RECOGNITION VS REVERSE ENGINEERING IN BOOLEAN CONCEPTS LEARNING

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ABSTRACT

This paper deals with two types of logical problems - recognition problems and reverse engineering problems, and with the interrelations between these types of problems. The recognition problems are modeled in the form of a visual representation of various objects in a common pattern, with a composition of represented objects in the pattern. Solving the recognition problem may therefore be understood as recognizing a visually-represented Boolean concept, with further formulation of the concept. The recognition problems can be perceived as a parallel process, so the recognition problems are considered a parallel type. Alternatively, solving a reverse engineering problem means reconstructing a Boolean function/concept implemented within a given “black box”. Since such a reconstruction is typically performed sequentially, step by step, this type of problem can be considered a sequential type. We study the above two types of problems for the same set of Boolean concepts and compare the corresponding solutions obtained by a group of students. The paper presents results of experiments that study how the complexity of Boolean concepts affects the students’ success in solving parallel and the sequential type problems respectively.

KEYWORDS

Boolean concepts, Recognition and Reverse engineering problems.

1. INTRODUCTION

Concepts are the atoms of thought and they are therefore at the nucleus of cognition science (Fodor, 1994). People begin to acquire concepts from infancy and continue to acquire and plan new concepts throughout their entire lives (Medin, Lynch, & Solomon, 2000; Medin & Smith, 1984). One way to create a new concept is by utilizing existing concepts in different combinations. One of the problems in learning concepts is determining the concept’s subjective difficulty. Logical thinking is the key to a wide variety of complex problem solving and decision making processes and therefore Boolean concepts are essential. An important aspect of concept learning theory is the ability to predict the level of difficulty in learning different types of concepts. In this respect, Boolean concepts are a fundamental topic in the literature. Boolean concepts can be defined by a Boolean expression composed of basic logic operations: negation, disjunction (“or”), and conjunction (“and”). These types of Boolean concepts have been studied extensively by Shepard, Hovland, and Jenkins-SHJ (1961), (Bourne 1966, 1974), (Nosofsky, Gluck Palmeri, McKinley, and Glauthier 1994). These studies focused on Boolean concepts with three binary variables, where the concept receives “1” for 4 out of 8 possible combinations and “0” for the remaining 4 combinations. Since some of the 70 possible Boolean concepts are congruent, they can be categorized as the same type into six subcategories. The six subcategories with structural equivalence can be described in a geometrical representation using cubes (Figure 1).

This notion of congruence seems to have been first introduced by Aiken and his colleagues (Aiken & the Staff of the Computation Laboratory at Harvard University, 1951) and subsequently became prevalent in the literature on the theory of switching circuits. It was introduced into psychology by Shepard, Hovland and Jenkins (1961).

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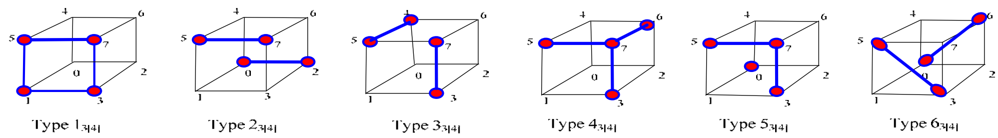

Figure 1. Shepard, Hovland, and Jenkins (SHJ) category types.

1.1 Cognitive Complexity of Boolean Concepts

Concept subcategories with structural equivalence belong to the same category and are defined as a Type. The study results pertaining to the six types of concepts presented in Fig1 showed that the concepts belonging to Type1 are the simplest to learn and the subgroup of concepts belonging to Type6 are the most difficult, according to the following order: Type 1<Type2<(Type3,Type4,Type5)<Type6. The results of this study are highly influential since SHJ proposed two informal hypotheses, the first being that the number of literals in the minimal expression predicts the concept's level of difficulty. The second hypothesis is that ranking the difficulty among the concepts in each type depends on the number of binary variables in the concept. The concept subcategory in Type1 has one variable, the subcategory concept in Type2 has only 2 variables, and concept subcategories in Types2 to 6 contain three variables.

