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MATHEMATICS IN THE LOWER AND MIDDLE COMMERCIAL AND INDUSTRIAL SCHOOLS

OF VARIOUS COUNTRIES REPRESENTED IN THE INTERNATIONAL COMMISSION ON THE TEACHING OF MATHEMATICS

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INTRODUCTION.

The object of this report is to give an account, in such a form as to allow comparisons, of the instruction in mathematics in the Lower and Middle Commercial and Industrial Schools in the countries reporting to the International Commission on the Teaching of Mathematics.

The data of this report have been taken entirely from the reports of the international commission, except in the cases of Great Britain, Germany and the United States. In these cases certain other sources of information, such as official school reports, have been consulted.

The countries included in this report are Austria, Belgium, Denmark, Finland, France, Germany, Great Britain, Holland, Hungary, Italy, Japan, Russia, Spain, Sweden, Switzerland, and the United States.

The reports of the international commission give a very brief treatment of the commercial and industrial schools in certain countries, and consequently the discussion of the schools of those countries is brief in the present report.

The first part of this report consists of an account of the schools considered, arranged according to countries. This account contains, so far as the facts are available, a brief statement of the organization of each type of school, the entrance requirements, length of the course, by whom supported, aim of the school and of the mathematical instruction, courses of study in mathematics, methods of instruction in mathematics, preparation of the teachers, examinations, and present tendencies.

The second part gives in tabular form, so far as the facts are available, for each type of school in each country the following facts: The number of years of previous school attendance required for entrance; the approximate age at entrance; the number of years in the course; the number of hours a week spent in the school; the number of hours a week given to the study of mathematics; the course of study in mathematics.

There is given, finally (pp. 94–96), a list of the kinds of schools here considered and the terms used as equivalents in this report.

An account of the teaching of mathematics in the commercial and industrial schools of the United States is given in the Reports of Committees I, II, III, IV, and V of the American Subcommission of the International Commission. As these reports are readily available,[1] it has been thought necessary in the present report to give only a brief summary of this work and to refer to the original committee reports for further details.

[1] In the bulletin series of the U. S. Bureau of Education. See Nos. 6–9, 12, 13, 16, of 1911, and 2, 4, 13, 14, 29, of 1912.

MATHEMATICS IN THE LOWER AND MIDDLE COMMERCIAL AND INDUSTRIAL SCHOOLS

OF VARIOUS COUNTRIES REPRESENTED IN THE INTERNATIONAL COMMISSION ON THE TEACHING OF MATHEMATICS.

AUSTRIA.

COMMERCIAL SCHOOLS.[1]

Organization.—The higher commercial schools, which are the ones discussed in the report, are State schools, and all, apparently, have the same course of study. When the report was written (1910) there were 25 of these schools. The course of study is four years. The age of entrance is 14 years. There is no unity of requirement as to preparation for entrance into the first year's work. Perhaps one may say that the average requirement is the completion of three years in a Gymnasium or a Realschule. This lack of unity in the entrance requirements is said to cause an unsatisfactory gradation of students.

Aim of the instruction.—Mathematics is to be taught both for its disciplinary and for its practical value. Special attention is given to applications to commercial problems.

COURSE OF STUDY IN MATHEMATICS.[2]

FIRST YEAR.[3]

Commercial arithmetic. (Three hours a week.) This course includes a review of the fundamental operations, abridged multiplication and division, accounts, the chain rule, and percentage.

Algebra. (Allgemeine und politische arithmetik.) (Two hours a week.) The fundamental operations, measures, multiples, fractions, equations of the first degree in one or more unknowns, and ratio and proportion are the principal topics studied in this course.

Geometry. (Two hours a week.) Plane and solid geometry.

SECOND YEAR. (THE STUDENT'S TENTH SCHOOL YEAR.)

Commercial arithmetic. (Three hours a week.) This includes interest, discount, equation of payments, accounts current with interest, stocks, exchange, and problems in buying and selling.

[1] Berichte über den math. Unterricht in Österreich, Heft 2: Der math. und phys. Unterricht an den höheren Handelsschulen, by M. Dolinski, pp. 29-41. Referred to hereafter as Heft 2.

[2] Heft 2, p. 36.

[3] In most cases this is probably the student's ninth school year. In some cases it is probably the eighth.

Algebra. (Two hours a week.) Powers, roots, equations of the first degree in one unknown, and logarithms.

THIRD YEAR. (THE STUDENT'S ELEVENTH SCHOOL YEAR.)

Commercial arithmetic. (Three hours a week.) Various problems in exchange, stocks, accounts current with interest, and calculation of prices of goods are taken up.

Algebra. (Two hours a week.) Exponential equations, progressions, compound interest and annuities, and sinking funds.

FOURTH YEAR. (THE STUDENT'S TWELFTH SCHOOL YEAR.)

Commercial arithmetic. (Two hours a week.) Calculation of prices of foreign goods, stocks, bonds, and exchange.

Algebra. (Two hours a week.) Probability, annuities, and insurance.

Methods of instruction[1].—The subject matter of the courses in mathematics, as the above program shows, is mainly vocational. The report gives no examples of the problems solved, and nothing is said of their origin. They are probably taken in the main from the texts, a list of which is given. Considerable freedom is given the teachers in administering the programs, but it is required that the work outlined be done. The method of teaching is heuristic.

Preparation of teachers.[2]—The problem of finding properly prepared teachers has not been solved. Some of those teaching mathematics are qualified to teach in the upper classes and some only in the lower classes of the secondary schools. There is, however, a law, passed 1907, that requires that all candidates to teach commercial arithmetic shall have completed the equivalent of the four lower classes of a middle school, and then the four years' course in a commercial school; this to be followed by two years of satisfactory commercial experience in a large commercial center, and finally the two years of work prescribed by the minister of education in some higher institution.

Examinations.[3]—Oral examinations are given from two to four times a semester. There is no final examination. Students who finish the course receive a leaving certificate.

Modern tendencies.[4]—The modern reform movement in the teaching of mathematics has affected the teaching in the commercial schools. It is recommended that the notion of function be introduced as early as possible, and that it be made the central point in the teaching of geometry. Graphical methods should be used. More hours should be given to algebra. The instruction in algebra and geometry should be connected more closely. The elements of trigonometry and the use of the slide rule should be taught. It is especially to be desired that mathematics and the other subjects be more closely connected.

[1] Heft 2, p. 87. [2] Ibid., p. 88. [3] Ibid., p. 35. [4] Ibid., pp. 39–41.

INDUSTRIAL SCHOOLS.

INDUSTRIAL CONTINUATION SCHOOLS.[1]

Organization and aim.—The industrial continuation schools have for their problem to give apprentices such necessary theoretical instruction as can not be given in the shop. The students are from 14 to 18 years of age. There is a preparatory class for those who have not completed the Volksschule, or who have not mastered the language in which the instruction is given. The course appears to be two years in length.

The teachers are mainly from the Volksschulen, Bürgerschulen, and Mittelschulen.

There are no final examinations.

COURSE OF STUDY IN MATHEMATICS.

Arithmetic. Only those parts are taught that have practical value. There is much use of tables in the solution of practical industrial problems.

Geometry. In the course in drawing some fundamental notions of geometry and projection are given.

Drawing. The course includes some work in freehand, geometrical, and projection drawing, with applications to the industries.

COURSES FOR MASTERS AND ASSISTANTS.[2]

These courses are given evenings and Sundays. The students must have completed their apprenticeship before entering the school, and must be 17 years of age or over.

The instruction in arithmetic follows the same lines as in the industrial continuation schools. The most important theorems and constructions of geometry are developed intuitively and concretely. Special emphasis is placed upon computations of surfaces, volumes, and weights. The instruction in projection is based upon observation and intuition.

Part of the teachers are from the secondary schools and part have technical training and practice.

GENERAL TRADE SCHOOLS.[3]

Organization and aim.—The aim of the general trade schools is to give the theoretical instruction and the practice which are necessary in learning a trade. The requirements for admission are an age of at least 12 years and evidence of having completed a course of study equivalent to the first six years of a Volksschule or the first year of a Bürgerschule. The students are as a rule from 12 to 15 years of age. As the compulsory school age in Austria is from 6 to 14, these

[1] Berichte über den mathematischen Unterricht in Österreich, Heft 4: Der mathematische Unterricht an den gewerblichen Lehranstalten, by Schulrat Wilhelm Rulf, pp. 58-59. Referred to hereafter as Heft 4.
[2] Ibid., pp. 59, 60.
[3] Heft 4, pp. 41, 42.

schools offer an opportunity for preparation for a trade during the last two years of compulsory school age.

The course is three years in length.

COURSE OF STUDY IN MATHEMATICS.

FIRST YEAR. (THE STUDENT'S SEVENTH SCHOOL YEAR.)

Industrial arithmetic. Applications of the fundamental operations with integers and fractions are made to simple practical problems.

Geometry. The geometry of this year is closely connected with the geometrical and projection drawing. From observation and examination of simple solids there is developed knowledge of their lines, angles, and surfaces, and also of the most important theorems of plane geometry. Representation of the principal sections of solids are made from models.

SECOND YEAR. (THE STUDENT'S EIGHTH SCHOOL YEAR.)

Industrial arithmetic. Positive and negative numbers are studied. Cost prices are computed from price lists. Selling prices are computed under given conditions as to capital involved and the cost of production.

Geometry. The course includes the graphical representation of geometrical bodies in orthogonal and oblique projection, and also of the plane sections of these bodies. Surfaces and volumes are computed and applications are made to practical industrial problems.

THIRD YEAR. (THE STUDENT'S NINTH SCHOOL YEAR.)

Industrial arithmetic. The subject matter of this course consists mainly of the solution of practical problems, and the discussion of such subjects as Government bonds, discount, exchange, and of the objects of savings banks and insurance companies.

Geometry. Plane sections and intersections of solids, and shades and shadows are studied.

Preparation of teachers.—The instruction is given, as a rule, by engineers and by specially prepared teachers from the Bürgerschulen.

TRADE SCHOOLS FOR PARTICULAR INDUSTRIES.

Organization and aim.—The aim of the instruction is to assist the students in learning a trade.[1] The emphasis here is placed upon drawing and the shop instruction. Students must be 14 years of age to enter. The course is three or four years in length.

COURSE OF STUDY IN MATHEMATICS.

Arithmetic. The object of arithmetic is to make the student familiar with the computations that arise in his trade, especially with those that involve the prices of materials and the values of foreign moneys.

Algebra is taught to a student if it is necessary for his trade. Emphasis is placed upon the evaluation of formulas, the solution of equations, and applications to the trades.

Geometry and geometrical drawing. Drawings of models are made to scale. The object of the work is to give the student ability to prepare and to read working drawings. Practice is given in computing surfaces and volumes.

Projective geometry. The emphasis is placed upon the representations of objects upon the three planes of projection and upon the development and computation of the surfaces of solids.

[1] These schools differ from the general trade schools just discussed in that the aim here is *gewerbliche Ausbildung*, and in the general trade schools the aim is *gewerbliche Vorbildung*.

Preparation of teachers.—The instruction is given by teachers from the Volksschulen, Bürgerschulen, and Mittelschulen, and by engineers.

SCHOOLS FOR THE BUILDING TRADES AND THE MECHANICAL ARTS.[1]

Organization and aim.—These schools have the problem of giving instruction to assistants in the building trades and the mechanical arts, and to prepare them for the master's examination.

For masons and carpenters there are two winter courses and a winter preparatory course, and for cabinetmakers, locksmiths, and workers in the mechanical arts there are two winter courses, a summer course, and a summer preparatory course.

Each course is of five months' duration.

In order to enter, a student must be at least 17 years of age, have completed his apprenticeship, and be in possession of such knowledge and skill as is expected of one who has completed two or three years in an industrial continuation school. Those who do not satisfy the last condition take a preparatory course.

COURSE OF STUDY IN MATHEMATICS.

Preparatory course. The instruction given here in industrial arithmetic and in geometry connected with the work in projection is the same, in the main, as that given in the first year of the industrial continuation school.

Courses for masons and carpenters.—Arithmetic and algebra. The work in arithmetic includes a review of squares and cubes, square and cube root, and applications to problems from the trades. The topics studied in algebra are the fundamental operations and the solution of equations so far as is necessary for the understanding and use of formulas.

Geometry. Areas and volumes are computed and applications are made to finding the volume and weight of various forms used in building. Some elementary work is done in trigonometry.

Courses for cabinetmakers, locksmiths, and workers in the mechanical arts.—Arithmetic and geometry. These courses are the same as those given above, except that the work in trigonometry is omitted. The arithmetic and geometry are taught with special reference to the needs of the trades. A second course in industrial arithmetic is given and is the same as that given in the third year in the general trade schools.

Projection drawing. The course includes plane sections and intersections of solids, shades and shadows, and practical applications to the building trades and to the mechanical arts.

Preparation of teachers.—The instruction is given by teachers who have taken both State examinations in the technical high school or by teachers from the Bürgerschulen.

SCHOOLS FOR MASTER WORKMEN.[2]

Organization and aim.—These are State industrial schools and have for their object to give theoretical and practical training to men who have had several years of practice and who wish to become master workmen or independent operators.

[1] Heft 4, pp. 44, 45.

[2] Ibid., pp. 45–49.

In order to enter the school, a student must be at least 17 years of age, must have completed the Volksschule, and must have had several years of practice in his trade.

The length of the course is two or two and one-half years.

COURSE OF STUDY IN MATHEMATICS.

The courses in mathematics are not the same in the different schools, and the report does not outline them in detail. The following statements are thought to give fair approximations of the general conditions as given in the report:

Arithmetic. There is usually one semester of industrial arithmetic.

Algebra. A course in algebra is usually given which includes the solution of equations of the first degree and in some cases quadratic equations.

Geometry. The course in geometry extends through one, and in some cases two, semesters. Geometrical drawing is sometimes given in connection with the geometry.

Trigonometry. The elements of trigonometry are given in part of the schools.

Projection drawing. In some schools the course in projection drawing extends through two semesters. Much attention is given to applications.

Preparation of teachers.—The teachers are engineers or teachers from the Mittelschulen.

THE HIGHER INDUSTRIAL SCHOOLS.[1]

Organization and aim.—The aim of the higher industrial schools is to prepare men to fill the important positions in the industries.

The course is four years in length and has three divisions—for the building trades, for the chemical industries, and for the mechanical industries.

Admission is given to those who are 14 years of age and have finished a three-class Bürgerschule, or with good standing a three-class trade school or the fourth class of a Mittelschule.

Aim of the instruction in mathematics.—The chief aim of the work in mathematics is to prepare for the study of technical subjects.

COURSE OF STUDY IN MATHEMATICS.

FIRST YEAR. (THE STUDENT'S NINTH SCHOOL YEAR.)

Algebra. (Five to eight hours a week.) Algebra through quadratics.

Geometry. (Four to five hours a week.) The course includes a study of rectilinear figures, the circle, similar figures, the computation of areas, the trigonometrical solution of right triangles, and an introduction to solid geometry and to the theory of projection.

Geometrical drawing. (Four hours a week.) Exercises in geometrical construction and an introduction to the theory of projection are given.

SECOND YEAR.[2] (THE STUDENT'S TENTH SCHOOL YEAR.)

Algebra. The subjects studied are powers and roots, imaginary numbers, logarithms, equations of the second degree with one or more unknowns, exponential and

[1] Heft 4, pp. 49–52.

[2] The courses outlined for the chemical section are slightly different from those outlined here. The average amount of time given to algebra, geometry, and trigonometry in the second year in the different schools seems to be about seven hours a week.

indeterminate equations, progressions, compound interest and annuities, and the binomial theorem with positive integral exponents.

Geometry. The work here includes a short review of plane geometry, the elements of graphical computation, and something of solid geometry.

Trigonometry. The extent of the course is not given.

Projective geometry. (Seven to eight hours a week.) The principal topics studied are plane sections, intersections of solids, the most important parts of the theory of shadows and perspective, and applications taken from the trades of the students.

THIRD YEAR. (THE STUDENT'S ELEVENTH SCHOOL YEAR.)

"In the third year mathematics comes only in the mechanical-technical division and has from two and one-half to four hours a week. Review and extension of the material previously studied. Theory of curves: The straight line, the conic sections (elements of analytical geometry), the cyclic curves, and the spirals."[1]

Methods of instruction.—Much attention is given to the discussion of formulas, to the notion of a function as it appears in the formulas of mechanics, to graphical representation, and to maxima and minima. Simpson's rule is studied in geometry. In the study of curves those properties are developed which are of most importance in practice. Special attention is given to the cyclic curves and spirals.

Preparation of teachers.—The instruction is given by teachers from the Mittelschulen or by engineers.

HIGHER SCHOOLS FOR THE TEXTILE INDUSTRIES.[2]

Organization and aim.—The object of the higher schools for the textile industries is to furnish the necessary general and technical instruction for those who wish to prepare to become operators or officers of textile manufactories. These schools have mechanical and chemical departments.

The length of the course is four years.

In order to enter, the student must be 14 years old and have completed the third class of a Bürgerschule, or the fourth class of a Mittelschule.

COURSE OF STUDY IN MATHEMATICS.

FIRST YEAR. (THE STUDENT'S NINTH SCHOOL YEAR.)

Algebra. To the solution of equations of the first degree. Logarithms.

Geometry. Including areas of surfaces and graphical computation.

Trigonometry. The elements of trigonometry.

Geometrical drawing. This course includes the construction of rectilineal figures, conics, cycloids, evolutes, and introduction to the projection of lines, surfaces, and solids.

SECOND YEAR. (THE STUDENT'S TENTH SCHOOL YEAR.)

Arithmetic. This is a course in industrial arithmetic.

Algebra. This includes a review, logarithms, and quadratic equations in one unknown.

Geometry. Computations of surfaces and volumes.

Trigonometry. This includes the solution of right triangles.

[1] Heft 4, p. 51.

[2] Ibid., pp. 53, 54.

Preparation of teachers.—The instruction is given by engineers and by teachers who are certificated to teach in the higher commercial schools.

INDUSTRIAL SCHOOLS FOR WOMEN.[1]

Organization and aim.—These schools give instruction to the women who are going into the garment-making trades, and they give also some instruction in the household arts.

The course is two years in length.

The only entrance condition given is that a student must have reached her fourteenth year.

COURSE OF STUDY IN MATHEMATICS.

FIRST YEAR. (THE STUDENT'S NINTH SCHOOL YEAR.)

Arithmetic. (Two hours a week.) There is a discussion of the lowest necessary income for women in the garment-making trade. A comparison is made between the wages commonly earned in the locality and the necessary expenses, including rent, heat, light, taxes, sickness, life and fire insurance, the use of a machine, and other items. There should be practice in mental arithmetic each recitation period.

SECOND YEAR. (THE STUDENT'S TENTH SCHOOL YEAR.)

Industrial arithmetic and bookkeeping. (Two hours a week.) There is a discussion of problems of buying and selling, loss and gain, cost of labor and general expenses, and the relations that exist between these quantities.

Preparation of teachers. The instruction is given by Volksschule and Bürgerschule teachers. Preference is given to applicants who have had courses in these industrial schools for women.

SCHOOLS FOR DOMESTIC SCIENCE.[2]

Organization and aim.—The aim of the *Koch- und Haushaltungsschulen* is to give courses in cooking and housekeeping. The length of the course is five months. In order to enter, a student must have reached her sixteenth year.

COURSE OF STUDY IN MATHEMATICS.

Arithmetic. (Two hours a week.) The principal topics studied are the metric system with applications to the cost of provisions and other household necessities; computation of household expenses by the day, week, month, and year; food values; cost of clothing, furniture, cooking utensils, and laundry; cost of light and heat; devices for saving; loss and gain; and insurance.

Preparation of teachers.—A teacher must hold a license to teach in a Volksschule or a Bürgerschule. Preference is given to applicants who have attended courses in these schools for domestic science.

[1] Heft 4, p. 55. [2] Ibid., p. 56.

BELGIUM.

COMMERCIAL AND INDUSTRIAL SECTION OF THE ATHÉNÉES ROYAUX.

Organization.—The athénées royaux are public secondary schools with a course of seven years. The pupils enter at the age of 11. One division of the school devotes the most of its time to the study of the modern humanities. The three upper classes of this division are divided into two sections—the scientific and the commercial and industrial.

COURSE OF STUDY IN MATHEMATICS.

The work outlined below includes only the special work given to the commercial and industrial section. The remainder of the work in mathematics is the same as that given other students in the athénées royaux.

FIFTH YEAR. (THE STUDENT'S TENTH SCHOOL YEAR.)

This course deals with financial operations with long periods of time, and includes compound interest, discount, exchange, annuities, and perpetuities. Logarithms are used.

SIXTH YEAR. (THE STUDENT'S ELEVENTH SCHOOL YEAR.)

Theory of annuities and sinking funds.

SEVENTH YEAR. (THE STUDENT'S TWELFTH SCHOOL YEAR.)

The study of long-term financial operations is continued, including loans, insurance, and public obligations.

The *écoles moyennes* of Belgium have general, commercial, industrial, and agricultural sections, but all of these sections seem to have the same mathematical programs.[1]

DENMARK.

The fifth chapter of the Danish Report[2] treats the elementary technical, commercial, and navigation schools. These schools were founded first by industrial organizations and technical societies. They later came to be controlled and supported wholly or in part by the State.

COMMERCIAL SCHOOLS.

"The[3] greatest of these schools is the so-called 'Kobmandsskolen,' which was erected in Copenhagen by the 'Verein für Ausbildung junger Handelsleute' (number of students, 2,480 in 1910–11). In

[1] Rapports sur l'enseignement des mathématiques, du dessin et du travail manuel dans les écoles primaires, les écoles normales primaires, les écoles moyennes, les athénées, les collèges belges, vol. 1, pp. 216–218.

[2] Der Mathematikunterricht in Dänemark, by Poul Heegaard, pp. 64–72. Referred to hereafter as Danish Report.

[3] Ibid., pp. 70, 71.

the three-year evening school (school year Sept. 1 to May 31) there are divisions for professional subjects and elective subjects. In the day school (one year, six hours instruction daily; 24 crowns [about $6.50] monthly) more preparation is assumed. There is gone through in the teaching of arithmetic (in the evening school two hours weekly, in the one-year school five hours weekly) pure elementary arithmetic (*Rechnen*), percentage and interest, reduction of moneys and measures, simple and complex calculation, stocks, exchange, and discount. There is connected with this institute a commercial school with a two-year course, the so-called 'Niels Brocks Handelsskole' (arithmetic two and three hours)."

The evening classes are being more and more abandoned.

TECHNICAL SCHOOLS.[1]

There is no common prescribed course of study for these schools, and the following discussion is limited to the technical school in Copenhagen. This school has six divisions: Evening school, day school for builders, machine constructors and electrotechnologists, school for painters, and institute for metal workers.

EVENING SCHOOL.

The instruction is given from 4 to 9.30 in the following subjects:

Arithmetic and algebra, two winter semesters, 6 hours.
Plane geometry, one winter semester, 6 hours.
Trigonometry, one winter semester, 6 hours.
Geometrical drawing, about 48 hours.
Projection drawing, 84 hours.
Perspective and vocational drawing.

DAY SCHOOL FOR BUILDERS. (TAGESSCHULE FÜR BAUHANDWERKER.)

This school has a course of four winter semesters. The tuition is 80 crowns ($21.60). "It is assumed that the students have the general common-school education and such knowledge of arithmetic, algebra, geometry, and geometrical drawing as can be obtained in the evening school. Instruction in mathematics is given in the first two courses—in the first, 12; in the second, 6 hours a week. After a short review of the previous work there are gone through: Jul. Petersen, Aritmetik og Algebra II; G. Juel, Trigonometri og Stereometri for tekniske Skoler."[2] Drawing is given 12 hours a week the first semester.

The instruction in mathmetics and drawing in the Tagesschulen der Maschinenkonstrukteure und Elektrotechniker is the same as that just given.

[1] Danish Report, pp. 65-70.

[2] Ibid., p. 65.

A number of examination questions from the above schools are given. These are of the usual textbook type. The following [1] are examples:

1. The following decimal fractions are to be written as common fractions and reduced: 0.4375; 0.592592·····; 1.1296296·····.

2. Find x in the equation: $\frac{x}{a-b}=\frac{x}{a+b}+2b$ and prove the correctness of the root found by substitution.

3. Compute: $x=\sqrt[5]{0.7809^{\sqrt{1.55}}}$.

4. Find the surface of a parallelogram with the sides a and b and an angle of 150°.

5. In a trapezoid ABCD, where BC and AD are the parallel sides given: AB=14m, BC=19.8m, AC=25.7m, and $\angle$D=67.38°. Find the surface of the trapezoid.

There exist special schools for certain trades. As an example a school for watchmakers is named, which gives a course of four years. The mathematics includes elementary arithmetic and algebra, geometry, trigonometry, the elements of analytical geometry, and of differential and integral calculus.

SCHOOLS FOR AGRICULTURE, FORESTRY, ETC.[2]

AGRICULTURE.

The higher instruction in agriculture is given in the Veterinary and Agricultural High School (Veterinär- und landwirtschäftliche Hochschule) in Copenhagen. Courses are given for veterinarians, farmers, surveyors, gardeners, and foresters. Instruction in mathematics is given only in the courses in surveying and forestry.

SURVEYORS' COURSES.[3]

There are three courses for surveyors, one of 1½ years, one of 1¼ years, and one of about 1 year. In the first course there are four or five hours of instruction in mathematics in the following courses:

Arithmetic. This course includes mental arithmetic, use of slide rule and tables.

Algebra. The principal topics are permutations and combinations, binomial theorem, elementary theory of equations, logarithms, compound interest, and annuities.

Trigonometry. This course includes the elements of both plane and spherical trigonometry.

Solid geometry. This course includes theorems concerning lines and planes, polyhedrons, cylinders, cones, and spheres.

Analytical geometry. This is a course in the elements of plane analytical geometry.

Differential calculus. Differentiation of algebraic, trigonometric, and exponential functions, indeterminate forms, complete differential, maxima and minima, and some applications to geometry are taken up in this course.

Integral calculus. This is a course of about the same extent as that in differential calculus.

[1] Danish Report, pp. 66-67. [2] Ibid., pp. 79-83. [3] Ibid., p. 80.

