TEDS-M

TEDS-M 2008 User Guide for the International Database

Supplement 4

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The International Association for the Evaluation of Educational Achievement, known as IEA, is an independent, international consortium of national research institutions and government research agencies, with headquarters in Amsterdam. Its primary purpose is to conduct large-scale comparative studies of educational achievement with the aim of gaining more in-depth understanding of the effects of policies and practices within and across systems of education.

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SUPPLEMENT 4:

TEDS-M Released Mathematics and Mathematics Pedagogy Knowledge Assessment Items

Overview

The goal for selecting the released set of test items was to have approximately 25% of each of the full item sets for mathematics content knowledge (MCK) and mathematics pedagogical content knowledge (MPCK) that would represent the full range of difficulty, content, and item format used in the TEDS-M study.

The initial step in the selection was to take a stratified random sample of the items, stratifying on both proportion correct and knowledge dimensions of MCK and MPCK. The next step was to consider if the selected items were part of item sets. If they were, an attempt was made to use the full set so that the full context of the items would be made public. Representation of the anchor points was another important consideration as was balance of item formats (MC, CR, CMC — see below for the explanation of item format abbreviations). The test items were then reviewed to determine if they efficiently and accurately represented the full item set. A reduced set of items was kept to make possible a link between TEDS-M and a future follow-up study.

The set of primary released items consists of:

- 24 MCK items (10 from the algebra domain, 6 from geometry, 6 from number, and 2 from data) including samples of the cognitive sub-domains of knowing (15), applying (8), and reasoning (1); and
- 10 MPCK items (2 from the algebra domain, 2 from geometry, 4 from number, and 2 from data) illustrating the two sub-domains of curriculum/planning (6) and enacting (4).

The set of secondary released items consists of:

- 23 MCK items (7 from the algebra domain, 7 from geometry, 8 from number and 1 from data) including samples of the cognitive sub-domains of knowing (6), applying (10), and reasoning (7); and
- 9 MPCK items (5 from the algebra domain, 0 from geometry, 3 from number, and 1 from data) illustrating the two sub-domains of curriculum/planning (4) and enacting (5).

For each set the TEDS-M ID number for the released Items and other relevant information is provided in a table preceding the items. These overview tables include the following information per item:

- Item ID: The TEDS-M item identifier
- Knowledge Dimension: MCK = mathematical content knowledge, MPCK = mathematics pedagogical content knowledge
- Content Domain: The content domain according to the TEDS-M conceptual framework
- Sub-domain: The sub-domain within the content domain
- Label: The item label

- Item format: MC = multiple-choice, CMC = complex multiple-choice, CR = constructed response
- Key: The correct answer item for multiple-choice and complex multiple-choice items
- Max Points: The maximum points assigned to a correct answer
- International Average: The percentage of future teachers answering correctly, separately stated for fully correct (FC) and partially correct (PC) answers if applicable.

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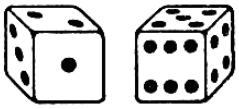
Section 1: Released Items Primary Schools

Item ID	Knowledge Dimension	Content Domain	Sub-domain	Label	Item format	Key	Max. Points	International Average
MFC106	MCK	Data	Applying	Fairness of two-dice game	MC	2	1	28%
MFC108	MPCK	Algebra	Enacting	Equation best representing [Amy's] pattern	MC	3	1	28%
MFC202A	MCK	Algebra	Knowing	Truth of algebraic statements	CMC	2	1	81%
MFC202B	MCK	Algebra	Knowing	Truth of algebraic statements	CMC	2	1	86%
MFC202C	MCK	Algebra	Knowing	Truth of algebraic statements	CMC	1	1	92%
MFC202D	MCK	Algebra	Knowing	Truth of algebraic statements	CMC	2	1	64%
MFC203	MCK	Geometry	Applying	Area of walkway around rectangular pool	MC	3	1	67%
MFC204	MCK	Geometry	Knowing	Interpreting student Venn diagrams about quadrilaterals	MC	3	1	61%
MFC206A	MCK	Number	Applying	Solving a rate problem about fuel use	MC	2	1	78%
MFC206B	MPCK	Number	Curric/Plan	Create a different problem about fuel used	CR	SG[1]	1	54%
MFC208A	MPCK	Number	Enacting	[Jeremy's] misconception in using a calculator	CR	SG	2	20% (FC)[2] 12% (PC)
MFC208B	MPCK	Number	Enacting	Visual representation to model 0.2 x 6	CR	SG	2	16% (FC) 16% (PC)
MFC303	MCK	Algebra	Applying	Unknown mass on a balance	MC	3	1	82%
MFC304	MCK	Number	Knowing	How many decimal numbers between two numbers?	MC	4	1	54%
MFC307A	MCK	Geometry	Knowing	Solving a volume problem about blocks	MC	1	1	78%
MFC307B	MPCK	Geometry	Curric/Plan	Reword a question about volume and blocks	CR	SG	2	38% (FC) 14% (PC)
MFC308	MCK	Algebra	Applying	Rule for the number of people around n tables	CR	SG	1	49%
MFC312	MPCK	Algebra	Curric/Plan	Equation not representable by a pan balance	MC	2	1	38%
MFC408	MCK	Geometry	Applying	Area of scalene triangle on grid	MC	1	1	60%
MFC410	MPCK	Data	Enacting	Similarities and differences in data presentation	CR	SG	2	29% (FC) 38% (PC)
MFC412A	MCK	Algebra	Knowing	Three consecutive even numbers - meaning of k	MC	1	1	56%
MFC412B	MCK	Algebra	Knowing	Three consecutive odd numbers - correct expression	MC	2	1	51%
MFC501	MCK	Geometry	Knowing	Net of triangular prism	MC	4	1	85%
MFC502A	MCK	Data	Reasoning	Unlabeled bar graph - interpreting information	MC	3	1	85%
MFC502B	MPCK	Data	Curric/Plan	Difficulty with a data representation problem	CR	SG	2	23% (FC) 51% (PC)
MFC503A	MCK	Number	Knowing	Numbers - rational or irrational	CMC	2	1	74%
MFC503B	MCK	Number	Knowing	Numbers - rational or irrational	CMC	1	1	89%
MFC503C	MCK	Number	Knowing	Numbers - rational or irrational	CMC	1	1	69%
MFC503D	MCK	Number	Knowing	Numbers - rational or irrational	CMC	1	1	42%
MFC505	MPCK	Number	Curric/Plan	Identify two most difficult number-story problems	CR	SG	2	77% (FC) 20% (PC)
MFC508	MCK	Algebra	Applying	Matchstick pattern - predict Figure 10	MC	2	1	74%
MFC509	MCK	Algebra	Knowing	The larger of 2n and n + 2	CR	SG	2	12% (FC) 21% (PC)
MFC511	MCK	Geometry	Applying	Length of ribbon of two gift boxes	CR	SG	2	19% (FC) 19% (PC)
MFC513	MPCK	Geometry	Curric/Plan	Two reasons for measuring with paper clips	CR	SG	2	9% (FC) 39% (PC)

[1] SG – See Scoring Guide provided with the item in this document.
[2] FC – Fully correct (2 score points); PC – Partially correct (1 score point)

ID: MFC106	MS Booklet: PM1, PM5	MS Block: B1PM	Item Format: MC	Max Points: 1
Knowledge Dimension: MCK	Content Domain: Data		Sub-domain: Applying	

MFC106 Two fair six-sided number cubes are thrown in a probability game and the two numbers at the top are recorded.



[Josie] wins if the difference between the two numbers is 0, 1 or 2.
[Farid] wins if the difference between the two numbers is 3, 4 or 5.

The students discuss whether the game is fair.

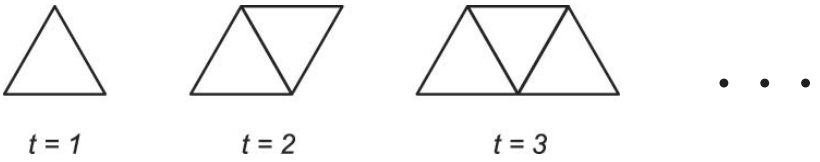
Which of the following statements is correct?

Check <u>one</u> box.

A.	Both have an equal chance of winning.	☐ 1
B.	[Josie] has the greater chance of winning.	☐ 2
C.	[Farid] has the greater chance of winning.	☐ 3
D.	As the game involves number cubes, it's not possible to say who has the greater chance of winning.	☐ 4

ID: MFC108	MS Booklet: PM1, PM5	MS Block: B1PM	Item Format: MC	Max Points: 1
Knowledge Dimension: MPCK	Content Domain: Algebra		Sub-domain: Enacting	

MFC108 [Amy] is building a sequence of geometric figures with toothpicks by following the pattern shown below. Each new figure has one extra triangle.
Variable *t* denotes the position of a figure in the sequence.



In finding a mathematical description of the pattern, [Amy] explains her thinking by saying:

I use three sticks for each triangle.

Then I see that I am counting one stick twice for each triangle, except the last one, so I have to remove those.

Variable *n* represents the total number of toothpicks used in a figure.

Which of the equations below best represent [Amy's] statement in algebraic notation?

Check <u>one</u> box.

A. $n = 2t + 1$ ☐ 1

B. $n = 2(t + 1) - 1$ ☐ 2

C. $n = 3t - (t - 1)$ ☐ 3

D. $n = 3t + 1 - t$ ☐ 4

ID: MFC202A MFC202B MFC202C MFC202D	MS Booklet: PM1, PM2	MS Block: B2PM	Item Format: CMC	Max Points: 4
Knowledge Dimension: MCK	Content Domain: Algebra		Sub-domain: Knowing	

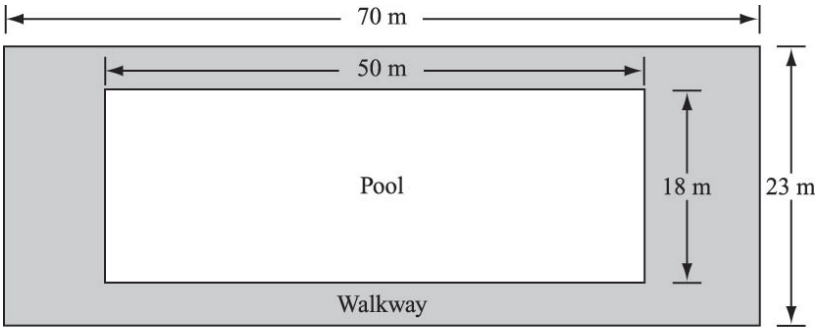
Indicate whether each of the following statements is true for the set of all whole numbers a, b and c greater than zero.

Check <u>one</u> box in <u>each</u> row.

			True	Not True
MFC202A	A.	$a - b = b - a$	☐ 1	☐ 2
MFC202B	B.	$a \div b = b \div a$	☐ 1	☐ 2
MFC202C	C.	$(a + b) + c = a + (b + c)$	☐ 1	☐ 2
MFC202D	D.	$(a - b) - c = a - (b - c)$	☐ 1	☐ 2

ID: MFC203	MS Booklet: PM1, PM2	MS Block: B2PM	Item Format: MC	Max Points: 1
Knowledge Dimension: MCK	Content Domain: Geometry		Sub-domain: Applying	

MFC203 A rectangular-shaped swimming pool has a paved walkway (shaded) around it as shown.



not to scale

What is the area of the walkway?

Check <u>one</u> box.

A. 100 m^2 ☐1

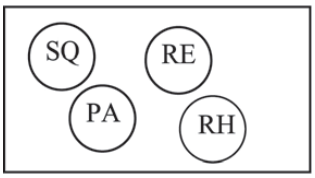
B. 161 m^2 ☐2

C. 710 m^2 ☐3

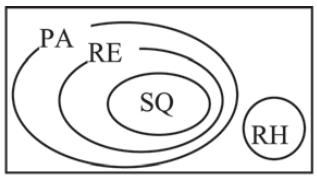
D. 1610 m^2 ☐4

ID: MFC204	MS Booklet: PM1, PM2	MS Block: B2PM	Item Format: MC	Max Points: 1
Knowledge Dimension: MCK	Content Domain: Geometry		Sub-domain: Knowing	

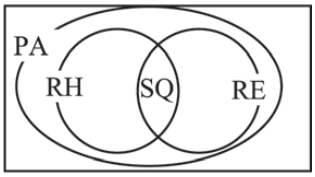
MFC204 Three students have drawn the following Venn diagrams showing the relationships between four quadrilaterals:
Rectangles (RE), Parallelograms (PA), Rhombuses (RH), and Squares (SQ).



[Tian]



[Rini]



[Mia]

Which student's diagram is correct?

Check <u>one</u> box.

