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THE TRAINING OF ELEMENTARY-
SCHOOL TEACHERS IN MATHEMATICS
IN THE COUNTRIES REPRESENTED IN
THE INTERNATIONAL COMMISSION ON
THE TEACHING OF MATHEMATICS

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THE TRAINING OF ELEMENTARY-SCHOOL TEACHERS IN MATHEMATICS

IN THE COUNTRIES REPRESENTED IN THE INTERNATIONAL COMMISSION ON
THE TEACHING OF MATHEMATICS.

INTRODUCTION.

The accompanying report deals with the mathematical training of prospective teachers in elementary schools as described in the reports submitted by the International Commission on the Teaching of Mathematics to the Fifth International Congress of Mathematicians, held at Cambridge, England, in August, 1912. A comparative study of the facts presented in these reports is of interest for those engaged in the training of teachers in this country, if only because they indicate that the standards elsewhere are as chaotic as they are here. The requirements of the normal schools, or corresponding institutions, vary from a review of the elementary-school arithmetic to the mathematics required for entrance to colleges and universities; or, from another point of view, from an emphasis on the purely professional needs, limited to the immediate requirements of the elementary schools, to academic and cultural aims founded on the principle that the more a teacher knows about the subject, in addition to the purely professional training, the more successful will be his teaching. But, great as is the variety of standards and aims in Europe, almost every standard finds a parallel in this country, owing to the absence of uniformity—a condition almost paralleled in England and Switzerland. But, if a generalization may be permitted, it would be true to say that the academic standards in the best systems are higher in the more advanced countries of Europe than they are in the United States.

Several reasons may be adduced to account for the condition here described. The training of elementary-school teachers is still obsessed with the traditions that are associated with a system of training by apprenticeship. More emphasis has always been placed on professional knowledge and technical ability than on general academic training; so that while the secondary-school teacher has been expected to be a master of subject matter, the elementary-school teacher has been narrowly trained in methods of instruction. Hence the candidate for the elementary-school branch has been considered to be sufficiently equipped if his knowledge of subject matter is equivalent to that given in a secondary school. Another important

factor, which has perhaps not received the attention in the reports that it merits, is that in general the instructors in normal schools are themselves not university trained, and the demand for university-trained instructors can not become strong until the principles upon which the training of elementary-school teachers is based become broader and more liberal. Finally, it may be that in most countries, and in the United States in particular, the standards in mathematics have been greatly affected by the fact that the great majority of candidates entering the teaching profession are women. It is almost universally the case in European normal schools that in mathematics lower attainments are required from the women than from the men students.

The general problem is, however, receiving considerable attention both in practical administration and in theoretical discussions. At present the European countries are all passing through a transition stage, which finds expression in dissatisfaction with the prevailing arrangements. The tendency generally is in the direction of raising the standards of the academic or cultural training—a tendency which reaches its culmination in the admission of certain elementary-school teachers to some university courses in Germany; in the provision of facilities to attract graduates of secondary schools to the elementary-school service, as in Hesse, England, Italy, and Switzerland; in the gradual separation of professional and academic training by such a provision as the introduction of a fourth year for students in the departments of education of universities in England; and in the United States in the continually increasing opportunities for the improvement of teachers in service. That the best thought in this country has not been backward in formulating the task that lies before those interested in the training of teachers is indicated in the recommendations of the American committees, which are quoted in the section on the United States.

BELGIUM.

The normal schools for men and women in Belgium are organized on the basis of a four-year course. The students are admitted at the age of 15, after an entrance examination on the subjects of the elementary school. Arithmetic is the only mathematical subject included in the examination, and consists of a written test (two questions of general arithmetic and two problems) and an oral test (two questions of general arithmetic and an exercise in mental arithmetic). The students must obtain 50 per cent of the marks to pass.

The mathematical subjects in the normal schools consist of arithmetic, algebra (in first and second years), and plane geometry (in second and third years) for men, and only arithmetic for women. The time allotment per week is as follows:

Time allotment per week in mathematical subjects.

	First year.	Second year.	Third year.	Fourth year.
Men.....	3	3	2	2
Women.....	2	2	1	1

Algebra and geometry are dropped in the fourth year of the men's course, and the time is given to a review of arithmetic with special reference to the requirements of the elementary school and the method of instruction. Promotion examinations take place at the close of each year on the subjects of that year. The final examination includes only mental and general arithmetic and the metric system.

The following aims are prescribed in the regulations of 1896: (1) The subjects are to be limited to what is necessary with special reference to the needs of the different stages of the elementary school; (2) attention is to be confined to the purely professional aspect without digressions into theory, except that instruction in everyday arithmetic must emphasize the practical utility of the subject and its content with reference to partnership, insurance, savings banks, annuities, etc.; in mental arithmetic the students are to be drilled in short methods and rules rather than principles; (3) the students must be trained by frequent handling of concrete objects, by questions, oral work, and practical exercises to promote their own development. It is found, however, that as a general rule the method of instruction is didactic, and the students tend to be passive recipients.

The outline of the work in the men's normal schools is as follows:

FIRST YEAR.

Arithmetic.—Integers. Preliminary introduction and definitions. Decimal system. Fundamental operations, their number and character; methodical explanation; principles of multiplication and division.

Decimal numbers. Numeration and its properties. Fundamental operations. Approximations.

Concrete and practical study of the legal system of weights and measures.

Divisibility and properties of numbers. Principles of divisibility, e. g., by 2 and 5, by 4 and 25, by 8 and 125, by 9 and 3, by 111. Remainders in divisions by these numbers. Test of multiplication and division by casting out the 9's.

Greatest common measure of two or more numbers by the method of successive divisions.

Common fractions; origin and definitions. Numeration. Simplification of fractions. Reduction of fractions. Reduction to the same denominator. Fundamental operations. Conversion into decimal fractions.

Exercises in mental arithmetic, chiefly with the aid of short method with whole numbers, decimals, and common fractions.

Solution (by the method of reduction to unity) of various problems bearing on everyday needs.

Discussion of the work in the lower stage of the elementary school.

Algebra.—Easy problems to illustrate the value of algebraic notation. Introduction and definitions. Fundamental operations. Factors. Fractions; definition; simplification by factors. Fundamental operations. Equations; definitions; general principles of the solution of equations. Solution of numerical equations of the first degree with one, two, and three unknowns. Methods of elimination by addition and subtraction, by substitution, by comparison of values. Problems. Principles of divisibility. Division of $x^m \pm a^m$ by $x \pm a$.

SECOND YEAR.

Arithmetic.—Properties of numbers. Least common multiple and greatest common measure of two or more numbers. Reduction of fractions to the same denominator. Divisibility by 6, 18, 15, 45, etc.

Recurring decimals.

Ratio and proportion. Numbers directly and inversely proportional. Problems.

Method of reduction to unity. Application to rule of three. Simple interest; simple discount at home and abroad; average maturity; revenue; contracts; savings banks and State annuities; insurance; proportional shares and partnership; rate, exchange, and commission; alligation; and alloys.

Mental arithmetic on these operations.

Discussion of the work of the intermediate stage of the elementary school, including the legal system of weights and measures.

Algebra.—Algebraic equations of the first degree with one unknown. Problems. Examples of indeterminate and impossible numerical equations.

Meaning of negative solutions of problems. Exercises on negative quantities.

Discussion of results in the forms: $\frac{A}{0}$, $\frac{0}{0}$, $\frac{0}{A}$.

Discussion of problems of moving bodies, etc., especially of geometric problems.

Algebraic equations of the first degree with two or three unknowns. Problems.

Discussions of the general principles for solving two equations with two unknowns.

Geometry.—Definitions. Axioms. Properties of triangles. Equality of triangles.

Properties of perpendiculars and oblique lines. Equality of right triangles. Parallels. The sum of the angles of a triangle and of some polygons. Properties of a parallelogram.

Properties of the circle and figures resulting from combinations of circles and straight lines. Relative positions of two circles.

Incommensurables in general. Measurement of angles. Inscribed and circumscribed quadrilaterals.

Noteworthy points of triangle; circumscribed, inscribed, and escribed circles; center of gravity, common point of the altitudes.

Problems of construction. Loci. Analysis and synthesis in geometric solutions. Applications.

Measurement of plane areas. Principal relations between the parts of a triangle.

THIRD YEAR.

Powers. Extraction of square and cube roots. Determination of these roots approximately.

Arithmetical and geometric progressions.

Logarithms. Use of tables. Application to compound interest and annuities.

Discussion of the work of the upper stage of the elementary school.

Geometry.—Proportional lines. Similarity of figures. Proportional lines in a circle. Area of the triangle as a function of the three sides.

Problems of construction based on the previous course.

Properties of regular polygons: Construction of regular polygons with 3, 4, 5, 6, 8, 10, 12, and 15 sides. Area of a regular polygon. Measurement of circumference, circle, and sector. Discussion of a method of determining the relation of the circumference to the diameter. Problems of construction.

Surveying. Description of use of surveying instruments. Measurement of the size of plots. Drafting of plans with the aid of the instruments and the plane table. Practical applications. Leveling.

Review.

FOURTH YEAR.

Arithmetic.—Thorough review of the work of the first two years. Review of the program of the elementary school as a whole with model lessons, especially on those parts less thoroughly studied in the normal schools.

Various problems and exercises in rapid calculation.

In addition to this program the students are given a course in special method by the professor of method. This course includes a discussion of the methods of the mathematical sciences—analysis and synthesis, induction and deduction, and of the theory and method to be followed in teaching arithmetic, the system of weights and measures, and the prescribed program. This course is supplemented by the professor of mathematics, who discusses the development and correlation of the programs of the elementary and adult school, and directs the model lessons in the subject and the subsequent criticisms. In the different classes of a practice school the students give practice lessons in computation, theoretic arithmetic, and the metric system.

The program of the normal schools is admittedly inadequate, but the tendency to adopt the suggestions of the modern reform movement is not yet appreciably strong. All that the reformers propose at present is an extension of the arithmetic program in the normal schools for women by one year and the introduction of two books of plane geometry, and in the normal schools for men the continuation of algebra up to the solution of equations of the second degree with one unknown and of geometry up to the chief principles of solid geometry.

DENMARK.

The normal schools of Denmark are public and private, but all are under State supervision; a few are coeducational; students, who must be over 18 years of age on entering, must have had one year's teaching experience before they are admitted. The minimum entrance requirements in mathematics consist of ability to handle the four fundamental operations with integral numbers and fractions (including decimals), to solve easy problems, and to work the four fundamental operations in algebra. In the normal schools the courses of the first two years are of a general cultural character, while the

third year is devoted to professional studies and practice teaching. Mathematics is taught only in the first two years, 8 hours a week in the first year and 7 in the second out of a total of 36 hours.

The standards attained in mathematics in the normal school are indicated in the scope of the final examination conducted by the State board of education:

Arithmetic.—Rule of three. Compound rule of three with application to problems including foreign coinage, measures, and weights; divisibility; alligation; compound interest; common business processes; decimal fractions. Extraction of roots; equations of the first and second degrees; calculation of surfaces and solids.

General arithmetic and algebra.—The four fundamental operations; powers and roots; factors; divisions of polynomials; greatest common measure and least common multiple of numbers and polynomials; proportion; equations of first and second degrees with one or more unknowns; simple equations of the second degree with several unknowns; decimal fractions; logarithms; compound interest and annuities; arithmetical and geometric progressions.

Geometry.—The chief principles of plane geometry; straight lines and circles; equality and similarity; calculation of areas and volumes; application of these principles to simple constructions.

It is probable that some reform will be introduced. Those who are especially interested—the normal-school instructors—demand some change in the content of the subjects and more attention to the special method of teaching arithmetic.

The State offers a one-year course and several shorter vacation courses which afford opportunities for the further education of elementary-school teachers. Here the students may select groups of special subjects. In mathematics 10 hours a week are given to arithmetic, algebra, plane and solid geometry, analytic geometry, trigonometry, and the elements of differential and integral calculus. While the underlying aim is to give the students a more thoroughgoing perspective of the school requirements, efforts are made also to impart some training in scientific methods and to develop independence.

ENGLAND.

A study of the training of teachers in England is somewhat complicated by the fact that the teachers may be recruited in several different ways. The central authority still recognizes uncertificated teachers and certificated teachers. The former may have received their preparation in pupil teacher centers, which are fast disappearing, or in secondary schools, with preliminary education in elementary schools. But in both cases the candidates to be recognized as uncertificated teachers must pass the preliminary examination for the elementary school teachers' certificate or some equivalent examination, usually the entrance or matriculation examination of one of the universities. The certificated teachers again are of two classes—those who have passed through a training college with a two years' course

or the education department attached to a university, and those who have been uncertificated teachers, have had some experience in schools, and have passed the certificate examination of the board of examination for teachers in elementary schools.

In 1911-12 the number of uncertificated teachers was 39,125; that of certificated teachers, 97,104, of whom only 55,497 had been professionally trained.

GENERAL CAREER OF CANDIDATES FOR TEACHING PROFESSION.

The most usual career of the certificated teacher, according to the standard at which the authorities are aiming, would be as follows: Entering the elementary school at the age of 6 or 7, he would remain and probably complete the course at the age of 12, when he would pass on to the secondary school. Here he would be a bursar or scholar maintained by public appropriations until 16 or 17, when he would become a student teacher for a year, combining practical training in an elementary school with academic training continued in the secondary school. At the close of this period he must pass the preliminary examination for the elementary school-teachers' certificate, or some equivalent examination, which admits him to a training college for a two-year course or the education department of a university for a three-year course. The training college gives both academic and professional training; the university education department gives the same and leads to a degree. Recently a new system has been introduced by which a student may devote three years in the university or college to academic subjects and spend a fourth year in the education department on professional work only.

ELEMENTARY SCHOOL MATHEMATICS.

The scope and character of mathematical teaching are changing rapidly in English schools, both elementary and secondary. In the elementary schools the subject has been broadened and now includes, in addition to arithmetic, some instruction in algebra to pupils in upper classes and those preparing for secondary schools, and geometry, covering practically the content of the first book of Euclid. The nature of the work in arithmetic is indicated in the following quotation from the Elementary School Code, 1912:

Arithmetic, including practical work in measuring and weighing, oral exercises, written exercises (which should be of a varied character and should not infrequently involve the application of more than one arithmetical operation), and, in the higher classes, practice in explaining the processes used. The principles and advantages of a decimal system of weights and measures should be explained to the older scholars, and the use of literal symbols in working simple problems may with advantage be taught in the higher classes. Practical instruction should be given in mensuration and should include drawing to scale, the older boys should learn the use of compasses and protractor, and such practical instruction should be correlated as far as possible with handwork.

