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CURRICULA IN MATHEMATICS

A COMPARISON OF COURSES IN THE COUNTRIES REPRESENTED IN THE INTERNATIONAL COMMISSION ON THE TEACHING OF MATHEMATICS

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INTRODUCTION.

The International Commission on the Teaching of Mathematics, created by the International Congress of Mathematicians at Rome, Italy, in 1908, submitted a large body of reports to the congress at Cambridge, England, in 1912. Those for the United States have been published as bulletins of the Bureau of Education (Bulletin, 1911, Numbers 6, 7, 8, 9, 12, 13, 16; 1912, Numbers 2, 4, 13, 14). The congress directed the commission to continue its activities and to submit further reports to the congress to be held at Stockholm, Sweden, in 1916. The following bulletin has been prepared as a part of the report of this country for the second period of the commission's labors. The work of this period will naturally consist, to a considerable extent, in a preparation by each nation of a digest of the results of the earlier work of the commission, as seen from the point of view of that nation.

No one believes in the bodily transplantation of a course of study or of methods of work from one nation to another; yet the very existence of the commission, and the phenomenal success of its work, are evidences of the widespread conviction that every nation can profit from a careful comparative study of the work of other nations.

The material that follows shows that the types of school organization and the curricula in mathematics of the leading nations of Europe have much more in common with each other than with those of the United States. European nations will therefore sometimes find in the practice of their sister nations encouraging confirmation of their own customs, whereas the United States will be confronted by the question as to whether she alone has found the path that is best, at least for herself. Some lines of divergence between the general practice in Europe and that of the United States are mentioned in the concluding remarks of this report (p. 87). Historical and other reasons for the existence of the divergencies are easily found. Reasons, sometimes of a theoretical, sometimes of a practical, nature, may doubtless also be found sufficient to warrant the present continuance of some or all of these divergencies. At the same time the possession of authoritative statements of the practice of the world's leading nations, as contained in the reports from which the material that follows is taken, can but prove helpful and stimulating to the educators of the United States. The efficient supervisor or the alert teacher will surely profit by a serious study of the curricula in other countries as set forth in this bulletin. He may find little or nothing

that is directly usable in his own environment, but he can not help being aided in his effort to improve his own work, according to local needs and conditions, by a wider knowledge of what is done in corresponding cases by the world at large.

All of the statements of fact in the bulletin are based upon reports of the international commission to the congress at Cambridge. A complete bibliography of the reports used is given on page 88. The data for the various nations have perforce been given with varying degrees of completeness, depending upon the information available in the reports. As the age at which pupils enter school varies somewhat in different countries, the age of the pupil, rather than the school year, was chosen as the basis of comparison.[1]

Unless otherwise stated, it is to be understood that the European schools are for boys only. Relatively little is as yet done in most of the European countries for the mathematical education of girls beyond the fundamentals of arithmetic. The scanty information that is available is given in appropriate connections.

[1] The public schools of the United States comprise, in most cases, an eight-year elementary course, followed by a four-year course in the secondary school. In some cases the course in the elementary school is seven years and in the secondary school five years in length. In rare instances both the elementary and the secondary school courses are six years in length.

The age of compulsory school attendance varies somewhat in the different States, but it is usually from the age of 6 or 7 to 14. Most pupils enter the elementary school at the age of 6 or 7. Both the elementary and the secondary schools are free to all pupils of the school district.

The courses in the elementary schools of the country are more nearly uniform than those of the secondary schools. In the elementary schools the attempt is made to lay the foundation for a good general education. Reading, writing, arithmetic, history, geography, elementary science, and spelling are the important subjects of the curriculum. There is but little opportunity for choice of courses in these schools. In the secondary school a pupil is sometimes permitted to select those subjects that he especially wishes. Certain subjects are often required, and the pupil is permitted to select a prescribed number of additional subjects. In many of the larger cities there are secondary schools of various types, such as commercial, manual training, and industrial.

A pupil who has completed the course in a good elementary school may enter a secondary school without examination. Many of the colleges and universities admit graduates of secondary schools upon the presentation of a diploma or a certificate, but some colleges and universities admit only by examination.

In this report the New York State course of study is rather closely followed in outlining the subject matter of the elementary school. This course is fairly typical of the best courses of the country.

Arrangement of school years in the United States.

Age of pupil.	School year.	Elementary school.	Secondary school.
6–7	First	1	
7–8	Second	2	
8–9	Third	3	
9–10	Fourth	4	
10–11	Fifth	5	
11–12	Sixth	6	
12–13	Seventh	7	
13–14	Eighth	8	
14–15	Ninth		1
15–16	Tenth		2
16–17	Eleventh		3
17–18	Twelfth		4

CURRICULA IN MATHEMATICS.

I. GENERAL ARRANGEMENT OF THE COURSES IN TYPICAL SCHOOLS OF THE VARIOUS COUNTRIES.

AUSTRIA.

Arrangement of school years.

Age of pupil.	School year.	Volks-schule.	Bürger-schule.	Gymna-sium.	Real-schule.	Real-gymna-sium.
6–7	First	1				
7–8	Second	2				
8–9	Third	3				
9–10	Fourth	4				
10–11	Fifth	5		1	1	1
11–12	Sixth		1	2	2	2
12–13	Seventh		2	3	3	3
13–14	Eighth		3	4	4	4
14–15	Ninth			5	5	5
15–16	Tenth			6	6	6
16–17	Eleventh			7	7	7
17–18	Twelfth			8		8

The minister of education has general supervision of education in all of the 14 Provinces into which Austria is divided. In each Province there is a superintendent of education, who stands midway between the minister and lower-school authorities. Each Province is divided into districts and each district has its school council. The members of this school council are nominated by the teachers of this district and appointed by the higher-school council. Each district is divided into smaller subdivisions and each of these has its local board. Compulsory education extends from the sixth to the fourteenth year. Coeducation is the rule in the rural districts and the exception in the cities.

The administration of secondary education is vested in the minister of instruction. Each Province has an inspector and a local board. All schools, whether public or private, are subject to the same regulations as the State schools. Most of the secondary schools are maintained by the State.

There are two kinds of elementary schools: (*a*) The common primary, usually called the Volksschule, and (*b*) the superior elementary school, called the Bürgerschule. Many of the Bürgerschulen in the rural communities are ungraded. In the cities the course in these schools comprises from two to eight grades. The Bürgerschulen usually consist of three classes, which follow the fifth year of the primary school. Pupils may enter directly from the primary school. Different courses are offered for boys and girls, and as far as possible the sexes are instructed in different schools.

Sometimes an intermediate course of one year follows the Bürgerschule. The nature of the course which a pupil follows during this year depends largely upon the

type of school for which he is preparing. It is now proposed to make the intermediate course an additional class of the Bürgerschule.

There are three general types of secondary schools: (*a*) The Gymnasium; (*b*) the Realschule; and (*c*) the Realgymnasium. The first has a course of eight, the second of seven, and the third of eight years, thus differing from the custom in Germany. The minimum age for admission to the Gymnasium is 10 years, so that the first year in the Austrian Gymnasium corresponds to the second year in the German Gymnasium, and four years of preliminary work are required for entrance. The average age at entrance is 10½ to 11 years.

The Gymnasium offers the traditional classical course and prepares the pupil for entering any university. In the Gymnasium more than 50 per cent of the school time is devoted to the study of Latin, Greek, history, and the mother tongue. About 25 per cent of the time is devoted to mathematics, the history of Austria, geography, physics, and chemistry. Latin and Greek are emphasized.

The Realschule attempts to furnish a thorough knowledge of the modern subjects, with special attention to the sciences. In these schools the pupil is prepared for the study of more advanced natural science and mathematics; no attempt is made to prepare him for any particular vocation.

The course in the Realgymnasium is intended for those who wish some secondary education but do not expect to enter a university or a higher technical school. A graduate of the Realgymnasium may enter certain classes of the Gymnasium or of the Realschule.

A type of secondary school called the Reform Gymnasium, standing intermediate in courses offered between the classical Gymnasium and the Realschule, was organized in 1908. It has an eight-year course. The four lower classes are identical with the corresponding classes of the Realschule. In the four upper classes Latin, a modern language, and free-hand drawing are required; in other respects the course is like the upper grades of the Gymnasium. Greek is not taught in the Reform Gymnasium.

BELGIUM.

Arrangement of school years.

Age of pupil.	School year.	Primary school.	Middle school.	Athénée royal.
6-7	First	1		
7-8	Second	2		
8-9	Third	3		
9-10	Fourth	4		
10-11	Fifth	5		
11-12	Sixth	6		
12-13	Seventh		1	1
13-14	Eighth		2	2
14-15	Ninth		3	3
15-16	Tenth			4
16-17	Eleventh			5
17-18	Twelfth			6
18-19	Thirteenth			7

The primary school in Belgium comprises the first six school years. To be admitted, a pupil must have passed the sixth year of his age by October of the year he enters school and must be less than 14 years of age.

The law provides that at least one primary school must be maintained in each commune. Children of indigent parents must be taught gratuitously. There are now more than 7,000 primary schools under State supervision. About one-third of the number are coeducational.

The King appoints an inspector for the primary schools of each of the Provinces of the country. The primary schools are usually divided into three subdivisions: (*a*) The elementary, (*b*) intermediate, (*c*) superior, each of which comprises two years. The primary schools are under the control of the commune, but the State reserves the right of inspection. After completing the prescribed work of the primary school—that is, at about the age of 12, a pupil may enter either the middle school or the Athénée Royal. The lowest class in each of these schools corresponds, therefore, to our seventh grade.

The course in the middle schools is three years in length, corresponding to our seventh, eighth, and ninth grades. These schools were created to meet the needs of the higher artisan and the commercial classes, and emphasis is placed upon immediate utility. The diploma is valuable in many business pursuits. Attendance at the middle school is compulsory for all who expect to take a Government position. Pupils leaving these schools are qualified for the lower public positions and for positions in commercial, industrial, and mechanical arts. The usual age of admission to the middle school is 12 years. The pupil must pass an examination before he is admitted.

The middle schools, like the primary schools, are under the control of the commune, but the State exercises the right of inspection. A pupil who has completed the course in the middle schools, and who desires to prepare for the university, may enter the fourth from the upper class in the Athénée. There are now about 80 middle schools for boys and 40 for girls.

The Athénées Royaux are the official schools of higher grade, and they have a course seven years in length. They are usually for boys only. To be admitted to the lowest class of these schools a pupil must be at least 11 years of age and must pass an entrance examination. In general, however, the lowest class corresponds to our seventh grade, the age of the pupil being about 12 years. The boy leaves at about the age of 19, after 13 years in school. These institutions are supported by the State and are independent of the commune. They correspond rather closely, in some respects, to the secondary schools of the United States.

There are three kinds of Athénées Royaux: (*a*) The humanistic, with seven years of Latin and five years of Greek; (*b*) the Latin humanities, with seven years of Latin and no Greek, but with an extensive course in mathematics; (*c*) the modern humanities with seven years devoted to a modern language. The course in the modern humanities, is frequently divided into two sections in the three upper classes. These sections are (*a*) scientific and (*b*) commercial.

Pupils who complete any of the three courses in the Athénée Royal and pass a final examination receive a diploma which admits them to the university. Graduates of the classical course are admitted to any faculty in any of the universities. Graduates of the Latin-scientific course are admitted to the higher schools of mining, engineering, and manufacturing. Graduates of the modern humanities are admitted to the commercial and consular sections in the Universities of Ghent and Liege.

The organization of parallel courses equivalent to the established classical course follows French rather than German precedent. In Germany the various types of courses are found in different schools: The Gymnasium, Realgymnasium, and Realschule.

The secondary schools of Belgium are a very important factor of the national life. The opportunity to secure a good education is offered to all, and no social or class distinctions determine the kind of education that a boy or girl shall receive. The great intellectual leaders of the country are usually products of the Athénée. In Belgium the classical course is still regarded as a dignified and scholarly course, but the utilitarian subjects are regarded as of equal importance and dignity with the classics.

DENMARK.

Arrangement of school years.

Age of pupil.	School year.	For-skole.	Mellem-skole or inter-mediate.	Real-classe.	Gym-nasium.
6–7	First	1			
7–8	Second	2			
8–9	Third	3			
9–10	Fourth	4			
10–11	Fifth	5			
11–12	Sixth	6	1		
12–13	Seventh	7	2		
13–14	Eighth	(8)	3		
14–15	Ninth		4		
15–16	Tenth			1	1
16–17	Eleventh				2
17–18	Twelfth				3

Subdivisions of the Gymnasium—Periods per week devoted to mathematics in each.

	First year.	Second year.	Third year.
Classical course	2	2	2
Modern language course	2	2	2
Mathematical-scientific course	6	6	6

The schools of Denmark have always been closely associated with the church, but in recent years the church control has been largely nominal. The clergy still continue to instruct the rural inhabitants in the sparsely settled sections of the country. The bishop and the clergy aid in the selection of teachers and in the general administration of the schools.

The schools are under the control of civil authorities. The minister of ecclesiastical affairs and public instruction has supervision of the entire educational system, including the university. Certain rights are vested in the local school authorities. It is the duty of the minister of public instruction to inspect and regulate the schools—to gather statistics and to apportion the school fund. Each of the 18 counties has its own school council, and each of the 60 districts has its school board, which appoints teachers, aids in selecting books, and arranges the course of study. Each commune and village has its school commission, which looks after the individual school.

The elementary schools are called Forskoler. Compulsory school attendance begins at the age of 7 and ends at 14. Most of these schools are free, and books are usually furnished without cost. In most of the rural districts the schools are coeducational, but this is not true of the schools in the larger cities.

The minimum length of the school year is 240 days of six periods each.

There are three divisions of the primary school. The first division is of three years; the second, two years; and the third, two years. The municipal school of Copenhagen is regarded as the best in the country, and the methods and courses of study used there are freely copied elsewhere. Coeducation has been introduced into some of the elementary schools of Copenhagen with marked success. The course at Copenhagen is seven years in length; in some parts of the country it is eight.

The middle and secondary schools are in process of development, and it is difficult to draw sharp distinctions between the various kinds of schools. In some of the municipal elementary schools the highest classes overlap some of the classes of the intermediate schools. In Copenhagen there are continuation schools where pupils

who have finished the first seven school years may secure further instructions in certain subjects. A short course in mathematics and its practical applications is given. These schools are growing rapidly.

The lowest four classes of the higher schools comprise the intermediate schools. A pupil enters the intermediate school at the age of 11 or 12 and completes the course at the age of 15 or 16. The intermediate schools are called "Mellemskole." They were established in 1903 in the attempt to eliminate the abrupt break between the elementary and the secondary schools. Most of the schools of this type outside of Copenhagen are coeducational.