A. [Tian] ☐1

B. [Rini] ☐2

C. [Mia] ☐3

ID: MFC206A	MS Booklet: PM1, PM2	MS Block: B2PM	Item Format: MC	Max Points: 1
Knowledge Dimension: MCK	Content Domain: Number		Sub-domain: Applying	

MFC206A (a) A machine uses 2.4 litres of fuel for every 30 hours of operation.
How many litres of fuel will the machine use in 100 hours if it continues to use fuel at the same rate?

Check <u>one</u> box.

A. 7.2 ☐1

B. 8.0 ☐2

C. 8.4 ☐3

D. 9.6 ☐4

ID: MFC206B	MS Booklet: PM1, PM2	MS Block: B2PM	Item Format: CR	Max Points: 1
Knowledge Dimension: MPCK	Content Domain: Number		Sub-domain: Planning	

MFC206B

(b) Create a different problem of the same type as the problem in (a) (same processes/operations) that is **EASIER** for <primary> children to solve.

Code	Response	Item: MFC206B
	Correct Response	
10	A different problem of the **same type** (same processes/operations) but is **easier** to solve. *Example:* • *A machine uses 3 litres of fuel for every 30 hours of operation. How many litres of fuel will the machine use in 100 hours?* • *A car uses 2.4 litres of fuel for every 50 km. How many litres of fuel will the car use in 100 km?*	
	Incorrect Response	
70	A different problem of the **same type** (same processes/operations) but is **NOT easier** to solve. (Note: Items judged to be of the **same** level of difficulty are NOT easier.) *Examples:* • *A machine uses 2 litres of fuel for every 30 hours of operation. How many litres of fuel will the machine use in 100 hours? (2 is not divisible by 3)* • *A tap drips 2 litres of water every day. How many ml is this per second? (the metric knowledge required and computational load is significantly higher)*	
79	Other incorrect (including crossed out, erased, stray marks, illegible, or off task) *Example:* • *Questions that are not meaningful/have no answer*	
	Non-response	
99	Blank	

ID: MFC208A	MS Booklet: PM1, PM2	MS Block: B2PM	Item Format: CR	Max Points: 2
Knowledge Dimension: MPCK	Content Domain: Number		Sub-domain: Enacting	

[Jeremy] notices that when he enters 0.2 × 6 into a calculator his answer is smaller than 6, and when he enters 6 ÷ 0.2 he gets a number greater than 6. He is puzzled by this, and asks his teacher for a new calculator!

MFC208A (a) What is [Jeremy's] most likely misconception?

Code	Response	Item: MFC208A
	Correct Response	
20	Responses that suggest the misconception is that multiplication always gives a larger answer **and** that division always gives a smaller answer. *Example:* • *He thinks that when you multiply the answer should be larger and when you divide the answer should be smaller.*	
	Partially Correct Response	
10	Responses that suggest the misconception is that multiplication always gives a larger answer **or** that division always gives a smaller answer **but not both**. *Examples:* • *He thinks that when you multiply the answer should be larger than either/both numbers.* • *He thinks that division should give an answer that is smaller than the numbers you started with.*	
11	Responses that suggest that Jeremy considers 0.2 as a whole number. *Example:* • *He thinks he is multiplying and dividing by 2 rather than by 0.2.*	
	Incorrect Response	
70	Responses relating to understanding of decimal numbers, decimal multiplication/division or use of a calculator. *Example:* • *He doesn't understand decimal multiplication (or division).* • *He doesn't know how to use his calculator.* • *Mathematical operations.* • *The decimal point.*	
79	Other incorrect (including crossed out, erased, stray marks, illegible, or off task)	
	Non-response	
99	Blank	

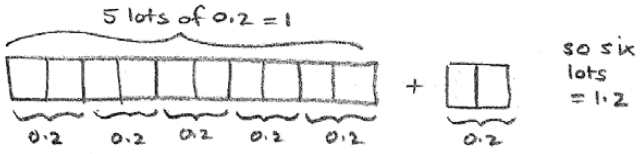
ID: MFC208B	MS Booklet: PM1, PM2	MS Block: B2PM	Item Format: CR	Max Points: 2
Knowledge Dimension: MPCK	Content Domain: Number		Sub-domain: Enacting	

MFC208B (b) Draw a visual representation that the teacher could use to model 0.2 × 6 to help [Jeremy] understand **WHY** the answer is what it is?

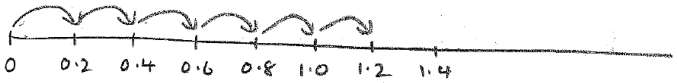
Code	Response	Item: MFC208B
	Correct Response	
20	A suitable visual representation that clearly shows why 0.2 × 6 is 1.2. *Example:* • *6 lots of 0.2 making it clear that 5 lots of 0.2 = 1, probably with some annotation. See Pictures 1, 2, 3 and 4 below.*	
	Partially Correct Response	
10	A visual representation that shows 6 lots of 0.2 but does NOT make it clear how this equals 1.2. Accept 0.2 shown as one-fifth or as two-tenths. *Example: See Picture 5 below.*	
11	A visual representation that shows how 5 lots of 0.2 make a whole but does NOT make it clear how 6 lots of 0.2 equals 1.2 *Example: See Picture 6 below.*	
12	A visual representation of an equation 0.2 × 6 = 1.2 without showing why it is true. *Example: See Picture 7 below.* • 0.2 + 0.2 + 0.2 + 0.2 + 0.2 + 0.2 = 1.2	
	Incorrect Response	
70	A visual representation showing 6 lots of 0.2 without showing what 0.2 is or how 5 lots of 0.2 equals 1. *Example: See Picture 8 below.*	
71	An example in words suggesting counting in lots of 0.2. *Example:* • *"Count 6 lot's of 0.2 as follows: 0.2, 0.4, 0.6, 0.8, 1.0, 1.2"* Note: This is a good teaching strategy but is not a visual representation.	
79	Other incorrect (including crossed out, erased, stray marks, illegible, or off task) *Example: An equation or written calculation of the form* 0.2 × 6 = 1.2	
	Non-response	
99	Blank	

Correct Responses (Code 20)

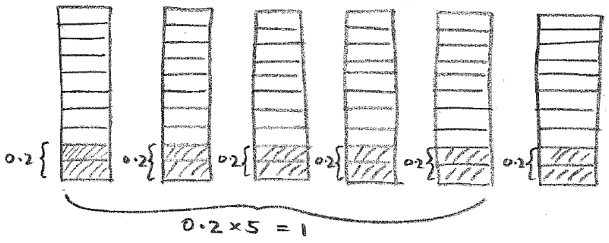
Picture 1



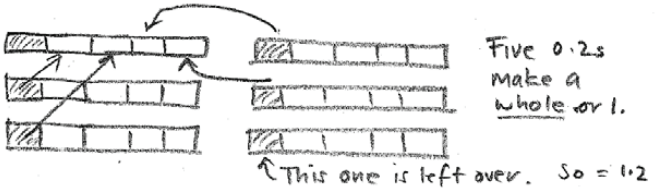
Picture 2



Picture 3

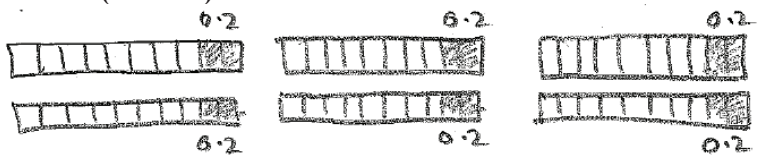


Picture 4

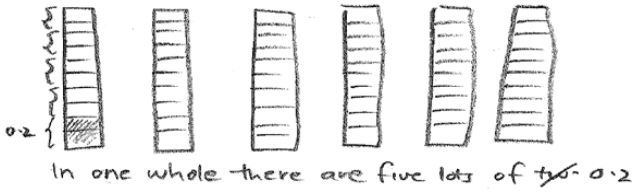


Partially Correct Responses

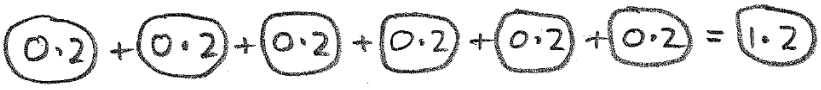
Picture 5 (Code 10)



Picture 6 (Code 11)



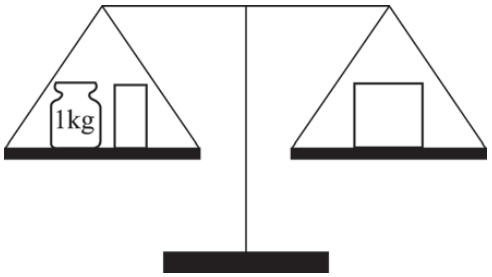
Picture 7 (Code 12)



Incorrect response (Code 70)
Picture 8

ID: MFC303	MS Booklet: PM2, PM3	MS Block: B3PM	Item Format: MC	Max Points: 1
Knowledge Dimension: MCK	Content Domain: Algebra		Sub-domain: Applying	

MFC303 The objects on the scale make it balance exactly. On the left pan there is a 1 kg mass and half a brick. On the right pan there is one whole brick.



What is the mass of one whole brick?

Check <u>one</u> box.

A. 0.5 kg ☐1
B. 1 kg ☐2
C. 2 kg ☐3
D. 3 kg ☐4

ID: MFC304	MS Booklet: PM2, PM3	MS Block: B3PM	Item Format: MC	Max Points: 1
Knowledge Dimension: MCK	Content Domain: Number		Sub-domain: Knowing	

MFC304 How many decimal numbers are there between 0.20 and 0.30?

Check <u>one</u> box.

A. 9 ☐1

B. 10 ☐2

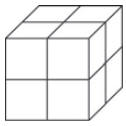
C. 99 ☐3

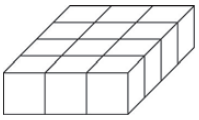
D. An infinite number ☐4

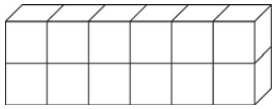
ID: MFC307A	MS Booklet: PM2, PM3	MS Block: B3PM	Item Format: MC	Max Points: 1
Knowledge Dimension: MCK	Content Domain: Geometry		Sub-domain: Knowing	

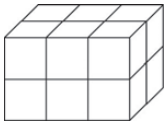
The following problem was given to <primary school> children.

All the small blocks are the same size. Which stack of blocks has a different volume from the others?

A. 

B. 

C. 

D. 

MFC307A (a) What is the correct answer to this question?

Check <u>one</u> box.

A. Stack A ☐1

B. Stack B ☐2

C. Stack C ☐3

D. Stack D ☐4

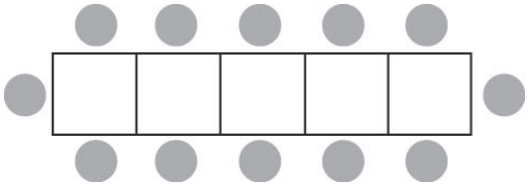
ID: MFC307B	MS Booklet: PM2, PM3	MS Block: B3PM	Item Format: CR	Max Points: 2
Knowledge Dimension: MPCK	Content Domain: Geometry		Sub-domain: Curriculum/Planning	

MFC307B (b) How could the question above be rewritten so that it assesses the same skills but **WITHOUT** using the word **VOLUME**?

Code	Response	Item: MFC307B
	Correct Response	
20	A reworded version of the question in (a) that assesses the **same** skills but **without** using the word 'volume'. *Examples:* • *Which stack of blocks is made from a different number of small blocks compared to the others?* • *All the small blocks are the same mass/weight. Which stack of blocks has a different mass/weight from the others?*	
	Partially Correct Response	
10	A question without the word 'volume' that assesses the **same** skills but is a **different question** to (a). Examples: • *Which stack of blocks has **less** blocks than any other?* • *Which stack of blocks takes up the **least** space?*	
	Incorrect Response	
70	A meaningful/answerable rewording of the question that assesses a skill **other than volume**. *Example:* • *Which stack of blocks has the largest **surface area**?*	
71	An unclear/ill-defined/unanswerable question *Examples:* • *Which stack of blocks is not equal in **size** to the others? ('Size' is too vague.)* • *Which stack of blocks takes up the **most** space? (There are 3 with the same volume.)* • *One of the stacks is different from the others. Solve the mystery! (Different in what way?)*	
79	Other incorrect (including crossed out, erased, stray marks, illegible, or off task, unsolvable)	
	Non-response	
99	Blank	

ID: MFC308	MS Booklet: PM2, PM3	MS Block: B3PM	Item Format: CR	Max Points: 2
Knowledge Dimension: MCK	Content Domain: Algebra		Sub-domain: Applying	

A square table can seat four people, one on each side. When 5 square tables are placed side by side, as shown below, 12 people can sit around them, 5 on each side and 2 on the ends.



