXAMINATION PAPERS.

Our college-entrance examinations have emphasized, by the test they present, the kind of work which has rendered the elementary mathematics difficult and unattractive. If the object of the examinations is to determine capacity and power, then the questions to be submitted might be distinctly more numerous but less complex. What we want to know after all is whether the student's previous attainment in mathematics has furnished him the tool for further expression, seem to be the main needs; and a mechanically smooth presentation of what has been memorized should be less valuable than a performance which may be unconventional in language but which reveals genuine grasp of the fundamental propositions. This recommendation would therefore culminate in this: That the examination papers should present a larger number of questions, but that in these questions matters of undue complexity should be altogether avoided.

SPECIMEN EXAMINATION.

In order that the nature of the entrance examinations to American colleges may be the better understood, the following specimen papers are appended, with the name of the colleges where they were set:

HARVARD.

(a) Plane Geometry.

(ONE HOUR AND A HALF.)

1. Prove that if two right triangles have the hypotenuse and a side of one respectively equal to the hypotenuse and a side of the other, the triangles are equal.

Draw a square ABCD. On the diagonal AC take the point E so that AE=AB, and draw through E a line perpendicular to AE cutting BC in F. Prove that

$$BF=FE=EC.$$

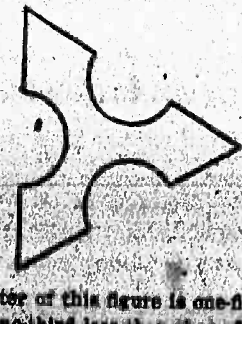
2. Prove that an angle formed by two chords of a circle intersecting each other within the circumference is measured by one-half the sum of the arcs intercepted by its sides and by the sides of its vertical angle.

Take five points, A, B, C, D, E on the circumference of a circle in the order given. Let the middle point of the arc ABC be F, and the middle point of the arc AED be G. Let the chord FG cut the chord AC in H and the chord AD in K. Prove that AH is equal to AK.

NOTE.—The points B and E are used only to indicate the arcs in which the points F and G lie.

3. Given a square with the side 8 inches long. Find the locus of a point P such that the distance from P to the nearest point of the perimeter of the square is 1 inch. Describe the locus accurately.

4. Semicircles are drawn with their centers at the middle points of the sides of an equilateral triangle, forming a figure as here shown.



Prove that if the perimeter of this figure is one-fifth greater than that of the triangle, its area is about one-third less

5. On the sides of an equilateral triangle ABC as bases equal isosceles triangles ABP, ACQ, BCR, are constructed; the first two are exterior to the given triangle, while R is on the same side of BC as A. Prove that APRQ is a rhombus.

Logarithms and Trigonometry.

1. The length of a side of a regular pentagon is 6 feet. Find the length of a diagonal.

2. A street slopes upward at an angle of 7° 12′. At the foot of the street an arc light hangs at a height of 18 feet above the pavement. How far from the light is a point up the street at which its rays meet the pavement at an angle of 12°?

3. The logarithm of 10 to the base 9 is 1.0480. Find without the use of tables the logarithm of 270 to the same base.

4. Prove that

$$\sin\left(45^\circ+\frac{x}{2}\right)-\cos\left(45^\circ+\frac{x}{2}\right)=\pm\sqrt{1-\cos x}.$$

5. From one corner of a cube lines are drawn in two of its faces making angles of 30° and 40° respectively with the common edge. Find the angle between these two lines.

(b) Elementary Algebra.

1. Find (*a*) the least common multiple and (*b*) the greatest common divisor of

$$(x^2+a^2)\ (x^3+a^3);\ (x^2+ax+a^2)\ (3x-a);\ 3x^2+2ax-a^2.$$

2. Solve the equation

$$\frac{9a^2}{x^2}+1-\frac{6a\sqrt{a}}{x}=\left(\frac{3a-\sqrt{a}}{\sqrt{x}}\right)^2.$$

3. Compute the larger of the two roots of the equation

$$x^2=0.100-0.200x$$

correct to three significant figures.

4. Show that $(a+b)^4$ will not be equal to a^4+b^4, unless

$$ab=2(a+b)^2$$

(assuming that neither a nor b is zero.)

5. In a certain town, the tax rate is 2 per cent on both real estate and personal property; under a new system the tax rate would be 5 per cent on real estate, and nothing on personal property. A certain citizen, p per cent of whose property is in real estate, would find his tax bill increased by $100 under the new system; but if the proportion of his real estate and his personal property had been reversed, he would have found his bill reduced one-half. Find the present amount of the bill.

(c) Advanced Algebra.

1. Prove that

$$\frac{a+c}{b+d}>\frac{c}{d}$$

2. In how many ways may a class of 10 students be seated in a room containing twelve seats?

3. Evaluate the determinant

$$\begin{vmatrix} a & b & c & d+x \\ a & b & c & d \\ a & b+x & c+x & d \\ a+x & b & c & d \end{vmatrix}$$

4. Find the value of

$$\frac{21\times143}{34}$$

the numbers being written in the scale of seven. Perform the indicated multiplication and division in the scale of seven, then change the numbers to the scale of ten and verify your results.

5. One root of the equation

$$2x^5-x^4+3x^3-5x^2-2x-6=0,$$

is $\sqrt{-2}$. Find all the other roots.

6. Find to three decimal places the negative root of the following equation. Use Horner's method.

$$x^3-x+7=0.$$

YALE (SHEFFIELD SCIENTIFIC SCHOOL).

(a) Elementary Algebra.

1. Solve each of the following equations by completing the square:

(a) $x^2+2x=ax+2a$; (b) $2x-7\sqrt{x}+3=0$; (c) $6x^2-5x-5=4\sqrt{6x^2-5x}$.

2. Each of two trains ran 200 miles. One ran 7 miles per hour faster than the other and required one hour and 45 minutes less time. Find the speed of each train. Explain the negative answer.

3. Solve the equation

$$\sqrt{2x-1}+\sqrt{x+8}-\sqrt{3x+2}=0.$$

Verify your answer.

4. Solve the simultaneous equations

$$y^2=2mx+m^2,\ x+y-m=9.$$

Arrange the answers in corresponding pairs and verify.

5. Simplify each of the following:

(a) $\left(\frac{m^3}{n^3}-\frac{n^3}{m^3}\right)\div\left(\frac{m^4+2m^2n^2+n^4}{m^2n-4mn^2}\right)$; (b) $\left(a^4+3a^2+2a^3+2a+1\right)^{\frac{1}{2}}$;

(c) $\sqrt{2}+\sqrt[3]{4}$; (d) $2\sqrt{\frac{y}{x}}-\sqrt{\frac{x}{y}}+\sqrt{2+\frac{x^2+y^2}{xy}}$;

(e) $\frac{2\sqrt{8-2x}-x(8-2x)^{-\frac{1}{2}}}{x^2}$; (f) $\frac{a^0b^{-1}c^{-2}}{a^{-2}b^0c^3}$.

6. Expand $(x^{-1}+2\sqrt{x})^5$ by the binomial theorem. Write the result in a form free of negative exponents.

7. The first and seventh terms of a geometrical progression are, respectively,

(b) Advanced Algebra.

1. Draw the graphs of the equations

$$\text{(a)}\quad 1-2x-3x^2=0; \qquad \text{(b)}\quad x^3-4x^2-8x+8=0.$$

Solve these equations and indicate clearly the connection of the roots and graphs.

2. Transform $24x^3-2x^2+6x+3=0$ into an equation for which the first coefficient is unity and the others integers.

3. What simple relation exists between the roots of the equation (a) $x^2+3x+5=0$? (b) of the equation $x^6+3x^3+5=0$?

4. Show that the equation $x^3+x^2-8x-5=0$ has three real roots and determine the first figure of each.

5. Calculate a negative root of the equation of 4 to two decimal places.

6. A certain target is divided into 5 concentric rings. Each ring is to be painted a different color. How many color schemes are possible with 8 colors of paint?

YALE.

Plane Geometry.

1. Two right triangles are equal if the hypotenuse and a leg of one are equal respectively to the hypotenuse and a leg of the other.

2. Upon a given straight line construct a segment of a circle which shall contain a given obtuse angle. Prove your construction correct.

3. If three or more nonparallel lines intercept proportional segments on two parallel lines, they pass through a point.

4. If two lines AB and CD cut at E so that AE×EB=CE×ED, prove that a circumference can be passed through A, B, C, and D.

5. Find the side of an equilateral triangle equivalent to a circle whose radius is 16.

6. Prove that the locus of the extremities of all the equal tangents that can be drawn to a given circle is a circle concentric to the given circle.

7. If the area of a rhombus is a and one diagonal is d, find the length of a side.

8. If a regular hexagon is inscribed in a given circle and another is circumscribed about the same circle, find the ratio of the areas of the hexagons.

9. If two circles are tangent internally, the chords of the greater circle drawn from the point of contact are divided into proportional segments by the smaller circle.

YALE (SHEFFIELD SCIENTIFIC SCHOOL).

Solid Geometry.

1. If two angles not in the same plane have their sides respectively parallel and extending from their vertices in the same direction, they are equal.

2. Prove the theorems regarding the lateral area of a prism and of a regular pyramid. State the corresponding theorems for cylinders and cones.

3. A spherical angle is measured by the arc of a great circle described with its vertex as a pole and included between its sides, produced if necessary.

4. Given two points A and B in space, to find the locus of the center of a sphere of given radius which shall pass through A and B.

5. A rifle shell has the shape of a cylinder surmounted by a hemispherical cap. The total length of the shell is four times its diameter. Compare the surfaces and also the volumes of the cylindrical and the spherical portions.

6. Given a sphere whose diameter is 10 inches. Find the volume and the surface of the inscribed cube.

PRINCETON.

(a) Algebra to Quadratics. A I.

1. Simplify

$$\cfrac{x^2}{\cfrac{x^2}{a+\cfrac{x^2}{a+\cfrac{x^2}{a}}}}$$

2. Multiply

$$x^{-\frac{1}{2}}y^{\frac{1}{3}}+x^{\frac{1}{2}}y^{-\frac{1}{3}} \text{ by } x^{\frac{1}{2}}-2y^{\frac{1}{3}}.$$

3. Reduce the following to their lowest common denominator:

$$\frac{1}{6x^2-5x-6},\quad \frac{2}{3+x-2x^2},\quad \frac{3}{2x^3-3x^2-2x+3}.$$

The candidate need not multiply out.

4. Factor $4a^2b^2-(a^2+b^2-c^2)^2$.

5. If $y=\frac{a+2t}{b-at}$, find the value of t in terms of the other letters.

6. Solve

$$\frac{3x+2}{x^2+x}+\frac{2(x-5)}{1-x^2}-\frac{x-3}{x^2-x}=0.$$

7. The sum of the three digits of a number is 16; the sum of the first and third digits is equal to the second; and if the digits in the unit's and in the ten's places be interchanged, the resulting number will be 27 less than the original number. What is the original number?

(b) Quadratics and Beyond. A II.

1. Solve the following equations:

$$\text{(a) } x+1+\frac{x^2}{x^2-1}=\frac{x^2}{x+1}-\frac{5x-4}{1-x^2};$$

$$\text{(b) } \sqrt{a-x}+\sqrt{a+x}=\sqrt{2a+2b};$$

$$\text{(c) } (x+2)^{\frac{2}{3}}-(x+2)^{\frac{1}{3}}-2=0.$$

2. Solve for x and y, writing the solutions so that the proper values are associated:

(a) $2x-3y+1=0,$ $x^2-6xy+1=0.$ (b) $xy=80,$ $\frac{1}{x}-\frac{1}{y}=\frac{1}{8}.$

3. The diagonal of a rectangle is $\sqrt{\frac{5}{2}}$; if each side were increased by 1, the area would be increased by 3. What are the sides?

4. In the expansion of $\left(\frac{2x^2}{3}-\frac{3}{2x}\right)$ [illegible], find the coefficient of [illegible].

5. Find the sum of all positive integers of three digits which are multiples of 7.

(c) Plane Geometry.

1. The line joining the middle points of the nonparallel sides of a trapezoid is parallel to the bases and equal to one-half of their sum.

2. An angle inscribed in a circumference is measured by one-half the arc intercepted by its sides.

3. Two triangles having an angle of one equal to an angle of the other are to each other as the products of the sides including the equal angles.

4. Given a parallelogram and a point outside of it, obtain a construction for finding the line which passes through the point and divides the parallelogram into two equal parts. Prove the construction.

5. Two parallel tangents of a circle are cut by a third tangent in the points A and B. If O be the center of the circle, prove that AOB is a right angle.

6. If through the point O within a parallelogram ABCD two straight lines are drawn parallel to the sides and making the parallelograms OB and OD equal, prove that the point O is on the diagonal AC.

(d) Solid Geometry.

1. There is one and but one common perpendicular to two straight lines not in the same plane.

2. If two parallel planes are cut by a third plane, the intersections are parallel.

In order to show that two lines are parallel, what must be proved?