FORESTERS' COURSE.[1]

The foresters have two courses, one of 1½ years, the other of about 2 years. In the first course they have four or five hours a week in mathematics. They have the same instruction in arithmetic, algebra, plane geometry, solid geometry, and analytical geometry as the surveyors. They have only the elements of calculus. A course in the theory of errors is given.

INSTRUCTION IN NAVIGATION.[2]

This instruction is given in private schools that are recognized and supported by the State. The examinations are under the examining commission appointed by the minister of commerce. There are four grades of examination: (1) For seamen making short journeys; (2) for fishermen; (3) the general pilot's examination, which is also taken by sea captains; (4) the extended pilot's examination. In all of these an elementary knowledge of mathematics is demanded. In (2) the examination includes practical computation with integers and fractions, proportion, the circle, triangles, the sphere, and lines on its surface. Knowledge of the theorems in arithmetic, plane geometry, trigonometry, and solid geometry are required in (3), but no proofs are required.

FINLAND.

COMMERCIAL SCHOOLS.[3]

The report mentions three kinds of commercial schools, the schools for commercial employees (Écoles pour les Employées de Commerce), schools of commerce (Écoles de Commerce), and commercial institutes (Instituts Commerciaux). These schools are under the control of cities or private associations, and if they fulfill certain conditions receive partial support from the State. It is said that about one-sixth of the time spent in the schools is given to the study of mathematics.

No entrance requirements are given for the Écoles pour les Employées de Commerce. The course of study gives at least 12 hours of work a week for 1 or 2 years. The age of entrance is about 14 or 15 years. In arithmetic a review of the elementary school courses is given, with special reference to the calculation of prices, percentages, and interest. These schools seem to correspond to the commercial continuation schools in other countries.

The Écoles de Commerce require for entrance the completion of the primary school course (six years). The age of entrance is about

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[1] Danish Report, p. 82.
[2] Ibid., p. 71.
[3] Rapport sur l'Enseignement des Math. dans les Écoles de Finlande, VIII, pp. 34, 35. Referred to hereafter as Rapport.

16 years. There is a two years' course. The work in mathematics includes commercial calculation, the more important properties of geometrical figures, and computations of areas and volumes.

Students may enter the Instituts Commerciaux if they have completed the first five classes of a lycée real,[1] or if they have completed the secondary school, a school for girls, or a school of commerce. The average age at entrance is 16 or 17 years. The courses are two or three years in length. These schools are coeducational.

The students enter the Instituts Commerciaux with some knowledge of arithmetic, of the fundamental operations in algebra, and the solution of the equation of the first degree in one unknown, and of the elements of plane geometry. The course in the school includes political and commercial arithmetic, algebra, with the solution of equations and applications to annuities and life insurance, and elementary geometry.

The above facts include practically all that is given concerning these schools in the report from Finland.

INDUSTRIAL AND TECHNICAL SCHOOLS.

LOWER[2] TECHNICAL SCHOOLS (ÉCOLES TECHNIQUES INFÉRIEURES.)

I. The manual training schools (Écoles de Travaux Manuels)[3] are usually situated in the country. These are schools for artisans of various sorts and give courses of one or two years. The work in arithmetic involves the calculation of prices and of surfaces and volumes.

II. The lower and higher trade schools (Écoles de Métiers Inférieures et Supérieures)[3] require for admission the completion of the elementary school. The lengths of the courses are one, two, and occasionally three years of eight months each. These are evening schools and have for their object to give artisans the necessary theory and practice for their trades.

The only mathematics given in the Écoles Inférieures is a review of the elementary school course in arithmetic.

The following courses are given in the Écoles Supérieures:

Arithmetic, two hours a week the first year. This gives a review of the elementary school course.

Algebra, two hours a week the second year. The fundamental operations and equations of the first degree in one unknown are studied.

Elementary descriptive geometry, two hours a week the second year.

VOCATIONAL SCHOOLS (ÉCOLES PROFESSIONELLES).[4]

Organization.—The requirement for admission is the completion of the elementary school. The students enter at the age of 13 or 14 years. The courses require from 12 to 20 hours a week. The

[1] Four years of primary school work are required for entrance to a lycée.
[2] Rapport, VIII, p. 36.
[3] Ibid., pp. 36, 37.
[4] Ibid., pp. 38, 39.

length of these courses is not given, but it is apparently two years. These schools, it seems, are supported by the cities. They have for their object to give the students the necessary practical and theoretical preparation for the practice of a trade. There are not many of these schools, and they are said to be new in Finland.

COURSE OF STUDY IN MATHEMATICS.

The following courses are given in the schools for boys. Only arithmetic is given in the schools for girls.

Arithmetic. (Classes I and II, five and three hours a week.) This course includes a review of the work of the elementary school, extraction of roots, proportion, percentage, and calculation of prices.

Algebra. This course extends through equations of the first degree in one unknown.

Geometry. (Classes I and II, four and two hours a week.) This course takes up the elementary theorems, calculation of surfaces and volumes, and problems concerning materials used in the shops.

Descriptive geometry. (Class I.) Elementary notions.

There are no leaving examinations. Most of the teachers in the boys' schools are technologists; in the girls' schools they are women primary and manual-training teachers.

INDUSTRIAL SCHOOLS (ÉCOLES INDUSTRIELLES).[1]

Organization.—To enter an industrial school a student must have finished the elementary school and have had one year of practice in a shop; or have finished four years of a secondary school, in which case no shop work is required. The minimum entrance age is 17 years, and the average age of the students is 23. To complete the course requires two years, six months a year, with 39 or 40 hours of work a week.

Aim of the instruction.—The aim of these schools is to educate skilled workmen and foremen, and prepare mechanics and chauffeurs to take the public examination.

COURSE OF STUDY IN MATHEMATICS.

Arithmetic and algebra. (Classes I and II, six and two hours a week.) This is a review of arithmetic with application to vocational problems. The algebra goes through equations of the second degree in one unknown.

Geometry. (Classes I and II, four and two hours a week.) This course includes the principal theorems of elementary geometry, calculations of areas and volumes, and applications to vocational problems.

Descriptive geometry. (Classes I and II, four and two hours a week.) This is a course in the elements of descriptive geometry.

Mechanics and theory of resistance. (Classes I and II, two and four hours a week.) This course deals with elementary mechanics and resistance of materials.

Books used.—The book used in mechanics is specially prepared for these schools. The books used in the other courses are the same as those used in the secondary schools.

[1] Rapport, VIII, p. 39.

Preparation of teachers.—The teachers must have had a course in a technical high school and have had at least one year of practical work.

FRANCE.

HIGHER PRIMARY SCHOOLS FOR BOYS.[1]

Organization.—These higher primary schools for boys are a part of the regular public elementary school system. In order to be admitted, the pupils must possess the certificate of primary studies and have had, in addition thereto, one year of instruction in the higher course of an elementary primary school. Hence, before entering the higher primary school the pupils have had seven years of schooling. The course is three years in length.

Aim of instruction.[2]—The students prepare in the higher primary schools to go into industry, commerce, agriculture, and similar vocations or prepare for certain State examinations. "One does not take up in the school the apprenticeship of any profession in particular, but prepares himself for the intelligent practice of many which have, from the point of view of technique, numerous points in common; in other words, we put the future worker on a good road, in a definite direction (agriculture, industry)."

COURSE OF STUDY IN MATHEMATICS.[3]

In the first year of the course the instruction for all students is the same, but during the last two years there are five sections—general, agricultural, industrial, commercial, and maritime.

FIRST YEAR. (THE STUDENT'S EIGHTH SCHOOL YEAR) COMMON INSTRUCTION.

Arithmetic.[4] (Three hours with algebra.) The course includes the fundamental operations, greatest common divisor, least common multiple, metric system, common and decimal fractions, and square root.

Algebra.[5] Fundamental operations, limited to applications to simple problems. (Part of the time allotted to algebra in the program is taken for topics of arithmetic, such as the rule of three, interest, and discount.)

Geometry.[6] (One hour a week.) The course includes the use of ruler, square, and compasses, study of triangles, parallelograms, the circle, constructions, projection—the projection of a point, line, plane, and the simpler solids. Applications are made to manual training, such as the work in wood and pasteboard.

Geometrical drawing.[7] Free-hand sketches and drawings to scale of simple geometrical solids, and later of other objects, such as the materials and tools used in the manual-training shop.

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[1] Rapports de la Commission Internationale, Sous-Commission française, Vol. I, L'Enseignement primaire, by M. Ch. Bloche, p. 17. Referred to hereafter as Vol. I.
[2] Ibid., p. 17.
[3] Ibid., pp. 23–38.
[4] Ibid., p. 23.
[5] Ibid., p. 23.
[6] Ibid., p. 25.
[7] Ibid., p. 27.

Experimental geometry.[1] (One hour for two-thirds of a year.) A large variety of geometrical models are made from cardboard. Plaster modeling and decoration.

SECTION OF GENERAL INSTRUCTION.

Arithmetic.[2] (Three hours with algebra.) Second year. (The student's ninth school year.) Ratio and proportion, quantities directly and inversely proportional.

Third year. (The student's tenth school year.) Review and supplementary work in preparation for examination.

Algebra.[2] Second year. (The student's ninth school year.) The course includes the fundamental operations with positive and negative numbers (the general case of multiplication and division of polynomials not included); equations of the first degree with one or more unknowns, and applications to problems of arithmetic and geometry; arithmetical and geometrical progressions, logarithms, compound interest, and annuities. Emphasis is placed on rapid calculation.

Third year. (The student's tenth school year.) Review and supplementary work in preparation for examinations.

Geometry.[3] (One hour.) Second year. (The student's ninth school year.) The topics studied include proportional lines and similar figures, regular polygons, value of π, definitions of sine, cosine, and tangent, and formulas relating to the right triangle, lines and planes in space, polyhedral angles.

Third year. (The student's tenth school year.) This year's work includes areas and volumes, similar solids, elementary problems in descriptive geometry, and review for examinations.

Geometrical drawing.[1] (Three hours.) Second and third years. (The student's ninth and tenth school years.) There is a continuation of the exercises of the first year (geometrical and perspective drawings). Review, with supplementary work, for examination.

AGRICULTURAL SECTION.

The programs for arithmetic, algebra, and geometry are the same as those for the general section, less the reviews and the special preparations for examinations.

Applied mathematics.[4] (One hour.) Second year. (The student's ninth school year.) The course includes a practical study of certain instruments, such as the chain, square, graphometer, compass, level, plane table; sketching, measuring the distance to an inaccessible point, study of the method of land surveying; practical measurements, as of wood, piles of stone or coal, wagons and reservoirs; practice in methods of weighing.

Drawing.[4] (One hour, except in spring.) Second year. (The student's ninth school year.) This course includes geometrical drawings and free-hand sketches of tools and parts of agricultural implements. Drawing to scale of sketches made in the field.

Third year. (The student's tenth school year.) There is a continuation of exercises for the second year. Applications are made to agricultural instruments and construction of buildings, such as sheds and stables.

Mechanics.[4] (One hour each year.) Same program as the Industrial Section.

INDUSTRIAL SECTION.

The programs for arithmetic, algebra, and geometry include those for the General Section, less the reviews and special preparations for examinations. Two hours a week in the second and third years

[1] Vol. I, p. 27. [2] Ibid., p. 24. [3] Ibid., p. 26. [4] Ibid., p. 29.

are given to this common instruction and one hour a week to special instruction in the following subject matter:

Arithmetic, algebra, and geometry.[1] (One hour.) Second year. (The student's ninth school year.) Graphical representations of algebraic operations. Definition of a function of a variable. Graphical representations of data, such as statistics and railroad time-tables. Arithmetical and geometrical progressions. Exponential function and its representation by a curve. Logarithms. Slide rule. Measures of surfaces and volumes. Applications to exercises from the shop.

Third year. (The student's tenth year school.) The course includes continued use of the slide rule, graphical study of curves of the second degree, construction and properties of other curves commonly used in building, and construction of diagrams from descriptive geometry for platbands, various kinds of arches, and other forms. Review.

Geometrical drawing.[2] Second year. (The student's ninth school year.) The course consists of some exercises in projection, with applications, according to the special work of the pupil, to work in metal or wood and sketching and drawing to scale.

Third year. (The student's tenth school year.) There is a continuation of preceding exercises and applications, according to the special work of the pupil, to mechanics, electricity, carpentry, modeling, and other subjects.

Mechanics.[3] Second year. (One hour.) (The student's ninth school year.) Statics, including balanced forces, moments of forces, couples, composition, resolution and substitution of forces, center of gravity.

Dynamics. There is a study of force, energy, work, friction, simple machines, and the steam engine.

Third year. (Two hours.) (The student's tenth school year.) Section for mechanics. The application of the principles studied in the second year to the study of the details of the machines used in the shops.

Section for builders. A study is made of strength of materials, resistance to flexion, crushing, pulling, and similar questions. There is a study in detail of problems of construction.

COMMERCIAL SECTION.

In both the second and the third year two hours a week are given to arithmetic and algebra in common with the general section, and one hour a week to rapid calculation.

Rapid calculation.[4] (One hour.) This is a course in commercial arithmetic, which includes simplifications and processes employed by accountants, numerous exercises for oral and written calculation taken from commercial practice, use of the slide rule as a check, and foreign moneys and measures.

MARITIME SECTION.

"The[4] special program of the maritime section includes certain complements to the program of the general section and the application of mathematics to the nautical operations."

Arithmetic.[5] (One hour each year to arithmetic, algebra, and geometry.) The course includes measures, with special reference to nautical units, and a consideration of the approximate errors in computation.

Algebra. "Practical[6] solution of biquadratic and some very simple irrational equations."

Geometry. "Theorems[6] concerning trihedral angles. Sum of the face angles of a polyhedron. Sphere. Angle of two great circles. Spherical triangles."

[1] Vol. I, p. 30. [2] Ibid., p. 34. [3] Ibid., pp. 30-34. [4] Ibid., p. 35. [5] Ibid., p. 36.

Trigonometry.[1] (One hour.) Third year. (The student's tenth school year.) This course includes elements of plane trigonometry, including the fundamental formulas and the solution of triangles, and the fundamental formulas of spherical trigonometry.

Nautical instruction.[2] (Two hours.) Second year. (The student's ninth school year.) A study is made of the elements of mathematical geography, nautical signals, maritime maps, nautical instruments, and tides, with practical exercises.

Third year. (The student's tenth school year.) This is an extension of the above, with applications to navigation.

Amount of vocational mathematics in the programs.—The program for the section of general instruction contains no distinctly vocational material. The applied mathematics and the design given in the agricultural section are vocational and the mechanics essentially so. One-third of the time given to arithmetic, algebra, and geometry in the industrial section in the second and third years is given to applications to industrial questions. The geometrical drawing and mechanics are both vocational in this section. Rapid calculation and business applications are distinctive features of the instruction in the commercial section. The supplementary work in arithmetic, algebra, and geometry in the maritime section contains some topics having special application to navigation; and the nautical instruction is applied mathematics so far as it is mathematics. It appears from this survey that all the special sections get some vocational mathematics, but that it either follows the conventional work in arithmetic, algebra, and geometry or is taught separately to the particular group of pupils desiring it.

Methods of instruction.—The official instructions demand that the subject matter be kept free from unnecessary theory and technicalities, that the instruction be concrete, and that the examples be practical. "The course[3] proper will be of extreme simplicity; it will be relieved in particular of the theoretical developments in numeration and the fundamental operations, and in finding the greatest common divisor (method of successive division)." "These problems[4] (of arithmetic) will be related to ordinary and social life, to commerce, to industry, to the arts, and to agriculture. Questions of a purely speculative order should be omitted." "The teaching of geometry[4] should be essentially concrete; its aim is to classify and to make precise certain notions acquired by observation, to deduce from these others, and to show their applications to some problems that are found in practice."

Examinations.[5]—After the completion of three years' work in the higher primary schools, the students may take an official examination given at the school for the certificate of primary studies. The report contains several questions in mathematics taken from these examinations. With the exception of a problem given to

[1] Vol. I, p. 36. [2] Ibid., p. 37. [3] Ibid., p. 22. [4] Ibid., p. 24. [5] Ibid., p. 38.

candidates from the commercial section, the problems are of a general nature and do not involve vocational questions.

Preparation of teachers.—The teachers in the higher primary schools must have the *brevet supérieur*, to obtain which they must spend three years in a normal school and then pass an examination. The examination in mathematics includes arithmetic, algebra, and geometry. The normal school course gives three hours to mathematics in the first year, four hours in the second year, and two hours in the third year.

HIGHER PRIMARY SCHOOLS OF THE CITY OF PARIS.[1]

The higher municipal schools of the city of Paris give a course of four years. With the exception of certain students in one school who pay tuition, all students are admitted by competitive examination and must be between the ages of 12 and 15 years. The examination in mathematics is in arithmetic, with some geometrical applications. The schools have two sections, general and commercial.

The course of study in arithmetic, algebra, and geometry in the first three years contains about the same material as the course for the higher primary schools, but less time is given to mathematics. The work of the fourth year is given to a general review and preparation for examinations.

These schools prepare for the certificate of primary studies, for the *brevet élémentaire*, and for admission to various professional schools.

PRACTICAL SCHOOLS OF COMMERCE AND INDUSTRY.[2]

Organization.—There are 28 of these schools in France. They are State schools and not compulsory. For admission students may present the certificate of primary instruction or pass an examination similar to the one taken for that certificate. Hence, before entering, the student has completed a course of six years in an elementary school. The age of entrance is from 12 to 13 years. The mathematical preparation is a study of arithmetic with some applications to mensuration. The students are said to lack proficiency in the operations of arithmetic. "Their knowledge[3] is vague, confused, and above all they do not possess in the practice of calculation skill sufficient to deal with accuracy with the usual applications of arithmetic." The majority of students spend a year in a preparatory class before entering the regular three years' course.

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[1] Vol. 1, p. 43.

[2] Rapports de la Commission internationale, Sous-Commission française, Vol. IV, L'Enseignement technique, by M. P. Rollet, pp. 7–71. Referred to hereafter as Vol. IV.

[3] Ibid., p. 53.

These schools have an industrial section and a commercial section.

Aim of instruction.—The aim of these schools is to furnish "an apprenticeship [1] in an industrial or commercial occupation under such conditions that on leaving the school the student may be immediately utilizable in industry or commerce."

The official instructions emphasize the idea that utility should be the aim of instruction. In particular, arithmetic should increase the ability to perform the necessary calculations of business; algebra should give ability to understand and apply formulas and to solve problems by algebraic methods; and geometry is to be the basis of design and to be used in the solution of practical problems.

INDUSTRIAL SECTION.

COURSES OF STUDY IN MATHEMATICS.

Arithmetic.[2] First year. (The student's seventh school year.) (Three hours a week.) This course includes the fundamental operations with integers and fractions, powers, square and cube root, metric system, measure of time, circumference of the circle, English measures of length, ratio and proportion, rule of three, and percentage and its applications. There is special emphasis throughout on rapid calculation.

Algebra.[3] Second year. (The student's seventh school year.) (Two hours a week.) This course includes the fundamental operations, equations of the first degree in one or two unknowns, quadratic equations, simple progressions, and logarithms.

Geometry.[4] First year. (The student's seventh school year.) (Three hours a week.) This course is a combination of plane and solid geometry, the corresponding notions in the two being developed side by side. It includes the properties of triangles, parallelograms, and circles, and of lines and planes in space.

Second year. (The student's eighth school year.) (Two hours a week.) This year's work includes ratio and proportion, similar figures, regular polygons, value of π, and length of circumference, surfaces of the usual plane figures and properties, surfaces and volumes of the usual solids. [One sees nothing in the outline of the course to differentiate it from the usual school course, so far as topics are concerned.]

Descriptive geometry applied to design.[5] Third year. (The student's ninth school year.) (One hour a week.) This course includes the projections of points, lines and the simpler geometrical solids, intersections of lines, surfaces, and solids; and the development of sections of the simple solids. Applications are made to the various trades.

Mechanics.[6] Third year. (The student's ninth school year.) (Two hours a week.) Principles of general mechanics: The course includes preliminary notions, composition and resolution of forces, moments, weight, center of gravity, equilibrium, motion, work; applications of the principles studied to simple machines; falling bodies, conservation of work, centrifugal force, simple and compound pendulum, friction, and study of certain mechanisms and simple machines.

Applied mechanics: The principal topics are the resistance of materials, hydraulic motors, and steam engines. Besides the above, the mechanicians receive instruction in their particular fields.

Geometrical drawing.[7] This is a course in geometrical drawing with special application to the different industries. To this course are given six hours a week during the first year, seven hours a week during the second, and seven hours a week during the third year.

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[1] Rapports de la Commission internationale, Sous-Commission francaise, Vol. IV, L'Enseignement technique, by M. P. Rollet, p. 34. Referred to hereafter as Vol. IV.
[2] Vol. IV, p. 10.
[3] Ibid., p. 11.
[4] Ibid., pp. 12–15.
[5] Ibid., pp. 16, 17.
[6] Ibid., pp. 18–24.
[7] Ibid., p. 24–28.

COMMERCIAL SECTION.

Arithmetic and algebra.[1]—Essentially the same subject matter is taken up as in the industrial section, plus a discussion of compound interest and annuities. Three hours a week are given in each the first and second years. More time is given to arithmetic than to algebra.

Commerce and bookkeeping.[2]—This is an extended course in the details of the theory and practice of commerce and bookkeeping. It deals with such subjects as general methods of commerce, exchange, transportation, post and telegraph, banking, boards of trade, insurance, and business organization.

Six hours a week in the first year and three in both the second and third are given to commerce and bookkeeping. In both the second and third years, six hours are given commercial exercises.

The limits of this report will not permit a statement of the details of the above courses as they are outlined. It may be said, however, that the outlines for the courses in arithmetic, algebra, and geometry contain in the main conventional material, but that much that is purely theoretical is omitted.

Methods of instruction.—In the report and in the official instructions emphasis is continually placed upon making the instruction simple and concrete. In the teaching of geometry [3] it is said that whenever "it is possible to simplify the instruction without injuring the precision or the rigor of the reasoning, the master will hasten to do it; experience will give the greatest possible contribution." Geometrical notions,[4] such as dihedral, perpendicular to a plane, solids and surfaces of revolution, are to be illustrated with familiar objects. The different plane and solid figures [5] are to be constructed from wood or cardboard. Theorems are demonstrated by the use of these solids.

In the algebra [6] the theory is reduced to the minimum strictly indispensable to the comprehension of the usual operations.

The principles of mechanics [7] are taught generally "not as demonstrable truths, but as incontestable principles of experience and observation." "The instruction [8] will be above all experimental." "For the dry abstract mathematical demonstrations,[9] of which the sacrifice is voluntarily made, we substitute simple and striking experimental verifications."

Problems.—The exercises [10] should be as far as possible practical. "It is above all in the choice of the exercises given that the professor of a technical school gives his instruction the really professional character." [11] "In a problem [11] the form of statement itself counts for much more than we are inclined to believe. The exercises

[1] Vol. IV, p. 28, 29.
[2] Ibid., pp. 29–32.
[3] Ibid., p. 41.
[4] Ibid., p. 42.
[5] Ibid., pp. 42, 43.
[6] Ibid., p. 61.
[7] Ibid., p. 66.
[8] Ibid., p. 18.
[9] Ibid., p. 67.
[10] Ibid., p. 28.
[11] Ibid., p. 44.

given ought then, as much as possible, to be based upon real conditions and be adapted to the needs of commercial, industrial, or everyday life." Two examples[1] accompanied by sketches are given to illustrate this point. This remark follows:

"The preceding statements,[1] with their practical appearance and the sketch given, interest the pupils to a degree quite different from that of the problems of the same difficulty, but with vague statements or without conditions really practical."

Some good suggestions[2] are given concerning possible sources of applied problems in the different trades.

The instruction[3] in mechanics is to be adapted to the needs of the locality, and the problems made as practical as possible.

Use of books.—The short time[4] given to the instruction and the large classes make it necessary to use a book as a basis for the work in geometry. The theory is supposed to be developed in class, the student taking notes. The book is used to supplement the notes and for exercises.

Examinations.[5]—At the end of the course the students may take an examination for the practical, industrial, or commercial certificate of studies. This includes a written examination on practical arithmetic and geometry, and an oral one on arithmetic, bookkeeping, geometry, mechanics, and technology.

Preparation of teachers.[6]—There are special normal sections in the School of Advanced Commercial Studies in Paris, and in the School of Arts and Crafts in Chalons-sur-Marne, for the preparation of teachers for the commercial and industrial schools. Students are admitted to these sections by examination, and must give evidence of a substantial and extended knowledge of mathematics. "During two years[7] the students follow the courses and exercises of the laboratory and the shops of the School of Advanced Commercial Studies, or of the School of Arts and Crafts, they receive the general instruction of the school to which they are attached, get the necessary professional preparation, and complete their studies with some special conferences having at the same time a pedagogic and educative character." This work is followed by an examination for the certificate to teach the commercial or industrial branches. To this examination are admitted, however, any candidates, who may have obtained their preparation in very different ways.

The teaching of arithmetic in these technical and commercial schools is said to be very satisfactory. "Accepting[8] the influence of the technical atmosphere in which they live, the professors of mathematics have succeeded in giving their instruction a distinctive character and in making it practical, without losing sight of the educative purpose which is the characteristic of mathematics."

[1] Vol. IV, p. 45. [2] Ibid., p. 68. [3] Ibid., p. 34. [4] Ibid., p. 4.
[5] Ibid., p. 46. [6] Ibid., p. 42. [7] Ibid., pp. 3-5. [8] Ibid., p. 5.

Tendencies.—In speaking of the instruction in geometry it is said that "the official instructions [1] and the new manuals of instruction are characterized by a twofold change: First, in the method; second, in the presentation.

"The reform in method consists in the abandonment of the Euclidian edifice for which we substitute a geometry in which the displacements (rotation and translation) play a preponderant rôle. That method has the advantage of being more intuitive and of arranging the geometrical facts in an order of increasing difficulty graphically.

"The reform in presentation is characterized by bringing into closer relation the principles relating to plane figures and those concerning figures of three dimensions. This method has for its aim to accustom the student to deal with space from the beginning of his study and to lead naturally to the reading in industrial design."

