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UNDERGRADUATE WORK IN MATHEMATICS IN COLLEGES OF LIBERAL ARTS AND UNIVERSITIES

INTERNATIONAL COMMISSION ON THE TEACHING OF MATHEMATICS

THE AMERICAN REPORT

COMMITTEE No. X

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THE AMERICAN REPORT.

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UNDERGRADUATE WORK IN MATHEMATICS IN COLLEGES OF LIBERAL ARTS AND UNIVERSITIES.

GENERAL REPORT.

The work of this committee has been done in large part by three subcommittees, as follows:

1. On men's colleges.
2. On women's colleges.
3. On coeducational colleges.

The facts presented in these reports have been gathered from 60 or more leading colleges by the Bureau of Education through the use of a questionnaire, and have been supplemented materially from the catalogues and by special correspondence carried on by the subcommittee on colleges for men, and by a special short list of questions circulated by the subcommittee on coeducational colleges. By these means over 100 colleges were included in our survey.

To these reports the committee as a whole desire to subjoin a brief commentary.

The practical uniformity found in the amount of mathematics required for admission to the colleges shows that in this particular we have nearly attained a condition of equilibrium; the secondary-school teacher is not hampered in his work by diversity of standards. Accordingly, most schools now fit students for many colleges, and very little direct criticism from the single college reaches or affects them. Some more effective and stimulative mode of interaction, as through associations, is yet to be developed. Standardization has been our aim, and we record the fact that, temporarily at least, the mathematics of the secondary school is standardized, and the prevalent minimum is two and one-half years—one year of plane geometry, one and one-half years of algebra. Some colleges have imposed and later have withdrawn a requirement of solid geometry. We may venture an inference that the coming decade will not see any increase in the total requirement.

Upon studies required of undergraduates there is at present nearly the same unanimity, an amount much greater than a century ago, but probably less than the average of 40 years ago. Very few students are allowed to graduate without a year's mathematics

in college, ordinarily at least solid geometry and trigonometry. To explain the requirement of analytic geometry and calculus, so frequent in small colleges prior to 1880, we need only to remember the paucity of courses that small colleges were able to offer, owing to their comparative poverty. Increasing resources have brought multiplication of elective studies, and so have permitted the lessening of the requirement in mathematics, as in other departments. Concerning mathematical studies offered as electives to undergraduates, from the nature of things uniformity is not to be expected; the predominance of two well-marked groups of subjects shows that college instructors are in substantial agreement upon the essentials for a student specializing in this department; beyond this, although it might be economically desirable, educational theory is not yet competent to draw a precise boundary between subjects of college grade and those of university grade.

One noteworthy thing is this, that branches of modern mathematics marking successive steps in the progress of this science still remain isolated as subjects of instruction, separate also from the older theories. Little blending has yet occurred or confluence of these tributaries into a homogeneous stream. Only within a decade is there found any indication of purposeful progress toward the combination and consolidation of accidentally severed lines of mathematical thinking. Probably not until Descartes, Newton, Lobacevsky, Galois, and Hamilton become dim names in the mists of antiquity will the characteristic thoughts of all pervade the whole of mathematical instruction. Yet toward such an end every college teacher of the present day might contribute his little. Habitual ways of thinking do influence habits in teaching; and the unremitting effort to recast and bring into relation the old and new in one's own ever-shifting horizon must produce, by slow integration, appreciable results in the doctrine or modes of thought that one imparts to students.

In the reports of two subcommittees some statistics are given of the number of men trained in universities and now teaching college mathematics. The very great increase in such teachers during two decades past indicates two things: A broader knowledge of the field of mathematics, and the acquisition of what has been called the research attitude, in the leading members of our profession in this country. These things make for stronger teaching, and it does not seem that they have been unduly emphasized as yet. But that we need at present to seek also intensification, to attach more importance to details, to increase where possible the content rather than the extent of our teaching, is believed by many. One evidence of this is the large output in recent years of mediocre textbooks. A deeper study of one's particular subject, with attention to vivid-

ness in presentation, should surely produce more radical departures from type, more extremes of specialized excellence. Further evidence is the scarcity of pedagogical papers in the programs of mathematical societies. Still, it is proper to repeat the positive conclusion, not to be obscured or belittled, that college teaching in the past 10 or 15 years has become rapidly permeated with more modern theories, has acquired the distinctive spirit of the natural sciences, and has tended to be conducted more and more in the research manner.

Finally, two other inferences may be gathered from the data and the reports of our subcommittees:

First. College departments of mathematics could be improved by closer internal organization and esprit de corps, by more frequent discussion over the details of elementary courses, and by systematic study of the specific results attained in each course; in short, by organic self-criticism.

Second. The secondary schools, and colleges no less, could achieve more permanent results if they insisted upon teachers having specific training of a pedagogic kind in those subjects in which large classes are to be expected.

SUBCOMMITTEE 1. MEN'S COLLEGES.

SOURCES OF INFORMATION.

The information on which the following report is based was obtained in part from the answers to the "Questionnaire on the Teaching of Mathematics in Universities, Colleges, and Technological Schools or Departments" sent out by the United States Commissioner of Education, Dr. Elmer Ellsworth Brown, and in part from the catalogues of the men's colleges considered. While replies to the questionnaire came from only 17 men's colleges, that number included institutions of so varied character as to make the list a fairly representative one. Catalogues were requested from 98 institutions; and the catalogues of 76 men's colleges and universities were received. These were carefully consulted for such appropriate data as they contained. Of these 76 institutions, all are of collegiate or university grade, to the extent of offering a 3 or 4 years' course of residential study leading to the degree of A. B., S. B., Litt. B., Ph. B., or other degree of similar rank which the institution confers, all admit men only to the undergraduate courses, and all are located in the United States. The list includes all of the larger and better-known men's colleges, but fails to comprise many of the smaller sectarian and, particularly, Roman Catholic institutions, since in these instances the requests for catalogues or other information brought few responses.

DEGREES HELD BY TEACHERS OF MATHEMATICS.