3. If an oblique cone of circular base be cut by a plane parallel to the base, this section is a circle.

4. The line AB is perpendicular to the plane a at B; CD is a line in a, and E is the foot of the perpendicular from A on CD. Prove that BE is perpendicular to CD.

5. The length of the altitude of a pyramid is a, and its base is a square, the length of whose edge is b. What is the area of a section parallel to the base whose distance from the vertex is s?

(e) Plane Trigonometry.

1. What is a radian? Express in radians the angles, 60°, −315°.

2. If $\cot\theta=\frac{1}{2}$ and θ is an angle in the third quadrant, find the values of:

$$\sin\theta,\ \cos 2\theta,\ \cos\tfrac{1}{2}\theta.$$

3. Prove geometrically
$\cos(A+B)=\cos A\cos B-\sin A\sin B$, where A, B, and $A+B$ are acute.

4. Prove the following identities:

(a) $\cos(A+B)\cos(A-B)=\cos^2 A-\sin^2 B$;

(b) $\dfrac{\cos^3 x+\sin^3 x}{\cos x+\sin x}=\dfrac{2-\sin 2x}{2}$;

(c) $\dfrac{\tan^2(45°+A)-1}{\tan^2(45°+A)+1}=\sin 2A.$

5. Find all values of x between 0° and 360° which satisfy:

(a) $4\sec^2 x-7\tan^2 x=3$;

(b) $\sin(x-60°)=\cos(x+60°)$.

6. Solve the triangle in which $A=57.15°$, $a=2418$, $b=2868$.

SUBCOMMITTEE 3. ENTRANCE TO COLLEGE BY COLLEGE ENTRANCE BOARD EXAMINATIONS.

HISTORY.

The college entrance examination board was organized at a meeting held at Columbia University, New York, November 17, 1900, for the purpose of fixing a standard of entrance to American colleges and technical schools.[1]

At this meeting the chief examiners for 1901 were appointed. The associate examiners were named January 12, 1901. The committee of examiners in mathematics consisted, like those in other subjects, of three members, two representing the colleges, and a third, a secondary-school principal or teacher.

The examiners were charged with the duty of framing the questions to be set, and with the preparation of a syllabus of instructions for the guidance of the readers who read and rated the answer books of the candidates. The definitions of the several subjects and topics upon which examinations were held were those recommended by the committee of the National Education Association upon college entrance requirements in cooperation with such bodies as the American Philological Association, the American Historical Association, and others.[2]

In 1904 the requirements in mathematics were modified by the adoption of the recommendations made by a committee of the American Mathematical Society. These recommendations were adopted by almost every college and university in the United States. They constituted a great step in advance toward uniformity and effected considerable simplification in the holding of mathematical examinations. Under this change, elementary algebra was made to include arithmetical and geometrical progressions. Permutations and combinations, and logarithms were excluded and transferred to advanced algebra and trigonometry, respectively.

Examiners.

An examination of the list of examiners in mathematics for the different years will show its representative character. At first the policy of the board was to change completely the personnel of the committee every year. Since 1904 the same members have been continued on the committee for three or four years.

In this way the board has profited more directly by the experience gained in previous years, and the result has been a greater uniformity in the standard set by the papers.

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[1] In Document No. 1, published by the board December 10, 1900, will be found a general statement of the action of the preliminary conferences on the subject of a college entrance examination board, together with a record of the organization of the board and an outline of its purposes and policy.

[2] Report of Secretary, 1901, p. 8.

EXAMINATION PAPERS.

The subjects of the papers in mathematics were elementary algebra, advanced algebra, geometry, and trigonometry.

During the first years, in order to meet the offerings of candidates with different degrees of preparation in each subject, the paper was subdivided into parts and groups, each group containing three or more questions. In the instructions to the candidate at the head of the paper were indicated the groups from which to choose the requisite number of questions to make up the set in the part of the subject he proposed to take.

Thus in 1901, the paper—Mathematics A—Elementary Algebra—contained 25 lines of instructions to the candidate. It consisted of 30 questions, 15 in part 1—to quadratics, and 15 in part 2—quadratics and beyond. Each part was subdivided into five groups: A, B, C, D, E, and F, G, H, J, K, respectively, of three questions each. Seven different classes of candidates were provided for in this paper, each class being instructed to choose the required number of questions from certain groups. A similar arrangement was made in the papers in geometry, providing for plane geometry only, solid geometry only, or plane and solid geometry; in advanced algebra; and in trigonometry, providing for plane trigonometry only or for both plane and spherical trigonometry.

As the years went by the increase in the number of candidates and more precise definitions of requirements brought about a continuous development in the number and subjects of the different papers, in the instructions to candidates, in the arrangement of the groups, and in the choice of questions allowed. This development has been generally toward simplicity, uniformity, and perhaps some restriction in the range of choice allowed the candidates.

At the present time, as for several years past, each candidate is provided with a separate paper on his particular subject and the instructions are reduced to a minimum. The board now prepares through its committee of examiners in mathematics the following papers:

	Weights.
A–I. Algebra to quadratics	1
A–II. Quadratics and beyond	½
A–I and II. Elementary algebra complete	1½
B. Advanced algebra	½
C. Plane geometry	1
D. Solid geometry	½
C D. Plane and solid geometry	1½
E. Trigonometry (plane and spherical)	½
F. Plane trigonometry	½

These values at the right are the weights of these examinations assigned by the Carnegie Institution for the advancement of teaching. In these papers the total number of questions ranges from 8 to 10, of which the candidate is required to take from 6 to 8. The instructions at the head of each paper are contained within two or three lines.

THE PLAN OF PREPARING THE PAPERS.

In preparing the examination papers the examiners have a free hand, but generally the work is distributed among the members of the committee; One member undertaking to frame the questions in the elementary algebra papers; another, in geometry; and the third, in advanced algebra and trigonometry. The appointment of subject is determined for the most part by personal preference and special aptitude.

In beginning the work the examiner has before him the papers of previous years. He has also the criticisms and suggestions that have been received during the year at the office of the board and which have been turned over to the committee by the secretary. Under his eyes are the definitions of requirements in each subject printed each year by the board.[1]

When an examiner has prepared his paper he sends a copy to the other members of his committee. After criticisms and suggestions have been offered, the committee meets to consider the papers in detail. No conclusion is adopted which is not agreed to by the whole committee.

The examiners feel that they must set a standard of knowledge and power that will test the fitness of the candidate for college work and satisfy the expectations and requirements of college teachers. The interests of the secondary school are kept in mind, too, and guarded by their representative on the committee and by the committee on revision. After the conference the changes decided upon are made in each paper, and the manuscripts are sent to the office of the board. The papers are then referred in proof to the committee on revision for final consideration and adoption.

The committee on revision under the rules of the board consists of the 15 chief examiners and the 5 representatives of the secondary schools in the board. "The proposed questions are submitted to a searching criticism in this committee with a view to harmonizing their standards of difficulty, and to shaping them toward a recognition of the *best methods of secondary school teaching*."[2] At this conference the discussion of particular questions is often animated and full, but in most cases conclusions satisfactory to all are reached.

The statement of President Nicholas Murray Butler, secretary of the board in 1901, in his annual report of that year, with reference

[1] Document No. 40. Jan. 1, 1909.
[2] Annual Report of Secretary, 1901, p. 7.

to the papers in all subjects may be quoted here as applying also in general and with aptness to the examination papers in mathematics in 1901 and in subsequent years:

A reading of the questions set in the examinations will show, I think, that they are much more thorough, better balanced, and more searching than those usually set by the colleges individually. This was to be expected, and was a result confidently counted upon to follow from the plan of bringing together several viewpoints, including that of the secondary schools, in the preparation of the questions. Every effort has been made and will continue to be made to obtain from teachers criticisms of the questions used and suggestions for their improvement in subsequent years. So far as answers to requests for criticisms and suggestions have yet reached the secretary, they are almost uniformly commendatory of the questions set, in many cases enthusiastically so. Some teachers are of the opinion that the question papers were in a few cases too long and in some cases too difficult. The one criticism that the board could not afford to face, namely, that the questions set were too easy, has not been made.[1]

METHOD OF READING AND GRADING PAPERS.

The readers are appointed from the colleges, universities, technical schools, and secondary schools, so that the interest will be as varied as possible. Preparatory schools are represented on the board so that the reading may be influenced by those who have had charge of the preparation of the candidate.

This staff of readers, 20 to 30 in number, sits in New York. The first task is to decide what weight shall be given to the various parts of each question. The examiners first indicate what they consider the relative importance of each part of a question, then the readers do so without knowledge of the previous estimate. To insure further uniformity in marking at the outset, each reader is asked to read and rate the same set of about a dozen papers. The ratings of the various readers are then compared and adjustment made until uniformity is reached. After this has been done, the readers are then ready to begin the reading independently.

Those papers which on first reading are rated above 60 are considered no further, but papers whose rating is 60 or below are read a second time by a different reader, and if the two ratings do not agree, the two readers then read the paper together question by question, and discuss it until an agreement is reached.

The papers which any one reader rates 60 or below are not always reread by the same second reader. This also tends toward producing uniformity. When all the precautions are taken into account, it is felt that the variation in rating is reduced to a minimum.

Most of the papers are disposed of in 8 or 10 days. The few papers which come in after that time are read by one of the readers who lives in New York, and those which have to be reread are sent

[1] First Annual Report of the Secretary, 1901, p. 20.

to the head reader for rereading. As soon as papers are completed, they are reported and tabulated. Frequently finished returns can be sent to candidates within a few days after the close of the examination.

ATTITUDE OF PREPARATORY SCHOOLS.

A letter was sent to every school that furnished 12 or more candidates for the examination set by the board in 1909, containing the following questions:

1. Are the examinations of the college entrance board more difficult than regents'?

2. Are special classes formed to meet the board's examinations?

3. Is there a tendency on the part of the pupils to have a different attitude toward these examinations than toward others?

The answers received from 25 schools were, in general, as follows: (1) Yes, (2) no, (3) no. Some schools regard the board examinations as slightly more serious or more worthy of respect than examinations held by the schools themselves. A changing sentiment was noted, tending toward less awe. Several commented upon the examinations of the board as fair.

At conferences of teachers it was stated that—

1. The examinations in algebra were more uniform than those in plane geometry.

2. The examinations vary according to the ratings previously obtained, e. g., in the year preceding. Thus while one year the examinations may be a little too difficult, the following year they are far too easy.

3. The examination returns tend to show the teaching of geometry in the schools reached by the board is either very good or very poor, for in the examination returns for 1909, of 1,425 pupils who took the examination in plane geometry 49 received 100 per cent, while less than three-fifths of the whole number received as much as 60 per cent.

ATTITUDE OF THE COLLEGES.

On account of the hearty cooperation of the 27 institutions participating in the activities of the board, together with that of the Carnegie Institution for the Advancement of Teaching, the Federation of Associations of Teachers of Mathematics and the Sciences, and the American Mathematical Society, the credentials of the examinations are accepted in practically every institution where they are offered. Most of the colleges have entirely discontinued holding entrance examinations in June on account of the board's examinations.

The New England colleges, with the exception of Harvard, accept the marks of the college entrance board: Yale college, 65; Sheffield Scientific School, Brown, Worcester Polytechnic Institute, University of Maine, 60; Amherst, Boston University, Bowdoin, Dartmouth, Wesleyan, 50. Smith College requires 60 for students who take mathematics in college and 75 for those who do not. Williams requires 50, except for preliminary credit, in which case 60 is ordinarily required, but for preliminary credit for *a* or *c* (or *b*, *d*, and *f*, if the admission group is IV or V) 75 is required. The Massachusetts Institute of Technology requires 60, except for the combined paper in elementary algebra, when 70 is required. A mark of 70 is also required for trigonometry, which is not an entrance subject for the institute. Harvard University does not accept the board marks at all, but allows students to take the board examinations on condition that the papers are sent to Harvard for marking. A few of the colleges require some papers sent to them for rereading. Some find that their own marks agree quite closely with the board marks, while others find the board marks much higher.

FINANCES.

To defray the expenses of the board each candidate is charged a fee of $5, which entitles him to take any of the examinations held that year and to receive a certificate of credit received in each. Moreover, an annual fee is paid by each of a number of institutions participating in the board. A further source of revenue is from the sale of examination questions and of printed documents. The annual budget is about $20,000. In addition to the direct expenses of conducting the examinations, reading papers, and preparing credentials, a permanent secretary-treasurer is maintained, who has general charge of all its administrative activities. The secretary is Prof. Thomas S. Fiske, post office substation 84, New York City.

PUBLICATIONS.

The annual report of the secretary appears in August of each year. It contains a summary of the action of the board, the names of the examiners and of the readers, a detailed study of the year's candidates, distribution as to residence and school last attended, as well as colleges the candidates expected to enter, a statistical analysis of the results of the examination, and the financial statement of the year. A number of other documents appear from time to time, particularly defining times, places, and other information concerning the examinations for the following year.

The publishing house of Ginn & Co. publishes the complete list of questions set by the board the preceding June, together with the names of examiners and readers and a time schedule of the examinations for the following years.