Much of the old formal and mechanical drill in numbers is being replaced by more insistence on clear and systematic thinking about number relations in practical situations; reasoning rather than mere mechanical figuring, logical setting out rather than working by rule, are the newer aims of instruction in this field. The following scheme indicates generally the amount of arithmetic, the chief portion of the mathematics of the school, that will have been acquired by a prospective school-teacher in the elementary school period. The practical work and the work correlated with handwork, drawing, and domestic subjects are not brought out in the scheme, but must be taken into account:

FIRST YEAR.

*Introductory.**First period.*

(Without notation or any ciphering abbreviations; e. g., the 4 in 43 must always be spoken of as forty or four tens, never as four.)

- (i) Numeration, as far as 100.
- (ii) Relations to each other of the 10 primary numbers comprise:
 - (a) The different ways in which each of them may be made up of any two which come before it.
 - (b) Those of the multiplication tables.
- (iii) Use of (a) to tell the sum or difference of two numbers, one of which is a primary.
- (iv) Addition and subtraction of two composite numbers.
- (v) Continued practice in making up the tables, using what has already been remembered.

Second period.

- (i) Continuation of above; increased speed to be looked for in addition and subtraction, especially in (iii) above. Counting and other practice with round numbers up to a few tens beyond 100.
 - (ii) Notation, and suitable exercises from what precedes, to be done in writing.
 - (iii) Application of the foregoing to simple practical calculations (money, length, and height, omitting fractional values, such as farthings).
- Nomenclature, none.

SECOND YEAR.

Chiefly experimental practice, oral and written, all operations without ciphering abbreviations.

First period.

- (a) Recapitulation.
- (b) Practical calculations as before, occasionally with two steps.
- (c) Introductory and experimental:
 - (i) Extension of numeration to 1,000.
(A few hundreds will be sufficient for all the practice necessary in this period.)
 - (ii) Continued practice with tables, looking for the results to be remembered, especially the first few multiples of 12.
 - (iii) Conversion of numbers expressed in tens to hundreds, and vice versa.
 - (iv) Simple multiplication and division; i. e., performing calculations similar to those of the tables with numbers outside the range of the tables. (The two forms of division to be kept separate until the children have had experience enough to recognize their identity when the working is set down in figures.)

Second period.

- (i) Continuation of above.
- (ii) Visual demonstration of the principle $a \times b = b \times a$, and its use especially for reckoning ten, twenty, or a hundred times a number.
- (iii) Easy exercises introductory to long division, such as number of thirties or twenty-eights in 200.
- (iv) Practical exercises in the four rules with simple numbers.
- (d) Ciphering: Additions, subtractions, and multiplications.
Nomenclature: *Add, subtract, multiply* (and *unit*, if required).

THIRD YEAR.

First period.

- (a) Recapitulation; all previous exercises to be frequently repeated at full length; i. e., without technical nomenclature or ciphering abbreviations.
- (b) Practical examples as before with larger, but in the main round, numbers.
- (c) Preparatory and experimental:
 - (i) Leading to the long rules.
 - (ii) Easy examples of repeated multiplications and divisions, with numbers and money.
 - (iii) Written form of the four rules, numbers and money, multipliers and divisors as a rule of not less than two figures.
 - (iv) The standard units; pounds sterling and avoirdupois, day, year, yard.

Second period.

- (v) Submultiples of these, attention being called to their insufficiency for the general purpose of measurement.
- (vi) Experimental practice in measurement (length and value) with fractional remainders as $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{9}$, $\frac{1}{10}$, fractional notation being withheld at first.
- (vii) Easy additions, etc., of such fractions, e. g., $\frac{1}{2}d. + \frac{1}{4}d.$, $1\frac{1}{2}in. + 1\frac{1}{4}in.$ half of $\frac{1}{2}d.$, $\frac{1}{2}in. + \frac{1}{4}in.$, $\frac{1}{3}in. \times 5$.
- (viii) Reductions to be worked visually without ciphering (e. g., $17s. = 12 \text{ times } 12 \text{ pence or } 12 \text{ times } 17d.$).
- (ix) Simple examples of factorial reductions, e. g., $27 \text{ times } 8d.$, $75 \text{ times } 16$, $9 \text{ times } 16 \text{ in.}$, $90 \text{ lbs.} \times 7$.
- (d) Ciphering, practice simple and compound rules and reductions.
Nomenclature, previous terms, *divide*, and words like *third*, in the sense of the third part.

FOURTH YEAR.

- (a) Recapitulation as before; simple explanations of suitable matters to be asked for in writing.
- (b) Practical exercises rather more searching from a manual and from blackboard.
- (c) Preparatory and experimental:
 - (i) Further practice in reduction, showing the form of the rules.
 - (ii) Further practice in repeated multiplications and divisions and factorial reductions.
(Children being shown the advantage to be derived from these by examples.)
 - (iii) Return to general problem of measurement; expression of the size of fractional remainder when a common measure is obtainable.
 - (iv) Conversion of fractional quantities into others of lower denomination (oral demonstration only).
 - (v) Easy additions, etc., of fractional quantities.
 - (vi) Tables of weights and measures, capacity, right angle, degrees.
 - (vii) Further development of the long rules.

- (d) Ciphering. All previous rules, reduction, with some practice in factorial multiplication and reductions.
Nomenclature as before; terms relating to fractions (as *numerator*, *denominator*, *common measure*, same name) to be withheld, and use of abstract fractional numbers to be avoided.

FIFTH YEAR.

First Period.

- (a) Recapitulation as before.
(b) Practical exercises in what precedes from a graduated collection and the black-board.
(c) (i) Further discussion of fractions with examples of their use in the same way as whole numbers for expressing relative magnitude, e. g., $30 = 2\frac{1}{2}$ times 12, $20 = 1\frac{2}{3}$ times 12, $2\frac{1}{3}$ times what = 100? etc.
(ii) Notation of fractions.
(iii) Conversion of a fraction to an equivalent one with any given denominator (by a rational process rather than by rule, e. g., $\frac{3}{8} = \frac{9}{24}$ of twenty twentieths), occasional examples with fractional numeration (e. g., $\frac{3}{10} = \frac{11}{2}$ may assist rather than perplex the class.
(iv) Practice in making up the four rules for fractional numbers; (least common denominator, where required by inspection); the two purposes of division to be treated separately. Special attention is recommended to the case which provides the rule for finding x from the datum times $nx = A$.

Second Period.

- (v) Calculation of values, amounts, etc., which may be expressed by simple fractional numbers in terms of quantities for which rates are given; e. g., 40 cwt. at 5s. for 12 cwt. (commonly called "proportion" sums).
(vi) Easy calculations in the same way involving shop discounts at so much in the shilling, rates and taxes at so many shillings in the pound, interest and discount at so much per £100, profits and losses expressed as simple fractions of the outlay.
(vii) The principle of practice:
(a) Short division preferable to compound multiplications, division by 12, etc.
(b) Cost of n things at $\frac{1}{3}$ of x sh. = $\frac{1}{3}$ of cost at x sh.
(d) Ciphering practice; all previous rules, including simple fractions, bills of parcels, and practice.

SIXTH YEAR.

First Period.

- (a) Recapitulation; increased use of questions to be answered in writing.
(b) Practice as before.
(c) (i) (a) Rules for greatest common measure and least common multiple, whole and fractional numbers.
(b) General form of the rules for multiplication and division with fractional numbers.
(ii) Square measure and cubic measure.
(iii) Practice in setting out the work of weight and measure problems with algebraic symbols.
(iv) The same where the quantities involved are fractional numbers.
(v) Purpose and meaning of the term "per cent": 3 per cent of quantity defined to mean three hundredths of it, and to find what per cent a quantity A is of another B , we must find how many hundredths of B there are in A .
(vi) Calculations involving percentage, profit and loss, attendances, strengths of mixtures, interest, etc., to be worked by fractional rules.

Second Period.

- (vii) Further considerations of problem of measurement; failure of method of vulgar fractions to meet the general case; decimal system of expressing fractional quantities and its advantages.
- (viii) As a new system of numeration (cf. fifth year (c) (i) above).
(A still further extended system will be propounded when algebra is begun, shortly.)
- (ix) Construction and practice of the fundamental rules with decimal fractions.
- (d) Ciphering; all previous rules, etc., processes, and elementary algebra, notation, further extension of numeration to negative values, etc.

SEVENTH YEAR.

- (a) and (b) as before, practice now largely preponderating.
- (c) (i) Banking and investments; explanation of terms and procedure; examples involving stocks and discounting of bills.
- (ii) Limitations of decimal system as described; recurring periods.
- (iii) Square root; explanation of rule and exercises.

SECONDARY SCHOOL MATHEMATICS.

All candidates for the teaching profession must attend a secondary school recognized by the board of education as efficient for at least three years. The standards of attainment in mathematics in such secondary schools are fairly well defined by the requirements for the entrance examinations conducted by the various universities, which are accepted as equivalent to the preliminary examination for the elementary school teachers' certificate. The University of London requires for its matriculation examinations the following attainments in elementary mathematics:

Arithmetic:

The principles and processes of arithmetic applied to whole numbers and vulgar and decimal fractions.
The metric system.
Approximations to a specified degree of accuracy.
Contracted methods of multiplication and division of decimals.
Ratio and proportion, percentage. Averages.
Practical applications of arithmetic.

Algebra:

Symbolical expression of general results in arithmetic.
Algebraic laws and their application.
Factors of simple binomial or quadratic expressions.
Equations of the first or second degree, and problems leading thereto.
Square root. Graphs of simple rational integral algebraic functions.
Arithmetic and harmonic progression.
Geometric progression.

Geometry:

The subjects of Euclid I-IV with simple deductions, including easy loci and the areas of triangles and parallelograms of which the bases and altitudes are given commensurable lengths. (All proofs of geometric theorems must be geometric. Euclid's proofs will not be insisted upon.)

This scheme represents the general scope of the work in elementary mathematics. Variations will, however, be found, and some schools will take their pupils beyond these standards. Thus, algebra may be taken up to binomial theorem, and geometry may include the ground covered by Euclid VI and XI (1-21). Mensuration is not infrequently added in many schools, and use is generally made of the opportunity for practice in logarithms. Trigonometry, again, usually forms part of the mathematical course in secondary schools for boys. Mechanics, statics, dynamics, and hydrostatics appear more rarely as part of the usual mathematical curriculum, although they may be included under the subjects of science. The general tendencies of the reform movement in mathematics is to do away with non-essentials and thus to find time for trigonometry or some other branch, while on the side of method more attention is given to training in mathematical thinking in place of mere imitation of processes.

The preliminary examination for the elementary school teachers' certificate, which is conducted by the board of education, consists of two parts. Part I, which is a qualifying test, includes arithmetic among the subjects of examination. In Part II, elementary mathematics forms one of eight optional subjects. The scope of the requirements in arithmetic and elementary mathematics is stated as follows:

PART I. ARITHMETIC.

Excluding Troy weight, apothecaries' weight, practice, ratio, proportion, except by the unitary or fractional method, stocks and shares, true discount, foreign exchange, scales of notation, recurring decimals and complicated fractions, and square and cube roots.

Candidates may be asked to find the square or cube roots of numbers that can readily be expressed as the product of the squares or cubes of small factors.

The metric system will only be applied to measuring length, area, and volume.

Questions may be set on the mensuration of rectangular surfaces and solids.

The use of algebraic symbols will be permitted.

As a rule, the questions set will not involve long operations or complicated numbers, and the answers to money sums will not be required beyond the nearest penny.

The papers will be sufficiently long to allow candidates some latitude in the selection of questions, but no limit will be placed on the number of questions which may be attempted.

PART II. ELEMENTARY MATHEMATICS.

The papers set will be sufficiently long to allow candidates some latitude in the selection of questions, and will permit, therefore, of latitude in the teaching schemes. Candidates will not be limited in the number of questions which they may attempt, nor will they be expected to answer the whole paper.

Every candidate should be provided with a ruler, graduated in inches and tenths of an inch and in centimeters and millimeters, a small set square, a protractor, compasses furnished with a hard pencil point, and a hard pencil. Squared paper will be provided when needed.

As a rule, the questions set will not involve long operations or complicated numbers, and the answers to money sums will not be required beyond the nearest penny.

ARITHMETIC.

Excluding Troy weight, apothecaries' weight, true discount, foreign exchange, scales of notation, recurring decimals, and cube roots.

Candidates may be asked to find the cube roots of numbers that can readily be expressed as the product of the cubes of small factors.

Candidates must understand the principles of the metric system, and should be able to decimalize money readily.

Questions on stocks and shares will be of a simple character and will not involve a knowledge of brokerage.

The use of algebraic symbols will be permitted.

ALGEBRA.

As far as and including simultaneous equations (one of which is linear) in two variables—least common multiple and highest common factor—by means of factors. Problems leading to the types of equations specified.

GEOMETRY—(1) PRACTICAL.

Bisection of angles and of straight lines. Construction of perpendiculars to straight lines. Construction of an angle equal to a given angle. Construction of parallels to a given straight line. Simple cases of the construction from sufficient data of triangles and quadrilaterals. Division of straight lines into a given number of equal parts or into parts in any given proportions. Construction of a triangle equal in area to a given polygon. Construction of tangents to a circle and of common tangents to two circles. Simple cases of the construction of circles from sufficient data. Construction of a fourth proportional to three given straight lines and a mean proportional to two given straight lines. Construction of regular figures of 3, 4, 6, or 8 sides in or about a given circle. Construction of a square equal in area to a given polygon.

GEOMETRY—(2) THEORETIC.

Angles at a point.—If a straight line stands on another straight line, the sum of the two angles so formed is equal to two right angles; and the converse.

If two straight lines intersect, the vertically opposite angles are equal.

Parallel straight lines.—When a straight line cuts two other straight lines, if

- (i) a pair of alternate angles are equal,
- or (ii) a pair of corresponding angles are equal,
- or (iii) a pair of interior angles on the same side of the cutting line are together equal to two right angles,

then the two straight lines are parallel; and the converse.