In an attempt to ascertain how much of graduate study had been done by the members of the mathematical staffs of these 76 institutions, as shown by the academic degrees in course held by them, it was found that 26 of the 76 institutions published in their catalogues either no degrees at all after the names of their teachers or a manifestly incomplete list. Thus, many Roman Catholic colleges, and such an institution as the United States Military Academy, had to be omitted. There were left 50 colleges and universities from the catalogues of which the following facts were obtained: The total number of teachers of mathematics with rank as high as instructor in these institutions was 192. Of these 192 men, 88 held the degree of doctor of philosophy, 69 had no higher degree than that of master, 29 had not gone further than the bachelor's degree, and 6 had no bachelor's degree but an engineering degree instead. Remembering, then, that the degree mentioned is in each instance the highest degree in course held by the man concerned, it was found that, of these 50 institutions, mathematics teachers with the doctor's degree were listed in 29 (58 per cent); with the master's degree in 40 (80 per cent); with a bachelor's degree in 21 (42 per cent); and with an engineering degree in 4 (8 per cent). Of these same 50 institutions, 29 (58 per cent), as above stated, had at least 1 mathematics teacher with a doctor's degree; 18 (36 per cent) had no teacher of mathematics with degree higher than the master's; and 3 (6 per cent) had no such teacher with degree higher than B. A. or B. S. Naturally the larger institutions, and particularly those offering graduate courses, have the larger proportion of teachers with high academic degrees. Thus Princeton has 13 men in its mathematical staff who have the doctor's degree, Yale (with the Sheffield Scientific School) has 11, Columbia 9, the University of Pennsylvania 8, Harvard 7, Johns Hopkins and Williams 4 each, and Brown 3. Of the 50 institutions considered, these 8 have 59 of the 88 teachers of mathematics who hold degrees as high as the doctor's.[1]

It appears to be true that the highest percentage of doctor's degrees is found among the teachers between 30 and 45 years of age. Many of the older teachers did their graduate studying in the days when the doctor's degree in course was very seldom conferred in American institutions; and many of the younger teachers have turned from their courses of graduate study to gain experience in teaching before proceeding to the point of the conferment of the coveted doctor's degree. Here and there one finds a mathematician of good standing who has chosen to scorn the prescribed course of study of which the goal is the doctoral thesis and examination, but such men are very

[1] These numbers will stand as follows for the year 1910-11: Princeton, 12; Yale, 8; Columbia, 13; the University of Pennsylvania, 10; Harvard, 10; Johns Hopkins, 4; Williams, 4; and Brown, 3.

few in the list of the American teachers of mathematics. As an asset for the young teacher who is aspiring to a position in an American college, the doctor's degree appears to have grown in value steadily. This growth will naturally continue if the American universities continue to offer, as leading to the doctor's degree, those wisely selected courses in the higher mathematics which tend to develop best the student's interest and powers.

VARIETY OF BACHELOR'S DEGREES.

While the degree of bachelor of arts is given in course for undergraduate work by practically every college and university considered, and has the advantageous position among bachelor's degrees which years and numbers secure, many of the institutions confer the degree of bachelor of science, and some—comparatively few—give in the same way the degrees of bachelor of letters and bachelor of philosophy. Mathematics is one of the subjects offered as contributing to the qualification for all of these bachelor's degrees and is required in almost every instance, at least to the extent of the usual freshman course, as a part of the work leading to the degrees.

DEPARTMENT OF MATHEMATICS AND RELATED DEPARTMENTS—ORGANIZATION.

In most institutions the department of mathematics is a separate department, and sometimes appears as two separate departments, mathematics and applied mathematics. The department with which it is combined, when combination occurs, as in the case of some small colleges, is most frequently either that of astronomy or that of physics. While the staff of instructors in mathematics makes up the department of mathematics, the members of the various departments constitute a faculty which has general control of those departments; this faculty is the only faculty in the institution in the case of most of the smaller colleges and is one of several in the universities and larger colleges, the faculties there being known as the faculty of arts and sciences, the faculty of engineering, etc. In every faculty giving undergraduate instruction there appears a department of mathematics of some form or other; and where the departments are rated as of varying prominence and of different grades, that of mathematics is regularly found among those of the largest prominence or highest grade; it is seldom, if ever, rated as of secondary importance as compared with the departments of other subjects of undergraduate study. In several instances it stands first in the catalogue arrangement of departments and in published lists of subjects in which instruction is given. It is fair to state that in the American colleges for men mathematics appears to be losing none of the importance which has been accorded to it since colleges were first established.

RELATION OF THE DEPARTMENT OF MATHEMATICS TO THE FACULTY.

Instances are rare where the department of mathematics is independent of the faculty, in which it has membership, to the extent of changing its policy widely or introducing important changes in the courses which it offers without submitting the proposed changes to the faculty for approval. While, however, marked departures from existing usages in the department of mathematics would seek approval in advance at the hands of the faculty, there is no evidence that the control of the department by the faculty is a dogmatic one or that it extends to matters of mere administrative detail. While the approval of the faculty is usually necessary for the offering of a new course or the extension in time or dropping of an old one, such approval is unnecessary in most colleges for the introduction of a new method of instruction in an old course or for alterations in the subject matter of a course. In a few institutions it appears that the department is wholly independent of the control of the faculty.

INTERNAL ORGANIZATION OF THE DEPARTMENT.

Where more than a single individual constitutes the staff of the department, somewhat of internal organization, either formal, or informal to the extent of being wholly undefined, exists. The senior member of the department staff is ordinarily its nominal head and takes the lead in determining the general arrangement of courses, the assignment of courses to their instructors, the method and the subject matter of the courses, the textbooks to be used, etc. In some cases the head of the department reports that he is "absolute," having full control of the department and its members; in others all teachers are "made to feel independent," while evidently subject to the authority of the head; in others "practically no serious influence or control is exercised by the senior teachers or by the head of the department over junior teachers." Probably the general usage is fairly characterized by the statement that the head of the department "may direct, but usually cooperates," or that he "controls by advice." The head of the department is, in one case at least, chosen by vote of the members of the staff of the department and is frequently not the senior member, but the unwritten rule which constitutes the senior professor the head of the department is the one most commonly followed.

The opinion and advice of the head of the department are generally sought and followed in the appointment of new instructors and assistants, as well as in the promotion of members of the staff, and this constitutes one of his chief sources of influence in his department.

RELATION OF ADMINISTRATIVE OFFICERS TO THE DEPARTMENT.

It is not customary for members of the department of mathematics to be instructed or directed at all by administrative officers in the con-

duct of their instruction as to either subject matter or methods. The decision of all questions of this sort is regularly left to the department, and it is very unusual for any occasion to arise for the enforcement of authority on the part of the head of the department; a conference of the staff of the department ordinarily leads to harmonious agreement on any question pertaining to the department. Yet it is reported from one college that administrative officers do direct the members of the department as to both subject matter and methods.

DISTRIBUTION OF TEACHING.