NATIONAL VOCATIONAL SCHOOLS.[2]

There are four of these schools in France. They are State schools under the control of the minister of commerce and industry. The admission is by competitive examination, and the candidates must be between the ages of 12 and 15. The examination in mathematics is limited to arithmetic, and includes only the work covered in the higher course of the elementary primary schools. The length of the course of study is four years.

These schools [3] have the same aims and employ the same means as the practical schools of commerce and industry. They have the advantage of offering a course of four years instead of three, and of requiring students to be a little farther advanced at entrance. The course in mathematics contains considerably more material than the course in the practical schools for commerce and industry. In algebra more work is given in logarithms and applications to compound interest and annuities, and maxima and minima. In geometry an extra year's work is given to geometrical applications and to the consideration of the conic sections and some higher plane curves. A course is given in plane trigonometry which includes the solution of oblique triangles. There are two years of mechanics giving special attention to applications to machines, in particular gas, hydraulic, and steam motors.

The work in geometrical drawing extends through the four years. In the last two years the students are divided into two sections, one for woodworkers and one for ironworkers, and special applications to the trades are made.

[1] Vol. IV, pp. 39, 40. [2] Ibid., pp. 87-97. [3] Ibid., pp. 89, 90.

THE VOCATIONAL INSTRUCTION OF GIRLS.[1]

Organization.—The schools giving this instruction are known in Paris as the *écoles professionelles et ménagères*, and in the country as the *écoles pratiques de commerce*. The students enter between the ages of 12 and 15. The length of the course is three years. The schools are under the control of the minister of commerce, and have industrial and commercial sections. The aim of these schools is practical, but the methods of instruction are expected to be such as to develop the general intelligence of the students.

Schedule of hours.[2]—In the country, where these schools are said to occupy a more important place than in Paris, the usual schedule of hours is as follows: In the first and second years there are 18 hours of class work and 24 hours of practical work; in the third year 10 hours of class work and 32 hours of practical work.

Hours given to mathematics in the industrial section.

Studies.	Practical schools (country).			Vocational schools (Paris).	
	First year.	Second year.	Third year.	First year.	Second year.
Arithmetic	1½	1½	.....	1½	1½
Geometry	.....	1½	1½	1½	1½

The commercial section gives three hours a week for three years to arithmetic.

COURSE OF STUDY IN MATHEMATICS.

Arithmetic.[3] Considerable latitude in the choice of material is said to be given the teachers in the country schools. The courses usually include the metric system, greatest common divisor, least common multiple, common and decimal fractions, applications of proportion, the rule of three, and practical problems. The programs for Paris name the additional topics: Interest and discount, foreign moneys and measures, problems about surface, volume, weight, and density. In the commercial section more time is given to commercial problems.

Geometry.[4] The course includes the elements of plane geometry, with many constructions and applications to practical problems.

Methods of instruction.—Both the topics chosen and the nature of the problems show that the work in mathematics is intended to be vocational. There are said to be no books well adapted to the instruction given, either in arithmetic or geometry.

Examinations.[5]—Oral and written examinations for a certificate of studies are given at the end of the course in certain schools.

Preparation of teachers.[5]—The teachers are prepared generally in the normal section of the Practical School of Havre.

[1] Rapports de la Commission internationale, Sous-Commission française, Vol. V, L'Enseignement professionel des jeunes filles, by Mlle. Ameux, pp. 65–76. Referred to hereafter as Vol. V.
[2] Ibid., pp. 66, 67.
[3] Ibid., p. 69.
[4] Ibid., pp. 75, 76.
[5] Ibid., p. 73.

GERMANY.

COMMERCIAL SCHOOLS.[1]

Kinds of commercial schools.—The following kinds of commercial schools exist in Germany:[2]

A. Commercial colleges (Handelshochschulen).
B. Commercial schools.
   1. Public higher commercial schools.
   2. One-year course for young persons with the certificate for one-year military service.
   3. Handelsrealschulen.
   4. Public commercial schools which give the certificate for one-year military service.
   5. Private commercial schools which give the certificate for one-year military service.
   6. One-year course for young persons without the certificate for one-year military service.
   7. Commercial classes in Realschulen.
C. Commercial continuation schools.
   1. Compulsory commercial continuation schools.
   2. Noncompulsory commercial continuation schools.
D. Commercial schools for female employees.
   1. Compulsory commercial continuation schools.
   2. Noncompulsory schools.
      (*a*) Commercial schools and commercial courses.
      (*b*) Free continuation schools and courses.

These schools may be grouped according to the time at which the vocational instruction is given into the following classes:[3]

1. Those that give the vocational instruction during the apprenticeship: Commercial schools for apprentices.
2. Those that give the vocational instruction before the apprenticeship:
   (*a*) After completing the general education: Commercial preparatory schools and higher commercial schools.
   (*b*) During the general education: Handelsrealschulen.
3. After the apprenticeship: Handelshochschulen.

COMMERCIAL CONTINUATION SCHOOLS.[4]

Organization.—These commercial continuation schools are supported in the main by the State, city, or community. Some are supported by commercial corporations and associations. Attendance is usually, though not always, required of merchants' apprentices during the three years of apprenticeship.

In order to enter a commercial continuation school a student must have completed the Volksschule, or be past the compulsory school

[1] Rechnen und Mathematik im Unterricht der kaufmännischen Lehranstalten, by Dr. B. Pennndorf. This is Band IV, Heft 6, of the German Reports, and is referred to hereafter as Band IV, Heft 6.
[2] Ibid., p. 4.
[3] Ibid., p. 5.
[4] Ibid., pp. 7-38.

age. Entrance examinations are given in German and arithmetic in some schools. Candidates not passing these examinations must go into a preparatory class.

The length of the course is three years. In most parts of Germany the average number of hours of instruction a week is 6. In the most of the commercial continuation schools of Baden 8 or 9 hours a week are given, and in Saxony the average is 12 hours a week.

Aim of the instruction.—These are vocational schools, and aim to give practical instruction to young people entering commercial life. The aim of the instruction in arithmetic is "the development of the ability to carry through the computations arising in commercial transactions, concisely, clearly, accurately, and independently."[1] Furthermore, by the discussion and solution of problems the students should be given a better understanding of certain phases of commercial life. Calculation must be made the central point of the instruction, and must be regarded as the most important part of commercial arithmetic.

COURSE OF STUDY IN MATHEMATICS.

The following work is prescribed for the Prussian commercial continuation schools.[2] The courses outlined for other States are very similar to this.

Preparatory course. (Three hours a week.) There is a review of the fundamental operations with special reference to multiplication, much practice with problems involving the German units of money, volume, and weight, and applications to problems from commercial life.

First year. (The student's ninth school year.) (Two hours a week.) The course includes applications of the fundamental operations in computing prices; the most important units of money, volume, and weight used in international commerce; examples arising from transactions with the railroads and the post office; the chain rule.

Second year. (The student's tenth school year.) (Two hours a week.) The principal topics studied are percentage, interest, discount, equation of payments, and the computation of prices of goods.

Third year. (The student's eleventh school year.) (One or two hours a week.) In this course are studied accounts current, exchange, stocks, and the computation of prices of goods.

"Besides this graded material [3] there are to be given occasionally examples from the administration of the municipality and the State, and from taxation and insurance."

"If the preparation of the students is good, an introduction to percentage can be given in the first year and accounts current in the second year. If the preparation of the students leaves something to be desired, it is necessary to reduce the material as much as possible."[3]

Nature of the instruction.—As the course of study shows, the subject matter in arithmetic is that which is supposed to be of most practical value to people who will engage in commercial work. It is made a

[1] Band IV, Heft 6, p. 2. [2] Ibid., pp. 28, 29. [3] Ibid., p. 29.

fundamental principle that only such examples be considered as have practical value. "Questions [1] requiring the finding of the time and capital in interest and discount, difficult computations with common fractions, examples from compound proportion, unusual computations with money, etc., are therefore eliminated, and problems concerning such things as prices must correspond as nearly as possible to actual conditions. From the practice, for the practice, that is the watchword."

The official regulations in Prussia recommend that about 10 minutes a day be given to mental arithmetic, and emphasize the importance of rapid calculation, mastery of short methods used in practice, and neatness of written work.

When the size of the classes makes it possible, sections are organized for the different branches of trade. In Munich,[2] for example, there are the following groups: Banking, transportation, insurance, the book trade; groceries, farm products, foodstuffs; textiles and other kinds of manufactured wares; iron and metal products, glass, porcelain, and others.

A school will, as a rule, give special attention to the important industries of the locality.

In some cases there is connection between the arithmetic and the bookkeeping; and in Prussia an effort is made to have the work in arithmetic supplement the commercial science (Handelskunde). The courses are so outlined that the topics discussed each year in these two subjects will be closely related.[3]

The methods of teaching in the commercial continuation schools are very similar to those in the Volksschulen, as most of the teachers in the continuation schools are, or have been, teachers in the Volksschulen.

Textbooks.—From some observations it would appear that rather general use is made of books as sources of problems, and it is said as a rule that these books furnish useful and practical problems.

Preparation of teachers.—"In scarcely any other kind of schools does the teaching force have so many different kinds of preparation as in the commercial schools—philologists, teachers of German, mathematicians, also lawyers, political economists, teachers of special subjects, all grades of teachers from the Volksschulen, merchants, etc., some teaching full time and some teaching part time." [4]

There seems to be no official requirement in Prussia for the preparation of teachers for the commercial continuation schools. In 1910,[5] out of 241 teachers teaching full time in these schools (im

[1] Band IV, Heft 6, p. 25.
[2] Ibid., p. 26.
[3] Ibid., p. 35.
[4] Ibid., p. 69.
[5] Verwaltungsbericht des königlich preussischen Landesgewerbeamts, 1912, p. 81.

Hauptamt), 207 were professional teachers and 34 were engaged in business; and of 2,137 who were teachers part time (im Nebenamt), 1,938 were teachers and 199 were engaged in business.

The official requirement for teachers in commercial schools in Baden [1] is either (*a*) the completion of the seventh year of a secondary school, two years of practice in business, and at least two years' attendance at a school for the preparation of commercial teachers; or (*b*) admission to candidacy to teach in the elementary schools, one year of practice in business, and at least two years' attendance at a school for the preparation of commercial teachers.

In Bavaria [2] the requirements are the completion of a nine-year secondary school, three years of study in a higher institution, at least one of which shall be in a commercial high school, and one year of practice in business. Some practice teaching is required during the three years of study.

COMMERCIAL PREPARATORY SCHOOLS.[3]

Organization and aim.—These schools, which have various names, give courses of one year (in Baden one and one-half years) for students who have not entered business. These are day schools giving from 24 to 33 hours of instruction a week. The students [4] are said to be about the same kind as in the schools for apprentices, and so is the aim of the instruction. The same subject matter may therefore be used, and the only difference arises in the methods of instruction. Since the students in the commercial preparatory schools have had no practical experience, it is recommended that special care be taken to make the presentation clear and concrete.

The entrance requirement is usually the equivalent of the completion of the Volksschule.

There were 40 of these schools in Germany in 1912.

COURSE OF STUDY IN MATHEMATICS.

Arithmetic. (From four to six hours a week.) [The student's ninth school year.] As an example of the work done in arithmetic, the report gives the Dresden plan,[5] which includes computations with integers and fractions, ratio, simple and compound proportion, discount, equation of payments, domestic and foreign exchange, and computation of prices.

HIGHER COMMERCIAL COURSES.[6]

Organization and aim.—The higher commercial courses are usually one-year courses, with approximately 35 hours of instruction a week.

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[1] Verordnungsblatt des grossherzoglichen Oberschulrats (Karlsruhe, 1907), p. 143.
[2] Band IV, Heft 6, p. 70.
[3] Ibid., pp. 39–41.
[4] Ibid., p. 41.
[5] Ibid., pp. 41–45.

The entrance requirement is, as a rule, the possession of the certificate for one-year military service, which means the completion of six years of a secondary school. Hence these students have a better preparation than those in the continuation schools, the mathematical preparation in particular, including algebra through quadratics and plane geometry. The students come, in the main, from well-to-do families, and the object of the instruction is to prepare them to fill responsible commercial positions.

COURSE OF STUDY IN MATHEMATICS.

Arithmetic. (From four to six hours a week, as a rule.) (The student's tenth school year.) The work done in Leipzig[1] is given as an example. It includes computation of prices, percentage, interest, discount, equation of payments, accounts current, stocks, and various methods of exchange.

COMMERCIAL REAL SCHOOLS (HANDELSREALSCHULEN).[2]

Organization.—These are secondary schools, and differ from other Realschulen in that in the last three years special attention is given to commercial subjects, such as bookkeeping, commercial arithmetic, commercial correspondence, and commercial science (Handelskunde). Elective subjects, such as stenography, are offered.

The course is six years in length, and leads to the certificate for one-year military service.

Aim of the instruction.—The aim of these schools is to give a general education, and at the same time some vocational preparation, to those who will later occupy important positions in commercial life.

The most important problem for the mathematical instruction is said[3] to be to develop in the student the ability to apply the knowledge and power he has gained to independent work.

The instruction in mathematics in the Handelsrealschulen differs from that in other Realschulen in giving commercial arithmetic in the last three years, and in giving more attention to commercial applications in the courses in algebra.

COURSE OF STUDY IN MATHEMATICS.

The following course of study[4] is the one outlined by the German society for commercial instruction. It seems to be followed closely by the schools of north Germany, and seems to give a better idea of the general practice than other courses. Class 1 is the highest class in the school.

CLASS 3. (THE STUDENT'S SEVENTH SCHOOL YEAR.)

Arithmetic. (Two hours a week.) "Comprehensive review of material previously studied with applications to commercial problems. Percentage. Interest. Discount and equation of payments."

[1] Band IV, Heft 6, p. 43. [2] Ibid., pp. 45-63. [3] Ibid., p. 53. [4] Ibid., pp. 62, 63.

Algebra. (Four hours a week with geometry.) "The fundamental operations with arithmetical numbers and introduction to positive and negative numbers. Theory of proportion. Equations of the first degree with one unknown."

Plane geometry. "Review of the material studied in Quarta.[1] Theory of parallelograms. Theory of the circle. Theorems concerning the areas of figures (Pythagorean theorem). Computation of the areas of polygons. Problems of construction."

CLASS 2. (THE STUDENT'S EIGHTH SCHOOL YEAR.)

Arithmetic. (Two hours a week.) "Simple exercises in calculating the prices of goods. Review and extension of the work in discount. Accounts current by different methods. Most important facts about reckoning with different moneys. Simple problems in stocks."

Algebra. (Four hours with geometry.) "Theory of powers and roots. Equations of the first degree in one or more unknowns. Simple quadratic equations in one unknown."

Plane geometry. "Theory of similarity. Proportionality of lines in the circle, division into parts having a constant ratio. Regular polygons, perimeter and area of the circle, problems of construction."

CLASS 1. (THE STUDENT'S NINTH SCHOOL YEAR.)

Arithmetic. (Two hours a week.) This course includes problems in stocks and bonds and problems dealing with the buying and selling of goods.

Algebra. (Three hours a week with geometry and trigonometry.) "Theory of logarithms. Practice in computation with logarithms. Quadratic equations. Arithmetical and geometrical series and their application to compound interest and annuities."

Plane geometry. "Continuation of the problems in construction."

Trigonometry. "Elements of trigonometry. Simple problems with triangles."

Solid geometry. "Introduction to the perspective drawing of space figures. The simple solids and the computation of their edges, surfaces, and volumes."

Preparation of teachers.—The preparation of the teachers in these schools is similar to that of the teachers in other secondary schools.

Methods of instruction.—The methods of instruction are similar to those used in other secondary schools.

Examinations.—At the end of the course the students take the examination for the certificate for one-year military service. Besides examinations in other mathematical subjects, there is included for the students from the Handelsrealschulen an examination in commercial arithmetic.

INDUSTRIAL SCHOOLS.

INDUSTRIAL CONTINUATION SCHOOLS.[2]

Organization.—The industrial continuation schools are generally organized and supported by the municipalities. The State has the right of supervision and sometimes furnishes aid.

The aim of these schools is to furnish theoretical instruction to apprentices during their apprenticeship. Many schools have well-

[1] Quarta is class 4, the student's sixth school year.

[2] Die math. Fächer an den gewerblichen Lehranstalten in Deutschland, by Dipl.-Ing. W. Trost. This is Band IV, Heft 5, of the German Reports. Referred to hereafter as Band IV, Heft 5.

equipped shops and supplement the theoretical instruction by shop-work under skilled mechanics. The length of the course is three years as a rule. In Prussia the number of hours of instruction a week is usually six. In southern Germany the number of hours is often more than this, being nine in many schools in Munich, for example.

Attendance in these schools is usually required of apprentices.

Usually when the number justifies it the students are classified for instruction according to trades. Berlin,[1] for example, is divided into 10 districts. The schools of each district are organized to take care of the industrial needs of the locality, and each school has a number of classes for different trades.

Aim of the mathematical instruction.—It is intended to make the mathematical instruction as practical as possible. The courses are planned to give the apprentices the instruction that is applicable to their particular trades.

COURSE OF STUDY IN MATHEMATICS.

The courses of study differ widely in different places and for different trades. Some typical courses are given.

The following work is outlined for machinists in Munich.[2] The outline divides the course in arithmetic into practical, geometrical, and technical arithmetic (bürgerliches, geometrisches, and technisches Rechnen).

FIRST CLASS. (THE STUDENT'S NINTH SCHOOL YEAR.)

Arithmetic. (One hour a week.) The principal topics studied are the earnings of apprentices and laborers, living expenses, savings institutions and the interest paid by them; quadrilaterals, triangles, polygons, circle, ellipse, and the Pythagorean theorem; problems concerning materials and problems from physics, if time permits.

SECOND CLASS. (THE STUDENT'S TENTH SCHOOL YEAR.)

Arithmetic. (One hour a week.) This course takes up problems concerning contracts; prism, pyramid, cylinder, cone; applications to machinery, such as cogwheels, pulleys, and other apparatus in the shop.

THIRD CLASS. (THE STUDENT'S ELEVENTH SCHOOL YEAR.)

Arithmetic. (One hour a week.) This year's work includes problems about contracts, taxes, and insurance; frustums of pyramids and cones, sphere and spherical segment; applications to machinery.

FOURTH YEAR. (THE STUDENT'S TWELFTH SCHOOL YEAR.)

Arithmetic. (One hour a week.) This course includes the computation of surfaces by transforming them approximately into simpler forms; simple determinations of the center of gravity and of the volumes of solids of revolution, and mechanical power and efficiency of machines.

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[1] Band IV, Heft 5, p. 28. [2] Ibid., p. 42.

The next following program[1] is given in the Gewerbeschule, in Carlsruhe, to tinners and plumbers.

FIRST YEAR. (THE STUDENT'S NINTH SCHOOL YEAR.)

Arithmetic. (One hour a week.) This work is divided into industrial arithmetic, which includes computation of the volumes and surfaces of forms met with in the trade, and use of technical tables; and business arithmetic, which takes up percentage, discount and rebates, profit and loss, wages, and cost of goods, including transportation and other expenses.

Applied geometry. (One hour a week.) This course includes a discussion of various solids, their lines, angles, and surfaces; examples of solids arising in practice, as the cylinder from rolling, turning, boring, molding, drawing, and bending together; practical use of instruments, as the straight edge, plumb line, water level, bevel, stencil, micrometer screw, calipers, and compasses; and the elementary geometrical constructions and their applications to problems coming from the trades.

Projection. (Two hours a week.) This course consists mainly of practical applications.

SECOND YEAR. (THE STUDENT'S TENTH SCHOOL YEAR.)

Arithmetic. (One hour a week.) The principal problem taken up in industrial arithmetic are computations of surfaces, with applications to roofs, heating apparatus, and various kinds of material; areas of cross sections; finding one dimension when the volume and two dimensions are given; square root and the use of technical tables; and in business arithmetic the principal topics are interest, moneys, commercial paper, and percentage applied to questions of profit and loss and expense.

Applied geometry. (One-half hour a week.) This course consists mainly of such problems of construction as the construction of ellipses, tangents to circles, dividing a line into parts proportional to given lines, drawing given figures to scale, and practical applications.

THIRD YEAR. (THE STUDENT'S ELEVENTH SCHOOL YEAR.)

Arithmetic. (One-half hour a week.) The principal topics studied are exchange and accounts current; estimates and operating expenses.

There is a course of bookkeeping running through the three years. The time given to it seems to be about one hour a week. A course of one hour a week in applied physics and mechanics is given in the third year.

The following courses are offered in the Gewerbeschule[2] in Frankfort on the Main. This is not a compulsory continuation school, but apprentices with approved preparation may attend this school in satisfaction of their requirement to attend a continuation school during apprenticeship. The courses outlined below are for such apprentices.

INDUSTRIAL ARITHMETIC.

First year. (The student's ninth school year.) This course gives practice in calculation with integers and fractions with applications to problems from industrial life.

Second year. (The student's tenth school year.) A study is made of percentage and its applications, insurance, taxes, partnership, alligation, and examples from statistics.

Third year. (The student's eleventh school year.) This course includes problems from the imperial insurance ordinance, fire, life, and accident insurance; wages; discounts, cost of materials, expenses; computation of surfaces and solids.

A course in algebra or geometry may be substituted for the last course.

[1] Band IV, Heft 5, pp. 46–49. [2] Lehrpläne der städt. Gewerbeschule in Frankfurt am Main, p. 12.

The following courses [1] in mathematics, open to masters, assistants, and apprentices, are also given in the Gewerbeschule in Frankfort on the Main. Each course is given two hours a week.

ALGEBRA.

I. "Introduction to algebra. Addition, subtraction, multiplication."

II. "Negative numbers. Division. Simple equations of the first degree in one unknown. Powers and square root."

III. "Review of powers, extension of the theory of roots. Equations of the first degree in several unknowns. Problems (eingekleidete Aufgaben). Pure quadratics."

IV. "Extension of the material treated in the third course. Quadratic equations. Powers with fractional and negative exponents. Logarithms."

GEOMETRY.

I. "*Plane geometry:* The straight line and the angle. Congruence of surfaces. The most important theorems concerning the triangle, parallelogram, and trapezium."

II. "The most important theorems concerning the circle and constructions involving the circle. Calculation of areas. Proportionality. The Pythagorean theorem and its applications. Problems of construction."

III. "Theorems concerning similar triangles and quadrilaterals. Proportion applied to the circle. Regular polygons. Circumference and area of the circle. *Solid geometry:* Relations of points, lines, and planes in space."

IV. "The most important theorems of solid geometry. Calculation of surfaces and volumes."

TRIGONOMETRY.

I. "The trigonometrical functions. Practice in the use of logarithmic and trigonometrical tables. Calculation of right angled and isosceles triangles, and rectangles. Sine theorem. Cosine theorem. Tangent theorem. Formulas for areas. Calculation of oblique triangles."

II. "Goniometry. Sums and differences of functions. Problems from practical geometry (geodesy, building, construction of machines) as well as from mechanics and physics."

MATHEMATICAL EXERCISES (MATHEMATISCHE ÜBUNGEN).

"Solution of problems from algebra, geometry, trigonometry, mechanics, chosen according to the callings and previous preparation of the students."

Nature of the instruction in mathematics.—The chapter entitled [2] "The method of the mathematical instruction in the industrial continuation schools," in Band IV, Heft 5, of the German reports, gives more attention to recommendations as to how mathematics should be taught than to descriptions of actual conditions, although it gives official directions from different places.

In this chapter emphasis is placed upon the need of mental arithmetic and of much practice in computation, the importance of clear, concrete presentation, and upon the care that should be used in the selection of practical problems.

"The theory must be made clear to the student by simple methods that are adapted to his stage of development.[3] The mathematical experiment must play an important rôle in the continuation school."

[1] Lehrpläne der städt. Gewerbeschule in Frankfurt am Main, p. 9. [2] Band IV, Heft 5, p. 64.
[3] Pp. 64–80.

"In the first place the student must learn to compute." [1] "In every recitation period in arithmetic about 10 minutes should be given to mental arithmetic." It is not intended, however, that only mechanical skill be developed. Emphasis is placed upon clear thinking.

The courses of study outlined above will give an idea of the topics from which problems are taken.

The Berlin program [2] states that the study of algebra should begin with the calculation of surfaces and volumes; that is, algebra should not be begun as an abstract science, but as an extension of arithmetic. The German report recommends that algebra be begun with the study of the equation, and that the fundamental operations be developed by using them in the solution of equations. Algebra is to be considered only as a tool, and is to be developed from that point of view.

Some schools attempt to make the instruction in mathematics more practical by giving courses called *mathematische Übungen*. The outline of such a course, which is offered in the Gewerbeschule in Frankfort on the Main, is given above. The present writer attended a recitation in this course. The students, about a dozen young men, were working on problems in different subjects—algebra, geometry, mechanics. The teacher passed around the room, assisting and correcting. The teaching was well done, and the problems seemed well chosen. As the students were working independently of each other, each had an opportunity to make whatever progress he was able. The exercise seemed to be a very valuable one.

The students in the continuation schools do little or no school work outside of the classroom. Hence all of the solutions of problems, as well as the presentation of theory, must be done in the classroom. The problems are usually worked in notebooks and kept by the students.

Preparation of teachers.[3]—Prussia is only beginning to attempt a solution of the problem of preparing teachers for the industrial continuation schools. In 1913 there was opened in Charlottenburg a school for the training of teachers for the industrial schools. Short courses for teachers and practical mechanics are given in different parts of the Kingdom. These courses usually continue for 14 days, and the most of the time is given to the discussion of methods of instruction.

In 1910 in the industrial continuation schools of Prussia, of 499 teachers who were teaching full time, 362 were professional teachers and 137 were practicing a trade; and of 14,823 who were teaching part time, 12,337 were professional teachers and 2,486 were practicing a trade.

[1] Band IV, Heft 5, p. 65. [2] Ibid., p. 71. [3] Ibid., pp. 128-140.

South Germany is far ahead in the preparation of teachers for the industrial schools. We will take as an example Baden.