Feldman (2000), based on the conclusions from the SHJ study, defined a quantitative relationship between the level of difficulty of learning Boolean concepts and the concept's Boolean complexity. Assuming that D is the number of binary features and P is the number of combinations out of all the combinations in which the Boolean concept receives "1" (SOP-Sum Of Products), D[P] indicates the family of Boolean concepts with D variables and P combinations where the concept receives "1". For example: the concepts that were examined by SHJ were represented as 3[4], 3 binary variables and 4 combinations in which the concept receives "1". In his study, Feldman examined the 3[2], 3[3], 3[4], 4[2], 4[3] concept family. Feldman also addressed the family of concepts where the number of combinations with "1" differs from the number of combinations with "0". For example, the concept 3[3] contains 3 combinations with "1" and 5 combinations with "0", unlike its mirror concept 3[5], which contains 3 combinations with "0" and 5 combinations with "1". The complexity measure of a Boolean concept as defined by Feldman is the number of literals in the most minimal expression that represents the concept's complete SOP. According to Feldman's definition of the complexity of the concepts and use of heuristic minimizing technique to the minimum literals, in SHJ's classification model the complexity measures in each category are: Type1: 1, Type2: 4, Type3: 6, Type4: 6, Type5: 6, Type6: 10. According to this complexity measure, it is possible to predict that concepts from Types3, 4, 5 have the same complexity and are learned more easily than Type6 but are more difficult to learn than Types1 and 2. These complexity measures predict difficulties in learning Boolean concepts, as examined by SHJ.

Since there are several techniques for reducing an expression to the minimum, Vigo (2006) presented use of the QM (Quine-McCluskey) technique to obtain a correct minimal description and showed that it is possible to minimize the expressions more correctly than what Feldman presented in the heuristic technique.

According to Vigo, based on Feldman's same definition as a measure of cognitive complexity in SHJ's classification model, the complexities of functions from each type are given in Table 1.

The definition of the Boolean concept's complexity as a minimal number of literals in a minimal expression creates several problems. The first: because the complexity is defined as the number of literals in the minimal expression and the expressions can be minimized using several techniques, a uniform complexity measure cannot be obtained. For example: according to Feldman's heuristics, Types3, 4, 5 have the same complexity. Contrary to Feldman, in a more correct minimal expression according to Vigo, concepts from Types2 and 3 have the same complexity.

The second problem: studies show that the Boolean concept "xor" as an operator is learned and acquired as a concept in human thought to the same degree or more easily than the Boolean concept "or" as an operator (Evans, Newstead, & Byrne, 1993). By using the "xor" operator, some of the Boolean expressions examined by Feldman and Vigo can be simplified significantly and therefore, the complexity decreases according to the definition as a minimum of literals in the minimal expression. In light of the problems presented, Feldman and Vigo developed alternative theories for Boolean complexity as a measure of predicting the difficulty in learning Boolean concepts.

Table 1. The six concepts in SHJ and their descriptions according to Feldmans heuristic, correct minimal descriptions, minimal descriptions using "xor" and Structural Complexity (SC)