How many people can sit around *n* square tables when they are placed side by side?

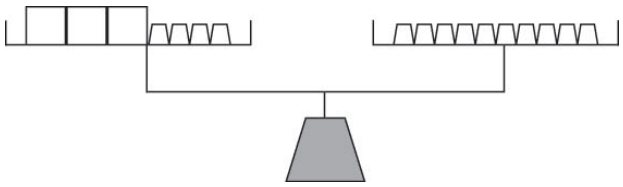
MFC308 Write your answer to the problem in terms of *n*.

Code	Response	Item: MFC308
	Correct Response	
20	$2n + 2$ or equivalent expression. *Examples:* • $(n \times 2) + 2$ • $2(n + 1)$ • $4n – 2(n – 1)$	
21	Correct rule in words in terms of n. *Example:* • n multiplied by 2, then add 2.	
	Partially Correct Response *[See Note below]*	
10	Correct rule but with variable confusion. **Variable(s) defined**. *Examples:* • $n = 2x + 2$ *where* x *= no. of tables*. • $p = 2t + 2$ *where* t *= no. of tables and* p *= no. of people.*	
11	Correct rule but with variable confusion. **Variable(s) NOT defined**. *Examples:* • $2x + 2$ • $4x – 2(x – 1)$	
12	Correct rule in words but **NOT** in terms of n. *Example:* • Multiply by two and add two.	
13	An iterative rule. *Examples:* • $P_n = P_{n-1} + 2$ • *Add* 2 *each time you add a table/square.*	
	Incorrect Response	
70	An incorrect rule, in words or symbols. *Examples:* $2n – 2$	
79	Other incorrect (including crossed out, erased, stray marks, illegible, or off task)	
	Non-response	
99	Blank	

Note: After psychometric analysis, this was recoded to a dichotomous item. Categories 20 and 21 were awarded one score point. Categories 10 – 13 were awarded zero score points.

ID: MFC312	MS Booklet: PM2, PM3	MS Block: B3PM	Item Format: MC	Max Points: 1
Knowledge Dimension: MPCK	Content Domain: Algebra		Sub-domain: Curriculum/Planning	

MFC312 If *B* represents the weight (in grams) of each box, □, pictured below, and ⏢ represents a one-gram weight, the equation $3B + 4 = 10$ can be pictured by the pan balance shown below.



An inequality such as $3B + 4 < 10$ or $3B + 4 > 10$ would show one side of the pan balance lower than the other.

Ms. [Clarke] is preparing to teach a unit on solving linear equations and inequalities.

If *X* represents the weight of a given box, which of the following sentences can **NOT BE REPRESENTED** by a pan balance?

Check <u>one</u> box.

A. $13 = 4X + 5$ ☐1

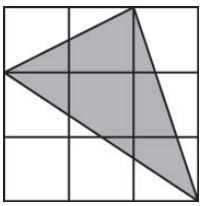
B. $3X + 10 = 4$ ☐2

C. $3X + 3 = 2X + 15$ ☐3

D. $9 + 6X < 21$ ☐4

ID: MFC408	MS Booklet: PM3, PM4	MS Block: B4PM	Item Format: MC	Max Points: 1
Knowledge Dimension: MCK	Content Domain: Geometry		Sub-domain: Applying	

MFC408 The area of each small square is 1 cm^2.



What is the area of the shaded triangle in cm^2?

Check <u>one</u> box.

A. 3.5 cm^2 ☐ 1

B. 4 cm^2 ☐ 2

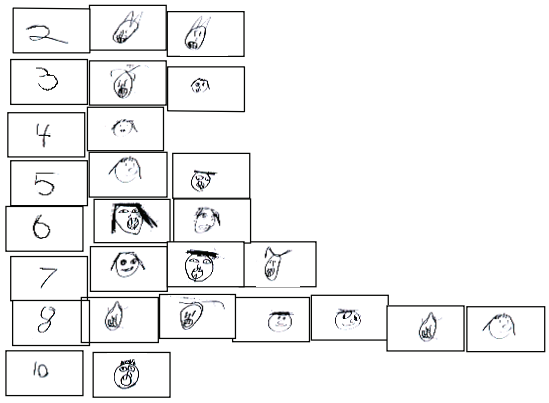
C. 4.5 cm^2 ☐ 3

D. 5 cm^2 ☐ 4

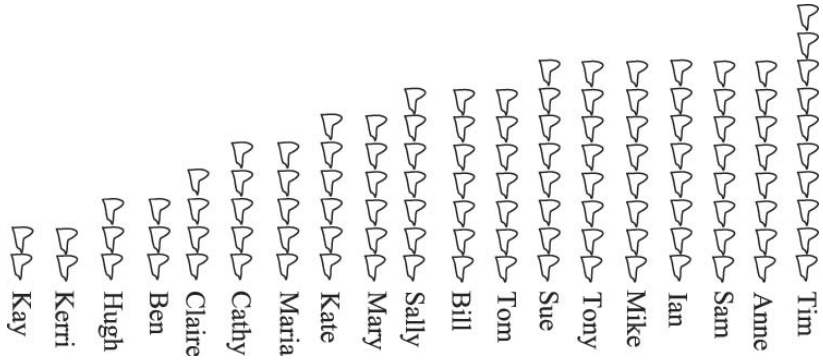
ID: MFC410	MS Booklet: PM3, PM4	MS Block: B4PM	Item Format: CR	Max Points: 2
Knowledge Dimension: MPCK	Content Domain: Data		Sub-domain: Enacting	

Imagine that two <primary> students in the same class have created the following representations to show the number of teeth lost by their classmates.[3]

[Mary] drew pictures of her classmates on cards to make this graph.



[Sally] cut out pictures of teeth to make this graph.



MFC410 From a data presentation point of view, how are the representations alike and how are they different?

Alike:

Different:

[3] This item was used with permission of the author, Dr. Maria Alejandra Sorto, and is based on her Ph. D. dissertation, *Prospective middle school teachers' knowledge about data analysis and its application to teaching*, completed in 2004 at Michigan State University.

Code	Response	Item: MFC410
	Correct Response	
20	Responses that indicate how the representations are **alike** AND how they are **different.** Alike: *Examples:* • *They both show the same data/same number of teeth lost.* • *They are both pictorial representations.* • *They are both forms of bar graphs.* • *They are both skewed in the same direction.* Different: *Examples:* • *Mary has grouped the data/done a frequency tally whereas Sally has not.* • *'In Mary's graph each bar or column represents the number of teeth lost, whereas in Sally's graph each column or stack represents a student.'* • *Mary's graph is categorized by the number of teeth lost whereas Sally's is person by person.*	
	Partially Correct Response	
10	**The 'alike' description is acceptable** but the 'different' description is not acceptable, trivial or is missing. Alike: *Example:* • *They both show the same number of teeth lost.* Different: *Examples:* • *Mary's is easier to comprehend than Sally's.*	
11	**The 'different' description is acceptable** but the 'alike' description is not acceptable, trivial or is missing. Alike: *Example:* • *They both made graphs about teeth.* (Trivial) Different: *Examples:* • *Sally made column for each student whereas Mary made a column for each number of teeth lost.*	
	Incorrect Response	
70	Responses that are insufficient or trivial. Alike: *Examples:* • *They are both graphs.* • *Both graphs are about teeth.* Different: *Examples:* • *Mary used numbers, Sally didn't.* • *Mary's is hard to read, Sally's is easier.*	
79	Other incorrect (including crossed out, erased, stray marks, illegible, or off task)	
	Non-response	
99	Blank	

ID: MFC412A MFC412B	MS Booklet: PM3, PM4	MS Block: B4PM	Item Format: MC	Max Points: 2
Knowledge Dimension: MCK	Content Domain: Algebra		Sub-domain: Knowing	

[Sam] wanted to find three consecutive **EVEN** numbers that add up to 84.
He wrote the equation $k + (k + 2) + (k + 4) = 84$.

MFC412A (a) What does the letter *k* represent?

Check one box.

A. The least of the three even numbers. ☐ 1

B. The middle even number. ☐ 2

C. The greatest of the three even numbers. ☐ 3

D. The average of the three even numbers. ☐ 4

MFC412B Which of the following expressions could represent the sum of three consecutive **ODD** numbers?

Check one box.

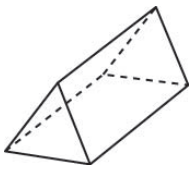
A. $m + (m + 1) + (m + 3)$ ☐ 1

B. $m + (m + 2) + (m + 4)$ ☐ 2

C. $m + (m + 3) + (m + 5)$ ☐ 3

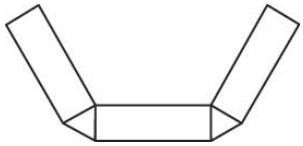
D. $m + (m + 4) + (m + 6)$ ☐ 4

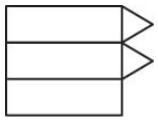
ID: MFC501	MS Booklet: PM4, PM5	MS Block: B5PM	Item Format: MC	Max Points: 1
Knowledge Dimension: MCK	Content Domain: Geometry		Sub-domain: Knowing	



MFC501 Which of the following could be folded to make a shape like the 3-D figure above?

Check <u>one</u> box.

A.  ☐1

B.  ☐2

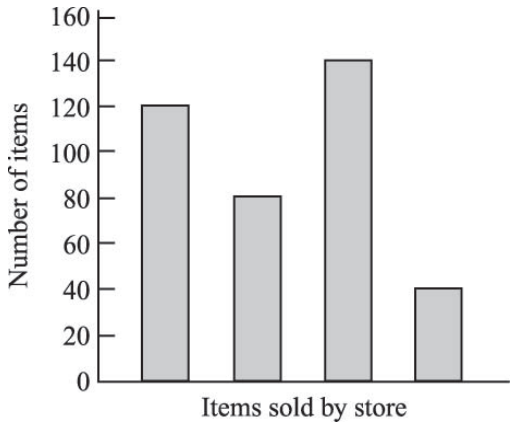
C.  ☐3

D.  ☐4

ID: MFC502A	MS Booklet: PM4, PM5	MS Block: B5PM	Item Format: MC	Max Points: 1
Knowledge Dimension: MCK	Content Domain: Data		Sub-domain: Reasoning	

The following problem was given to children in <primary> school.

The graph shows the number of pens, pencils, rulers and erasers sold by a store in one week.



The names of the items are missing from the graph. Pens were the item most often sold. Fewer erasers than any other item were sold. More pencils than rulers were sold.

MFC502A (a) How many pencils were sold?

Check <u>one</u> box.

A. 40 ☐1

B. 80 ☐2

C. 120 ☐3

D. 140 ☐4

ID: MFC502B	MS Booklet: PM4, PM5	MS Block: B5PM	Item Format: CR	Max Points: 2
Knowledge Dimension: MPCK	Content Domain: Data		Sub-domain: Curriculum/Planning	

MFC502B (b) Some <primary> students would experience difficulty with a problem of this type. What is the main difficulty you would expect? Explain clearly with reference to the problem.

Code	Response	Item: MFC502B
	Correct Response	
20	Responses that refer to reading and comprehension difficulties related to the **complexity of the language** used in the question with **reasons** and/or **references to specific examples.** Examples: • *The language used is quite challenging. Example, "fewer than any other" and "more pencils than rulers".* • *Students would be challenged by the difficulty/complexity of the wording in the question such as 'most often' 'fewer'. There is a considerable load on their 'higher order' skills as they are required to organise, interpret and relate back to the graph.* • *The items described in the text are listed in a different order to the bars on the graph creating logistic or sequencing challenges.*	
	Partially Correct Response	
10	Less detailed responses that recognize that the language is likely to be a difficulty for children but **without reasons or examples.** *Examples:* • *They would have trouble with the language used in the question.* • *Reading and comprehending the text would be difficult for many children.* • *There is a considerable amount of information to read, organize, sequence and relate to the graph.*	
11	A statement describing difficulties attributable to the graph rather than the text. *Examples:* • *They would have trouble reading the graph.* • *The names are missing from the graph and they wouldn't have experienced this before.*	
12	A statement attributing difficulties to the level of **problem-solving** or **analysis** required without explaining how/why. *Examples:* • *They would have trouble analyzing the information in the problem.* • *The problem requires problem-solving strategies and they would have trouble with that.*	
	Incorrect Response	
79	Incorrect (including crossed out, erased, stray marks, illegible, or off task)	
	Non-response	
99	Blank	

ID: MFC503A MFC503B MFC503C MFC503D	MS Booklet: PM4, PM5	MS Block: B5PM	Item Format: CMC	Max Points: 4
Knowledge Dimension: MCK	Content Domain: Number		Sub-domain: Knowing	

Indicate for each number whether it is rational or irrational.