STATISTICS.

In 1909 examinations were held at 167 points, at which 3,466 candidates participated, the number of books examined in mathematics being 4,324. While these figures represent the actual cases treated by the board, its influence is in fact very much wider. With continued cooperation of the various interests concerned it will soon be the dominant factor in defining the standard for entrance to college.

SPECIMEN EXAMINATION QUESTIONS.

In order that the nature of the examinations should be understood, specimen papers in elementary algebra, plane geometry, plane and solid geometry, advanced algebra, and trigonometry are given herewith. Two specimens of each of the first three are given, being those set in 1909 and 1910. Since relatively few candidates try the examinations in advanced algebra and trigonometry, and since many schools do not prepare students in these subjects, only a single specimen of each is given.

Mathematics A I and II—Elementary Algebra Complete.

Six questions are required; two from Group A, two from Group B, and both questions of Group C. No extra credit will be given for more than six questions.

GROUP A.

1. Factor (a) $32a^3b^3-4b^6$,
   (b) $x^2+2xy-a^2-2ay$,
   (c) $a^4-14a^2b^2+b^4$.

2. Reduce to their simplest forms the following expressions:
   (a) $\sqrt[n]{\frac{4}{2^{n+2}}}$
   (b) $2x^2\sqrt{9x^3+81}+27\sqrt{4x^3+36}$,
   (c) $\frac{ax(a^{-1}x-ax^{-1})}{x^{\frac{1}{2}}-a^{\frac{1}{2}}}$.

3. Show that if $c^2=a^2+b^2$,
$$a+b+c:b+c-a=a+c-b:a+b-c.$$

GROUP B.

4. (a) Solve $(a-b)x^2-a^2x+a^2b=0$.
   (b) Solve $\left.\begin{aligned} x-y&=4, \\ \frac{1}{y}-\frac{1}{x}&=\frac{4}{117}, \end{aligned}\right\}$
   and associate properly the values of x and y.

5. The sum of two numbers multiplied by the greater is 126 and their difference multiplied by the less is 20. Find the numbers.

6. (a) Construct with respect to the same axes of reference the graphs of $y-x=2$, $y=x^2-4$.
   (b) Solve these equations and show the relations between the roots and the graphs.

GROUP C.

7. Show that the coefficient of the middle term of $(1+x)^{16}$ is equal to the sum of the coefficients of the 8th and 9th terms of $(1+x)^{15}$.

8. (a) Derive in terms of a, n and d, from the fundamental formulas or otherwise, a formula for the sum of n terms of an arithmetic progression in which a is the first term and d is the common difference.

(b) The sum of the first eight terms of an arithmetic progression is 64 and the sum of the first eighteen terms is 324. Find the series.

(c) Sum to infinity the series $1.35+0.045+0.0015+$etc.

MATHEMATICS A I AND II—ELEMENTARY ALGEBRA COMPLETE.

Six questions are required. They must include two questions from Group A, two from Group B, and both questions of Group C. No extra credit will be given for more than six questions.

Group A.

1. Express $\frac{1}{x^2-4x+4}$, $\frac{1}{8-x^3}$ and $\frac{1}{x^4+4x^2+16}$ as fractions having the least common denominator in the form of a product of factors.

2. A man drives to a certain place at the rate of $8\frac{1}{2}$ miles an hour. He returns by a road that is $2\frac{1}{4}$ miles longer at the rate of 9 miles an hour and takes 10 minutes longer than in going. How long is each road?

3. (a) Find the value of x from the equation $5x=\sqrt{3}(1+2x)$ and express it as a fraction having a rational denominator.

(b) Simplify $(a^4+x^4)\,(a^2-x^2)^{-\frac{1}{2}}-(a^2-x^2)^{\frac{1}{2}}$.

(c) Find the square root of $11-6\sqrt{2}$.

Group B.

4. An audience of 360 persons is seated in rows each containing the same number of people. They might have been seated in four rows less had each row contained three more persons. How many rows were there?

5. (a) Write down the rth term of the expansion of $(a-x)^n$.

(b) In the expansion of $\left(x-\frac{2}{3x^2}\right)^9$ find the value of the term which does not contain x.

6. (a) For a certain pamphlet it costs \$100 to prepare the type and 10 cents to produce each copy. If y be the cost in dollars of x copies write down the value of y.

(b) Construct a graph—on the scale of one inch to 1,000 copies and the same unit to \$100—to show the total cost of any number of copies up to 3,000. Find from the figure the cost of 2,500 copies and the number of copies costing \$275.

Group C.

7. Solve $\left.\begin{aligned} 2x-3y&=12 \\ 4x^2+9y^2+21y&=30 \end{aligned}\right\}$

Write the results so that with each value of x the proper value of y is associated.

8. (a) Derive a formula for the sum of n terms of a geometric progression whose first term is a and common ratio r.

(b) The first term of a geometric progression is 225 and the fourth term is $14\frac{2}{5}$. Find the series and sum it to infinity.

Mathematics C—Plane Geometry.

Six questions are required; four from Group A and two from Group B. No extra credit will be given for more than six questions.

GROUP A.

1. Prove that the locus of points equidistant from the sides of an angle is the bisector of the angle.
2. Prove that the perimeter of a convex quadrilateral is less than twice the sum of the two diagonals.
3. Prove that the medians of a triangle meet in a point which trisects each.
4. Prove that in a right triangle the square on the hypotenuse is equivalent to the sum of the squares on the two sides.
5. Prove that two regular polygons of the same number of sides are similar.
6. The diameters of two circular pulleys are, respectively, 12 feet and 2 feet, and the distance between their centers is 10 feet. Find the length of the shortest string which will go around the pulleys, correct to three significant figures.

GROUP B.

7. Choose two points, A and B, upon a given straight line, and two other points, C and D, upon a straight line perpendicular to AB. Prove that the hypotenuse of a right triangle whose legs are equal to AC and BD is equal to the hypotenuse of a right triangle whose legs are equal to AD and BC.
8. On a semicircle whose diameter is AC take two points, D and E. Draw AD and CE, and let them meet in F; draw AE and CD, and let them meet in G; draw FG.

Then prove that FG is perpendicular to AC.

9. Let ABC be a triangle with a right angle at C. Draw CD and CE equally inclined to CB, and meeting AB (or AB prolonged) in D and E, respectively. Let M be the mid-point of AB. Prove that MB is a mean proportional between MD and ME.

Mathematics C—Plane Geometry.

Answer four questions from Group A, and two from Group B. No extra credit will be given for more than six questions.

GROUP A.

1. (a) Define a parallelogram.

   (b) Prove that the opposite sides of a parallelogram are equal.
2. Prove that in the same circle or equal circles, the less of two chords is at the greater distance from the center.
3. Prove that if from a point without a circle a tangent and a secant be drawn, the tangent is a mean proportional between the whole secant and its external segment.
4. Prove that the perpendiculars from the vertices of a triangle to the opposite sides meet in a point.
5. The lengths of the circumferences of two concentric circles differ by 6 inches. Compute the width of the ring between the two circles, correct to three significant figures.
6. Prove that the area of the triangle formed by joining the middle point of one of the non-parallel sides of a trapezoid to the extremities of the opposite side is equal to one-half the area of the trapezoid.

GROUP B.

7. The three sides of a triangle are, respectively, 4 feet, 13 feet, and 15 feet long. Show that the length of the altitude upon the side of length 15 is 3.2 feet.
8. Through the vertex A of the parallelogram ABCD draw a secant. Let this line cut the diagonal BD in E, and the sides BC, CD (or these sides produced) in F and G, respectively. Prove that AE is a mean proportional between EF and EG.
9. Given two indefinitely long intersecting straight lines, and a point. Find all the points which are equally distant from the two given lines, and at the same time 1 inch away from the given point.

Discuss the number of solutions for different positions of the given point.

Mathematics CD—Plane and Solid Geometry.

This paper will be rated as a whole; separate credits will not be given on this paper for plane geometry and solid geometry.

Eight questions are required; four from Group A and four from Group B. No extra credit will be given for more than eight questions.

GROUP A.

1. Prove that the locus of points equidistant from the sides of an angle is the bisector of the angle.
2. Prove that in a right triangle the square on the hypotenuse is equivalent to the sum of the squares on the two sides.
3. Prove that the medians of a triangle meet in a point which trisects each.
4. On a semicircle whose diameter is AC take two points, D and E. Draw AD and CE, and let them meet in F; draw AE and CD, and let them meet in G; draw FG.

Then prove that FG is perpendicular to AC.
5. Let ABC be a triangle with a right angle at C. Draw CD and CE equally inclined to CB, and meeting AB (or AB prolonged) in D and E, respectively. Let M be the mid-point of AB. Prove that MB is a mean proportional between MD and ME.

GROUP B.

6. State and prove a formula for the lateral area of the frustum of a regular pyramid.
7. Prove that two symmetrical spherical triangles on the same sphere are equivalent.
8. Given a material sphere, find its diameter.
9. Prove that the edge of a regular octahedron is approximately 2.45 times the radius of the inscribed sphere.
10. A sphere whose radius is 10 inches is cut into two parts by a plane whose distance from the center of the sphere is 6 inches. Find, correct to three significant figures, (a) the volume of the sphere, (b) the volume of the smaller segment.

Mathematics CD—Plane and Solid Geometry.

This paper will be rated as a whole; separate credits will not be given on this paper for plane geometry and solid geometry.

Eight questions are required, four from Group A and four from Group B.

GROUP A.

1. (a) Define a parallelogram.
(b) Prove that the opposite sides of a parallelogram are equal.
2. Prove that if from a point without a circle a tangent and a secant be drawn, the tangent is a mean proportional between the whole secant and its external segment.
3. Prove that the perpendiculars from the vertices of a triangle to the opposite sides meet in a point.
4. Prove that the area of the triangle formed by joining the middle point of one of the non-parallel sides of a trapezoid to the extremities of the opposite side is equal to one-half the area of the trapezoid.
5. Through the vertex A of the parallelogram ABCD draw a secant. Let this line cut the diagonal BD in E, and the sides BC, CD (or these sides produced) in F and G, respectively. Prove that AE is a mean proportional between EF and EG.

GROUP B.

6. (a) Define parallel lines.
(b) Prove that if a straight line is parallel to a plane, the intersection of the plane with a plane passed through the line is parallel to the line.
7. State and prove a formula for the volume of a triangular pyramid.
8. Prove that the area of the surface of a sphere is equal to the product of its diameter by the circumference of a great circle. State a rule for the area of a zone.
9. The corner of a cube is cut off by a plane passing through the mid-points of the edges which terminate at that vertex, and the process is repeated for each corner of the cube. Prove that the volume of the solid that remains is five-sixths of the volume of the cube.
10. Show that the area of a spherical triangle drawn on a sphere whose radius is 10 inches is $(200\pi-270)$ square inches, if the lengths of the sides of its polar triangle are, respectively, 8, 9, and 10 inches.

Mathematics B—Advanced Algebra.

Six questions are required. No extra credit will be given for more than six questions.

1. (a) Prove the formula for the number of permutations that can be made of n different things taken r at a time.
(b) How many different words of four letters each, any arrangement of letters being regarded as a word, can be formed from the letters in *Cambridge?*
(c) From a school board of 10 men and 6 women how many different committees can be formed composed of 3 men and 2 women?
2. (a) Find the value of the determinant:

$$\begin{vmatrix} 3 & 2 & 1 & 4 \\ 15 & 29 & 2 & 14 \\ 16 & 19 & 3 & 17 \\ 33 & 39 & 8 & 38 \end{vmatrix}$$

(b) Prove: If two adjacent columns of a determinant are interchanged the sign of the determinant will be changed.
3. (a) Give and prove the expressions for the coefficients of an equation of degree n, in terms of the roots.
(b) In the equation $x^4+7x^3+x^2-63x-90=0$ the sum of two of the roots is -8, and the product of the other two is -6; find the roots.

4. (*a*) Find to one place of decimals the approximate values of the roots of the equation $x^3-4x^2+x+4=0$.
   (*b*) Construct the graph of the function and verify the roots obtained.
5. (*a*) Determine the character of the roots of the equation $x^3+15x^2+7x-11=0$.
   (*b*) Show that the equation $x^n-1=0$ has two real roots when n is even; and but one real root when n is odd.
6. (*a*) Transform the equation $10x^4+20x^3-4x^2+25x-30=0$ into another one in which the coefficient of the first term is unity and the other coefficients are integral.
   (*b*) Transform the equation $x^3-5x^2+4x-3=0$ into another one whose roots are 3 greater than those of the given equation.
7. The cube of a number plus six times the number equals 100, find the number to two decimal places.

Mathematics B—Trigonometry (Plane and Spherical).

Six questions are required; three from Group A and three from Group B. No extra credit will be given for more than six questions.

GROUP A.