In Baden [1] a position as a teacher in an industrial school can only be obtained after passing a preliminary and a principal examination. In order to be admitted to the preliminary examination, the candidate must have completed a course equivalent to the completion of the seventh year of a secondary school (which means 10 years of schooling); and also the first three semesters of the course for industrial teachers in the Baugewerkeschule in Carlsruhe; to be admitted to the principal examination he must have had one, or in some cases two years of practice in a trade, and have completed the course of three and one-half years for industrial teachers in the Baugewerkeschule in Carlsruhe. The mathematics in this course includes:[2] Algebra through quadratic equations, arithmetical and geometrical series, logarithms, compound interest and annuities, combinations, the binomial theorem, and elements of maxima and minima; plane, solid, descriptive, and practical geometry; trigonometry, including the solution of the general triangle and applications; elements of analytical geometry; geometrical drawing; and mechanics.

In the last two years there is one hour a week given to introduction to the practice of teaching in industrial schools.

Some graduates of the middle technical schools become teachers. There is in Stuttgart a one-year course to prepare such persons for teaching. Another source of teachers for the industrial schools is the technical high school.

Textbooks.—Books are rather generally used as sources for problems. Some of these texts are said to be good and to furnish material that is really vocational. A list of texts is given in Band IV, Heft 5, page 149.

TRADE SCHOOLS FOR THE METAL INDUSTRIES.[3]

These are schools with courses of two or three years. They differ from continuation schools in that they give more theoretical instruction, occupy all of the student's time, and in two or three years give the apprentice the theoretical and practical instruction that a trade demands. "The schools[4] are, in the proper use of the word, schools for apprentices, as we find them in America, where the students get thorough theoretical instruction along with their shop training. They take the place at the same time of the apprenticeship period and the continuation school."

As the students have only the preparation of the Volksschule, the methods of instruction must be simple and concrete as in the continuation school.

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[1] Verordnungsblatt des grossherzoglichen Oberschulrats (1907), p. 147.
[2] Grossherzoglich Badische Baugewerkeschule, Programm (1909–10), pp. 30–36.
[3] Band IV, Heft 5, pp. 101–106.
[4] Ibid., p. 101.

The subject matter of the courses in mathematics is said to be limited to the things that are most essential for the mechanic.

The trade school in Schmalkalden [1] offers the following courses in mathematics. This school gives a course of two years.

Arithmetic. (Two hours a week the first year, one hour a week the second year.) (The student's ninth and tenth school years.) This includes a review of the fundamental operations with integers and fractions and applications to technical problems; units of money, weight, and volume; simple and compound rule of three; percentage, interest, discount, alligation; insurance; commercial paper.

Five hours a week the first year are given to the following subjects:

Algebra. Algebra through quadratics with many problems, particular attention being given to the use of formulas.

Geometry. This includes the elementary notions of plane geometry so far as they are important for the trade instruction or for practice; applications which involve the calculation of surfaces, volumes, weights, and the use of tables.

Trigonometry. This includes the trigonometrical functions and the solution of the right triangle.

A course in elementary mechanics is given three hours a week for one and one-half years.

FACTORY SCHOOLS (WERKSCHULEN).[2]

The Werkschulen are schools maintained by various industries for the purpose of furnishing theoretical and practical instruction to their apprentices and other employees. In 1910 there were 75 such schools, with 7,647 students.

There are many varieties of these schools as to the grade and the nature of the instruction. The most of them belong to a group "in which is given along with general practical education, comprehensive trade instruction."

The courses [3] outlined below are those for the machine makers in the Werkschule of the firm of Ludw. Löwe & Co., in Berlin. The length of the apprenticeship of the machine makers is four years, and there are courses in the Werkschule corresponding to each of these years which the apprentices must attend.

CLASS III.

Arithmetic. (One hour.) This includes a review of the fundamental operations, with applications to cost of materials, wages, savings, and insurance.

Geometry. (One hour.) This deals mainly with theorems concerning triangles and quadrilaterals and with constructions.

CLASS II.

Arithmetic (One hour.) There is a review of decimal fractions and the metric system, and then a study of such topics as interest, wages, expenses, cost and transportation of material, and the size, lighting, heating, and ventilation of the workroom.

Geometry. (One hour.) This course includes theorems concerning the circle, areas of polygons, and the Pythagorean theorem.

Algebra. (One hour.) The fundamental operations.

[1] Band IV, Heft 5, pp. 102-103. [2] Ibid., pp. 49-64. [3] Ibid., pp. 59, 60.

Geometry. The number of hours is not given. The principal topics are the areas of quadrilaterals, proportion, division of a line into proportional parts, and similar figures.

Algebra. The number of hours is not given. This includes a study of fractions, proportion and its applications, equations of the first degree, powers, square root, irrational and imaginary numbers, equations of the second degree in one unknown, and applications of these topics to practical problems.

SELEKTA.

Arithmetic and algebra. The number of hours is not given. This is a course in practical calculation and the application of formulas.

EVENING AND SUNDAY CLASSES FOR ARTISANS.[1]

The trade schools so far discussed have been for apprentices. The evening and Sunday classes now under discussion are for men who have finished their apprenticeship and wish to extend the lines of instruction started in the continuation school. In the small cities these classes may be connected with the continuation schools, but in the larger cities they will generally be in separate schools or be connected with middle technical schools.

The student can generally choose whatever course he desires, provided he has the necessary preparation. The tuition is usually very small. As to previous schooling, the student will, as a rule, have completed an eight-year course in the Volksschule and a three-year course in the continuation school.

The methods of instruction are similar to those of the continuation school.

The courses outlined below are offered in the Handwerkerschule in Berlin.[2] Each course seems to be one-half a year in length.

Arithmetic. (Four hours a week.) There is a review of common and decimal fractions with applications to business problems and to the calculation of areas and volumes. The more advanced students are given problems from the trades and the use of mathematical tables.

Algebra. (Three successive courses, each two hours a week.) First course: The fundamental operations and easy equations of the first degree. Second course: Equations of the first degree, with one or more unknowns, positive integral powers, square roots. Third course: Powers, roots, equations of the second degree, logarithms.

Geometry. (Three successive courses, each two hours a week.) First course: Plane geometry—triangle, parallelogram, and circle. Second course: Similar polygons and calculation of areas. Third course: Lines and planes; solids, their surfaces and volumes.

Trigonometry. (Two hours a week.) This includes a study of trigonometrical functions and the calculation of the areas of plane figures.

Higher mathematics. (Two hours a week to each course.) First course: Introduction to analytical geometry, differential and integral calculus.

Mathematical exercises. These are courses which take up the solution of problems from mathematics, physics, mechanics, and the mathematical treatment of problems from practice.

[1] Band IV, Heft 5, pp. 106–128.

[2] Ibid., pp. 115, 116.

Geometrical drawing. (Four hours a week.) This is an elementary course in this subject.

Projection. (Four hours a week.) This includes the representations of solids in various positions, plane and curved sections of solids, and simple cases of the intersections of solids.

Projective geometry. (Four hours a week.) This is a course in the fundamentals of projective geometry with applications.

MIDDLE TECHNICAL SCHOOLS.[1]

Organization and aim.—There is a great variety of middle technical schools in Germany, and the report states that only certain typical ones are chosen for discussion.

Of the middle technical schools, some are State, some are private, and some are municipal schools.

These schools prepare their students to take such positions as heads of technical industries, officials in technical bureaus and in technical departments of the Government, and industrial engineers.[2] The greater part of the graduates go into private industry.[3]

Tuition is charged in these schools.

The official programs of Prussia, Hamburg, Nuremburg, and Wurzburg provide for a course of four half years; those of Bremen, Carlsruhe, and Stuttgart, five; and that of Strassburg, three.

Entrance requirements.[4]—The entrance requirements are, as a rule, the following: The student must have the certificate for one year military service or have completed a certain class in a secondary school; or have completed the Volksschule and a preparatory class in the middle technical school; or take an entrance examination which includes the elements of mathematics and science and free-hand drawing. In all cases one or two years of practical shopwork is required.

The students [5] who have the certificate for one-year military service usually have as mathematical preparation algebra through quadratic equations, and plane geometry, including similar figures and the circle. Some of them have studied logarithms, plane trigonometry, including the solution of the right triangle, and the calculation of the surfaces and volumes of the simpler solids.

The other group of students,[6] those coming from the Volksschule, have had, as a rule, longer practice in the shop and have had considerable technical arithmetic in the continuation school. The elements of algebra and geometry are taught in some of the Volksschulen, and, as we have seen, there are usually courses in algebra and geometry offered in the continuation schools. In certain middle technical

[1] Der math. Unterricht an den deutschen mittleren Fachschulen der Maschinenindustrie, by Dr. Heinrich Grünbaum, being Band IV, Heft 1 of the German Report. Referred to hereafter as Band IV, Heft 1.
[2] Ibid., p. 2.
[3] Ibid., p. 27.
[4] Ibid., p. 34.
[5] Ibid., p. 49.

schools the students coming from the Volksschulen must go into a preparatory class, and in other schools the students from the Volksschulen and those from the secondary schools are put into separate classes.

Aim of the instruction in mathematics.—Mathematics[1] is regarded throughout as a tool. It is simply to furnish the knowledge necessary for the solution of technical problems. The student must be put into a position to be able to attack and solve the problems that he will meet in his work. The choice of subject matter and of methods of instruction must be determined by this end. It does not follow, however, that the instruction is to be purely formal.

"The highest and most important aim of the teaching of mathematics in the technical school will always be to bring up the student to that power and maturity of thought necessary to recognize the quantitative relations affecting the solution of a given practical problem."[2]

COURSE OF STUDY IN MATHEMATICS.

The course of study of the Prussian Higher Maschinenbauschulen is given as an example.[3] The courses in the other technical Fachschulen are said to agree with this in the main. Out of 170 week-hours of instruction, 18 are given to pure mathematics.

"I. *Algebra:* Powers, roots, logarithms, equations of the first degree in one or more unknowns, exponential equations, equations of the second degree in one or more unknowns, arithmetical and geometrical series, compound interest and annuities, convergent and divergent series, binomial theorem for integral, positive, negative and fractional exponents, exponential and logarithmic series, natural logarithms, maxima and minima, graphical solutions of numerical equations, other methods of approximation."

"II. *Plane geometry:* The most important theorems and constructions of elementary geometry; theory of the circle, calculation of the areas of polygons and the circle, proportions in polygons and in the circle, problems of construction."

"III. *Trigonometry:* The trigonometrical functions and their relations, use of trigonometrical tables, calculations of triangles, quadrilaterals, and regular polygons."

"IV. *Solid geometry:* Lines and planes in space, the trihedral angle, the regular solids, volume and surface of the prism, pyramid, cylinder, cone, sphere, frustums, portions of the sphere, and general methods for the calculation of volumes: Guldin's rule, Simpson's rule, applications to the calculation of volumes and weights."

"V. *Theory of curves:* Introduction to coördinate geometry, the straight line, transformation of coordinates, the equations of the circle, ellipse, and hyperbola; the equations of their tangents and normals, particular properties of the conic sections, conjugate diameters, asymtotes, quadrature of the parabola, ellipse and hyperbola, Simpson's rule, parabolas of higher order, cycloids, curvature of curves."

Courses in mechanics and descriptive geometry are also offered. Certain schools offer courses in the calculus. The following courses[4] are given in the Royal Maschinenbauschulen in Cologne.

[1] Band IV, Heft 1, p. 32.
[2] Ibid., p. 37.
[3] Ibid., p. 42.
[4] Programm der Königlichen vereinigten Maschinenbauschulen zu Cöln, 1912, pp. 23–26.

Descriptive geometry. (Ten hours a week, the first half year.)
Calculus. (Two hours a week, the second half year.)
Mechanics. (*a*) (Six hours a week, the first half year.); (*b*) (Six hours a week, the second half year); (*c*) (Six hours a week, the third half year.)

Nature of the instruction.—In discussing the teaching of applied mathematics Ott[1] says that there are two methods used, the lecture method, brought from the university, and the heuristic method, which is used in the elementary and secondary schools. The lecture method, pure and simple, is thought not to be a proper method for students with the preparation of those in the middle technical schools. Because of lack of time, and because the greater part of the material in applied mathematics can not be worked out by question and answer, a purely heuristic method can not be used. Hence the method generally used is a combination of lecturing with questions and answers.

Along with the discussion of theory exercises are given, which are solved in the classroom and copied in the notebooks. Not infrequently the class exercise extends through two 45 or 50 minute periods. In many cases the students are in the classroom or the workshop practically all day. Hence there is little opportunity for outside study, and all theory as well as applications must be worked out in the classroom.

The reports recommend that the presentation of theory be made simple and concrete, and be reenforced by many examples. While the students must know and understand the fundamental theorems, it is not thought necessary that all the formal proofs be given. It is recommended that, in applied mathematics at least, use be made of models and other similar aids.

"We have pointed out that there must be developed first of all in the technical school skill and ability to do."[2] This aim determines the choice of subject matter and of method. It is said that in the choice of subject matter the only question to be raised is, "What parts of mathematics are necessary or important for the study of technology, and moreover can be taught in an elementary way suitable to the development of the student?"[3] "What the technologist can not use will be unhesitatingly omitted."[4]

Methods of approximation form an important topic in mathematics, as these often give the best methods for the solution of problems, both because of brevity and because the limited mathematical knowledge of the students makes it impossible to develop a complete theoretical treatment.

[1] Die angewandte Mathematik an den deutschen mittleren Fachschulen der Maschinenindustrie, by Dipl.-Ing. Karl Ott, p. 6. This is Band IV, Heft 2 of the German Report.
[2] Band IV, Heft 1, p. 36.
[3] Ibid., p. 41.
[4] Ibid., p. 35.

Grünbaum[1] criticizes the teaching of algebra for not doing more with the notion of a function, which is of great importance in applied mathematics.

Graphical methods have at present, and have had for a long time, an important place in the teaching of mathematics in the middle technical schools.

The slide rule is much used in some institutions, but apparently not much used in others.

Preparation of teachers.—In speaking of the instruction in applied mathematics, Ott says:[2] "At present the instruction in this field is with increasingly few exceptions in the hands of engineers, and there appears to be no prospect that this condition will be essentially changed."

The proportion of engineers among the teachers of pure mathematics is probably not so great, as Grünbaum says that the question of the course of preparation for the teachers in the technical schools is not settled.[3] At present the instruction in pure mathematics is given by engineers and mathematicians. The first may lack theoretical preparation and good methods of instruction, and the second may be deficient in technical knowledge and experience. There has been some discussion of special instruction in the technical high schools for teachers in the middle technical schools, and also of requiring a year of practice teaching; but present opinion is not in favor of these movements.[4]

Books.—There is no lack of texts in mathematics for the middle technical schools, and they are of various grades of excellence as to presentation of theory and choice of problems.[5] Many of these books show the influence of the secondary schools. In not a few cases the books prepared for the technical schools are merely revisions of books written for the secondary schools, and contain more than is necessary of the purely formal exercises of the old books, as well as large numbers of conventional problems that have no technical applications.

NAVIGATION SCHOOLS.[6]

Organization.—The report on the navigation schools has to do with the schools which prepare pilots and seamen for ocean navigation.

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[1] Band IV, Heft 1, p. 68.
[2] Band IV, Heft 2, p. 24.
[3] Band IV, Heft 1, p. 93.
[4] Ibid., p. 99.
[5] Ibid., pp. 50–65.
[6] Der mathematische Unterricht an den deutschen Navigationsschulen, by Dr. C. Schilling and Dr. H. Maldau. This is Band IV, Heft 4, of the German Reports. Referred to hereafter as Band IV, Heft 4.

When the report was written (1911) there were 18 such schools[1] in Germany, of which 12 were in Prussia, 1 in Oldenburg, 2 in Mecklenburg, and 1 in each of the Hanseatic cities of Hamburg, Bremen, and Lubeck. All of these are State schools, except one in Mecklenburg, which is under the control of the city in which it is located. There are, furthermore, 6 Prussian preparatory navigation schools. There are no private navigation schools, properly speaking, but some candidates do get their preparation for taking the seaman's examination from private sources.

Aim of the schools.—The chief aim of the schools seems to be to prepare for examinations the pilots and seamen who will engage in ocean navigation. The following outline of the requirements for these examinations shows what is, apparently, the extent of the mathematical instruction.

COURSE OF STUDY IN MATHEMATICS.[2]

Algebra. The examination covers the fundamental operations in arithmetic with common and decimal fractions, equations of the first degree, powers, roots, and logarithms.

Plane geometry. This includes the simpler theorems about angles, congruence, similarity, and equality of polygons, the simpler theorems about the circle, simple constructions, and easy computation of areas.

Solid geometry. This includes the simpler theorems about lines and planes in space, and about the sphere and spherical triangles; and the computation of the volumes of prisms, cylinders, and casks, and of the ship's hold by Simpson's rule.

Plane trigonometry. This covers the simplest relations of the trigonometrical functions and the solutions of triangles.

Spherical trigonometry. (Required only of seamen *auf grosser Fahrt.*) This includes the solution of spherical triangles.

Methods of teaching.—The authors of the report would have the geometry[3] made intuitional as far as possible, proving only those important theorems which do not appear at once evident. There should be many applications to nautical problems. The rules for the computation of areas should be taught in the simplest concrete way.

The trigonometry seems to be taught much as in other schools, except that the development is simpler, and special emphasis is placed on the parts which are of the greatest importance to navigation.

"In contrast to the technical schools, drawing and methods of graphical solution are in the background in the navigation schools."[4] The reasons given are that the students in the navigation schools

[1] Band IV, Heft 4, p. 19.
[2] Ibid., pp. 10–12.
[3] Ibid., pp. 35–36.
[4] Ibid., p. 42.

lack previous practice in drawing, and that conditions on board in small, poorly lighted cabins are not favorable to accurate drawing.

It seems that little is done with the notion of a function except in trigonometry. The authors of the report favor a more extended use of this idea, and give some examples the solution of which would be simplified by its use.

Preparation of teachers.—"To-day [1] the question of the preparation of the teachers for the navigation schools has not found a thoroughly satisfactory answer." The problem is to get teachers with the proper academic and practical training; and because of the small number of teachers in navigation schools, there is no prospect that a university or technical school will take up the problem of preparing these teachers.

In the schools outside of Prussia part of the teachers are academically prepared and part are seamen. In general, the first class have had experience of a year or more in navigation and the second have received their mathematical preparation by private study or by attending a university or technical high school.

"In the Prussian navigation schools there are employed at present, besides 3 directors, 29 navigation-school teachers, 17 preparatory-school teachers, and 5 candidates. It is a general principle here that there will be given positions as teachers only former seamen who have passed the examination for pilots and sailors *auf grosser Fahrt*. There are no further requirements as to school preparation." [2]

An examination is generally necessary before a position as teacher in a navigation school can be obtained.

Examinations.—As is mentioned above, the students attend the navigation schools to prepare for the nautical examinations. Since 1870 these examinations have been uniform in the German Empire. They have also, according to the report, become stereotyped, and are criticized by many teachers in the navigation schools, who say that before 1870 the schools were interested in teaching the students something, but now are forced to prepare the students to pass the examinations. "If, however, this criticism can not be truly denied, yet there is no doubt that on the whole the introduction of the new system of examinations has a good influence on the preparation of the German seamen." [3]

Books.—In many schools no books are used. In some schools *Aufgabensammlungen* or *Leitfaden* that are specially prepared for the navigation schools are used.

Modern tendencies.—There is some discussion of the question of requiring attendance of the navigation schools from those who take the nautical examinations.

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[1] Band IV, Heft 4, p. 49. [2] Ibid., p. 55. [3] Ibid., p. 17.

The authors of the report wish a change in the examinations, so that they will not be so stereotyped, and so that they will require more thorough preparation on the part of the candidates.

Modern improvements in ship construction and equipment, and the use of modern instruments also, require changes in the school instruction.

GREAT BRITAIN.

Some of the most noteworthy work that has been done in recent years in the improvement of the teaching of mathematics, especially in technical schools, has been done in England. The effect of this work has been felt in many other countries. The reports of the British Subcommission do not, however, give a sufficiently detailed account of the teaching of mathematics in the vocational schools to make it possible to discuss the subject here for Great Britain as is done for other countries. But it has been thought useful to give such an account of present tendencies as is furnished by the "Memorandum on the Teaching of Engineering in Evening Technical Schools," published by the board of education.

This memorandum does not attempt to give an account of present conditions. Its aim may be seen in the following extract from the prefatory note.[1]

> The following memorandum has been drawn up with the object of furnishing suggestions to teachers and organizers of schools which provide evening classes in mechanical and electrical engineering. It is not in the least intended to lay down a scheme of instruction suitable for universal application; it is obviously necessary and desirable that there should be great variety both in methods of teaching and in organization to meet the needs of different types of students and the varying industrial conditions of different areas. Further, the last thing which the board desires in making these or any other suggestions is to fetter the liberty of the teachers or discourage individuality in teaching. The object of the memorandum is simply to assist teachers and organizers to work out for themselves the schemes of instruction best suited to the conditions of their classes.
>
> The memorandum has been prepared by a number of the board's inspectors, many of whom have had recent experience as teachers.
>
> *Organization.*—A complete curriculum [2] of evening instruction falls naturally into three stages, which may for administrative purposes be classified as follows: (*a*) The junior course; (*b*) the senior course; (*c*) the advanced course.

The junior course usually occupies two years and is intended for boys who leave the public elementary schools at the age of 14. It gives a general preparation for technical courses of all types, in particular for the senior course, and comprises instruction in mathematics, drawing, science, and English.

It is assumed that all students entering a senior course have had the equivalent of the junior course. The senior course normally

[1] P. 4. All references are to the above memorandum. [2] P. 6.

occupies a period of not less than three years, with attendance of three evenings a week.

The advanced course includes work of a more advanced and specialized character and normally extends over at least two years.

For purposes of administration it is convenient to speak of a major course and a minor course. A major course will usually extend over not less than four years and will include senior and advanced courses. A minor course will probably be completed in two or three years, so that it will often consist of a senior course only.

"The present [1] memorandum deals in detail with the engineering courses which are subsequent to the junior course."

Nature of the instruction.—The minor course is intended primarily to help workers to a more intelligent understanding of their craft. The practical side of the instruction is the important one. It is desirable, but not necessary, that the teachers have technical training.

The major course is intended for persons who expect to become foremen, designers, heads of departments, and the like. "Its [2] curriculum will center around the scientific basis of engineering and will include the training in mathematics and drawing which is necessary for sound progress." "Teachers in the advanced course will be specialists in one or other of the branches into which engineering is divided and will probably confine themselves to their special subjects."

The work of the first three years [subsequent to the junior course] may be planned on lines which are generally similar for each year, while the later treatment will usually be on more specialized lines.

In each of the earlier years the work should be organized to include the treatment of a number of branches or aspects of the subjects, which may be roughly classified as follows: (i) Mensuration; (ii) manipulation of algebraic expressions, evaluation of formulæ and solution of equations; (iii) the idea of functionality or the dependence of one quantity upon another, and the use of graphs; (iv) trigonometry; (v) variation of functions leading up to the calculus.

The aim of teaching should be to develop a habit of dealing quantitatively with material objects and physical phenomena, and of expressing the results symbolically and in graphical form; also to impart the power to interpret the relationship between quantities when so expressed.

COURSE OF STUDY IN MATHEMATICS.[3]

SENIOR COURSE. FIRST YEAR. (THE STUDENT'S ELEVENTH SCHOOL YEAR.)

(*a*) *Arithmetic and mensuration.* Review of the work of the junior course. Rules for areas and volumes. Checks.

(*b*) *Logarithms.* Use of common logarithms.

(*c*) *Measurement of angles.* Notion of an angle. Construction of angles with ruler and compasses. Radian. Sine, cosine, and tangent defined and computed from measurements. Variation and graphs of these functions from 0° to 90°. Solutions of right triangles.

(*d*) *Graphs.* Finding the areas of irregular figures by the use of the midordinate rules. Graphs of statistics and of familiar functions.

[1] P. 9 [2] P. 6. [3] Pp. 30-33.

(e) *Algebra.* Solution of simple equations in one or two unknowns. Factoring. Simple examples in indices, and much practice in simplifying expressions and in solving formulas for any letter involved.

(f) *Slope of a curve.* The notion of the slope of a curve and its measurement.

(g) *Practical work.* The use of rulers, scales, micrometers, calipers, planimeter, spring balances, and weighing scales. Much measuring of the usual geometrical solids and of such objects as washers, rings, plates, disks, wires, bars, rods, pipes, and short lengths of standard rolled sections. Drawing to scale.

SENIOR COURSE. SECOND YEAR. (THE STUDENT'S TWELFTH SCHOOL YEAR.)

(a) *Mensuration.* More advanced work. Irregular plane and solid figures.

(b) *Slide rule.*

(c) *Logarithms.*

(d) *Trigonometry.* Review. Solution of triangles by reducing them to sums or differences of right triangles. Projections of lines and surfaces.

(e) *Vectors.* Introduction, and simple examples in addition and subtraction.

(f) *Algebra.* Review and extension of the work of the previous year. Quadratic equations. Simple exercises in the graphical solution of equations.

(g) *Graphs.* "More practice in plotting graphs should be given. This should lead to exercises involving interpolation, the calculation of average rates of increase, the determination of areas, maxima and minima values, the finding of probable laws connecting varying quantities, and the solution of equations."

(h) *Calculus.* The notion of a derivative with simple examples.

ADVANCED COURSE. FIRST YEAR. (THE STUDENT'S THIRTEENTH SCHOOL YEAR.)

(a) *Mensuration.* "More advanced work may be taken in mensuration, including the theorems of Guldinus, and a number of exercises on the areas and volumes should be given."

(b) *Trigonometry.* Trigonometric functions for any angle, the most useful formulas, and the solution of triangles.

(c) *Algebra.* "The work in algebra may include further exercises in the solution of quadratic equations, and the roots of such equations; further practice in manipulating expressions containing indices; the simplification of fractional expressions; products of binomial expressions, including the consideration of approximations."

(d) *Graphs.* Plotting of certain useful types of curves such as $pv^n = constant$, $y = ae^{bx}$, $y = a \sin(x+c)$, $y = a \sin(x+c) + d \sin(x+e)$.

(e) *Calculus.* The notion of a derivative, and of an indefinite and a definite integral, and simple applications. Much care is to be taken to make the ideas clear.

ADVANCED COURSE. SECOND YEAR. (THE STUDENT'S FOURTEENTH SCHOOL YEAR.)