The SHJ six concepts	Feldman's heuristic	correct minimal descriptions	minimal descriptions using "xor"	SC
Type1$_{3[4]}$	$\bar{a}$ (1)	$\bar{a}$ (1)	$\bar{a}$ (1)	1.66
Type2$_{3[4]}$	$ab+\bar{a}\,\bar{b}$ (4)	$ab+\bar{a}\,\bar{b}$ (4)	$\overline{a\oplus b}$ (2)	2.00
Type3$_{3[4]}$	$\bar{a}\left(\overline{b\,c}\right)+a\,\bar{b}\,c$ (6)	$\bar{a}\,\bar{c}+\bar{b}\,c$ (4)	$\bar{a}\,\bar{c}+\bar{b}\,c$ (4)	2.14
Type4$_{3[4]}$	$\bar{a}\left(\overline{b\,c}\right)+a\,\bar{b}\,\bar{c}$ (6)	$\bar{c}\left(\bar{a}+\bar{b}\right)+\bar{a}\,\bar{b}$ (5)	$\bar{c}\left(\bar{a}+\bar{b}\right)+\bar{a}\,\bar{b}$ (5)	2.14
Type5$_{3[4]}$	$\bar{a}\left(\overline{b\,c}\right)+a\,b\,c$ (6)	$\bar{a}\left(\overline{b\,c}\right)+a\,b\,c$ (6)	$\overline{a\oplus(bc)}$ (3)	2.34
Type6$_{3[4]}$	$a\left(\bar{b}\,c+\bar{c}\,b\right)+\bar{a}\left(\bar{b}\,\bar{c}+b\,c\right)$ (10)	$a\left(\bar{b}\,c+\bar{c}\,b\right)+\bar{a}\left(\bar{b}\,\bar{c}+b\,c\right)$ (10)	$\overline{a\oplus b\oplus c}$ (3)	4.00

Aware of these difficulties, Feldman (2006) has recently introduced his spectral decomposition model. In this updated model, the complexity of a concept is driven by its decomposition into a set of underlying regularities. The basic idea is that learning from examples involves the extractions of patterns and regularities. The formal model describes how a pattern (expressed in terms of a Boolean rule) may be decomposed algebraically into a "spectrum" of component patterns, each of which has a simpler or more "atomic" regularity. Regularities of a higher degree represent more idiosyncratic patterns while regularities of a lower degree represent simpler patterns in the original decomposed pattern. There are two kinds of simple concepts: those that consist of a single constant value of a variable, and those that consist of an implication between the values of two variables. These two basic types of concepts can be algebraically combined to represent more complex linear concepts. This step is based on an analysis of a concept's ''power spectrum''. Thus, any concept can be decomposed into a set of underlying rules, each of differing degrees of complexity, depending on the number of variables that they instantiate. The weighted mean of the complexity of these underlying rules then provides an overall index of the algebraic complexity of the concept. The model yields a complexity index for any Boolean concept. This model is characterized by its ability to explain unique phenomena of the learning process of concepts. The algebraic complexity makes it possible to rank each concept combination according to its structure.

Vigo (2009), developed an alternative theory for calculating the complexity measure of a Boolean concept, defined as structural complexity. The theory is based on a Boolean derivative. The Boolean derivative was introduced by Reed (1954) in a discussion of error-correcting codes in electrical circuits. The basic concept has been mainly relegated to this very specialized domain of applied Boolean algebra.

The question that the theory is supposed to address is: What is it about the internal structure of Boolean concepts from any category that makes them harder to learn than concepts from a different category? For the purpose of quantitative definition of the structural complexity, the degree of categorical invariance must be calculated. In order to quantify the structural complexity, several calculations that will be presented below must be conducted. For a Boolean function with n variables derived from the Boolean part similar to the partial derivative of a mathematical function defined as follows:

$$\frac{\partial F\left(x_1,x_2,..x_i,..x_n\right)}{\partial x_i}=F\left(x_1,x_2,..x_i,..x_n\right)\oplus F\left(x_1,x_2,.....\overline{x_i},...x_n\right)$$

Let F be a Boolean expression that defines a Boolean category. The logical manifold or Lm of F in respect to x is defined as:

$$Lm=\frac{\left(\left|F_0\cap F_p\right|\right)}{\left|F_0\right|}$$

were:

$$F_0 - \min terms\ sum\ of\ \left(F\left(x\right)\right)\ ,\ F_{pi} - \max terms\ \text{sum}\ of\ \left(\frac{\partial F\left(x\right)}{\partial x_i}\right) = \left(\frac{\overline{\partial F\left(x\right)}}{\partial x_i}\right)$$