Straight lines which are parallel to the same straight line are parallel to one another.

Triangles and rectilinear figures.—The sum of the angles of a triangle is equal to two right angles.

If the sides of a convex polygon are produced in order, the sum of the angles so formed is equal to four right angles.

If two triangles have two sides of the one equal to two sides of the other, each to each, and also the angles contained by those sides equal, the triangles are congruent.

If two triangles have two angles of the one equal to two angles of the other, each to each, and also one side of the one equal to the corresponding side of the other, the triangles are congruent.

If two sides of a triangle are equal, the angles opposite to these sides are equal; and the converse.

If two triangles have the three sides of the one equal to the three sides of the other, each to each, the triangles are congruent.

If two right-angled triangles have their hypotenuses equal, and one side of the one equal to one side of the other, the triangles are congruent.

If two sides of a triangle are unequal, the greater side has the greater angle opposite to it; and the converse.

Of all the straight lines that can be drawn to a given straight line from a given point outside it, the perpendicular is the shortest.

The opposite sides and angles of a parallelogram are equal; each diagonal bisects the parallelogram; and the diagonals bisect one another.

If there are three or more parallel straight lines, and the intercepts made by them on any straight line that cuts them are equal, then the corresponding intercepts on any other straight line that cuts them are also equal.

Areas.—Parallelograms on the same or equal bases and of the same altitude are equal in area.

Triangles on the same or equal bases and of the same altitude are equal in area.

Equal triangles on the same or equal bases are of the same altitude.

Illustrations and explanations of the geometric theorems corresponding to the following algebraic identities:

$$\begin{aligned} k(a+b+c+\dots) &= ka+kb+kc+\dots, \\ (a+b)^2 &= a^2+2ab+b^2, \\ (a-b)^2 &= a^2-2ab+b^2, \\ a^2-b^2 &= (a+b)(a-b). \end{aligned}$$

The square on a side of a triangle is greater than, equal to, or less than the sum of the squares on the other two sides, according as the angle contained by those sides is obtuse, right, or acute. The differences in the cases of equality is twice the rectangle contained by one of the two sides and the projection on it of the other.

Loci.—The locus of a point which is equidistant from two fixed points is the perpendicular bisector of the straight line joining the two fixed points.

The locus of a point which is equidistant from two intersecting straight lines consists of the pair of straight lines which bisect the angles between the two given lines.

The circle.—A straight line drawn from the center of a circle to bisect a chord which is not a diameter is at right angles to the chord; conversely, the perpendicular to a chord from the center bisects the chord.

There is one circle, and one only, which passes through three given points not in a straight line.

In equal circles (or, in the same circle) (i) if two arcs subtend equal angles at the centers, they are equal; (ii) conversely, if two arcs are equal, they subtend equal angles at the center.

In equal circles (or, in the same circle) (i) if two chords are equal, they cut off equal arcs; (ii) conversely, if two arcs are equal, the chords of the arcs are equal.

Equal chords of a circle are equidistant from the center; and the converse.

The tangent at any point of a circle and the radius through the point are perpendicular to one another.

If two circles touch, the point of contact lies on the straight line through the centers.

The angle which the arc of a circle subtends at the center is double that which it subtends at any point on the remaining part of the circumference.

Angles in the same segment of a circle are equal; and if the line joining two points subtends equal angles at two other points on the same side of it, the four points lie on a circle.

The angle in a semicircle is a right angle; the angle in a segment greater than a semicircle is less than a right angle; and the angle in a segment less than a semicircle is greater than a right angle.

The opposite angles of any quadrilateral inscribed in a circle are supplementary; and the converse.

If a straight line touch a circle, and from the point of contact a chord be drawn, the angles which this chord makes with the tangent are equal to the angles in the alternate segments:

If two chords of a circle intersect either inside or outside the circle, the rectangle contained by the parts of the one is equal to the rectangle contained by the parts of the other.

Proportion—Similar triangles.—If a straight line is drawn parallel to one side of a triangle, the other two sides are divided proportionally; and the converse.

If two triangles are equiangular their corresponding sides are proportional; and the converse.

If two triangles have one angle of the one equal to one angle of the other and the sides about these equal angles proportional, the triangles are similar.

The internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle, and likewise the external bisector externally.

The ratio of the areas of similar triangles is equal to the ratio of the squares on corresponding sides.

Elementary solids.—Candidates will be expected to be acquainted with the forms and simple properties of the cube, rectangular block, cylinder, and cone.

The board's preliminary examination or its equivalent is the main avenue of entrance to training college, but it must be borne in mind that a certain percentage of the elementary school teachers in England and Wales do not attend a training college, but may become certificated by passing the necessary examinations. The usual length of the course in the training colleges is two years. The best candidates, however, are usually selected for admission to the departments of education of universities, where they are required to take one or other of the courses leading to a degree, with education as one branch of the required work. A still higher type consists of those students who take a four-year course, the first three being devoted to some course leading to a degree and the last to purely professional work.

THE TRAINING COLLEGES.

The various ways by which a candidate can become a teacher in an elementary school have already been indicated above. It will be necessary here to deal only with the course provided in the two-year training college and in the university departments of education. The two-year training colleges are institutions established by private and mainly religious bodies, and recently in an increasing number by local education authorities. They provide in the two years both academic and professional courses, and, up to the present, have placed more emphasis on the academic than on the purely professional side.

The standards of admission have, so far as mathematics is concerned, already been described. Elementary mathematics is continued as one of the required subjects of study, and here, again, the require-

ments of the final examination conducted by the board of education and leading to the elementary school teachers' certificate, indicate the scope of the subject. Two papers are given in the final examination. The first paper includes arithmetic and algebra, Part I, which all candidates are required to take; a few questions are also set in geometry, but women are not required to answer them unless they desire to obtain distinction. The second paper also contains questions in arithmetic and algebra, Part II, and in geometry; all men students must take this paper, but women only on the conditions just described. The following scheme of study is outlined in the board's regulations for students taking the final examination in 1913 and 1914:

EXAMINATION IN MATHEMATICS FOR ELEMENTARY-SCHOOL TEACHERS' CERTIFICATE.

ARITHMETIC AND ALGEBRA

Part I.—Arithmetic, excluding Troy weight, apothecaries' weight, true discount, cube root, foreign exchange, and scales of notation. Questions on stocks will not involve a knowledge of "brokerage." Candidates must be acquainted with the principles of the metric system. Algebraic symbols and processes will be generally permitted.

Algebra as far as, and including, simple equations of one unknown, with easy problems leading up to such equations.

Part II.—Quadratic equations of one unknown, simple simultaneous equations of two unknowns and easy simultaneous equations involving the squares of the unknowns and problems leading up to these equations. Arithmetical progressions; geometrical progressions to a finite number of terms; square root, highest common factors, and lowest common multiples, ratio, and proportion. Permutations and combinations. The binomial theorem for positive integral exponents. The use of logarithmic tables.

N. B.—Questions involving graphic methods may be set in both parts, and when such questions are proposed squared paper will be provided.

GEOMETRY.

Every candidate must be provided with a ruler graduated in inches and tenths of an inch, and in centimeters and millimeters, a small set square, a protractor, compasses furnished with a hard pencil point, and a hard pencil.

Figures should be drawn accurately with a hard pencil.

Questions may be set in which the use of the set square or of the protractor is forbidden.

Any proof of a proposition will be accepted which appears to the examiners to form part of a logical order of treatment of the subject. In the proof of theorems and deductions from them, the use of hypothetical constructions is permitted.

Practical geometry.—The following constructions and easy extensions of them: Bisection of angles and of straight lines. Construction of perpendiculars to straight lines. Simple cases of construction from sufficient data of triangles and of quadrilaterals. Construction of parallels to a given straight line. Construction of angles equal to a given angle. Division of straight lines into a given number of equal parts. Construction of a triangle equal in area to a given polygon. Construction of tangents to a circle. Construction of common tangents to two circles. Construction of circumscribed, inscribed, and escribed circles of a triangle. Construction of a segment of a

circle containing an angle of given magnitude. Division of straight lines into parts in any given proportion. Construction of a fourth proportional to three given straight lines and of a mean proportional to two given straight lines. Division of straight lines in extreme and mean ratio. Division of a straight line internally or externally into segments, so that the rectangle under the parts is equal to a given square. Construction of regular polygons in and about circles. Construction of a circle from sufficient data of the following character: (1) Radius given, (2) point on the circle given, (3) contact with a given straight line or circle, (4) contact with a given straight line at a given point. Construction of a rectilinear figure to a specific scale or of specified area, and similar to a given figure. Construction of a square equal in area to a given polygon. (In cases where the validity of a construction is not obvious, candidates may be required to indicate the reasoning by which it is justified.)

Illustration and explanation by means of rectangular figures of the following identities:

$$\begin{aligned}
 k(a + b + c + \dots) &= ka + kb + kc + \dots \\
 (a + b)^2 &= a^2 + 2ab + b^2, \\
 (a - b)^2 &= a^2 - 2ab + b^2, \\
 a^2 - b^2 &= (a + b)(a - b)
 \end{aligned}$$

Theoretic geometry—Candidates should be acquainted with the fundamental propositions concerning angles, parallel straight lines, and the congruence of triangles, such as are contained in the substance of Euclid, Book I, Propositions 4-6, 8, 13-16, 18, 19, 26-30, 32. Easy deductions from these theorems will be set, and arithmetical illustrations will be included.

The substance of the theorems contained in Euclid, Book I, Propositions 33-41, 43, 47, 48; and Book III, Propositions 3, 14-16, 18-22, 31, 32, 35-37; Book VI, Propositions 1-8, 19, 20, 33, together with Propositions A and D. Questions upon these theorems, easy deductions from them, and arithmetical illustrations will be included.

In dealing with proportion it may be assumed that all magnitudes of the same kind can be treated as commensurable.

Candidates will be expected to be acquainted with the forms of the cube, the rectangular block, the tetrahedron, the sphere, the cylinder, the wedge, the pyramid, and the cone.

In addition to the ordinary prescribed subjects, students may also offer an examination in 2 optional subjects selected from 20 subjects, including mathematics. The scope of the examinations in optional elementary mathematics is shown in the following syllabus:

Elementary mathematics up to, and including: Geometry of lines, circles, and of the simpler solid bodies, but excluding conic sections.

Coordinate geometry of lines and circles.

Algebra: Progressions. The binomial theorem for positive integers. Logarithms and their use. Probabilities.

Plane trigonometry: The solution of triangles

Mechanics: Friction. Virtual work. Center of gravity. Simple machines. Motion of pendulums and projectiles. Motion in a circle. Impulsive forces acting on elastic and inelastic particles.

The board of education, however, does not require training colleges to undertake the whole of this syllabus, nor is it intended that the examination shall be a test of the knowledge of the whole of this ground so much as a test of mathematical power.

The professional work is included in a course on principles of teaching and a course of six weeks' practice teaching. The board of education offers a number of suggestive syllabi on principles of teaching which include "Numbers and elementary mathematics; methods and apparatus; practical instruction and its relation to handwork; use of literal symbols and graphs by older children; mensuration and geometrical drawing." Special provision is made for candidates who intend to teach in infants' schools. At the London Day Training College a course of lectures, extending over two terms, is given on general principles of mathematical teaching, with special reference to the more elementary parts of the subject. Demonstration lessons are also given by the members of the staff in the practice schools of the college. The system varies, and it is impossible to make any general statement of the amount of time given to the methods of teaching mathematics. Similarly, in the case of practice teaching, in which each student must engage for six weeks, there is no requirement that mathematics shall be taught; although it is very probable that some time will be given to this subject. But at best the time devoted to practice teaching is limited. It is becoming more usual now for the lecturer in mathematics of the college to include the methods of teaching in his course, to give demonstration lessons, and supervise the practice of the students in their field.

The majority of students who pass through a course of training for the elementary school positions attend the two-year training colleges. Provision is, however, made for three-year courses in training departments closely connected with the universities. The number who can avail themselves of these courses is limited, since, so far as is possible, it is proportioned to the total number of students in the respective universities. The three-year courses include both academic and professional subjects, and students are expected to have met the ordinary requirements for entrance to the universities. Since the students themselves select the group of academic subjects which they will study for their degree, there is no compulsion that mathematics shall be included. Hence the general standard of attainment in this field is the elementary mathematics required for the entrance examinations. A few students, of course, may include mathematics in their course or may even take an honors course, that is, specialize intensively in the subject. It is intended that in future the training of teachers at the universities shall be given in a four-year course—three to be devoted to academic subjects and one to the professional. Among the professional subjects are included principles of education which cover instruction in the methods of teaching all the subjects of a public elementary school. The practical work consists of eight weeks of practice teaching in an elementary school.

The three-year course, therefore, does not provide for higher attainments in mathematics than are found in the examination for the board's certificate; it does, however, afford better opportunities than the two-year course to those students who are interested and desire to carry forward their study of mathematics.

FRANCE.

The training of teachers in France is entirely under the control of the State. There are 85 normal schools for men and 84 for women teachers. Students are admitted to the normal schools by a competitive examination. Candidates for these examinations must possess the *brevet élémentaire*, a certificate which is itself a qualification for teaching in the *écoles maternelles*, *classes enfantines*, and *écoles primaires élémentaires*. Candidates for the *brevet élémentaire* must be 16 years of age. The examination for this certificate consists of questions divided into three series. The mathematical questions are somewhat simple and consist (1) of a question in arithmetic and the metric system and the analytic solution of a problem covering the four operations with integral numbers and fractions and the measurement of surfaces and volumes, and (2) of oral questions in arithmetic and the metric system. The exercise in drawing may, for boys, include geometric drawing of a simple object with ground plan, cross section, and elevation. The examination is based on the following standards, which may serve to indicate also the attainments of the elementary schools:

Review covering principles and theory of arithmetic, and short processes in mental and written work.

Primary numbers; the most important cases of divisibility.

Factors.

Greatest common divisor, interest, accounts, partnerships, averages, etc.

Metric system applied to the measurement of volume and weight.

First notions of bookkeeping.

As a general rule, the examinations show that the reasoning ability of the candidates is defective and indicate no powers of analytic thinking or acquaintance with simplified methods. Considering that the *brevet élémentaire* is accepted as a qualification for appointment as temporary teacher (*stagiaire*) in the schools mentioned above, it is felt that these standards are by no means too high.