After completing the four years of the Mellemskole, a pupil may enter the Realclasse, which continues for one year, or he may enter the three-year Gymnasium, which prepares for the university.

The Gymnasium course is composed of three subdivisions: (*a*) The classical course, (*b*) the modern language course, (*c*) the mathematical–scientific course. In all secondary schools the recitation period is 50 minutes. The summer vacation begins early in July.

There are a number of private schools in Denmark, called Folkehoiskole. They follow the intermediate school and are largely cultural in aim. The mathematical instruction in these schools is usually limited to facility in calculation and correlation with the natural sciences.

FINLAND.

Arrangement of school years.

Age of pupil.	School year.	Primary school.	Lycée.
7–8	First	1	
8–9	Second	2	
9–10	Third	3	
10–11	Fourth	4	
11–12	Fifth	5	1
12–13	Sixth	6	2
13–14	Seventh	7	3
14–15	Eighth		4
15–16	Ninth		
16–17	Tenth		
17–18	Eleventh		7
18–19	Twelfth		(8)

The elementary primary schools of Finland are coeducational; the higher primary schools are not. A small fee is charged at all primary schools. Education is not compulsory beyond the elementary school.

The primary schools in cities are usually divided into three periods: (*a*) The elementary primary school of two years; (*b*) the ordinary primary school of four years; (*c*) the complementary school of one year. A pupil must be at least 9 years of age before he can enter the ordinary primary. The age for entering the complementary school varies from 14 to 17 years.

As the population of Finland is largely rural, and exceptionally homogeneous, the courses in all primary schools are practically the same.

There are four types of secondary schools: (*a*) The classical lycée, (*b*) the real lycée, (*c*) the preparatory schools, (*d*) finishing schools (for girls only).

There are 26 State lycées for boys and 16 for girls. The pupils who enter the lycées are from 9 to 12 years of age. In the real lycée one more hour a week is given to mathematics than in the classical lycée, and much more emphasis is placed upon the study of physics.

FRANCE.

Arrangement of school years.

Age of pupil.	School year.	Primary school.	Higher primary.	Lycée.
6–7	First	1		
7–8	Second	2		
8–9	Third	3		
9–10	Fourth	4		
10–11	Fifth		1	1
11–12	Sixth		2	2
12–13	Seventh		(3)	3
13–14	Eighth			4
14–15	Ninth			5
15–16	Tenth			6
16–17	Eleventh			7
17–18	Twelfth			(8)

Primary education in France is free, compulsory, and secular. The law requires that every commune must have at least one primary school for boys, and every commune of more than 5,000 population must have a primary school for girls. Education is compulsory from the sixth to the thirteenth year.

Primary instruction is given up to about the age of 10 or 11; then more or less differentiation in training occurs. There is an elementary course of two years; the pupils in this course range in age from 6 to 9 years. This is followed by an intermediate course of two years; the pupils are from 9 to 11 years of age. Finally, there is a higher primary school of two or three years. Most of the pupils take only the first two of these courses and then enter a secondary school. The higher primary course is not offered in all schools.

Higher primary instruction is of two kinds: (*a*) The higher primary course, and (*b*) the complementary course. The first differs from the second in that it is entirely separate from the elementary school and is under a different director. The complementary course is connected with the elementary school and is under the same direction. The higher primary school usually has a course of two, sometimes of three, years. The complementary course is one year. Pupils who complete the course in the higher primary school are prepared for agriculture, industry, and commerce. However, the aim in these schools is not primarily to prepare the pupil for a particular vocation, but to furnish a general practical knowledge of several vocations. Those who wish to prepare for examinations for entrance to more advanced schools are put into a special section. To be admitted to either of the above divisions, a pupil must have a certificate from the elementary school and must have taken at least one year of additional work.

The school day lasts from 8.30 to 11.30 a. m. and from 1 to 4 p. m. Sessions are held daily except Sunday and Thursday. The latter day is set apart for religious instruction outside of the school. Vacation begins about August 15 and lasts until December 1. Coeducation is the exception throughout the French system.

The minister of education nominates an inspector of primary schools for each district. Superior in authority to these are the inspectors of academies; these men are really heads of departments of primary schools. There are 10 general inspectors of primary education, who stand next to the minister in authority in these schools.

Secondary education is not compulsory; nor is it free. The State charges a small fee, but numerous scholarships are offered. The present organization of the secondary schools dates back to 1902. The curricula of these schools were somewhat modified in 1905 and 1909.

There is a course of seven years. The pupil usually enters at the age of 10 or 11 and graduates at the age of 17 or 18. The secondary school is divided into two cycles, the first of four and the second of three years. Pupils who enter the first section are required

to study Latin and they may elect Greek; in the second section particular emphasis is put upon the study of French and the sciences; Latin and Greek are not offered.

Electives are offered in the second and third years of the second cycle. In one course Greek and Latin are continued. In a second course Latin and either English or German are studied. In a third course the sciences and modern languages predominate, and but little Latin is offered. In the fourth no Latin is offered; emphasis is put upon the sciences and upon modern languages.

At the end of the second cycle the pupil presents himself for the first part of the bacalaureate degree. He must be at least 16 years of age. An additional year is then given to preparation for the second part of the degree. This degree is a sufficient passport for entering the higher schools.

The secondary schools of France are called lycées. In some communes institutions called colleges are maintained. These are usually of a lower grade than the lycées and are financed by the communes. All private secondary schools are subject to State inspection.

The lycées for girls usually have courses of five or six years.

GERMANY.

Arrangement of school years.

Age of pupil.	School year.	Volksschule.	Bürgerschule.	Gymnasium.	Realgymnasium.	Oberrealschule.
6–7	First	1				
7–8	Second	2				
8–9	Third	3				
9–10	Fourth	4	1	1	1	1
10–11	Fifth	5	2	2	2	2
11–12	Sixth	6	3	3	3	3
12–13	Seventh	7	4	4	4	4
13–14	Eighth	8	5	5	5	5
14–15	Ninth		6	6	6	6
15–16	Tenth		7	7	7	7
16–17	Eleventh		8	8	8	8
17–18	Twelfth		9	9	9	9

Classes in a nine-year secondary school.

	Classes.	Usual age of pupils.
		Years.
Lower stage	Sexta—VI	9
	Quinta—V	10
	Quarta—IV	11
Intermediate stage	Untertertia—UIII	12
	Obertertia—OIII	13
	Untersekunda—UII	14
	Obersekunda—OII	15
Upper stage	Unterprima—UI	16
	Oberprima—OI	17

The following table indicates the average number of periods per week allotted to the study of mathematics in each type of secondary school:

Periods per week in mathematics.

	Class.									Total.
	VI	V	IV	UIII	OIII	UII	OII	UI	OI	
Gymnasium	4	4	4	3	3	4	4	4	4	34
Realschule	4	4	4	4	5	5	5	5	5	41
Oberrealschule	5	5	6	6	5	5	5	5	5	47

In Germany centralization of educational authority is not carried to so high a degree as in France. Each of the German States is independent in all local affairs, and the educational system is characterized by flexibility rather than rigidity.

In the United States a pupil may enter the elementary school and continue his course through the university. The "educational ladder" is unbroken. In Germany there are two distinct systems, the lower or elementary school and the higher school system. After a pupil has passed his fourth school year it is almost impossible to transfer from one to the other. Elementary education is usually based on State laws; secondary education is usually regulated by ordinances of a local character.

In the United States the direction of the internal organization of the schools rests with the local communities. In Germany it is in the hands of the various States. The official to whom is delegated the highest authority in educational affairs always has some other State duties. In Prussia the highest official in education is the minister of public worship and education; in Bavaria he is the minister of the interior, of public worship and education; in Wurttemberg he is the minister of ecclesiastical and school affairs.

The various subdivisions of the schools are under the immediate direction of subordinate ministers and of directors and school councils. All of these authorities are State officials. The community looks after the external administration of the school, such as the erection, equipment, and sanitation of buildings. In some of the large city systems a school superintendent is appointed as professional advisor. The appointment of all teachers must be confirmed by the State.

Education is compulsory from the sixth to the fourteenth year (sixth to the thirteenth year in Wurttemberg), and there are practically no illiterates in Germany.

The elementary schools are called Volksschulen, and are all free. The majority of the teachers are men, but the percentage of women is steadily increasing. Most of the Volksschulen have an eight-year course. The school day consists of four or five periods. It begins at 7 or 8 in summer and at 8 or 9 in winter. The number of classes depends upon the size of the school. All Volksschulen must conform to certain minimum requirements prescribed by the State.

Most of the German States make a special provision in the elementary schools for the education of backward and of exceptionally gifted children. The schools are divided not only vertically, but horizontally. The brighter children have a richer curriculum, usually including a foreign language. Normal pupils require eight years to complete the course in the Volksschule. Very able pupils, after two years' attendance, are put into special classes which prepare them for the Gymnasium in one and one-half instead of two years. For backward pupils, courses of from 7 to 10 years are offered.

The middle schools of Germany are called Mittelschulen, higher elementary schools, or Bürgerschulen. They are intermediate between the lower elementary and the secondary school from the point of view of courses offered. Attendance at these schools is not a prerequisite for admission to the secondary schools. Mittelschulen are especially common in the States of southern Germany. Some of these are for boys, some for girls, and some are attended by both boys and girls. The course is usually nine years in length, and the lower grades overlap the primary school. The curricula in these schools are adapted as far as possible to local and individual needs. The schools are largely utilitarian in aim. Usually not more than one foreign language is studied, but exceptional pupils may study a second foreign language after the seventh year.

SECONDARY SCHOOLS.

The public secondary schools are almost all undenominational or interdenominational. The elementary schools are usually denominational, except in Baden and Hesse, and the Volksschulen in some of the cities.

Separate schools are usually maintained for boys and girls, except in Wurttemburg, Baden, and Hesse, where mixed classes are common. In all of the States, mixed classes are found in the small schools in rural communities. Most of the secondary schools for boys are public, and many of those for girls are private. The maintenance of the elementary school usually falls upon the community. Cities and towns maintain their own secondary schools. There is a tendency to provide more liberally for public secondary education for girls.

A few secondary schools have preparatory schools (Vorschulen), but entrance is usually made from the third or fourth grade of the elementary schools. Most of the secondary schools have a nine-year course, but some have a course of six years.

Promotion is by classes, not by subjects. A pupil who fails in two major subjects is not promoted. The maximum size of classes is 50 in the lower stage, 40 in the intermediate, and 30 in the upper.

The school day begins at the same time as in the elementary school and consists of five or six periods of 40 or 50 minutes each.

There are three kinds of higher schools with nine-year courses: (*a*) Gymnasium, (*b*) Realgymnasium, (*c*) Oberrealschule.

The Gymnasium is the classical secondary school. Both Latin and Greek are taught.

In the Realgymnasium no Greek is taught. Latin, the modern languages, and mathematics are emphasized.

In the Oberrealschule Latin and Greek are not taught, but emphasis is placed upon the modern languages and science.

In all three types of schools German, history, religion, and mathematics are taught.

The fundamental idea in organizing the Realschule was to prepare for the commercial professions, as the Gymnasium prepares for the learned professions. The curricula in most of the Realschulen have been enlarged to meet the demands for broader culture.

After the first three years in a secondary school it is almost impossible for a pupil to transfer to another type of school. This means that parents must decide by the time their children are 9 or 10 years of age which type of school they wish them to enter. In order that this decision, which is almost irrevocable, may be postponed until the child is a few years older, institutions called Reformgymnasia and Realgymnasia have been established. In these schools a common foundation for all three types of secondary schools is laid in the first three years. At the end of this period one section begins the study of English, and emphasis is later put upon the natural sciences (Realschule and Oberrealschule). Another section begins the study of Latin, and two years later this section is subdivided, one division (Gymnasium) beginning the study of Greek and the other section (Realgymnasium) beginning the study of English. This general plan is sometimes called the Frankfort system.

Most of the States now maintain separate high schools for girls. The curricula of these schools are not unlike those of the Realschule, except that less emphasis is placed upon science and mathematics. In the boys' schools all of the teachers are men. In the girls' schools some of the teachers are men.

For a description of the conducting of classes in the various types of schools the reader may consult "Mathematics in the Schools of Prussia," by J. W. A. Young, published by Longmans, Green & Co., New York. See also "The present Teaching of Mathematics in Germany" (Bureau of Publications, Teachers College, New York City), for details in regard to courses in mathematics in the secondary schools of the more important German States.

HOLLAND.

Arrangement of school years.

Age of pupil.	School year.	Primary school.	Bürger-schule.	Middle school.	Gym-nasium.
6–7	First	1			
7–8	Second	2			
8–9	Third	3			
9–10	Fourth	4			
10–11	Fifth	5			
11–12	Sixth	6			
12–13	Seventh		1	1	1
13–14	Eighth		2	2	2
14–15	Ninth			3	3
15–16	Tenth			4	4
16–17	Eleventh			5	5
17–18	Twelfth				6

There are many large cities in Holland, and to a considerable extent they determine the general character of the primary and secondary education. Although complete religious liberty prevails throughout the Kingdom, the various religious denominations exercise great influence in educational matters.

The minister of the interior is in charge of all State administration schools. There are 3 general inspectors and 25 district inspectors, and these have several subordinates. These officials are appointed by the sovereign and are paid by the State.

The local civil authorities, the communal burgomaster, and the council have charge of the local administration of the schools. All needs and conditions are reported to the communal council by the school boards.

The primary school comprises the first six school years. The pupils usually enter at the age of 6. Boys and girls attend the same school and are instructed together. Primary instruction is given in the day schools, the evening schools, and in the continuation schools. Attendance at the day school is obligatory for all children from 7 to 13 years of age.

There are between five and six thousand public day schools in Holland, and almost two thousand private primary schools that have been subsidized by the State.

The primary schools are open all the year except on holidays. Promotion from grade to grade is upon the basis of examinations. A pupil who completes the course in the primary school in a satisfactory manner is given a certificate of honor.

The secondary schools may be public or private. There are four general subdivisions:

1. Bürgher schools.
2. Higher Bürgher schools.
3. Industrial, trade, and technical schools.
4. Agricultural schools.

These schools are sometimes called the middle schools, to distinguish them from the Gymnasia.

The immediate supervision of all public secondary schools is exercised by local committees approved by the minister of the interior, who is supreme in all matters relating to the secondary schools.

Pupils may enter these schools either by examination or upon presentation of a certificate from the primary school.