The degree of structural complexity (SC) of a Boolean category defined by the Boolean expression F and belonging to the family D[P] is indirectly proportional to its degree of categorical invariance and directly proportional to its cardinality SOP(F):

$$SC = \left|F_0\right|\left[\left(\sum_{i=1}^{n}\left(Lm\,i\right)^2\right)^{\frac{1}{2}} + 1\right]^{-1}$$

Vigo’s (2009) account of the invariance of concepts, as he acknowledges, does not specify how individuals learn concepts. He suggests only that cognitive processes could detect invariances by comparing a set of instances to the set yielded by the partial derivative of each variable. For SHJ’s six categories, the complexity measures described above appear in Table1.

1.2 Recognition Problems

The recognition problems are modeled in the form of visual representation of various objects in a common pattern, with composition of thus represented objects in the pattern. Solving the recognition problem may thus be understood as recognizing a visually-represented Boolean concept, with further formulation of the concept. The recognition problems can be perceived as a parallel process, so the recognition problems are considered of a parallel type.

One of the important roles of human consciousness is to reveal patterns and find data sequences. Not all the patterns leave the same impression on people as a basis for identification and perhaps subsequently, identical patterns are not equally observed by different people.

1.3 Reverse Engineering (RE)

The process of finding and reconstructing operating mechanisms in a given functional system of a digital electronic apparatus is defined as Reverse Engineering (RE) (Chikofsky & Cross, 1990). RE is applied in a wide variety of fields: competition in manufacturing new products, from electronic components to cars, among competing companies without infringing upon the competing company’s copyrights, replacing system components with refurbished or spare parts (Ingle, 1994), solving problems and defects in a system (Eilam, 2005). RE can be referred to as a certain type of problem solving.

A reverse engineering problem means reconstructing a Boolean function implemented within a given “black box”. Since such a reconstruction is typically performed sequentially, step-by-step, this type of problem can be considered a sequential type.

When the values in the variables in a Boolean function express declarations or claims in specific combinations, the result of the expression is not a Boolean concept but “true” or “false”. For example: “Switch one and switch two are on or switch three is off”. People are capable of making such claims by examining the combinations. Sequentially examining a series of combinations makes it possible to reveal all the possible combinations that give a “true” or “false” value. A mental model for the possibilities that give a “true” value (a mental model demonstrates the “true” principle) has been built based on a discovery of the combinations. The mental model is translated into a representation of the system using literals, by minimizing the variables that do not affect the “true” result (Johnson-Laird, 2006).

This paper deals with two main questions. The first: What effect does the cognitive complexity of a Boolean concept have on the success of solving recognition and reverse engineering type problems? What is the ratio between the cognitive complexity of a Boolean concept and the complexity of solving recognition and reverse engineering type problems?

Reverse engineering problems were represented as a black box that can be used to control the lighting of a bulb using independent switches. Recognition problems were given as a pattern containing geometric shapes.

We study the above two types of problems for the same set of 9 Boolean concepts and compare the corresponding solutions obtained by a group of students.

2. EXPERIMENT

The experiment was conducted in two stages for 9 concepts, where each concept was described by means of a Boolean expression in Table 2.

During the first stage, RE problems were examined using a black box that could be used to control the lighting of a bulb using three independent switches for seven concepts with three variables (concepts 1-7, Table2) and using four independent switches for two concepts with four variables (concepts 8 and 9, Table2). The participants were required to try the different switch combinations that light the bulb and describe the combinations that light the bulb for each of the 9 concepts using a Boolean expression. A maximum of 5 minutes was allocated for each of the 9 tasks. The tasks were presented as a simulation on a computer monitor and the time taken to complete each task was measured. Successful completion of the task was measured based on correct solving during the allotted time.

During the second stage, recognition problems were examined using a questionnaire with nine patterns, where each pattern represents one of the nine concepts examined, respectively, Figure 2 presents patterns for concepts 1, 5 and 8. The participants were asked to describe each of the patterns with as few literals as possible. A maximum of 45 minutes was allocated to completing the questionnaire for each of the nine patterns. The two stages of the experiment were conducted two days apart.