The competitive examination for admission to the normal schools requires approximately the same standards. Since 1910 definite syllabi are issued for each school every four years in certain subjects, including mathematics, and it is hoped that as a result the work of the pupils will be less vague and more accurate than hitherto. The candidates come from rural schools, higher primary schools, and the

supplementary courses (*cours complémentaires*). The preparation and attainments of the candidates are accordingly not uniform. It is proposed, therefore, that the following standards be accepted as sufficiently satisfactory:

- (a) Detailed study of integers and decimals and their application in mental arithmetic.
- (b) Definitions, theories, and rules of the following operations in written work: Addition, subtraction, and multiplication of integers. Definitions and rules of the divisions of integers and the extraction of the square root.
- (c) The same for fractions and decimals.
- (d) Properties of sums, differences, products, and exact quotients. Simplified processes.
- (e) Theory and rules of mental arithmetic.
- (f) Characteristics of divisibility by 2, 3, 4, 5, 9, 25. Tests of multiplication and division by the excess of 9's.
- (g) Metric system.
- (h) Problems on the four fundamental operations. Percentage. Rules of interest and commercial accounts.
- (i) Solution of simple numerical equations with one and two unknowns.

In geometry the following outline is suggested:

Experimental study by folding, drawing and measurement, of the chief properties of the following figures: Straight line, plane figures, circumference, angle, triangle, parallelogram, rectangle, rhomboid, square, rectangular parallelepiped, cube, cylinder, cone, and sphere. The use of the rules, square, compasses, protractor, measuring gauge. Rules for measurement of surfaces and simple volumes.

Candidates for the competitive examination must be between the ages of 16 and 18. The normal school course consists of three years. Since 1905 the first two years have been devoted to general instruction and the last year to professional work. The schedule in mathematics is as follows:

Normal school course in mathematics.

	First year.	Second year.	Third year.	
			General.	Professional.
Total hours per week.....	31	32	23	9
Mathematics.....	3	4	1	1

There is an examination for promotion at the end of each year. At the end of the second year the students must pass the *brvet supérieur*; at the end of the third year comes the final examination (*l'examen de fin d'études normales*).

The aims of mathematical instruction (including geometric drawing) in the normal schools are declared to be (1) intellectual training through habits of clear and precise thinking, logical analysis, discrimination of the true and false, and accuracy in reasoning; (2) to

furnish a certain amount of useful and definite knowledge and to clarify and complete such knowledge as the students already have; (3) instruction in the methods of teaching arithmetic and geometry.

The course of study for the three years is as follows:

FIRST YEAR.

Practical and mental arithmetic, and algebra (one hour): Algebraic calculation; positive and negative numbers; operations limited to such applications as can be employed in the normal schools. Simple equation. Problems.

Geometry (2 hours): Straight lines, circumference; angles, triangles; rules of equality; parallels; parallelograms; chords and arcs of circles; tangents, measurement of angles; construction.

Straight lines and planes; parallel straight lines and planes; straight lines perpendicular to planes; dihedral angles; perpendicular planes; parallel planes. Introduction to trihedral angles. Definitions of simple polyhedrons, prisms, parallelepiped, pyramid.

Proportional lines. Similar triangles and polygon. Metric properties of triangles and circles. Exercises. Measurement of areas. Regular polygons. Length of circumference. Area of circle. Measurement of volume, parallelepiped, prisms, pyramid.

Methods employed for solution of problems or demonstrations of theorems. Methods of deduction. Analysis and synthesis. Examples.

SECOND YEAR.

Algebra (1 hour): Review. Quadratic equations with one unknown with simple problems. Compound interest and annuities.

Geometry (1 hour): Review. Cylinder, cone, sphere. Elementary properties of solids. Mensuration.

Arithmetic (2 hours): Theoretic arithmetic; proportions; rule of three; partnership; bonds; insurance; discount; average maturity; proportional division; problems of alligation and alloys.

THIRD YEAR.

Surveying (10 hours). Cosmography (10 lessons).

Method of mathematical sciences (mathematical deduction, definitions, axioms and postulates, propositions, examples). Methods of demonstration and examples. Outline of the development and progress of mathematical sciences (3 lessons).

Serious criticisms are brought against the present arrangement of putting algebra in the first year, to be followed by arithmetic in the second year. It is objected that (1) the students, through insufficient knowledge of arithmetic, do not see its connection with algebra, which in any case can not be studied without a knowledge of arithmetical terms; (2) mental work is based largely on such knowledge, while the practical arithmetic lacks the solid foundation which should be laid in the first year; (3) the study of proportional lines and equalities demands a knowledge of ratio and proportion which are not taken up until later. The program of the third year is further criticized as too ambitious; too much is attempted in too little time. It is suggested, for example, that, instead of teaching the history of

mathematical sciences, an introduction might be given to elementary trigonometry, which is at present neglected entirely. The standards of attainment for women are in all cases lower than for men students.

METHODS OF INSTRUCTION.

The regulations recommend that the Socratic method of instruction be used in the normal schools. It is conceded that this recommendation has much in its favor, but it neglects the time element, for much more ground could be covered by the didactic method. Instruction of this character, however, is in agreement with the general aim of the teaching of mathematics. It also enables the teachers to emphasize the characteristic methods of mathematics, of deduction, analysis and synthesis, and induces the students to consider the theory underlying the methods employed and so to discern for themselves the appropriate method in each case. It is admitted, however, that in spite of the prescription of the regulations to emphasize principles and theory, the students are not sufficiently mature to look upon arithmetic as anything but an accumulation of facts and a collection of operations or to recognize the value, use, or connection of these. Hence, while they can perform the four operations, they fail to arrive at their true meaning or at exact definitions and rigorous proofs. The charge is as usual laid at the door of the elementary schools, for too often the instructor is compelled to repeat the work of the lower stage. It is, however, suggested that such repetition can be made valuable if it is done in such a way as to throw new light on the general theory and principles of the subject, the aim suggested in all the regulations since 1881.

On the practical side the regulations recommend an emphasis on mental arithmetic in order to secure readiness and flexibility in working with numbers. Problems are selected from the operations of everyday life—commerce, industry, manufacture, and agriculture. The impossible or improbable exercises of former years are excluded.

The same spirit permeates the suggestions on the teaching of algebra. Long and complicated exercises are to be avoided and those parts of the subjects are to receive attention which can be of some service in other subjects of the normal-school course. At the same time from six to seven hours are to be devoted to the theory of algebra.

Geometry is, so far as possible, made practical, and little attention is given to theory. The subject is correlated with drawing, and the figures that are studied in the latter connection are made the objects of study in the geometry lessons.

The instructor is not limited in any way in the selection of a textbook, but the textbook is only to be used for the study of details.

The work of the classroom is intended to bring out the essential points and to develop the most difficult parts.

At the close of the second year of the normal-school course the students are given an examination in the general academic work of the first two years, leading to the *brevet supérieur*. This examination is required as a qualification for appointment as a director of a *cours complémentaire* or as teacher in such a course or in the higher elementary school. It is also one of the preliminary requirements for the *certificat d'aptitude au professorat des écoles normales* and certificates for teaching special subjects. Candidates for the *brevet supérieur* must be over 18 and hold the *brevet élémentaire*. The examination consists of two parts, written and oral. In the written part, mathematics is grouped with science and the examination consists of (a) a problem in arithmetic or geometry, applied to practical operations, (b) a question on the theory of arithmetic, and, for men only, (c) a question on physics and natural science as applied to hygiene, industry, and agriculture. In the oral part, questions are given on arithmetic and problems to be performed mentally, and for men, on algebra and geometry. It will easily be seen that little can be expected in an examination of this character, and the importance of the subject is still further minimized by the small proportion of the marks allotted to it.

The third year of the course is devoted to the professional preparation of the students. The work includes a study of the principles of education and methods of instruction, with observations of model lessons and practice in an elementary school of the locality or attached to the normal school.¹ The topics studied in connection with the special methods of teaching arithmetic include the following:

General principles governing the teaching of elementary arithmetic.

Study of courses in arithmetic in elementary schools with reference to such special problems as the teaching of fractions or the introduction of decimals.

Knowledge of numbers. Importance of the first 10 numbers.

Arithmetical operations and their introduction at different stages of the elementary school course.

Solution of problems. Development of reasoning ability. Principles underlying mental arithmetic. Critical study of the textbook.

Model and criticism lessons are conducted in connection with this study. The students themselves are expected to do two months of practice work during the year in two periods of a month each. There is no provision for the time to be allotted to the teaching of arithmetic. The practice work is not regarded with much seriousness, since teachers are expected to have at least two years' actual experience before they receive their permanent appointments as *titulaires*.

¹ General method is included under pedagogy and principles of education.

The final examination for the *certificat de fin d'études normales* consists of three parts: (1) A written theme on a pedagogical topic selected from a list, for which two months are allowed; (2) a practice lesson after one hour's preparation; (3) an oral examination with questions on the organization of a class, school programs, methods of instruction, with special reference to the practice lesson.

In general the work of the normal schools is criticized on three grounds, over the first of which the normal school has obviously no control: (1) The poor preparation of the students; (2) too much subject matter is to be covered and too many class periods are required to enable the students to do any genuine independent study; and with reference to mathematics, (3) the proportion of the marks allotted to mathematics in the various examinations is small.

GERMANY.

No country is making greater progress in the training of its teachers than Germany, and in no subject is this more true than in mathematics. Wedded as Germany, and more particularly Prussia, has been to the immediate demands of professional training, nothing less than a revolution is taking place in the whole conception of the training of teachers.¹ In place of the narrow restrictions which the traditions of 50 years have imposed on the normal school, the realization is gaining ground that a good teacher must be broadly educated, with a liberal grasp of subject matter, and not merely the master of a few tricks of the trade. But the influences of tradition are not to be swept away in a few years. All that can be said for the present is that the tendencies are liberal and the recent reforms in mathematical study are gradually being introduced. It is not intended, however, that the mathematical curriculum of the normal schools shall be more extensive than that of the secondary school, nor again that the students of the normal schools shall be encouraged to proceed to the universities. But, as will be indicated later, while there is progress universally throughout Germany, it is not equally marked in all the States. The smaller States of central Germany, for example, are far more liberal than Prussia or Bavaria; while Saxony may be said to stand midway.

There are, in general, two main types or systems for training teachers—that of Prussia and that of Saxony, the remaining States following more or less closely the one or the other. Hesse and Bavaria until recently stood alone in having a five-year course, but in 1912 the Bavarian course was changed to the usual six years. The chief differ-

¹ The present account deals solely with the training of men; the women who intend to teach in elementary schools in the majority of cases pass through the secondary schools for girls (*Höhere Mädchenschulen*), but where this is not the case, as in Württemberg, the requirements in mathematics are always lower for women than for men.

ences of organization between these two systems are that the Prussian course is divided into two periods—the preparatory course of three years in the *Präparandenanstalt* and the three-year course in the normal school (*Volksschullehrerseminar*). The other system provides a course of six years in the same institution (*Volksschullehrerseminar*). This distinction is slight, however, for the standard of work does not vary, although in Prussia the qualifications of teachers in the preparatory institutions are not as high, for example, as those of the instructors in the normal schools of Saxony. The first five years of the whole course whether given in one or two institutions, are devoted mainly to general academic training, the professional preparation being introduced in the fourth year and given special emphasis in the final year.

The training career of the students begins at the age of 14, when they enter the preparatory course, which, like the normal-school course, is intended solely for future teachers. The majority enter from the elementary schools, the few exceptions coming from the "Real Schools" (*Realschulen*) and occasionally from a secondary school with full nine-year course. Hesse alone is trying the interesting experiment of providing a one-year professional course for graduates of the nine-year secondary schools.

The students entering the training course are selected on the basis of an entrance examination on the subjects of the elementary schools. The outline prescribed in mathematics by the Prussian Government will serve to indicate the scope of the work:

Arithmetic.—Lower stage: Operations with concrete and abstract numbers from 1 to 100. Middle stage: Similar operations without any limit; averages; factors and reductions; simple rule of three. Upper stage: Fractions and applications to arithmetic of everyday life, and decimals.

So far as possible, it is required that the work in the lower stage should be mental and preparatory with work on the blackboard, and that the problems should be practical and related to the needs of everyday life. At all stages it is desired that emphasis should be placed on clear thinking and correct expression, with ability to do independent, accurate, and rapid work as the ultimate goal.

Geometry.—Lines, angles, triangles, quadrilaterals, regular figures, the circle and its parts, regular solids. To this are added, in graded schools, principles of lines and angles, and equality and similarity of plane figures.

The work in drawing is closely related both to geometry and arithmetic. Algebra has practically disappeared entirely from the elementary school program.

The elementary school programs in arithmetic are very similar in the other States. The larger cities are offering somewhat more extensive courses, plane and solid geometry, for example, being added to the descriptive geometry.

THE TRAINING COURSE.

It will serve for the present purpose to disregard the division between the preparatory institutions and the normal schools which prevails in Prussia, and to consider the six-year course as a unit. The work of the first few years is devoted to a review of the elementary school course and the introduction of algebra. The aim of the whole course may be indicated by several quotations:

Prussia.—The aim is to secure clear comprehension, a readiness in solving problems and ability to impart instruction in arithmetic and geometry in the elementary school. The preparatory institutions serve to lay a sound foundation by reviewing and extending the work of the upper grades of the elementary school.

Saxony.—The aim is to secure accuracy and ability to solve problems in arithmetic and geometry that are appropriate to the elementary and continuation schools, with clear insight into the essence and principles of the methods necessary for professional work; and to secure the possession of such mathematical knowledge as will furnish a grasp of the problems of daily life beyond the standards of the elementary and continuation schools, or as may be essential for more thorough work in other subjects, e. g., physics and geography.

Bavaria.—The chief aims of instruction are the thorough grasp of the relations of number and space, correct understanding of operations and methods in calculations, accuracy, and rapidity in estimating and in solving problems.

Württemberg.—The aim is to secure clear insight into the most important principles of elementary mathematics, their relation, application, and significance to other sciences; readiness in the solution of problems.

The time allotted to mathematics (arithmetic, algebra, geometry, and, in some States, trigonometry) shows considerable variation. In some States very little is done in the last, or sixth, year of the course; in general, no new work is then taken up, except where there are elective courses. The following table represents the number of hours per week given to the mathematical subjects in each year of the course:

Number of hours per week in mathematical subjects.

