SOME ASPECTS OF MATHEMATICAL MODEL OF COLLABORATIVE LEARNING

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ABSTRACT

There are some mathematical learning models of collaborative learning, with which we can learn how students obtain knowledge and we expect to design effective education. We put together those models and classify into three categories; model by differential equations, so-called Ising spin and a stochastic process equation. Some of the models do not contain interactions among students and we discussed the possibility of improvement of the models for collaborative learning.

KEYWORDS

Mathematical learning model, Collaborative learning, Differential equation, Ising spin, Stochastic process

1. INTRODUCTION

In recent years, the theoretical study of teaching-learning processes has attracted the attention of an increasing number of scientists. Early studies on those processes have been conducted by psychologists and sociologists (Piaget, 1929; Vygotsky, 1978). Although the topic is so difficult and complex that there are still many open questions, the study of cognitive processes has developed into an active area of multidisciplinary investigation as a number of physicists have got interested in research areas such as economy, social science, biology and so on. Since Hake reported that the performance of the students can be enhanced using a teaching approach involving collaborative group work, in contrast to the traditional non-interactive lectures (Hake, 1998), the processes of learning and understanding physics and mathematics have become the focus of cognitive research. In order to precede the research, mathematical model of teaching-learning process have been proposed and studied.

The aim to study teaching-learning process by the use of mathematical model is to investigate the influence of the structure of the group works on the achievement of the students and to design effective education. In this reflection paper, we put together mathematical models of learning process and summarize some aspects of those models.

2. MATHEMATICAL LEARNING MODEL

There are some mathematical models of teaching-learning process and we classify these models into three categories in the following subsections.

2.1 Differential Equation Modeling

Pritchard et al proposed models of teaching-learning process by the use of differential equation (Pritchard, 2008). The models are based on various theories of learning: tabula rasa, constructivist, and tutoring. They predict the improvement of the post-test as a function of the pre-test score depending on the type of instruction. One of the models is the connectedness model combined with tabula rasa and constructivist. The connectedness parameter β measures the degree to which the rate of learning is proportional to prior knowledge; $dU_T / dt = -\beta\alpha K_T U_T - (1-\beta)\alpha' U_T$. U_T and K_T are fraction of unknown and known in a test domain T respectively and they are dependent of the amount of instruction t. α and α' are control parameters. The first term of the right hand side (rhs) is motivated by the constructivist view that students learn new knowledge by constructing an association between it and some prior knowledge. The second term of rhs is motivated by the tabula rasa theory of learning.

Although the model itself is quite simple and an exact solution can be obtained analytically, it fits existing data by sharply determining a parameter. However, these models do not describe a collaborative learning among students.

2.2 Ising Spin Modeling

Bordogna and Albano proposed a mathematical model of teaching-learning process in a classroom by the use of a constructive approach (Bordogna and Albano, 2001). In the model, interactions between students and teachers are described with a set of equations similar to those that describe magnetism in materials. They used so-called generalized Ising spin σ_j that is the knowledge of the student *j* and is defined as a dynamic variable such as $-1 \le \sigma_j \le 1$. Students play the role of spins and their knowledge of a subject is similar to the orientation of the spin. A teacher behaves like an external magnetic field trying to align the student's knowledge to the right direction. The model is described as follows. The cognitive impact of the teacher on the student *j* is $CI_j^{TS} = P_{jT}(1-\sigma_j\sigma_T)$, where σ_T and P_{jT} are the knowledge of the teacher and his/her ability to persuade the student *j*, respectively. Within groups of *N* students, the cognitive impact of the student-student interaction is given by $CI_j^{SS} = \sum_{i=1, i\ne j}^{N} [P_{ij}(1-\sigma_i\sigma_j) - S_{ij}(1+\sigma_i\sigma_j)]\mathrm{sign}(\sigma_i / \sigma_T)$, where P_{ij} and S_{ij} are mutual persuasiveness and support of student *i* to *j* respectively. The knowledge σ_j changes according to the total cognitive impact stochastically by the use of Metropolis algorithm.

The simulation results of the model is consistent with well-established empirical results, such as the higher achievements reached by working in collaborative groups and the influence of the structure of the group on the achievements of the individuals. Yasutake et al introduced network structures formed by students into the model and investigated some effects of the network structures (Yasutake, 2011).

2.3 Stochastic Process Modeling

Nitta developed a phenomenological theory of peer instruction (Nitta, 2010). His model describes rather a short-time process of learning, in contrast to the models presented in the above subsections that describe long-term learning gain. He modeled the transition of the number of students answering correctly for multiple-choice questions (MCQ) after discussions among students by the use of a master equation. By denoting the normalized number of students choosing the answer *a* for the MCQ *q* before discussion and after discussion as $\rho_1(q;a)$ and $\rho_2(q;a)$, respectively, $\rho_2(q;a)$ is described as $\rho_2(q;c) = \rho_1(q;c) - \sum_{d(\ne c)} T_{dc}(q)\rho_1(q;c) + \sum_{d(\ne c)} T_{cd}(q)\rho_1(q;d)$, where *c* and *d* represent the correct answer and distractors, respectively and $T_{ab}(q)$ is the transition matrix that represents the normalized transition rate of students from answer *b* to *a* on MCQ *q*. Then the master equation was simplified analytically and he showed that the number of correct answers after peer discussion is approximately given by a simple function of the number of correct answers before discussion as $\rho_2 = \rho_1 + \rho_1(1-\rho_1)$.

The theoretical curve agrees with data obtained from lectures implementing the peer instruction. In addition, it was shown that a differential equation derived from the model corresponds to the simple connected model of Pritchard et al. However details of the peer instruction is not included in the model.

3. REFLECTIONS

Although Ising spin model describes the mutual interaction among students and teachers, differential equation model and stochastic process model do not include elements of collaborative learning. In this section, we would like to suggest the possibility to extend those models in order to take into account collaboration among students.

There is a simple extension in the differential equation model described in section 2.1. The unknown fraction of a student *i* in a test domain U_i should decrease with the help of other wiser students and could rather increase because of confusion caused by discussion with less wise students. One possibility to implement the collaboration is to add a term $-\alpha'' \sum_{j \neq i} (K_j - K_i)$, where K_i is a known fraction of a student *i* and the subscript *T* for test domain is omitted here.

It is not straightforward to improve the stochastic process model described in section 2.3. Collaboration should emerge as a result in elements of the transition matrix $T_{ab}(q)$. Therefore we have to model each student's behavior intended to choose an answer for MCQ before and after discussion and make up the result statistically to construct the transition matrix. This extension requires further consideration.

4. SUMMARY

We briefly summarized three kinds of models for teaching-learning process; differential equation modeling, Ising spin modeling and stochastic process modeling. Mathematical modeling approach to design effective teaching-learning environments has just started and the study hovers at a level of applying models to each individual case. Further investigation and more general model would be expected.

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