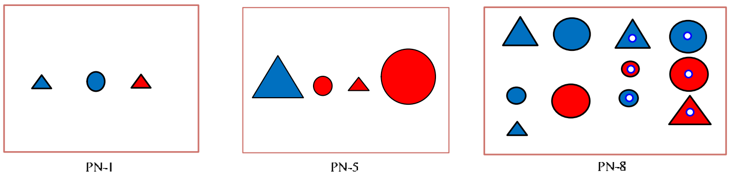

Figure 2. Patterns for concepts 1, 5 and 8 respectively tested during the experiment.

Successful completion of the tasks solving both RE problems and the recognition questionnaire was measured based on correct solving in the allotted time. All the solutions were examined compared to two complexity measures: complexity according to a minimal representation of the expression (including using the “xor” operator) and complexity according to structural complexity (see Table2)

An example for calculating structural complexity for concept 1 (PN-1 in Table2):

$$F_{10}(a,b,c) = \bar{b}(a+c) = \sum(1,4,5)$$

$$\frac{\partial F_1}{\partial a} = F_1(a,b,c) \oplus F_1(\bar{a},b,c) = \sum(0,4) \Rightarrow F_{ap} = \overline{\frac{\partial F_1}{\partial a}} = \sum(1,2,3,5,6,7)$$

$$\frac{\partial F_1}{\partial b} = F_1(a,b,c) \oplus F_1(a,\bar{b},c) = \sum(1,3,4,5,6,7) \Rightarrow F_{bp} = \overline{\frac{\partial F_1}{\partial b}} = \sum(0,2)$$

$$\frac{\partial F_1}{\partial c} = F_1(a,b,c) \oplus F_1(a,b,\bar{c}) = \sum(0,1) \Rightarrow F_{cp} = \overline{\frac{\partial F_1}{\partial c}} = \sum(2,3,4,5,6,7)$$

$$Lm(a,b,c) = \left(\frac{|F_{10}| \cap |F_{ap}|}{|F_{10}|}, \frac{|F_{10}| \cap |F_{ap}|}{|F_{10}|}, \frac{|F_{10}| \cap |F_{ap}|}{|F_{10}|} \right) =$$

$$\left(\frac{\sum(1,4,5) \cap \sum(1,2,3,5,6,7)}{3}, \frac{\sum(1,4,5) \cap \sum(0,2)}{3}, \frac{\sum(1,4,5) \cap \sum(2,3,4,5,6,7)}{3} \right) = \left(\frac{2}{3}, 0, \frac{2}{3} \right)$$

$$SC = |F_0| \left[\left(\sum_{i=1}^{n} (Lmi)^2 \right)^{\frac{1}{2}} + 1 \right]^{-1} = 3 \times \left[\left(\left(\frac{2}{3}\right)^2 + 0 + \left(\frac{2}{3}\right)^2 \right)^{\frac{1}{2}} + 1 \right]^{-1} = 1.544$$

2.1 Method

The research population included thirty 1st year students studying for a Bachelor of Engineering degree at a college. Twenty students studied in the Department of Electric and Electronic Engineering and 10 students studied in the Software Engineering Department. All students studied the Logic Design course in the same study group and with the same lecturer. All students completed the course successfully with the final exam average grade of 75.

Table 2. The 9 concepts were tested during the experiment and their descriptions according to correct minimal descriptions-MD, minimal descriptions using "xor"-MD (xor) and Structural Complexity (SC)