1. (*a*) Prove $\sec^2 A=1+\tan^2 A$.
   (*b*) Express in terms of $\cos x$ all the other trigonometric functions of the angle x.
   (*c*) Given the right triangle ABC in which $a=8$, $b=15$, $C=90°$; find the trigonometric functions of the angle A.
2. (*a*) Prove the formula

$$\cos (A+B)=\cos A \cos B-\sin A \sin B$$

   in which A, B, and $A+B$ are all angles in the first quadrant.
   (*b*) Prove $\cos x+\cos y=2 \cos \frac{1}{2} (x+y) \cos \frac{1}{2} (x-y)$.
   (*c*) Derive formulas for $\sin 2A$, $\cos 2A$ and $\sin \frac{1}{2}A$.
3. Two sides of a triangular field measure 243.6 feet and 184.5 feet respectively, and the angle between them is found to be 105° 36′.
   (*a*) Find the perpendicular distance from one end of the unmeasured side to the opposite side.
   (*b*) Calculate the area of the field.
4. In the oblique triangle ABC, given $a=872.3$, $b=632.7$, $A=60° 45'$, find B, C, and c.

GROUP B.

5. In a right spherical triangle, $C=90°$, prove by a figure any two of the following formulas:

$$\cos c=\cos a \cos b,$$
$$\sin a=\sin c \sin A,$$
$$\cos A=\cos a \sin B.$$

6. (*a*) State Napier's rules of circular parts.
   (*b*) Prove the following relation between the parts of a right spherical triangle, $C=90°$:

$$\cos B \sin b=\tan a \cos c.$$

7. In the right spherical triangle ABC, given $a=132° 6'$, $b=77° 51'$, $C=90°$, find A and c.
8. In an isosceles spherical triangle the equal sides are each 48° 10′, and the angle between them is 100° 6′ 40″, find the third side.

SUBCOMMITTEE 4. ENTRANCE TO COLLEGE BY STATE EXAMINATIONS.

While several other States have made some progress in this direction, New York is the only State in the American Union that maintains under State control a fully developed system of examinations leading to college entrance. These examinations cover the entire high-school curriculum, and therefore include mathematics through trigonometry. Under these circumstances a report on State examinations for admission to college necessarily becomes a report on the New York State system of examinations.

The history of this system of examinations is interesting and instructive. The general direction of the educational activities of the State of New York is in the hands of a board of regents of the University of the State of New York. This board is entirely nonpartisan. Its members are elected by the State legislature for a term of 11 years, the terms of one or two members expiring each year. The executive officer of the board is the commissioner of education, elected directly by the board of regents to serve during their pleasure. This organization leaves the education department unhampered by any untoward influences, political, sectarian, or otherwise. Early in the history of the State a fund was established for the encouragement and support of secondary education, and this distribution was intrusted to the regents of the university. This fund was distributed among the secondary schools of the State on the basis of the attendance of the academic students. Originally the report of the principal of the school as to the number of such students attending each school was accepted as final and was made the basis of the apportionment, but after a time it became evident that there was a wide divergence of opinion as to what constituted an academic student. In other words, the question of qualification for admission to the academic school was raised, and an examination in preacademic subjects was instituted under the direction of the regents of the University of the State of New York, for the sole purpose of determining who were academic students entitled to be counted in making the apportionment of the academic fund. No other purpose than the one mentioned was then in the mind of anyone, and no extension of the system was contemplated; but as soon as these preacademic examinations had been put upon a substantial basis it was discovered that an increased zeal and an increased interest resulted on the part of both pupils and teachers, and requests for the extension of the system to high-school subjects began to be made by those engaged in secondary school work. In 1877 the regents were by statute directed to establish academic examinations and to furnish a suitable standard of graduation and of admission to college. In 1880 the first academic syllabus or summary of requirements was issued, and the examinations have been regularly conducted since that time.

The methods of managing the examination system have developed as the examinations have extended and as experience has indicated the needs of change. At the present time the general direction of the system is in the hands of the New York State Examinations Board. This board is composed of 20 members who are appointed by the board of regents on the recommendation of the commissioner of education. Five of the members are representatives of the state education department, five representatives of colleges, five representatives of the secondary schools, and five school superintendents representing the elementary schools. This board determines questions of general policy and also appoints committees to prepare the examination question papers in the various subjects.

Each question committee appointed by this board consists of three persons, all of whom must be actively engaged in educational work, one being a representative of a college, one a representative of a high school, and one a representative of the educational department. It is the settled policy of the board that at least one member of each question committee shall be retired each year and a new member shall be appointed in his place. These question committees meet at the capitol in Albany at appointed times and confer on the preparation of the questions to be used at the examination and prepare such questions in manuscript.

After this first draft of each question paper has been thus prepared, the papers are all submitted to a committee of final revision, which is selected from the members of the examinations board, this committee having full power to modify any question paper to any extent that seems desirable or to send back a question paper to the committee that prepared it for revision or for reconstruction.

The question papers, when thus finally prepared, are printed by the education department and distributed to the schools, and the examinations are conducted under the direction of the principals of the high schools of the State in accordance with regulations that have stood the test of 30 years' experience.

The scope of the examinations is quite accurately defined by the academic syllabus. This syllabus is most carefully prepared by committees consisting of the leading educators of the State, drawn from the colleges and the secondary schools, and is regularly revised once in five years to keep it in line with the best educational thought.

Before a student can be regularly admitted to one of these examinations, he must furnish evidence that he has pursued his studies for an adequate length of time in a regularly organized and regularly inspected academic school. When a student has passed examinations for four school years, an academic diploma is issued by the state department of education, provided the subjects covered meet fixed requirements as to distribution in English, mathematics, history, and science. The college entrance diploma, in two forms, covers the specific requirements for admission to college.

The following important facts are to be noted concerning this system of examinations:

1. The system is not devised by State authorities and imposed upon the schools. Its origin, its growth, and its development are in response to a demand coming from the secondary schools and colleges of the State.

2. The general direction of the system is in the hands of a selected body of representative college and high school men, with the result that the interests of the colleges, of the preparatory schools, and the students themselves are all zealously guarded.

3. The question papers are prepared by committees in which the colleges and the preparatory schools are equally represented, thus insuring standards satisfactory to the colleges and possible of attainment by the high schools.

4. The change in the personnel of each question committee by retiring one of its three members each year provides for the continual infusion of new life and new ideas without the danger of sudden and radical changes in the standards or methods.

5. The schools in which these examinations are held must conform to fixed requirements as to material equipment, teaching force, and course of study, and each of the schools is regularly inspected by an officer of the department.

6. Admission to the examination carries with it the certificate of the principal that the student has satisfactorily pursued the subject in an approved school for an adequate length of time.

7. Each answer paper is examined and rated first by the local school authorities, and if, in the estimation of such authorities, it is found to reach a passing mark, it is forwarded to the central office and there reread and rerated by competent examiners employed by the State.

Moreover, the rating of any paper is subject to reconsideration on appeal, thus insuring fairness to the student and to the school as well as to the college.

From the above statements it appears that a diploma issued in accordance with these regulations virtually carries with it a certificate that the student has pursued the subject satisfactorily under competent teachers for an adequate length of time in a properly equipped school, the work of which is regularly inspected by officers of the State department, and that after such study the student has the diploma has inscribed upon its face a list of the subjects in which the student has so passed, with the standing attained in each subject.

Such a system of examinations seems theoretically to meet all the essential requirements for a system of tests for admission to college that shall be absolutely just and that shall conserve the interests of all concerned.

No statistics are available bearing on the results attained by these examinations as compared with the results attained by other methods of admission to college and covering an extensive field. So far as we are able to learn, Cornell University is the only institution where such a study has been made. In the report of the president of that institution for the year 1894–95 it appears that 5.8 per cent of the students admitted on regents credentials were found deficient in their studies in the university compared with 8.98 per cent of those admitted on certificates, and 11.67 per cent of those admitted through entrance examinations conducted by the university authorities, which, of course, includes some who failed to enter in any other way. This study included all the students admitted to that institution for the preceding six years.

The following table shows the growth in mathematical examinations by the regents of New York State by decades:

	1880	1890	1900	1909
Algebra:				
Examined	262	8,771	16,850	31,639
Allowed	242	4,493	10,226	23,540
Advanced algebra:				
Examined	(1)	523	787	1,177
Allowed	(1)	76	423	909
Plane geometry:				
Examined	180	4,561	9,958	18,562
Allowed	106	2,720	5,048	11,986
Solid geometry:				
Examined	(1)	695	1,394	2,676
Allowed	(1)	545	1,132	1,908
Trigonometry:				
Examined	(1)	298	427	1,541
Allowed	(1)	240	316	1,172

[1] Data not available.

SPECIMEN EXAMINATION QUESTIONS.

The following examination papers were set in January, 1911, and show the general nature of the work required by the New York State Education Department:

Advanced Arithmetic.

Write at the top of the first page of your answer paper (a) the name of the

Two recitations a week for a school year (or four recitations a week for half a school year), in a recognized academic school, is the regular requirement, and any statement showing less or other than this should be accompanied by a satisfactory claim or explanation made by the candidate and certified by the principal; otherwise such paper will be returned.

Answer eight questions. No credit will be allowed unless all operations (except mental ones) necessary to find results are given; simply indicating the operations is not sufficient.

1. Prove that 8 is an exact divisor of a number if the number expressed by its three right-hand figures is divisible by 8.

2. Prove that the greatest common divisor of any two numbers is a divisor of their sum.

3. At the same rate per cent, is the 6 per cent method or the exact interest method the more favorable to the borrower? How much more favorable? Explain.

4. On a certain village proposition 112 votes are cast; $\frac{4}{11}$ of the affirmative votes equals $\frac{4}{5}$ of the negative votes. Find the number of votes for the proposition and the number opposed to the proposition.

5. A and B have watches set to Greenwich time. At noon A finds that his watch indicates 25 minutes past 7 a. m.; B finds that his indicates 15 minutes past 4 p. m. Find (a) the longitude of each, (b) their difference of longitude.

6. If $\frac{1}{3}$ of the time to midnight is the time past noon, what is the time? Solve by arithmetic and give analysis.

7. After 10 pounds of sugar of a certain grade are mixed with 6 pounds of another grade worth only $\frac{2}{3}$ as much per pound, the mixture is worth a cent a pound less than the better sugar; find the price per pound of each grade of sugar and the price per pound of the mixture.

8. The diagonal of a rectangular field is 100 rods; its length is to its width as 4:3. Find the number of acres in the field.

9. A cube contains 4,913 cubic inches; find the volume of a cone the diameter of whose base is equal to the length of the cube and whose height is equal to the height of the cube.

10. At what price apiece must steel pens be bought that a dealer may make 100 per cent by selling them at 144 cents a gross?

11. A man sells a certain amount of 5 per cent stock at 86 and invests the proceeds in 6 per cent stock at 103; by so doing his income is increased $1. Find the amount of stock he sold.

12. John Doe borrows from the Albany National Bank $300 for 90 days; he discounts the note at 6 per cent on the day it is dated. Find the face of the note. Write the note with Richard Roe's indorsement.

Elementary Algebra.

Write at the top of the first page of your answer paper (a) the name of the school where you have studied, (b) the number of weeks and recitations a week that you have had in algebra.

Five recitations a week for a school year, in a recognized academic school, is the regular requirement, and any statement showing less or other than this should be accompanied by a satisfactory claim or explanation made by the candidate and certified by the principal; otherwise such paper will be returned.

Answer the first six questions and two of the others. No credit will be allowed unless all operations (except mental ones) necessary to find results are given; simply indicating the operations is not sufficient.

1. Simplify $\left(\frac{1}{x+1}+\frac{1}{x-1}\right)\div\left(\frac{1}{x-1}-\frac{1}{x+1}\right)$

2. Find the prime factors of *each* of the following: $27x^3-64$; $16a-25ab^2$; $16x^2+25y^2+40xy$; x^6+y^6; $x-1+x^3-x^2$

3. Solve for x and y: $\begin{cases} ax-by=0 \\ x-\ y=c \end{cases}$

4. Reduce *each* of the following expressions to its simplest form: $\sqrt{\frac{1}{7}}+\sqrt{63}+5\sqrt{7}$; $(4\sqrt[6]{7}-8\sqrt[6]{21}+6\sqrt[6]{42})\div2\sqrt[6]{7}$; $\sqrt[4]{\frac{9}{625}}+6\sqrt{\frac{1}{3}}-\sqrt{12}$; $a\sqrt{\frac{x}{a}}+x\sqrt{\frac{a}{x}}$

5. Solve $\sqrt{(3+x)}+\sqrt{x}=\dfrac{6}{\sqrt{(3+x)}}$

6. Solve $\begin{cases} x^2-xy=3 \\ xy-y^2=2 \end{cases}$

7. The perimeter of a rectangle is 22 inches; if the cube of its length is added to the cube of its width the result is 407 inches. Find the area of the rectangle.

8. A man sold 2 acres more than $\frac{3}{8}$ of his farm and had 4 acres less than half of it left; find the number of acres in the farm.

9. A man invests $\frac{2}{3}$ of his capital at $4\frac{1}{2}$ per cent and the rests of it at 4 per cent; his annual income is \$176. Find his capital.