Check <u>one</u> box in each <u>row</u>.

			Rational	**Irrational**
MFC503A	A.	π	☐ 1	☐ 2
MFC503B	B.	2	☐ 1	☐ 2
MFC503C	C.	$\sqrt{49}$	☐ 1	☐ 2
MFC503D	D.	$-\frac{3}{2}$	☐ 1	☐ 2

ID: MFC505	MS Booklet: PM4, PM5	MS Block: B5PM	Item Format: CR	Max Points: 2
Knowledge Dimension: MPCK	Content Domain: Number		Sub-domain: Curriculum/Planning	

A <Grade 1> teacher asks her students to solve the following four story problems, in any way they like, including using materials if they wish.

Problem 1: [Jose] has 3 packets of stickers. There are 6 stickers in each pack. How many stickers does [Jose] have altogether?
Problem 2: [Jorgen] had 5 fish in his tank. He was given 7 more for his birthday. How many fish did he have then?
Problem 3: [John] had some toy cars. He lost 7 toy cars. Now he has 4 cars left. How many toy cars did [John] have before he lost any?
Problem 4: [Marcy] had 13 balloons. 5 balloons popped. How many balloons did she have left?

The teacher notices that two of the problems are more difficult for her children than the other two.

MFC505 Identify the **TWO** problems which are likely to be more **DIFFICULT** to solve for <Grade 1> children.

Problem _____ and Problem _____

Code	Response	Item: MFC505
	Correct Response	
20	Problem 1 and Problem 3 (or Problem 3 and Problem 1)	
	Partially Correct Response	
10	Problem 1 only correct (with or without Problems 2 and 4) *Examples:* • *Problem 1 and Problem 2 (or 2 and 1)* • *Problem 1 and Problem 4 (or 4 and 1)* • *Problem 1 and Problem ___ (blank)*	
11	Problem 3 only correct (with or without Problems 2 and 4) *Examples:* • *Problem 3 and Problem 2 (or 2 and 3)* • *Problem 3 and Problem 4 (or 4 and 3)* • *Problem 3 and Problem (blank)*	
	Incorrect Response	
70	At least one problem selected but **neither Problem 1 nor Problem 3.** *Examples:* • *Problem 2 and Problem 4 (or 4 and 2)* • *Problem 2 and Problem ___ (blank)* • *Problem 4 and Problem ___ (blank)*	
79	Other incorrect (including crossed out, erased, stray marks, illegible, or off task)	
	Non-response	
99	Blank	

ID: MFC508	MS Booklet: PM4, PM5	MS Block: B5PM	Item Format: MC	Max Points: 1
Knowledge Dimension: MCK	Content Domain: Algebra		Sub-domain: Applying	

MFC204 Matchsticks are arranged as shown in the figures.

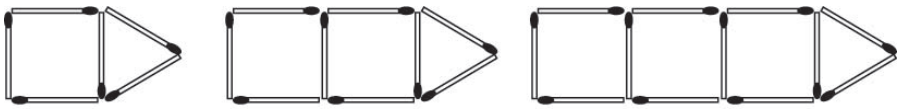


Figure 1 Figure 2 Figure 3

If the pattern is continued, how many matchsticks would be used to make Figure 10?

Check <u>one</u> box.

A. 30 ☐1

B. 33 ☐2

C. 36 ☐3

D. 39 ☐4

E. 42 ☐5

ID: MFC509	MS Booklet: PM4, PM5	MS Block: B5PM	Item Format: CR	Max Points: 2
Knowledge Dimension: MCK	Content Domain: Algebra		Sub-domain: Knowing	

Students who had been studying algebra were asked the following question:

For any number n, which is larger, $2n$ or $n + 2$?

MFC509 Give the answer and show your reasoning or working.

<table>
<tr><th>Code</th><th>Response</th><th>Item ID: MFC509</th></tr>
<tr><td></td><td colspan="2">Correct Response</td></tr>
<tr><td>20</td><td colspan="2">A correct general solution written in words or using inequalities.
Examples:
• Correct inequality statements OR without the $n = 2$ case
$If\ n > 2,\ then\ 2n > n + 2.$ $If\ n > 2,\ then\ 2n > n + 2.$
$If\ n = 2,\ then\ 2n = n + 2.$ $If\ n < 2,\ then\ 2n < n + 2.$
$If\ n < 2,\ then\ 2n < n + 2.$

• In words, such as, “$n + 2$ is larger when n is less than 2 and $2n$ is larger when n is greater than 2.”</td></tr>
<tr><td>21</td><td colspan="2">A correct general solution using graphs.
• Responses that construct a graph of $y = n + 2$ and $y = 2n$ AND show on the graph where one is greater than the other OR conclude in words that $n + 2 > 2n$ when $n < 2$ and $2n > n + 2$ when $n > 2$.</td></tr>
<tr><td>22</td><td colspan="2">A correct, ordered, specific-value solution.
Examples:
• A table (or sequential list of ordered pairs) with values of n and evaluations of $2n$ and $n+2$ AND from the table/list conclude that $n + 2 > 2n$ when $n < 2$ and $2n > n + 2$ when $n > 2$.

<table>
<tr><th>n</th><th>$2n$</th><th>$n + 2$</th></tr>
<tr><td>1</td><td>2</td><td>3</td></tr>
<tr><td>2</td><td>4</td><td>4</td></tr>
<tr><td>3</td><td>6</td><td>5</td></tr>
<tr><td>4</td><td>8</td><td>6</td></tr>
</table>

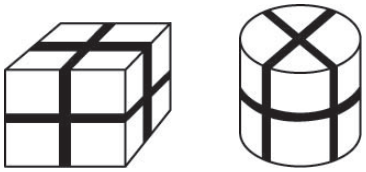
“The table shows that $2n$ is less than $n+2$ when n is less than 2 and that $2n$ is greater than $n+2$ when n is greater than 2.”</td></tr>
</table>

Continued next page

	Partially Correct Response
10	**General responses** that are 'on the right track' but incomplete or are limited in some way. *Examples:* • *One correct inequality without the other.* *E.g. If* $n > 2$*, then* $2n > n + 2$. *E.g.* $2n$ *is greater than* $n + 2$ *when n is greater than* 2. • *Two inequalities but only one is correct.* *E.g. (a) If* $n < 2$*, then* $2n > n + 2$ *(incorrect) and if* $n > 2$*, then* $n + 2 < 2n$ *(correct).* *E.g. (b) If* $n < 2$*,* $n + 2$ *is larger (correct) and if* $n > 2$*,* $n + 2$ *is larger (incorrect).*
11	**Graphical solutions** that are 'on the right track' but incomplete or are limited in some way. *Examples:* • *Two correct graphs* ***without*** *showing on the graph where one is greater than the other OR* ***without*** *concluding in words that* $n + 2 > 2n$ *when* $n < 2$ *and* $2n > n + 2$ *when* $n > 2$. • *Two graphs but* ***only one is correct****. The conclusion or annotation with the graphs must be correct for the two graphs shown.*
12	**Specific-value solutions** that are 'on the right track' but incomplete or are limited in some way. *Examples:* • *Responses that use trial-and-error and* ***more than one*** *specific value of n but* ***do not generalize*** *them into the same categories as shown under code 20.* • *Responses that say it* ***depends on the value of n*** *with* ***more than one*** *supporting example. For example, "It depends. When* $n = 1$*,* $n + 2$ *is larger, when* $n = 5$*,* $2n$ *is larger."*
	Incorrect Response
70	Responses that indicate that: • it cannot be known which is larger because the value of *n* is not known*; or* • 'it depends on the value of *n',* **with no (or only one)** supporting example or with no other valid argument.
71	One correct inequality only and an additional error. *Examples:* • $2n > n + 2$ *when* $n > 1$ • $n + 2$ *is greater than* $2n$ *when n is* 1 *or less* (Has assumed *n* is integral)
72	Conclusion reached on the basis of **only one specific value of n**. *Example: If* $n = 10$*,* $2n = 20$ *and* $n + 2 = 12$ *so* $2n > n + 2$
73	Responses that select $2n$ with no correct qualifying inequality (e.g. without 'when $n > 2$')
79	Other incorrect (including crossed out, erased, stray marks, illegible, or off task).
	No Response
99	Blank.

ID: MFC511	MS Booklet: PM4, PM5	MS Block: B5PM	Item Format: CR	Max Points: 2
Knowledge Dimension: MCK	Content Domain: Geometry		Sub-domain: Applying	

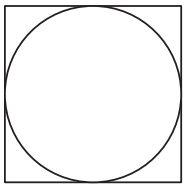
Two gift boxes wrapped with ribbon are shown below. Box A is a cube of side-length 10 cm. Box B is a cylinder with height and diameter 10 cm each.



A B

MFC511 Which box needs the longer ribbon? _____

Explain how you arrived at your answer

Code	Response	Item: MFC511 and MFC703
	Correct Response	
20	Box A with a correct and complete explanation involving calculations of ribbon lengths *Examples:* • *Box A needs* $6 \times 20 = 120$ cm *ribbon. Box B needs* 4 × 20 = 80 *cm plus the circumference which is* 10π. $10\pi < 40$ *so Box A needs more ribbon.* • *Box A. Box A needs* 120 *cm but box B needs about* 110 *cm (using* $\pi = 3$*).*	
21	Box A based upon a complete argument (with or without calculation) comparing the square and circumference (both of equal 'width') together **with a statement that the other lengths of ribbon are equal.** *Examples:* • *Box A because the circumference of a circle diameter 10 is less than the perimeter of a square of side 10 and the other lengths of ribbon are the same.* • *Box A. As shown in the diagram, the ribbon around the cylinder is less than the ribbon around the square. The other lengths of ribbon are equal on each box. Therefore Box B needs less ribbon altogether than Box A.*  • *Box A. The circumference is about 31.4 but the perimeter of the square is 40. So Box A needs more ribbon because the other ribbon is the same (80) on both boxes.*	

*Note: Accept reasonable approximations of π such as 3.14, 3.1, 3, 22/7 etc.

Continued next page

	Partially Correct Response
10	Box A with a correct and complete explanation as in Code 20 but with **one identifiable calculation error** (or use of a wrong formula) logically leading to Box A. *Example:* • *Box A because Box A needs 120 cm and Box B needs* **60** + 10π < 120.
11	Box B with a correct and complete explanation as in Code 20 but with **one identifiable calculation error** (or use of a wrong formula) logically leading to Box B. *Examples:* • 80 + 10π = 120.4 (rather than 111.4) > 120. • *Box B because Box A needs 120 cm of ribbon and box B needs 80 +* 25π *> 120. (Used area formula instead of circumference formula but intending to compare perimeter.)*
12	Box A with an explanation that correctly calculates and compares the lengths of ribbon on each box that are **different** but **fails to mention that the other lengths of ribbon are the same.** *Example:* • *Box A needs more ribbon because the circumference of the cylinder is* 10π *which is less than the perimeter of the square,* 40.
13	Box A with an explanation that correctly supports the choice of Box A but that is **limited** and/or **lacking the detail** of a Code 20 or 21 response. *Examples:* • *Box A because Box B can fit inside Box A.* • *Box A because the circumference is less than the perimeter.* • *Box A. You can see it's bigger. Its ribbon is 120 cm but Box B would be less.*
	Incorrect Response
70	Box A **without** any explanation or calculation. *Example: Box A*
71	Box A or B with an explanation based on a **conceptual error**. *Examples:* • *Box A but with an explanation based upon surface* ***area or volume****.* • *Box A because it has more sides.*
72	Box A or B with an explanation based on **incorrect and/or incomplete** ribbon lengths for **both** boxes. *Example:* • *Box B because Box A needs* 60 *cm but box B needs more than* 80.
73	Neither. The length of ribbon needed is the **same.** *Example:* • *Length width and height are the same therefore they need the same amount of ribbon.*
79	Other incorrect (including crossed out, erased, stray marks, illegible, or off task) *Example:* • *Box B* **without** any explanation or calculation.
	Non-response
99	Blank

ID: MFC513	MS Booklet: PM4, PM5	MS Block: B5PM	Item Format: CR	Max Points: 2
Knowledge Dimension: MPCK	Content Domain: Geometry		Sub-domain: Curriculum/Planning	

When teaching children about length measurement for the first time, Mrs. [Ho] prefers to begin by having the children measure the width of their book using paper clips, then again using pencils.

MFC513 Give **TWO** reasons she could have for preferring to do this rather than simply teaching the children how to use a ruler?