In the distribution of teaching it is customary for every qualified member of the mathematical staff to have part from time to time in both the elementary and the advanced courses. The junior teachers give the larger part of the elementary instruction and the senior teachers give the larger part of the advanced instruction; but no member of the department whose training fits him for some of the more advanced courses is required to confine himself to freshman or sophomore work permanently. The reasons for this general distribution of teaching are that, for the welfare of the student, the more experienced teachers should share in the elementary work and that, for the sake of the pleasure and the progress of the junior teacher himself, some of the advanced work should fall at least occasionally to his lot. Still, some institutions report that "no principles are followed in the distribution of instruction; in others "as a rule, senior teachers take the advanced subjects," and in still others the "rule of seniority" is "well observed," implying undoubtedly either a first choice on the part of the senior teacher or else the allotment to him of the advanced courses. Experience seems to have established that the best welfare of all concerned demands that there be frequent interchange of courses between teachers and that the junior teachers have their share of the advanced courses.

DEPARTMENT MEETINGS.

When the number of those teaching mathematics in the institution is large enough to make the holding of meetings feasible, it is customary for such meetings to be held. Sometimes these are regular and stated meetings, but more often they are informal and appointed only when circumstances seem to demand. While scientific matters are likely to be discussed on all such occasions, the conferences are usually called for the discussion of current teaching, the adoption of methods, the selection of textbooks, etc. The formal presentation of scientific papers is usually reserved for the meetings of some larger scientific organization within the college, such as the science association, where all the departments of science unite, the mathematical-physical society, where the physics teachers join the mathematics teachers, or some other similarly made-up organization.

SEPARATION OF ENGINEERING AND NON-ENGINEERING STUDENTS.

Collegiate and technological students are ordinarily taught in the same classes in the smaller institutions. While this arrangement seems the result of a desire for economy in some instances, it is explained in one case as justified by the fact that "a class for either class of students should include both the practical and theoretical." In one of the large universities "undergraduates intending to study engineering may take their elementary mathematical courses in the mathematical department or the engineering department at pleasure, and so may students not intending to become engineers. For would-be engineering students, there is supposed to be a saving of time in taking the latter courses rather than the former, and the result is that a large proportion of them choose accordingly." In most institutions where the equipment permits the offering of separate courses for engineering and non-engineering students, the two classes are assigned to their work in mathematics accordingly.

MECHANICS, DESCRIPTIVE GEOMETRY, ETC.

Among the subjects taught by the mathematical department, mechanics and descriptive geometry are often included, to the extent of at least one course in each, while in many instances surveying and mechanical drawing also are found. In one small college the department of mathematics gives three semesters of work in descriptive geometry and a semester course in mechanical drawing; in another, the professor of surveying and mechanical drawing is not a member of the department of mathematics; and in numerous small colleges no courses at all in mechanics or descriptive geometry are given.

METHOD OF APPOINTMENT OF TEACHERS OF MATHEMATICS.

It is the well-nigh invariable usage for vacancies in the mathematical staff to be filled by the vote of the governing corporation, generally known as the board of trustees, corporation, or board of overseers. The official nomination is brought to the board by the president and comes to him in some cases on his own initiative, but more often at the suggestion of the head of the department, or sometimes by the formal ballot of the department. An appointment is seldom made without a personal conference between the candidate and the president or some member of the department or both. Ordinarily, as the college grows, the president depends more and more on the judgment of the members of the mathematical staff for the selection of new teachers of mathematics.

CHANGES IN RECENT YEARS.

Among the changes that have come during the past few years the most marked is the offering of more work in the applications of mathematics. New courses in mechanics, descriptive and projective geometry, surveying, and mechanical drawing, have been

introduced in many colleges. To give these courses, more teachers have been needed and the mathematical staff has accordingly grown appreciably in size. Furthermore, the growth of nearly every one of the American colleges in the number of students in attendance has been marked during the past decade and has resulted in a large increase in the number of teachers of mathematics, in addition to the increase due to the offering of new courses in applied mathematics.

The reports of the United States Commissioner of Education show that the 76 men's colleges here considered had an attendance of 16,672 students in 1898 and 25,412 in 1908. This shows an increase of 52 per cent for the chosen period of 10 years. The 76 institutions contained an average of 222 students each in 1898 and 338 students each in 1908. Furthermore, only 12 of the 76 institutions show a decrease in students during that decade, and the decrease is so slight as to be disregarded in all but three or four cases.

CHANGES STILL TO BE DESIRED.

No well-defined notion of the changes still to be desired in the directions thus far discussed exists in the minds of the average mathematical staff. The majority of those formulating any ideal express it as involving still further enlargement of the teaching staff to enable more courses to be given, particularly in applied mathematics, and to make possible the giving of all mathematical instruction to divisions of small size. One professor suggests 15 men as the maximum that should ever be taught mathematics in a single division, since no larger number can receive the requisite amount of individual attention in the classroom for the securing of the best results.

The size of many of the departments of mathematics appears to warrant the holding of more frequent department meetings. The esprit de corps of the department and the productivity of its members in lines of research will both gain much advantage from the holding of regularly appointed meetings for the discussion of general mathematical questions. The help which the young instructor obtains from a presentation to his associates of the results of his own investigations is very great; and every instructor will be much surer of keeping his mathematical interests alive if he knows that an opportunity will be afforded him every month to give an account of his progress to the other members of his department. It seems desirable, therefore, that every department of mathematics which has as many as 3 members hold regular conferences for the encouragement of mathematical research.

ADMISSION REQUIREMENTS IN MATHEMATICS.

The catalogues of the 76 institutions considered show that 37 varieties of admission requirements in mathematics are made at those 76 colleges and universities. Accepting as the norm of admission requirements algebra through quadratics (including ratio and

proportion, progressions, and the binomial theorem with positive integral exponents) and the usual textbook amount of plane geometry, it was found that only 8 of the 76 institutions hold precisely that norm for their admission requirement. Nine institutions require this norm with the omission from the algebra of ratio and proportion, progressions, and the binomial theorem; 5 institutions require the norm thus restricted in the algebra, but add the usual textbook amount of solid geometry; and 10 institutions demand the full norm with solid geometry added. The lightest admission requirement made consists of algebra to (but not including) quadratics, with no geometry; this is held by two institutions. The heaviest minimum admission requirement made consists of the norm with solid geometry, plane trigonometry, and advanced algebra (including Horner's method and determinants) added; this requirement was found in only a single instance as the lowest which the institution holds. The remaining 31 varieties of admission requirements range between the two extreme lists just given and are held in every instance by a single institution only. It is true that the differences are in many cases very slight and may sometimes amount actually to nothing more than a variety in the form of statement. That so great diversity of statement of requirements should exist seems to many instructors unfortunate, and a movement has recently made rapid progress in New England which has for its goal the securing of uniformity of definition of admission requirements in mathematics in all the New England colleges and universities, the form of definition adopted being that published (in December, 1909) by the College Entrance Examination Board.