The Bürgher schools are especially for the children of the tradesman, mechanic, and the agriculturist. The law requires that such a school must be maintained in every community having a population of more than 10,000.

The Higher Bürgher schools train those who expect to become engineers, architects, and technologists. Those who expect to enter the service of the State also attend

these schools. The diploma admits the pupil without examination to certain of the special higher schools.

The Higher Bürgher schools have two courses, one of five years and the other of three. The tendency is to extend the course to six years. These schools offer especially thorough instruction in the sciences and modern languages. Entrance is by examination. A candidate for admission must be at least 12 years of age. A pupil who successfully completes a course is given a certificate which is of value to him if he seeks a civil or commercial appointment. There are about 100 Higher Bürgher schools in Holland. Both boys and girls may be enrolled in these schools. Only male teachers are employed.

The Higher Bürgher schools offer the majority of the pupils the best opportunity for a good, general secondary education. A graduate of these schools can not enter a university until he has studied one year of Latin and of Greek. In Belgium the sciences and the modern languages are considered as equivalent to the ancient languages in dignity and in importance, but the classical idea still prevails quite largely in Holland.

There are numerous types of industrial and technic schools. The course in these schools varies from one to five years. The Bürgher schools make provision for commercial education. There are several schools of industrial and household arts for girls. Women frequently predominate on the boards in these schools. In all of the technical schools a good deal of attention is devoted to the subject of drawing.

Agricultural education receives a great deal of attention. A special inspector is in charge of the agricultural schools. Most of the schools of this type offer thorough courses in colonial agriculture. A special State inspector has supervision of the agricultural schools.

In addition to the schools mentioned above, every community of 20,000 inhabitants must have a Gymnasium. These prepare especially for the universities. The age of admission is 12 to 13 years, and the candidate for admission must pass an examination in the mother tongue, French, reading, writing, history, and arithmetic. The course is six years in length. There are about 70 Gymnasia in Holland, public and private.

The inspection and examination of the schools is very rigid and effective throughout the country.

HUNGARY.

Arrangement of school years.

Age of pupil.	School year.	Volks-schule.	Bürger-schule.	Gym-nasium.	Realschule.
6-7	First	1			
7-8	Second	2			
8-9	Third	3			
9-10	Fourth	4			
10-11	Fifth	5	1	1	1
11-12	Sixth	6	2	2	2
12-13	Seventh		3	3	3
13-14	Eighth		4	4	4
14-15	Ninth		(5)	5	5
15-16	Tenth		(6)	6	6
16-17	Eleventh		(7)	7	7
17-18	Twelfth		(8)	8	8

The number of hours devoted each week to the study of mathematics and of geometric drawing in the Gymnasium and Realschule is indicated in the following:

Periods per week in mathematics and geometric drawing.

Classes.	Gymnasium.			Realschule.		
	Mathematics.	Geometric drawing.	Total.	Mathematics.	Geometric drawing.	Total.
First	4	3	7	4	4	8
Second	4	3	7	4	4	8
Third	3	2	5	3	2	5
Fourth	3	2	5	4	2	6
Fifth	3	.....	3	5	.....	5
Sixth	4	.....	4	4	.....	4
Seventh	3	.....	3	4	.....	4
Eighth	2	.....	2	3	.....	3
Total	26	10	36	31	12	43

In Hungary, as in Austria, the minister of education is the highest school official. He is assisted by a staff of subordinate ministers and numerous school inspectors.

The elementary schools include the national, private, and city schools, but there is a certain unity in the courses, because all pupils are required to pass the same examinations. Boys and girls are usually taught in separate institutions, but coeducational elementary schools are not uncommon in the less densely populated districts. The course in the elementary school is six years in length. The law provides for a minimum school year of eight months in the country and nine in the city. Education is compulsory from the sixth to the twelfth year.

The Bürgerschulen are much the same as the corresponding schools in Austria. The course is usually four years in length, but some schools offer six or eight year courses. Some of the Bürgerschulen are supported by the State, some by religious organizations, and others by the communities. The girl and boy may enter these schools after completing the fourth class of the Volkschule. The entering age is usually 10. Both male and female teachers are employed.

In the Bürgerschule for boys the same subjects are usually taught in the first four years as in the first four years of the Gymnasium and Realschule, with the exception of Latin and French. The practical aim of the Bürgerschule is indicated by the introduction of some subjects not taught in the Gymnasium and Realschule. The Bürgerschule does not attempt to prepare the pupil for higher education, but seeks to emphasize the practical subjects and to lead to the higher vocations. Attendance at these schools does not excuse one from service in the army. Pupils may be transferred from certain classes of the Bürgerschule to the Gymnasium and Realschule and vice versa by passing examinations. The change that is gradually taking place in the Bürgerschule is evidenced by the fact that Latin is now taught in some of the boys' schools, and some of these schools are not unlike the Gymnasium and Realschule.

The secondary schools of Hungary are of two kinds: (*a*) The Gymnasium, (*b*) the Realschule. Both types of school have an eight-year course. A pupil may enter by examination after completing the first four classes of the elementary school.

In the Gymnasium Latin is studied in each of the eight classes and Greek in the last four. The study of Latin is obligatory. In many Gymnasia a pupil is permitted to elect some subject instead of Greek. German is taught after the second class.

Latin is not a required subject in the Realschule, but it is sometimes offered in the last four classes. In the Realschule German is taught in all classes and French

in the last six. Mathematics is treated more extensively and more intensively in the Realschule than in the Gymnasium.

Separate secondary schools of a distinctive type have recently been established for girls. These schools offer a six-year course and may be entered by examination after the completion of the sixth year of the elementary school. There are now two kinds of high schools for girls. In one of these especial emphasis is placed upon the modern languages and domestic science; in the other the course closely resembles that of the Gymnasium for boys. Both men and women are employed as teachers in the high school for girls.

ITALY.

Arrangement of school years.

Age of pupil.	School year.	Elementary school.	Secondary school.		
			Classical school.		Modern school.
			Ginnasio.	Liceo.	
6–7	First	1			
7–8	Second	2			
8–9	Third	3			
9–10	Fourth	4			
10–11	Fifth	5	1		1
11–12	Sixth	6	2		2
12–13	Seventh		3		3
13–14	Eighth		4		4
14–15	Ninth		5		5
15–16	Tenth			1	6
16–17	Eleventh			2	7
17–18	Twelfth			3	

The minister of public instruction is at the head of the national educational system and is a member of the cabinet. There is a higher council of 36 members, 12 of whom are nominated by the minister, 12 are designated by the ordinary and extraordinary professors of the universities, and the remainder are elected by the senate and the chamber. Of these 36 members, 15 are appointed by the minister as a special committee in charge of all matters pertaining to higher education. Another group of members has charge of the secondary schools, and a third group has charge of the elementary schools. There are also several permanent committees to act as advisers on particular subjects.

There is an official (Prooveditore) in each Province who has charge of the matters relating to public instruction within the Province. Each Province has also an educational council for the elementatry schools and one for the secondary schools.

The elementary school usually consists of six grades. The first three grades comprise the inferior and the next three the superior course. A pupil may enter school at the age of 6, and education is compulsory from the sixth to the twelfth year. After completing the fourth year of the elementary school, the pupil who is going to a higher school may take an examination, and if he passes this he may enter the secondary school. No pupil is allowed to remain in the elementary school after he is 15 years of age or in the inferior course after he is 12 years of age.

Separation of pupils on the basis of sex occurs wherever the number of pupils in a school is sufficient to necessitate a duplication of classes and courses. Women teachers predominate in mixed schools and in elementary schools for girls. Men usually teach in the schools for boys.

The State assumes a share of the expenses for the elementary schools, and the remainder of the expense is borne by the communes.

Secondary schools are usually erected and equipped by the local authorities, and the other expenses are shared by the State. There are two types of secondary schools besides the normal schools:

I. The classical school and the modern school with Latin—
   (*a*) Ginnasio—5-year course.
   (*b*) Liceo—3-year course.

II. Modern school without Latin—Technical school—
   (*a*) Scuola tecnica and scuola complementare—3-year course.
   (*b*) Instituto tecnico—4-year course.
   Instituto nautico—3-year course.

Secondary schools are divided into first and second grade. The first-grade schools are the Ginnasio, Scuola Tecnica, and Scuola Complementare. The second-grade schools are the Liceo, Instituto Tecnico, Instituto Nautico, and Scuola Normale.

No pupil is admitted to the secondary schools until he has passed an examination (maturita).

The complete classical course covers eight years, and the modern course covers seven years.

Girls are admitted to secondary schools upon the same conditions as boys. The Scuola Complementare is for girls only.

The secondary schools are subject to rather rigid inspection by officials from the office of the minister of education.

JAPAN.

Arrangement of school years.

Age of pupil.	School year.	Ordinary primary.	Higher primary.	Middle school.
6–7	First	1		
7–8	Second	2		
8–9	Third	3		
9–10	Fourth	4		
10–11	Fifth	5		
11–12	Sixth	6		
12–13	Seventh		1	1
13–14	Eighth		2	2
14–15	Ninth		3	3
15–16	Tenth			4
16–17	Eleventh			5
17–18	Twelfth			(6)

Periods per week in mathematics.

	First year.	Second year.	Third year.	Fourth year.	Fifth year.	Sixth year.
Ordinary primary	5	6	6	6	4	4
Higher primary	4	4	4			
Middle school	4	4	4	4	2	

The State has entire control of education, and school attendance is compulsory from the sixth to the fourteenth year. The minister of education is a member of the imperial cabinet.

The school year extends from the first of April to the first of March of the following year. The actual school year, after all vacations have been deducted, is about 40 weeks.

The elementary school consists of two subdivisions. The first extends over the first six grades and is called the ordinary primary. The second, called the higher elementary, extends over the next two or three years. Education is compulsory

in the ordinary primary, but not in the higher elementary school. In some localities supplementary education is now provided for those who can not pursue their regular education beyond the compulsory stages.

Boys and girls are usually taught in the same school and in the same class during the elementary school period, but the middle and higher schools are not coeducational. Frequently, if the number of girls in one school year of the ordinary elementary schools or the number of girls in all the classes of a higher elementary school is enough to organize one class, the boys and girls are separated.

After completing the course in the ordinary primary school, the pupil who expects to enter a higher school goes at once to the middle school instead of the higher primary.

Applicants for admission to the middle school must be male graduates of the ordinary elementary schools, not less than 12 years of age, or must have attainments equal or superior to those of the graduates of ordinary elementary schools. Graduates of the ordinary elementary school are given preference over others.

The course in the middle school is usually five years in length; sometimes a supplementary year is added. The graduates of middle schools are qualified to enter higher special schools of various kinds and special industrial schools. They may enter military, naval, or navigation schools, or the higher normal school.

The middle schools were established to give boys a higher common education, but many of these schools have become virtually preparatory schools. The authorities are going to close the high schools preparing for the imperial universities and to establish new higher middle schools, for the purpose of imparting to those who have finished the middle school course a higher common education more thorough than before.

After graduating from a middle school, a boy who expects to enter the university may enter a higher school, having a course of three years, which prepares for the university. A boy who enters the university after preparing in this manner is 20 or 21 years of age. Admission to the higher middle schools is usually on the basis of competitive examinations. During the first year five hours per week are devoted to mathematics; during the second and third years, four hours per week. The course in mathematics in these schools includes trigonometry, algebra, analytic geometry, and the calculus.

After a girl finishes the ordinary primary school, she may enter a girls' high school or she may enter the higher primary school. The course in a girls' high school is four or five years. This is sometimes supplemented by two additional years. This supplementary course is the only provision for the higher education of women except the normal school and certain technical schools. No girl is allowed to enter the imperial university.

ROUMANIA.

Arrangement of school years.

Age of pupil.	School year.	Primary school.	Secondary school.	
			Gymnasium.	Lycée.
6-7	First	1		
7-8	Second	2		
8-9	Third	3		
9-10	Fourth	4		
10-11	Fifth	(5)	1	
11-12	Sixth		2	
12-13	Seventh		3	
13-14	Eighth		4	
14-15	Ninth		(5)	1
15-16	Tenth			2
16-17	Eleventh			3
17-18	Twelfth			4

The schools of Roumania are divided into three groups: (*a*) Primary; (*b*) commercial, technical, and private; (*c*) secondary and higher.

The minister of public instruction has general supervision of all education and is officially advised by a general educational council. This council is divided into three groups. Each group has general supervision over one of the types of schools enumerated above. There are also general and district inspectors for both the primary and the secondary schools.

The primary and the secondary schools are free, and education is compulsory between the ages of 7 and 14 years.

In the cities the primary school course is 4 years in length, and the school year is 10 months. In rural communities the course is 5 years in length, and the school year is 9 months.

The secondary schools are of two kinds: The gymnasium and the lycée. In some cities both types are found in the same school. Pupils may be admitted to the gymnasium by examination or upon presentation of a certificate from the primary school. Admission to the lycée is by certificate from the gymnasium.

There are three parallel courses in the lycée, somewhat as in the French schools. A pupil may elect the mathematics-science course, the Latin-science course, or the classical course. Only a few of the public secondary schools are for girls.

A pupil who does not expect to go to a university spends five years in the gymnasium instead of entering the lycée.

RUSSIA.

Arrangement of school years.

Age of pupil.	School year.	Primary school.	Real-schule.	Gymnasium.
7–8	First	1		
8–9	Second	2		
9–10	Third	3		
10–11	Fourth	4	1	1
11–12	Fifth	(5)	2	2
12–13	Sixth	(6)	3	3
13–14	Seventh		4	4
14–15	Eighth		5	5
15–16	Ninth		6	6
16–17	Tenth		7	7
17–18	Eleventh		(8)	8

Primary instruction in Russia is under either the minister of public instruction or the Holy Synod.

The schools under the direction of the Holy Synod bear about the same relation to the other schools that the church schools of England bear to the public schools. The synod manages the schools through the bishop and the clergy.

The primary schools usually have courses of three or four years, but in some cities the course is five or six years in length. Five periods a week are devoted to arithmetic.

The secondary schools are of two types, the classical gymnasium and the modern gymnasium.

Gymnasia for girls are quite numerous, and the course of study indicates that quite a high standard is sought. Most gymnasia for women have courses seven years in length. There are some gymnasia with three and four year courses.

The following table indicates the number of periods per week devoted to mathematics in each of the types of secondary schools:

Periods per week in mathematics.

	First year.	Second year.	Third year.	Fourth year.	Fifth year.	Sixth year.	Seventh year.	Eighth year.
Classical gymnasium	4	4	4	4	5	4	3	4
Modern gymnasium	4	4	4	4	4	4	3	3

SWEDEN.