Reason 1:

Reason 2:

Code	Response	Item: MFC513
Note: Significant and acceptable reasons Reason 1: (Understanding of what measurement is) Using familiar/different units enables understanding of what measurement is, that any object/unit can be used to measure, that the scale on a ruler is just the repetition of a basic unit. Reason 2: (Need for standard units) Use of non-standard units can, by creating uncertainty about length, show the **need** for standard/formal units and possibly create opportunities to discuss the (historical) development of measurement. Reason 3: (Choosing most appropriate unit) Using objects of different lengths helps children learn how to decide which unit/object is the most appropriate to measure a given length.		
	Correct Response	
20	Responses that give **any TWO** of the three significant and acceptable reasons noted above.	
	Partially Correct Response	
10	Responses that give **Reason 1 only**: (Understanding of what measurement is) *Examples:* • *Using familiar objects to measure enables young students to focus just on the idea of measurement before they have to deal with formal units and the skill of using a ruler.* • *Using everyday objects to measure shows that anything can be used to measure and makes measurement easier to understand because there is no abstract scale to read.*	
11	Responses that give **Reason 2 only**: (Need for standard units) *Examples:* • *Using non-standard units of different length to measure gives differing numbers of units for the same length and shows that we need standard units.* • *Using different units like paper clips and pencils to measure means that students will get different answers for the same length and through discussion about what measurement is can come to realize the need for a common unit and more formal system of measurement.*	
12	Responses that give **Reason 3 only**: (Choosing most appropriate unit) *Examples:* • *The teacher wants the students to see that they should think about which unit is most appropriate for different lengths. Pencils would be more efficient for larger lengths than paperclips, for example. Paperclips would better for shorter lengths. Paces would be better for very long lengths.* • *This would show that long lengths are best measured with large units (pencils) and short lengths are best measured by small units (paper clips).*	

Continued next page

	Incorrect Response
70	Responses that focus on motivation, enjoyment, etc. *Examples:* • *Using concrete materials is more fun, motivating, interesting and engaging.* • *It is not as boring for the students if the teacher uses a variety of methods and aids* • *The teacher knows that the students will enjoy their work more if they can use hands-on materials*
71	Responses that focus on other unrelated or insignificant aspects. *Examples*: • *Using familiar objects such as pencils encourages estimation skills.* • *The teacher wants to encourage creativity by getting students to measure with paper clips and pencils.* • *So that her children will know how to measure with paperclips and pencils.*
79	Other incorrect (including crossed out, erased, stray marks, illegible, or off task)
	Non-response
99	Blank

Section 2: Released Items Secondary Schools

Item ID	Knowledge Dimension	Content Domain	Sub-domain	Label	Item format	Key	Max. Points	International Average
MFC604A1	MCK	Algebra	Applying	Solve a word problem about linear relations	CR	SG[1]	1	72%
MFC604A2	MCK	Algebra	Applying	Solve a word problems about linear relations	CR	SG	1	50%
MFC604B	MPCK	Algebra	Enacting	Analyze why one word problem is more difficult than another.	CR	SG	1	39%
MFC610A	MCK	Number	Knowing	Determine whether a number is irrational.	CMC	1	1	44%
MFC610C	MCK	Number	Knowing	Determine whether a number is irrational..	CMC	1	1	54%
MFC610D	MCK	Number	Knowing	Determine whether a number is irrational.	CMC	3	1	37%
MFC703	MCK	Geometry	Reasoning	Determine length of ribbon of two boxes	CR	SG	2	33% (FC)[2] 20% (PC)
MFC704	MCK	Geometry	Applying	Determine lengths of segments in a figure.	CR	SG	2	32%(FC) 25%(PC)
MFC705A	MCK	Geometry	Knowing	Describe solution to an equation in a plane	CMC	2	1	53%
MFC705B	MCK	Geometry	Knowing	Describe solution to an equation in space.	CMC	3	1	51%
MFC709A	MPCK	Number	Enacting	Determine whether student's response is a valid proof.	CMC	1	1	75%
MFC709B	MPCK	Number	Enacting	Determine whether student's response is a valid proof.	CMC	2	1	46%
MFC709C	MPCK	Number	Enacting	Determine whether student's response is a valid proof.	CMC	2	1	60%
MFC710A	MCK	Algebra	Applying	Determine whether a situation can be modeled by an exponential function.	CMC	2	1	41%
MFC710B	MCK	Algebra	Applying	Determine whether a situation can be modeled by an exponential function.	CMC	2	1	39%
MFC710C	MCK	Algebra	Applying	Determine whether a situation can be modeled by an exponential function.	CMC	1	1	60%
MFC711	MCK	Algebra	Reasoning	Write a proof about the sum of two functions.	CR	SG	2	11%(FC) 8%(PC)
MFC712A	MPCK	Algebra	Curriculum & Planning	Determine if knowledge is needed to prove the quadratic formula.	CMC	1	1	78%
MFC712B	MPCK	Algebra	Curriculum & Planning	Determine if knowledge is needed to prove the quadratic formula.	CMC	1	1	78%
MFC712C	MPCK	Algebra	Curriculum & Planning	Determine if knowledge is needed to prove the quadratic formula.	CMC	1	1	49%
MFC712D	MPCK	Algebra	Curriculum & Planning	Determine if knowledge is needed to prove the quadratic formula.	CMC	2	1	64%
MFC802A	MCK	Number	Reasoning	Decide if argument is a proof.	CMC	2	1	46%
MFC802B	MCK	Number	Reasoning	Decide if argument is a proof.	CMC	1	1	63%
MFC802C	MCK	Number	Reasoning	Decide if argument is a proof.	CMC	2	1	58%

[1] SG – See Scoring Guide provided with the item in this document.
[2] FC – Fully correct (2 score points); PC – Partially correct (1 score point).

Item ID	Knowledge Dimension	Content Domain	Sub-domain	Label	Item format	Key	Max. Points	International Average
MFC802D	MCK	Number	Reasoning	Decide if argument is a proof.	CMC	2	1	54%
MFC804	MCK	Number	Knowing	Find number of ways to choose 2 students from 10 and 8 students from 10.	MC	3	1	35%
MFC806A	MCK	Data	Applying	Determine whether student's interpretation of histogram is right or wrong.	MC	2	1	71%
MFC806B	MPCK	Data	Enacting	Explain student's thinking about histogram.	CR	SG	1	69%
MFC808A	MCK	Geometry	Applying	Correct students' answers about lines of symmetry in a regular hexagon.	CMC	1, 2	1	70%
MFC808B	MCK	Geometry	Applying	Correct students' answers about lines of symmetry in a regular pentagon.	CMC	1, 2	1	61%
MFC808C	MCK	Geometry	Applying	Correct students' answers about lines of symmetry in a rhombus.	CMC	2, 1	1	53%
MFC814	MCK	Algebra	Reasoning	Determine if a statement about an operation with matrices is correct, and justify response.	CR	SG	2	19% (FC) 2% (PC)

ID: MFC704	MS Booklet: SM1, SM2	MS Block: B2SM	Item Format: CR	Max Points: 2
Knowledge Dimension: MCK	Content Domain: Geometry		Sub-domain: Applying	

On the figure, *ABCD* is a parallelogram, $\angle BAD = 60^0$, *AM* and *BM* are angle bisectors of angles *BAD* and *ABC* respectively. If the perimeter of *ABCD* is 6 cm, find the sides of triangle *ABM*.

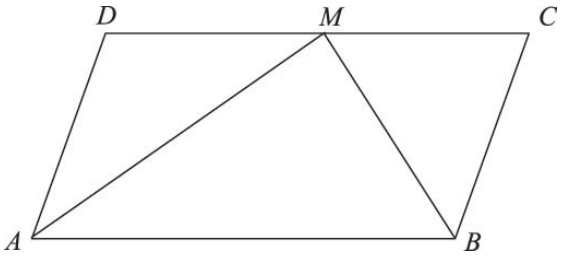
Write your answers on the lines below.

MFC704

AB = _____cm

AM = _____cm

BM = _____cm



Code	Response	Item ID: MFC704
	Correct Response	
20	Responses that indicate all three correct entries below: *AB* = 2 cm *AM* = √3 cm or equivalent *BM* = 1 cm	
	Partially Correct Response	
10	Any two entries correct and one incorrect (or blank).	
11	Any one entry correct and two incorrect (or blank).	
	Incorrect Response	
79	Incorrect mathematical statements or statement of no value (including crossed out, erased, stray marks, illegible, or off task).	
	No Response	
99	Blank	

ID: MFC705A MFC705B	MS Booklet: SM1, SM2	MS Block: B2SM	Item Format: CMC	Max Points: 2
Knowledge Dimension: MCK	Content Domain: Geometry		Sub-domain: Knowing	

We know that there is only one point on the real line that satisfies the equation $3x = 6$, namely $x = 2$.

Suppose now that we consider this same equation in the plane, with coordinates x and y, and then in space with coordinates x, y, and z. What does the set of points that satisfy the equation $3x = 6$ look like in these settings?

Check <u>one</u> box in each <u>row</u>.

			One point	One line	One plane	Other
MFC705A	A.	The solution to $3x = 6$ in the plane	☐ 1	☐ 2	☐ 3	☐ 4
MFC705B	B.	The solution to $3x = 6$ in space	☐ 1	☐ 2	☐ 3	☐ 4

ID: MFC709A MFC709B MFC709C	MS Booklet: SM1, SM2	MS Block: B2SM	Item Format: CMC	Max Points: 3
Knowledge Dimension: MPCK	Content Domain: Number		Sub-domain: Enacting	

Some <lower secondary school> students were asked to prove the following statement:

When you multiply 3 consecutive natural numbers, the product is a multiple of 6.

Below are three responses.

[Kate's] answer

A multiple of 6 must have factors of 3 and 2.
If you have three consecutive numbers, one will be a multiple of 3.

Also, at least one number will be even and all even numbers are multiples of 2.

If you multiply the three consecutive numbers together the answer must have at least one factor of 3 and one factor of 2.

[Leon's] answer

$1 \times 2 \times 3 = 6$

$2 \times 3 \times 4 = 24 = 6 \times 4$

$4 \times 5 \times 6 = 120 = 6 \times 20$

$6 \times 7 \times 8 = 336 = 6 \times 56$

[Maria's] answer

n is any whole number

$$n \times (n+1) \times (n+2) = (n^2+n) \times (n+2)$$

$$= n^3 + n^2 + 2n^2 + 2n$$

Canceling the n's gives $1 + 1 + 2 + 2 = 6$

Determine whether each proof is valid.

Check <u>one</u> box in each <u>row</u>.

			Valid	Not valid
MFC709A	A.	[Kate's] proof	☐ 1	☐ 2
MFC709B	B.	[Leon's] proof	☐ 1	☐ 2
MFC709C	C.	[Maria's] proof	☐ 1	☐ 2

ID: MFC710A MFC710B MFC710C	MS Booklet: SM1, SM2	MS Block: B2SM	Item Format: CMC	Max Points: 3
Knowledge Dimension: MCK	Content Domain: Algebra		Sub-domain: Applying	

Indicate whether each of the following situations can be modeled by an exponential function.

Check <u>one</u> box in each <u>row</u>.

			Yes	**No**
MFC710A	A.	The height h of a ball t seconds after it is thrown into the air.	☐ 1	☐ 2
MFC710B	B.	The amount of money A in a bank after w weeks, if each week d zeds are put in the bank.	☐ 1	☐ 2
MFC710C	C.	The value V of a car after t years if it depreciates d % per year.	☐ 1	☐ 2

ID: MFC711	MS Booklet: SM1, SM2	MS Block: B2SM	Item Format: CR	Max Points: 2
Knowledge Dimension: MCK	Content Domain: Algebra		Sub-domain: Reasoning	

Prove the following statement:

If the graphs of linear functions

$$f(x) = ax + b \text{ and } g(x) = cx + d$$

intersect at a point *P* on the *x*-axis, the graph of their sum function

$$(f + g)(x)$$

MFC711 must also go through *P*.