10. If from a certain number the square root of half that number is subtracted, the result is 28; find the number.

Intermediate Algebra.

Write at the top of the first page of your answer paper (*a*) the name of the school where you have studied, (*b*) the number of weeks and recitations a week that you have had in algebra.

Two recitations a week for a school year (or four recitations a week for half a school year) in addition to the five recitations a week for a school year required for elementary algebra, in a recognized academic school, is the regular requirement, and any statement showing less or other than this should be accompanied by a satisfactory claim or explanation made by the candidate and certified by the principal; otherwise such paper will be returned.

Answer eight questions, selecting two from each group. No credit will be allowed unless all operations (except mental ones) necessary to find results are given; simply indicating the operations is not sufficient.

Group I.

1. Prove that a^{-1}, a^0, and $a^{\frac{3}{4}}$ equal, respectively, $\frac{1}{a}$, 1, and $\sqrt[4]{a^3}$

2. If $x-y$ is a mean proportional between y and $y+z-2x$, prove that x is a mean proportional between y and z.

3. Determine, without extracting roots, which one of the following is the greatest: $\sqrt[3]{10}$, $\sqrt{6}$, $\sqrt[4]{17}$

Group II.

4. Solve $x\sqrt{x}-2\sqrt{x}=x$

5. Solve $\begin{cases} 2x+\sqrt{xy}=10 \\ 3y-2\sqrt{xy}=-1 \end{cases}$

6. Plot the graph of $x^2-4x+3=y$

GROUP III.

7. In two years the population of a city increased from 64,000 to 81,000; the rate per cent of increase during the first year was equal to the rate per cent of increase during the second year. What was this rate?

8. Separate the number n into three parts such that the first is to the second as a is to b and the second is to the third as c is to d.

9. Find the square root of $5x^2-23x^4+12x+8x^5-22x^3+16x^6+4$

GROUP IV.

10. Show that the fraction $\frac{x^n-\sqrt{x^{2n}-4}}{2}$ is the reciprocal of the expression $\left(\frac{\sqrt{x^n+2}+\sqrt{x^n-2}}{2}\right)^2$

11. Simplify $\frac{\frac{a^2+b^2}{b}-a}{\frac{1}{b}-\frac{1}{a}}\times\frac{a^2-b^2}{a^3+b^3}\times\left(\frac{a+b}{a-b}+\frac{a-b}{a+b}\right)\times\left(\frac{a}{a+b}+\frac{b}{a-b}\right)$

12. Write in its simplest form the quadratic equation whose roots are $a+b\pm(a-b)\sqrt{-1}$

Advanced Algebra.

Write at the top of the first page of your answer paper (a) the name of the school where you have studied, (b) the number of weeks and recitations a week that you have had in algebra.

Five recitations a week in algebra for two school years, in a recognized academic school, is the regular requirement, and any statement showing less or other than this should be accompanied by a satisfactory claim or explanation made by the candidate and certified by the principal; otherwise such paper will be returned.

Answer eight questions. No credit will be allowed unless all operations (except mental ones) necessary to find results are given; simply indicating the operations is not sufficient.

1. How many planes are determined by 30 points if no four of the points lie in the same plane?

2. Divide $8-4\sqrt{-1}$ by $3+4\sqrt{-1}$. Find the 6th term and the sum of the series of $\sqrt{\tfrac{2}{3}}$, 1, $\frac{\sqrt{3}}{\sqrt{2}}\cdots$

3. Prove that $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$

4. Show by the binomial theorem that

$$\sqrt{10}=3\left\{1+\frac{1}{2\times 9}-\frac{1}{8\times 9^2}+\cdots\right\}$$

and continue the series *three* terms further.

5. Apply the method of undetermined coefficients to develop the following fraction to *five* terms of a series in ascending powers of x: $\frac{1+x}{1-x-x^2}$

6. Prove that if two integral series, arranged to ascending powers of x, are equal for all values of x which make them both convergent, the coefficients of like powers of x are equal.

7. Prove that $\log_a x=\log_b x\times\log_a b$, and that $\log_b a\times\log_a b=1$.

8. Prove that an equation of the nth degree has n roots and only n roots.

9. If $a+b\sqrt{-1}$ is a root of an equation with real coefficients, prove that $a-b\sqrt{-1}$ is also a root of the equation.

10. One root of the equation $x^4-2x^3-9x^2+10x+7=0$ lies between -2 and -3; find by Horner's method the value of this root to *two* places of decimals.

11. Construct the graph of $\frac{1}{2}x^2-x+2=y$ from $x=-4$ to $x=+4$. Determine approximately by graphic methods (a) a negative root, (b) the positive value of x that will produce the least value of $\frac{1}{2}x^2-x+2$.

12. Prove that $x^7+2x^5-5x^3-4=0$ has *one* positive real root and *six* complex roots.

Plane Geometry.

Write at the top of the first page of your answer paper (a) the name of the school where you have studied, (b) the number of weeks and recitations a week that you have had in geometry.

Five recitations a week for a school year, in a recognized academic school, is the regular requirement, and any statement showing less or other than this should be accompanied by a satisfactory claim or explanation made by the candidate and certified by the principal; otherwise such paper will be returned.

Answer eight questions, selecting two from each group.

Group I.

1. Prove that the sum of the exterior angles of any polygon, made by producing the sides in succession, is four right angles.

2. Prove that every point in the bisector of an angle is equally distant from the sides of the angle.

3. Prove that the medians of a triangle intersect in a point which is two-thirds of the length of each median from its vertex.

Group II.

4. Prove that if two circles intersect, their line of centers is perpendicular to their common chord at its middle point.

5. Prove that a tangent to a circle from an exterior point is a mean proportional between a secant from the same point and the external segment of the secant.

6. Prove that the area of a triangle is equal to half the product of its base by its altitude.

Group III.

7. Find the area of a circle inscribed in an equilateral triangle whose side is 10 inches.

8. The area of a trapezoid is 204 square inches; its altitude is 17 inches; and one base is 16 inches. Find the other base.

9. With the vertices of a square as centers, four equal circles are drawn so that each circle touches two others; the part of the square not covered by the circles equals 85.84 square inches. Find the radius of each circle.

Group IV.

10. Construct a square equivalent to the sum of two squares whose areas are 81 and 144, respectively. Prove your construction.

11. Construct a triangle equivalent to a given pentagon. Prove your construction.

12. Construct a mean proportional between two given lines. Prove your construction.

Solid Geometry.

Write at the top of the first page of your answer paper (a) the name of the school where you have studied, (b) the number of weeks and recitations a week that you have had in solid geometry.

Two recitations a week for a school year (or four recitations a week for half a school year), in a recognized academic school, is the regular requirement, and any statement showing less or other than this should be accompanied by a satisfactory claim or explanation made by the candidate and certified by the principal; otherwise such paper will be returned.

Answer eight questions, selecting two from each group.

Group I.

1. Prove that if two trihedral angles have the face angles of one equal to the face angles of the other, their homologous dihedral angles are equal.

2. In how many ways may squares or equilateral triangles be grouped about a point to form a convex polyhedral angle? Explain.

3. Prove that if a straight line is parallel to a plane, the intersection of the plane with any plane passing through the given line is parallel to the given line.

Group II.

4. Prove that two tetrahedrons having a trihedral angle of one equal to a trihedral angle of the other, are to each other as the products of the edges including the equal trihedral angles.

5. Prove that the volume of a triangular pyramid is equal to one-third of the product of its base by its altitude.

6. Find the altitude of a triangular pyramid, every edge of which is 10 inches.

Group III.

7. Find the volume of a spheric shell whose inner radius is 10 and whose outer radius is 20.

8. Prove that the volume of a spheric sector is equal to the area of the zone which forms its base multiplied by one-third the radius of the sphere.

9. If the earth is a sphere of radius r, what is the area of the zone visible from a point whose height above the surface of the earth is h?

Group IV.

10. The sides of a triangle are each 12.49 inches; what is the volume generated if the triangle revolves about one side?

11. Prove that a section of a sphere made by a plane is a circle.

12. What is the locus of the centers of spheres whose surfaces are tangent to all three sides of a given triangle?

Trigonometry.

Write at the top of the first page of your answer paper (a) the name of the school where you have studied, (b) the number of weeks and recitations a week that you have had in trigonometry.

One recitation a week for a school year (or two recitations a week for half a school year), in a recognized academic school, is the regular requirement for admission to the examination in plane trigonometry or spheric trigonometry, and any statement showing less or other than this should be accompanied by a satisfactory claim or explanation made by the candidate and certified by the principal; otherwise such paper will be returned.

Candidates for plane and spheric trigonometry will answer five questions, selecting one question from each group except Group III. Answers 20 credits each.

A, B, and C represent the angles of a triangle, a, b, and c the opposite sides. In a right triangle C represents the right angle.

Give special attention to arrangement of work.

Plane Trigonometry.

Candidates for plane trigonometry will answer five questions from Groups I, II, and III. Answers 20 credits each.

Group I.

1. Prove the identities $\frac{\sin x}{1+\cos x}+\frac{1+\cos x}{\sin x}=2\csc x$

$$\cos x \cos y = \tfrac{1}{2}\{\cos(x+y)+\cos(x-y)\}$$

2. Given $\tan x=\frac{b}{a}$, $\tan y=\frac{a-b}{a+b}$; show that $\tan(x+y)=1$

Group II.

3. In a plane triangle given two sides and an angle opposite one of them; explain with diagrams when there are two solutions, one solution, and no solution.

4. In a plane triangle show that the ratio of any side to the sine of the opposite angle is equal to the diameter of the circumscribed circle.

Group III.

5. A lighthouse was observed from a ship to bear N. 34° E.; after the ship had sailed due south 3 miles the lighthouse bore N. 23° E. Find the distance from the lighthouse to the ship in each position.

6. In order to find the distance between two objects A and B separated by a swamp, a station C was chosen and the distances $CA=382.5$, $CB=347.6$, together with the angle $ACB=62°\ 31'$, were measured; what is the distance AB?

Spheric Trigonometry.

Candidates for spheric trigonometry will answer five questions from Groups IV, V, and VI. Answers 20 credits each.

Group IV.

7. Deduce the following formulas for the solution of right spheric triangles:

$$\cos c = \cos a \cos b$$

$$\sin a = \sin c \sin A$$

8. Prove that in a right spheric triangle an oblique angle and the side opposite are either both less or both greater than 90°.

GROUP V.

9. In a right spheric triangle given $B=83° 50'$, $a=108°$; find A and c.
10. In a right spheric triangle given $B=80°$, $b=67° 40'$; find A and a.

GROUP VI.

11. In a spheric triangle $A=132°$, $B=140°$, $b=127°$; find a.
12. In a spheric triangle $a=64° 24'$, $b=42° 30'$, $C=58° 40'$; find A and B.

SUBCOMMITTEE 5. ENTRANCE TO COLLEGE BY CERTIFICATION.

HISTORY.

The first suggestion looking toward the admission of students on certificate from the high schools is contained in the report of Acting President Frieze, of the University of Michigan, for the year 1869–70. In this we read:

As a means of strengthening, consolidating, and elevating the whole State system, some of our best educators both in the local schools and in the university have proposed that a commission of examiners from the academic faculty should visit annually such schools as may desire it and give certificates to those pupils who may be successful in their examinations, entitling them to admission, without further examination, to the university.

At a meeting of the faculty of the department of literature, science, and the arts of the University of Michigan held November 14, 1870, a resolution was passed looking to the appointment of a committee—

to see if some system can not be devised by which the high schools of the State can be brought into closer connection with the university, the committee to report so early that their report can be placed in the next edition of the catalogue.

At a meeting of the faculty held February 27, 1871, on recommendation of this committee the following statement was ordered introduced into the catalogue:

SPECIAL NOTICE TO PREPARATORY SCHOOLS.

Whenever the faculty shall be satisfied that the preparatory course in any school is conducted by a sufficient number of competent instructors and has been brought up fully to the foregoing requirements, the diploma of such schools, certifying that the holder has completed the preparatory course and sustained the examination in the same, shall entitle the candidate to be admitted to the university without further examination.

At a meeting of the faculty held May 1, 1871, a resolution was adopted to the effect—

that the high schools of the State which informed that, after they have made application through their board with a statement as to their courses and the number of teachers employed, a visitor from the faculty will be sent to them and this department will act on his report.

In his report for the year 1870–71 the acting president says:

The effect of this plan of annual examination, which is, of course, to be matured and perfected by experience, will be to stimulate the schools to a higher grade and bring them to a more perfect uniformity of preparation, and thus make it possible to elevate the scholarship of the lower classes in the university. But more than this, it creates at once a reciprocal interest between the schools and the university, and also wins for the university a livelier interest on the part of the citizens whose schools are brought into such close connection with the institution. * * * The principle of this movement is obvious. We go back to the schools and aid their instructors in devising correct plans and laying solid foundations of scholarship, instead of waiting until pupils present themselves at the university prepared under dissimilar and, perhaps, erroneous systems, often imperfectly prepared, and sometimes rejected for deficiencies which could have been obviated by this previous interchange of views between the faculty and the preparatory teachers.