Arrangement of school years.

Age of pupil.	School year.	Primary school.	Real-schule.	Gymnasium.
6–7	First	1		
7–8	Second	2		
8–9	Third	3		
9–10	Fourth	4		
10–11	Fifth	5	2	
11–12	Sixth	6	3	
12–13	Seventh		4	
13–14	Eighth		5	
14–15	Ninth		6	1
15–16	Tenth			2
16–17	Eleventh			3
17–18	Twelfth			4

Periods per week in mathematics.

Gymnasia.	Classes.			
	First.	Second.	Third.	Fourth.
Realgymnasium	7	6	6	6
Latin gymnasium	5	4	4	5

Pupils usually enter school at the age of 6 or 7, and attendance is compulsory up to the age of 14. The school year is 34½ weeks in length. Both the church and the school are under the supervision of the same department of the Government.

The primary school has a six years' course. The parish is usually the school district, and instruction is carried on under the supervision of local inspection and of inspectors appointed by the Government. Coeducation is common in the primary schools of the rural districts. From five to seven periods per week are devoted to mathematics in the primary school.

The higher school is divided into the Realschule and the Gymnasium. The aim of the Realschule is to provide a general practical education beyond the primary school. The course of six years is concluded without an examination. In all Realschulen four or five periods per week are devoted to mathematics.

After completing the first five years of the Realschule, a pupil may enter the first year of the Realgymnasium or of the Latin Gymnasium, instead of taking the sixth year of the Realschule. In the Gymnasium pupils are urged to take subjects for which they have special talent, and considerable freedom to discontinue a subject is permitted.

The school day for secondary schools begins at 7.45 in the morning and must not be longer than six periods of 45 minutes each.

With the exception of a few of the public Realschulen, all of the secondary schools are for boys only, and a small fee is charged. Most of the higher schools for girls are private, and they are taught almost exclusively by women.

Any pupil who is successful in passing the examinations at the close of the last year in the Gymnasium is permitted to enter the university.

SWITZERLAND.

Arrangement of school years.

Age of pupil.	School year.	Primary school.	Lower middle school (Sekundarschule).	Higher middle school.	
				Gymnasium.	Realschule.
6–7	First	1			
7–8	Second	2			
8–9	Third	3			
9–10	Fourth	4			
10–11	Fifth	5	1		
11–12	Sixth	6	2		
12–13	Seventh		3	1	
13–14	Eighth		4	2	
14–15	Ninth		(5)	3	1
15–16	Tenth			4	2
16–17	Eleventh			5	3
17–18	Twelfth			6	4
18–19	Thirteenth			7	5

The Cantons differ greatly in the number of periods per week devoted to mathematics. In the primary school one period daily is the general rule.

The following table shows the number of periods per week devoted to mathematics, including bookkeeping, in the literary Gymnasium, Realgymnasium, and Realschule of Zurich.

Periods per week in mathematics.

	Years.						
	First.	Second.	Third.	Fourth.	Fifth.	Sixth.	Seventh.
Literary Gymnasium	5	5½	3	3	3½	4	1½
Realgymnasium	5	5½	6	5½	5½	4½	2½
Realschule	9½	8	9	9	5½	.....	.....

Although each of the 25 Cantons of Switzerland is autonomous in all matters pertaining to education, there are some similar features in the various school systems.

In most of the Cantons the legislature appoints a general director of education, but in some Cantons this authority is vested in an educational council. Most of the Cantons have special inspectors for each type of school.

The chief characteristic of education might be said to be its variability. This is caused by the great geographical, economic, and religious differences prevailing in the various Cantons. There is considerable dissatisfaction in Switzerland because of the great variety in educational systems, and there is a tendency toward centralization of authority in education. The school year varies from 38 to 48 weeks.

The primary schools are said to be unusually efficient in the training they give. The entering age varies from 6 to 7 years in the different Cantons, and the course is six years in length. All primary schools are free, and coeducation predominates except in the large cities.

In many Cantons a pupil who has completed the fourth year of the primary school may enter an advanced school called the lower middle school or the Sekundarschule. In this school one or more foreign languages and algebra are taught. In some Cantons a pupil enters this type of school after completing the sixth year of the primary school. The course is then from three to five years in length. These schools correspond to the Bürgerschulen in Germany and Austria and to the upper primary schools in France.

A pupil who has completed the six-year course of the primary school may enter the higher middle school. There are many names used in the various Cantons for the

different types of schools of secondary grade, but most of these schools may be called either Gymnasia or Realschulen. The Realschulen of Switzerland correspond to the Oberrealschulen of Germany. In French Switzerland there is even more differentiation of courses than in German Switzerland, and many of the secondary schools resemble the French lycées.

Zurich is the largest of the Cantons, and the system there may be taken as roughly typical. There are three types of secondary schools in Zurich: (*a*) Literary Gymnasium, (*b*) Realgymnasium, (*c*) Realschule. A pupil who has completed a six-year course in the primary school may enter either type of Gymnasium. To enter the Realschule a pupil must have completed not only a six-year course in the primary school, but he must have had two additional years in some secondary school.

[For arrangement of school courses in the United States, see note on page 6.]

TABLE 1.

Table 1 is a graphic representation of the arrangement of the school years in different countries. A heavily dotted line indicates that in *some* cases the type of school under consideration extends over the period represented.

The table portrays general conditions and not exceptional cases.

TABLE 1.—*Arrangement of school years.*

Age	6–7	7–8	8–9	9–10	10–11	11–12	12–13	13–14	14–15	15–16	16–17	17–18	18–19
School year	1	2	3	4	5	6	7	8	9	10	11	12	13
Austria:													
Volksschule, 5-year													
Bürgerschule, 3-year													
Gymnasium, 8-year													
Realschule, 7-year													
Realgymnasium, 8-year													
Belgium:													
Primary, 6-year													
Middle, 3-year													
Athénée, 7-7½-year													
Denmark:													
Folkeskole, 8-year													
Intermediate, 4-year													
Realklasse, 1-year													
Gymnasium, 3-year													
Finland:													
Primary, 7-year													
Lycée, 8-year													
France:													
Primary, 4-year													
Higher Primary, 3-year													
Lycée, 8-year													
Germany:													
Volksschule, 8-year													
Bürgerschule, 9-year													
Gymnasium, 9-year													
Realgymnasium, 9-year													
Oberrealschule, 9-year													

TABLE 1.—*Arrangement of school years*—Continued.

Age	6-7	7-8	8-9	9-10	10-11	11-12	12-13	13-14	14-15	15-16	16-17	17-18	18-19
School year	1	2	3	4	5	6	7	8	9	10	11	12	13
Holland:													
Primary, 6-year	—	—	—	—	—	—							
Bürgerschule, 2-year							—	—					
Middle, 5-year							—	—	—	—	—		
Gymnasium, 6-year							—	—	—	—	—	—	
Hungary:													
Volksschule, 6-year	—	—	—	—	—	—							
Bürgerschule, 4-6-year					—	—	—	—					
Gymnasium, 8-year					—	—	—	—	—	—	—	—	
Realschule, 8-year					—	—	—	—	—	—	—	—	
Italy:													
Elementary, 6-year	—	—	—	—	—	—							
Ginnasio, 5-year					—	—	—	—	—				
Liceo, 3-year										—	—	—	
Modern, 7-year					—	—	—	—	—	—	—		
Japan:													
Ordinary Primary, 6-year	—	—	—	—	—	—							
Higher Primary, 3-year							—	—	—				
Middle, 6-year							—	—	—	—	—	—	
Roumania:													
Primary, 5-year	—	—	—	—	—								
Gymnasium, 5-year					—	—	—	—	—				
Lycée, 4-year									—	—	—	—	
Russia:													
Primary, 3-5 year	—	—	—	—	—								
Realschule, 7-8 year				—	—	—	—	—	—	—	—		
Gymnasium, 8 year				—	—	—	—	—	—	—	—	—	
Sweden:													
Primary, 6-year	—	—	—	—	—	—							
Realschule, 6-year				—	—	—	—	—	—	—			
Gymnasium, 4-year									—	—	—	—	
Switzerland:													
Primary, 6-year	—	—	—	—	—	—							
Lower Middle, 5-year					—	—	—	—	—				
Gymnasium, 7-year							—	—	—	—	—	—	—
Realschule, 5-year									—	—	—	—	—
United States of America:													
Elementary, 8-year	—	—	—	—	—	—	—	—					
Secondary, 4-year									—	—	—	—	

II. THE WORK IN MATHEMATICS IN THE FIRST SCHOOL YEAR.

AUSTRIA.—The pupils are taught to read and write numbers from 1 to 10, and the four fundamental operations are usually taught within these limits. In some schools the pupils learn to read and write numbers from 1 to 100 during the first school year. Simple problems involving denominate numbers familiar to the children are given. Most of the work is oral.

BELGIUM.—Pupils are taught to count and to read and write numbers from 1 to 20. Simple exercises involving numbers within these limits are given. The reading and writing of numbers is often extended to 100. Objects are freely used to develop the number concepts. From four to five periods a week are usually devoted to arithmetic.

DENMARK.—The first year's work includes the reading and writing of numbers from 1 to 10, and the four fundamental operations involving numbers within these limits. In a few schools the limit is extended to 20. The concepts ½, ⅓, and ¼ are taught. Almost all the work is oral and objects are freely used. Arithmetic is taught five periods a week to boys and four to girls.

ENGLAND.—The third grade of the infant department corresponds to the first school year in the United States. Pupils are taught to count from 1 to 10 and from 10 to 1. The various numbers from 1 to 10 are factored. No formal addition or subtraction is taught. Only the simplest exercises are given. The pupils play store, dominos, and various other games. Measurement, using the foot, inch, and half-inch is taught, and pupils learn to estimate small distances. The terms half and quarter are taught by means of paper folding and by the use of numerous objects.

FINLAND.—In the city schools the pupils are taught to count and to read and write numbers from 1 to 100. The most common measures are taught.

In the country schools the reading and writing of numbers from 1 to 100 are taught and addition and substraction involving numbers less than 20 are studied. The meaning and use of the terms meter, liter, kilogram, and mark are also taught.

FRANCE.—The course includes the reading and writing of numbers from 1 to 100, and the four operations involving the numbers to 10. The terms meter, liter, and franc are taught and extensively used. The fractions ½, ⅓, and ¼ are taught. Almost all of the work is oral.

GERMANY.—The number scale from 1 to 20 is taught, and addition and subtraction of numbers within this limit.

In some of the German States the multiplication table of twos is taught during the first school year. The terms meter, decimeter, pound, and mark are usually taught.

In a few of the States the number scale is taught only from 1 to 10, but all four operations are taught within these limits.

From three to four periods a week are devoted to arithmetic.

HOLLAND.—The course is practically the same as in Belgium. The numbers from 1 to 20 are studied synthetically. From 20 to 100 the tens are studied before the intermediate numbers. Four to five periods a week are given to the subject.

HUNGARY.—The course is practically the same as in Austria. The simple number relations involved in the activities of the home and the school are emphasized.

ITALY.—The pupils are taught to read and write numbers from 1 to 100 and to perform the four operations on numbers from 1 to 20. Most of the work is oral.

JAPAN.—The course includes the reading and writing of numbers from 1 to 100 and the four operations on numbers less than 20. Most of the exercises are in addition and subtraction. A few simple exercises with numbers between 20 and 100 are given.

Five periods a week for 40 weeks are usually devoted to arithmetic.

NORWAY.—(No report is available.)

ROUMANIA.—(The report does not include the work of this school year.)

RUSSIA.—(Details of the work of the first school year are not available; see the third school year.)

SPAIN.—(There is no report on the work of the elementary school.)

SWEDEN.—The pupils are taught to read and write numbers from 1 to 100, and to solve simple oral problems involving numbers of one digit.

SWITZERLAND.—The pupils are taught to read and write numbers to 100, and to perform the four operations on small numbers.

UNITED STATES.—In many of the schools of the United States only incidental number work is given during the first school year. In some schools regular number work is begun the latter half of the first year. In a few schools incidental number work is given during the first two school years.

A great variety of courses exists in those schools which provide special periods for number work during all or a part of the first school year. The following outline is from the New York State course of study. It represents one of the most advanced courses.

Pupils are taught to count, read, and write numbers to 100 and to memorize the 45 addition combinations. The drill in these combinations is given in such a way as to prepare for subtraction as well as addition. Pupils learn to count 100 by twos, fives, and tens. The children are taught to carry in addition. No attempt is made to teach the science of numbers; the art of computation is emphasized. Oral work greatly predominates, but a good deal of seat and blackboard work is given.

GENERAL SUMMARY OF THE FIRST YEAR'S WORK.

There is not much divergence between the courses in arithmetic in the most progressive schools of the various countries. In general it may be said that the aim is to teach the children to count and to read and write the numbers to 100; to perform easy additions and subtractions within these limits; to know the fractions ½, ⅓, and ¼; and to make a few easy multiplications and divisions involving numbers less than 20. Practically all the work is oral, and objects are freely used.

III. THE WORK IN MATHEMATICS IN THE SECOND SCHOOL YEAR.

AUSTRIA.—Drill upon addition, subtraction, multiplication, and division is continued, and the exercises involve numbers from 1 to 100. The reading and writing of numbers is extended to 1,000. The pupils are taught the use of the common measures.

BELGIUM.—Knowledge of the number scale is extended to 100, and pupils frequently learn to write numbers to 1,000. The four operations involving numbers not greater than 100 are taught, and place value is emphasized. The multiplication tables through 10 times 10 are built up and learned. Considerable attention is devoted to the small fractions in common use. Both oral and written exercises are given, but the oral work largely predominates.

In boys' schools four hours a week and in girls' schools three hours are devoted to the study of arithmetic.

DENMARK.—The four fundamental operations are taught simultaneously for numbers up to 100. The pupils usually learn to read numbers to 1,000. The work in division includes divisors of only one figure.

Danish money is studied, and the following terms are learned and used: Meter, decimeter, centimeter, kilogram, liter, dozen, year, month, week, and day. Simple reductions in the metric system are taught. The fractions ½, ⅓, ¼, and $\frac{1}{10}$ are taught objectively and are used in simple computations.

ENGLAND.—The year that most nearly corresponds to our second school year is called "Standard I." This is usually preceded by at least two years in the infant department. During this period a good deal of incidental number work is given.

The general practice is to teach the reading and writing of numbers from 1 to 100. The course includes addition and subtraction of two numbers of one digit each, but no formal multiplication or division is given. Rapid addition and subtraction are

emphasized. These processes are taught on the "completion of the ten" system. The fractions ½, ⅓, and ¼ are taught.