Code	Response	Item ID: MFC711
	Correct Response	
20	Response carefully lays out the steps of the proof in a general way, without using the given formulas of $f(x)$ and $g(x)$. *Example: Suppose f(x) and g(x) intersect at point (p,* 0*) on the x-axis.* *Then f(p) =* 0*, g(p) =* 0*. Then (f + g)(p) = f(p) + g(p) =* 0 + 0 = 0*.* *Therefore f+g also goes across point (p,* 0*).*	
21	Response has carefully laid out the steps of the proof using the given formulas of $f(x)$ and $g(x)$. *Example: Suppose f(x) and g(x) intersect at point (p,* 0*) on the x-axis, then the following inferences can be made:* *(1) f(p) =* 0 → *ap + b =* 0 → *p = –b/a;* *(2) g(p) =* 0 → *cp + d =* 0 → *p = –d/c;* *(3) f(p) = g(p)* → *b/a = d/c* → *ad = bc;* *(4) f(p) = g(p)* → *ap + b = cp + d* → *p = –(b + d)/(a + c);* *Since (f + g)(p) = f(p) + g(p), together with two or more of the above inferences, one can show that (f + g) (p) =* 0*. Therefore (f + g)(x) also goes across point (p,* 0*).*	
22	Response has carefully laid out the steps of the proof using a graphical argument. *Example: A graph of two lines intersecting on the x-axis is shown. Suppose f(x) and g(x) intersect at point (p,* 0*) on the x-axis. The value of (f + g)(x) is the sum of f(x) and g(x) for each x. But at x = p,* 0 + 0 = 0*, so f + g also goes through the point (p,* 0*).*	
	Partially Correct Response	
10	Response shows evidence of a chain of reasoning about general functions without using the given formulas of $f(x)$ and $g(x)$, but some mistake is made or the response stops before the proof is complete. *Example: Understands f(p) =* 0*, g(p) =* 0*, and (f + g)(p) = f(p) + g(p), but doesn't arrive at the fact that (f + g)(p) =* 0 *and/or the conclusion that (f + g)(x) also goes through (p,* 0*).*	
11	Response shows evidence of a chain of reasoning using the given formulas of $f(x)$ and $g(x)$, but some mistake is made or the response stops before the proof is complete. *Example: Makes one or more of inferences (1) – (4) under code 21, also states that (f + g)(x) = f(x) + g(x) = (a + c)x + (b + d), even is able to show (f + g)(p) =* 0*, but there is major flaw in logical reasoning.*	
12	Response shows evidence of a chain of reasoning about general functions using an intuitive/graphical proof, but some mistake is made or the response stops before the proof is complete. *Example: Response is able to show graphically that f(x) and g(x) go through the same point on x-axis, also points out the meaning of the sum function, but isn't able to conclude that the sum function goes through the same point.*	
	Incorrect Response	
79	Incorrect mathematical statement or other incorrect work (including crossed out, erased, stray marks, illegible, or off task)	
	No Response	
99	Blank	

ID: MFC712A MFC712B MFC712C MFC712D	MS Booklet: SM1, SM2	MS Block: B2SM	Item Format: CMC	Max Points: 4
Knowledge Dimension: MPCK	Content Domain: Algebra		Sub-domain: Planning	

A mathematics teacher wants to show some <lower secondary school> students how to prove the quadratic formula.

Determine whether each of the following types of knowledge is needed in order to understand a proof of this result.

Check <u>one</u> box in each <u>row</u>.

			Needed	**Not needed**
MFC712A	A.	How to solve linear equations.	☐ 1	☐ 2
MFC712B	B.	How to solve equations of the form $x^2 = k$, where $k > 0$.	☐ 1	☐ 2
MFC712C	C.	How to complete the square of a trinomial.	☐ 1	☐ 2
MFC712D	D.	How to add and subtract complex numbers.	☐ 1	☐ 2

ID: MFC802A MFC802B MFC802C MFC802D	MS Booklet: SM2, SM3	MS Block: B3SM	Item Format: CMC	Max Points: 4
Knowledge Dimension: MCK	Content Domain: Number		Sub-domain: Reasoning	

You have to prove the following statement:

If the square of any natural number is divided by 3, then the remainder is only 0 or 1.

State whether each of the following approaches is a mathematically correct proof.

Check one box in each row.

		Yes	**No**
MFC802A	A. Use the following table:		

Number	1	2	3	4	5	6	7	8	9	10
Square	1	4	9	16	25	36	49	64	81	100
Remainder when divided by 3	1	1	0	1	1	0	1	1	0	1

		☐ 1	☐ 2
MFC802B	B. Demonstrate that $(3n)^2$ is divisible by 3 and for all other numbers, $(3n \pm 1)^2 = 9n^2 \pm 6n + 1$ which always has a remainder of 1 once it has been divided by 3.	☐ 1	☐ 2
MFC802C	C. Choose a natural number n, find its square n^2, and then check whether the statement is true or not.	☐ 1	☐ 2
MFC802D	D. Check the statement for the first several prime numbers and then draw a conclusion based on the Fundamental Theorem of Arithmetic.	☐ 1	☐ 2

ID: MFC804	MS Booklet: SM2, SM3	MS Block: B3SM	Item Format: MC	Max Points: 1
Knowledge Dimension: MCK	Content Domain: Number		Sub-domain: Knowing	

MFC804

A class has 10 students. If at one time, 2 students are to be chosen, and another time 8 students are to be chosen from the class, which of the following statements is true?

Check <u>one</u> box.

A. There are more ways to choose 2 students than 8 students from the class. ☐1

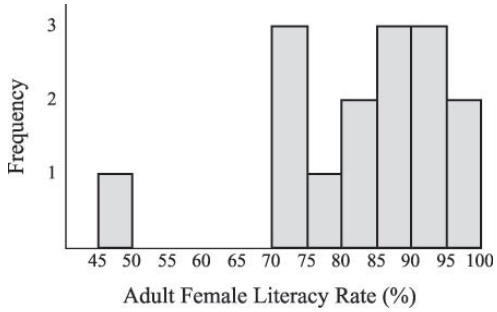
B. There are more ways to choose 8 students than 2 students from the class. ☐2

C. The number of ways to choose 2 students equals the number of ways to choose 8 students. ☐3

D. It is not possible to determine which selection has more possibilities. ☐4

ID: MFC806A	MS Booklet: SM2, SM3	MS Block: B3SM	Item Format: MC	Max Points: 1
Knowledge Dimension: MCK	Content Domain: Data		Sub-domain: Applying	

The following graph gives information about the adult female literacy rates in Central and South American countries.[3]



Suppose you ask your students to tell you how many countries are represented in the graph. One student says, "There are 7 countries represented."

		Check <u>one</u> box. **Right**	**Wrong**
MFC806A	a) Is the student right or wrong?	☐ 1	☐ 2

[3] This item is copyright 2004 by Maria Alejandra Sorto as part of her Ph. D. dissertation *Prospective Middle School Teachers' Knowledge about Data Analysis and its Application to Teaching* at Michigan State University. It is used with her permission..

ID: MFC806B	MS Booklet: SM2, SM3	MS Block: B3SM	Item Format: CR	Max Points: 1
Knowledge Dimension: MPCK	Content Domain: Data		Sub-domain: Enacting	

MFC806B

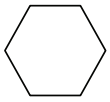
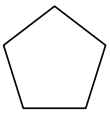
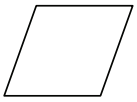
b) In your opinion, what was the student thinking in order to arrive at that conclusion?

Code	Response	Item: MFC806B
	Correct Response	
10	Response indicates that the student thought that each bar represented one country. *Example:* *The student counted the number of bars, and concluded that the answer (7) represented the number of countries.*	
	Incorrect Response	
79	Incorrect response (including crossed out, erased, stray marks, illegible, or off task).	
	No response	
99	Blank	

ID: MFC808A MFC808B MFC808C	MS Booklet: SM2, SM3	MS Block: B3SM	Item Format: CMC	Max Points: 3
Knowledge Dimension: MCK	Content Domain: Geometry		Sub-domain: Applying	

Your students have been working on symmetry. They were given the task below requiring them to decide the number of lines of symmetry for three different shapes.

Answers of [Sam] and [Michael] are shown in the table. Correct the answers of each student by checking correct or incorrect.

			Students and their answers about the number of the lines of symmetry	
	Shape	**Shape name**	**[Sam]**	**[Michael]**
MFC808A		regular hexagon	6 ☐₁ Correct ☐₂ Incorrect	12 ☐₁ Correct ☐₂ Incorrect
MFC808B		regular pentagon	5 ☐₁ Correct ☐₂ Incorrect	10 ☐₁ Correct ☐₂ Incorrect
MFC808C		rhombus	4 ☐₁ Correct ☐₂ Incorrect	2 ☐₁ Correct ☐₂ Incorrect

Note: This CMC question originally was considered as six items. After psychometric analysis, it was recoded as three items and scored as follows.

MFC808A: Score 1 if answers of both Sam and Michael are correctly checked (1 and 2); otherwise, score 0.
MFC808B: Score 1 if answers of both Sam and Michael are correctly checked (1 and 2); otherwise, score 0.
MFC808C: Score 1 if answers of both Sam and Michael are correctly checked (2 and 1); otherwise, score 0.

ID: MFC814	MS Booklet: SM2, SM3	MS Block: B3SM	Item Format: CR	Max Points: 2
Knowledge Dimension: MCK	Content Domain: Algebra		Sub-domain: Reasoning	

Let $A = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$ and $B = \begin{bmatrix} t & u \\ v & w \end{bmatrix}$. Then $A \otimes B$ is defined to be $\begin{bmatrix} pt & qu \\ rv & sw \end{bmatrix}$.

MFC814

Is it true that if $A \otimes B = \mathrm{O}$, then either $A = \mathrm{O}$ or $B = \mathrm{O}$ (where O represents the zero matrix)? Justify your answer.

Code	Response	Item ID: MFC814
	Correct Response	
20	Response indicates that the statement is false (or not necessarily true) and provides a correct (and specific) counterexample. *Example: No, it is not true. If* $A = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$ *and* $B = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$, *then* $A \otimes B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.	
21	Response indicates that the statement is false (or not necessarily true), and provides a ***general*** description of a counterexample using words. *Example: Let's assume that all elements in the first column of the matrix A is 0, and all elements in the second column of the matrix B is 0. When we apply the operation defined in the question to matrix A and matrix B, we get the 0 matrix at the end.* **Note:** As indicated in the example above, even though the response does not indicate that the second column of matrix *A* and the first column of matrix *B* must have non-zero entries, we code such solutions as correct.	
29	Other correct responses.	
	Partially Correct Response	
10	Response indicates that the statement is false (or not necessarily true), and provides a counterexample that is *not sufficiently* described.	
	Incorrect Response	
70	Response indicates that the statement is false or (not necessarily true), but provides no justification or a justification that is incorrect or irrelevant.	
71	Response indicates that the statement is true.	
79	Other incorrect (including crossed out, erased, stray marks, illegible, or off task).	
	No Response	
99	Blank	

Section 2: Released Items Secondary Schools

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MFC610D	MCK	Number	Knowing	Determine whether a number is irrational.	CMC	3	1	37%
MFC703	MCK	Geometry	Reasoning	Determine length of ribbon of two boxes	CR	SG	2	33% (FC)[2] 20% (PC)
MFC704	MCK	Geometry	Applying	Determine lengths of segments in a figure.	CR	SG	2	32%(FC) 25%(PC)
MFC705A	MCK	Geometry	Knowing	Describe solution to an equation in a plane	CMC	2	1	53%
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MFC711	MCK	Algebra	Reasoning	Write a proof about the sum of two functions.	CR	SG	2	11%(FC) 8%(PC)
MFC712A	MPCK	Algebra	Curriculum & Planning	Determine if knowledge is needed to prove the quadratic formula.	CMC	1	1	78%
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[2] FC – Fully correct (2 score points); PC – Partially correct (1 score point).

Item ID	Knowledge Dimension	Content Domain	Sub-domain	Label	Item format	Key	Max. Points	International Average
MFC802D	MCK	Number	Reasoning	Decide if argument is a proof.	CMC	2	1	54%
MFC804	MCK	Number	Knowing	Find number of ways to choose 2 students from 10 and 8 students from 10.	MC	3	1	35%
MFC806A	MCK	Data	Applying	Determine whether student's interpretation of histogram is right or wrong.	MC	2	1	71%
MFC806B	MPCK	Data	Enacting	Explain student's thinking about histogram.	CR	SG	1	69%
MFC808A	MCK	Geometry	Applying	Correct students' answers about lines of symmetry in a regular hexagon.	CMC	1, 2	1	70%
MFC808B	MCK	Geometry	Applying	Correct students' answers about lines of symmetry in a regular pentagon.	CMC	1, 2	1	61%
MFC808C	MCK	Geometry	Applying	Correct students' answers about lines of symmetry in a rhombus.	CMC	2, 1	1	53%
MFC814	MCK	Algebra	Reasoning	Determine if a statement about an operation with matrices is correct, and justify response.	CR	SG	2	19% (FC) 2% (PC)

ID: MFC604A1 MFC604A2	MS Booklet: SM1, SM3	MS Block: B1SM	Item Format: CR	Max Points: 2
Knowledge Dimension: MCK	Content Domain: Algebra		Sub-domain: Applying	

The following problems appear in a mathematics textbook for <lower secondary school>.