The system actually wrought out does not in every respect conform to the plan here suggested. The university sends a commission to examine schools, and to approve such as are found worthy, for periods of from one to three years according to the merits of the schools. The student who comes with proper certificate from one of these approved schools is admitted at once. The examination of the school includes the courses of study, the teaching, the equipment of the school in the way of laboratories and libraries, and its discipline and spirit.

The schools have been stimulated to a higher grade of work; the preparation for the university has been made more uniform, and the scholarship of the lower classes has been elevated; a closer reciprocal interest between the schools and the university has been created, and a livelier interest in the university awakened in the public mind. To be sure, other causes have worked to the same ends, but this one has been pronounced and unmistakable. On the side of the school, the superintendent, the principal and teachers look to the visits of the examiners with interest, as occasions for comparing notes, rectifying errors, discussing policies and methods, and receiving fresh stimulation. In a word, they receive from the university the most practical and useful report on their work that it is possible for them to receive. The pupils, too, feel the significance of the inspection. More than this, boards of education often, if not generally, interest themselves in the visitation. Seeking out the visitors, they have questions to ask about studies, teachers, and the organization and spirit of their schools, in order that they may more fully bring them into line with university ideas. On the whole the schools desire the visits to be more frequent rather than less frequent; and if it were seriously proposed to discontinue them altogether, strong opposition would come from this quarter. On the side of the university, the faculty, in the most practical and direct of ways, renews its acquaintance with the schools. Faculty discussions and decisions are guided in no small degree by the observations of the professors eral that the university must keep in touch with the schools; and when forward steps are proposed the question is sure to be asked, What are the schools prepared to do?[1]

As the list of schools seeking approval grew larger and larger the burden of inspecting these schools became too great for the faculty committees so that now there is a man under regular appointment on the faculty as assistant professor of education and inspector of schools, whose attention is largely given to the work of inspection in which other members of the faculty take no part save on very rare occasions when their service is specifically called for.

To satisfy a natural curiosity as to the working of the system of admitting students on certificate from approved schools a committee appointed in March, 1876, made a careful study of the results of the semester examinations of all the freshmen admitted since 1871. In 1880 a still more exhaustive study was made of all the data bearing on the question from 1871 to 1880. One of the items of the report of the committee was the following:

> The average percentage of success of 351 diploma students for the first seven years that the diploma system was in operation was 88.36. The average percentage of success of 442 students admitted on examination for the same period was 86.50. The corresponding figures for the last two years, 1878–1880, were, respectively, 90.56 and 89.61. The percentages are very slightly in favor of the students admitted on diploma.

Since that time the question has not been reopened. The relative number of students admitted on certificate has gone on steadily increasing until now but very few are admitted on examination. The plan originated at the University of Michigan is now in force, with modifications, of course, in all the leading colleges and universities of Michigan, Ohio, Indiana, Illinois, Wisconsin, Minnesota, North and South Dakota, Iowa, Kansas, Nebraska, Colorado, Montana, Wyoming, Missouri, and Oklahoma.

At a comparatively early period in the history of the movement the authorities of the University of Michigan were invited to inspect schools outside of the State and the invitations were accepted to a considerable extent. Other institutions followed the example till finally certain high schools of the first class were examined by inspectors representing several different States. The situation became embarrassing.

In the year 1895 a meeting was held at the Northwestern University, in Evanston, Ill., which led to the organization of the North Central Association of Colleges and Secondary Schools, whose avowed object is to establish closer relations between the colleges and showed vigor and enthusiasm. Its discussions were stimulating and instructive. At the meeting held in 1901 Prof. Whitney, of the University of Michigan, read a paper on "The problem of harmonizing State inspection by numerous colleges so as to avoid duplication of work and secure the greatest efficiency." As a result of the discussion which followed a committee was appointed which brought in the following recommendations:

We recommend that the association do now proceed to the establishment of some definite form of affiliation and credit, as fixed, comprehensive, and uniform as may be, between the colleges and universities of this association and the secondary schools of the North Central States, and to this end we make the following further recommendations:

(1) That a permanent commission be formed to be called the commission on accredited schools and to consist, first, of 12 members to be appointed by the chair, 6 from the colleges and 6 representing the secondary schools; and second, of additional or delegate members, one from each college or university belonging to the association which has a freshman class of at least 50 members and which may appoint such representatives, together with a sufficient number of members from the secondary schools, to be appointed by the chair, to maintain a parity of representation as between the secondary schools and the colleges. * * *

(2) That it be made the duty of this commission to define and describe unit courses of study in the various subjects of the high-school program, taking for the point of departure the recommendations of the national committee of thirteen; to serve as a standing committee on uniformity of admission requirements for the colleges and universities of this association; to take steps to secure uniformity in the standards and methods, and economy of labor and expense in the work of high-school inspection; to prepare a list of high schools within the territory of this association which are entitled to the accredited relationship; and to formulate and report methods and standards for the assignment of college credit for good high-school work done in advance of the high-school requirement.

The number of high schools accredited to the association for the year 1909, whose graduates may be admitted on certificate to any of the colleges or universities belonging to the association, is 595, distributed as follows: Colorado, 31; Illinois, 97; Indiana, 42; Iowa, 50; Kansas, 17; Michigan, 74; Minnesota, 44; Missouri, 28; Montana, 1; Nebraska, 24; North Dakota, 5; Ohio, 93; Oklahoma, 2; South Dakota, 15; Wisconsin, 71; Wyoming, 1.

Each State university inspector reports favorably on quite a number of schools not yet of sufficiently high standing to be put upon the list of the North Central Association. In this way encouragement is given to schools whose condition is steadily improving.

At the University of Michigan very close watch ...

This history of admission on certificate to the University of Michigan has been related for two reasons:

First. Because the system had its origin at that institution.

Second. Because its development throughout the Middle West has been along the same lines.

The next event in this connection worthy of attention is the establishment of the certificate system at the University of California.

From the opening of the university in 1869 to 1884 entrance was by examination. The accrediting of secondary schools in California was authorized by the regents of the university in 1884. For the following 19 years those schools which desired to avail themselves of the accrediting system made annual application to the university. Four or more examiners each year went to each school and investigated the work. The result was a better understanding of conditions and relations, which led to the naming of the majority of the high-school teachers by the university.

In 1891 the State authorized the establishing of union district high schools, and in the act it required that at least one course of study should prepare the student for entering the State university.

By 1893 the number of high schools had so increased in number that the method of visiting was changed. A university examiner, who is a professor in the department of education, undertook the work of general supervision of the 200 high schools of the State, of which 150 are now on the accredited list.

The method that has been pursued by the University of California has been so distinctive and thorough that in many parts of the world it is known as the California method. No other State university has exercised so much control over the high schools, and none but a State university could.

The principal difference between the California and the Michigan plans was the thoroughness of inspection at the former university. There, at first, each department of instruction in the high school was examined by a member of the same department of the college faculty. Instead of approving the school and accepting the diploma, departments of the school were approved. As the number of schools increased, this method was modified first by allowing one member of the faculty to examine for some other subject kindred to his own, and finally, as stated above, by delegating the whole inspection to a special member of the department of education. His work in this respect has so much increased that he has requested an assistant, as the amount of inspecting that he can do has fallen below what he regards as a minimum.

In the part of the country so far considered the growth of the certificate system was closely connected with that of the State universities. The basis of it was the idea of a State system of education should be no more difficult in such a system to pass from the high school to the university than from one grade to another. The denominational or endowed college was largely overshadowed by the State university and compelled to accept its lead in this respect.

In the East the situation was entirely different. The denominational and endowed colleges and universities had the field almost entirely to themselves. They had no such power over the public schools, or over the distinctly preparatory schools, as the State universities of the West had over the secondary schools of that region. The theory in the East was that the college set the standard of admission and was chiefly concerned as to the best method of enforcing it. It was believed that the high-school teacher, who had known the pupil for three or four years, was better able to say whether or not the standard of the college had been met than the examiner, who had only the written work of a few hours to depend upon. In practice, here as elsewhere, some institutions seemed to be swayed too much by a desire for increased numbers, but this was by no means generally true.

Notwithstanding the difference in the point of view, the certificate system appeared in New England as early as 1874 and has now spread over practically the whole of New England and the Middle States. Only three institutions in New England—Harvard, Radcliffe, and Yale—admit only by examination. All the others admit both by examination and certificate. One, the Institute of Technology, of Boston, accepts certificates in only such subjects as are not regarded as preparatory for courses required after entering the institute. Amherst and Williams refuse to accept certificates in French and German. The Rhode Island State College requires that the record of the applicant in the school at which he was prepared be submitted, and admits or rejects him on the basis of this standing. Some practically accept the certificate of any school, while others have an approved list from which they receive certified pupils. There was no attempt at uniformity and in most cases no systematic scrutiny of the approved list until the formation of the New England college entrance certificate board.

The commission of colleges in New England on admission examinations, at its annual meeting in May, 1900, *Voted:* That a committee of three be appointed to gather information upon the method of administering the certificate system by the colleges represented on the commission which employ it, and to report what in their judgment may render it more efficient and uniform.

This committee made an extensive report[1] at the annual meeting in 1901. The commission passed the vote recommended by the committee and also, *Voted:* That the colleges that shall adopt the above

[1] Fifteenth Annual Report of the Commission of Colleges in New England on Admis-

recommendation be requested to appoint, before the close of the college year, delegates, one from each college, to meet at such time as may be arranged for by the executive committee of this commission, and to form a definite plan for the organization and work of the board herein provided for.

In response to this request delegates from nine New England colleges met on the 31st of January, 1902, and voted to recommend to the colleges the establishment of a board, composed of one member from each college, for the purpose of receiving, examining, and acting upon all applications of schools that ask for the privilege of certification. They also adopted certain general provisions [1] under which the board should be organized. One of these provided that the assent of seven colleges should be necessary for the establishment of the board.

On May 16, 1902, eight colleges having assented to the general provisions, their delegates met and organized the board. Since then five more colleges have been admitted. The board limits its jurisdiction to schools located in New England, and the colleges agreed not to accept the certificate of the principal of any school in New England not approved by the board. The first list of schools was adopted October 22, 1903, others were added May 20, 1904, and the first complete list was used by the colleges for the class entering in 1904.

In preparing the list of approved schools, the board considers: First, whether the school in respect to the number and training of its teachers, its curriculum, and its equipment, is able to prepare properly a student for admission to some course leading to a degree in some one of the colleges represented on the board; secondly, whether the certificate of the school can be depended upon. In the opinion of the board, the second question is the more important. If the students certified by the school successfully pass the work of the first term in college, the board retains the school on the approved list. The board desires to encourage individuality on the part of the school and emphasizes results rather than methods. For this reason, when a school applies for the first time, the board will not consider the application until at least two have entered on examination within three years some one or more of the colleges belonging to the board. If accepted, the school is first placed on a trial list for one year, so that the board may be able to judge of what the school considers a proper preparation for college. If satisfactory, the school will then be approved for a period of three years. On application the approval will be extended for a period not exceeding three years if the record remains satisfactory. The board three years, a warning notice will be sent to it on receipt of the first bad report, and if the work is not improved during the next year the school is dropped. Even though the reports received from all the colleges except one are good, if the school continues to send unsatisfactory pupils to that one, it will be dropped. Also, if the reports are poor in only one subject and no improvement is shown, the school is dropped. The board makes some allowance for the fact that a boy may go to pieces under new and strange surroundings, but it believes that this should be the exception and not the rule.

For the year 1909–10 there were on the approved list of the New England board 286 schools, and on the trial list 34 schools, or 320 schools from which pupils could be received on certificate. These were distributed among the States as follows: Maine, 54; New Hampshire, 33; Vermont, 25; Massachusetts, 158; Rhode Island, 17; Connecticut, 33.

In the Middle States conditions are now about as they were in New England before the formation of the board. Three colleges—Princeton, Columbia, and Bryn Mawr—receive only by examination. The others accept certificates. Efforts have been made within the last two years to establish a college entrance certificate board of the Middle States and Maryland, with functions similar to those of the corresponding New England board. So far the attempt has been unsuccessful.

RESULTS.

In the part of the country west of the Middle States general satisfaction with the certificate system seems to prevail. Some connected with the endowed institutions complain that they have little influence over the secondary schools and are compelled to accept certificates of schools approved by the State universities, whether they wish to or not. Some teachers in the State institutions also complain of the results. One of them says, "I am continually fighting the faculty, local board, and State board for a higher standard in mathematics, mechanics, and physics. But as the State board controls secondary and State institutions, we are required to take what is sent." Another correspondent, a teacher in an endowed college with a preparatory department, says, "Church schools are too weak to dictate terms, and we have practically no influence in shaping high-school courses. The State superintendent and State university control us. Those we train in preparatory work surpass those trained in high schools."