A good deal of attention is given to the drawing of the simplest geometrical figures and to estimates of length. Such objects as cubes, bricks, and coins are extensively used.

FINLAND.—The course includes the four fundamental operations involving numbers not greater than 100, and the reading and writing of numbers to 1,000.

The common measures of length, weight, volume, and capacity are also taught.

FRANCE.—The pupils are taught to count and write numbers to 1,000. The four fundamental operations involving easy combinations with numbers less than 100 are taught. Division is limited to divisors of one figure.

The terms meter, liter, gram, franc, and their easy multiples and submultiples are learned. Pupils learn to construct squares, rectangles, and triangles. A good deal of emphasis is put upon simple estimates and measurement.

Forty minutes a day are usually devoted to the study of arithmetic.

GERMANY.—The reading and writing of numbers is extended to 1,000, and numerous oral exercises involving the four operations with small numbers are given. The operations are proved by reversing. Special emphasis is put upon the multiplication and division tables up to five. The fractions ½, ⅓, and ¼ are taught and used in easy problems. The terms mark, pound, meter, and centimeter are taught. The pupils are required to do a good deal of measuring and estimating. Rhymes and games are used to fix the sequence of numbers and their combinations.

In all the States oral arithmetic greatly predominates. In a few States special emphasis is put upon the fractional parts of small numbers, and upon counting to 100 by fives and tens.

HOLLAND.—The pupils are taught to count, read, and write numbers to 1,000. The four fundamental operations are quite thoroughly taught for numbers less than 100. A good deal of emphasis is placed upon the common denominate numbers.

In Amsterdam four and one-half hours a week are devoted to arithmetic.

HUNGARY.—(See the course in Austria.)

ITALY.—The course includes the reading and writing of numbers to 1,000, and oral exercises involving the four operations on numbers less than 100. In multiplication and division the multiplier and divisor are limited to one-digit numbers. The fractions ½, ⅓, and ¼ are taught objectively. The elementary notions of units of weight, length, and capacity are taught. Numerous easy practical problems are given.

JAPAN.—Notation and numeration are taught to 1,000. Pupils are taught to count by tens to 1,000. Oral addition and subtraction involving numbers less than 100 and multiplication and division by easy two-figure numbers are taught. The use of the abacus makes the learning of the tables beyond the nines useless. The tables and the inverse operations are begun in this grade.

The school year is 10 months, and 6 periods a week are devoted to arithmetic.

NORWAY.—(No data are available for this year.)

ROUMANIA.—(No data are available for this year.)

RUSSIA.—(See the report for the third school year.)

SWEDEN.—Pupils learn to count, read, and write numbers to 1,000; to perform the four operations orally on numbers less than 50, and to perform them in writing on numbers less than 100. The multipliers and divisors are always single digits or multiples of 10. The simplest common measures are studied and the easiest fractions are taught.

SWITZERLAND.—Details of the course for the second year are not given in the reports. Oral arithmetic is the basis of the work throughout the primary grades, and great emphasis is put upon easy practical problems within the experience of the child.

UNITED STATES.—In the New York State course there is continued drill on the use of the 45 combinations in addition and subtraction. There is also drill on series in addition and counting by twos, threes, fours, and fives. The addition method is used in subtraction. There is continued drill in rapid additions. The pupils memorize the 45 combinations in multiplication. These are so taught as to prepare for division at the same time. The process of carrying in multiplication is taught. Good model forms are extensively used. No explanation of the processes is attempted.

GENERAL SUMMARY OF THE SECOND YEAR'S WORK.

The course of the second school year varies more than that of the first year. In general, the aim of the work may be said to be to teach the children to count, read, and write numbers to 1,000; to perform the fundamental operations on numbers less than 100; and to learn the simple units of measure. In several countries, multipliers and divisors are limited to one figure. The pupils are taught to count to 100 by twos, fives, and tens. The 45 addition combinations are learned in this year and the multiplication tables involving products up to 10 times 10 are usually studied.

Subtraction is usually taught by the addition method and is studied at the same time as addition. The fractions ½, ⅓, and ¼ are taught, and objects are very extensively used. The simple denominate numbers are studied and much attention is devoted to measures and estimates. Oral work predominates. Numerous concrete problems involving the experiences of the pupils are given.

The course in the most progressive schools of the United States compares favorably with the most advanced courses in Europe. In the great majority of the schools of the United States, however, not so much is attempted in arithmetic as in the best schools of Europe during the second school year. The longer school year and the longer school day enable the European teachers to devote more time to drill in fixing the number facts, and the pupils leaving the second grade there are probably more thoroughly grounded in the fundamentals than is the case in this country.

IV. THE WORK IN MATHEMATICS IN THE THIRD SCHOOL YEAR.

AUSTRIA.—A great deal of emphasis is put upon speed and accuracy in the four fundamental operations. A large amount of oral drill is given, and numerous exercises are solved at the blackboard or on paper.

The fractions include all those with denominators less than 10. Simple exercises involving these fractions are given. Exercises involving simple estimates and measurements are frequently given.

BELGIUM.—The pupils are given an intuitive and practical knowledge of the terms meter, liter, gram, and franc, and of their multiples and submultiples. The units of measure are put into the hands of the pupils whenever practicable. The four operations are explained, and computation is extended to large numbers. The decimal notation is introduced, and the various operations are involved in problems. Emphasis is put upon easy, practical problems. The textbook is first used in this grade. In the boys' schools four hours a week and in the girls' schools three hours are devoted to the study of arithmetic.

DENMARK.—Notation and numeration are extended to 10,000. Place value, in the reading and writing of numbers, is emphasized. The four operations are taught, using both abstract and concrete numbers. Multipliers and divisors not exceeding two figures are used, except in the case of powers of 10. The terms day, week, hour, minute, second, meter, kilometer, millimeter, hectometer, decileter, gram, kilogram, and ton are taught, and are used in numerous problems. Simple exercises

involving easy fractions are given. There is daily oral and written drill on the multiplication tables.

Five hours a week are devoted to arithmetic.

ENGLAND.—In Standard II the course includes the reading and writing of numbers from 1 to 1,000. The four operations are extended to include addition of hundreds, multiplication of tens by units and of hundreds by easy tens; subtraction of tens and easy hundreds; division of tens and easy hundreds by units.

The fractions ½, ⅓, and ¼ are used in easy additions, subtractions, multiplications, and divisions. The tape measure and ruler are extensively used, and the pupils are taught the following terms: Yard, foot, inch, one-half inch, pound, and one-half pound. They are also taught the values of the various English coins.

A good deal of attention is devoted to the drawing of the simplest geometrical figures. The pupils are required to draw rectangles and squares of given dimensions. Squared paper is rather extensively used in these drawings. The pupils are taught the right angle and how to divide angles by means of paper folding.

FINLAND.—Place value is carefully explained. The four operations are extended to 1,000, for both abstract and concrete numbers. The common units of length, weight, and capacity are taught, and numerous exercises involving these measures are given.

Four hours a week are given to arithmetic.

FRANCE.—A thorough review of the work of the previous grade is given, and the operations are extended to large numbers. The general ideas of fractions are introduced. Simple reasoning problems are given, the data being selected from matters within the experience of the child. The pupils make numerous estimates and comparisons of distances, the metric units being used. Exercises involving the most elementary plane figures are given, and the various kinds of angles are studied. The simplest solids are studied from models.

Forty minutes a day are devoted to the study of arithmetic.

GERMANY.—The number scale is extended to 1,000 and the pupils are taught to count by 10, 50, and 100. Oral addition and subtraction are extended to numbers of two digits, and written addition and subtraction to numbers of six digits. Multiplication up to four-place numbers by two-place numbers is taught. Division of numbers containing from two to six figures by numbers of one figure and by easy tens is taught. The terms mark, pound, meter, centimeter, liter, hectoliter, gram, kilogram, and kilometer are taught, and are used in numerous exercises. The four operations involving units of length and weight are given a good deal of attention.

There are in Germany numerous collections of problems designed to give facility in oral computation.

In most of the German States, five hours a week are devoted to the study of arithmetic.

HOLLAND.—The course is practically the same as in Belgium. Exercises involving the four operations with all numbers are given. Much emphasis is placed upon the various weights and measures.

Five hours a week are given to arithmetic.

HUNGARY.—(For details, see the course in Austria.) The problems are chosen almost entirely from practical life. There is a good deal of freedom and variation in courses.

ITALY.—Notation and numeration are extended to 10,000. Numerous oral exercises on numbers less than 100 are given. The written exercises involve numbers up to 1,000. Multipliers and divisors are usually small or else a multiple of 10. The decimal notation is introduced and easy common fractions are reduced to decimals. A good deal of attention is given to practical exercises involving the metric system. Some of the elementary notions of geometry are derived by intuition. Free-hand drawing is emphasized.

JAPAN.—Notation and numeration are extended to 10,000. Oral and written exercises involve all four operations. The chief aim of the year is to teach written addition, subtraction, multiplication, and division with numbers less than 10,000.

From five to ten minutes' oral drill is given daily to help fix the number facts firmly in mind.

NORWAY.—(No data are available on the work of this year.)

ROUMANIA.—(No data are available on the work of this year.)

RUSSIA.—The course for the first three school years includes the four operations for abstract and concrete numbers and the easiest fractions. The tables of weights and measures and of Russian money are taught. There are numerous collections of good problems for use in the primary schools. The fundamental ideas of geometry are taught by means of paper folding and cutting, using squares, rectangles, and simple solids. The pupils do not use a textbook.

Five hours a week are given to arithmetic.

SWEDEN.—(The course is not given in detail. See the sixth school year.)

SWITZERLAND.—Notation and numeration are taught to 1,000. Mental arithmetic is the basis for all work. Calculation, and not reasoning, is emphasized. All the operations are taught.

UNITED STATES.—(New York State course.) Drill in counting is continued. The pupils are taught to count by fives to 100, beginning with 0, 1, 2, 3, or 4, and by sixes, beginning with each of the numbers from 0 to 5, inclusive. In short division, 2, 3, 4, 5, 6, 7, 8, and 9 are used as divisors. Multiplication, with two or more figures in the multiplier, is taught. Definitions of the terms addend, sum, minuend, subtrahend, remainder, multiplicand, multiplier, product, dividend, divisor, and quotient are learned. Pupils are taught to measure, using the inch and the foot. Square inch and square foot are also taught. The fractions ½ and ¼ are applied to the use of the linear unit in measuring.

During the second half of the year the following topics are taught: Long division, multiplication tables of the tens, elevens, and twelves, and their use as divisors in short division; tests for divisibility by 2, 3, 5, 9, and 10; the definition of factor and prime factor. The pupils memorize the prime factors up to 25; linear and square measurement of objects in the school room, and liquid measure are taught.

A great deal of attention is given to oral drill and written work for accuracy and rapidity in the four operations. At the close of the year the pupil is expected to be able to add, subtract, multiply, and divide integers with accuracy and facility.

GENERAL SUMMARY OF THE THIRD YEAR'S WORK.

There is greater variety in the third-year courses than in those of the first and second school years. In a few of the countries—for example, Belgium and Italy—the notation of decimal fractions is introduced. This is usually not done in the United States until the latter part of the fourth or the early part of the fifth year. It is a common practice abroad to introduce fractions with denominate numbers. In all of the European countries and in Japan oral arithmetic greatly predominates. In Japan a special part of the recitation is set apart for this oral drill. The textbooks in several of the countries, notably Germany, Austria, and Italy, are collections of problems rather than expositions of number.

In most of the schools subtraction is taught by the addition method.

At the end of his third school year the German boy may either (1) continue in the Volksschule, (2) pass to the Bürgerschule, or (3) enter a secondary school (Gymnasium, Realgymnasium, or Oberrealschule). The Russian pupil may enter either the Realschule or the Gymnasium. If he prefers he may continue in the primary school a year or two more. In Sweden the primary school lasts six years, but a pupil may enter the Realschule after he has completed the third school year.

V. THE WORK IN MATHEMATICS IN THE FOURTH SCHOOL YEAR.

AUSTRIA.—Much attention is still given to speed and accuracy in the four fundamental operations. A large amount of oral drill is provided. The idea of fractions is extended, and decimal fractions are introduced. Numerous exercises involving easy operations with common and decimal fractions are given. The common units of weight, length, surface, capacity, and money receive a good deal of attention.

BELGIUM.—Common fractions are formally introduced, and the reduction of common fractions to decimal fractions is explained and practiced. Numerous practical problems involving the systems of weights and measures are given. The subject of decimal fractions is correlated with that of the metric system. The fundamental ideas of simple proportion are taught. Simple interest is begun.

DENMARK.—The four operations are extended to exercises involving large numbers. Multipliers with three and divisors with two figures are used. Addition, subtraction, multiplication, and division of denominate numbers are taught.

The concept of fraction is extended to include all fractions with denominators not greater than 10. The pupils are required to find fractional parts of a number.

The fundamental ideas of simple proportion are taught. The metric units are used in many of the problems.

ENGLAND.—The tens, hundreds, and thousands groups are studied. Much emphasis is laid upon the factors of numbers less than 100. Addition is extended to numbers of four figures, and subtraction is extended to hundreds and thousands. Multiplication by easy hundreds is taught, and division by easy factors is studied. The four operations are employed in problems involving money, yards, feet, inches, gallons, quarts, pints, pounds, and ounces. Ascending and descending reductions are presented. The pupils are taught to use the fractions $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{8}$, $\frac{1}{9}$ to $\frac{1}{9}$, and $\frac{1}{12}$ to $\frac{11}{12}$. Numerous problems involving these fractions are given, and they are represented on lines and by means of paper folding. The use of the measuring tape is taught.

The pupils are made familiar with the idea of bisection by paper folding. A study is made of squares and rectangles. The pupils are required to measure and compare the opposite sides and the diagonals. The equilateral and isosceles triangles are briefly studied. Figures are drawn to easy scales and simple geometric patterns are constructed.

FINLAND.—Both common and decimal fractions are taught during the year. Most of the common fractions have small denominators, and most of the exercises do not involve decimals of more than the third order. The pupils are taught how to reduce a common to a decimal fraction. The fundamental ideas of ratio and proportion are taught. Some very elementary problems are given in simple interest.

FRANCE.—The ideas of common fractions are extended, and decimal fractions are introduced. The four operations are applied to decimal fractions. The fundamental ideas of proportion are studied, and a few problems are given involving simple interest. There is a good deal of attention given to problems involving the legal systems of weights and measures, and the four operations are involved in many mental problems. Pupils are required to represent at the board and on paper the simple geometric figures and to study from models the fundamental properties of the cube, prism, cylinder, and sphere. The texts include numerous problems relating to losses caused by alcoholic drinks.