1. [Peter], [David], and [James] play a game with marbles. They have 198 marbles altogether. [Peter] has 6 times as many marbles as [David], and [James] has 2 times as many marbles as [David]. How many marbles does each boy have?

2. Three children [Wendy], [Joyce] and [Gabriela] have 198 zeds altogether. [Wendy] has 6 times as much money as [Joyce], and 3 times as much as [Gabriela]. How many zeds does each child have?

(a) Solve each problem.

MFC604A1

Solution to Problem 1:

MFC604A2

Solution to Problem 2:

Note: The correct answers to MFC604A1 and MFC604A2 follow:
Problem 1: David has 22 marbles, Peter has 132 marbles, and James has 44.
Problem 2: Wendy has 132 zeds, Joyce has 22 zeds, and Gabriela has 44 zeds.

The following methods are considered in the scoring guide:

1) Using *one variable*, setting up *one equation* and solving.
   Example (Problem 1): Let m = the number of marbles that David has. Then Peter has $6m$ and James has $2m$. Therefore, $6m + 2m + m = 198$, and $m = 22$.
2) Using *more than one variable*, establishing a *system of equations*, performing substitutions, and solving.
   Example (Problem 1): Let p = the number of marbles that Peter has, d = the number of marbles that David has, and j = the number of marbles that James has
   $p = 6d$ and $j = 2d$, $p + d + j = 198$.
3) Trial and error or guess and check
4) Ratio or other arithmetic methods
5) Representation/diagram

Code:	Response	Item ID: MFC604A1
11	Response uses Method 1 correctly to solve Problem 1 and get the correct answers.	
12	Response uses Method 2 correctly to solve Problem 1 and get the correct answers.	
13	Response uses Method 3 correctly to solve Problem 1 and get the correct answers.	
14	Response uses Method 4 correctly to solve Problem 1 and get the correct answers.	
15	Response uses Method 5 correctly to solve Problem 1 and get the correct answers and get the correct answers.	
19	Response uses a valid but different method from the list above to solve Problem 1 and get the correct answers.	
	Incorrect Response	
70	Response uses one of Methods 1 - 5 to start Problem 1, but arrives at an incorrect answer or cannot complete the solution because of a computation or algebra error.	
71	Response uses a correct but different method from the list above to solve Problem 1, but arrives at an incorrect answer or cannot complete the solution because of a computation or algebra error.	
79	Other incorrect (including crossed out, erased, stray marks, illegible, or off task).	
	No Response	
99	Blank	

Code	Response	Item ID: MFC604A2
	Correct Response	
11	Response uses Method 1 to solve Problem 2.	
12	Response uses Method 2 to solve Problem 2.	
13	Response uses Method 3 to solve Problem 2.	
14	Response uses Method 4 to solve Problem 2.	
15	Response uses Method 5 to solve Problem 2.	
19	Responses use a correct but different method from the list above to solve Problem 2 and get the correct answers.	
	Incorrect Response	
70	Response uses one of Methods 1 - 5 to start Problem 2, but arrives at an incorrect answer or cannot complete the solution because of a computation or algebra error.	
71	Response uses a correct but different method from this list to solve Problem 2, but arrives at an incorrect answer or cannot complete the solution because of a computation or algebra error.	
79	Other incorrect (including crossed out, erased, stray marks, illegible, or off task).	
	No Response	
99	Blank	

ID: MFC604B	MS Booklet: SM1, SM3	MS Block: B1SM	Item Format: CR	Max Points: 1
Knowledge Dimension: MPCK	Content Domain: Algebra		Sub-domain: Enacting	

(b) Typically Problem 2 is more difficult than Problem 1 for <lower secondary> students. Give one reason that might account for the difference in difficulty level.

MFC604B

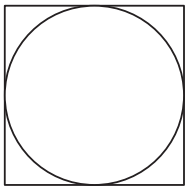
Code	Response	Item ID: MFC604B
	Correct Response	
10	Reason clearly expresses a difference in the mathematical or cognitive complexity of the two problems. *Examples:* *1) In Problem 1 it is easier (in comparison to Problem 2) to choose the base variable, and see the relations between the variables. In Problem 1, the number of marbles that both Peter and James have is in direct relationship to the number of marbles that David has. However, in Problem 2, the relation between the number of zeds that Joyce and Gabriela have is not directly stated.* *2) Problem 2 is phrased in such a way that the respondent seems more likely to use fractional equations than whole number equations. Fractional equations can be more challenging to solve, making calculations more prone to error.*	
	Incorrect Response	
79	Incorrect reason (including crossed out, erased, stray marks, illegible, or off task).	
	No Response	
99	Blank	

ID: MFC610A MFC610C MFC610D	MS Booklet: SM1, SM3	MS Block: B1SM	Item Format: CMC	Max Points: 3
Knowledge Dimension: MCK	Content Domain: Number		Sub-domain: Knowing	

Determine whether each of the following is an irrational number always, sometimes or never.

Check <u>one</u> box in each <u>row</u>.

			Always	**Sometimes**	**Never**
MFC610A	A.	The result of dividing the circumference of a circle by its diameter.	☐ 1	☐ 2	☐ 3
MFC610C	C.	The diagonal of a square with side of length 1.	☐ 1	☐ 2	☐ 3
MFC610D	D.	Result of dividing 22 by 7.	☐ 1	☐ 2	☐ 3

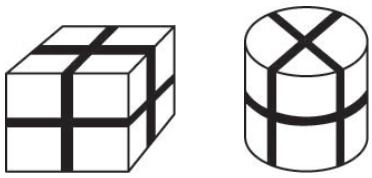
Code	Response	Item: MFC703 and MFC511
	Correct Response	
20	Box A with a correct and complete explanation involving calculations of ribbon lengths *Examples:* • *Box A needs* $6 \times 20 = 120$ cm *ribbon. Box B needs* 4 × 20 = 80 *cm plus the circumference which is* 10π. $10\pi < 40$ *so Box A needs more ribbon.* • *Box A. Box A needs* 120 *cm but box B needs about* 110 *cm (using* $\pi = 3$*).*	
21	Box A based upon a complete argument (with or without calculation) comparing the square and circumference (both of equal 'width') together **with a statement that the other lengths of ribbon are equal.** *Examples:* • *Box A because the circumference of a circle diameter 10 is less than the perimeter of a square of side 10 and the other lengths of ribbon are the same.* • *Box A. As shown in the diagram, the ribbon around the cylinder is less than the ribbon around the square. The other lengths of ribbon are equal on each box. Therefore Box B needs less ribbon altogether than Box A.*  • *Box A. The circumference is about 31.4 but the perimeter of the square is 40. So Box A needs more ribbon because the other ribbon is the same (80) on both boxes.*	

*Note: Accept reasonable approximations of π such as 3.14, 3.1, 3, 22/7 etc.

Continued next page

ID: MFC703	MS Booklet: PM4, PM5	MS Block: B5PM	Item Format: CR	Max Points: 2
Knowledge Dimension: MCK	Content Domain: Geometry		Sub-domain: Applying	

Two gift boxes wrapped with ribbon are shown below. Box A is a cube of side-length 10 cm. Box B is a cylinder with height and diameter 10 cm each.



A B

MFC511 Which box needs the longer ribbon? _____

Explain how you arrived at your answer

	Partially Correct Response
10	Box A with a correct and complete explanation as in Code 20 but with **one identifiable calculation error** (or use of a wrong formula) logically leading to Box A. *Example:* • *Box A because Box A needs 120 cm and Box B needs* **60** + 10π < 120.
11	Box B with a correct and complete explanation as in Code 20 but with **one identifiable calculation error** (or use of a wrong formula) logically leading to Box B. *Examples:* • 80 + 10π = 120.4 (rather than 111.4) > 120. • *Box B because Box A needs 120 cm of ribbon and box B needs 80 +* 25π *> 120. (Used area formula instead of circumference formula but intending to compare perimeter.)*
12	Box A with an explanation that correctly calculates and compares the lengths of ribbon on each box that are **different** but **fails to mention that the other lengths of ribbon are the same.** *Example:* • *Box A needs more ribbon because the circumference of the cylinder is* 10π *which is less than the perimeter of the square,* 40.
13	Box A with an explanation that correctly supports the choice of Box A but that is **limited** and/or **lacking the detail** of a Code 20 or 21 response. *Examples:* • *Box A because Box B can fit inside Box A.* • *Box A because the circumference is less than the perimeter.* • *Box A. You can see it's bigger. Its ribbon is 120 cm but Box B would be less.*
	Incorrect Response
70	Box A **without** any explanation or calculation. *Example: Box A*
71	Box A or B with an explanation based on a **conceptual error**. *Examples:* • *Box A but with an explanation based upon surface* ***area or volume****.* • *Box A because it has more sides.*
72	Box A or B with an explanation based on **incorrect and/or incomplete** ribbon lengths for **both** boxes. *Example:* • *Box B because Box A needs* 60 *cm but box B needs more than* 80.
73	Neither. The length of ribbon needed is the **same.** *Example:* • *Length width and height are the same therefore they need the same amount of ribbon.*
79	Other incorrect (including crossed out, erased, stray marks, illegible, or off task) *Example:* • *Box B* **without** any explanation or calculation.
	Non-response
99	Blank

ID: MFC704	MS Booklet: SM1, SM2	MS Block: B2SM	Item Format: CR	Max Points: 2
Knowledge Dimension: MCK	Content Domain: Geometry		Sub-domain: Applying	

On the figure, *ABCD* is a parallelogram, $\angle BAD = 60^0$, *AM* and *BM* are angle bisectors of angles *BAD* and *ABC* respectively. If the perimeter of *ABCD* is 6 cm, find the sides of triangle *ABM*.

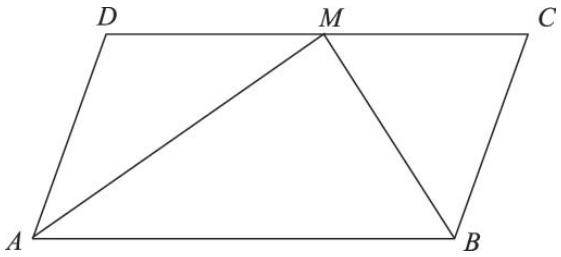
Write your answers on the lines below.

MFC704

AB = _____cm

AM = _____cm

BM = _____cm



Code	Response	Item ID: MFC704
	Correct Response	
20	Responses that indicate all three correct entries below: *AB* = 2 cm *AM* = √3 cm or equivalent *BM* = 1 cm	
	Partially Correct Response	
10	Any two entries correct and one incorrect (or blank).	
11	Any one entry correct and two incorrect (or blank).	
	Incorrect Response	
79	Incorrect mathematical statements or statement of no value (including crossed out, erased, stray marks, illegible, or off task).	
	No Response	
99	Blank	

ID: MFC705A MFC705B	MS Booklet: SM1, SM2	MS Block: B2SM	Item Format: CMC	Max Points: 2
Knowledge Dimension: MCK	Content Domain: Geometry		Sub-domain: Knowing	

We know that there is only one point on the real line that satisfies the equation $3x = 6$, namely $x = 2$.

Suppose now that we consider this same equation in the plane, with coordinates x and y, and then in space with coordinates x, y, and z. What does the set of points that satisfy the equation $3x = 6$ look like in these settings?

Check <u>one</u> box in each <u>row</u>.

			One point	One line	One plane	Other
MFC705A	A.	The solution to $3x = 6$ in the plane	☐ 1	☐ 2	☐ 3	☐ 4
MFC705B	B.	The solution to $3x = 6$ in space	☐ 1	☐ 2	☐ 3	☐ 4

ID: MFC709A MFC709B MFC709C	MS Booklet: SM1, SM2	MS Block: B2SM	Item Format: CMC	Max Points: 3
Knowledge Dimension: MPCK	Content Domain: Number		Sub-domain: Enacting	

Some <lower secondary school> students were asked to prove the following statement:

When you multiply 3 consecutive natural numbers, the product is a multiple of 6.

Below are three responses.

[Kate's] answer

A multiple of 6 must have factors of 3 and 2.
If you have three consecutive numbers, one will be a multiple of 3.

Also, at least one number will be even and all even numbers are multiples of 2.

If you multiply the three consecutive numbers together the answer must have at least one factor of 3 and one factor of 2.

[Leon's] answer

$1 \times 2 \times 3 = 6$

$2 \times 3 \times 4 = 24 = 6 \times 4$

$4 \times 5 \times 6 = 120 = 6 \times 20$

$6 \times 7 \times 8 = 336 = 6 \times 56$

[Maria's] answer

n is any whole number

$$n \times (n+1) \times (n+2) = (n^2 + n) \times (n+2)$$

$$= n^3 + n^2 + 2n^2 + 2n$$

Canceling the n's gives $1 + 1 + 2 + 2 = 6$

Determine whether each proof is valid.