States.	First year.	Second year.	Third year.	Fourth year.	Fifth year.	Sixth year.
Prussia.....	5	5	5	5	5	1
Saxony.....	4	4	5	4	4	3
Bavaria.....	4	4	4	3	3	12
Württemberg.....	5	5	5	4	2	4
Hesse.....	6	6	5	5	5	4
Baden.....	5	5	5	4	4	4

¹ Elective.

In presenting an outline of the mathematical studies, it will be convenient to give in parallel columns the work of the normal schools in Prussia and Saxony, as the chief representatives of the two types, and to indicate the variations found in other States.

ARITHMETIC AND ALGEBRA.

*First year.**Prussia.*

Fundamental operations with integral numbers, decimals, and vulgar fractions; averages; percentage; profit and loss; taxes and customs; alloys; approximations.

Saxony.

Operations with vulgar and decimal fractions; practical applications.

Second year.

Interest; profit and loss; discount; social arithmetic; alligation; insurance, especially workmen's compensation; notes and bills.

The four fundamental operations in algebra. Extension of the idea of number to negative and compound numbers.

Third year.

Introduction to algebra; fundamental operations with integral numbers; equations of the first degree with one unknown.

Fractions, factors; proportions; equations of the first degree with one unknown quantity; practical problems solved by means of equations and proportions; powers.

Fourth year.

Powers; roots; logarithms; equations of the first degree with several unknowns.

Roots; extension of the idea of numbers to irrational and imaginary numbers; equations of the first degree with two or more unknown quantities; practical problems in arithmetic; introduction to logarithms.

Fifth year.

Equations of the second degree; arithmetical and geometric progressions; compound interest and annuities.

Logarithms; equations of the second degree; progressions; compound interest and annuities.

Sixth year.

Methods of instruction in arithmetic and geometry.

Continuation of compound interest and annuities review; discussion of problems of method.

GEOMETRY.

First year.

Lines, angles, and triangles. Theory.

Plane geometry; fundamental ideas; straight lines; the circle in general; angles and triangles.

Second year.

Parallelograms; trapezoid; regular polygons; circles.

Continuation of triangles; quadrilaterals and polygons; elements of the theory of the circle; constructions by geometric analysis.

Third year.

Equality of figures; areas of figures of equal size, regular polygons and circles.

Comparison of areas; measurement of lines and surfaces; proportionality of lines; theory of equality.

Fourth year.

Proportionality of straight lines and similarity of figures; solid geometry.

Proportional lines about a circle; constructions by algebraic analysis; measurement of lines, regular figures, and circles.

Fifth year.

Solid geometry; constructions of algebraic expressions; trigonometric functions; mensuration of plane figures.

Plane trigonometry; solid geometry.

Sixth year.

Arithmetic and geometry included in the one hour per week devoted to the methods of instruction in these subjects.

Conclusion of solid geometry.

These programs form the core of the mathematical studies throughout the German normal schools. More extended courses are found in some of the normal schools of the smaller States of central Germany and of Wurttemberg. Thus the use of graphs has been introduced as early as the second year at Sondershausen and Cöthen and in the third year in Wurttemberg. At Hildburghausen and Gotha algebra is carried up to quadratic equations; in Wurttemberg the elective course in the sixth year includes the binomial theorem and some study of functions. In geometry Cöthen introduces the theory of projections in the third year, and solid geometry is begun in Wurttemberg in the fourth year. Trigonometry finds a place in the programs of Coburg, Greiz, Cöthen, and Sondershausen in the fourth year, while spherical trigonometry is found in the fourth-year course at Cöthen, in the fifth year in Wurttemberg, and in the sixth year in Baden, with applications to geography and astronomy. Surveying with practical work in the open has been introduced in many schools, but it is objected that too many students are required merely to look on while the few gain the practical experience.

The suggestions and recommendations of the *Deutscher Ausschuss für den mathematischen und naturwissenschaftlichen Unterricht* will indicate at once the criticisms that are leveled against the work of the normal schools and the direction in which the reformers are moving. In its *Vorschläge für den mathematischen, naturwissenschaftlichen und erdkundlichen Unterricht an Lehrerseminarien* the committee sets up the following theses: (1) The mathematical course should be six, even seven, years in length. (2) Preparation for the teaching profession covers both academic and professional training; each is to be treated separately; the professional training should come at the close of the academic. (3) The six-year course should be given in one institution under one director and one staff with the same spirit throughout. (4) The normal school should in every way approximate more closely to the secondary schools, but should not attempt to prepare for the university. (5) Those subjects which are not of great value to all students should be limited in treatment, as well as those which can not be made up with the help of books, and more emphasis should be placed on training in the methods of independent study, the use of books, apparatus, etc., which are as a rule not accessible to the teacher in a small elementary school. (6) The students should be taught methods of study, trained in habits of thinking, and given a grasp of the broader phases of education rather than be crammed in subject matter and methods of the elementary school. (7) The normal school staff should be in a position to give instruction in all stages of the school, with full mastery of the subject. (8) Opportunity should be afforded in the last year of the course for specialization in some group of subjects or some special subject.

The committee recommends five hours a week in the first five years and two hours a week in the last, for mathematics. The problem at present is to secure not only logical training, ability to handle numbers, and preparation for teaching arithmetic and mensuration, but also adaptation to the problems of the modern school. Greater emphasis is necessary on the thought content of mathematics and its connection with the actual facts and needs of practical life. Ability must be developed to think mathematically about the phenomena of the world and the relation of mathematics to life, and to develop the idea of change and function in the most general sense. In general the standard set up in the Meran proposals for the secondary schools should be carried over to the normal schools with the addition that the future calling of the teacher must be borne in mind. The teacher, however, must be in a position to take a broad view of his subject from a higher standpoint. The Prussian program, for example, should be supplemented by the addition of spherical trigonometry, with application to mathematical geography and astronomy, and a little of conic sections. The study of complex numbers and infinitesimal calculus should be offered as an elective only. Algebra could well be begun in the first year. Arithmetic should deal with the problems of family, community, and State management, and economic life in general, with the emphasis as much on content as on arithmetical ability. The five hours of the course should be divided so that two hours a week are given to algebra, two hours to geometry, and one hour to arithmetic, but the different branches should be taught by one teacher.

COURSE IN MATHEMATICS RECOMMENDED BY THE COMMITTEE.

FIRST YEAR.

Arithmetic: Problems of household management and the vocations with applications of the rule of three and percentage.

Algebra: Algebraic formulas for the fundamental operations with integral and fractional numbers as an introduction to general arithmetic. Concrete representation of numbers by lines. Evaluation of algebraic expressions. Numerical solution of simple equations with one unknown. The use of brackets and their application to mental arithmetic and abbreviated methods.

Geometry.—Straight lines and angles. Triangles and quadrilaterals, particularly parallelograms, trapezoid, and deltoid (motion, interdependence of the parts, symmetry, congruence, area). Measurement and plotting of lines and angles in the field. Geometric drawing in connection with field observations and the construction of simple triangles and quadrilaterals.

SECOND YEAR.

Arithmetic: Community and State management.

Algebra: The idea of relative magnitudes with practical examples, concretely illustrated by directed numerical line segments. Rules for calculation with relative magnitudes. Simple polynomials. Theory of proportion in connection with frac-

tions. Pure and applied equations of the first degree with one or several unknowns. Simple inequalities.

Geometry.—Chief principles of the circle. Equivalence of figures bounded by straight lines. Computation of the areas of such figures and their transformation into equivalent figures. Approximate calculation of figures bounded by curved lines. Such constructions as are closely connected with the course. Applications of the theory of triangles and quadrilaterals to simple problems in surveying and measurement of altitudes in the field. Geometric drawing of the circle and straight line in ornamental design. Drawing of plane figures. Examples from surveying and technical sciences.

THIRD YEAR.

Arithmetic. The money market and international exchange. The decimal system. The simplest rules of divisibility. Decimal fractions. Approximations in working with fixed numbers.

Algebra. Graphic representation to illustrate ordinary relations. Graphic representations of linear functions and their application to the solution of equations. Powers with positive integral exponents. Graphic representations of the functions $y=ax^n$ (parabolic curves), $n=2, 3, 4$. Powers with negative integral exponents. Graphic representation of the functions $y=ax^{-n}$ (hyperbolic curves), $n=1$ and 2 . Roots.

Geometry.—Theory of similitude, and the circle. Measurement of circle. Practical exercises in mensuration. Geometric drawing to scale, especially of sketches made in the field by the students; drawing of curves and use of squared paper.

FOURTH YEAR

Arithmetic.—Selected parts of commercial arithmetic.

Algebra.—Equations of the second degree with one unknown. Relation between coefficients and roots. Graphic representation of an expression of the second degree depending on a varying quantity. Graphic solution of equations of the second degree with one unknown, also by the intersection of a fixed parabola and a movable line, or of a fixed line and a movable circle. Equations of the second degree with two unknowns in simple examples, to be solved numerically and graphically.

Geometry.—Simplest propositions about lines and planes in space; plane representations and measurement of solids. Conic sections as plane sections of the right circular cone; Geometric drawing; introduction to descriptive geometry. Representation of simple solids in perspective, as well as ground plan and elevation. Plane sections of simple solids. Plane diagrams of the surface of such solids. Construction of ellipses. Spirals.

FIFTH YEAR.

Arithmetic.—Simplest ideas of insurance. Review of elementary school arithmetic, with observations on method.

Algebra.—Extension of the idea of powers. The function $y=ax^2$ and its graphic representation. Idea and use of logarithms; simple method of calculating logarithms; representation of logarithmic functions. Theory and use of the slide rule. Arithmetical progressions of the first order. Geometric progressions and their application to compound interest and annuities. Comprehensive review of the extension of numbers to irrational numbers.

Geometry.—Trigonometry in connection with constructive plane geometry. Graphic representation and calculation of trigonometric functions. Application of the measurement of triangles to practical problems in connection with independent measurement by the students in the field. Sine and cosine in spherical trigonometry and their application to fundamental problems of mathematical geography and astronomy;

determination of time and position. Comprehensive review of the scientific structure of geometry. Geometric drawing as in third year, with the simplest penetrations and their practical applications; shading.

NINTH YEAR.

Comprehensive review of the functions studied, with examples from geometry and physics, especially mechanics.

Geometry.—Constructive treatment of conic sections, with indications of their analytic representation.

With reference to the method of instruction in the normal schools the committee emphasizes the need of reform. The pupils should be discouraged from mechanical work and memorization. In place of the dogmatic and didactic teaching, which is a characteristic of the Prussian schools in particular, more encouragement should be given to independent and experimental work, especially in geometry. Great importance should be attached to oral expression and mental arithmetic. The students should be trained to construct their own problems, and should be restricted to problems of a practical character rather than the mere elaboration of fictitious exercises. More attention should be given to graphs and graphic representation, and to some extent to the history of important theories and principles. In the final year the instructor should be given free scope and should aim to give the students some acquaintance with the problems of higher mathematics and an insight into mathematical method to serve as a preparation for further study.

In the training of the students in the methods of the teaching, two principal types are again found. A number of States, following the practice of Prussia, assign instruction in methods of each subject to the specialist. Thus, while one teacher gives the general method and principles of teaching, the teacher of mathematics has charge of the instruction in the special methods of his subject. In the second type one teacher has charge of all the purely pedagogic work of the school and gives courses in both general and special methods. It is felt, however, that in both types when methods courses are handled by two instructors, there may be not only an absence of uniformity, but even that contradictory principles may be enunciated. This danger is all the greater in those States in which the specialist in subject matter has had no experience in the elementary school, while on the other hand it is feared that too often the special teacher of method may for example, be inadequately trained in mathematics. To avoid these dangers a system of cooperation has been adopted in Saxony between the specialist and the teacher of general method. While the specialist throughout the course aims to show the development and interconnection of the different parts of the subject with special reference to the elementary

school, the method teacher in a brief course of 15 hours gives a historical outline of the special method and discusses the aim and problems of mathematical instruction and the arrangement and distribution of the material for the elementary schools. On this subject the committee already referred to recommends that the course on method should include the methods of teaching in all grades of the elementary schools, a critical study of methods and theories, and an examination of the most usual apparatus and local textbooks. To give a comprehensive view of the subject, the course should not omit reference to the methods both of the kindergarten and of the secondary schools, while some assistance and advice should be given to the students for their own further study.

The permanent appointment of German teachers can only be secured by passing a second examination for which candidates become eligible as a rule two years after they have passed the first examination at the close of their normal school career. The second examination, however, is rather of a practical or professional character and is intended to test the ability of the candidate as a teacher rather than as a student. While a test is, as a rule, given in a special elective subject, it is not of so much importance as professional skill. Since the first appointments are in most cases in rural districts, the teachers are thrown on their own resources and any study that they undertake must necessarily be done independently and without guidance. Bavaria has instituted district extension courses to prepare for the second examination, but this practice appears to be isolated. A stimulus is afforded to further study by the examinations for promotion to principalships or positions in the middle schools and normal schools. Increasing opportunities are being offered, as in Saxony, Hesse, Wurttemberg, and Bavaria, to students who acquire themselves excellently in the final examination at the normal schools to proceed to the universities, but as students who do so rarely return to the elementary schools, a discussion of these facilities and examinations is not appropriate here.

HUNGARY.

The Hungarian normal schools are maintained by the State or the different religious denominations, schools of the latter type being in the majority. Separate schools exist for the training of men and women. All the schools follow more or less closely the program prescribed by the State. These have been recently revised (1911) and brought into closer agreement with modern reform movements. The normal schools furnish a four-year course to candidates who are admitted at the age of 14 from the lower classes of the secondary schools and the intermediate schools (Bürgerschulen).

The aim of instruction in mathematics is declared to be—

the study of algebraic foundations of the common arithmetical operations, the knowledge and accurate application of practical arithmetical problems taken from daily life, and the study of the most important principles and the simplest practical application of elementary geometry, with particular attention to the needs of elementary school instruction.

The following outline indicates the scope of the course in mathematics:

FIRST YEAR (four hours).

Arithmetic and elementary algebra: Introduction to the nomenclature of algebra; fundamental operations of algebra. Negative numbers, the number system; the fundamental operations in the decimal system. Divisibility of integral numbers. Fundamental operations with common fractions and decimals. Simple equations with one unknown; graphic equations with two or three unknowns.