The investigation of the admission requirements shows further that advanced algebra (algebra beyond quadratics) is made a requirement for admission in 8 institutions, a requirement for admission to the scientific course in 2, and an optional admission subject in 2. Solid geometry, similarly, is required by 32 institutions, while it is necessary for admission to the scientific course in 6 other institutions, and is an optional admission requirement in 6. Finally, plane trigonometry is required by 17 institutions, while 4 additional make it necessary for admission to the scientific course and 7 make it an optional admission subject.

QUALITY OF PREPARATION IN MATHEMATICS.

The men's colleges and universities maintain in general that the preparation in mathematics shown by their freshmen is very unsatisfactory. While an occasional individual professor feels that it is "fair" and that it "has improved within the last 10 years," the great consensus is to the effect that the students are "lamentably weak" in algebra, and are "in general but poorly prepared to do college work in mathematics." One of the more kindly criticisms

is as follows: "The failures in the prescribed algebra are due chiefly to ignorance of the ordinary processes of algebra, and to lack of skill in dealing with ordinary algebraic expressions. In geometry, the difficulty is not that candidates can not solve hard originals, but that they can not solve easy ones, and that they do not know the book work." Again, "In algebra many students not only lack facility in algebraic transformations, but seem not to have an intelligent grasp of the principles underlying such operations." There appears to be general agreement that the student lacks "power in the manipulation of radical forms and fractional and negative exponents" and that he is not able, as he may fairly be expected to be, to solve the quadratic equation. Many are wondering whether the preparatory-school teacher, in a desire to find entertainment for the student, is not employing too much of his time in useless problems and far too little in thorough drill in the fundamental operations of algebra. Plainly, it can hardly be too much to ask of the preparatory-school graduate that he be able to solve a quadratic equation twice out of three trials; yet few can do so well as that.

No general change in the scope of the admission requirements in mathematics has taken place within 10 years. Some colleges have increased their requirement by the addition of solid geometry and few indicate as desirable in the near future any change except that solid geometry should be so added to the present requirements.

UNDERGRADUATE REQUIRED COURSES IN MATHEMATICS.

It is difficult to find any combination of subjects that can serve as a norm for the requirement in mathematics made of undergraduates in the 76 institutions or for the elective courses offered in that subject. The only case of entire agreement as to the requirement in mathematics in college is that found in the cases of two institutions that make no requirement whatever in that subject. The remaining 74 institutions announce in their catalogues 74 different requirements, different in the subjects required or, if the subjects are nominally the same, materially different as to the ground covered in them. One institution prescribes no mathematics and offers no elective work in that subject in its B. A. course. In another institution, the entire course in mathematics is of so elementary a character as to bring the student to trigonometry as the most advanced subject of senior year. A third college includes no other mathematics in its curriculum than a two-hour course for one year in plane trigonometry. If, for a measure of the time devoted to a course, year-hour be defined to mean the equivalent of an hour a week for a year, a fourth institution requires a total of 13½ year-hours of mathematics in its B. S. course and a fifth institution prescribes as a requirement of the B. A. course the following amounts of collegiate courses in mathematics: plane trigonometry, 2 year-hours, algebra 2, plane analytic geometry 3,

calculus 5, spherical trigonometry 2, and solid analytic geometry 1, making a total of 15 year-hours. This amount comprises about one-fourth of the total requirement for the degree and is the largest requirement in mathematics found among all the institutions considered. Plane trigonometry is required more generally than any other mathematical subject; it is prescribed in the curricula or portions of the curricula of 67 institutions and the amount of time devoted to it varies in the different colleges from one-half to 3 year-hours, the various amounts of time in year-hours being ½, 1, 1¼, 1⅓, 1½, 1⅔, 2, 2½, 3. Advanced algebra is a required subject in 54 institutions, with the amount of time in year-hours devoted to it ranging thus, 1, 1¼, 1⅓, 1½, 1⅔, 2, 2¼, 2½, 3. Solid geometry is a prescribed course in 48 institutions with its amounts of time the same as those given above for plane trigonometry.

NORM OF COLLEGIATE REQUIREMENT.

The most commonly found combination of college subjects in mathematics is that of advanced algebra, solid geometry, and plane trigonometry. Exactly half of the 76 institutions prescribe this combination and the amounts of time in year-hours devoted to it are 3 (including spherical trigonometry), 3, 3⅔, 4, 4⅙, 4⅓, 5, 5½, 6, 8 (including spherical trigonometry). If a norm of college requirements in mathematics is to be selected, it must undoubtedly comprise these three subjects and these alone.

The following table shows, for the 76 institutions, the number that require the mathematical subjects indicated, in the curriculum leading to the degree of B. A. and perhaps other degrees also, or in the curriculum leading to some other degree than B. A. while not requiring them for the B. A.

Subject.	Required for B. A., etc.	Required for other degrees, but not for B. A.
	Institutions.	*Institutions.*
Plane analytic geometry	41	16
Spherical trigonometry	37	6
Calculus	19	18
Surveying	9	1
Mechanics	7	8
Solid analytic geometry	7	3
Differential equations	2	2
Navigation	1	.....
Theory of equations	1	2
Descriptive geometry	.....	3
Determinants	.....	2
Projective geometry	.....	1

As a requirement in college, plane analytic geometry takes precedence over spherical trigonometry, while either is more commonly prescribed than calculus. These three subjects form a combination standing as a distant second to the group, advanced algebra, solid geometry, and plane trigonometry, in the frequency of its use as a college requirement. Another combination of three subjects which stand remotely next in the order of their requirement is made up of surveying, mechanics, and solid analytic geometry. There is no general recognition whatever of any other mathematical subjects as suitable ones for a requirement in undergraduate study.

UNDERGRADUATE ELECTIVE COURSES IN MATHEMATICS.