These are exceptional reports. Most of those reporting seem satisfied with present results. Probably 80 or 90 per cent are admitted on certificate throughout this part of the country.

The number of students now entering the University of Caliby certificate, that comparisons of relative efficiency can not be well made. However, a careful examination covering a period of ten years, from 1884 to 1894, during which time the average number admitted by certificate was equal to the number admitted by examination, has shown: (*a*) That those who were admitted upon examination, without condition, did better work in their freshman year than those admitted on certificate. (*b*) That those admitted on examination with one condition did better work than those admitted on certificate. (*c*) That those admitted on examination and having two conditions did not do such good work as those admitted on certificate. (*d*) That the few who were admitted with three conditions did very little satisfactory work.

A member of the faculty reports that he was opposed to the plan at the time it was adopted. He foresaw the evils more clearly than the benefits. After 25 years of experience with accrediting he is ready to say that the benefits outweigh the disadvantages.

As far as reports have been received by the committee, most college men in New England and the Middle States, outside of the institutions which admit only by examination, are fairly well satisfied with the results obtained by the certificate system. This is especially true of those connected with colleges forming the New England board. The secondary school principals are divided. Many object to the responsibility which they claim has been thrust upon them.

The only statistics available concerning the results obtained from the certificate system in New England are those published in the annual report of the New England College Entrance Certificate Board. From these, it is seen that the per cent of unsatisfactory pupils has greatly decreased in the principal subjects offered for admission since the establishment of the board. This is shown in six subjects by the following table taken from the seventh annual report:

Students received on certificate.

	English.	Latin.	Greek.	French.	German.	Mathematics.
Number certified in 1908-9	1,161	960	275	846	621	1,104
Number who failed first term 1908-9	85	33	4	39	39	136
Per cent who failed first term—						
1908-9	7.3	3.4	1.5	4.6	6.3	12.3
1907-8	7.5	3.2	2.9	6.6	7.3	13.6
1906-7	8.4	4.0	2.0	5.8	2.9	15.5
1905-6	7.4	3.7	2.5	4.3	4.3	10.6
1904-5	9.5	4.4	1.9	3.8	7.8	12.8
Per cent unsatisfactory first term 1903-4[1]	20.2	14.0	11.8	13.4	18.4	24.6

[1] The last year before the formation of the approved list. This year the colleges reported those whose preparation was unsatisfactory. In subsequent years they reported those who failed during the first term. This may account for a part of the difference, but not for a large part.

This table also shows that the per cent of failures among students in mathematics greatly exceeds that in each of the other subjects for every year reported. The same difference appears among those entering by examination. It is not, therefore, connected with the certificate system.

The following table made up from the same report compares the results for 1908–9 obtained from the certificate with those obtained by examination:

	Received by examination.	Received on certificate.
English:		
Number admitted	393	1,161
Number who failed during first term	35	85
Per cent of those admitted who failed	8.9	7.3
Latin:		
Number admitted	259	960
Number who failed during first term	11	33
Per cent of those admitted who failed	4.2	3.4
Greek:		
Number admitted	43	275
Number who failed during first term	1	4
Per cent of those admitted who failed	2.3	1.5
French:		
Number admitted	326	846
Number who failed during first term	34	39
Per cent of those admitted who failed	10.4	4.6
German:		
Number admitted	174	621
Number who failed during first term	6	39
Per cent of those admitted who failed	3.4	6.3
Mathematics:		
Number admitted	436	1,104
Number who failed during first term	84	136
Per cent of those admitted who failed	19.3	12.3

In every subject except German the results are in favor of admission by certificate. In 1907–8 the per cents in German were 10.1 on examination and 7.9 by certificate. In either method the mathematical subjects fare the worst. In comparing the two systems, it should be borne in mind that in these colleges the large majority of the best students are admitted on certificate and for the most part only those who are unable to obtain certificates present themselves for an examination.

Some friends of the certificate system have been disappointed by one result of the administration of the system by the New England board. They hoped to be able to obtain the benefit of the individual judgment of the principals in the case of men who, under the examination system, might fail to pass the set examinations, but who might be able to take up college work successfully. In this they have been grievously disappointed. The principals almost invariably establish a standing which entitles the recipient to a certificate. If the number reported as failing after entering college brings a warning from the board, the principal simply raises the per cent required for the certificate. The certificate has therefore become a prize for good scholarship, which entitles the successful ones to exemption from college examinations. The effect of this is that an increasing number of doubtful men appear at the examination and the examiner receives no aid from the principal in determining whether or not to admit them.

The figures are as follows:

	Examined.	Certified.	Received.	Per cent of whole examined.
English:				
1904-5	239	829	1,068	22.4
1908-9	393	1,161	1,554	25.3
Latin:				
1904-5	184	711	895	20.6
1908-9	259	960	1,219	21.2
Greek:				
1904-5	71	309	380	18.6
1908-9	43	275	318	13.5
French:				
1904-5	155	590	745	20.8
1908-9	326	846	1,172	27.8
German:				
1904-5	70	370	440	15.9
1908-9	174	621	795	21.9
Mathematics:				
1904-5	237	819	1,056	22.4
1908-9	436	1,104	1,540	28.3

Of the two methods of administering the certificate system as practiced in the East and in the West, each appears to be best adapted to the region where it is used. Each has been of great benefit to the secondary school. In the East, the board has been of great benefit in raising the standards, since the school boards seem anxious that their schools shall have the certificate privilege. The curricula are therefore broadened and greater care is taken to obtain competent teachers.

SUBCOMMITTEE 6. STATE AND LOCAL EXAMINATIONS OF TEACHERS.

Although the circulars sent out to the different States brought in a great deal of information pertaining to teachers' certificates and examinations, this information is, so much of it, indefinite that it is impossible to present it in statistical form.

Almost all States hold examinations for teachers, either through the State departments or through the accredited normals of the States. The majority of the States recognize college or university diplomas from other States when the applicants present proper credentials of experience. Many of the States issue temporary certificates on these recommendations. In some States a certain amount of professional work and experience is rewarded by a permanent certificate. Some of these certificates are for life or during good behavior, while others are permanent only while the teacher retains the position for which the certificate was granted.

The examining board varies. For State certificates, this board is usually appointed by the State superintendent of public instruction, and it has the privilege of appointing assistant examiners. The county superintendents or local commissioners are responsible for the grading.

In Massachusetts the certificates are all local. The State department has nothing to do with the examinations.

In Connecticut a State examination may be held in any town upon the request of the town school officers. The manuscripts are examined and the certificates issued by the State department.

In the State of New York the elementary certificates are issued by local school commissioners. The secondary certificates are issued by city superintendents and the higher grades by the State commissioner of education.

Iowa and a number of the other States have a system of examinations very similar to that of Indiana. Questions made by the State board of education are sent to the county superintendents who hold examinations in the counties. The examination manuscripts are sent sealed to the State department where they are passed upon by a board of examiners. The certificates issued by the State department are valid in any part of the State. In Colorado the superintendent of public instruction, who by the way is a woman, makes all the examination questions unassisted.

Three-fourths of the States issue first, second, third, and fourth grade certificates. Added to these often are temporary or trial, professional, and life certificates.

In Maine the rank of certificates is determined by examination grades. Usually scholastic and professional attainments are determining factors.

The first grade certificates in the various States range from five to ten years; the second, three to five years; the third, one to two; the fourth is usually a trial or one-term certificate. Few States advance teachers without examinations in professional studies or credits in the pedagogical courses of accredited normal schools and universities.

Exemptions from examination are based more on scholastic and professional attainments than on teaching experience. Nearly every State exempts the graduates of its best universities and collethe best colleges and universities of other States, and after the experience test, admits these teachers to the exemption privileges of its own graduates.

Pennsylvania exempts college graduates with three years' experience. New Jersey exempts graduates of State normal schools after they have finished a certain amount of postgraduate work.

Wisconsin exempts graduates of its State institutions; Utah, college graduates with two years' experience; Montana, college or normal graduates with from two to five years' successful experience are generally granted certificates, without examination, good for six years.

The school law of Arkansas forbids the issuance of certificates or licenses to teach based upon licenses or diplomas from other States.

In North Dakota graduates from reputable colleges and normal schools receive certificates without examination, and teachers who are not graduates, but who hold first or second grade county certificates, may have these same certificates renewed in case they do the work prescribed by the teachers' reading circle board for a period of two years successfully, or if they attend summer schools.

Wyoming has no uniform examinations for elementary or secondary schools. The local boards apply any requirement they may decide upon.

In the largest cities of the different States there is a great variety of standards, but few city boards accept State certificates. The city superintendent, assisted by several appointed assistants, constitute an examining board. In Boston promotional examinations are held twice per year and a teacher who fails to pass these tests twice consecutively loses her position. These examinations consist of three parts: (1) Success in school during the previous year; (2) professional study; (3) academic study in some one line.

The superintendent of the Grand Rapids, Mich., schools, says: "I find I can secure graduates of normal schools for vacancies in our schools, and consequently the examination of teachers who are not eligible has become almost unknown."

New York City and Chicago demand a great deal in educational fitness from their teachers and hold city examinations regardless of State certificates.

A small per cent of the States report that examinations in algebra and plane geometry are required of applicants for elementary or grade certificates. At least three-fourths of the States and cities require applicants for high school certificates to pass in algebra, plane and solid geometry, regardless of whether or not mathematics is the subject to be taught. Texas, California, Kansas, and Missouri include trigonometry in the mathematical list.

Few States advance teachers' license grades without additional professional work.

SUBCOMMITTEE 7. EXAMINATIONS OF ACTUARIES.

Up to the present time the only organization that has conducted examinations for actuaries in the United States is the Actuarial Society of America. This society, as its name implies, does not confine its activities to the United States. A large and influential percentage of its membership is in Canada, where, too, it holds examinations. The Institute of Actuaries of Great Britain also conducts examinations in Canada, but they no doubt will be considered in the report from England.

The Actuarial Society of America was founded in 1889, but prior to 1897 did not require the passing of examinations for admission. It limited its membership to those in high actuarial positions and who were known to have thorough qualifications. Since 1897 admission without examination has not been absolutely abolished, but restrictions have been so thrown about such admission that up to the present only a few distinguished foreigners and a bare half dozen actuaries from this continent have been so admitted. All of the half dozen of this continent are men who had attained a standing in the actuarial world before the examination system was established and were in responsible positions at the time of admission.

In a certain sense the examinations of the Actuarial Society may be compared with those for admission to the bar, in the fact that the passing of them stamps the man as technically qualified to be a member of the profession. In another sense they are entirely different in that legally the examination of the Actuarial Society has no effect, since a person, whether he be a member of the society or a dry-goods clerk, has the legal right to call himself an actuary. The Actuarial Society is not even incorporated.

It might also be added that the passing of the actuarial examinations is not sufficient to make a fully qualified actuary any more than the passing of the bar examinations makes a lawyer fully qualified. In each case practical experience is absolutely necessary.

The examinations of the Actuarial Society are four in number and are divided into two sets of two examinations each. The first two examinations admit to associateship in the society. This is not membership, nor even qualified membership, but entitles persons to the right to attend the meetings and is recognized, both in and out of the society, as showing thorough qualification in certain of the fundamentals of actuarial science and as indicating the younger men who will ultimately pass the examinations and be entitled to the full privileges of the society.

The remaining two of the examinations may be taken by those who have become associates and lead to fellowship in the society, the

the examination for "Fellow" to give to a considerable extent a practical turn to questions, so that they will test the candidate's familiarity with and understanding of the practical questions which an actuary meets.

The subjects covered by the examinations are best shown by the syllabus of the society, which is herewith quoted:

SYLLABUS OF EXAMINATIONS.

For Enrollment as Associate.

SECTION A.

Part I:

1. Arithmetic.
2. Algebra, including—
   a. Permutations and combinations.
   b. Binominal theorem.
   c. Series.
   d. Theory and use of logarithms.
3. The elements of the theory of probabilities.
4. Elementary plane geometry.

Part II:

1. Elements of the calculus of finite differences.
2. Elements of differential and integral calculus.
3. The principles of double entry bookkeeping.
4. Compound interest and annuities certain.

SECTION B.

Part I:

1. The application of the theory of probabilities to life contingencies.
2. Theory of annuities and assurances, including the theory and use of commutation tables and the computation of premiums.
3. The use of life tables (including application to population statistics) and the construction of monetary tables based thereon.
4. The outlines of the history of life insurance.

Part II:

1. The application of the calculus of finite differences and of the differential and integral calculus to life contingencies.
2. Valuation of policies.
3. The general nature of insurance contracts.

For Admission as Fellow.

Part I:

1. The sources and characteristics of the principal mortality tables.
2. Methods of constructing and graduating mortality tables.
3. Methods of loading premiums to provide for expenses and contingencies.
4. Valuation of the liabilities and assets of life insurance companies.
5. Insurance of under-average lives and extra premiums for special hazards.
6. Practical treatment of cases of alteration or surrender of life insurance contracts.
7. Valuation of life interests and reversions.