Geometry: Measurement of length and angles. Angles. Parallel straight lines. The chief properties of the triangle, quadrilateral, polygon, and circle.

SECOND YEAR (three hours).

Arithmetic and elementary algebra: Powers and roots. The second and third power of algebraic and decimal expressions. Fractional numbers; pure and mixed quadratic equations with one unknown; imaginary and complex numbers.

Geometry: Principles of equality. Problems of construction with reference to the triangle, quadrilateral, and regular polygons. Similarity of figures; similar triangles; the theorem of Pythagoras. Inscribed and circumscribed triangles and quadrilaterals. Calculation of the side of an equilateral triangle and the regular hexagon. The circumference. Calculation of area of these figures.

THIRD YEAR (two hours)

Arithmetic and elementary algebra: Proportion; rule of three and proportional division. Percentage in commercial and statistical applications; coinage; alloys; national and foreign money systems.

Geometry: Analytic geometry of the point, straight line, and triangle, with corresponding development of the prerequisite algebraic knowledge, based upon the function concept and graphic methods.

Method: Discussion and explanations of the syllabus and suggestions for teaching arithmetic and geometry in the elementary schools, including a consideration of manuals, textbooks, and apparatus.

FOURTH YEAR (three hours).

Arithmetic and elementary algebra: Simple interest and discount. Arithmetical and geometric progressions. Principles of compound interest (savings banks, annuities, redemption of loans); compound-interest tables. The most important features of commercial practice and exchange.

Geometry: Relation of lines and planes in space. Simple exercises in leveling. Lines of equal height. Reading of topographic, especially military, charts; simple exercises in surveying. Construction, surface, and volume of the prisms, cylinder, pyramid, cone, and conic sections, and the sphere. Definition and construction of the ellipse, parabola, and hyperbola.

The chief criticism that is made against this syllabus is on the ground of arrangement. While it is thought that the addition of

logarithms and the elements of plane trigonometry would be a great gain, it is felt that this is the only criticism that can be made on the ground of subject matter. Considerable improvement could, however, be made if all the arithmetic were completed as a foundation for the study of principles and of algebra, and if the algebra and geometry were more closely correlated.

The professional study in the normal schools begins with the observation of instructions in the model school from the third year on. Practice teaching is commenced in the third year, but is not fully developed until the fourth year, when each student teaches six hours a week.

ITALY.

The training of teachers for Italian elementary schools is provided in normal schools, which are classed with secondary schools. The normal schools are organized on a three-year basis, and until recently boys and girls were taught in separate institutions. Co-educational schools have, however, sprung up within the last few years. Since the normal schools can not supply the demand for teachers created by the establishment of new elementary schools and the extension of the elementary-school period, special two-year courses (*corsi magistrali*) have been established for students who have passed through the *ginnasi*, which give a five-year secondary-school course. The *corsi magistrali* are mainly of a professional character.

Students are admitted to the normal schools at about the age of 15 from higher elementary or intermediate schools with three-year courses beyond the elementary schools—*scuole tecniche* for boys and *scuole complementari* for girls. Admission is by certificate from these schools; entrance examinations are given only in special cases. Each institution is equipped with a complete elementary school for practice and observation, while the girls' normal schools have, in addition, kindergartens and complementary schools attached to them.

Normal-school instructors must be graduates of universities and are appointed on the basis of a competitive examination. In the boys' normal school the same instructor has charge of mathematics, physics, and natural sciences; in the girls' schools these subjects are in different hands, but the instructor of mathematics is also required to teach in the complementary school.

A striking feature of the mathematical program, which is given below, is the omission of arithmetic in the first year of the course and the introduction of algebra in its place. Special attention is also given to method in the second year, a survival probably from

the time when a teacher's lower certificate could be obtained at the close of that year. The program is as follows:

FIRST YEAR (three hours boys, two hours girls).

Algebra: Introductory ideas. The four operations with integral quantities; equations of the first degree with one unknown. Square and cube roots with approximations.

Geometry: Definitions and introduction to plane geometry. Angles, triangles, and quadrilaterals. Regular and irregular polygons. Circle. Equality of polygons. Measurement of straight lines, angles, polygons, and circles. Equality of plane figures and principal theorems concerning plane figures.

SECOND YEAR (two hours).

Arithmetic: Magnitudes; numbers; numeration; analysis of the four operations. Methods of teaching numbers and the four operations in the elementary schools. Ratio and proportion.

Geometry: Proportional lines and similar polygons. Methods of teaching the notions of plane geometry in the elementary schools.

Bookkeeping: Inventory; trial balance; accounts rendered.

THIRD YEAR (two hours).

Arithmetic: Magnitudes in direct and inverse ratio; rule of three, simple and compound; solution of related problems by proportion and reduction to unity. Methods of teaching the rule of three in the elementary schools.

Geometry: Straight lines and surfaces and their relative position in space. Dihedral and polyhedral angles; polyhedrons; prisms, cylinder, pyramid, cone, sphere. Fundamental notions of congruence and similarity. Methods of teaching solid geometry and the metric system in elementary schools.

Bookkeeping: Daybook; ledger; cashbook.

This is by no means an ambitious program. Its critics object that the time allotted to it is insufficient, and that the arrangement is illogical, since much of the early work depends on a knowledge of matter which is at present postponed to a later year. Furthermore, the students under the present arrangement fail to obtain a comprehensive and systematic view of the methods of instruction and of the program of the elementary schools. The regulations prescribe that arithmetic must be taught with scientific rigor, while instruction in geometry is to be deductive in the first year and inductive in the second and third—a distinction which the reformers regard as vague and meaningless.

The normal schools have been under fire for some time. It is felt that the course of three years is too short a time in which to train cultured and efficient teachers. The practice of crowding the academic and cultural subjects and the professional subjects together is not only dangerous, but results in inefficiency. Demands are now being made for a longer course; for a separation of the two main purposes of the normal schools—the general and professional training; for better coordination between the various subjects; and for a reorganization of the program to the exclusion of the superfluous and an emphasis on the new elements demanded by modern culture.

RUSSIA.

The Russian public elementary school system provides for three types of schools: The ungraded elementary school with a course of three years, gradually being extended to four years, receiving pupils at the age of 7; the two-class elementary school which gives a five-year course, divided into two sections of three and two years, and which receives pupils at the age of 7; and the municipal elementary school, which in general gives a four-year course to pupils coming from the elementary school at the age of 10 or 11. The first two types of school are taught by a single teacher. The mathematical work of these schools is necessarily simple. In the elementary schools arithmetic only is taught, with the briefest outlines of geometry. The arithmetic here covers the four operations with abstract and concrete numbers and simple fractions, and geometry includes simple ideas of form and measurement. Five hours a week for the whole school are allotted to the subject. In the elementary schools with two classes the program is somewhat extended, but there is much variation both in selection of subject matter and in the time allotment, which ranges from three to six hours for arithmetic and from one to three hours for geometry. Arithmetic is carried up to common fractions, decimals, problems in the rule of three, alligation, alloys, partnership, and interest. Instruction in geometry is limited to a concrete basis.

The mathematical work in the municipal elementary schools is still organized on the basis of regulations issued in 1877, and includes arithmetic, geometry, and algebra. Arithmetic, to which are devoted from 8 to 16 hours a week in all the classes together, covers recurring decimals and proportions. Geometry, from 6 to 9 hours a week, is restricted to a fairly complete study of the problems occurring in plane geometry. The work in algebra, from 2 to 4 hours a week, is carried as far as simple equations.

It is obvious from the above that the preparation of students who enter the normal schools is very slight. Two types of normal schools are maintained, corresponding to the two main types of elementary schools. The normal seminaries are intended for the training of teachers (men and women) for the lower elementary schools—the ungraded and the two-class schools—and the normal institutes for the training of teachers (men only) for the municipal schools. Students are admitted to these institutions on the basis of a competitive examination. Candidates for the normal seminaries are drawn from among the more able pupils of the two-class schools, who, on completing their course, usually remain for review and practical work under the direction of their teachers until they reach the eligible age of 14. Candidates for admission to the normal institutes must have

completed the course of a municipal elementary school and be 16 years of age.

The length of the normal seminary course is three years, but efforts are successfully being made to extend it to four years. The aim of the mathematical course is closely adjusted to the demands of the future career of the students, i. e., the curriculum of the lower elementary schools. The course is as follows:

- (1) Theoretic arithmetic with solution of problems. Five to eight hours a week are devoted to the subject according to the previous preparation of the students.
- (2) Principles of algebra. Here there is much divergence, both in standards, which vary from simple equations to quadratic equations, progressions, and logarithms, and in time which ranges from one to five hours a week.
- (3) Six to eight hours a week are given to geometry, the program in which is identical with that of the secondary schools (given below) with the exception that more attention is given to surveying.

The two upper classes have instruction in the methods of teaching arithmetic and geometry, one hour a week being given to each. The graduating class has practice teaching in a two-class school.

The aim of the work of the normal institutes is defined in the same way as that of the normal seminaries. The standards are accordingly higher, both for this reason and because the preparation of the students has been more extensive. The following is an outline of the mathematical course:

- (1) Theory of arithmetic (one-half to three hours a week).
- (2) Algebra up to binomial theorem, recurring decimals, etc. (seven to eight hours).
- (3) Geometry (four to five hours) follows the program of the secondary schools, viz: Straight lines; angles; parallels; triangles, quadrilaterals and polygons in general; circumference. Fundamental problems of construction and numerical examples. Measurement of straight lines and angles. Proportionality of segments. Similarity of triangles and polygons. Numerical relation of the sides of a triangle. Regular polygons. Limits. Length of circumference and calculation of it. Simple problems of constructions and numerical applications to each figure studied. Relative position of lines and planes in space. Dihedral and polyhedral angles. Regular polyhedrons—areas and volume of prisms and pyramids—round bodies—cylinder, cone, and sphere, and calculation of their area and volume.
- (4) Plane trigonometry with a little of spherical trigonometry (one or two hours). This subject, however, is not prescribed and may be introduced optionally.

The upper classes have instruction in methods of teaching mathematics in the municipal elementary schools, to which the graduating class is also assigned for practice teaching.

The regulations recommend that the method of teaching in the normal seminaries should be didactic without neglecting, however, the importance of developing habits of independent work. In the normal institutes the method of the recitation is employed with drill reviews about every term. The textbooks play a secondary part and are used merely for purposes of reference. The same books are used in the secondary schools.

The staffs of the normal seminaries are recruited from among university graduates and in mathematics from those who have graduated in the faculty of science. But since the supply of these is short, graduates of the normal institutes are not infrequently appointed. In the normal institutes the instructors are exclusively university graduates. It is objected that such training is not adequate qualification for normal school positions, since the university graduates have no acquaintance whatever with the elementary schools and have had no pedagogic training, while the courses of the normal schools are not definitely prescribed. It is claimed, however, that these disadvantages are offset by the introduction of a broad and scholarly spirit into the training of the future teachers.

Since the supply of normal school graduates is small, candidates for teaching positions are also accepted who pass examinations in the theory and practice of the work of the elementary schools without any special training in one type or other of the normal schools.

SWEDEN.

The system of normal schools of Sweden belongs to that class which limits itself somewhat closely to the actual needs of the future teacher. Two types of schools are maintained—the regular four-year normal school for men and women, maintained by the State, and institutions for training infant school teachers (Kleinschulseminar), attended almost entirely by women, and giving a course varying in duration from one to two years.

Candidates for admission to the normal schools must be between 16 and 26 and must pass an entrance examination on the subjects of the elementary school curriculum. Since the number of candidates is usually greater than the number of vacancies, this examination is practically competitive. Preparatory institutions have been established to cram students for these examinations. Some of the students are graduates of "Real schools." Students from the universities who desire to teach in elementary schools are admitted to short courses, usually of one year's duration. The instruction in the normal schools is given by university-trained specialists who have the same qualifications as candidates for teaching appointments in secondary schools.

The mathematical course of the normal school is at present still unsatisfactory, for the reason that it does not furnish a much more extensive course than that found in elementary schools. The regulations bearing on the subjects date back to 1886, with some amendments in 1894, and do not contemplate an extensive course. Their prescriptions, however, are regarded as minimal, and several normal schools have adopted broader standards. Thus they have intro-

duced algebra up to arithmetical and geometric progressions, geometry of as high a standard as that taught in the secondary schools, and a little trigonometry, branches which are not required by the regulations. Four hours a week are devoted to mathematics in the first three years and two hours in the fourth. Professional work is begun in the second year with observation and practice teaching under the direction of the specialist instructors.

The following general outline of the course in mathematics represents the letter rather than the spirit of the present day; the latter is being strongly influenced by modern reform tendencies:

Exercises to attain speed and accuracy in handling the four operations with integral numbers and fractions, with applications and problems on the most important practical needs; the knowledge of theory underlying these operations; the theory of numerical equations of the first degree with one unknown; mental arithmetic, especially with the numbers from 1 to 100; extraction of square and cube root; and introduction to bookkeeping.

Principles of geometric bodies, the general properties of the most important plane figures, with the method of their measurement and calculation.

The institutions for training infant-school teachers (Kleinschul-seminar) are governed by the regulations of 1897. The scope of the mathematical course is even more restricted than of that given above. It includes: Speed in the four operations with integral numbers and fractions, and their application to simple practical problems with exercises in mental arithmetic.

While geometry is not referred to in the regulations, it has been introduced in several institutions.

The whole question of the normal-school curriculum has been under discussion for several years, and it is highly probable that changes will be made that will bring the curriculum into conformity with the demands of recent reform movements.

SWITZERLAND.

The educational system of Switzerland presents an interesting parallel with that of the United States. So far as the training of teachers is concerned, the system is entirely local, though slight aid is given by the Federal Government. Since, as in the United States there is a great variety of students without the minimum requirements usually established under systems that are centrally organized, it is practically impossible to present a comprehensive account of the work of the schools.

The normal schools are public and private; some are independent institutions, others are departments of secondary or "middle" schools; a few are coeducational; others admit only one sex. In a few instances special short courses of two or three semesters are

organized for graduates of the "Real schools" and gymnasiums, but as a general rule the normal schools are intended for students entering between the ages of 14 and 16 from a higher elementary school (Sekundarschule). With a few exceptions the duration of the courses is four years. The divergence of standards of attainment is very great; some institutions offer the same standard of work as the gymnasiums, while others hardly surpass those of the higher elementary schools. In the best schools the mathematical subjects include arithmetic and algebra, geometry, surveying, theory of projections, commercial arithmetic and bookkeeping, and mathematical drawing. The total number of hours devoted to these subjects ranges from 10 to 28 a week, with an average of about 19 for the four classes.