(a) *General.* The greater part of the time may be spent on the calculus and its applications, and some time on exercises in the binomial theorem and with trigonometric formulas.

(b) *Differential calculus.* "Differential coefficients for standard functions; differentiation of products and quotients of functions and of the functions of a function; partial differentiation; Taylor's and Maclaurin's theorems; exponential, logarithmic, and Fourier's series and their applications; curvature; construction of curves from law of slope."

(c) *Integral calculus.* "Standard integrals; integration by parts and by substitution; graphical methods of integration; simple differential equations."

(d) *Engineering applications.* "It is recommended that the teacher of mathematics cooperate with the teacher of engineering and that applications of the calculus be made to such topics as areas, volumes, velocities, accelerations, vibrations, centers of pressure, centers of gravity, moments of inertia, theory of beams, thermodynamics, and problems in electrical engineering."

(e) *Plane analytical geometry.* If time permits, it is recommended that the equations of the conic sections be studied.

HOLLAND.

COMMERCIAL AND INDUSTRIAL SCHOOLS.

The lower and middle vocational schools discussed in the report from Holland [1] are the "Burgeravondscholen," the vocational schools (écoles professionelles), schools of design (écoles de dessin), technical schools, and vocational schools for girls. The first three kinds have the same course of study in mathematics and can be considered together. There is little or no instruction in mathematics in the schools for girls.

BURGERAVONDSCHOLEN, SCHOOLS OF DESIGN, VOCATIONAL SCHOOLS, AND TECHNICAL SCHOOLS.

Organization.—The Burgeravondscholen are evening schools organized by the communes, but in some cases partially supported and supervised by the Government. Their aim is to give theoretical instruction to persons going into commerce and industry. The course is from two to four years in length, with from 6 to 12 or 15 hours of instruction a week.

Schedule of hours per week of a Burgeravondschole of four years.

Classes.	Geometrical drawing.	Free-hand drawing.	Technical drawing.	Mathematics.	Physics.	Mechanics.	Dutch.
First	5	3	.....	4	1	.....	2
Second	.....	2	6	4	(1)		2
Third	.....	2	7	3	(2)		1
Fourth	.....	2	8	2	(3)		1

"For the most part there are no examinations for admission to these schools.[2] A declaration by the head of the primary school that the student has followed the primary instruction with success is sufficient." This statement applies also to the schools of design and to the vocational schools.

The schools of design are "in the main the same as the Burgeravondscholen;[3] the course of study is from three to five years in length and from 8 to 15 hours of instruction are given a week." The most of these schools have been founded by private societies, but in some communes there are schools of this kind which receive some support from the State or the Province.

The vocational and technical schools give both theoretical and practical instruction. The most of these schools give a course of three years with from 40 to 44 hours of instruction a week. These

[1] Rapport sur l'Enseignement mathématique dans les Pays-Bas, publié par la Sous-Commission nationale, pp. 11–29. Referred to hereafter as Rapport.
[2] Ibid., p. 20.
[3] Ibid., p. 14.

schools are in session throughout the year, except five or six weeks. The following schedule of hours a week is given.[1]

Schedule of hours per week of vocational and technical schools.

Classes.	Practice.	Geometrical and technical drawing.	Freehand drawing.	Mathematics.	Dutch.	Physics and mechanics.
First	24	9	4	4	2	1
Second	26	8	3	3	2	2
Third	27	8	3	3	1	2

COURSE OF STUDY IN MATHEMATICS.[2]

The following is the work usually given in the vocational schools and in the Burgeravondscholen:

Arithmetic. (First year.) The subjects studied include the fundamental operations, the metric system, foreign weights and measures, ratio and proportion, percentage, square root, and applications.

Algebra. (First year.) Fundamental operations.

Second year. Review of the fundamental operations and equations of the first degree in one unknown.

Third year. Linear equations in two unknowns and the solution of problems.

Geometry. (First year.) Rectilinear figures, areas, similar figures, and problems.

Second year. The circle, regular polygons, the ellipse; planes, and regular polyhedrons.

Third year. Prisms, pyramids, cylinders, cones, sphere; calculation of surfaces and volumes.

Mechanics. (Third year.) Elementary mechanics.

Descriptive geometry. "In the teaching of that branch we make as many applications as possible, relating them to the specialty of the student."[3] Many of the purely mathematical examples of the books are discarded and practical applications are substituted.

Methods of instruction.—The above outline of the work in mathematics shows no distinctly vocational material. The examination questions given in the report as examples of those given at the conclusion of the course in the Burgeravondscholen are of the usual textbook type.

In some schools models are used in the instruction. As most of the teachers of mathematics come from the elementary schools, the methods of instruction of those schools are probably used in the vocational schools. There appears to be a tendency to make the instruction more practical, and, in particular, to unify the theoretical development of a topic and its vocational applications.

Preparation of teachers.—Some of the teachers are engineers, but the most of them are elementary school teachers.

[1] Rapport, p. 15. [2] Ibid., pp. 18, 19. [3] Ibid., p. 19.

MIDDLE TECHNICAL INSTRUCTION.

"By middle technical instruction is meant that which is necessary for those who are to be the intermediaries between the employer, the engineer, etc., and the worker."[1] Instruction of this kind in some trades is given in certain Burgeravondscholen. There is but one school devoted entirely to middle technical instruction, the middle technical school of Amsterdam, which at the time the report was written was being reorganized. The courses in this school are specially for machinists, electricians, and in general, for those who are to be engaged in the metal industries. No course of study is given for this school.

It is said that a middle school for architects will soon be opened in Utrecht.

NAVIGATION SCHOOLS.[2]

Three such schools are named, two in Amsterdam and one in Rotterdam. The entrance requirements for one school are the same as for the fourth year, and for the other two the same as for the second year of the middle school, thus requiring respectively nine and seven years of previous schooling. The age of entrance is from 14 to 17 years, except that in the case of candidates for positions as pilots there is no age limit, and they need have only an elementary school education (six years of schooling).

Aim of the instruction.—The aim of these schools is to educate captains, officers, and pilots for the merchant marine, and masters and pilots of fishing vessels. "The teaching of mathematics has a tendency essentially practical."[3]

COURSE OF STUDY IN MATHEMATICS.[4]

Thirteen hours a week are given to mathematics and the sciences of navigation, and two hours a week to physics in each of the two years.

FIRST YEAR.

Plane trigonometry. This course includes the elements of plane trigonometry with some applications to practical geometrical problems.

Solid geometry. Enough of solid geometry is studied to enable the student to compute volumes, and to understand spherical trigonometry and the coördinate systems of astronomy.

Spherical trigonometry. This course includes the solution of spherical triangles and applications to problems of navigation.

Analytical geometry. The elementary properties of conics are studied.

Navigation. In this course are taken up the usual problems of navigation, such as calculation of direction and distance, determination of the place from observation, and navigation along a great circle.

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[1] Rapport, p. 10. [2] Ibid., pp. 33–37. [3] Ibid., p. 35. [4] Ibid., pp. 34, 35.

SECOND YEAR.

Astronomy and navigation. This course includes the following: Determination of time and place at sea; influence of errors of observation; instruments of navigation; compass variations; tides.

Examinations.—The examinations for pilots for ocean navigation cover about the material given above in arithmetic, algebra, geometry, trigonometry, astronomy, and navigation.

SCHOOLS FOR FISHERMEN.

There are schools for fishermen which are more elementary than the schools of navigation just mentioned. "As to the schools for fishermen, they are of very recent date.[1] The students in these schools have generally only the education of the elementary schools, which is such that the instruction in mathematics and navigation must be very simple."

HUNGARY.

HIGHER COMMERCIAL SCHOOLS.[2]

Organization and aim.—In the year 1911–12 there were 51 of these schools, of which 20 were supported by the State, 11 by societies, 15 by cities, 3 by religious denominations, and 2 by private parties. Five of these schools are for girls. Students are admitted if they have completed the fourth year of a Gymnasium, Realschule, or a Bürgerschule, and are 14 or 15 years of age. The course is 3 years in length. The completion of the course in a higher commercial school admits the student to a maturity examination, the certificate from which gives the right of one-year volunteer military service.

The particular problem of these schools is said to be the training and education of able tradesmen and commercial officials.

COURSE OF STUDY IN MATHEMATICS.

The following is the schedule of hours a week:

Hours per week in mathematics in higher commercial schools.

Years.	Commercial arithmetic.	Mathematics.
First year (the student's ninth school year)	4	2
Second year (the student's tenth school year)	3	2
Third year (the student's eleventh school year)	4	2

Arithmetic. The subject matter is said to be carefully chosen and to be practical. No outline of the subject matter is given.

[1] Rapport, p. 37.

[2] Der mathematische Unterricht an den Handelsschulen, by Max Havas and Samuel Bogyó, pp. 3–7.

Algebra. The principal topics studied are equations of the first degree in one or two unknowns, equations of the second degree, arithmetical and geometrical series, combinations, probability, compound interest, annuities, loans, lotteries, and the fundamentals of life insurance.

Geometry. Enough of geometry is taught so that the students can compute the surfaces and the volumes of the simple solids.

Preparation of teachers.—Dissatisfaction is expressed with regard to the teachers. Teachers of other subjects are often required to teach the commercial arithmetic.

The preparation of the regular professors in the higher commercial schools is usually made in a school for commercial school professors (Handelsschulprofessoren-Bildungsanstalt) which is closely connected with the commercial academy in Budapest.[1]

Most of the students enter this school for commercial teachers with the maturity certificate from a higher commercial school and take a course of four years. Then follow examinations and one year of practice teaching before a teacher's certificate is granted.

Recommendations for improving the instruction.—It is said that the work in mathematics can not be done satisfactorily in the time now given to it, and it is recommended that the course be extended to four years. If this extension is made, it is thought that the results of the work in mathematics will be much improved.

INDUSTRIAL AND TRADE SCHOOLS.[2]

Organization.—The Hungarian report discusses two kinds of industrial schools, the *höhere Gewerbeschulen*, which will be called here "higher industrial schools," and the *gewerbliche Fachschulen*, which will be called "trade schools." When the report was written (1912) there were in Hungary 4 higher industrial schools and 21 trade schools. These schools are under the ministry of commerce.

These schools have similar aims. They have the problem of preparing skilled workmen and minor official for technical undertakings. The principal difference is that the graduates of the higher schools have a better social position and become, in the main, technical officials, while the graduates of the trade schools find positions as foremen and master workmen.

Entrance requirements.—In order to enter the higher industrial school, the student must have finished the fourth class of a Mittleschule or of a Bürgerschule and have had one year's practice in his chosen trade. The students are in general 16 years old when they enter the school.

In order to enter the trade school, the student must be 13 years of age, and have completed the sixth year of a Volksschule or the second year of a Mittelschule or of a Bürgerschule.

[1] Der mathematische Unterricht an den Handelsschulen, by Max Havas and Samuel Bogyó, pp. 12, 13.
[2] Der math. Unterricht an den höheren Gewerbeschulen und gewerblichen Fachschulen, by Daniel Arany.

COURSE OF STUDY IN MATHEMATICS IN THE HIGHER INDUSTRIAL SCHOOLS.

The following program is in force in the two schools in Budapest, and the differences between it and the programs in the other schools are noted.

Algebra. The course includes the fundamental operations, equations of the first degree in one or more unknowns, powers and roots, logarithms, equations of the second degree in one unknown, arithmetical and geometrical series, compound interest and annuities.

In both the other schools exponential equations are included; and the program of the school in Szeged contains also the binomial theorem, equations of the second degree in more than one unknown, and approximate solutions of equations of higher degree.

Geometry. The principal topics in plane geometry are the angle, triangles, the principal properties of quadrilaterals, polygons, the circle, similar figures, and the computation of the surfaces of polygons and the circle.

The work in solid geometry includes the principal properties and the computation of the surfaces and volumes of the prism, pyramid, cylinder, cone, and sphere.

Trigonometry. This course gives the fundamentals of goniometry and the solution of triangles.

Constructive and descriptive geometry. This course begins with a review of the elementary constructions of plane geometry and goes so far as to include the representation of the prism, pyramid, cylinder, cone, and sphere in different positions by orthogonal projection and parallel perspective, and their development and plane sections. The most important cases of the intersections of solids are taken up.

COURSE OF STUDY IN MATHEMATICS IN THE TRADE SCHOOLS.

The official course of study for the trade schools contains the following work in mathematics.

The following work is outlined for all the trade schools except those for watchmakers and certain other highly skilled mechanicians (*Feinmechaniker*):

First year. (The student's seventh school year.) (Two hours a week.) The course includes the fundamental operations with integers, common fractions, decimal fractions, factoring, squares and square root, the metric system, and the finding of areas and volumes.

Second year. (The student's eighth school year.) (Two hours a week.) This year the students study ratio, proportion and its use in the solution of problems, percentage and its applications, partnership, alligation, and exchange.

Third year. (The student's ninth school year.) (One hour a week.) This is a course in commercial arithmetic, including problems from industrial life, exchange, discount, revenues, stocks, and bonds.

The watchmakers and *Feinmechaniker* have the following courses:

First year. (The student's seventh school year.) (Four hours a week.) *Algebra and geometry.* The course in algebra, which also includes a review of arithmetic, involves a review of the fundamental operations with common and decimal fractions, powers and roots, equations of the first degree in one or more unknowns, simple and compound proportion, and the solution of practical problems.

The principal topics studied in geometry are the measurement of lines and angles, the congruency and similarity of triangles, quadrilaterals, polygons, the circle, the ellipse, and spirals.

Second year. (The student's eighth school year.) (Two hours a week.) *Algebra* and *geometry*. The subjects taken up in algebra are logarithms and their applications to problems in watchmaking, and the solution of practical problems.

The course in geometry includes a study of the cube, prism, cylinder, pyramid, cone, and sphere, and the computation of their surfaces and volumes. Some work in plane trigonometry is given in this course.

Third year. (The student's ninth school year.) (Two hours a week.) *Arithmetic, algebra*, and *geometry*. Some simple commercial arithmetic is given, including percentage, interest, and exchange.

The topics studied in algebra are equations of the first and second degree in one and two unknowns, and simple cases of the binomial theorem.

The applications of trigonometry to watchmaking are discussed.

There is a course in geometrical drawing and descriptive geometry in the first year in all the trade schools. The time given to this course is three hours a week in the schools for the textile industries and six hours a week in all other schools. A course of two hours a week in these subjects is given in the second year in the schools for the building trades.

Methods of instruction.—The time given to a subject is divided officially into lecture hours and examination hours. During the first the professor lectures without interruption, and in the last he gives tests to see if the students have mastered the lectures. There are, furthermore, certain review hours in mathematics, which are devoted to the solution of practical problems.

Examinations.—There are reviews of a month in length at the end of each semester. These, with the examination hours mentioned above, include everything that is given in the way of examinations.

Preparation of teachers.—There is no special preparation for teachers in industrial schools. An engineer's diploma is required of the teachers of most subjects. Mathematics may be taught either by engineers or those prepared to teach in the secondary schools.

ITALY.

INDUSTRIAL SCHOOLS.

The part of the Report [1] of the Italian Subcommission that deals with the industrial and commercial schools gives a brief account of the organization of these schools, but gives no outlines of the courses of study in mathematics except in the case of the commercial schools.

The following table [2] gives the kinds of commercial and industrial schools and the number of each. These statistics are for the school year 1908:

Schools of agriculture:	Number.
Practical	28
Special	7
Higher	3

[1] Commissione internazionale dell'insegnamento matematico, Atti della Sottocommissione italiana: L'Insegnamento della matematica nelle scuole industriali professionali e commerciali, by G. Lazzeri. Referred to hereafter as Lazzeri.

[2] Ibid., p. 6.

	Number.
Schools of mining	8
Industrial schools:	
Arts and crafts	47
Professional	19
Industrial schools proper	19
Schools of industrial art:	
Of design	112
Intermediate	87
Higher	7
Commercial schools:	
Lower	34
Intermediate	12
Higher	5
Vocational schools for women	29

All of the above schools are under the control of the minister of agriculture, industry, and commerce. These schools are of many types. Full freedom is given to the development of these schools, so that they may best meet the needs of the communities in which they are located.

SCHOOLS OF AGRICULTURE.[1]

The practical schools of agriculture give practical instruction in farming. The students prepare for such positions as that of vine-dresser.

The condition for admission is the possession of the certificate of maturity from the elementary school or passing an examination which covers the same ground. It should be said that this certificate of maturity, to which references are frequently made in this report, may be obtained at the end of the fourth year of the elementary school by passing an examination.

A certificate is granted at the completion of the course in the practical schools.

The special schools of agriculture have a lower and a higher course. The admission to the lower course is the same as to the practical schools. The certificate of the lower course admits to the higher course.

The graduates of the lower course obtain positions as skilled laborers, and those of the higher course as superintendents.

The higher schools of agriculture require for admission the diploma of a secondary school, or equivalent preparation. The aim of these schools is to give the students such knowledge of agriculture as will enable them to take charge of large farms, to teach agriculture, and to promote agricultural research. At the end of the regular course the degree of doctor of agriculture is granted.

Course of study in mathematics.—The instruction in mathematics in the first two classes of schools is limited to arithmetic and elementary

[1] Lasseri, pp. 7, 8.

geometry. In the higher schools the work in mathematics is extended to some further applications in mechanics and practical geometry.

SCHOOLS OF MINING.[1]

The admission is by examination. The aim is to train mining experts and head mechanics.

The course in mathematics includes algebra, elementary geometry, descriptive geometry, and in some schools theoretical and applied mechanics.

A diploma is given at the end of the course.

INDUSTRIAL SCHOOLS.[2]

The schools of arts and crafts require for admission the certificate of maturity of the elementary school. Their aim is to prepare skilled artisans. They give a diploma which certifies to the ability of the graduate in certain arts and crafts. This certificate sometimes gives admission to the higher courses in the industrial schools.

"The professional schools belong to the preceding type, but are superior, as they furnish more complete training, both theoretical and practical."

The industrial schools, "properly so called," are of a higher grade than the preceding. They require for admission to the superior course the equivalent of the diploma of the schools of arts and crafts. They correspond, apparently, to the middle technical schools of other countries.

SCHOOLS OF INDUSTRIAL ART.[3]

The instruction in the schools of design seems to be limited to drawing and art. The lessons are given in the evenings and on holidays.

The intermediate schools are of a higher grade than the preceding. Some of them are said to be devoted to particular industries, as coral cutting. Some instruction in elementary mathematics is given.

The higher schools aim to turn out skillful artisans who will devote themselves to such trades as watchmaking, china making, the graphic arts, carving, and the like. Something of the theory of shades and shadows and of the theory of perspective is taught.

VOCATIONAL SCHOOLS FOR WOMEN.

The vocational schools for women require for admission the certificate of maturity of the elementary school. These schools prepare their students to take various positions, as cooks, nurses, designers, and also such positions as clerks, cashiers, and in the mail and telegraph service.

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[1] Lasseri, p. 8. [2] Ibid., pp. 8, 9. [3] Ibid., pp. 9, 10.

The course in mathematics includes arithmetic and the elements of geometry.

COMMERCIAL SCHOOLS.[1]

In order to enter the lower commercial schools the student must possess the certificate of maturity of the elementary school. The aim of these schools is to train clerks and accountants. The instruction in mathematics is limited to arithmetic, elementary algebra, and intuitional geometry.

The requirement for admission to the intermediate commercial schools is the possession of a diploma of the technical school, or one from a gymnasium or a lower school of commerce, or the passing of an examination.

The aim of these schools is to prepare men to take responsible commercial positions. A diploma is granted at the end of the course.

The study of mathematics extends through two, three, or four years according to the school. The instruction is planned to prepare the student for the study of the mathematics of finance.

As an example of the work done in mathematics in the intermediate commercial schools the report gives the following, which is offered in the R. Scuola Media di studi applicati al commercio of Florence. This school has a course of four years.

First year. This year's work includes arithmetic, elementary algebra, and the elements of geometery.

Second year. This course is called *matematica finanziaria*, and includes arithmetical and geometrical progressions, logarithms, compound interest and annuities, amortization, elements of the theory of probability, and insurance.

The higher commercial schools are higher institutions of learning, and grant diplomas of the same grade as the universities.

JAPAN.

COMMERCIAL SCHOOLS AND COLLEGES.

COMMERCIAL SCHOOLS.

The commercial schools are State schools. There are two classes of them, class A and class B. The following discussion applies to both classes except as otherwise indicated.

Organization.—Class B. To enter the schools the student must be at least 12 years of age and be a graduate of an elementary school. The average age of the students in the first class is 13. The course is three years or less.

Class A. To enter a school of class A the student must be at least 14 years of age and be a graduate of a higher elementary school, or

[1] Lasserl, pp. 10, 11.

have equivalent preparation. The average age of entrance is 15. The following courses of study are offered:

Preparatory course 1 year; principal course 3 years.
Preparatory course 2 years; principal course 3 years.
Preparatory course 2 years; principal course 4 years.
Preparatory course 0 year; principal course 5 years.

The school year is divided into three terms.

Aim of instruction.—The aim of instruction "is the training in the practical use of commercial calculations, and the development of mathematical mental power." [1]

COURSE OF STUDY IN MATHEMATICS.[2]

Class B. Arithmetic. This course includes mental, soroban, and written calculation, compound numbers, fractions, proportion, percentage, and commercial calculations. Arithmetic is usually given four or five hours a week throughout the course.

The following courses are given only in schools of class A:

Arithmetic. (Usually given in the preparatory course.) The principal topics are integers and decimals, compound numbers, measures and multiples, fractions, ratio and proportion, percentage, square and cube root, and mensuration.

Commercial arithmetic. This includes rapid calculation, money, systems of measures, various applications of percentage, exchange, invoices, accounts, and annuities.

Algebra. Algebra through quadratics, square and cube root, proportion, progressions, logarithms.

Geometry. A study is made of straight lines, rectilinear figures, circles, proportion, and solids.

In some schools there is less algebra and geometry than is outlined, and in some schools there is no geometry.

Methods of instruction.[3]—The methods of instruction are said to be similar to those of the middle schools, but special emphasis is placed on mental, soroban,[4] and written calculations, and upon applications. There is general use of geometrical models, weighing machines, and of Japanese, English, and metric measures. Textbooks are used.

Examinations.[5]—Besides the annual examination there are, in some schools, term examinations and tests at irregular intervals. The mark for the year depends upon the examinations and the daily marks.

Preparation of teachers.—There is a commercial teachers' training institute in connection with the Tokyo Commercial College. The course there is four years. The course of study in mathematics is given below.

[1] The Teaching of Mathematics in Japan: Art. XI, The Teaching of Mathematics in Commercial Schools and Colleges, p. 12. Referred to hereafter as Art. XI.
[2] Ibid., pp. 15–17. The courses are not outlined by years.
[3] Art. XI, p. 18.
[4] The soroban is a kind of abacus.
[5] Ibid., p. 18.

COMMERCIAL COLLEGES.

Organization.—Six of these commercial colleges are private, five Government, and one municipal (Osaka). In order to enter these schools the students must be 17 years of age and be graduates of a middle school or of a commercial school of class A or have equivalent scholarship. Admission is either by examination or by fulfilling the conditions for free admission in the different colleges. The average age of students in the first-year class is 20. Two of the Government colleges have courses of four years. The others have courses of three years.

Aim of instruction.—Besides a general educational aim, the "principal aim of mathematical instruction is to impart knowledge of the principles and methods of the calculations required in the different branches of commerce, and to drill the students in practical accounting." [1]

COURSE OF STUDY IN MATHEMATICS.[2]

This is the program of the Tokyo Commercial College, which has a course of four years. After the second year the course is given in English.

First year. (The student's twelfth school year.[3]) (Three hours a week.) Mental and soroban calculation, foreign weights, measures, and moneys, percentage, interest, logarithms, summation of series, probability, graphs of functions.

Second year. (The student's thirteenth school year.) (Two hours a week.) Business arithmetic, including short cuts and approximations, percentage, interest, discount, equation of payments, current accounts, alligation, taxes, freight, customs, and similar topics.

Third year. (The student's fourteenth school year.) (Three hours a week.) Monetary standards and values, exchange in its various phases, fire and marine insurance, foreign trade.

Fourth year. (The student's fifteenth school year.) (One hour a week.) Mathematics applied to financial, actuarial, statistical, and economical problems, such as annuities, mortality tables, theory of errors, demand and supply curves.

Methods of instruction.—As the above program shows, the subject matter of the mathematical instruction is vocational. The instruction is given by means of "lecture,[4] practice in calculation, solution of problems and making out of business forms and documents." Books are not generally used. Formulas, problems, and summaries are sometimes printed or mimeographed.

Examinations.[4]—The year's mark depends upon the term examinations and the annual examination. The latter is given at the end of each year in each subject. There is generally no final examination.

Preparation of teachers.—There is no special preparation.

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[1] Art. XI, p. 4.
[2] Ibid., pp. 4–9.
[3] In many cases the thirteenth or fourteenth school year.
[4] Art. XI, p. 9.

TECHNICAL SCHOOLS AND COLLEGES.[1]

TECHNICAL SCHOOLS OF MIDDLE GRADE.

Organization.—The technical schools of middle grade are State schools. The entrance requirements are the completion of the first six years of the elementary school. The average age of the first-year students is 15 years. The courses are of three or four years, with sometimes a preparatory course of two years.

Aim of instruction.—The aim "is to enable the students to understand the lectures on engineering and other like subjects, and also to train them in practical applications." [2]

COURSE OF STUDY IN MATHEMATICS.

No program is given, only the following schedule of hours.[3]

Schedule of hours.

Studies.	First year.	Second year.	Third year.	Fourth year.	Total.
Arithmetic	145	0	0	0	145
Algebra	78	156	39	0	273
Geometry	50	78	39	0	167
Trigonometry	0	0	78	0	78

Nature of the instruction.—An excellent list of topics for applied problems is given at the beginning of the report. Otherwise there is nothing stated as to how far the work is really vocational. It is recommended that textbooks be used and that special texts in mathematics be written for these schools.

Examinations.[4]—There are special, term, and annual examinations. The grade for the year depends upon these and upon daily marks.

Preparation of teachers.[5]—There is a class for training teachers in connection with the Tokyo High Technical School. Most of the students in this class are from the normal school and come to the technical school to learn the technical subjects.