In the matter of elective courses, there is general agreement on the calculus, and elective courses in it are offered by 53 of the 76 institutions. Next in order, as subjects of elective study, stand plane analytic geometry, offered in 36 institutions (and required in others), differential equations, offered in 26, mechanics in 23, and solid analytic geometry in 22. For the rest of the elective courses, 15 institutions offer theory of equations, 14 descriptive geometry, 14 surveying, 13 determinants, 8 algebra, 8 spherical trigonometry, 8 projective geometry, 7 mechanical drawing, 5 plane trigonometry, 5 modern analytic geometry, 5 history of mathematics, 3 theory of functions, 3 quaternions, 2 modern higher algebra, 2 solid geometry, 2 vector analysis; and single institutions offer courses in many other subjects. Altogether the 76 institutions offer 123 varieties of elective courses.

INFLUENCE OF OTHER DEPARTMENTS.

While some attempt is made in most mathematical departments to adjust the work to the needs of other departments, particularly the departments of physics and astronomy, it is not frequently found necessary to alter any mathematical course in any marked way for this purpose. The elementary training which is found to be best for the student of pure mathematics is regarded by most teachers as best for the student of applied mathematics also. Some attempt is made in most courses, and particularly in the calculus, to find such problems as have immediate application in physics and astronomy, if such problems suit the mathematical needs of the case; and in this way the interests of the kindred departments are served. But, while indirectly contributing thus to the teaching of other sciences, it is the belief of most teachers of mathematics that astronomy and physics, as such, should be taught by the professors of those sciences rather than made a part of the courses in mathematics. The applications used in the mathematical courses have the emphasis placed on the mathematics of the problem rather than its physics or its astronomy;

and an application sought too far from the field of mathematics may fail of its purpose as an illustration of mathematical principles.

Some correlation of courses in mathematics and engineering is attempted in many institutions where both are taught, but this attempt is not carried so far as materially to alter the ground covered. There is no ground for believing that the mathematical curriculum is restricted by the congestion of subject matter in the engineering courses. In a few institutions the departments of applied science prefer to give the instruction in mathematics which their students need. In the majority of cases the mathematical courses, as far as to include the calculus and differential equations, are taken by the technological students.

RECENT CHANGES.

The changes of the past 10 years have involved the substitution of the elective for the required system in mathematics in some institutions, thereby lessening the number of students in the mathematics courses; the introduction of more courses in the application of mathematics, surveying, mechanical drawing, etc.; and the acceptance of a wider range of admission subjects, with the consequent adjustment of the college courses to a greater variety of preparation in mathematics.

AIMS OF TEACHING.

The aims in the teaching of mathematics are stated by the teachers as comprising, first, the development of analytical power and logical keenness, and, secondly, the advancement in mathematical dexterity and accuracy in computation. The making of expert mathematicians is avowedly regarded as subordinate, in some of the colleges, to the development of mathematical power as a means of culture. Complaint is offered in some quarters on the ground that the average student who does not find mathematics an attractive subject obtains little advantage of any sort in the required study of it. Some teachers aim at the development of different powers in the study of different mathematical questions, laying emphasis on accuracy in computation in logarithms, on mathematical dexterity in analytical trigonometry and parts of the calculus, on analytical power in analytical geometry and mechanics, etc. It is interesting to find that many teachers of mathematics attach great importance to the cultural value of the study of their subject. One professor writes thus: "Students who go far enough in the study of mathematics and who give it their interest as well as their time gain immensely in resourcefulness, and I am inclined to believe that the gain in general culture (especially if the teaching is done somewhat with this aim) is far more than is generally supposed." One notes that the study of mathematics, with emphasis on its analytical reasoning, influences the whole habit of mind of the student and affects his entire education.

CLASSROOM METHODS.

In one of the large universities which have reported their methods, it is found that "oral recitations and blackboard work for the student are little used. Lectures with problem work and written tests are used in most of the courses." It is perhaps because it is difficult in very large classes to apply methods of oral recitation and of much solving of problems on the blackboard in the classroom by the student, that the large institutions have so little of that work In the colleges there is ordinarily no formal giving of lectures in any of the lower courses, but much time is devoted to the quizzing of the students and to the solving of problems by the students at the blackboard. In some of the leading colleges a portion of the exercise is ordinarily devoted to a presentation of the subject of the next lesson, with much questioning of the students after such a fashion as to convince the student that he is assisting at the rediscovery of mathematical truths; in the more advanced courses this method develops into a form of lecturing, which is very informal, but suffices even in courses where no textbook is used. This encouragement of the attitude of discovery on the part of the student is one of the most hopeful signs of the present time in the mathematical world. In most of the institutions there appears to be very little use made any longer of formal reciting of rules committed to memory from the textbook; the recitation is regularly in the words of the students.

In very recent years a strong movement has made itself felt toward lessening the size of divisions in mathematics classes. In some institutions as many as 12 divisions are made of a class of 180 freshmen for their work in mathematics, with the consequent possibility of having the entire division at the blackboards at once. The enthusiasm of the teachers as to the advantages of this arrangement of very small divisions in all work in mathematics, in the institutions where it has been tried, makes one feel that it is a misfortune, if not a great mistake, to have to teach mathematics to divisions numbering more than 20. While the possibility of much informal conference between teacher and pupil is regarded as most desirable, few institutions make any use of so-called seminar methods of teaching mathematics to undergraduates; such methods are in use in the teaching of graduates. With the lessening of the requirement of the formal recital of rules from memory, there has come a more widely prevalent custom of the assignment of problems to be handed in by the student as evidence of the work done outside the classroom, and of the imposition of frequent written tests in the classroom.

PROGRAMS.

Definite programs of elementary courses are prepared in advance in somewhat more than half the institutions from which reports have come. In perhaps half of the institutions where such programs

are prepared, duplicate copies are placed in the hands of the students; in the other institutions, such programs are for the use of teachers only. Where no programs are prepared, textbooks are regularly used as the basis of the classroom work.

DETERMINATION OF FINAL GRADES.

The tendency of recent years has been to attach more and more weight to the term work, as compared with the final examination, in determining the final grade. While some of the institutions considered attach equal weight to the term work and to the final test, none gives greater weight to the latter than to the former, and the great majority of institutions let the term work count for considerably more than half in determining the final standing. In some institutions the apportioning of weights differs greatly in the various mathematical courses according to the subject studied and the methods used. The term work regularly includes, as the basis for determining its mark, not only the oral recitations, but also the blackboard work, the problems solved outside the classroom, and the occasional tests.

INDEPENDENCE IN PROBLEM WORK.