GERMANY.—In Germany a pupil generally spends his fourth school year in the Volksschule, the Gymnasium, the Realgymnasium, or the Oberrealschule, although, as in other countries, there are other types of schools which he may attend.

Course in the Volksschule.—Oral multiplication by 12, 15, and 25 is presented. The four operations with denominate numbers are studied. Simple problems involving frac-

tions are given. Notation and numeration are extended up to seven places. Continual practice is given in the four operations. Multiplication involves multipliers of four digits, and division involves three place divisors.

Decimal fractions are taught to three decimal places, and the subject is closely correlated with the metric system. Denominate numbers are frequently written in decimal form. Ascending and descending reduction are applied to the metric system. Addition and subtraction of common fractions are taught, and aliquot parts are freely used.

In some of the German Volksschulen formal written arithmetic begins with this grade. Applications are postponed until the processes are fixed.

Course in the Gymnasium.—The four fundamental operations with abstract and denominate numbers are extensively drilled upon. The simple measures of weight, length, capacity, surface, and volume are taught. Problems involving parenthesis are given. The ideas of common fractions are extended and decimals are given some attention.

Course in the Realgymnasium.—The course in this type of school is the same as for the Gymnasium.

Course in the Oberrealschule.—Four hours a week are given to arithmetic. The course includes drill in the four operations with abstract and denominate numbers; practice in decimal notation; the rules for divisibility by one place numbers; prime factors; multiples and divisors. The ideas of common fractions are extended. Oral arithmetic receives a good deal of emphasis.

HOLLAND.—The four operations with integers are thoroughly reviewed. Emphasis is placed upon denominate numbers and the various common units of measure.

The concept of fractions is extended; decimal fractions are taught and pupils reduce common to decimal fractions. The rectangle, cube, and parallelopiped are studied.

Four and a half hours a week are given to arithmetic.

HUNGARY.—The attempt is made to secure a high degree of accuracy and a fair degree of speed with the four operations. A thorough study is made of denominate numbers and the simple units of measure. The decimal notation is taught and the concept of common fraction is extended. The pupils learn how to reduce a common to a decimal fraction.

ITALY.—The written work includes the four operations with integers and decimal fractions; the reduction of a common to a decimal fraction; a study of the Roman numerals; and numerous practical exercises on the metric system.

The pupils are taught the fundamental properties of the simple geometric figures. Free-hand drawing forms an important part of the work. The rules for the mensuration of the common plane figures and the names of the solids are taught.

JAPAN.—The chief aim of the work of the fourth school year is to secure a high degree of accuracy and speed in computations involving numbers less than 100,000,000 and to teach computation with compound and decimal numbers.

Multipliers are usually of two or three digits, and divisors and quotients are usually of not more than three digits. The pupils are taught to find a fractional part of a number.

In compound numbers the units of length, distance, weight, capacity, area, time, and money are taught.

Decimal notation is taught only to thousandths in this school year.

NORWAY.—(No report.)

ROUMANIA.—The four operations are extended to larger numbers; the decimal notation is introduced, and the relation between common and decimal fractions is emphasized. The various metric units are studied.

RUSSIA.—The Gymnasium and the Realschule begin with the fourth school year. A pupil may continue his course in the primary school. All courses for the fourth year are much alike. The four operations are emphasized. Common and decimal fractions are introduced. In the Gymnasium four hours a week are devoted to

arithmetic. Measurements and easy drawing to scale form an important part of the course.

SWEDEN.—The Realschule begins the fourth school year and lasts for six or seven years. Four hours a week are devoted to mathematics.

There is continued attention given to the four operations. Common and decimal fractions are introduced, and easy examples involving them are solved. Only integers are used as multipliers and divisors, and no remainders occur in the divisions. The study of the metric system is continued.

SWITZERLAND.—The attempt is made to develop speed and accuracy in the four operations with integers and denominate numbers.

The decimal notation is introduced, and the ideas of common fractions are extended. The four operations are taught for both integers and common fractions; only easy problems are considered. A good deal of attention is put upon estimates and drawing to scale.

UNITED STATES.—(New York State course). Roman numerals are taught from 1 to 100 and by hundreds to 1,000. The pupils learn to read and write United States money, to use cancellation, when possible, in the solution of problems; and to use the terms pint, quart, peck, and bushel. Common fractions are developed objectively. The pupils are taught to change fractions to equivalent fractions of higher and lower denominations; to add and subtract fractions the denominators of which do not contain more than two digits; to multiply a fraction by an integer and by a fraction, and to multiply an integer by a fraction; to divide a fraction by an integer and by a fraction, and to divide an integer by a fraction. The principles for multiplying or dividing a fraction by the proper operation upon its numerator or denominator are taught; also the effect of multiplying both terms of a fraction by the same number or dividing both by the same number.

There is continued drill throughout the year on the four operations with integers. Addition and subtraction of mixed numbers are taught. The pupils learn how to factor and to find the least common multiple of numbers to 100. Problems are carefully stated before being solved. Cubic measure is taught. Volumes studied include cubic inches, cubic feet, and cubic yards. Simple problems are given in bills and accounts.

GENERAL SUMMARY OF THE FOURTH YEAR'S WORK.

At the close of the fourth school year (in Germany, the third school year) the pupil may enter a "higher" school of some sort; in Austria the Gymnasium, the Realgymnasium, or the Realschule; in France, the higher primary or lycée; in Germany, the Gymnasium, the Realgymnasium, or the Oberrealschule; in Hungary, the Bürgerschule, the Gymnasium, or the Realschule; in Italy, the Ginnasio, or the modern school; in Roumania, the Gymnasium; and in Switzerland, the middle school. Conditions are such that pupils can not well enter a "higher" school except at the beginning; so that the fourth (or the third) school year is the last year of the primary for many European pupils.

The attempt is made to fix the four operations for abstract and denominate numbers firmly in mind by the time the pupil has completed his fourth school year. In all of the countries a large amount of both oral and written drill is provided. Speed and accuracy in the fundamentals are watchwords everywhere. In addition, the concept of common fractions is much extended, and the decimal notation is introduced. In several of the countries, only addition and subtraction of common and decimal fractions are taught, but in others multiplication and division are also included in the course. When this is done, only the easy cases are usually considered.

The general use of the metric system in the countries of Continental Europe makes the introduction of decimals practicable at an earlier date than in the United States. Usually the subject of decimal fractions is closely correlated with the metric system.

Most courses in the United States are not so advanced as the New York State course. In general, it may be said that the courses in European countries include all that is offered in the United States during the fourth school year and a good deal of the work of the fifth school year. The formal study of common or decimal fractions is seldom begun in the United States before the fifth school year.

In most of the European countries emphasis is put upon computation rather than upon reasoning during the first four school years.

VI. THE WORK IN MATHEMATICS IN THE FIFTH SCHOOL YEAR.

AUSTRIA.—The fifth school year is the last year of the Volksschule and the first of the Gymnasium, Realgymnasium, and Realschule.

Course in the Gymnasium.—The four operations with abstract and denominate numbers are continued. Roman notation is introduced. Austrian money, weights, and measures are studied. Common fractions are studied in connection with denominate numbers, most of the fractions being of small denomination; and the study of decimal fractions is continued.

The work in geometry is of a propædeutic nature. The cube and sphere are studied. Exercises with ruler, compasses, triangle, and protractor are given. A good deal of emphasis is placed upon the measuring and drawing of easy geometrical figures. The pupils are made familiar with the common solids. The constructions include angles of 30, 60, and 90 degrees; isosceles, equilateral, and right triangles; parallels; and perpendiculars. The pupils find the areas of squares and rectangles, and the volumes of cubes and right prisms.

Course in Realschule and Volksschule.—The course is practically the same as in the Gymnasium. Even more attention is put upon the securing of speed and accuracy in the four operations with integers and fractions.

Arithmetic is taught in very close connection with geometry in all types of schools. The two subjects complement each other and form one instruction unit.

The pupils make numerous simple models of pasteboard and sticks. Planes and solids are taken up in close connection, since the study is made from models.

Three hours a week are usually devoted to the study of arithmetic and one to geometry.

Course in Realgymnasium.—The course is the same as in the Realschule, except that there is no special period for the study of geometry.

BELGIUM.—The four processes with integers, common and decimal fractions are explained. Quotients are obtained correct to one-thousandth. Cancellation is extensively used. Pupils are taught the tests of divisibility for 2, 3, 4, 5, 8, 9, 25, and 125. Prime numbers are studied and applied to tests of divisibility by 6, 12, 15, 18, 21, and 35. Unitary analysis is taught. The problems are based largely on local industries, trades, and agriculture. The study of the metric system is continued. The mensuration of the rectangle, parallelogram, triangle, trapezoid, circle, and polygon is studied. Oral computation is emphasized, and short processes are encouraged.

Most of the teaching of geometric forms is done in connection with the metric system, manual training, and drawing.

DENMARK.—The pupils are taught to resolve numbers into prime factors. Addition and subtraction of common fractions are studied, and the study of decimal fractions is extended. Proportion is taken up, the fractional form of writing the proportion being used. The metric system is studied as a practical application of decimal fractions, and addition and subtraction of decimal fractions and multiplication and division of a decimal by an integer are taught.

Special periods are not devoted to the subject of geometric forms, but much is done in the drawing classes. Mensuration is taught in connection with arithmetic. The compasses are but little used.

ENGLAND.—Notation and numeration are extended to tens of thousands. Factors and easy long division are taught. Common fractions are studied, the denominators being restricted in most cases to numbers less than 12. Addition and subtraction of fractions are taught. Decimal fractions up to three decimal places are introduced, and the four operations involving decimals are taught by means of divided squares, rectangles, and rulers. Numerous problems involving distances and heights expressed in yards, feet, and inches are solved. The terms ton, hundredweight, chain, pound, ounce, gallon, quart, pint, month, day, hour, minute, and second are used in exercises. The method of unitary analysis receives a good deal of attention. The area of the square and the rectangle is taught, and the term perimeter is used. An attempt is made to give the pupil the idea of a standard pound and yard. Metric rulers are used, and pupils learn some of the equivalents. Calipers and wedges are used to determine internal and external diameters.

The work in geometry includes the bisection of lines, angles, and triangles by folding; the superposition of triangles; the use of compasses; folding circles to get angles of 180, 90, 45, and 22½ degrees; the drawing of parallels by means of the set square and by the eye; the drawing of parallelograms, and drawing to scale.

FINLAND.—The course in arithmetic includes decimal and common fractions, tests of divisibility, simple proportion, introductory work in percentage, and simple interest.

One hour a week is given to the study of geometry. The course includes the study of lines, angles, rectilinear figures and their areas, curvilinear figures and their areas.

FRANCE.—The fifth school year is the first year of the higher primary and the first of the lycée. The higher primary is not found in all schools.

The course in the higher primary includes the following subjects: The study of prime numbers and tests for divisibility by 2, 3, 4, 5, 9, and 25; the resolution of numbers into factors; greatest common divisor and least common multiple; unitary analysis; the metric system; the four operations with common and decimal fractions. Letters are used in the solution of some simple problems.

The work in geometry is given in connection with drawing, and is frequently taught by the same teacher. The following figures are studied: Square, rectangle, triangle, circle, perpendiculars, obliques, parallels, and parallelograms. The pupils learn the relation of circumference and radius, the terms chord, arc, tangent, and secant. Angles are measured and there is a good deal of graphic construction. Very elementary projections are introduced.

In the first year of the lycée of France the pupils may enter either the classical or the scientific section.

In the classical lycée two hours a week are devoted to mathematics, and the course includes the study of integers, common and decimal fractions.

In the scientific lycée three hours a week are given to arithmetic and one to geometric drawing. The work in arithmetic includes the study of common and decimal fractions, the rule of three, problems in interest, and alligation.

In the work in geometrical drawing the pupils are taught to use the ruler and compasses, and to make simple designs.

GERMANY.—*Volksschule.*—The subjects of denominate numbers and common fractions are closely related, and pupils are given a good deal of practice in computation in both topics. All four processes are studied with both common and decimal fractions, and the pupils learn how to reduce a common to a decimal fraction. In some schools decimal fractions are studied before common fractions, and in others this order is reversed. The problems involve a good deal of computation with time. In a few of the States abridged multiplication and division are being experimented with.

Course in the Gymnasium.—The course includes the study of common and decimal fractions, simple proportion, tests of divisibility for numbers of one digit, prime numbers, factors, greatest common divisor and least common multiple, and the applications of fractions to simple proportion.

The instruction in geometry is of a propædeutic nature, and includes the study of straight lines, angles, and triangles. The pupils are taught to use compasses and ruler and to make easy constructions.

Course in Realgymnasium.—The course is practically the same as in the Gymnasium. More emphasis is put upon the subject of decimal fractions. Three hours a week are given to the work in mathematics.

Course in the Oberrealschule.—The course is the same as in the Gymnasium except that four hours a week are given to the work and more attention is devoted to oral arithmetic.

HOLLAND.—The course in arithmetic includes exercises and problems involving computations with integers, common and decimal fractions; simple problems in percentage; profit and loss; the elements of proportion; and the metric system.

HUNGARY.—The pupil in Hungary who has completed his fourth school year may continue for two years more in the Volksschule, or he may enter the Bürgerschule, Gymnasium, or Realschule.

The course in the Volksschule and in the Bürgerschule is the same. Four hours a week are devoted to the study of mathematics.

The four operations with decimal and common fractions are taught. The study of decimal usually precedes that of common fractions. Numerous problems are based on school and city statistics. The various units of measure are studied, and problems involving time, rate, and distance are based upon time tables. Originality is encouraged, and speed and accuracy are greatly emphasized. The heuristic method is extensively used.

The work in geometric drawing includes the use of the ruler and compasses; drawing from objects; a study of the simple plane and solid figures, such as the various quadrilaterals, polygons, triangles, circles, and right prisms. Pupils are taught to make easy designs and ornaments.

Three hours a week are usually devoted to the subject of geometric drawing.

Course in the Gymnasium and the Realschule.—The course is practically the same as the one above. Mathematical instruction in general is more extensive and more intensive in the Realschule than in the Gymnasium.

ITALY.—The pupil in Italy who has completed his fourth year may continue in the elementary school for two years, or he may enter the first year of the Ginnasio or of the modern school.

The course in mathematics is practically the same in all types of schools. In the Ginnasio one hour a week is specifically set aside for "practical arithmetic."