HIGH TECHNICAL SCHOOL.

Organization.—For entrance the students must have the equivalent of the work of the middle schools. The course is three years of about 33 weeks each. This is a State school.

Aim of instruction.[6]—The school aims to train practical engineers.

[1] The Teaching of Mathematics in Japan; Art. XII, The Teaching of Mathematics in Technical Schools and Colleges.
[2] Ibid., p. 24.
[3] Ibid., p. 27.
[4] Ibid., p. 28.
[5] Art. XII, p. 42.
[6] Ibid., p. 29.

COURSE OF STUDY IN MATHEMATICS.[1]

Algebra. The course includes imaginaries, inequalities, permutations and combinations, maxima and minima, indeterminate forms, convergence and divergence of series, determinants, and the binomial theorem with any index.

Geometry. This takes up the computation of areas and volumes.

Trigonometry. This course includes the solution of trigonometrical equations and inverse trigonometrical functions.

Plane analytics. A study is made of conics, including the general equation of the second degree.

Solid analytics. This course extends through surfaces of the second degree.

Calculus. This is a fairly extended course in differential and integral calculus

Differential equations. Elementary course.

Schedule of hours in mathematics.

Branch courses.	Architecture.	Textile technology.	Electrical engineering.	Mechanical engineering.	Chemical engineering.
Algebra	66	66	66	66	.....
Analytics	99	99	99	99	.....
Mathematics	.....	.....	.....	.....	66
Calculus and differential equations.	66	66	66	66	.....
Total	231	231	231	231	66

The remarks made concerning the nature of the instruction, examinations, and preparation of teachers under Technical Schools of Middle Grade apply here.

TRAINING SCHOOLS FOR LAND SURVEYORS.[2]

LOWER COURSE.

This is a course of one year. The preparation for admission seems to be completion of the middle school. These are State schools and aim to prepare practical surveyors. The average age of the students is 25 years.

COURSE OF STUDY IN MATHEMATICS.

Algebra. (Thirty-five hours.) The principal topics are series, binomial theorem, partial fractions, logarithms, differential coefficient, maxima and minima.

Solid geometry. (Forty hours). The course includes the usual material of elementary solid geometry, through spherical triangles.

Plane trigonometry. (Forty-five hours.) This is a course in elementary plane trigonometry.

Spherical trigonometry. (Twenty-five hours.) This course includes the solution of spherical triangles.

Analytics. (Forty hours.) This course extends through the equation of the second degree in the plane.

Method of least squares. (Forty hours.)

The instruction is given mainly by lectures. No textbooks are used.

[1] Art. XII, pp. 32-34. This program is recommended in the report, but it is not said to be in use.
[2] Art. XIII, pp. 44-48.

Examinations are given in some subjects. The teachers are selected from the best land surveyors, graduates of the Imperial University, and specialists in mathematics.

HIGHER COURSE.

This is a two-year course. The students are selected from the best practical surveyors. The average age of the students is 33 years. The aim is to train specialists "to guide common surveyors." [1]

COURSE OF STUDY IN MATHEMATICS.[2]

Algebra. (Sixty hours.) Series, determinants, probability, and theory of equations are studied.

Plane and solid analytics. (One hundred ten hours.)

Differential calculus. (One hundred hours.) This course extends through expansion of functions and maxima and minima.

Integral calculus. (One hundred forty hours.)

Differential equations. (One hundred twenty hours.) This is a course in ordinary and partial differential equations.

Dynamics. (Ninety hours.) This is a course in statics and dynamics of rigid bodies.

Method of least squares. (Two hundred hours.)

RUSSIA.

INDUSTRIAL SCHOOLS.[3]

SCHOOLS FOR APPRENTICES.[4]

Organization.—The course is three years in length. The age of entrance is from 12 to 15 years. The purpose of the schools is to give technical instruction during apprenticeship. In 1909 there were 28 of these schools.

COURSE OF STUDY IN MATHEMATICS.

Arithmetic. (Four hours a week the first year, two hours the second year, and one hour the third year.) The principal topics studied are fractions, ratio, proportion, and interest.

Geometry. This course gives the geometry needed in practice. No attempt is made to give rigorous proofs. This course is followed by geometric drawing.

ELEMENTARY TRADE SCHOOLS.[5]

Organization.—The length of the course is four years, but the last year is given to practice. The students enter from the primary schools at the age of 12 to 15 years. The aim of the schools is to educate for a trade. In 1909 there were 131 of these schools.

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[1] Art. XIII, p. 44.

[2] Ibid., p. 47.

[3] Rapports présentés à la Délégation Russe: Sur l'organization de l'enseignement des mathématiques dans les écoles industrielles du ressort du Ministère de l'Instruction Publique, par P. Kolournitzki et A. Hartzouck. Referred to hereafter as Rapport.

[4] Rapport, pp. 29–31.

[5] Ibid., pp. 30, 30.

COURSE OF STUDY IN MATHEMATICS.

Arithmetic. (Two hours a week the first year, one hour a week the second year.) This course includes integers, common and decimal fractions, the metric system, and applied problems.

TRADE SCHOOLS.[1]

Organization.—The length of the course is three years. In order to enter, the students must present the certificate of studies from an *école préparatoire primaire.* There were 30 such schools in 1909. Their purpose is to prepare their students for the trades.

COURSE OF STUDY IN MATHEMATICS.[2]

Arithmetic. (The course extends through two years.) The principal subjects studied are divisibility of numbers, common and decimal fractions, ratio and proportion, interest, and the extraction of roots.

Geometry. (Three hours a week the first year, two hours the second year, and one hour the third year.) The object of this course is to teach the geometry necessary for the trades.

Geometrical drawing. (Two hours a week the first year and three hours the second year.)

Industrial drawing. (Two hours a week the second year and five hours a week the third year.)

LOWER TECHNICAL SCHOOLS.[3]

Organization.—These schools require for entrance the completion of the four-year course in the municipal schools. (This is preceded by a three-year course in an elementary school.) The course of study covers three years. There were 19 of these schools in 1909. Their object is to prepare master workmen and foremen.

COURSE OF STUDY IN MATHEMATICS.[4]

Arithmetic. (Three hours a week the first year, three hours with algebra and geometry the second year.) In the first year the subjects studied are fractions, and ratio and proportion; in the second year the rule of three, interest, and exchange.

Algebra. Two hours a week are given to this course in the first year, and the fundamental operations are studied; in the second year, through quadratics.

Geometry. Plane geometry is studied the first year; the second year, the theory of proportion, some parts of solid geometry, and the trigonometrical ratios.

Mechanics. This course includes the mechanics of a point, the mechanics of solids, applications of mechanics to simple machines, and resistance of materials.

Projection drawing. (One hour a week the first year, two hours the second year, and six hours the third year.)

SECONDARY TECHNICAL SCHOOLS.[5]

Organization.—The entrance requirements are the completion of the five lower classes of the Realschule. The course is four years or less. There were 32 of these schools in 1909. Their aim is to give an industrial education directed toward the special calling of the student, and in particular to educate engineers' assistants.

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[1] Rapport, pp. 27–32.
[2] Ibid., pp. 31, 32.
[3] Ibid., p. 27.
[4] Ibid., pp. 33, 34.
[5] Ibid., pp. 27, 28.

COURSE OF STUDY IN MATHEMATICS.[1]

Algebra. (One hour a week the first year.) This is mainly a study of logarithms in preparation for trigonometry.

Geometry. (One and one-half hours a week the first year and one hour the second year.) Plane and solid geometry.

Trigonometry. (One and one-half hours a week the first year.) Plane trigonometry.

Analytics. (Two hours a week the second year.) The fundamental notions of plane and solid analytics.

Drawing. Geometrical drawing, six hours a week the first year; machine design second and third years.

Mechanics. First year, general mechanics and simple machines; second year, graphical statics and resistance of materials.

SPAIN.[2]

COMMERCIAL SCHOOLS.

The course of study in the commercial schools is divided into three parts—preparatory, elementary, and higher.

In order to be admitted to the preparatory division a student must be at least 10 years of age and must pass an examination in grammar, arithmetic, geometry, geography, religion, and writing. The age of admission into the elementary department is 14 years.

These schools prepare their students to be merchants, skilled accountants, and commercial teachers. In 1907 there were 14 of these schools, with 3,242 students.

The course for merchants includes the study of the elements of arithmetic and algebra, bookkeeping, commercial practice, and industrial technology.

The course for commercial teachers includes algebra, advanced commercial calculation, bookkeeping, and public administration.

SCHOOLS OF ARTS, INDUSTRIES, AND TRADES.

There are many institutions of an elementary grade, some of which prepare for the fine arts or the arts in general and others for the industries.

The aim of the schools of arts and industries is: (*a*) To prepare skilled mechanics, who study arithmetic, algebra, geometry, descriptive geometry, trigonometry, physics, general and applied mechanics, and machines; (*b*) to prepare skilled electricians, chemists, and workers in various other industries.

The schools of arts and trades are night schools and are open to workers. There are 26 of these schools. In order to enter, the students must have an elementary education.

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[1] Rapport, p. 26.

[2] L'Enseignement Mathématique en Espagne, 2ème rapport de la Sous-commission espagnole, par le délégué Dr. Zoel G. de Galdeano, pp. 3, 4.

NAVIGATION SCHOOLS.

Navigation is taught in five general technical schools and in two navigation schools. The number of students is about 300. The students obtain the pilot's certificate.

The subjects studied are arithmetic, algebra, geometry, trigonometry, topography, geography, physics, design, astronomy, and piloting.

SWEDEN.

INDUSTRIAL SCHOOLS.[1]

ELEMENTARY INDUSTRIAL SCHOOLS.

Organization.—There were at the time that the report was written (1911) about 60 elementary industrial schools in Sweden, 54 of which had State support. These schools began with private foundations, but have gradually come under State or community control.

The school year has two semesters—13 weeks in the fall and 20 weeks in the spring. The instruction is usually given from 7 to 9 in the evening, on weekdays, and from 8 to 10 on Sunday mornings.

The course is two or three years in length.

The entrance requirements are an age of at least 14 years and the certificate of dismissal from the Volksschule.

A small fee of from 1 to 2 crowns (27 cents to 54 cents) is charged.

These schools are not compulsory, but a commission is working over the plan of instruction, and it is hoped that attendance will be required between the ages of 14 and 18. In schools where the numbers justify it the students are divided into classes according to occupation, and the instruction in mathematics is adapted to the needs of the different trades.

Aim of the instruction.—The aim of these schools is to give practical industrial training.

COURSE OF STUDY IN MATHEMATICS.

Arithmetic. The course in arithmetic usually extends through three semesters, two hours a week. "In arithmetic,[2] where the object of the teaching is to give the students skill in the computations that they will meet in their special trades, the course given includes whole numbers, common and decimal fractions with applications, percentage, interest, stocks and bonds, discount, and general problems that can be solved by the aid of equations of the first degree."

Algebra. This is a course of two hours a week, but the number of semesters is not given. The topics studied include integral and fractional expressions and equations of the first and second degree.

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[1] Der mathematische Unterricht in Schweden: Der mathematische Unterricht an den elementartechnischen Gewerbeschulen Schwedens, by Dr. K. L. Hagström.

[2] Ibid., p. 3.

Geometry. There is usually a course of one year, with one or two double periods a week. Six books of Euclid are studied, particular attention being given to problems of construction. Practice is given in the computation of areas and volumes.

In some industrial schools trigonometry and calculus are taught.

Methods of instruction.—There are short lectures by the teacher, followed by the solution of problems by the students.

The same books are used as in the secondary schools.

MIDDLE TECHNICAL SCHOOLS.[1]

Organization.—Students who have finished the Realschule enter without examination. These schools are said to belong to group C in the plan of the International Commission. The average age of entrance is about 18 years. The length of the course is three years.

COURSE OF STUDY IN MATHEMATICS.

There is much freedom given the teacher as to the nature of the instruction. The official plan requires only that certain subjects be taught. These are arithmetic, plane and solid geometry, algebra including computations with series and logarithms, plane trigonometry, and the elements of analytical, descriptive, and practical geometry. Calculus is taught in some schools.

Aims and methods of instruction.—The official instructions recommend that the teacher keep constantly in mind the needs of the industries. Mathematics is to be regarded as a tool. Hence the parts that have only a theoretical value will be omitted, such as certain formal parts of algebra. Geometry is of great importance in design. Therefore it should be developed so as to contribute as much as possible to practical applications. The principal aim of the calculus is to prepare the student to read technical literature.

Examinations.—Examinations are held at the option of the teacher to determine the progress and promotion of the students. There is no final examination. A diploma is granted at the completion of the course.

SWITZERLAND.

COMMERCIAL SCHOOLS.[2]

COMPLEMENTARY COMMERCIAL INSTRUCTION.

Organization.—The complementary commercial instruction is both public and private. Courses are organized by commercial socities, by communes, and by one Canton (Fribourg). The students are

[1] Der mathematische Unterricht in Schweden: Die Mathematik an den technischen Lehranstalten in Schweden, II: Der mathematische Unterricht an den technischen Mittelschulen, by O. Gallander, pp. 14-21.

[2] L'Enseignement mathématique en Suisse, No. 6: Les mathématiques dans l'enseignement commercial suisse, by L. Morf. Referred to hereafter as No. 6.

apprentices and adult employees. The programs given in the report are for a course of three years.

Aim of the instruction.[1]—The aim of the mathematical instruction is to give the students accuracy and speed in the calculations involved in commercial affairs.

COURSE OF STUDY IN MATHEMATICS.[2]

In the courses of study given in the report, from one to two hours a week for three years are given to elementary and commercial arithmetic, and the same to bookkeeping.

Arithmetic. The following are the principal topics taken up in the different years:

First year. Computing by the hundred and by the thousand, English money and measures, and a review of fractions.

Second year. Interest, discount, exchange, ratio and proportion, alligation, and partnership.

Third year. Accounts current, accounts involving public funds, oral calculation, and a review of the work of the three years.

Bookkeeping. Some of the topics given in this course are simple and double entry by American and Italian methods, accounts of a society, loss and gain accounts, balances, partnership accounts, accounts of an industry and of a hotel.

Methods of instruction.[3]—The method used is that of exposition by the teacher, followed by the solution of numerous problems in and outside of the recitation.

Books.—A list is given of the books most widely used. For arithmetic these books are mainly collections of problems.

Examinations.[4]—At the end of the last year of the course an examination is given to which any person who has served two years' apprenticeship is eligible, whether or not he has attended the *cours complementaires*. This is an examination for a diploma and is required of apprentices in 10 Cantons. The tendency is to make it compulsory everywhere. This examination requires, in arithmetic, a written examination lasting from 2 to 2½ hours, and a test in oral calculation lasting from 6 to 7½ minutes; in bookkeeping, a written examination lasting from 3 to 3½ hours, and an oral one lasting from 6 to 7½ minutes.

The examinations are set by the Cantons or by the commercial societies, according to the school. They are under the supervision of the Federal department of commerce and industry, and in certain Cantons under that of the cantonal department of commerce and industry.

The examples of examination questions given in the report[5] are in the main practical ones, such as would occur in business practice.

[1] L'Enseignement mathématique en Suisse, No. 6: Les mathématiques dans l'enseignement commercial suisse, by L. Morf. Referred to hereafter as No. 6, p. 3.
[2] Ibid., p. 5.
[3] Ibid., p. 11.
[4] Ibid., p. 6.
[5] Ibid., pp. 6–11.

Preparation of teachers.[1]—The teachers belong to three groups—elementary school teachers, secondary school teachers, and persons engaged in business. Apparently no special preparation is required. These teachers are usually paid by the hour, at the rate of from 3 to 5 francs. In the larger places there are permanent positions which may pay as much as 4,800 or 6,000 francs for work of 28 hours a week.

SECONDARY COMMERCIAL INSTRUCTION.

1. The Lower Secondary Schools of Commerce.[2]

There are few of these schools. They have had support from the Federation since 1909. They have a large attendance of girls. The length of the course is one or two years. From five to six hours a week are given to arithmetic and bookkeeping in each year in a course of study outlined in the report. This is a two-year course and is used in a school in Basel. The subject matter in this course is similar to that already given on page 74.

2. Higher Schools of Commerce.[3]

Organization.—These schools are partly supported by the Federal Government. In some of the Cantons students of the ages of 14 or 15 who are normal in their studies are admitted without examination, and some schools require an examination of all students. Of 31 official schools of commerce, 6 are attended by boys only, 11 by girls only, and 14 are coeducational.

Aims of the mathematical instruction.[4]—The aims of the mathematical instruction are said to be to develop ability to reason, to give precision in language, to give ability in abstract and applied calculation, and to prepare the students for advanced commercial work in the university.

COURSE OF STUDY IN MATHEMATICS.[5]

The department of commerce of the Federal Government has issued a type program for courses of three and of four years. In the former, 10 hours a week are prescribed for arithmetic and 11 for bookkeeping and commercial work; and in the latter, 13 hours for arithmetic and 16 hours for bookkeeping and commercial work. Besides these, courses in algebra, geometry, and trigonometry are usually given.

Arithmetic. (*a*) Elementary arithmetic. "Review of arithmetic from the point of view, especially, of rapid calculation." This includes a review of the fundamental operations with integers and fractions, the metric system, square root, ratio and proportion, percentage, interest, discount, proportional division, alligation, and partnership, and oral calculation.

[1] No. 6, p. 12. [2] Ibid., p. 13. [3] Ibid., p. 14. [4] Ibid., p. 18. [5] Ibid., pp. 19–21.

(b) Commercial arithmetic. This includes a study of notes, bills, and accounts; the moneys and measures of the principal countries; methods of exchange and values of the precious metals; and buying and selling of stocks and bonds and other securities.

Algebra. (a) Elementary. This course begins with the fundamental operations and extends through square and cube root, equations of the second degree in one or more unknowns, arithmetical and geometrical progressions, logarithms, and permutations and combinations.

(b) Financial. This includes a study of compound interest, annuities, sinking funds, and public loans.

(c) Insurance. A study is made of probabilities and of mortality tables and their uses.

Elementary geometry. "Study of plane surfaces. Surfaces and volumes of the principal bodies."

Plane trigonometry. This course includes the solution of triangles.

Bookkeeping. This is a fairly comprehensive course in bookkeeping, including sets of books for banks and for mercantile and industrial establishments.

The courses in mathematics in certain schools include a few topics not mentioned above.

Methods of instruction.[1]—"The method employed is the method of exposition; it is reduced, generally, to a strict minimum; it is possible to devote much time to the solution of numerous and varied problems drawn, usually, from the domain of practical affairs." Little use is made of models. There is a list given of the textbooks in general use.

Examinations.[2]—"The examinations for promotion at the end of the year are giving way more and more to quarterly examinations." The grade for the quarter is obtained by combining the examination grade with the grade made in recitation. The average of the three quarterly grades is the grade for the year. A diploma is given students who finish the course and pass an examination. This examination is both oral and written and is based mainly upon the work of the last year of the course. The question is being considered of creating a Federal diploma for these schools.

About 20 pages[3] of the report are taken up with questions used in the examinations. These questions are in the main in arithmetic and bookkeeping, some in algebra and geometry. They are almost entirely applied problems, those in arithmetic dealing with somewhat complicated questions of accounts current, exchange, stocks and bonds, insurance, and profit and loss.

Preparation of teachers.[4]—The teachers prepare themselves either in a university, by private study, or, in a few cases, by commercial practice. It is recommended that some practical experience be made a part of the preparation of those who expect to teach in the commercial schools. A part of the teachers of mathematics in the commercial schools are selected from those who have prepared to teach in the secondary schools. It appears, then, that no definite require-

[1] No. 6, p. 47. [2] Ibid., pp. 25, 26. [3] Ibid., pp. 26–46. [4] Ibid., p. 48.

ments are made of teachers of mathematics except the passing of a special examination.

Present tendencies.—"The actual experiences are too recent for it to be possible to indicate a definite tendency in the teaching of mathematics in the higher schools of commerce." [1] It is of interest, however, to note the growth of these schools. In 1891, when the Swiss Federation first began to give them aid, there were 8 schools of commerce, with about 700 students. In 1910 there were 31 official schools of commerce, with 3,477 students—2,249 male and 1,228 female.

SCHOOLS OF ADMINISTRATION AND FOR THE RAILWAY SERVICE.[2]

Organization.—"By a school of administration is meant a school that prepares the youth to enter the Federal post office, telegraph, telephone, or customs service." [2]

The schools of administration and for the railway service have courses of two or three years. Students enter at the age of 14 or 15 years. "Students who have completed their previous studies normally are admitted without examination; each school reserves the right, however, to examine all students." The entrance examination includes the mother tongue, arithmetic, geography, and national history.

COURSE OF STUDY IN MATHEMATICS.[4]

The time given to arithmetic varies from four hours a week for one year to three hours a week for three years; to algebra, from one hour a week for six months to two hours a week for two years; to geometry, from nothing to one hour a week for three years; to bookkeeping, from nothing to two hours a week the first year and one hour a week the second year.

The following outline of the course is given:

Arithmetic. (*a*) Elementary. "Review and extension of the four fundamental operations with integers and fractions. Common fractions. Ratio and proportion. Simple and compound rule of three. Interest and discount. Proportional division. Partnership and alligation. Metric system. Square root."

(*b*) Commercial. "Calculation of interest by different methods. Stocks and bonds. Accounts current. Bills and notes."

Elementary geometry. "Calculation of surfaces and volumes."

Elementary algebra. "The four fundamental operations with positive and negative numbers. Fractions. Square and cube root. Equations of the first and second degree. Progressions and logarithms."

Elementary bookkeeping. This is an elementary course.

Methods of instruction.[5]—The methods used are similar to those used in the commercial schools.

Books.—A list of the books in general use is given.[6]

[1] No. 6, p. 53. [2] Ibid., pp. 64–69. [3] Ibid., p. 64. [4] Ibid., pp. 65, 66. [5] Ibid., p. 66.

Examinations.[1]—Examinations are given under conditions similar to those of the higher commercial schools. A diploma is given at the completion of the course.

Preparation of teachers.[2]—The teachers in these schools should have the preparation required of the teachers in the secondary schools.

INDUSTRIAL SCHOOLS.[3]

ELEMENTARY TECHNICAL SCHOOLS.

Under the head of elementary technical instruction are discussed the trade courses (cours professionels) and the permanent trade schools (écoles professionelles permanentes).

VOCATIONAL COURSES.[4]

These trade courses are for apprentices and give from four to seven hours of instruction a week. The law of Bern, which is cited, requires the attendance of apprentices in places where schools exist. "These courses [5] are supported by particular corporations, by cities, and by the cantonal and Federal authorities, and they are regularly inspected by their delegates. The Federal support is 50 per cent of that of the corporations plus one-third of the net expenses."

The trade courses are a regular part of the system of continuation schools of Switzerland. The continuation schools usually give courses of two or three years.[6] Attendance is required in 13 Cantons, is voluntary in 5, and in the hands of the municipalities (Gemeinden) in 3 Cantons. The general aim of these schools is to supplement the common-school education, and also to furnish the apprentices in the different trades the necessary theoretical instruction.

From two to four hours are given to mathematics in the trade courses. The time given to the different mathematical subjects is in general as follows:[7]

Arithmetic, 1 or 2 hours a week, one course each year.
Geometry, 1 or 2 hours a week, one course each year.
Bookkeeping, 1 hour a week, one course each year.
Geometrical drawing, 2 to 4 hours a week, one course each semester.
Projection, 2 to 4 hours a week, one course each semester.

A discussion of the aims and methods of instruction is given under the permanent trade schools.

Examinations.—The only examination mentioned is the one at the end of the apprenticeship, which is obligatory in most Cantons.

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[1] No. 6, pp. 66, 67.
[2] Ibid., p. 69.
[3] L'Enseignement mathématique en Suisse: No. 5, L'Enseignement des mathématiques dans les Écoles techniques moyennes suisses, by Dr. L. Crelier. This report is referred to hereafter as No. 5.
[4] Ibid., pp. 11-15.
[5] Ibid., p. 11.
[6] L'Enseignement mathématique en Suisse: No. 2, Der mathematische Unterricht an den schweizerischen Primarschulen, by Just. Stocklin, p. 13.
[7] No. 5, p. 26.

PERMANENT VOCATIONAL SCHOOLS.[1]

Organization.—"Certain cities have gone further than the trade courses.[2] Assisted by various trade associations and by the cantonal and Federal authorities they have organized trade schools in which young people serve their complete theoretical and practical apprenticeship. Only those apprentices may enter who are not required to attend the public schools. They are then at least 14 or 15 years of age and remain two, three, or four years, according to the obligatory duration of the apprenticeship. These institutions are inspected by the Confederation and are supported by it to the same extent as the trade courses. In these trade schools we develop the superior workmen, capable of becoming excellent foremen after some years of service in the shops. This is the intermediate stage between the elementary and the middle school instruction."

Aim of the instruction.—These schools are organized to assist in the development of the industries of the places in which they are located. The larger cities have schools for different trades.

Number of hours spent in the schools.—As a rule, the apprentices are in the schools 56 hours a week (in some only 52), 9½ hours daily except Saturday, when the time is 8½ hours. "The purely practical or purely vocational instruction takes from 33 to 48 hours, according to the school and the trade.[3] There remains then from 8 to 19 hours for general and technical culture. The largest part of this time is given to mathematics."

The courses usually given in mathematics are the following:[4]

	Hours per week.
Arithmetic, one course a year	2
Geometry, one or two courses a year	2
Algebra, one or two courses a year	2
Trigonometry, one course a year	2
Mechanics, one or two courses a year	2
Bookkeeping, one course a year	1
Geometrical drawing, one course a semester	4
Projection, one course a semester	4

COURSE OF STUDY IN MATHEMATICS.

Report No. 5 gives outlines of the courses in mathematics offered in four of the middle trade schools. The following abstract of the courses offered in the section for mechanics in the School of Arts and Crafts of Geneva[5] will give a fairly good notion of the nature of the work done in mathematics as shown in the report:

FIRST YEAR. (PROBABLY THE STUDENT'S NINTH SCHOOL YEAR.[6])

Numerical calculation and algebra. (Two hours a week.) This course includes calculations involving simple literal expressions, the solution of easy equations from geometry, mechanics, and physics, technical tables, and graphical representation.