In some institutions very great care is taken to encourage independence in the solving of problems; this is carried to the extent of assigning to each student a different set of problems for outside work and of criticising with much thoroughness and returning all written work, all this being done by the instructor in charge of the class, without the help of any assistant. The honor system is sometimes made to include the solving of problems outside the classroom, so that the student works under a pledge of honesty. Rigorous disciplinary measures are used in a few colleges to secure independent work, while in others the statement of the student as to the number of the assigned problems which he has solved is accepted without further verification than that afforded by the recitation exercise.

The work of examining the problems submitted in writing by the student falls in some instances to an assistant, sometimes himself an undergraduate, but it is still the general rule that the man in charge of the course attends to all this work. Although in nearly all the small institutions problems are assigned for daily work outside the classroom and are carefully examined by the instructor, yet some of the instructors in the larger institutions assign no problems whatever for outside work, appear to attach no importance to independence in problem work, and take no steps to secure it.

PERSONAL CONFERENCES WITH STUDENTS.

The attitude of passive consent on the part of the instructor to a personal conference, when one is sought by the student, has been

changed of late years to a position of definitely requiring conferences, either with each student individually, or with such students as choose to come in response to a general invitation. Some teachers are regularly available in the classroom at suitable hours each week for "consultation" and find those hours in great demand on the part of their pupils; other teachers make the same attempt but meet with little response from their classes. That an instructor's time should be held so largely at the disposal of any earnest student marks somewhat of change in recent years. It is generally believed that this investment of an instructor's time and energy is a productive one. Its chief danger lies in the fact that, in the intimacy of the personal conference, a kind-hearted instructor often finds it difficult to refrain from doing the lazy student's work for him.

AMOUNT OF TEACHING.

The average number of hours of classroom instruction given by the individual teacher is 11; the largest found in the reports is 20, and the smallest regular number, in some of the larger institutions, is 9. Members of the mathematical staff are ordinarily given a generous share in administrative work and the duties of the average professor of mathematics are much increased from that source. Few, if any, other subjects are so largely represented in the administrative offices of the colleges and universities.

SIZE OF CLASSES.

The sizes of divisions in the classes in mathematics range from 60 to 20 (with 30 for the average) in the elementary courses and from 20 to 3 or 4 in the advanced courses. In general, the larger divisions are found in the larger institutions.

USE OF MODELS.

Models are used to some extent in nearly all institutions, while in a very few they are made by the students. Some professors of mathematics object to involved or intricately fashioned models, feeling that thereby the student has his attention drawn away from the mathematical point at issue. Although few institutions possessed any models at all 20 years ago, it is encouraging to discover that excellent collections of them are now found in many of the smaller colleges.

LABORATORY METHODS.

Courses in mechanical drawing are now given in many colleges and universities, and it is in these courses only that anything of the nature of laboratory work is reported as required in mathematics.

Tables of formulas are allowed to the student in the classroom in many institutions. The more simple and frequently used formulas

only are learned, and the student is encouraged to devote his attention to becoming expert in applying the more difficult formulas in the tables without memorizing them.

PRESENT TENDENCIES.

The increase of conference methods, the elimination of mere memorizing, the attempt to make problems and illustrations more than mere puzzles, and to show the widest application of the mathematics taught—these are the lines of change of the past 10 years and of the present. Less praiseworthy changes mentioned are the use of such methods of presentation of the mathematical subjects as to secure less concentration of mind and less of patient labor than formerly—a criticism evidently intended to apply to the so-called lecture method, which, fortunately, is not in general use. A tendency to make the connection between different mathematical subjects plain and close, amounting in some instances to the teaching of higher algebra, analytical geometry, and calculus in a single course with a single textbook prepared for the purpose is one of the noteworthy movements of the present time in mathematics. Another movement of perhaps still greater promise is that which aims at the development in the student s mind of the attitude of the explorer, through teaching him to discover mathematical facts for himself, rather than to learn them from some textbook.

PREPARATION OF TEACHERS.

There is general agreement that a good undergraduate course in arts, followed by a graduate course in mathematics leading to the degree of Ph. D., provides the best preparatory study for the work of teaching mathematics. Much importance is attached to experience in teaching, with experience in college usually preferred to that of the preparatory school. Some colleges refuse to appoint an inexperienced teacher, and few institutions will fill any, save the very lowest places, with those who have never taught. Many believe that the course of graduate study should not be completed until there has been at least a year devoted to teaching, since "the successful teacher must know two things, his subject and his pupil. He should not try to learn all of the one before he begins the earnest study of the other." Although great weight is given to scholarship in the filling of most professorships, one professor writes: "I would not belittle research work, but the most of our teachers should be devoted to teaching and not to research primarily. A teacher should have broad training, an invigorating personality, and broad sympathies. I believe great devotion, save in rare instances, to research makes one narrow and selfish." Fortunately, it is generally believed that devotion to research keeps the enthusiasm of

the teacher alive and insures a certain vitality in his instruction which is of the greatest value to his students. The teacher who years ago ceased to study is not likely to kindle in others a wholesome enthusiasm for intellectual work. A subject becomes dull when presented by a man who is not participating in its active development but only marking time. While actual engineering experience is regarded desirable, it is declared by the majority not essential; only a single professor, himself an engineer, states that it is absolutely necessary to the best training for the mathematics teacher's life. The report of the head of a department which contains no men with doctor's degrees implies that a single year of graduate study affords ample opportunity for preparation. One worthy professor maintains that the chief qualities necessary to success in the teaching of mathematics are "good sense and good citizenship." A single professor speaks of graduate study abroad as though it contained any special merit; would not many have thought a score of years ago that it must be "abroad" to be graduate study at all?

The establishment in this country, under the leadership of the Johns Hopkins University, of several excellent graduate schools is one of the most helpful movements of recent years for the work of our colleges in mathematics. Not only are the courses which these institutions offer comparable in their scope and in the ability of the men who give them with the best to be found in other lands, but the teacher of mathematics who has done his graduate work in an American institution has the great advantage of being able to maintain a close relation with his graduate teachers throughout his life. This intimate connection with a graduate school is of very great help to the teacher.

Opinion is divided regarding the wisdom of a special attempt on the part of technological schools and departments to prepare students for mathematical teaching and to encourage those technological students who show exceptional ability to specialize in mathematics. Most heads of departments feel that the course in arts affords the best preparation for the mathematics teacher; but the dearth of good candidates for the many vacancies in mathematics which occur each year offers a strong argument for the encouragement of the technological students in preparation for teaching.

COURSES IN PEDAGOGY.