At least three types of courses can be distinguished: (1) Those limited to a review of the elementary school courses and instruction in method; (2) those which aim to give a deeper and better command of the mathematical work of the elementary, higher elementary, and "middle" (secondary) schools with an emphasis on practical and mental arithmetic—the idea of number, numeration, common and decimal fractions, and general insight into the content side of practical arithmetic; (3) those in which mathematics is taught as in the gymnasiums as a cultural subject with training in mathematical thinking.

In general, the courses in geometry confine themselves to the study of surface and volume. But attempts are being made to extend this and to include the theory of projections, conic sections, mathematical geography, and physics.

The methods of instruction naturally vary with the scope of the courses, so that while drill is emphasized in the lower stage the higher stage introduces a scientific treatment of the subject.

Under these circumstances it will perhaps be most useful for the purpose in hand to present an outline of one of the fullest courses, premising, however, that this represents the goal to which most of the normal institutions (for men at any rate) aspire rather than conditions widely prevalent. The course given is that of the normal school of the Canton of Zurich, at Kusnacht.

Aim of mathematics course: To give training in clear comprehension of space and number in the use of critical and logical processes of numerical and measurable quantities; to develop ability in terse, clear, and logical expression; to train in the recognition of the quantitative relations and problems of practical life, nature, and technology, and to solve these with insight and accuracy by calculation and drawing.

FIRST YEAR (five hours).

(a) *Arithmetic:* Review of rule of three and simple bookkeeping; profit and cost accounts. Mental arithmetic. Short methods. Ratio and proportion.

(b) *General arithmetic:* Operations of the first and second stages with general numbers; rational numbers; powers with integral exponents.

- (c) *Algebra*: Equations of the first degree with one unknown. Problems.
- (d) *Plane geometry*: Review of ideas of space and the establishment of fundamental ideas. Line, circle: measurement of distance and angles; parallels and central and axial symmetry; congruence. Translation and rotation. Construction of triangles. Principles of general and special quadrilaterals, secants, tangents, and circumscribed triangles. Comparison, computation, and measurement of the areas of triangles and polygons.

SECOND YEAR (six hours).

- (a) *Bookkeeping*: Accounts current with payment of interest and use of interest tables; elements of double entry limited to simple commercial processes.
- (b) *Arithmetic*: Extraction of square and cube root. Irrational numbers. General involution. Prime numbers.
- (c) *Algebra*: Equations of first degree with several unknowns.
- (d) *Plane geometry*: Similarity of plane figures. Scale drawing. Regular polygons and measurement of circle. Construction of simple algebraic expressions. Line and circle as loci. Methods of solving problems in planimetric construction. Practical exercises in use of simple instruments. Application to surveying of small plats of land.
- (e) *Trigonometry*: Definition of the functions of acute angles; study of right angles and equilateral triangles. Problems with use of numerical values of functions.

THIRD YEAR (five or six hours).

- (a) *Arithmetic*: Theory of common logarithms. Arithmetical and geometric progressions. Compound interest and annuities.
- (b) *Algebra*: Solution and theory of equation of the second degree with one unknown.
- (c) *Trigonometry*: Principles of obtuse angles, triangle and geometric application. Continuation of definitions. Problems, especially of triangulation. Physics and solid geometry. General definitions and chief principles of goniometry. Construction of trigonometric expressions and examples of the trigonometric analysis of geometric constructions.
- (d) *Solid geometry*: Relative positions of objects in space, and especially parallels and perpendiculars. The idea of projections, measurement of distances and space, angles, symmetry. Uniquely determined construction of trihedral and polyhedral angles. The sum of the sides and angles of convex polyhedrons. Euler's theorem on polyhedrons and regular polyhedrons.
- (e) *Theory of projections*: Inclined parallel projections as illustrative material. Representation of points, straight lines, plane polygons, and simple objects in ground plan and elevation. Folding. The ellipse as projection of a circle and its focal determination.

FOURTH YEAR (six hours).

- (a) *Arithmetic*: The principal notions of permutations. Elements of probability applied to insurance.
- (b) *Coordinate geometry*: Rectangular and polar coordinates in plane and space. Graphic representation of the simplest functions of one variable. Graphic solution of numerical equations.
- (c) *Solid geometry*: The sphere and its related surfaces. Plane sections of these surfaces. Spherical triangles. Areas and volume of simple solids and their parts. Application to determination of weight.
- (d) *Theory of projections*: Representation of polyhedra and elementary curved solids in ground plan and elevation. Plane diagrams of the surfaces of solids. Making of models. The chief map projections. Simple problems of intersection with appli-

cation to shading. Fundamental ideas of geometric perspective, especially of the vanishing point and foreshortening.

(e) *Methodic and historical survey* in the field of pure and applied arithmetic and the study of forms in the elementary school.

The certification of the students as teachers depends on a final examination which consists of two parts, general-academic and professional. Both parts of the examination may be taken at the same time, or the professional examination in theory and practice of teaching may be postponed for one or two years during which the candidates are expected to teach in an elementary school. No provision is made, however, for the supervision of the candidates during this period.

UNITED STATES.

In dealing with educational topics concerning the United States it is difficult to do more than present a general picture of tendencies, since the standards in different parts of the country vary so greatly and since they are at different stages in their development. But this variety is perhaps no greater than in Switzerland and, in the matter of training of teachers, than in England. In some States the qualifications demanded of teachers in elementary schools are still no higher than the attainments of one or two years beyond the elementary schools, while at the other end of the scale teachers may have the qualifications of college graduates. The most general standard may, however, be said to be graduation from high school, with two years of normal school training. The mathematical attainments will thus have been acquired in the eight grades of an elementary school, four years of high school, and two years of normal school.

The work of the elementary grades covers the four fundamental operations, fractions, decimals, and percentage, with business applications. Mensuration and algebra are included in some systems in the eighth grade. There is more uniformity in the first six grades than in the last two. The variations in the seventh and eighth grades are due not only to differences in amount of subject matter taught, but also to the employment of the department system. Departmental teaching in these grades is quite common in the larger school systems and is being extended as rapidly as circumstances permit. Under this system teachers are employed who are not only more experienced, but have had special training for the work and are frequently college graduates. Where algebra and geometry are not introduced, more intensive work is given in the practical and business applications of arithmetic. There is a tendency in the better schools to emphasize thoroughness and practical utility. In the schools where algebra is taught, there is again some variation in scope and method; some courses are merely based on high-school texts; in

others the subject is introduced as part of the work in arithmetic. The amount of geometry taught in the elementary schools has in recent years tended to decrease. The mensuration of geometric figures has always been included in arithmetic textbooks. The reform movement in geometry, however, while not widespread, aims to cultivate habits of geometric study and to train such powers of observation and generalization as pertain to geometric data.

In the high schools the curriculum in mathematics is determined in general by the admission requirements of the colleges, especially in those States which have a complete and State-wide organization of education, crowned by the university. This is true, also, in other States and even in the smaller communities, where few, if any, of the pupils plan to go to college, and where the local school committee disclaims the intention of following university guidance. And even where this is not true of other subjects of study, it holds for the mathematical curriculum, because, apart from the colleges, the subject is not defined by any other authority that the schools are willing to accept.

The ordinary mathematical course in high schools includes algebra, plane geometry, solid geometry, and trigonometry, and, less frequently, arithmetic. The algebra courses are usually divided into three groups (1) up to quadratics; (2) quadratics up to the binomial theorem; and (3) an advanced course, including various disconnected subjects and intended to furnish preparation for further mathematical study. In plane geometry the ordinary sequence is that of Legendre, including geometric constructions and original exercises ("riders"). Under solid geometry are included the usual topics up to the surface and volume of the sphere. The course in trigonometry includes definitions, the formulas and their proofs, logarithms, and triangles with practical applications to surveying and navigation. Where arithmetic is included, not more than half a year is devoted to it, usually at the beginning of the course, to make up any deficiencies in the elementary school preparation, or in order to insure facility and accuracy in routine operations. It must be noted that under the elective system no student in high school is required to take all the courses offered in mathematics, with the result that the mathematical equipment of the future teachers varies considerably both in scope and depth.

THE TRAINING OF TEACHERS.

The institutions for training teachers for the elementary schools are in a preponderating majority normal schools maintained by the State. A few of the larger cities maintain their own training institutions, and there exist in addition a number of private normal schools. More recently there have been established training classes in connection with high schools, for the purpose of training teachers for rural

schools. The high-school training classes usually offer one year of professional study either as part of or at the end of the regular high-school course. There is great variation in the entrance requirements, in the quality and amount of the work offered, and in the length of the course. With the exception of the high-school training classes, the training institutions as a rule offer a two-year professional course based on graduation from high schools. But, as usual, it is impossible to generalize, for some normal schools offer five-year courses beyond the elementary school; others one-year courses beyond the high school. The State normal schools with a two-year course beyond the high school may be taken as the type. They are generally supported by appropriations by the State legislatures. A small number of these schools are supported by a specified State tax, supplemented by appropriations by the State legislatures, and in a few cases they receive some income from the sale or rental of public-land grants. A few State normal schools have small permanent endowments. Tuition is free, except for slight incidental fees, to persons declaring an intention to teach in the State in which the school is located. With the exception of a few schools in the South, the State normal schools are coeducational.

The curriculum of these schools includes academic and professional subjects. The quality of the academic work varies greatly, partly because of the variety of preparation of the students, partly because of the lack of training on the part of the teachers and the heavy programs that they too frequently carry. But the graduates of the better normal schools usually receive full credit for two years' work on entrance to college and universities. The professional studies include psychology, history of education, methods, observation and criticism of instruction, and practice teaching. All normal schools have facilities for practice teaching, either in schools under their immediate control or in schools of the regular public system. The practice or training school is usually under the charge of a member of the normal school faculty, whose task it is to coordinate the work of the normal schools and practice schools. The practice teaching of the students is done under his supervision and that of critic teachers.

There are as yet no definite qualifications required from normal-school teachers. Accordingly, while the majority of the teachers who have charge of mathematics are college trained, there is still a large number with only high-school or normal-school training. Most of the teachers of mathematics would prescribe work through the calculus as the minimum academic preparation for teachers of mathematics in normal schools; a few would include more advanced courses as the minimum. It is generally admitted, too, that such teachers should have had some professional preparation as well as experience in teaching in elementary and high schools. With the gradual extension of the scope of the work of the normal schools, as, for example, the

inclusion of analytics and calculus in the curriculum offered, there is a tendency also to require at least one year of graduate work from the teachers of these subjects.

It is usually assumed that students have had a high-school course in arithmetic of one term or semester, one year or more of algebra, and one year of geometry, usually plane. In some normal schools arithmetic is required of students "found wanting" in its subject matter. In many schools solid geometry, advanced arithmetic, advanced algebra, and trigonometry are offered as electives.

It is thus apparent that the teacher of mathematics in the first six grades has a knowledge of subject matter in formal mathematics far beyond any of the actual needs arising within these grades. That this knowledge of subjects beyond elementary arithmetic is effective in the teaching of the arithmetic, it would be very difficult to show. The formal and isolated character of the work in algebra and geometry, as they are usually taught, leaves them barren of any content values having a bearing upon anything which appears in the usual work of the first six grades. Even in the best normal schools there is little evidence that the work in algebra and geometry is any less academic than in the classical high schools. In the normal school "humanistic" values should certainly most fully reveal themselves. There is but little specific preparation for the teaching of seventh and eighth grade mathematics. There is a failure to realize the truth of the statement that "this (seventh) grade marks the approach of the time when the pupil should pass from mere control and manipulation to understanding and investigation," and to provide the kind of instruction needed to lead the pupil out into the broader field of mathematical knowledge.

The following table shows for 64 schools the different courses given, the number of schools offering each course, and the average number of hours given to each course:

Number of schools offering certain courses.

Subjects.	Number of schools offering.	Average number of hours.
Arithmetic.....	All.	100
Elementary algebra.....	All.	171
Plane geometry.....	All.	148
Advanced algebra.....	9	108
Solid geometry.....	18	72
Methods of teaching.....	23	68
Mathematics.....	40	77
Plane trigonometry.....	4	80
Spherical trigonometry.....	25	96
Analytics.....	23	87
Calculus.....	3	62
Theory of equations.....	1	48
Differential equations.....	11	57
Surveying.....	1	48
Analytical mechanics.....	8	70
History of mathematics.....		

The professional courses and work include a course in arithmetic methods or in arithmetic with some attention to methods in all normal schools, observation and practice teaching in some form in most of them, and a course in the history of mathematics in very few. The subcommittee of Committee No. I, that dealt with the preparation of elementary branches in the preparation of the American report to the international commission, came to the conclusion that great differences in ideals and practice exist with reference to the following points: There is variation from method as a mere incident to subject matter to the use of full time for method in "methods" courses; from no use whatever of current literature on the teaching of mathematics, to "very extensive" use of such literature; from no consideration of games and recreational devices to very careful consideration and testing of these; from no consideration of the course of study in arithmetic for the grades to the full development of such a course and the organization for presentation of certain of its units; from no mention at all of the history of the development and teaching of mathematics to the establishment of well-organized courses in this subject; from no observation of lessons in arithmetic in the grades in connection with methods courses to one observation lesson each week during the course; from no practice teaching at all required in mathematics to practice work in arithmetic for all in at least two grades; from no supervision of practice work and the teaching of mathematics in the training school by the teacher of mathematics to close, jointly responsible supervision with the grade supervisors; from positive discouraging of departmental teaching of mathematics in the grades to positive advocating of it in middle and upper grades; and from no differentiation in training for the grades to courses in detailed special methods for primary grades.

There are also evident the following points of general uniformity: Entrance requirements to methods courses in arithmetic are quite uniformly high-school courses of about one-half year's work in arithmetic, one year in algebra, and one-half year or one year in plane geometry. Some kind of methods or teacher's course in mathematics is found in all. Some form of observation work, either in connection with the course in arithmetic methods or with general method or practice teaching, is advocated by all. In general, departmental work below the seventh is discouraged. No school is fully satisfied with its present practice.