The course in the schools includes the following: The four operations with integers, common and decimal fractions; the metric system; business applications; simple proportion; simple interest; discount; profit and loss; rules for surfaces and volumes.

The course in geometry includes free-hand drawing and the making of easy constructions and models.

JAPAN.—The chief aim of the work of the fifth year is to teach computation with integers, decimal and compound numbers; to make the pupils proficient in the solution of applied problems, easy mensuration, and to acquaint the pupils with the metric system, English and American money, and the systems of measure of certain foreign countries.

The work in mensuration includes quadrilaterals, triangles, polygons, circles, and a few of the easier solids.

ROUMANIA.—The Gymnasium begins with the fifth school year. The four operations with integers, decimal and common fractions are emphasized. Geometric

system is taught, and the common tests for divisibility of small numbers are introduced. The rule of three is taught and some easy problems in simple interest are given. Pupils learn the rules for the mensuration of quadrilaterals, triangles, circles, parallelopipeds, pyramids, cylinders, and cones. The course includes but little theory.

RUSSIA.—*Course for Gymnasium.*—Four hours a week are devoted to the study of mathematics. A good deal of emphasis is put upon the theoretical.

The course includes the four operations with common and decimal fractions; the metric system; tests for divisibility by 2, 3, 4, 5, 6, 8, 9, 10, 12, and 15; highest common factor, and least common multiple.

The course in the Realschule is the same as above, and the same number of hours is devoted to mathematics.

SWEDEN.—The course includes the four operations with common and decimal fractions, reduction of fractions, the metric system, and simple problems in percentage.

The course is practically the same in the second year of the Realschule and the fifth year of the primary school.

SWITZERLAND.—A pupil who has completed the fourth school year may continue in the primary school for two years more or he may enter the middle school.

The course is practically the same in both. Four hours a week are given to the subject.

The four operations with integers, common and decimal fractions, and reduction of fractions are taught. The fundamental ideas of ratio and proportion are studied. Simple problems in percentage are given. Interest and simple accounts are studied.

The work in geometry includes the study of lines, angles, quadrilaterals, triangles, polygons, circles, linear and surface measure.

UNITED STATES. (New York course of study).—The pupils are taught how to read and write decimal fractions and to reduce common to decimal and decimal to common fractions. The four fundamental operations with decimal fractions are presented, and the common aliquot parts are studied. The tables for linear, square, and cubic measure are reviewed. Ascending and descending reductions involving the various tables of denominate numbers are taught. Square measure is applied to finding the area of squares, triangles, rectangles, and to problems of painting, papering, and plastering. Cubic measure is applied to finding the volume of rectangular solids, the capacity of bins and cisterns, and the cost of masonry. Many problems are given involving avoirdupois, dry and liquid measure, English money, time and circular measure. Numerous problems involving reductions are given. The value of the franc, pound, and mark in United States money is learned. Problems involving simple bills and accounts are given.

GENERAL SUMMARY OF THE WORK OF THE FIFTH SCHOOL YEAR.

The work of the fifth school year in most European countries is decidedly more extensive than in most of the schools of the United States. It is unusual in this country for the general ideas of percentage to be introduced before the sixth school year. In several of the European countries this work is introduced during the fifth school year. Proportion is rarely introduced in this country in the fifth school year, but it is not uncommonly introduced in that year abroad. Tests of divisibility and prime factors are given much more attention in the European countries than here.

The general use of the metric system gives the European teacher an excellent field of application for decimal fractions, and the two topics are closely related.

Common fractions and denominate numbers are more closely related than is usually the case in the United States.

Probably the most marked difference between fifth-grade work here and abroad is the large amount of time and attention put upon the propædeutic study of geometry in the European countries. Very frequently this work is given under the subject

of drawing, and when this is the case the arithmetic and drawing are usually taught by the same teacher.

It may be said, in general, that the work of the fifth year in the European schools is considerably in advance of the work in the schools of the United States. The courses abroad include all that is included in the fifth-year courses in this country and a good deal that is not included here.

VII. THE WORK IN MATHEMATICS IN THE SIXTH SCHOOL YEAR.

AUSTRIA.—*Course in Gymnasium.*—The topics of highest common factor and lowest common multiple are studied. Prime factors of large numbers are found and the general rules for fractional computation are explained and used in numerous problems. Pupils learn how to reduce a common fraction to a decimal and the reverse. Direct and inverse proportion are studied as an introduction to functional thinking. Unitary analysis is extensively used. There is almost daily drill in computing with denominate numbers and decimal fractions. Simple computations in interest are made.

Geometry.—The symmetry of plane and solid bodies is studied and the congruency of plane figures is established intuitively. Extensive use is made of indoor and outdoor measurements. Pupils learn to construct triangles, quadrilaterals, regular polygons, and circles. The fundamental properties of the right prism, cylinder, cone, and sphere are studied. The study of the sphere is closely related to the work in geometry. The study of geometric drawing is not obligatory in the Gymnasium.

Realschule.—The most important rules for divisibility and factoring are studied. The pupils learn how to find the highest common factor and the lowest common multiple of two or more numbers. All operations with common and decimal fractions are taught and used in numerous exercises. Ratio and proportion are studied as exercises in functional thinking. Problems in interest are solved by formulas and by proportion. The work in geometry includes the study of the symmetry of plane and solid figures, and the fundamental constructions of plane geometry, such as the bisecting of angles and the erecting of perpendiculars. The pupils learn how to construct the various kinds of triangles, quadrilaterals, and polygons. The terms cord, tangent, and secant are taught and used in constructions. A brief study is made of the fundamental properties of pyramids, prisms, cones, and spheres. The symmetry of the equilateral triangle is also considered. Some attention is devoted to familiarising the pupils with the drawing instruments, and simple exercises are given involving constructions and designs. Three hours a week are devoted to instruction in arithmetic and two hours to geometric drawing. The instruction in both subjects is given by the same teacher.

Realgymnasium.—The course in the Realgymnasium is the same as in the Realschule, except that no particular hours are prescribed for the study of geometric drawing.

BELGIUM.—A thorough review of the four processes with integers, common and decimal fractions is given, and the study of the tests for divisibility is continued. The pupils learn how to find the highest common factor and the lowest common multiple of two or more numbers by the process of successive division. The study of unitary analysis is continued, and many practical problems are solved. A good deal of attention is given to problems proposed by the pupils. The study of quadrilaterals, polygons, and circles is continued. The pupils learn how to find the volume of prisms, cylinders, cones, pyramids, and spheres. Attention is directed to the relation between and measures of capacity and of volume. Oral arithmetic is given a good deal of atten-

tion, and abbreviated processes are encouraged. The utilitarian aim of arithmetic is kept constantly in mind, but the formal study of the subject is not neglected. Most of the teaching of geometry is done in connection with the work on the metric system, drawing, and manual instruction. Three hours a week are given to arithmetic.

DENMARK.—The pupil who has completed the first five school years may continue two years longer in the Folkeskole, or he may enter the intermediate school.

Course in the Folkeskole.—The four operations with common and decimal fractions are studied. Multiplication and division of fractions include only easy multipliers and divisors. The pupils learn how to reduce common fractions to decimal fractions and vice versa. Ratio and proportion are taught, and many of the exercises involve decimals. The fundamental ideas of percentage are presented, and pupils learn how to compute simple interest. The subject of profit and loss is the only other application of percentage presented in this grade. The mensuration of all plane figures is studied intuitively.

The course in the intermediate school is practically the same as in the Folkeskole. Four hours a week are devoted to the subject of mathematics.

ENGLAND.—The four operations with common and decimal fractions and their application to denominate numbers are given considerable attention. Unitary analysis is taught and cancellation is emphasized. The pupils are taught to find the highest common factor and least common multiple by inspection. The work in mensuration includes the area of the rectangle and parallelogram and the volume of the simplest rectangular solids. The callipers are used for determining internal and the wedge for external diameters. The use of the metric ruler is taught, and the equivalents for the common metric units are learned. The areas of parks and public grounds are found, and squared paper is used. Pupils are taught how to make simple invoices and bills, and to receipt bills.

The work in geometry includes the following: The intersection of planes, study of the surface and volume of the cone, cylinder, hemisphere, prism. The plumb line and spirit level are extensively used. Lines and angles are bisected by means of the compasses and by paper folding. The pupils cut circles to show angles of 180, 90, 45, 22½, 67½, 157½, 270, 315, and 330 degrees. The protractor is also used.

A few simple problems in loci are given. These usually include the following:

(*a*) The locus of points equidistant from the extremities of a given line.

(*b*) The locus of points equidistant from three given points.

From these, pupils learn how to inscribe and circumscribe a circle. Equilateral and isosceles triangles are constructed, and the pupils discover the important facts relative to these by means of cutting and folding. Set squares are used for constructing parallels and parallelograms. The constructions for erecting a perpendicular at a given point and dropping a perpendicular from a given point are taught.

FINLAND.—The study of common and decimal fractions is continued. Simple and compound proportion are taught. The subject of percentage is introduced. The algebraic equation is introduced in connection with the work in arithmetic.

FRANCE.—*Higher primary school.*—In many schools the sixth school year is the last year of the higher primary. In some schools an additional year is given. Ratio and proportion are taught. Both direct and inverse proportion are studied. Abundant provision is made for drill in oral and written computation.

The elementary ideas of algebraic calculation are applied to positive and negative numbers, and simple algebraic problems in multiplication and division of polynomials are included in the course. Simple equations of the first degree are solved, and the applications of algebra to arithmetical problems are emphasized. Very easy equations of the second degree in one unknown are given some attention. The principles of arithmetical and geometrical progression are taught, and the pupils learn to use tables of logarithms to four and five places. These tables are used in computations in interest progressions and annuities.

Geometry.—The course includes the study of proportional lines and similar figures. A good deal of attention is devoted to graphic constructions. The pupils learn how to divide lines into proportional parts, to find the mean proportional and the fourth proportional. The method of constructing a triangle and a polygon similar to a given triangle and polygon is taught. Regular polygons, including equilateral triangles, squares, pentagons, and hexagons, are studied. The mensuration of the circle is taught, and the pupils learn how to find the approximate value of π (pi). The trigonometric functions sine, cosine, and tangent are taught, and a few simple formulas are used. The pupils learn the terms dihedral, trihedral, and polyhedral angles. The Euclidian geometry has been abandoned in some elementary schools, and the method of Meray is used. Three hours a week are devoted to geometric drawing and designs.

In the various types of special schools, such as the agricultural, industrial, and commercial schools, the mathematics is closely correlated with the practical subject emphasized in the school.

[NOTE.—The subject matter in this school year seems somewhat advanced and excessive. However, the above outline may be found on pages 24-26, Vol. I, Rapports Enseignement Primaire, Paris.]

Lycée.—In the classical lycée two hours a week are devoted to arithmetic, and in the modern language lycée four hours are given to this subject. One of these four hours is usually devoted to geometric drawing. The course in arithmetic includes the metric system, measures of time and velocity, the rule of three, some generalized arithmetic, and simple formulas. The course in algebra includes only simple problems involving equations of the first degree. In both types of lycée a great deal of attention is given to the subject of geometric drawing. The course in this subject includes the geometric representation and perspective drawing of the common geometric solids and numerous useful simple objects. Some attention is given to the representation of ornaments. By means of the ruler and compasses, architectural designs are made.

GERMANY.—*Volksschule.*—Drill in oral and written computations involving integers, common and decimal fractions is continued. Numerous problems involving prices, comparison of money values, and simple profit and loss are given. The subject of exchange is taught, and simple alligation is given some attention. Discount and simple interest are studied. Ratio is taught. Numerous problems are given which involve finding the approximate cost.

Geometry.—The serious study of geometry begins with the sixth school year. The pupils are taught to estimate the length of straight lines, to compute the cost of fences, rails, borders, etc. The terms angles, vertex, right, acute, obtuse, vertical, and adjacent are taught. By means of protractors pupils measure the sizes of angles. The theorems for the congruency of triangles and the theorem for the sum of the angles of any triangle are discovered intuitively. The pupils are taught how to compute the areas of triangles and the various kinds of quadrilaterals. Some attention is given to the study of the circle. The methods of drawing parallels and perpendiculars are taught, and the pupils learn how to construct the various kinds of triangles and quadrilaterals. These constructions are made with care, and are kept in good notebooks.

Gymnasium.—The course in arithmetic in the Gymnasium includes the multiplication and division of decimal fractions and the reduction of common to decimal fractions, and vice versa. A good deal of attention is devoted to abridged computation. Proportion is taught, and numerous problems involving the various systems of weights and measures are given. Percentage and simple interest are presented. Daily drill in oral arithmetic is usually provided.

The course in geometry is the same as the course in the Oberrealschule, which is given below.

In the Realgymnasium from three to six hours a week are devoted to the study of mathematics. The content of the course is the same as in the humanistic Gymnasium.

In the Realschule four hours a week are usually devoted to arithmetic and two to geometry. The course is practically the same as in the Oberrealschule.

Oberrealschule.—The time devoted to mathematics varies, but it is usually five or six hours a week. The course in arithmetic is practically the same as in the Gymnasium. Special attention is devoted to discount and business practice. Some practice is given in using letters in the solution of arithmetical problems.

Geometry.—The pupils are made familiar with the fundamental concepts of plane and solid forms. The course provides abundant practice with the ruler, compasses, and protractor. Simple problems involving surfaces and volumes are given. The pupils learn how to construct triangles from given parts. The congruency theorems are discovered intuitively. Special attention is devoted to the study of the right, isosceles, and equilateral triangles and to the volumes of the parallelogram and trapezoid. The properties of the circle are also considered, and elementary ideas of loci are presented. There is no sharp distinction between plane and solid geometry because of the propædeutic instruction in the subject. Two hours a week are usually devoted to geometry.

HOLLAND.—The sixth year is the last year of the primary school. A comprehensive review of the arithmetic of previous school years is given, and numerous practical applications are made. The pupils learn how to compute the surfaces and volumes of the simple solids. Unitary analysis is emphasized. The content of the course is very similar to that of the corresponding school year in Belgium.

HUNGARY.—The sixth school year is the last school year of the Volksschule. The course in the Volksschule and the Bürgerschule includes the study of the fundamental operations, with common and decimal fractions. Short cuts and numerous problems involving statistics of the home, the city, agriculture, and the industries are taught. Two hours a week are usually devoted to the subject.

Gymnasium.—Abridged multiplication and division are taught, and short cuts are given considerable attention. The subject of ratio and proportion is presented, and numerous problems involving percentage are based on economic conditions. Four hours a week are usually devoted to the study of arithmetic.