There is a universal feeling that courses in the pedagogy of mathematics are of very small advantage to the future college teacher. Those professors who are willing to see such courses introduced specify either th they should be in addition to all the present courses in mathematics and not a substitute for any purely mathematical subject, or that they should be introduced only to satisfy the imperative demand of the schools.

A wiser view seems to be that which argues that the college teacher can well afford to spend some time in learning the best methods of teaching, whatever his subject; and that a course in the pedagogy of mathematics will be of very great value to the teacher of mathematics when the graduate school or university shall have developed a course suited to his needs. There appears to be sufficient material at hand, and it is strange that no American institution has solved the problem of a course in mathematical pedagogy in such a manner as to appeal to the professor of mathematics.

CHANGE OF PERSONNEL.

The rate of change in the mathematical staff of a college or university appears not to be large. In a fairly representative list of 12 institutions which have 52 men in their departments of mathematics, only 101 different men have been in service during the past 10 years. One college which ordinarily has a staff of 3 men has had 10 different men during that period, while a large university with a staff of 11 men has had only 15 different men since 1899. The rate of change appears to be considerably larger in the institutions with low salaries than in those where the salaries are more nearly adequate to a man's need.

PERMANENCY IN POSITIONS OF LOW RANK.

The great majority of the men answering the questionnaire believe that there should be no permanent or protracted retention of teachers below the rank of assistant professor. No man should be continued in college teaching who fails to prove himself well fitted for his task. Surely a man who is well fitted should be given no less in rank or salary than the full professor now receives.

SUBCOMMITTEE 2. WOMEN'S COLLEGES.

INTRODUCTORY.

The following report is based upon replies to a set of 45 questions sent to the following colleges for women: Bryn Mawr College, Bryn Mawr, Pa.; Elmira College, Elmira, N. Y.; Goucher College, Baltimore, Md.; Radcliffe College, Cambridge, Mass.; Rockford College, Rockford, Ill.; Smith College, Northampton, Mass.; Vassar College, Poughkeepsie, N. Y.; Wellesley College, Wellesley, Mass.; Wells College, Aurora, N. Y.

ORGANIZATION.

1. *Mathematical staff.*—The number of teachers engaged in mathematical instruction in each college varies from 1 to 7. In general there is one professor in charge of the department, with associate professors, instructors, and assistants. In colleges not affiliated with

universities the professors are women (with two exceptions), and all other instructors (with one exception) are women.

2. *Method of appointment.*—Members of the department are appointed by the trustees on the recommendation of the president, and, in some cases, of the head of the department.

3. *Restrictions on work.*—A general supervision of courses is exercised by the governing board of the college, which in some cases limits the amount of work that can be required of students, passes upon new courses proposed by the department, makes regulations as to examinations, etc.

4. *Responsibility of the head of the department.*—In most cases the professor in charge of the department is responsible to the college authorities for the conduct of the work of the entire department, but confers with the other members on questions of method, textbooks, etc.

5. *Division of work.*—The fact that a one-year course in mathematics is required in almost every college of this group necessitates a far larger proportion of students in elementary courses than in advanced courses. The elective work is generally carried by the senior teachers, though in some colleges an effort is made to divide the required and elective work evenly among all the teachers.

6. *Department meetings.*—Department meetings seem to be employed mainly for discussion of work.

7. *Changes in organization.*—Practically no changes in organization have been made lately or are now contemplated.

CURRICULUM.

1. *Requirements for admission.*—The usual requirement for admission to these colleges is two and one-half or three units, including elementary algebra through quadratics, ratio, proportion, the progressions; and plane geometry with much work in the solution of original exercises. These requirements have been practically unchanged for many years.

2. *Age at entrance.*—The age of the students at entrance is 16 to 18½ years.

3. *Defects in preparation.*—The chief defects noted are lack of independent thinking and of accuracy, and rustiness in algebra, which seems to be less satisfactorily taught in preparatory schools than geometry.

4. *College courses.*—Every college offers work in solid geometry, advanced algebra, trigonometry, analytical geometry, and calculus. Other subjects offered in all but some of the smaller colleges are projective geometry, modern geometry, curve tracing, differential equations, advanced calculus. Some also offer courses in quaternions, theory of functions, modern algebra, modern geometry, rational

mechanics, celestial mechanics, history of mathematics; two colleges report some work in mathematical drawing.

5. *Textbooks.*—The textbooks mentioned as in use are the same that are found in similar lists prepared for men's colleges. Textbooks in mathematics for women are unknown in the United States.

6. *Applications of mathematics.*—In only a few colleges is an effort made to meet the needs of other departments dependent on mathematical preparation. There seems to be little systematic effort made to apply the principles of pure mathematics to physics, etc., though applications made in ordinary textbooks are studied. Three colleges report work in applied mathematics.

7. *Changes in curriculum.*—The standard courses have changed little; new courses are introduced as opportunity is offered.

AIMS AND METHODS OF TEACHING.

1. *Place of emphasis.*—Emphasis is laid upon independence of thought, self-reliance, accuracy, the cultivation of the reasoning powers, insight or discernment; the stage of advancement of the student and the individuality of the teacher determining which shall be foremost in any given course.

2. *Conduct of classes.*—Wide divergence of method is reported; some emphasize oral recitations and blackboard work, some lectures. The former seems to be oftenest used for immature students, the latter for advanced students, while with graduate students the seminar method is employed. Work submitted by the students, often at each appointment, is returned to them after being criticized by the instructor. The importance of independence in such work is generally strongly emphasized.

3. *Tests.*—Tests are made by frequent written work and by final written examinations, and the final record of the students' work is determined both by these tests and by the daily work.

4. *Conferences.*—Conferences with students are not arranged according to any regular system, but instructors hold office hours or arrange for interviews as needed for advice or assistance.

5. *Amount of work.*—The amount of work carried by each teacher varies in different institutions from 9 to 18 hours each week.

6. *Size of classes.*—So far as reports are given, the number of students in a single division of required work varies from 20 to 33. In advanced classes the variation is still greater, but the numbers are in general smaller.

7. *Use of models.*—Half the total number of colleges report that rough models are made by the students. Half report regular use of a collection of models in all geometrical work.

PREPARATION OF TEACHERS.

Those colleges which replied to this question report unanimously on the requirement of graduate study. Some emphasize the necessity for successful experience in secondary schools.

In general little divergence is found between the men's colleges and the women's colleges as regards the above points, and particularly requirements for admission and general content of courses. The most marked difference is probably in short courses planned for engineering and scientific students. In the women's colleges the only reason for electing work in mathematics is a genuine interest in the subject; it is rarely studied as a means to an end.