PN	FCN	MD	MD (xor)	SC	Success (%) RE problems	Success (%) Rec. problems
1	$Type1_{3[3]}$	$\bar{b}(a+c)(3)$	-	1.54	100	100
2	$Type3_{3[4]}$	$\bar{a}\ c + b\ \bar{c}\ \ (4)$	-	2.14	90	70
3	$Type3_{3[4]}$	$\bar{a}\ \bar{b} + ab\ \ (4)$	$\overline{a \oplus b}\ \ (2)$	2.14	100	80
4	$Type4_{3[4]}$	$a(b+c)+bc\ \ (5)$	-	2.14	100	90
5	$Type5_{3[4]}$	$(\bar{a}+\bar{b})\bar{c}+a\ b\ c\ \ (6)$	$\overline{c \oplus (ab)}\ \ (3)$	2.34	60	40
6	$Type6_{3[4]}$	$a(\bar{b}\ c+\bar{c}\ b)+\bar{a}(\bar{b}\ \bar{c}+b\ c)\ (10)$	$\overline{a \oplus b \oplus c}\ \ (3)$	4.00	40	30
7	$Type6_{3[4]}$	$a(b\ c+\bar{c}\ \bar{b})+\bar{a}(\bar{b}\ c+b\ \bar{c})\ (10)$	$a \oplus b \oplus c\ (3)$	4.00	60	30
8	$Type4_{4[5]}$	$a(b+c+d)+b(d+c)+cd\ \ (9)$	-	4.48	90	80
9	$Type4_{4[5]}$	$\bar{a}(\bar{b}+\bar{c}+\bar{d})+\bar{b}(\bar{d}+\bar{c})+\bar{c}\ \bar{d}\ \ (9)$	-	4.48	90	70

An experimental environment using Lab View was developed on a computer monitor for "black box" RE problems. The monitor displayed a black box with switches and a bulb. The state of the switches could be changed by clicking on the appropriate key with the mouse. A change in the switch's state resulted in a color change from black to red and the written indication "off" (black) and "on" (red). According to the switch's state, the light bulb is either on or off. A lit bulb is green and has the word "on", and a bulb that is off is with the word "off". As soon as the participants reached the conclusion that they knew the appropriate logical function for the system of switches and the bulb, they were asked to write the states of the switches that light the bulb using a Boolean expression, written using an equation generator, and to press "finish". Time was measured from the moment the first switch in the box is pressed until the task was completed. The course of the experiment was filmed and a brief interview was conducted at the end of the task. The objective of the interview was to examine the solution strategy employed by the participants to discover the functionality regarding each of the nine tasks. The video clips were analyzed to compare the strategy that the students stated during the interview and the solution strategy as observed in the video clips.

A two-part questionnaire was developed for recognition problems. The first part presented a pattern with eight geometric shapes. Large and small triangles in red and blue, and large and small circles in red and blue. The shape was defined using binary variable *a*, size was defined as binary variable *b* and color was defined using binary variable *c*. Seven patterns were then presented, where each of the patterns matched the examined Boolean concept (Table2), respectively. The second part presented a pattern with 16 geometric shapes. Large and small triangles in red and blue and large and small circles in red and blue, with a nucleus in the center of the shape and without a nucleus. The shape was defined using binary variable *a*, the size was defined using binary variable *b*, the color was defined using binary variable *c*, and the nucleus was defined using binary variable *d*. Time was measured from the commencement of solving the questionnaire to when each participant submitted his questionnaire.

3. RESULTS AND CONCLUSION

By examining the results and conclusions, we will assess the effect the various complexity measures as presented in Table 2 have on the success, or lack thereof, in solving the two types of problems – recognition and RE problems, and the relationship between them. Additional aspects that we will examine are the effect of the “xor” operator on complexity, the “true principle” in RE problems, meaning whether the tendency is to focus on combinations that give “1” also for problems in which the number of combinations whose result is “0” is significantly less than the combinations that give “1”.

Table 2 presents the success rates for solving RE problems and recognition problems for all the nine concepts presented in the Table. The results show that participants are more successful in solving RE problems than solving recognition problems. Not a single participant that did not succeed in solving RE problems managed to solve recognition problems for the same concept. However, not all the participants that managed to solve RE problems were also successful in solving the recognition problems. For example, for problem 3, 27 participants managed to solve the RE problem, compared to 24 of them that also successfully solved the recognition problem. Difficulty in comprehending and learning a Boolean concept cannot be predicted based on Boolean complexity measures of the concept alone, but rather it also depends on the type of problem representing the concept. It can be concluded from the results that for problems where the information is absorbed concurrently, recognition problems are more difficult to learn than problems where the information is sequentially obtained, in this case RE problems. Therefore, the difficulty not only depends on the concept’s complexity but also on the complexity of the manner in which the problem is presented.