Part II:

1. The assessment of expenses and the distribution of surplus.
2. Life insurance bookkeeping, office practice, preparation of schedules, statements, and reports.
3. The investments of life insurance companies.
4. The principles of banking and finance.
5. Laws of the United States and Canada relating to life insurance.
6. Calculation of premiums for and valuation of pension funds.
7. Method of compiling and graduating sickness, accident, and disability statistics; construction of premiums therefrom.

It should be stated that while the scope and the difficulty of the examinations have been rapidly increasing, the wording of the syllabus has been but little changed except in the addition of a few subjects and the rearrangement of subjects as between the different examinations. In reading the syllabus it should be borne in mind that a broad interpretation is given to the topics, and especially that in the examination for admission to fellowship practically any question which is properly an actuarial question and which does not distinctly belong to the examination for the associates is admissible.

The foundation for actuarial education rests, as is generally recognized, upon the subject of mathematics and especially upon the theory of probabilities. Mathematics, however, is only the foundation, and even a thorough knowledge of the mathematics of actuarial science falls far short of making a competent actuary. An inspection of the topics of the examination for fellowship shows how large a part of the preparation lies entirely outside of the field of mathematics and how important are such subjects as law, finance, investments, and accounting. In certain phases of each of these subjects, particularly law and accounting, actuarial knowledge is necessary to make one competent to advise his company, since questions arise which can not be properly understood or dealt with without such a knowledge of the foundations of life insurance as the actuary alone has. Thus, to a certain extent, the actuary must be a lawyer, a financier, an accountant, and also, last, but not necessarily least, a man with a practical knowledge of affairs.

It is natural to compare the examinations of the Actuarial Society with those of the Institute of Actuaries, which is the older society and has always maintained the highest standard. A few years ago those of the Actuarial Society were much easier than those of the institute. They have, however, gradually become more difficult, until to-day it is the aim of the examiners to place them on a par with those of the older society. The type of question in the two organizations is sometimes quite different. This is due partly to the difference in practice in the two countries and partly to the effort to make the examinations of the society as thoroughly practical as possible.

One of the reasons why the earlier examinations of the society were easier than those of the institute is that in England the coaching system of preparation with first-class instruction and directions for reading and studying was available to candidates, whereas, in this country until quite recently, there has been no instruction or guidance available for the majority of those coming up for the examinations except as they could get it from their superior officers in the companies by which they were employed, and there were not even satisfactory books to help in the preparation. In more recent years the situation has been much better, both in regard to coaching, if desired, and in regard to books and papers. While the Textbook of the Institute of Actuaries has been for years the standard work and covers a large part of the theoretical work which the student should know, it is written in difficult style, and its chapters and subjects are arranged in logical order rather than in the order to assist the student. Consequently, it is not a book for beginners. About five or six years ago there appeared a well-written elementary textbook (Moir's Primer) covering the foundations and being an introduction to most of the subjects taken up in the Textbook. This bridged the way over many of the difficulties for the beginner. There have also appeared in England certain books which gave great help to the student in the more advanced work, covering some of the parts which he had heretofore to dig out of actuarial papers as best he could, and there have also appeared in the Journal of the Institute of Actuaries and in the Transactions of the Actuarial Society various papers of a character to be of great assistance.

In regard to the practical subjects for the fellowship examinations the situation has not changed much to the advantage of the student, but the demands of the society have nevertheless been made greater, requiring a higher grade of knowledge in these subjects recently than in former years.

We include below a partial list of works on actuarial subjects now in print. Works on law, finance, accounting, administration, and other subjects, with which actuaries must be familiar, are omitted from the list, as they do not have reference to the mathematics of insurance.

BOOKS ON TECHNICAL ACTUARIAL SCIENCE.

Life Assurance Primer, H. Moir. C. C. Hine's Sons, New York.
Practical Lessons in Actuarial Science, Dawson. Spectator Company, New York.
Notes on Life Insurance, E. B. Fackler. Spectator Company, New York.
Institute of Actuaries Textbook:
Part I, R. Todhunter. C. & E. Layton, London.
Part II, G. King. C. & E. Layton, London.
Insurance. T. E. Young. Sir Isaac Pitman & Sons, London.

Actuarial Science, Graduated Exercises, Ackland and Hardy. C. & E. Layton, London.

Mathematisch-technische Kapitel zur Lebensversicherung, C. Landré. Gustav Fischer, Jena.

Die Lehre von der Mortalität und Morbilität, Westergaard. Gustav Fischer, Jena.

Die Mathematischen Rechnungen bei Lebens- und Rentenversicherungen, A. Zillmer.

Théorie des assurances sur la vie. H. Poterin du Motel. L. Warnier et Dulac, Paris.

Théorie et pratique des assurances sur la vie, H. Laurent.

Actuarial Theory. Robertson & Ross. Spectator Company, New York.

Theory of the Construction of the Tables of Mortality, G. F. Hardy. C. & E. Layton, London.

Annuities and Reversionary Payments, 2 vols., Jones. Baldwin, London.

Annuities, Treatise on, with Tables. Davies. C. & E. Layton, London.

Theory of Finance, G. King. C. & E. Layton, London.

Journal of the Institute of Actuaries, vol. 1–42.

Transactions of the Actuarial Society of Edinburgh.

Transactions of the Faculty of Actuaries.

Transactions of the Actuarial Society of America.

Proceedings of the International Congress of Actuaries. (Separate sets for each of the six congresses.)

Encyclopädie der mathematischen Wissenschaften; Article Lebensversicherungs Mathematik, G. Bohlmann.

British Offices Life Tables; Account of the Principles and Methods adopted in the compilation of the data, the Graduation of the Experience, and the Construction of the Deduced Tables. C. & E. Layton, London.

Bulletin de l'Association des Actuaires Belges.

Bulletin Trimestriel de l'Institut des Actuaires Français.

SPECIMEN EXAMINATION QUESTIONS.

Below are given a number of typical problems for each of the four examinations. These are all taken from the examination questions of years subsequent to 1906, as these later years give the best representations of the examinations as they now are. There are also attached to this report copies of the last three pamphlets giving in full the examinations of the Actuarial Society for 1907, 1908, and 1909, and certain other information with regard to the society, including lists of fellows and associates.

Section A.—Associate.

Part I.

1. An army column 10 miles long is moving forward at a uniform rate. An aid rides from rear to the head of the column and back to the rear at a uniform rate. When he reaches the rear it is where the head was when he started. How far did he ride?
2. (a) Expand to four terms $(9-6x)^{-\frac{3}{2}}$.
   (b) Write in its simplest form the coefficient of x^8 in $(2+\frac{1}{3}x)^{17}$.
   (c) Is the binomial expansion always correct for negative and fractional exponents? Discuss briefly.

3. (a) Five men toss a coin in order until one wins by tossing head. What is the probability that neither the second or third man will eventually win?

(b) A coin is drawn at random from a purse known to contain either silver or gold coins, nine in all, and proves to be a silver one. What is the probability that at least seven of the coins were silver before the drawing took place?

4. (a) There are two sets of counters, each set numbered from 1 to 8, inclusive. In how many ways can four counters be chosen so that none of them will be duplicates?

(b) In how many different ways may 20 recruits be drafted into five regiments, four to each; and in how many different ways may 15 oranges be divided into 3 equal groups?

Part II.

1. (a) Show that the mth difference of an integral function of n of the rth degree is an integral function of the $(r-m)$th degree if $m<r$, a constant if $m=r$, zero, if $m>r$.

(b) Having given the following net premiums, find by interpolation the net premium for a 31-year endowment at age 34.

Age.	Endowment— 25 years.	Endowment— 30 years.	Endowment— 35 years.
30	32.376	26.748	23.118
33	32.807	27.827	23.880
36	33.406	28.130	24.127

2. A loan of $100,000 bearing interest at 5 per cent is to have the principal repayable as follows:

$1,000 at the end of 5 years,
1,250 at the end of 6 years.
1,500 at the end of 7 years. etc.

What price must be paid for the loan so that the purchaser may realize 6 per cent on his investment?

Given $V^5=.7473$ $V^{25}=.2330$ at 6 per cent.

3. A loan of $15,000 is to be repaid by monthly payments of $100, which include interest on principal outstanding at the rate of 6 per cent per annum compounded monthly. How many monthly payments would be made?

Given $\log 2=.3010300$; $\log 3=.4771213$; $\log 67=1.8260748$.

4. A company starts business October 1 with $150,000 capital stock, paid in cash; $100,500 is immediately invested in 4 per cent bonds (interest payable semiannually) at 100 and accrued interest for 1½ months. During the three months the company collects $35,000 in annual premiums, pays $27,000 for commissions and other expenses, and has no death claims. On December 31 the reserve and other liabilities are computed at $20,000. Indicate the entries to be made on the ledger and give a statement of financial condition December 31, so far as obtainable from the data given.

Section B.

Part I:

1. Derive the formula for the net annual premium for a continuous installment endowment which provides that at maturity (at end of endowment period or by death) 5 per cent of the face shall be paid each year for 20 years and that these payments shall continue as long beyond the 20 years certain as either the assured or the beneficiary survive.

2. Prove that

(a) $n\,V\,x:\overline{m|} = dx + n:\overline{m-n|}\,(Px + n:\overline{m-n|} - P\,x\,\overline{m|})$

(b) $n:m\,V\,x = n\,V\,x + A\,x\,(n\,V\,x:\overline{m|} - n\,V\,x)$

(c) $t\,P\,x:\overline{m|} = \dfrac{t\,P\,x\,.\,P\,x:\overline{m|}}{P\,x:\overline{m|} + d}\left(1 + \dfrac{d}{Px}\right)$

and give a verbal interpretation of any one.

Part II:

1. (a) Explain select and ultimate reserves.
   (b) Compare the select and ultimate method with preliminary term.
   (c) How can the present value of expected mortality saving be easily obtained from select and ultimate tables?
   (d) What is the formula for a forborne temporary annuity? Name one or two important uses of forborne annuities.
2. Describe briefly the origin and peculiar features of the following mortality tables, and state for what purposes, if any, each has been or may be used:
   (a) Farr's healthy districts.
   (b) 30 American offices.
   (c) O^M.
   (d) McClintock's.

Part I. Fellow.

1. (a) What methods were adopted in the construction of the British offices life tables, 1893, for the elimination of duplicate lives, in the case of aggregate and select tables, respectively?
   (b) Among a body of lives selected five years ago, how could you determine theoretically how many damaged lives would be included among the survivors? Discuss whether this calculation could be used reliably for any practical purpose.
2. (a) Why is it necessary to graduate mortality tables?
   (b) What are the tests of a good graduation?
   (c) Outline briefly the methods adopted in the graduation of the O^M, $O^{M(5)}$, and $O^{[M]}$ tables.
3. (a) State four different methods of valuing for an annual report the marketable securities of an insurance company.
   (b) Which would you choose, and why? (Be explicit in showing why the method you choose is the proper one.)
4. (a) Describe the method which you consider the best for granting insurances on under-average lives, distinguishing risks that are under-average on account of family or personal history and on account of occupation.
   Mention any statistics with which you are acquainted which would be useful in rating substandard risks.

(b) Specify how you would treat the following cases in granting policies:
(1) Army officer, age 30; age 50.
(2) Bartender, age 30.
(3) Bookkeeper, age 30. Mother died at age 25 of consumption; father alive at age 65; no brothers or sisters; 20 per cent underweight.
(4) Application for large amount (company's limit) on capitalist, age 60; 25 per cent overweight. Mother died age 30, consumption; father died age 60, Bright's disease.

Part II.

1. (a) Explain fully how you would assess expenses for the purpose of distribution of surplus:
(1) As between new and renewal business.
(2) As between life and endowment policies.
(b) In distributing profits from mortality would you distinguish between various plans of assurance?
(c) On what basis would you allot to participating policy holders profits derived from nonparticipating insurance and annuity business?
In each case state your reasons fully.
2. (a) Describe clearly and briefly the economic functions of a stock exchange
(b) Describe the workings of a clearing house. Does a clearing house effect any important economic saving, or is it merely a convenience to the members and a means of saving clerk hire and wages of messengers?
(c) Give a satisfactory reason that a more or less serious crisis should occur about every 10 years.
3. State in tabular form and without discussion the most important advantages and disadvantages of each of the following classes of securities as an investment for life companies, supposing no legal limitations to investments. Indicate which you would reject entirely and which you consider doubtful:
(1) City mortgages.
(2) Suburban mortgages.
(3) Farm mortgages.
(4) Railroad stocks.
(5) Railroad bonds.
(6) City gas stocks.
(7) Telephone stocks and bonds.
(8) Street-railway stocks and bonds.
(9) Interurban electric-railroad bonds.
4. (a) State fully the provisions of the New York law as to limitation of expenses. Do these provisions apply in all respects to every life insurance company doing business in New York State? If not, to what cases and in what respects do they not apply? What amendments to this section of the New York law would you propose, and why?
(b) In what respects does the Wisconsin law differ from that of New York in regard to limitation of expenses?

INDEX.

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