Check <u>one</u> box in each <u>row</u>.

			Valid	Not valid
MFC709A	A.	[Kate's] proof	☐ 1	☐ 2
MFC709B	B.	[Leon's] proof	☐ 1	☐ 2
MFC709C	C.	[Maria's] proof	☐ 1	☐ 2

ID: MFC710A MFC710B MFC710C	MS Booklet: SM1, SM2	MS Block: B2SM	Item Format: CMC	Max Points: 3
Knowledge Dimension: MCK	Content Domain: Algebra		Sub-domain: Applying	

Indicate whether each of the following situations can be modeled by an exponential function.

Check <u>one</u> box in each <u>row</u>.

			Yes	**No**
MFC710A	A.	The height h of a ball t seconds after it is thrown into the air.	☐ 1	☐ 2
MFC710B	B.	The amount of money A in a bank after w weeks, if each week d zeds are put in the bank.	☐ 1	☐ 2
MFC710C	C.	The value V of a car after t years if it depreciates d % per year.	☐ 1	☐ 2

ID: MFC711	MS Booklet: SM1, SM2	MS Block: B2SM	Item Format: CR	Max Points: 2
Knowledge Dimension: MCK	Content Domain: Algebra		Sub-domain: Reasoning	

Prove the following statement:

If the graphs of linear functions

$$f(x) = ax + b \text{ and } g(x) = cx + d$$

intersect at a point P on the x-axis, the graph of their sum function

$$(f + g)(x)$$

MFC711 must also go through P.

Code	Response	Item ID: MFC711
	Correct Response	
20	Response carefully lays out the steps of the proof in a general way, without using the given formulas of $f(x)$ and $g(x)$. *Example: Suppose f(x) and g(x) intersect at point (p,* 0*) on the x-axis.* *Then f(p) =* 0*, g(p) =* 0*. Then (f + g)(p) = f(p) + g(p) =* 0 + 0 = 0*.* *Therefore f+g also goes across point (p,* 0*).*	
21	Response has carefully laid out the steps of the proof using the given formulas of $f(x)$ and $g(x)$. *Example: Suppose f(x) and g(x) intersect at point (p,* 0*) on the x-axis, then the following inferences can be made:* *(1) f(p) =* 0 *→ ap + b =* 0 *→ p = –b/a;* *(2) g(p) =* 0 *→ cp + d =* 0 *→ p = –d/c;* *(3) f(p) = g(p) → b/a = d/c → ad = bc;* *(4) f(p) = g(p) → ap + b = cp + d → p = –(b + d)/(a + c);* *Since (f + g)(p) = f(p) + g(p), together with two or more of the above inferences, one can show that (f + g) (p) =* 0*. Therefore (f + g)(x) also goes across point (p,* 0*).*	
22	Response has carefully laid out the steps of the proof using a graphical argument. *Example: A graph of two lines intersecting on the x-axis is shown. Suppose f(x) and g(x) intersect at point (p,* 0*) on the x-axis. The value of (f + g)(x) is the sum of f(x) and g(x) for each x. But at x = p,* 0 + 0 = 0*, so f + g also goes through the point (p,* 0*).*	
	Partially Correct Response	
10	Response shows evidence of a chain of reasoning about general functions without using the given formulas of $f(x)$ and $g(x)$, but some mistake is made or the response stops before the proof is complete. *Example: Understands f(p) =* 0*, g(p) =* 0*, and (f + g)(p) = f(p) + g(p), but doesn't arrive at the fact that (f + g)(p) =* 0 *and/or the conclusion that (f + g)(x) also goes through (p,* 0*).*	
11	Response shows evidence of a chain of reasoning using the given formulas of $f(x)$ and $g(x)$, but some mistake is made or the response stops before the proof is complete. *Example: Makes one or more of inferences (1) – (4) under code 21, also states that (f + g)(x) = f(x) + g(x) = (a + c)x + (b + d), even is able to show (f + g)(p) =* 0*, but there is major flaw in logical reasoning.*	
12	Response shows evidence of a chain of reasoning about general functions using an intuitive/graphical proof, but some mistake is made or the response stops before the proof is complete. *Example: Response is able to show graphically that f(x) and g(x) go through the same point on x-axis, also points out the meaning of the sum function, but isn't able to conclude that the sum function goes through the same point.*	
	Incorrect Response	
79	Incorrect mathematical statement or other incorrect work (including crossed out, erased, stray marks, illegible, or off task)	
	No Response	
99	Blank	

ID: MFC712A MFC712B MFC712C MFC712D	MS Booklet: SM1, SM2	MS Block: B2SM	Item Format: CMC	Max Points: 4
Knowledge Dimension: MPCK	Content Domain: Algebra		Sub-domain: Planning	

A mathematics teacher wants to show some <lower secondary school> students how to prove the quadratic formula.

Determine whether each of the following types of knowledge is needed in order to understand a proof of this result.

Check <u>one</u> box in each <u>row</u>.

			Needed	**Not needed**
MFC712A	A.	How to solve linear equations.	☐ 1	☐ 2
MFC712B	B.	How to solve equations of the form $x^2 = k$, where $k > 0$.	☐ 1	☐ 2
MFC712C	C.	How to complete the square of a trinomial.	☐ 1	☐ 2
MFC712D	D.	How to add and subtract complex numbers.	☐ 1	☐ 2

ID: MFC802A MFC802B MFC802C MFC802D	MS Booklet: SM2, SM3	MS Block: B3SM	Item Format: CMC	Max Points: 4
Knowledge Dimension: MCK	Content Domain: Number		Sub-domain: Reasoning	

You have to prove the following statement:

If the square of any natural number is divided by 3, then the remainder is only 0 or 1.

State whether each of the following approaches is a mathematically correct proof.

Check <u>one</u> box in each <u>row</u>.

		Yes	**No**
MFC802A	A. Use the following table:		

Number	1	2	3	4	5	6	7	8	9	10
Square	1	4	9	16	25	36	49	64	81	100
Remainder when divided by 3	1	1	0	1	1	0	1	1	0	1

		☐ 1	☐ 2
MFC802B	B. Demonstrate that $(3n)^2$ is divisible by 3 and for all other numbers, $(3n \pm 1)^2 = 9n^2 \pm 6n + 1$ which always has a remainder of 1 once it has been divided by 3.	☐ 1	☐ 2
MFC802C	C. Choose a natural number *n*, find its square n^2, and then check whether the statement is true or not.	☐ 1	☐ 2
MFC802D	D. Check the statement for the first several prime numbers and then draw a conclusion based on the Fundamental Theorem of Arithmetic.	☐ 1	☐ 2

ID: MFC804	MS Booklet: SM2, SM3	MS Block: B3SM	Item Format: MC	Max Points: 1
Knowledge Dimension: MCK	Content Domain: Number		Sub-domain: Knowing	

MFC804

A class has 10 students. If at one time, 2 students are to be chosen, and another time 8 students are to be chosen from the class, which of the following statements is true?

Check <u>one</u> box.

A. There are more ways to choose 2 students than 8 students from the class. ☐1

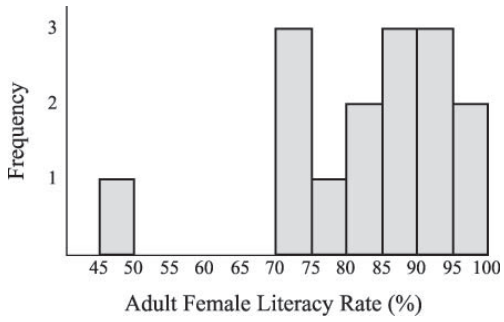
B. There are more ways to choose 8 students than 2 students from the class. ☐2

C. The number of ways to choose 2 students equals the number of ways to choose 8 students. ☐3

D. It is not possible to determine which selection has more possibilities. ☐4

ID: MFC806A	MS Booklet: SM2, SM3	MS Block: B3SM	Item Format: MC	Max Points: 1
Knowledge Dimension: MCK	Content Domain: Data		Sub-domain: Applying	

The following graph gives information about the adult female literacy rates in Central and South American countries.[3]



Suppose you ask your students to tell you how many countries are represented in the graph. One student says, "There are 7 countries represented."

		Check <u>one</u> box.	
		Right	**Wrong**
MFC806A	a) Is the student right or wrong?	☐ 1	☐ 2

[3] This item is copyright 2004 by Maria Alejandra Sorto as part of her Ph. D. dissertation *Prospective Middle School Teachers' Knowledge about Data Analysis and its Application to Teaching* at Michigan State University. It is used with her permission..

ID: MFC806B	MS Booklet: SM2, SM3	MS Block: B3SM	Item Format: CR	Max Points: 1
Knowledge Dimension: MPCK	Content Domain: Data		Sub-domain: Enacting	

MFC806B

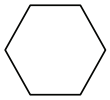
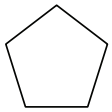
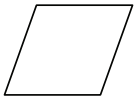
b) In your opinion, what was the student thinking in order to arrive at that conclusion?

Code	Response	Item: MFC806B
	Correct Response	
10	Response indicates that the student thought that each bar represented one country. *Example:* *The student counted the number of bars, and concluded that the answer (7) represented the number of countries.*	
	Incorrect Response	
79	Incorrect response (including crossed out, erased, stray marks, illegible, or off task).	
	No response	
99	Blank	

ID: MFC808A MFC808B MFC808C	MS Booklet: SM2, SM3	MS Block: B3SM	Item Format: CMC	Max Points: 3
Knowledge Dimension: MCK	Content Domain: Geometry		Sub-domain: Applying	

Your students have been working on symmetry. They were given the task below requiring them to decide the number of lines of symmetry for three different shapes.

Answers of [Sam] and [Michael] are shown in the table. Correct the answers of each student by checking correct or incorrect.

			Students and their answers about the number of the lines of symmetry	
	Shape	**Shape name**	**[Sam]**	**[Michael]**
MFC808A		regular hexagon	6 ☐$_1$ Correct ☐$_2$ Incorrect	12 ☐$_1$ Correct ☐$_2$ Incorrect
MFC808B		regular pentagon	5 ☐$_1$ Correct ☐$_2$ Incorrect	10 ☐$_1$ Correct ☐$_2$ Incorrect
MFC808C		rhombus	4 ☐$_1$ Correct ☐$_2$ Incorrect	2 ☐$_1$ Correct ☐$_2$ Incorrect

Note: This CMC question originally was considered as six items. After psychometric analysis, it was recoded as three items and scored as follows.

MFC808A: Score 1 if answers of both Sam and Michael are correctly checked (1 and 2); otherwise, score 0.
MFC808B: Score 1 if answers of both Sam and Michael are correctly checked (1 and 2); otherwise, score 0.
MFC808C: Score 1 if answers of both Sam and Michael are correctly checked (2 and 1); otherwise, score 0.

ID: MFC814	MS Booklet: SM2, SM3	MS Block: B3SM	Item Format: CR	Max Points: 2
Knowledge Dimension: MCK	Content Domain: Algebra		Sub-domain: Reasoning	

Let $A = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$ and $B = \begin{bmatrix} t & u \\ v & w \end{bmatrix}$. Then $A \otimes B$ is defined to be $\begin{bmatrix} pt & qu \\ rv & sw \end{bmatrix}$.

MFC814

Is it true that if $A \otimes B = \mathrm{O}$, then either $A = \mathrm{O}$ or $B = \mathrm{O}$ (where O represents the zero matrix)? Justify your answer.

Code	Response	Item ID: MFC814
	Correct Response	
20	Response indicates that the statement is false (or not necessarily true) and provides a correct (and specific) counterexample. *Example: No, it is not true. If* $A = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$ *and* $B = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$, *then* $A \otimes B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.	
21	Response indicates that the statement is false (or not necessarily true), and provides a ***general*** description of a counterexample using words. *Example: Let's assume that all elements in the first column of the matrix A is 0, and all elements in the second column of the matrix B is 0. When we apply the operation defined in the question to matrix A and matrix B, we get the 0 matrix at the end.* **Note:** As indicated in the example above, even though the response does not indicate that the second column of matrix *A* and the first column of matrix *B* must have non-zero entries, we code such solutions as correct.	
29	Other correct responses.	
	Partially Correct Response	
10	Response indicates that the statement is false (or not necessarily true), and provides a counterexample that is *not sufficiently* described.	
	Incorrect Response	
70	Response indicates that the statement is false or (not necessarily true), but provides no justification or a justification that is incorrect or irrelevant.	
71	Response indicates that the statement is true.	
79	Other incorrect (including crossed out, erased, stray marks, illegible, or off task).	
	No Response	
99	Blank