Suggestions for reform are of four types: Greater knowledge of subject matter; a reorganization of arithmetic material, giving it more vital relationship to the child's life and to social life; a more intelligent knowledge of the pedagogy of arithmetic; and a closer, more vital relationship between the department of mathematics and the teaching of arithmetic in the training school. Three ways

are suggested for bringing about this last relationship: By more systematic observation work in the grades, by more responsibility for the course of study and the methods of teaching in the grades, and by the closer supervision of practice teaching in arithmetic by the department of mathematics.

From inferences based upon the foregoing, and from reflection upon the general problem, the committee believe that the best theory and practice of to-day point to the following conclusions: That a foundation in subject matter as a basis for the professional study of mathematics for teaching the subject in the first six grades of the elementary school should include a minimum of one-half year in high-school arithmetic, one year of algebra, and one year of geometry; that, exclusive of all courses in psychology, pedagogy, principles of teaching, general method, or history of education, a minimum of one-half year's professional study of arithmetic should be required to include the following: A course in "special method," the teaching of elementary mathematics which should consider the more elementary phases of the psychology of number; principles of general method in their application to arithmetic; educational values of arithmetic and the place of arithmetic in the general educational scheme; the organization of the elementary school curriculum in arithmetic; the organization of typical units of subject matter for presentation to appropriate grades; the development and writing of typical plans for teaching; the utilization of local and general economic studies for number application; the observation and discussion of typical lessons in the grades showing concrete applications of principles developed; the place of games and other recreational devices in lower grade work in number, and the historical development of the teaching of arithmetic, showing the place and value of certain "methods," as the Grube, Speer, etc.

The committee believe, further, that every school engaged in the preparation of teachers of mathematics should develop a museum or teaching collection of materials—apparatus, books, pamphlets, papers, etc.—which will aid in interpreting the historic development of the subject, present-day practice, textbooks, etc.; that the head of the department of mathematics should be largely responsible for the organization of the course of study in mathematics in the training school in cooperation with the department of education and the supervisors in the training school; that the head of the department of mathematics should aid in the supervision of the teaching of mathematics in the training school; and that he, as well as the supervisors or critic teachers, should be able to give demonstration lessons in the training school illustrative of principles of teaching developed in the "methods" class, and that the points of emphasis in all obser-

vations, discussions, plans, and criticisms should be upon the basis of fundamental principles rather than upon petty details.

The aim in the whole professional consideration of mathematics for teachers of these grades is, broadly, to give acquaintance with the fundamental principles of teaching arithmetic, of the organization of its subject matter, of its place in the educational scheme, and of its historical development. The teacher should be given the pedagogical outlook and perspective of arithmetic, as well as the ways and means of teaching its details. He should know enough of the psychology of number to enable him to secure healthy interest and adequate drill and to sacrifice neither at the expense of the other.

The greatest problem of all at the present time would seem to be to find teachers for departments of mathematics in normal schools who themselves have the wide pedagogic outlook desired for such work. When teachers can be found who have this larger perspective and who will regard the training school as the laboratory for developing insight, intelligence, and a minimum of skill in the teacher to be sent out into the field at large, this vital, daily union of theory and practice will do much to increase the efficiency of the prospective teacher in elementary mathematics.

To this may be added the recommendations received by the subcommittee that inquired into the conditions which a rational preparation of teachers for the seventh and eighth grades should fulfill. The replies received were as follows: Massachusetts—(a) normal or college diploma, (b) summer school course at least every third year, (c) membership in local mathematics club, (d) membership in New England Mathematical Association; Virginia—(a) high-school course, (b) good normal course, with practice work; Georgia—(a) teaching knowledge of subject, (b) practice course, (c) normal course in pedagogy; Indiana—inexperienced teachers should not be allowed in seventh or eighth grade work; Iowa—good knowledge of arithmetic, algebra, and geometry; Missouri—more attention to subject matter, but method not neglected.

IMPROVEMENT OF TEACHERS IN SERVICE.

The agencies for the further training of teachers in service are far more numerous in the United States than in any other country. Among these may be mentioned teachers' institutes, summer schools, reading circles, and the ordinary routine supervision. Teachers' institutes are classified into State, county, and city, the county being the most usual unit. The meetings are generally held during the school year, and the teachers are compelled to attend. The length of the sessions varies from half a day, when the institutes are held several times during the year, to five days or longer, when the insti-

tute assumes more the character of a regular training school at which the regular lessons are assigned and recitations are conducted. There is at present a widespread demand for reform in the method of conducting the institutes to provide for genuine professional growth instead of being what has been termed "inspirational." It is proposed to replace the discussion of general topics with a class of pupils and discussions by the teachers. In such a program arithmetic often finds a place, from two to three hours in a five-day session being devoted to the subject. In this time the instructor may give his time wholly to the detail of special method, or, perhaps, set before the institute the latest reforms and tendencies in the teaching of arithmetic. The topics of a more general nature discussed by institute instructors may be those concerned with the enriching of the course of study, the making of arithmetic practical, the elimination of waste material, the relative value of the topical and spiral plans in the arrangement of subject matter, the place of algebra and geometry in the elementary school, and others. In presenting special method, the instructor may relate it to devices in the lower grades, the teaching of the fundamental operations, explanations of problems, special topics like percentage, etc.

Summer schools are conducted by universities and colleges and normal schools, or are specially organized by State educational authorities. Their duration varies from 2 to 12 weeks. The work is, in general, intended for teachers in service. Courses in mathematics have been regarded in most cases as of special importance in these summer curricula. In fact, in several instances the work began with courses in mathematics and perhaps one other subject, and was gradually extended to include courses in all the regular departments as the number of students increased and the demand became apparent.

The courses offered in mathematics may be roughly divided into two classes: (1) Those intended primarily to emphasize the pedagogic aspect of the subject, and (2) those intended primarily to develop the subject matter for its own sake or as a prerequisite to other courses. The pedagogic courses include critical studies of the various elementary and secondary branches with reference to scientific interpretation and methods of presentation and also studies in the history of mathematics, with special reference to the needs of teachers in elementary and secondary schools.

In general, it may be said that a large number of those in attendance at the summer sessions are teachers spending a part or all their vacation in study, but it does not follow that all are seeking the strictly pedagogic courses. In fact, very many are pursuing courses for degrees, and are therefore filling out requirements or choosing elective work in subject matter in which they are interested. It is

true, however, that in most cases where special courses of a pedagogic character are given they are well attended and fully appreciated. On the other hand, there is a wide difference of opinion among institutions offering summer work as to the usefulness of the pedagogic courses as compared with the content courses. It is believed by many that the best pedagogic training comes through careful and diligent study of the subject matter under the guidance of an inspiring teacher who knows how to exhibit good methods and to impress them by example, rather than precept, upon the students.

BIBLIOGRAPHY.

The account of the training of mathematical teachers here presented is based on the following reports made to the International Commission on the Teaching of Mathematics, and a few other works of a general character.

BELGIUM.

Commission Internationale de l'Enseignement Mathématique. Sous-Commission Belge. Sur l'enseignement des mathématiques . . . dans les écoles primaires, les écoles normales primaires, etc. Rapports. Brussels, 1911.

DENMARK.

Heegaard, Poul. Der Mathematikunterricht in Dänemark. Basel & Geneva, 1912.

ENGLAND.

Board of Education. Suggestions for the consideration of teachers. London, 1912.

— The teaching of mathematics in the United Kingdom. Special reports on educational subjects. Vol. 26, part II. London, 1912.

The red code, 1912.

FRANCE.

Bioche, Ch. Enseignement primaire. In Commission Internationale de l'Enseignement Mathématique. Sous-Commission Française. Rapports. Paris, 1911.

GERMANY.

Deutscher Ausschuss für den mathematischen und naturwissenschaftlichen Unterricht. Vorschläge für den mathematischen, naturwissenschaftlichen und erdkundlichen Unterricht an Lehrerseminaren. Leipzig & Berlin, 1912.

Dressler, H., and Körner, K. Der mathematische Unterricht an den Volksschulen und Lehrerbildungsanstalten in Sachsen, Thüringen und Anhalt. Leipzig & Berlin, 1914.

Körner, K., and Lietzmann, W. Die Organisation des mathematischen Unterrichts in den Lehrerbildungsanstalten in Preussen. (In preparation.)

Lietzmann, W. Die Organisation des mathematischen Unterrichtes in der Preussischen Volks- und Mittelschule. Berlin, 1914.

Treutlein, P. Der mathematische Unterricht an den Volksschulen und Lehrer- und Lehrerinnenbildungsanstalten in Süddeutschland. Baden, Hesse, Württemberg, Bavaria.

HUNGARY.

Goldziher, K. Der mathematische Unterricht an den Lehrerbildungsanstalten. Budapest, 1912.

ITALY.

Gonti, A. L'enseignement della matematica nelle scuole normali. Rome, 1912.

RUSSIA.

Commission Internationale de l'Enseignement Mathématique. Sous-Commission Russe. Rapports présentés à la Délégation Russe. St. Petersburg, 1911.

SWEDEN.

Dahlgren, H. Die Mathematik an den Volksschulen und Volksschuleseminaren Schwedens. Stockholm, 1911.

SWITZERLAND.

Fehr, H. Der mathematische Unterricht in der Schweiz. Basle and Geneva, 1912.

UNITED STATES.

United States. Bureau of Education. Influences tending to improve the work of the teachers of mathematics. Washington, Government Printing Office, 1912. (Bulletin, 1912, No. 13.)

Mathematics in the elementary schools of the United States. Washington, Government Printing Office, 1911. (Bulletin, 1911, No. 13.)

Mathematics in the public and private secondary schools of the United States. Washington, Government Printing Office, 1911. (Bulletin, 1911, No. 16.)

Training of teachers of elementary and secondary mathematics. Washington, Government Printing Office, 1911. (Bulletin, 1911, No. 12.)

BULLETIN OF THE BUREAU OF EDUCATION.

[NOTE.—With the exceptions indicated, the documents named below will be sent free of charge upon application to the Commissioner of Education, Washington, D. C. Those marked with an asterisk (*) are no longer available for free distribution, but may be had of the Superintendent of Documents, Government Printing Office, Washington, D. C., upon payment of the price stated. Remittances should be made in coin, currency, or money order. Stamps are not accepted. Numbers omitted are out of print.]

1906.

- *No. 3. State school systems: Legislation and judicial decisions relating to public education, Oct. 1, 1904, to Oct. 1, 1906. Edward C. Elliott. 15 cts.

1908.

- *No. 5. Education in Formosa. Julian H. Arnold. 10 cts.
- *No. 6. The apprenticeship system in its relation to industrial education. Carroll D. Wright. 15 cts.
- No. 8. Statistics of State universities and other institutions of higher education partially supported by the State, 1907-8.

1909.

- *No. 1. Facilities for study and research in the offices of the United States Government in Washington. Arthur T. Hadley. 10 cts.
- No. 2. Admission of Chinese students to American colleges. John Fryer.
- *No. 3. Daily meals of school children. Caroline L. Hunt. 10 cts.
- No. 5. Statistics of public, society, and school libraries in 1908.
- *No. 6. Instruction in the fine and manual arts in the United States. A statistical monograph. Henry T. Bailey. 15 cts.
- No. 7. Index to the Reports of the Commissioner of Education, 1867-1907.
- *No. 8. A teacher's professional library. Classified list of 100 titles. 5 cts.
- *No. 9. Bibliography of education for 1908-9. 10 cts.
- No. 10. Education for efficiency in railroad service. J. Shirley Eaton.
- *No. 11. Statistics of State universities and other institutions of higher education partially supported by the State, 1908-9. 5 cts.

1910.

- *No. 1. The movement for reform in the teaching of religion in the public schools of Saxony. Arley B. Show. 5 cts.
- No. 2. State school systems: III. Legislation and judicial decisions relating to public education, Oct. 1, 1908, to Oct. 1, 1909. Edward C. Elliott.
- *No. 5. American schoolhouses. Fletcher B. Dresslar. 75 cts.

1911.

- *No. 1. Bibliography of science teaching. 5 cts.
- *No. 2. Opportunities for graduate study in agriculture in the United States. A. C. Monahan. 5 cts.
- *No. 3. Agencies for the improvement of teachers in service. William C. Ruediger. 15 cts.
- *No. 4. Report of the commission appointed to study the system of education in the public schools of Baltimore. 10 cts.
- *No. 5. Age and grade census of schools and colleges. George D. Strayer. 10 cts.
- *No. 6. Graduate work in mathematics in universities and in other institutions of like grade in the United States. 5 cts.
- No. 7. Undergraduate work in mathematics in colleges and universities.
- No. 9. Mathematics in the technological schools of collegiate grade in the United States.
- *No. 13. Mathematics in the elementary schools of the United States. 15 cts.
- *No. 14. Provision for exceptional children in the public schools. J. H. Van Sickle, Lightner Witmer, and Leonard P. Ayres. 10 cts.
- *No. 15. Educational system of China as recently reconstructed. Harry E. King. 10 cts.
- No. 19. Statistics of State universities and other institutions of higher education partially supported by the State, 1910-11.

1912.

- *No. 1. A course of study for the preparation of rural-school teachers. F. Mutchler and W. J. Craig. 5 cts.
- *No. 2. Report of committee on uniform records and reports. 5 cts.
- *No. 4. Mathematics in technical secondary schools in the United States. 5 cts.
- *No. 5. A study of expenses of city school systems. Harlan Updegraff. 10 cts.
- *No. 6. Agricultural education in secondary schools. 10 cts.
- *No. 7. Educational status of nursing. M. Adelaide Nutting. 10 cts.
- *No. 8. Peace day. Fannie Fern Andrews. 5 cts. [Later publication, 1912, No. 12. 10 cts.]

- *No. 9. Country schools for city boys. William S. Myers. 10 cts.
- No. 11. Current educational topics, No. I.
- *No. 12. Influences tending to improve the work of the teacher of mathematics. 5 cts.
- *No. 14. Report of the American commissioners of the international commission on the teaching of mathematics. 10 cts.
- *No. 17. The Montessori system of education. Anna T. Smith. 5 cts.
- *No. 18. Teaching language through agriculture and domestic science. M. A. Lelper. 5 cts.
- *No. 19. Professional distribution of college and university graduates. Bailey B. Burritt. 10 cts.
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