EVALUATION OF MATHEMATICAL SELF-EXPLANATIONS WITH LSA IN A COUNTERINTUITIVE PROBLEM OF PROBABILITIES

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ABSTRACT

In this paper different type of mathematical explanations are presented in relation to the mathematical problem of probabilities Monty Hall (card version) and the computational tool Latent Semantic Analyses (LSA) is used. At the moment the results in the literature about this computational tool to study texts show that this technique is appropriate for the case of expository and narrative texts, but there is not evidence with mathematical texts. The technique could help us to identify which are the better explanations and if this is relevant to explain correct responses.

KEYWORDS

Self-explanation, LSA, implicit inference, reasoning

1. INTRODUCTION: DIFFERENT EXPLANATIONS FOR THE MHD

In this paper different type of mathematical explanations are presented in relation to the mathematical problem of probabilities Monty Hall dilemma (MHD) (card version, see Tubau and Alonso 2003). We will see that there are several ways of arriving at the correct response to the dilemma without resorting to the application of the theorem of Bayes. We will explore these explanations in order to introduce the explanations provided by the participants.

A simple and completely correct explanation for the participants' continued belief in the probabilities after the elimination appeals to the invariability of the probabilities of the initial sets. The elimination is not informative from the point of view of the probabilities of the initial sets: since the beginning we have known that at least 1 of the 2 cards held by the informant is different from the ace; the elimination is informative from the point of view of the individual probabilities of the cards (since the cards of the informant change from having equal probabilities of 1/3 to having probabilities of 0 and 2/3), but these do not affect the probabilities of the initial sets.

The illusory response and a possible explanation

The initial probability of each card being correct is the same, 1/3. Once the informant has showed one of his cards (situation of elimination), we tend to make the illusory inference that "if there are only two cards, the probability of each of them of being the ace is the same (1/2)", when the probabilities are in fact the same as they were at the beginning: for the decision-maker, p (ace) = 1/3, and for the informant, p (ace) = 2/3.

A possible explanation for such an illusory response could draw from a heuristic of similarity, where in situations in which there are two alternatives, we think the probabilities are equal.

Necessary and sufficient information for the correct resolution of the dilemma

It is important to realise that the key point in the explanation of the dilemma is the situation of elimination. Any of the factors that facilitate the resolution of the dilemma in fact facilitate the comprehension of the implications of the situation of elimination. It is at this moment that it is necessary to bear in mind that the performance of the informant is conditioned, and this is relevant to their response.

Let us consider, furthermore, that to overcome the strong illusory inference, it is not enough only to consider the variety of possible cases of performance of the informant (that is to say, to think that "if John has the ace and the 7, he will always show the 7, whereas if he has the 7 and the 8 he will show half of the

time the 7 and half of the time the 8), but it is necessary also to consider the globality (or generic sum) of these and to compare the number of times in which there is the ace and the 7 and the number of times in which there is the 7 and the 8. For in the case where the informant has the 7, for example, the reasoning would be as follows: "Every 3n/2 times that he shows 7, n will be with the ace and n/2 with 8.", and specifying for a natural number n=2, "Every 3 times that he shows 7, 2 of them will be with the ace and 1 with the 8." Therefore, if the informant shows the 7, it will be with the ace more times than with the 8; this reasoning have been explored in several questionnaires (see, for example, Tubau, 2008).

2. EXPERIMENT

A version of the problem was designed (see Appendix for details of the game) and the experiment was separated among 3 conditions; they were administered at different consecutive stages.

The first condition was a control condition to be compared with other versions in which an explanation is asked. Condition A included a "Help-sentence" that shows to them the ways in which the informant might perform (following the line suggested by Johnson-Laird, Legrenzi, Girotto, Legrenzi, & Caverni (1999)) that was also included in the other versions.

In the conditions B and C, a "Double question" was designed in order to help participants to answer correctly the dilemma that asked them for an explanation (justification) for their responses. The aim of this experiment was to see the efficacy of self-explaining the problem and to see, in condition C, if the same correct explanation given by the participants in condition B would be enough to correctly understand the problem[1].

2.1 Method

Material. See Annex of Majà, 2009. In condition B, question "(a)" will always be correctly answered if the statement was correctly comprehended. Question "(b)", which in fact already implies the illusory inference, is the one that theoretically could imply serious problems; more specifically, it could be similarly difficult to the two questions (Decision and Probability) that finally were considered in order to evaluate the correct resolution of the dilemma. Participants. 22 students of the University of Barcelona participated in exchange for course credit (one of the participants was eliminated due to previous familiarity with the problem); in condition B, 52 students; in condition C, 58 students of the University of Balearic Islands participated.
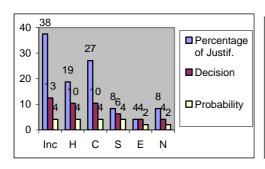
2.2 Results

The results show a surprisingly high percentage of switches (correct responses) in the first illusory question and a strong decrease in the later questions, despite all of them inviting the same illusory inference.

As was expected, in the Decision question (in which the participant was only given the option of switching or sticking with the initially selected card), better results than in the Probability question (in which the exact probabilities were also asked) were found in both the B and C conditions (following the tendency that there is more difficulty expressing the answers by exact numbers rather than relative probabilities).