[1] No. 5, pp. 15-27.
[2] Ibid., p. 15.
[3] Ibid., p. 26.
[4] Ibid., p. 27.
[5] Ibid., pp. 24, 25.
[6] The number of years of school attendance before entering the trade school is not the same in all the Cantons.

Geometry. (Three hours a week.) The topics taken up in this course are geometrical constructions, curves used in mechanics; reduction of figures, the pantagraph; and practical applications to solid geometry, such as the calculation of surfaces, volumes, and weights of pieces of machines.

Mechanics. (Two hours a week.) This course includes elementary statics with applications to machines and the elements of kinematics and dynamics.

Technical drawing. (Six hours a week.) The principal topics in this course are making sketches of the tools in the shop, general principles of machine design, and the elements of elementary and descriptive geometry applicable to machine design.

SECOND YEAR. (THE STUDENT'S TENTH SCHOOL YEAR.)

Trigonometry. (Two hours a week during one semester.) This is a course in plane trigonometry with applications to mechanics.

Mechanics. (Three hours a week.) This course includes applied kinematics, the problem of falling bodies, and the study of such mechanisms as cams, connecting rods and cranks, and gearing.

Resistance of materials. (Two hours a week during one semester.) There is a study of simple problems; applications to such cases as belts and gearing; the use of tables.

Technical drawing. (Six hours a week.) General principles of machine design, with applications to the machines used or constructed.

THIRD YEAR. (THE STUDENT'S ELEVENTH SCHOOL YEAR.)

Mechanics. (Four hours a week.) This includes further study of dynamics and of certain types of motors.

Technical drawing. (Six hours a week.) Sketches, working drawings, copies, and blue prints.

Methods of instruction.—The following remarks on the methods of instruction apply to the trade courses as well as the permanent trade schools unless otherwise stated:

Arithmetic. "The teaching of arithmetic in the trade courses and in the shops for apprentices requires an annual course of one or two hours and has for its principal aim the solution of vocational applications.[1] We mean here by vocational applications all the practical problems which may be met in the trade of the apprentice. The carpenter calculates surfaces, volumes, and the buying and selling prices of materials used in his trade. The locksmith deals with the surfaces, volumes, weights, and prices of pieces of iron of various kinds.

"The instruction, however, is systematic and is developed as in the other schools—complete knowledge and facility in the four operations: Metric system; rule of three; calculation of costs and selling prices; rules of interest and discount."

"As to programs and methods they may be summed in this:[2] To arrive at a practical and useful result by the simplest and most intuitive means."

"The method[1] of work usually adopted consists in the solution of numerous problems, either on the blackboard or in the notebook, and always, if possible, in class under the the supervision of the master. Little or nothing of books and nothing of written theory."

We find here the not unusual complaint concerning the poor work done in arithmetic in the elementary schools:

"In all cases the fact is this:[1] The young people who enter the technical schools do not know sufficiently well how to calculate." "The teaching of arithmetic in the elementary technical schools reveals another general defect: The students who come

[1] No. 5, p. 74. [2] Ibid., p. 27.

to us do not know how to prepare the solution of a problem in arithmetic. Now, such a task is at once a matter of reasoning, of order, and of good taste upon which it is absolutely necessary to insist. It is not the writer who is speaking here, but a great number of colleagues who replied to the questionnaire."

Geometry. The work begins with a review of the properties of lines, surfaces, and solids which the students have learned in the elementary school. "We continue with purely intuitive instruction.[1] The knowledge of geometrical figures is developed by the aid of drawings, and that of solids by the examination of models in wood or cardboard. The principal rules are established by the use of decomposable cardboard figures." "While being intuitive, the instruction remains systematic; the study of perpendiculars and obliques, of the equalities of triangles, the circle, parallelograms, surfaces, the Pythagorean proposition, and regular polygons lends itself to this method of treatment. Such is, in its large outlines, the geometric basis of the knowledge of our future artisans."

"We may add that, without question, it is impossible to put a manual of geometry into the hands of the students. First of all, there does not exist to our knowledge a French or German text that fulfills completely the requirements of the technical schools. On the other hand, it is not in thumbing a book that a young person learns geometry."

The method of instruction is to have the necessary theory dictated by the teacher and copied by the students in notebooks. Many exercises are solved. Plane and solid geometry are developed together. "The notions of geometry are terminated with the principal properties of the regular pyramid, cone, right prism, cylinder, by utilizing the construction of these to aid in their development."[2]

Algebra. Algebra is seldom taught in the trade courses. The following remarks apply to the permanent trade schools:

"As in arithmetic, this course is directed toward a utilitarian end."[3] "As in arithmetic, little of theory, each new question, each step in advance is explained by corresponding examples. The solution of numerous exercises by the students on the blackboard, and in the class notebook, such is the method recommended and adopted by the most of the schools."

The books most used are:[4]

C. Morf et S. Tzaut, exercices et problèmes d'algèbre.
E. Bardey, Aufgabensammlung.

MIDDLE TECHNICAL SCHOOLS.

Organization.—The most of the middle technical schools are under the control of the Cantons. The lengths of the courses are from three to nine semesters, with five and six semesters as the most frequent lengths.

Entrance requirements.—"The technical schools[5] receive, after examination, youths of 15 years who have attended the last two classes of a secondary school and who possess the knowledge required by its regulations.

"This knowledge includes: Elementary arithmetic. The elements of algebra and geometry. The mother tongue (preparing a simple composition with ease and confidence). Free-hand and elementary geometrical drawing."

[1] No. 5, p. 70. [2] Ibid., p. 77. [3] Ibid., p. 79. [4] Ibid., p. 81. [5] Ibid., p. 32.

A part of the students have several years of shop experience before entering the technical schools.

Aims.—"Between the engineer,[1] the thought that conceives, and the workman, the hand that executes, a 'technician' is necessary, capable first of grasping easily the projects and plans of the engineer or the architect, and then directing their execution with intelligence."

As the principal aim of these schools is to educate these technicians the principal aim of the mathematics is a purely practical one. It is intended, however, that the mathematics be taught so as to develop logical thought.[2] The young technician must know enough mathematics to organize and solve his problems mathematically, and to be able to read the literature of his subject.

COURSE OF STUDY IN MATHEMATICS.

"The number of hours weekly (spent in the school) is in the neighborhood of 40 throughout the course.[3] In the first year 12 or 15 hours are given to pure mathematics, in the second year 5 or 6, and in the third year only 2 or 4. In the second year the course in applied mathematics requires 8 or 10 hours."

The following course[4] in mathematics is given in the section for mechanicians in the *Technicum* of Berthoud. The length of the course in this school is five semesters.

FIRST SEMESTER. (THE STUDENT'S NINTH SCHOOL YEAR.)

Arithmetic. (Six hours a week.) The principal subjects studied are the fundamental operations, systems of measures, fractions, ratio, and proportion, percentage and its applications, partnership, and alligation.

Algebra. (Four hours a week.) This course includes the fundamental operations with integral expressions, equations of the first degree in one unknown, proportion, and logarithms.

Geometry. (Three hours a week.) The theorems of plane geometry, the constructions, and computations are taken up.

Geometrical drawing. (Five hours a week.)

SECOND SEMESTER. (THE STUDENT'S NINTH SCHOOL YEAR.)

Algebra. (Four hours a week.) The topics studied in this course are equations of the first degree in more than one unknown, equations of the second degree, powers and roots, logarithms, the slide rule, arithmetical and geometrical series, and compound interest.

Geometry. (Four hours a week.) This is a course in solid geometry and includes the most important theorems, and computations of the surfaces and volumes of the elementary solids and also of the prismatoid and of solids of revolution. There is a short introduction to trigonometry which is used in the computations of volumes.

Projective geometry. (Four hours a week.) This course begins with the projection of the point and the straight line and extends through the study of the plane sections of pyramids, prisms, cylinder and cones, and the intersections of bodies bounded by plane and curved surfaces.

[1] No. 5, p. 28. [2] Ibid., p. 99. [3] Ibid., p. 42. [4] Ibid., pp. 50–55.

THIRD SEMESTER. (THE STUDENT'S TENTH SCHOOL YEAR.)

Plane trigonometry. (Three hours a week.) This includes the solution of oblique triangles and applications.

Analysis. (Four hours a week.) This course includes the binomial theorem, the notion of a function, and an introduction to differential calculus and its application to the theory of curves and to mechanics, and to the development of functions into power series.

Descriptive geometry. (Three hours a week.) Intersections of solids and applications to numerous problems in the construction of machines. Light and shade and parallel perspective.

Mechanics. (Four hours a week.) This is a course in the elementary theory of mechanics, with many practical applications.

Resistance of materials. (Four hours a week.)

FOURTH SEMESTER. (THE STUDENT'S TENTH SCHOOL YEAR.)

Analysis. (Three hours a week.) Introduction to integral calculus and applications to geometry and mechanics.

Mechanics. (Six hours a week.) This course includes something of the dynamics of rigid bodies, hydrostatics, and hydraulics with applications.

Graphical statics. (Two hours a week.)

FIFTH SEMESTER. (THE STUDENT'S ELEVENTH SCHOOL YEAR.)

Mathematics. (One hour a week.) Review with selected examples. Short introduction to practical geometry.

Graphical statics. (One hour a week.)

Methods of instruction.—What has been said concerning the general aims and methods of instruction in the trade schools applies here. The instruction is expected to be concrete, to involve many practical problems, and in point of view of theory to be limited to what is absolutely necessary. "All the purely theoretical developments are suppressed." [1]

The work in descriptive geometry and in geometrical and technical drawing is said to be of great importance, is given much time and attention, and every effort is made to keep the instruction thoroughly practical.

Much importance is attached to the study of mechanics in the middle technical schools. It is sometimes taught by mathematicians and sometimes by engineers. "We will only add that the choice of examples is made in absolute conformity with the technical needs of the students, and that this course is characterized by a great number of carefully chosen graphical and numerical problems." [2] It is urged that in geometry and in mechanics "the professor should prepare the examples according to the necessities of the school.[1] He ought to know the questions of which the technologists have the greatest need; his problems ought to relate the mathematical theory to the other branches of the program."

[1] No. 5, p. 101. [2] Ibid., p. 88.

Examinations.[1]—At the end of the course the students take an examination to obtain the *diplôme de technicien*. In certain schools there is an examination at the end of the second year which includes the examination in pure mathematics.

The examination in mathematics is oral and written, on algebra, geometry, trigonometry, analytics, mechanics, and the resistance of materials. A considerable number of examination questions are given in the report.[2] The most of these examples are theoretical or involve such applications as are found in the books.

Preparation of teachers.[3]—In general the teachers in the middle technical schools are engineers and architects with diplomas from the higher technical schools, and professors of branches of general culture who have the *diplôme de maître de gymnase.*

Some of the instructors of technical branches are technologists who after leaving the *Technicum* have gone into practice, and later have gone into teaching. Some of the teachers of nontechnical subjects have only the *diplôme de maîtres secondaires.* "We have then no special preparation for the professors of technical instruction. There is found the engineer with no pedagogical knowledge, and the mathematician with no technical knowledge." It generally happens, however, that the engineer has a taste for teaching, and the mathematician is interested in mechanics or architecture; so that the results seem to be good. It is recommended that some special preparation be given instructors in technology, just as is now done in the case of the teachers in the secondary schools. "It is not sufficient for a future professor to be instructed in his subject; it is necessary that he should have practice teaching, show what he can do, and by some well directed trials improve and perfect his individual methods."

UNITED STATES.

The vocational schools of the United States have no general State or national organization, and present great diversity of organization, aims, and methods. As a rule these schools have grown up to meet the needs of particular vocations or of particular localities. The types are not clearly defined, and it is practically impossible to give detailed descriptions that will apply to a large number of schools.

The data used here are taken almost entirely from the American reports to the International Commission on the Teaching of Mathematics. Only some of the principal facts are given here, and references are made to the reports for details.

[1] No. 8, p. 97. [2] Ibid., pp. 91–97A. [3] Ibid., p. 102.

COMMERCIAL SCHOOLS.[1]

Private commercial schools.—There is generally no definite entrance requirement. The aim is to prepare young persons, in the shortest possible time, to take such positions as that of clerk, bookkeeper, accountant, and stenographer. The courses are from a few months to two years in length. Commercial arithmetic is usually the only work in mathematics.

Commercial departments in high schools.—These are ordinary high schools, and the course offered by the commercial department is only one of several that may be elected by the students. The work in mathematics in the commercial course differs from that of the other courses in including commercial arithmetic and bookkeeping, and less of algebra and geometry.

Commercial high schools.—These have the same organization as other high schools, and hence require for entrance the completion of the eighth grade of the elementary school, and the pupils are as a rule 14 years old when they enter. The aim of the commercial high school, as well as of the commercial department in the general high school, is to give a general education and at the same time to give vocational training that may be utilized immediately after leaving the school. Both of these classes of schools prepare for the same kinds of positions as those mentioned under private commercial schools.

The courses in mathematics offered are commercial arithmetic, algebra, geometry, bookkeeping, and sometimes trigonometry.

The length of the course is four years.

INDUSTRIAL SCHOOLS.

Intermediate industrial, preparatory trade, or vocational schools.[2]—The aim of these schools is to prepare for a trade. In order to enter these schools the student must be at least 14 years of age and have completed at least the first six years of the elementary school. These schools are as a rule supported by public funds. "In schools of this class the course of study commonly combines bookwork and shopwork in almost equal proportions. The bookwork generally includes English, shop mathematics, industrial history, and civics, together with the elements of physics and chemistry."

Trade schools with day courses.[3]—"In these schools the aim is to prepare pupils for actual work in the trades in the shortest possible time. The length of the course varies with the needs of the student. In some schools of this class very little attention is given to mathematics; but other schools "giving longer courses frequently give very

[1] International Commission on the Teaching of Mathematics: The American Report: Mathematics in the Technical Secondary Schools of the United States, pp. 21–29.
[2] Mathematics in the Elementary Schools of the United States, pp. 154, 155.
[3] Ibid., pp. 155, 156.

thorough instruction in English, mathematics, and science." The work in mathematics sometimes includes the elements of arithmetic, algebra, geometry, and trigonometry.

Some of these schools are supported by private foundations and others by public funds.

Apprenticeship schools.[1]—"Apprenticeship schools are provided by a number of large industrial corporations for the education of boys who are learning their trade." "Through training in mathematics, drafting, English, and science there are recruited from the ranks of those who work in the shops a group of men much more capable than the ordinary worker." "The fact that in most of these schools shop time is taken for the class work, and that this time is paid for at the regular rate, is an indication of the value attached to such exercises." These schools offer courses in arithmetic, and in some cases courses in algebra, geometry, and trigonometry.

Evening schools.[2]—These schools are supported by private foundations, by public funds, and by societies such as the Young Men's Christian Association. "They are intended to give a type of education, both theoretical and practical, that will give the worker at the trade a better understanding of his work and of its relationship to the industry toward which his work contributes." Some of these evening schools give free courses. Certain schools give courses extending over a number of years. The free night school of the Cooper Union in New York, for example, offers a five-year course in general science. The mathematics offered in this course includes algebra, plane and solid geometry, trigonometry, analytical geometry, calculus, and mechanics. A student will generally be admitted to a course in an evening school if he gives evidence of sufficient preparation to profit by the course.

Part-time schools.[3]—"The part-time plan, as carried on at Fitchburg, Mass., is an arrangement by which boys in the high school give half of their time to work for wages in the commercial shops in the city and half to school work." Like other high schools, these schools require the completion of the eight grades of the elementary school for admission. "For the first year the boy devotes his entire time to school work, and for the next three years equal groups of boys alternate between shop and school, so that one group is always at work in the shops and one in the school."

Trade schools for the colored races.[4]—"The object of these schools is to teach a skilled trade to Indians and Negroes. In some of these schools there is a course of four years. The entrance requirements vary. In some of these schools the relationship between industrial

[1] Mathematics in the Elementary Schools of the United States, pp. 156, 157.
[2] Ibid., pp. 157–159.
[3] Ibid., p. 159.
[4] Ibid., pp. 159, 160.

and ordinary academic work is exceptionally well handled. This is peculiarly true in the subject of mathematics. The problems of the shop and farm are quite commonly treated in the classes in arithmetic, algebra, and geometry."

Technical schools with day courses.[1]—"The schools included under this classification are of secondary grade and do not give instruction of higher engineering rank. The courses are designed to give the pupil such acquaintance with scientific and mathematical principles underlying commercial processes as will enable him to qualify in time for the work of foreman, master mechanic, inspector, etc." The completion of the high school is usually required for entrance. The length of the course is usually four years. Some of these confer degrees. The work in mathematics includes algebra, plane and solid geometry, trigonometry, analytical geometry, calculus, and mechanics.

Manual training and technical high schools.[2]—Like other public high schools, these schools require for admission the completion of the eight grades of the elementary school, and give a course of four years. The aim of these schools is to give a general education which touches more closely the industrial life of the community than does the conventional classical and literary courses, and to prepare for higher technical training. Some direct training for the trades is given.

"In schools [3] where curricula are arranged with more or less reference to the requirements of the colleges, algebra, geometry, and trigonometry receive about the same amount of time as in general secondary schools, viz, one year each of elementary algebra and plane geometry, and one half-year each of advanced algebra, solid geometry, and trigonometry. A year's work ordinarily represents five exercises per week for 33 to 40 weeks, i. e., a total of 165 to 200 exercises. The length of the exercises is from 40 to 50 minutes." "A few schools present courses in so-called 'shop mathematics.'"

Agricultural high schools.[4]—These schools are generally public high schools, and have the same entrance requirements and length of course as other high schools. "Their object is to provide such an education for the youth of the rural community as will tend to retain him in that community as an efficient member thereof." The required work in mathematics usually consists of arithmetic and one year each of algebra and plane geometry. It is reported that an attempt is made to make the work in mathematics more practical.

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[1] Mathematics in the Elementary Schools of the United States, p. 156.
[2] Mathematics in the Technical Secondary Schools of the United States, pp. 1-21.
[3] Ibid., p. 9.
[4] Ibid., pp. 29-32.

TABULAR STATEMENTS OF COURSES IN MATHEMATICS IN COMMERCIAL AND INDUSTRIAL SCHOOLS.

The following tables contain, so far as the facts are available, for each type of school in each country the following facts:

The number of years of previous school attendance required for entrance.

The approximate age at entrance.

The number of years in the course.

The number of hours a week spent in the school.

The number of hours a week given to the study of mathematics.

The branches of mathematics studied.

In determining the number of years of previous school attendance and the approximate age at entrance, free use has been made of the tables prepared by Mr. J. C. Brown and published in Bulletin, 1914, No. 45, of the United States Bureau of Education.

Commercial and industrial schools in Austria.

Requirements.	Higher commercial schools.	Industrial continuation schools.	Courses for masters and assistants.	General trade schools.	Trade schools for particular industries.	Schools for the building trades and the mechanical arts.
Years of previous school attendance required.	8	8	8 or more.	6	8	8
Approximate age at entrance.	14	14 or over.	17 or over.	12 or over.	14 or over.	17 or over.
Number of years in course	4	2	.....	3	3 or 4	1½
Hours per week in school	Full time.	Part time.	Part time.	Full time.	Full time.	Full time.
Hours per week for mathematics.	About 5 each year.	.....	.....	.....	.....	.....
Courses in mathematics	Arith., alg., geom.	Arith., geom.	Arith., geom.	Arith., geom.	Arith., alg., geom., projection drawing.	Arith., geom., projection drawing.

Requirements.	Schools for master-workmen.	Higher schools for the textile industry.	Higher industrial schools.	Industrial schools for women.	Schools of domestic science.
Years of previous school attendance required.	8 or more.	8	8	8	.....
Approximate age at entrance.	17 or over.	14 or over.	14	14	16
Number of years in course	2 or 2½	4	4	2	5 months.
Hours per week in school	.....	Full time.	Full time.	.....	.....
Hours per week for mathematics.	.....	.....	1st year, 9-13; 2d, about 7; 3d, 2½ to 4.	2 each year.	2
Courses in mathematics	Arith., alg., geom., trig., projection drawing.	Arith., alg., geom., trig., geometrical drawing.	Alg., geom., trig., analytical geom., projection drawing.	Arith.	Arith.

Commercial and industrial schools in Denmark.

Requirements.	Evening commercial schools.	Day commercial schools.	Evening industrial schools.	Day schools for mechanics.	Courses for surveyors and foresters.	Navigation schools.
Years of previous school attendance required.	.....	.....	.....	7	.....	.....
Approximate age at entrance..	.....	.....	.....	.....	.....	.....
Number of years in course.....	3	1	4 winter semesters.	1-2	.....	.....
Hours per week in school.....	.....	30	.....	Full time, apparently.	Full time, apparently.	.....
Hours per week for mathematics.	2	5	.....	1st semester 12; 2d, 6.	.....	.....
Courses in mathematics.....	Commercial arith.	Commercial arith.	Arith., alg., geom., trig., projection drawing.	Arith., alg., trig., geometrical drawing.	Arith., alg., plane and solid geom., analytics calculus.	Arith., geom., trig.

Commercial and industrial schools in Finland.

Requirements.	Schools for commercial employees.	Schools of commerce.	Commercial institutes.	Manual training schools.	Lower and higher trade schools.	Vocational schools.	Industrial schools.
Years of previous school attendance required.	6	6	9	.....	6	6	6, and 1 year in shop.
Approximate age at entrance.	14	16	16 or 17	.....	13 or 14	13 or 14	17 or over.
Number of years in the course.	1 or 2	2	2 or 3	1 or 2	1, 2, or 3	2	2 years of 6 mos. each.
Hours per week in school...	12	.....	.....	.....	Part time.	12 to 20	39 or 40.
Hours per week for mathematics.	.....	.....	.....	.....	.....	.....	.....
Courses in mathematics.....	Arith.	Arith., geom.	Arith., alg., geom.	Arith.	Arith., alg., descriptive geom.	Arith., alg., geom., descriptive geom.	Arith., alg., geom., descriptive geom., mechanics.

Commercial and industrial schools in France.

Requirements.	Higher primary schools for boys.		Practical schools of commerce and industry.		National vocational schools.	Vocational schools for girls.
	Commercial section.	Industrial section.	Commercial section.	Industrial section.		
Years of previous school attendance required.	7	7	6 or 7	6 or 7	6	6
Approximate age at entrance..	13	13	12 or 13	12 or 13	12 to 15	12 to 15
Number of years in course.....	3	3	3	3	4	3
Hours per week in school.....	28 1st year, 32 2d, 32 3d.	28 1st year, 35 2d, 37 3d.	Full time.	Full time.	40 1st year, 44 2d, 47 3d, 50 4th.	Full time.
Hours per week for mathematics.	4 1st year, 3 2d, 3 3d.	4 1st year, 3 2d, 3 3d.	6 1st year, 5 2d, 3 3d.	6 1st year, 5 2d, 3 3d.	6 1st year, 6 2d, 6 3d, 3½ 4th.	1½ 1st year, 1 2d, 1½ 3d.
Courses in mathematics.....	Arith., alg., geom.	Arith., alg., geom., mechanics.	Arith., alg.	Arith., alg., geom., descriptive geom., mechanics.	Arith., alg., geom., trig., descriptive geom.	Arith., geom.

Commercial and industrial schools in Germany.

Requirements.	Commercial continuation schools.	Commercial preparatory schools.	Higher commercial courses.	Commercial Realschulen.	Industrial continuation schools.	Trade schools for the metal industries.
Years of previous school attendance required.	8	8	9	6	8	8
Approximate age at entrance	14	14	15	13	14	14 or over.
Number of years in course	3	1	1	3	3	2 or 3.
Hours per week in school	6 to 12	24 to 33	35	30	6 to 9	Full time.
Hours per week for mathematics.	2	4 to 6	4 to 6	1st and 2d years, 6; 3d year, 5.	1 or 2 each year.	Probably 4 each year.
Courses in mathematics	Commercial arith.	Commercial arith.	Commercial arith.	Arith., alg., plane and solid geom., trig.	Arith., alg., geom.	Arith., alg., geom., trig.

	Factory schools.	Evening and Sunday classes for artisans.	Middle technical schools.	Navigation schools.
Years of previous school attendance required	8	8 plus continuation school.	9	8
Approximate age at entrance	14 or over	17 or over	17 or over	.....
Number of years in course	2 or 3	.....	1½ to 2½	.....
Hours per week in school	Part time.	Part time.	Full time.	.....
Hours per week for mathematics	Probably 2 for 2 years.	.....	4 or 5 for 2 years.	.....
Courses in mathematics	Arith., alg., geom.	Arith., alg., geom., trig., projection drawing.	Alg., plane and solid geom., trig., analytical geom.	Alg., plane and solid geom., plane and spherical trig.

Commercial and industrial schools in Holland.

Requirements.	Burgeravondscholen.	Schools of design.	Vocational and technical schools.	Navigation schools.
Years of previous school attendance required	6	6	6	7 or 9
Approximate age at entrance	12	12	12	14 to 17
Number of years in course	2 to 4	3 to 5	3	2
Hours per week in school	6 to 15	8 to 15	40 to 44	Full time.
Hours per week for mathematics	1st year 4, 2d 4, 3d 3, 4th 2.	Same as Burgeravondscholen, apparently.	1st year 4, 2d 3, 3d 3.	13, each year.
Courses in mathematics	Arith., Alg., mechanics.	Plane and solid geom., descriptive geom.		Solid geom., plane and spherical trig., analytical geom., navigation.

Commercial and industrial schools in Hungary.

Requirements.	Higher commercial schools.	Trade schools.	Higher industrial schools.
Years of previous school attendance required	6	8	8, plus 1 year of practice.
Approximate age at entrance	14 or 15	13	16
Number of years in course	3	3	
Hours per week in school	Full time.	Full time.	Full time.
Hours per week for mathematics	1st year 6, 2d 5, 3d 6.	1st year 2, 2d 2, 3d 1.	
Courses in mathematics	Arith., alg., geom.	Arith., descriptive geom.[1]	Alg., plane and solid geom., trig., descriptive geom.

[1] The courses for watchmakers and *Feinmechaniker* in the trade schools give the same courses in mathematics as the higher industrial schools.

Commercial and industrial schools in Italy.