SUBCOMMITTEE 3. COEDUCATIONAL COLLEGES.

This report on the teaching of mathematics in coeducational colleges and universities is based on an examination of the catalogues of a large number of such schools and on the replies to questions sent out to the heads of mathematical departments in institutions of various ranks. An effort has been made to secure information from institutions which are widely representative, and to this end inquiries were sent to the larger State universities, all of which are coeducational, several large institutions on independent foundations, and many smaller colleges. Two sets of questions were submitted: First, a general list intended to cover the facts of the preparation and experience of teachers, the organization of departments, the plan of the courses of study, and the methods of instruction; and second, a list designed to elicit information on the special topic of coeducation.

To the first series of questions replies were received from 32 institutions, of which 5 were State universities, while the remaining 27 included colleges and universities of all ranks and widely distributed. The information obtained may thus be said to fairly represent conditions of mathematical instruction in the class of institutions covered by this report. From the data at hand it would appear that the general conditions of mathematical instruction in coeducational colleges and universities do not differ materially from those in institutions for men alone. The staff ordinarily consists of a professor with supervisory powers and responsibilities, appointed for an indefinite period, and associated with him one or more of the same rank but with less responsibility. Next are associate and assistant professors appointed for a period of three or five years; then follow instructors on annual appointments and with little responsibility other than the immediate work of teaching. With minor variations

this is the organization of the department staff in all American colleges. In some instances, however, the staff selects its own administrative head, but such instances are rare. Appointments are made by the governing board of the university or college on the nomination of the president after consultation with the head of the department.

Of the preparation of the teaching staff it may be noted that in the 32 institutions reporting the aggregate staff was 97, and of the whole number, 37, or slightly more than one-third, had attained the doctor's degree. The distribution of men of the best preparation is quite uneven, some being found in small colleges with few students, but the largest percentage of those with doctor's degrees, as might be expected, are in the largest and most aggressive institutions. There is wide difference of opinion among those reporting as to the importance of pedagogical training for the young instructor, the larger institutions apparently putting the emphasis on scholarly attainment and the smaller colleges in general attaching importance to teaching ability and skill in classroom manipulation. All agree, however, that a year or two of experience as teacher in a secondary school is a valuable asset in the college or university instructor, though perhaps not in the minds of all the most valuable.

The method of instruction employed varies with the topic and with the advancement of the pupil. For elementary courses classroom exercises and recitations prevail. This method is ordinarily carried through the introductory courses in analytical geometry and the calculus, but for more advanced courses the usual procedure is for the instructor to lecture on discrete topics, accompanying his lectures by quizzes and written exercises. In elementary courses the classes are usually limited in number to 25 or 30 persons, and where more than this number are taking the same subject duplicate sections are formed. The seminary method is used in most advanced courses, and especially in graduate courses.

For admission to college classes the mathematical requirement is almost uniformly a knowledge of plane geometry and of algebra so far as to include quadratic equations. This knowledge may be acquired in a secondary school in two and one-half years, one and one-half years being given to algebra and one year to geometry. A few coeducational colleges require also solid geometry for admission. After the student is admitted to the college or university there is more divergence in the requirement. About two-thirds of the institutions reporting require mathematical study of all students during their first year, the remaining one-third permit a substitution of physics, astronomy, or some other science. Of the subjects required, plane trigonometry is most common; in fact, is almost universal. With it appear solid geometry when not required for admission, a

more advanced course in algebra, and, in a few instances, an elementary course in analytical geometry. There is no instance in a coeducational school of the calculus being required of all students.

The special inquiries on the relative attitude of men and women toward mathematical study show that in the 23 institutions reporting on this subject, there was an average of 58 per cent of men and 42 per cent of women in the courses required of all students, and in elective courses there were 62 per cent of men and 38 per cent of women. Though these percentages of required and elective courses show no marked difference, they seem to indicate that while a fair proportion of women left to their own choice will select mathematical studies in preference to others, the proportion is not so great as among men.

The selection of courses in any science is a fair index to the tastes of students. Even though it may be said that the courses are followed for utilitarian purposes, for example, for engineering on the one hand or for teaching on the other, still the native taste of the student, whether for this science or that, influences the choice of occupation. Except for a fondness for mathematical reasoning, the young man will seldom choose the profession of astronomer or engineer in preference to that of historian or linguist, and neither man nor woman chooses to teach mathematics in preference to languages except for pleasure in the subject matter itself. On this basis, then, though it may be claimed that mathematics is studied chiefly for utilitarian purposes, we find little difference between men and women in their natural inclinations toward the science.

As to ability in the pursuit of mathematics, the consensus of those reporting again seems to be that no marked difference exists, so far at least as is shown in the more elementary courses of the ordinary college curriculum. The mode of approach is sometimes different, the women being more bookish, the men more independent and original, and this condition tends to carry men further into mathematical study and to develop a difference in later and more advanced courses, but the evidence even on this point is not clear. One reporter in a State university says: "In recent years the greater number of students in advanced mathematics have been women. The women have, moreover, averaged better than the comparatively few men, but the best student in nearly every class has been a man." Another in a similar institution says, "Formerly the men were plainly stronger on the average than the women. Now in my classes the women probably three times out of four lead the men."

In response to an inquiry as to a preference by women for geometry over analysis or the contrary, the replies are practically unanimous that no such preference is shown. One person reporting states that

"women do in some measure prefer geometry," but in the main there is an agreement of "no difference between men and women. Sex has nothing to do with it," as far as choice of subject is concerned. Men and women for the most part meet in the same classes and receive the same instruction. If for convenience or for temperamental reasons, classes are organized for the sexes separately, as is done in some colleges, the methods of instruction and the subject matter covered are identical.

It may be truly said that this investigation has developed nothing that would indicate a tangible difference between men and women in mathematical ability or taste, and the colleges are treating them alike in methods of instruction and in quality and quantity of work demanded. Still, it remains that most teachers are conscious of a certain difference in the mental attitude as shown by a woman or by a man toward a mathematical problem. What that difference is can not be easily defined, but a man's approach has in it a certain ruggedness which is lacking in the approach of a woman. The latter is sometimes more effective, and the woman will frequently surpass the man in mental acumen, but still there is a difference, and this fact leads naturally to the inquiry, now that it has been demonstrated by many years of education side by side that women can do mathematically what man can do, if there should not be in the near future some differentiation in college courses, in subject-matter or in method, which will emphasize and develop that proper sex difference which is so generally recognized.