The difference between the questions of condition A and condition B was significant in the Decision question; it was not significant in the Probability question. In the condition B, by seeing the difference between questions, significant differences comparing the Decision question and the Probability question were also found (45% vs. 20%; x^2 (1, N=102) = 7.57, p<.01). Therefore, the fact of giving explanations seem to especially helpful for the correct comprehension o the problem.

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[1] In Condition C, instead of the Help sentence, the complete explanation given for participants in condition B was included: i.e., the correct response, followed by the connective 'because', and followed by the explanation provided by most of the participants in condition A.
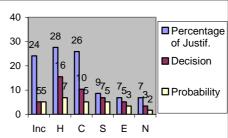
2.3 Analyses of Justifications

Figure 1. Type of justification and correct responses (%) in the Decision and Probability questions for Condition B and C

Comparing the justifications of the conditions B (in which the justification 'C' was the most common one) and C, there was an increase in the condition C of the type of justifications 'H' (not being significant). There was a significant difference between the type of justification and correct responses to the 'Decision' and 'Probability' questions, being the correctness of both the Decision and the Probability questions more predictable from the type of justifications "S" (perhaps more creative).

Table 1. Classification of the justifications of the participants (conditions B and C)

Types of justification[2]	Examples of justifications (of the participants)	LSA coherence[3]	LSA (with implicit inference) coherence	Difference between LSA_i and LSA
C: Repetition of the consideration of the *Cases* of the Help-sentence	"It's more probable that John shows the ace because if he has the ace and the 7 he always shows the 7 [and if he has the 7 and the 8 he only shows it half the time]"	0.391	0.438	0.047
N: Consideration (explicit) of a consecutive *Number* of trials	Long version exemplified with 6 cases "It's more probable that John shows the ace when he has the ace and 7	0.47	0.491	**0.021**
	because if they play several times and always show the 7 it would be that hides the ace."	0.295	0.34	0.045
E: Double sense (*Equivalency*) of the relationship between the cards that John has and the card that he shows	"If John shows the 7 it's more probable that he has the ace because if John has the ace and the 7 it's more probable that shows the 7."	0.36	0.412	0.052
S: Invariability of the probabilities of the initial *Sets*	"If John shows the 7 it's more probable that he has the ace because John always has 2 cards"	0.311	0.33	**0.019**
H: John *Hides* the ace.	"If John shows the 7 it's more probable that he has the ace because the ace, because he is forced to hide it."	0.284	0.333	0.049
	"If John shows the 7 it's more probable that he has the ace because he hides the 8 only half of the time whereas he always hides the ace."	0.29	0.339	0.049

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[2] Considering only the correct responses ('Inc' expresses 'Incorrect Responses')

[3] It is considered the value LSAassa 'LSA, Sentence to Sentence, adjacent, mean', the one that show more differences in the results.

2.4 Analyses with LSA

There are several authors (for example, Louwerse, 2010) that hold that LSA is not only a computational tool for text analyses but also it has also implications in order to understand the process of cognition. One reason to argue this would be that when the computational technique LSA estimates that the coherence of a sentence is high, it says that on the basis of a mechanism of semantic activation similar to a connectionist one in which the activation (or coherence) of a sentence happens in relation to the activation of another (a mechanism of co-ocurrence) (as developed in Kintsch 1988, 1998, for example).

Therefore, LSA could be used in order to study these justifications (explanations) and to see if a specific implicit inference ('i') would be in the mind of some of the participants that correctly answered the problem until the end of the questionnaire; the LSA values obtained in a specific text (which includes both (a) a specific justification and (b) the implicit inference 'i') could show that this implicit inference is involved in the comprehension of the participants.

It is hypothesized that there is an implicit inference on the basis that the literature (Gigerenzer and Hoffrage, 1995) show that, in probabilistic reasoning, natural frequencies are easier to understand than relative frequencies, and this is in relation to the fact that a number of consecutive trails in probability have to be assumed, as it was emphasized at the beginning of the paper. The implicit inference therefore would be: "If I consider, on a consecutive number of trials, a sum of the diverse possible cases in which John shows 7, I see that John shows more times 7 when it is with the Ace than when it is with 8."

In order to obtain the LSA cosine values, the informatic tool Coh-Metrix is used; the full long texts with justification with/without an implicit inference were introduced. The analyses showed that the difference between LSA and LSA_i was marginally significant for the comprehension question Decision, $t(95)=2.465$; $p=0.016$, and was significant for the question Probability, $t(95)=3.648$; $p<0.01$.

The difference between LSA and LSA_i cosine values seem to show that the implicit inference could be more present in the case of the participants who would give the longer explanations N and S. Comparing the results of condition C with condition A, there is a significant improvement in correct responses in the Probability question; as this increase is only with the increase of number of justifications 'H', it cannot be said that the overcome of an associative effect is accompanied with the processing of the implicit inference. Looking at the large difference in values for the justification H between LSA *vs* LSA_i, LSA would predict that the implicit inference is not necessarily involved in the correct comprehension of the problem.

LSA seem to able to evaluate more creative explanations as well as both short and large explanations. LSA seem to be a helpful tool to evaluate understandable explanations for education.

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