The greater the complexity measure, the lower the success rates, except for PN4, PN8 and PN9, which we will elaborate on later. It cannot be unequivocally concluded that the greater the complexity measure the lower the success rate for solving the problem. For problems 1, 2-3, 5, 6-7, the success rates for solving both types of problems indeed decreases as the complexity measure increases, also according to the MD and SC measures. For problem 1, the complexity is the lowest and the success rate the highest. For problems 2 and 3, the complexity is slightly higher than problem 1 and the success rates are consequently smaller. The situation is the same for problems 6, 5, and 7, respectively. The inverse relationship between the complexity and success rate also manifests in the average time required to solve the problems. As the complexity increases, the average time it takes to solve the problem increases accordingly.

Complexity with minimal literals when the “xor” operator is involved is a measure that is the least predictive, relative to MD and SC, of the subjective difficulty of successfully solving the problem. The majority of participants did not recognize the “xor” operator in both types of problems. Participants that grasped the “xor” concept as an operator to the same degree as the “or” and “and” concepts were more successful in solving the problem and their average time was substantially lower. Eight out the thirty participants consistently used “xor” in problems 6, 3, and 7. All of the eight participants arrived at the correct solution for both types of problems at an average time of 30 seconds, which is much lower than the average time it took the total number of participants. Out of twenty two participants that did not use “xor” but that did use the other operators, sixteen participants succeeded in solving problem 3 as a recognition problem. For problem 6, four out of twenty two participants managed to solve the RE problem and nine out of twenty two managed to solve the recognition problem. For problem 7, ten out of twenty two managed to solve the RE problem and one managed to solve the recognition problem. It can be concluded from the results that if the “xor” operator is acquired as a concept to the same degree as the operators “or” and “and”, the concepts’ level of complexity decreases, the success rates increase, and the difficulty in solving the problem decreases. However, not everyone acquires the three concepts – “xor”, “or” and “and” to the same degree. For most, the “xor” concept is more difficult to grasp than the other two concepts.

Although the level of complexity of the concepts in problems 4, 8, 9 are higher relative to problems 1, 2, 3, the success rates for solving both types of problems is significantly higher and the average time needed to solve them is much lower. It can be hypothesized that the reason for this is that the concepts in problems 4, 8, 9 fulfill qualities of symmetric functions. Apparently, the MD and SC complexity measures are not sufficiently reliable in predicting the level of difficulty in solving the problems for symmetrical functions. Based on the hypothesis, it is interesting to examine the effect of qualities of Boolean functions such as symmetry, linearity, etc. on the complexity of Boolean concepts.

Two RE problem solving strategies were observed. The first, adopted by twenty seven out of the thirty participants, was to attribute a logical value to one of the variables and conduct fewer checks of the combinations to a check times 2 consistently, building a mental model for the combinations in which the transitions among them, a change in one of the variables does not influence the state of the lit bulb. For problems 8 and 9, only ten out of twenty two participants reached the solution for the states in which the bulb is not lit and managed to solve the problem, since the number of combinations in which the bulb is not lit is 4, versus 12 states in which the bulb is lit. They took the same approach with recognition problems in solving problems 8 and 9, and succeeded. For the remaining participants, the "true principle" guided them in solving all the states, including the ones where it is more effective to examine the states in which the bulb is not lit, which were significantly lower than the number for states in which the bulb is lit. Three students tended to use the strategy of covering all the possible states. They managed to reach the correct solution only for RE problems for concepts 1, 3, 4, and did not succeed in reaching the required solution for the other states.

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