Requirements.	Practical schools of agriculture.	Higher schools of agriculture.	Schools of mining.	Lower commercial schools.	Intermediate commercial schools.
Years of previous school attendance required.	4	12	Admission by examination.	4	8
Number of years in the course					2, 3, or 4.
Courses in mathematics	Arith., geom.	Arith., alg., geom., mechanics.	Alg., geom., descriptive geom., mechanics.	Arith., alg., geom.	Arith., Alg., geom.

Commercial and industrial schools in Japan.

Requirements.	Commercial schools, class B.	Commercial schools, class A.	Commercial colleges.	Technical schools of middle grade.
Years of previous school attendance required.	6	9	11	6
Approximate age at entrance.	12	14	17	Average age, 15
Number of years in the course.	3 or less.	4 or 5	3 or 4	3 or 4
Hours per week in school.	Full time.	Full time.	Full time.	Full time.
Hours per week for mathematics.	4 or 5 each year.		1st year, 3; 2d, 2; 3d, 3; 4th, —.	(See table, p. 67.)
Courses in mathematics	Commercial arith.	Commercial arith., alg., geom.	Math. of finance.	Arith., alg., geom., trig.

Requirements.	High technical schools.	Schools for land surveyors, lower course.	Schools for land surveyors, higher course.
Years of previous school attendance required.	11	11	
Approximate age at entrance.		Average age, 25	Average age, 33
Number of years in the course.	3	1	2
Hours per week in school.	Full time.	Full time.	Full time.
Hours per week for mathematics.	(See table, p. —.)	(See table, p. —.)	(See table, p. 69.)
Courses in mathematics.	Alg., geom., trig., plane and solid analytical geom., calculus, differential equations.	Alg., solid geom., plane and spherical trig., analytical geom., least squares.	Alg., plane and solid analytics, calculus, differential equations, dynamics, least squares.

Commercial and industrial schools in Russia.

Requirements.	Schools for apprentices.	Elementary trade schools.	Trade schools.	Lower technical schools.	Secondary technical schools.
Years of previous school attendance required.				7	
Approximate age at entrance.	12 to 15	12 to 15			
Number of years in the course.	3	4	3	3	4 or less.
Hours per week in school.	Part time.				
Hours per week for mathematics.		1st year, 2; 2d, 1.			
Courses in mathematics.	Arith., geom.	Arith.	Arith., geom., geometrical drawing.	Arith., alg., geom., mechanics, projection drawing.	Alg., geom., trig., analytical geom., mechanics, geometrical drawing.

Commercial and industrial schools in Spain.

Requirements.	Prepatory commercial schools.	Elementary commercial schools.	Higher commercial schools.	Schools of arts and industries.	Navigation schools.
Courses in mathematics.	Arith., alg.	Arith., alg.	Alg., commercial calculation.	Arith., alg., geom., descriptive geom., trig., mechanics.	Arith., alg., geom., trig.

Commercial and industrial schools in Sweden.

Requirements.	Elementary industrial schools.	Middle technical schools.
Years of previous school attendance required	6	9
Approximate age at entrance	14	18
Number of years in the course	2 or 3	3
Hours per week in school	Part time.	Full time.
Hours per week for mathematics	Probably 2 for 2 years.	.....
Courses in mathematics	Arith., alg., geom.	Arith., alg., plane and solid geom., trig., analytical geom., descriptive geom.

Commercial and industrial schools in Switzerland.

Requirements.	Complementary commercial instruction.	Secondary commercial instruction.	Higher schools of commerce.	Schools of administration and for the railway service.
Years of previous school attendance required.	6	6	8	8
Approximate age at entrance	14 or over.	14 or over.	14 or over.	14 or 15
Number of years in the course	3	1 or 2	3 or 4	2 or 3
Hours per week in school	Part time.	Part time.	26	Full time.
Hours per week for mathematics	1 or 2 for 3 years.	3	1st year, 3; 2d, 4; 3d, 3; 4th, 3.	1st year, 6; 2d, 3; 3d, 3.
Courses in mathematics	Commercial arith.	Commercial arith.	Arith., alg., geom., trig.	Arith., alg., geom.

Requirements.	Vocational courses.	Permanent vocational schools.	Middle technical schools.
Years of previous school attendance required.	6 to 8	6 to 8	9
Approximate age at entrance	14	14 or 15	15
Number of years in the course	2 or 3	2, 3, or 4	2½ or 3
Hours per week in school	4 to 7	52 to 56	40
Hours per week for mathematics	2 to 4	4 each year, approximately.	1st year, 12 to 15; 2d, 13 to 16; 3d, 2 to 4.
Courses in mathematics	Arith., geom., projection drawing.	Arith., alg., geom., trig., mechanics, projection drawing.	Arith., alg., geom., trig., analytical geom., calculus, mechanics, descriptive geom., projection drawing.

Commercial and industrial schools in the United States.

Requirements.	Private commercial schools.	Commercial departments in high schools.	Commercial high schools.	Intermediate industrial, preparatory trade, or vocational schools.	Trade schools with day courses.	Apprenticeship schools.
Years of previous school attendance required.	Usually 8	8	8	6 to 8	6 to 8	6 to 8
Approximate age at entrance.	14 or over.	14	14	14	14	14 or over.
Number of years in the course.	1 to 2.	.....	4	.....	.....	.....
Hours per week in school.	Full time.	Full time.	Full time.	Full time.	Full time.	.....
Hours per week for mathematics.	5 for 1 year.	5 for 3 years.	5 for 4 years.	.....	.....	.....
Courses in mathematics.	Commercial arithmetic.	Commercial arith., alg., geom.	Commercial arith., alg., geom.	Shop mathematics.	Arith., alg., geom., trig.	Arith., alg., geom., trig.

Requirements.	Evening schools.	Part-time schools.	Trade schools for the colored races.	Technical schools with day courses.	Manual-training and technical high schools.	Agricultural high schools.
Years of previous school attendance required.	6 or 8.	8	.....	12	8	8
Approximate age at entrance.	Various.	14 to 16.	17	18	14	14
Number of years in the course.	.....	4	4	4	4	4
Hours per week in school.	.....	.....	Full time.	Full time.	Full time.	Full time.
Hours per week for mathematics.	.....	.....	4 for 3 years.	5 for 3 years.	5 for 3 or 4 years.	5 for 3 years.
Courses in mathematics.	Alg., arith., geom., trig., analytical geom., mechanics.	.....	Arith., alg., geom., applied math.	Alg., trig., analytical geom., calculus, mechanics.	Alg., plane and solid geom., trig.	Arith., alg., geom.

NAMES OF TYPES OF SCHOOLS, WITH EQUIVALENTS USED IN THIS REPORT.

In reading such a report as the present, it is often desirable to be able to determine the exact name, in the foreign language, of the schools under consideration. In order to avoid the frequent repetition, in the body of the report, of the names of the schools, the following tables of equivalents have been prepared.

There are given below the names, as they appear in the reports of the International Commission, of the types of schools in the different countries, and the terms used as equivalents in this report:

AUSTRIA.

Höhere Handelsschulen	Higher Commercial Schools.
Gewerbliche Fortbildungsschulen	Industrial Continuation Schools.
Spezialkurse für Meister und Gehilfen	Special Courses for Masters and Assistants.
Allgemeine Handwerkerschulen	General Trade Schools.
Fachschulen für einzelne gewerbliche Zweige.	Trade Schools for Particular Trades.
Bau- und Kunsthandwerkerschulen	Schools for the Building Trades and the Mechanical Arts.
Werkmeisterschulen	Schools for Master Workmen.
Höhere Gewerbeschulen	Higher Industrial Schools.
Höhere Fachschulen für Textilindustrie.	Higher Schools for the Textile Industry.
Frauengewerbeschulen	Industrial Schools for Women.
Koch- und Haushaltungsschulen	Schools for Domestic Science.

DENMARK.

Elementarschulen für Technik, Handel, und Seefahrt.	Elementary Technical, Commercial, and Navigation Schools.
Tagesschule für Bauhandwerker	Day School for Builders.
Tagesschulen für Maschinenkonstrukteure und Elektrotechniker.	Day Schools for Machine Constructors and Electrotechnologists.
Veterinär- und landwirtschaftliche Hochschule.	Veterinary and Agricultural High School.

FINLAND.

Écoles pour les Employeés de Commerce	Schools for Commercial Employees.
Écoles de Commerce	Schools of Commerce.
Instituts Commerciaux	Commercial Institutes.
Écoles Techniques Inférieures	Lower Technical Schools.
Écoles de Travaux Manuels	Manual Training Schools.
Écoles de Métiers Inférieures et Supérieures.	Lower and Higher Trade Schools.
Écoles Professionelles	Vocational Schools.
Écoles Industrielles	Industrial Schools.

FRANCE.

Écoles Primaires Supérieures de Garçons..	Higher Primary Schools for Boys.
Écoles Pratiques de Commerce et d'Industrie.	Practical Schools of Commerce and Industry.
Écoles Nationales Professionelles.....	National Vocational Schools.
Écoles Pratiques de Commerce.....	Practical Schools of Commerce.
Écoles Professionelles et Ménagères.....	Écoles Professionelles et Ménagères.

GERMANY.

Kaufmännische Fortbildungsschulen.....	Commercial Continuation Schools.
Handelsvorschulen.....	Commercial Preparatory Schools.
Höhere Handelskurse.....	Higher Commercial Courses.
Handelsrealschulen.....	Commercial Realschulen.
Gewerbliche Fortbildungsschulen.....	Industrial Continuation Schools.
Fachschulen für die Metallindustrie.....	Trade Schools for the Metal Industries.
Werkschulen.....	Factory Schools.
Abend- und Sonntagsklassen für Handwerker.	Evening and Sunday Classes for Artisans.
Technische Mittelschulen.....	Middle Technical Schools.
Navigationsschulen.....	Navigation Schools.

HOLLAND.

Burgeravondscholen.....	Burgervondscholen.
Écoles Professionelles.....	Vocational Schools.
Écoles de Dessin.....	Schools of Design.
Écoles Techniques.....	Technical Schools.
Écoles de Marine.....	Navigation Schools.
Enseignement Moyen Technique.....	Middle Technical Instruction.
Écoles Professionelles pour Filles.....	Vocational Schools for Girls.

HUNGARY.

Höhere Handelsschulen.....	Higher Commercial Schools.
Gewerbliche Fachschulen.....	Trade Schools.
Höhere Gewerbeschulen.....	Higher Industrial Schools.

ITALY.

Scuole Agraria Pratiche.....	Practical Schools of Agriculture.
Scuole Agraria Superiori.....	Higher Schools of Agriculture.
Scuole Minerarie.....	Schools of Mining.
Scuole di Commercio Inferiori.....	Lower Commercial Schools.
Scuole di Commercio Superiori.....	Higher Commercial Schools.

RUSSIA.

Écoles d'Apprentis.....	Schools for Apprentices.
Écoles Élémentaires de Métiers.....	Elementary Trade Schools.
Écoles de Métiers.....	Trade Schools.
Écoles Techniques Inférieures.....	Lower Technical Schools.
Écoles Techniques Secondaires.....	Secondary Technical Schools.

SWEDEN.

Elementartechnische Gewerbeschulen..	Elementary Industrial Schools.
Technische Mittelschulen.....	Middle Technical Schools.

SWITZERLAND.

Enseignement Commercial Complémentaire.	Complementary Commercial Instruction.
Enseignement Commercial Secondaire..	Secondary Commercial Instruction.
Écoles Supérieures de Commerce.....	Higher Schools of Commerce.
Écoles d'Administration et de Chemins de Fer.	Schools of Administration and for the Railway Service.
Cours Professionels.....	Vocational Courses.
Écoles Professionelles Permanentes....	Permanent Vocational Schools.
Écoles Techniques Moyennes.....	Middle Technical Schools.

BULLETIN OF THE BUREAU OF EDUCATION.

[NOTE.—With the exceptions indicated, the documents named below will be sent free of charge upon application to the Commissioner of Education, Washington, D. C. Those marked with an asterisk (*) are no longer available for free distribution, but may be had of the Superintendent of Documents, Government Printing Office, Washington, D. C., upon payment of the price stated. Remittances should be made in coin, currency, or money order. Stamps are not accepted. Numbers omitted are out of print.]

1908.

*No. 3. State school systems: Legislation and judicial decisions relating to public education, Oct. 1, 1904, to Oct. 1, 1906. Edward C Elliott. 15 cts.

1908.

*No. 5. Education in Formosa. Julean H. Arnold. 10 cts.
*No. 6. The apprenticeship system in its relation to industrial education. Carroll D. Wright. 15 cts.
No. 8. Statistics of State universities and other institutions of higher education partially supported by the State, 1907-1908.

1909.

*No. 1. Facilities for study and research in the offices of the United States Government in Washington. Arthur T. Hadley. 10 cts.
No. 2. Admission of Chinese students to American colleges. John Fryer.
*No. 3. Daily meals of school children. Caroline L. Hunt. 10 cts.
No. 5. Statistics of public, society, and school libraries in 1908.
*No. 6. Instruction in the fine and manual arts in the United States. A statistical monograph. Henry T. Bailey. 15 cts.
No. 7. Index to the Reports of the Commissioner of Education, 1867-1907.
*No. 8. A teacher's professional library. Classified list of 100 titles. 5 cts.
*No. 9. Bibliography of education for 1908-9. 10 cts.
No. 10. Education for efficiency in railroad service. J. Shirley Eaton.
*No. 11. Statistics of State universities and other institutions of higher education partially supported by the State, 1908-9. 5 cts.

1910.

*No. 1. The movement for reform in the teaching of religion in the public schools of Saxony. Arley B. Show. 5 cts.
No. 2. State school systems: III. Legislation and judicial decisions relating to public education, Oct. 1, 1908, to Oct. 1, 1909. Edward C. Elliott.
*No. 5. American schoolhouses. Fletcher B. Dressler. 75 cts.

1911.

*No. 1. Bibliography of science teaching. 5 cts.
*No. 2. Opportunities for graduate study in agriculture in the United States. A. C. Monahan. 5 cts.
*No. 3. Agencies for the improvement of teachers in service. William C. Ruediger. 15 cts.
No. 4. Report of the commission appointed to study the system of education in the public schools of Baltimore. 10 cts.
*No. 5. Age and grade census of schools and colleges. George D. Strayer. 10 cts.
*No. 6. Graduate work in mathematics in universities and in other institutions of like grade in the United States. 5 cts.
No. 7. Undergraduate work in mathematics in colleges and universities.
No. 9. Mathematics in the technological schools of collegiate grade in the United States.
*No. 13. Mathematics in the elementary schools of the United States. 15 cts.
*No. 14. Provision for exceptional children in the public schools. J. H. Van Sickle, Lightner Witmer, and Leonard P. Ayres. 10 cts.
*No. 15. Educational system of China as recently reconstructed. Harry E. King. 10 cts.
No. 19. Statistics of State universities and other institutions of higher education partially supported by the State, 1910-11.

1912.

*No. 1. A course of study for the preparation of rural-school teachers. F. Mutchler and W. J. Craig. 5 cts.
*No. 3. Report of committee on uniform records and reports. 5 cts.
*No. 4. Mathematics in technical secondary schools in the United States. 5 cts.
*No. 5. A study of expenses of city school systems. Harlan Updegraff. 10 cts.
*No. 6. Agricultural education in secondary schools. 10 cts.
*No. 7. Educational status of nursing. M. Adelaide Nutting. 10 cts.

*No. 8. Peace day. Fannie Fern Andrews. 5 cts. [Later publication, 1913, No. 12. 10 cts.]
*No. 9. Country schools for city boys. William S. Myers. 10 cts.
No. 11. Current educational topics, No. 1.
*No. 13. Influences tending to improve the work of the teacher of mathematics. 5 cts.
*No. 14. Report of the American commissioners of the international commission on the teaching of mathematics. 10 cts.
*No. 17. The Montessori system of education. Anna T. Smith. 5 cts.
*No. 18. Teaching language through agriculture and domestic science. M. A. Leiper 5 cts.
*No. 19. Professional distribution of college and university graduates. Bailey B. Burritt. 10 cts.
No. 22. Public and private high schools.
*No. 23. Special collections in libraries in the United States. W. D. Johnston and I. G. Mudge. 10 cts.
No. 26. Bibliography of child study for the years 1910–11.
No. 27. History of public-school education in Arkansas. Stephen B. Weeks.
*No. 28. Cultivating school grounds in Wake County, N. C. Zebulon Judd. 5 cts.
No. 29. Bibliography of the teaching of mathematics, 1900–1912. D. E. Smith and Chas. Goldziher.
No. 30. Latin-American universities and special schools. Edgar E. Brandon.

1913.

No. 1. Monthly record of current educational publications, January, 1913.
*No. 2. Training courses for rural teachers. A. C. Monahan and R. H. Wright. 5 cts.
*No. 3. The teaching of modern languages in the United States. Charles H. Handschin. 15 cts.
*No. 4. Present standards of higher education in the United States. George E. MacLean. 20 cts.
No. 5. Monthly record of current educational publications, February, 1913.
*No. 6. Agricultural instruction in high schools. C. H. Robison and F. B. Jenks. 10 cts.
*No. 7. College entrance requirements. Clarence D. Kingsley. 15 cts.
*No. 8. The status of rural education in the United States. A. C. Monahan. 15 cts.
No. 11. Monthly record of current educational publications, April, 1913.
*No. 12. The promotion of peace. Fannie Fern Andrews. 10 cts.
*No. 13. Standards and tests for measuring the efficiency of schools or systems of schools. 5 cts.
No. 15. Monthly record of current educational publications, May, 1913.
*No. 16. Bibliography of medical inspection and health supervision. 15 cts.
*No. 18. The fifteenth international congress on hygiene and demography. Fletcher B. Dresslar. 10 cts.
No. 19. German industrial education and its lessons for the United States. Holmes Beckwith.
*No. 20. Illiteracy in the United States. 10 cts.
No. 21. Monthly record of current educational publications, June, 1913.
*No. 22. Bibliography of industrial, vocational, and trade education. 10 cts.
*No. 23. The Georgia club at the State Normal School, Athens, Ga., for the study of rural sociology. E. C. Branson. 10 cts.
*No. 24. A comparison of public education in Germany and in the United States. Georg Kerschensteiner. 5 cts.
*No. 25. Industrial education in Columbus, Ga. Roland B. Daniel. 5 cts.
*No. 26. Good roads arbor day. Susan B. Sipe. 10 cts.
*No. 28. Expressions on education by American statesmen and publicists. 5 cts.
*No. 29. Accredited secondary schools in the United States. Kendric C. Babcock. 10 cts.
*No. 30. Education in the South. 10 cts.
*No. 31. Special features in city school systems. 10 cts.
No. 32. Educational survey of Montgomery County, Md.
*No. 34. Pension systems in Great Britain. Raymond W. Sies. 10 cts.
*No. 35. A list of books suited to a high-school library. 15 cts.
*No. 36. Report on the work of the Bureau of Education for the natives of Alaska, 1911–12. 10 cts.
No. 37. Monthly record of current educational publications, October, 1913.
*No. 38. Economy of time in education. 10 cts.
No. 39. Elementary industrial school of Cleveland, Ohio. W. N. Hailmann.
*No. 40. The reorganized school playground. Henry S. Curtis. 10 cts.
*No. 41. The reorganization of secondary education. 10 cts.
No. 42. An experimental rural school at Winthrop College. H. S. Browne.
*No. 43. Agriculture and rural-life day; material for its observance. Eugene C. Brooks. 10 cts.
*No. 44. Organized health work in schools. E. B. Hoag. 10 cts.
No. 45. Monthly record of current educational publications, November, 1913.
*No. 46. Educational directory, 1913. 15 cts.
*No. 47. Teaching material in Government publications. F. K. Noyes. 10 cts.
*No. 48. School hygiene. W. Carson Ryan, jr. 15 cts.
No. 49. The Farragut School, a Tennessee country-life high school. A. C. Monahan and Adams Phillips.
*No. 50. The Fitchburg plan of cooperative industrial education. M. R. McCann. 10 cts.
*No. 51. Education of the immigrant. 10 cts.
*No. 52. Sanitary schoolhouses. Legal requirements in Indiana and Ohio. 5 cts.
No. 53. Monthly record of current educational publications, December, 1913.
No. 54. Consular reports on industrial education in Germany.

No. 55. Legislation and judicial decisions relating to education, Oct. 1, 1909, to Oct. 1, 1912. James C. Boykin and William R. Hood.
No. 58. Educational system of rural Denmark. Harold W. Foght.
No. 59. Bibliography of education for 1910–11.
No. 60. Statistics of State universities and other institutions of higher education partially supported by the State, 1912–13

1914.

*No. 1. Monthly record of current educational publications, January, 1914. 5 cts.
No. 2. Compulsory school attendance.
*No. 3. Monthly record of current educational publications, February, 1914. 5 cts.
No. 4. The school and the start in life. Meyer Bloomfield.
No. 5. The folk high schools of Denmark. L. L. Friend.
No. 6. Kindergartens in the United States.
No. 7. Monthly record of current educational publications, March, 1914.
*No. 8. The Massachusetts home-project plan of vocational agricultural education. R. W. Stimson. 15 cts.
No. 9. Monthly record of current educational publications, April, 1914.
No. 10. Physical growth and school progress. B. T. Baldwin.
*No. 11. Monthly record of current educational publications, May, 1914. 5 cts.
*No. 12. Rural schoolhouses and grounds. F. B. Dresslar. 50 cts.
No. 13. Present status of drawing and art in the elementary and secondary schools of the United States. Royal B. Farnum.
No. 14. Vocational guidance.
No. 15. Monthly record of current educational publications. Index.
No. 16. The tangible rewards of teaching. James C. Boykin and Roberta King.
No. 17. Sanitary survey of the schools of Orange County, Va. Roy K. Flannagan.
No. 18. The public-school system of Gary, Ind. William P. Burris.
No. 19. University extension in the United States. Louis E. Reber.
No. 20. The rural school and hookworm disease. J. A. Ferrell.
No. 21. Monthly record of current educational publications, September, 1914.
No. 22. The Danish folk high schools. H. W. Foght.
No. 23. Some trade schools in Europe. Frank L. Glynn.
No. 24. Danish elementary rural schools. H. W. Foght.
No. 25. Important features in rural school improvement. W. T. Hodges.
No. 26. Monthly record of current educational publications, October, 1914.
*No. 27. Agricultural teaching, 15 cts.
No. 28. The Montessori method and the kindergarten. Elizabeth Harrison.
No. 29. The kindergarten in benevolent institutions.
*No. 30. Consolidation of rural schools and transportation of pupils at public expense. A. C. Monahan. 25 cts.
*No. 31. Report on the work of the Bureau of Education for the natives of Alaska. 25 cts.
No. 32. Bibliography of the relation of secondary schools to higher education. R. L. Walkley.
No. 33. Music in the public schools. Will Earhart.
No. 34. Library instruction in universities, colleges, and normal schools. Henry R. Evans.
No. 35. The training of teachers in England, Scotland, and Germany. Charles H. Judd.
*No. 36. Education for the home—Part I. General statement. B. R. Andrews. 10 cts.
*No. 37. Education for the home—Part II. State legislation, schools, agencies. B. R. Andrews. 30 cts.
No. 38. Education for the home—Part III. Colleges and universities. B. R. Andrews.
No. 39. Education for the home—Part IV. Bibliography, list of schools. B. R. Andrews.
No. 40. Care of the health of boys in Girard College, Philadelphia, Pa.
No. 41. Monthly record of current educational publications, November, 1914.
No. 42. Monthly record of current educational publications, December, 1914.
No. 43. Educational directory, 1914–15.
No. 44. County-unit organization for the administration of rural schools. A. C. Monahan.
No. 45. Curricula in mathematics. J. C. Brown.
No. 46. School savings banks. Mrs. Sara L. Oberholtzer.
No. 47. City training schools for teachers. Frank A. Manny.
No. 48. The educational museum of the St. Louis public schools. C. G. Rathman.
No. 49. Efficiency and preparation of rural school-teachers. H. W. Foght.
No. 50. Statistics of State universities and State colleges.

1915.

No. 1. Cooking in the vocational school. Iris P. O'Leary.
No. 2. Monthly record of current educational publications, January, 1915.
No. 3. Monthly record of current educational publications, February, 1915.
No. 4. The health of school children. W. H. Heck.
No. 5. Organization of State departments of education. A. C. Monahan.
No. 6. A study of colleges and high schools.

No. 7. Accredited secondary schools in the United States. Samuel P. Capen.
No. 8. Present status of the honor system in colleges and universities. Bird T. Baldwin.
No. 9. Monthly record of current educational publications, March, 1915.
No. 10. Monthly record of current educational publications, April, 1915.
No. 11. A statistical study of the public school systems of the southern Appalachian Mountains. Norman Frost.
No. 12. History of public school education in Alabama. Stephen B. Weeks.
No. 13. The schoolhouse as the polling place. E. J. Ward.
No. 14. Monthly record of current educational publications, May, 1915.
No. 15. Monthly record of current educational publications. Index, Feb., 1914-Jan., 1915.
No. 16. Monthly record of current educational publications, June, 1915.
No. 17. Civic education in elementary schools as illustrated in Indianapolis. Arthur W. Dunn.
No. 18. Legal education in Great Britain. H. S. Richards.
No. 19. Statistics of agricultural, manual training, and industrial schools, 1913-14.
No. 20. The rural school system of Minnesota. H. W. Foght.
No. 21. Schoolhouse sanitation. William A. Cook.
No. 22. State versus local control of elementary education. T. L. MacDowell.
No. 23. The teaching of community civics.
No. 24. Adjustment between kindergarten and first grade. Luella A. Palmer.
No. 25. Public, society, and school libraries.
No. 26. Secondary schools in the States of Central America, South America, and the West Indies. Anna T. Smith.
No. 27. Opportunities for foreign students at colleges and universities in the United States. Samuel P. Capen.
No. 28. The extension of public education. Clarence A. Perry.
No. 29. The truant problem and the parental school. James S. Hiatt.
No. 30. Bibliography of education for 1911-12.
No. 31. A comparative study of the salaries of teachers and officers.
No. 32. The school system of Ontario. H. W. Foght.
No. 33. Problems of vocational education in Germany. Geo. E. Myers.
No. 34. Monthly record of current educational publications, September, 1915.

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