The course in geometry includes the study of cubes, prisms, pyramids, cylinders, cones, and spheres. The areas and surfaces of cubes, pyramids, cones, and spheres are determined, and pupils make models of all of these solids.

Realschule.—Four hours a week are usually devoted to arithmetic, and four hours to geometric drawing. The courses in both subjects are practically the same as in the Gymnasium.

ITALY.—The sixth school year is the last year of the elementary school. Unitary analysis, and simple and compound proportion are taught. Commercial arithmetic is especially emphasized. Numerous problems involving the various money systems are given. A comprehensive review of rules and principles learned in previous years is given. The pupils learn how to use the ruler and compasses, and many theorems of geometry are discovered intuitively.

JAPAN.—The sixth year is the last of the ordinary primary. The chief aim of the course is to give simple computations in common and decimal fractions, ratio and percentage, and to afford a comprehensive review of the arithmetic of the first five school years. The review is given the latter part of the year.

NORWAY.—(No report is available.)

ROUMANIA.—The course in arithmetic includes the four operations with integers, common and decimal fractions. The metric system, powers and factors, and the reasons underlying the processes are explained to the pupils. Applications of the metric system are especially emphasized during the sixth school year.

The course in geometry includes the intuitive study of angles, triangles, quadrilaterals, circles, parallels, and perpendiculars. The heuristic method is used, and most of the important theorems preceding the subject of similar figures in the Euclidean geometry are discovered. Simple constructions are given a good deal of attention.

RUSSIA.—Ratio, simple and compound proportion, interest and partial payments are taught. Four hours a week are usually devoted to the subject of arithmetic.

The course in algebra includes the solution of simple arithmetical problems by means of letters, the rules for signs, the four operations with monomials and easy exercises with polynomials.

Realschule.—Direct and inverse proportion, compound proportion, percentage and its practical applications, and alligation are taught. A comprehensive review of arithmetic is given.

The course in algebra includes all that is offered in the Gymnasium, and in addition to this, the squares and cubes of binomials of the types $(a\pm b)$,^{2} $(a\pm b)^3$ and easy equations of the first degree.

SWEDEN.—The sixth school year is the last year of the primary school. Arithmetic and geometry are closely correlated. Emphasis in arithmetic is placed chiefly on rapid and accurate computation with integers, common and decimal fractions, and their direct applications. Five or six hours a week are usually devoted to the subject.

Realschule.—The course in arithmetic includes the four operations with integers, common and decimal fractions, ratio and proportion, percentage and its application to interest, and the metric system. Proportion is usually introduced in connection with simple geometric problems.

The course in algebra includes the removal of parentheses, easy factoring, and simple reductions.

The course in geometry is of a propædeutic nature and includes mensuration and simple constructions. Five hours a week are devoted to the study of mathematics.

SWITZERLAND.—The sixth school year is the last of the primary school. A pupil must complete the work of the primary school before he can enter the Gymnasium.

The course in arithmetic includes the review of the four operations with integers, common and decimal fractions, the application of percentage to business, and applications of proportion to easy geometrical problems.

The course in geometry includes the intuitive study of the triangle, rhombus, rhomboid, and trapezium.

UNITED STATES.—(New York State course.) The subject of common fractions is reviewed, especial emphasis being placed upon the three problems:

(1) To find a fractional part of a number.
(2) To find what fractional part one number is of another.
(3) Given a fractional part of a number and its relation to the whole, to find the whole.

Denominate numbers are reviewed, and drill is given on industrial problems demanding their use. The idea of percentage is introduced, and percentage is applied to profit and loss, trade and cash discount, commission, simple interest, and the making of promissory notes. Some problems are given in simple interest in which three of the elements principal, rate, time, and interest are given to find the fourth. The simple equation is introduced and used in the solution of some of the problems.

In some schools of the country the course includes, in addition to the above, the keeping of simple accounts, the making out and receipting of bills, and some simple measurements. These measurements are usually made in connection with the study of denominate numbers.

In many schools special provision is made for daily drill in oral computation.

GENERAL SUMMARY OF THE SIXTH YEAR'S WORK.

The course in mathematics in practically all of the European countries is decidedly more advanced than in the United States.

The sixth school year is the last year of primary instruction in many of the countries.

The courses abroad include all that is given during the corresponding school year in the United States and also many subjects that are not included in the course in this country.

One marked contrast to the work in this country is found in the emphasis that is put upon the function concept. This feature of the work is especially emphasized in the schools of Germany, Austria, and Hungary, but it is also given some attention in several other countries. In some countries the function concept is introduced in connection with the work in mensuration or graphs; in others, it is first presented with direct and inverse proportion. The tendency abroad is to increase still further the emphasis already placed upon the idea of function.

Another marked contrast to the work in the United States is found in the emphasis that is put upon the propædeutic study of geometry. The courses abroad, almost without exception, provide for the study of intuitive or observational geometry. The amount of time given to this work varies, but the general prevalence of such work is indicative of the importance attached to it. In several of the countries provision is made for the study of geometric drawing. The pupils learn to use the ruler, protractor, compasses, and triangle, and to make the simple geometric constructions. This work is closely correlated with the work in intuitive geometry and the classes are usually taught by the same teacher. In Germany the systematic and serious study of geometry begins with the sixth school year. In Germany and England easy loci problems are introduced.

Short methods and abbreviated processes receive more emphasis abroad than in this country. This is especially true in Austria, Belgium, Germany, and Hungary.

Alligation is taught in several of the countries; for example, in Germany and Russia. The subject is seldom taught in the United States.

The elementary ideas of algebra and of algebraic computation are introduced during the sixth school year in a few of the European countries; for example, in France, Russia, and Sweden.

The time devoted to the study of mathematics abroad is about the same, on the average, as in this country. In some of the countries the time devoted to mathematics is somewhat in excess of that in the United States, if we consider the course in drawing as a part of the course in mathematics.

VIII. THE WORK IN MATHEMATICS IN THE SEVENTH SCHOOL YEAR.

AUSTRIA.—The seventh school year is the second year of the Bürgerschule and the third year of the Gymnasium, Realgymnasium, and Realschule.

In the course in the Bürgerschule five hours a week are devoted to mathematics, three hours being given to arithmetic, one to observational geometry, and one to geometric drawing and constructions.

The pupils are taught how to find the greatest common divisor and the least common multiple of two or more numbers. The study of common fractions is completed, and the method of reducing common to decimal fractions is taught. The four operations with recurring decimals are taught. Fractions are applied to direct and inverse proportion, and the functional idea is emphasized. Some simple problems in interest are given. The fundamental ideas of planimetry are presented. The pupils learn how to bisect angles and certain regular figures. The idea of the symmetry of certain plane figures is presented, and a special study is made of triangles, quadrilaterals, polygons, and circles. Formulas for the surface and volume of a pyramid, prism, cone, cylinder, and sphere are learned. The study of the sphere is correlated with the work in geometry. The work in constructions includes the drawing of angles, triangles, quadrilaterals, polygons, and a few simple solids.

Gymnasium and Realgymnasium.—A comprehensive survey of the arithmetic of previous years is made. Rules are studied both in words and in letters, and simple transformations of formulas are made. The pupils are taught the use of parenthesis, and how to make substitutions in formulas and in equations. The ideas of negative number are presented.

The course in geometry includes the computation of simple surfaces, and the volume of the right prism, cylinder, pyramid, and cone. The pupils measure numerous objects both in and out of doors. The Pythagorean theorem is presented and is applied to plane and solid figures. Formulas for the surface and volume of the sphere are taught without proof. A good deal of attention is given to simular figures, and the idea is emphasized by means of reductions and enlargements. The pupils draw many figures to scale. The instruction in arithmetic and geometry is very closely related.

The pupils are taught the graphic representation of the four fundamental operations and of $(a+b)^2$, $(a-b)^2$, $(a+b)$, $(a-b)$, $(a+b)^3$, and $(a-b)^3$. Abbreviated and approximate computations are given a good deal of attention, and square and cube root are presented. The pupils are taught to estimate their results and to check these estimates by measuring and weighing. Functional thinking receives continual emphasis. A few of the simplest equations are studied. Geometric drawing may be studied, but it is not obligatory. In some of the Realgymnasia it is required.

Realschule.—Three hours a week are usually devoted to mathematics and two to geometric drawing. The drawing is usually taught by the mathematics teacher, and this is considered a decided advantage. Arithmetic is always taught in close connection with geometry.

Approximate computation of decimals is presented and applied to the finding of surfaces and volumes. The pupils make approximate measurements of various objects, distances, and heights, and use the data in problems. General arithmetic is taught, and a summary of the work of previous school years is given. Rules are studied both in words and in letters, and the pupils are taught to generalize rules whenever it is possible. Simple transformations and checks are given a good deal of attention. The terms "coefficient," "powers," and "exponent" are taught. The pupils learn the rules for the square and cube of a binomial, and they represent graphically $(a\pm b)^2$ and $(a\pm b)^3$. Graphic representations of roots are also made. The idea of the negative number is presented, and the four operations with algebraic numbers are taught. The course in geometry is quite similar to the course in the Gymnasium. The pupils learn the fundamental laws regarding areas and volumes. The functional idea is especially emphasized during this school year. Figures are usually drawn to scale, and results are frequently checked by drawing. When solids are involved, results are usually checked by weighing. The course in geometric drawing relates principally to the mensuration and transformation of areas. Some attention is given to decorative forms using circles or arcs.

BELGIUM.—The seventh school year is the first year of the middle school and of the Athénée Royal.

Course for the middle school.—The four fundamental operations with integers, common and decimal fractions are explained and extensively drilled upon. The pupils are taught the tests of divisibility for 2, 3, 4, 5, 8, 9, 11, 25, and 125. The casting out of nines is used as a check for multiplication and division. The greatest common divisor of two numbers is found by successive divisions. The changes effected by adding, subtracting, multiplying, or dividing both terms of a common fraction by the same number are studied. The subject of legal weights and measures is given a good deal of attention. Numerous problems involving the rule of three, simple interest, profit and loss, and commission are solved by means of unitary analysis.

Geometry.—The fundamental concepts of geometry are presented and the pupils are led to discover the conditions under which triangles are congruent. The theorem

for the sum of the angles of a triangle is also developed. The principal properties of triangles and quadrilaterals are studied.

Athénée Royal.—Practically the same course is offered in both the classical and the modern-language schools. The fundamental operations with integers, common and decimal fractions are explained and drilled upon. Tests of divisibility for 2, 3, 4, 5, 6, 9, and 11 are taught without proof. The method of reducing a common to a decimal fraction and the various principles of common fractions are taught without proof. Only small denominators are used. The study of the metric system is continued.

DENMARK.—The seventh school year is the last year of the Folkeskole and the second year of the intermediate or Mellemskole.

In the Folkeskole four hours a week are devoted to mathematics. The study of proportion is continued, and percentage is applied to profit and loss. A comparative study of foreign coins is made.

Geometry.—The course in geometry includes the study of triangles, quadrilaterals, and circles. The pupils learn to compute the volume of right prisms and cylinders with given altitudes and bases. Drawings and models of plane and solid figures are made. Instead of making these drawings, girls are taught the method of keeping simple accou[illegible]

Intermedi[illegible]hool.—Five hours a week are devoted to mathematics. Provision is made for almost daily drill in oral arithmetic. Simple and compound proportion, percentage, simple interest, partnership, simple bookkeeping, and easy mensuration are studied. Geometry is usually taken up before algebra. One or two hours a week are devoted to the study of intuitional or observational geometry. The fundamental properties of straight lines and planes, the measurement of angles, the congruency and similarity of triangles, and the principal properties of quadrilaterals and circles are taught.

ENGLAND.—The four fundamental operations for common and decimal fractions are reviewed, and in a few schools some facts relating to recurring decimals are presented. Numerous problems are worked by means of unitary analysis. The pupils receive a good deal of practice in the drawing up of invoices and the making out of bills. The subject of simple interest is taught, most of the problems being worked by means of formulas. The applications of percentage include profit and loss, discount, and taxes. Some easy problems are given in generalized arithmetic. Many rules are expressed with letters. The algebraic equation is introduced, and easy equations are solved. Simple substitution is also taught, and the meaning and use of negative numbers are briefly presented. Pupils are taught to use rulers marked in centimeters and millimeters. Volumes are found by the use of cubic blocks, by graduated vessels, and by emptying or displacement. The rules for the mensuration of various kinds of triangles, of the rhombus and rhomboid are derived. Simple volumes are studied. Squared paper is extensively used for finding the area of irregular figures. The relation between the length of the circumference and the diameter is determined. Pupils graph simple statistics, such as the school attendance, temperature, and prices.

The work in geometry includes the finding of the locus of points equidistant from a given point, the locus of points at a given distance from a given line, and the locus of points equidistant from three given points. The pupils prove in several ways that the sum of the angles of a triangle is 180 degrees. The relation of interior and exterior angles of triangles is discovered, and the method of dividing a line into equal parts or into parts having a given ratio is taught. The equality of triangles is shown by means of superposition. Some attention is given to the study of circles and to inscribed and circumscribed squares.

FINLAND.—The seventh school year is the last year of the primary school. From one to two hours are devoted to arithmetic. The work of previous years is reviewed, and proportion and discount are studied. The study of algebra is begun in the lyceum.

FRANCE.—In some schools the higher primary lasts but six years. In others there is an additional year which corresponds to our seventh school year. The seventh school year is also the third year of the first cycle of the lycée.

Higher primary.—In view of the examinations, there is a comprehensive review of the mathematics of previous years.

Geometry.—The pupils are taught how to compute the areas of rectangles, parallelograms, triangles, trapezoids, polygons, circles, sectors, and segments of circles. Comparisons of the areas of similar polygons are made, and the formulas for the volumes of prisms, pyramids, and the frustrum of pyramids are developed. The volumes of similar polyhedra are compared. The formula for the volume of the cylinder, cone, and frustrum of the cone are developed. The rules for finding the surface and volume of a sphere and the surface of a zone are taught. The fundamental ideas of descriptive geometry and the elementary notions of rotation are presented.

Three hours a week are devoted to the subject of geometric drawing. The course is a review and a slight extension of the course of the previous year.

Classical lycée.—Two hours a week are devoted to the subject of arithmetic. The course includes factoring, tests for divisibility, prime numbers, proportion, and the square root.

Geometry.—The ruler, square, compasses, and protractor are used. Straight lines, angles, triangles, quadrilaterals, circles, cords, arcs, and tangents are studied. The congruency of triangles, and the theorems for the sum of the angles of a triangle and of a polygon are developed intuitively. Elementary constructions involving straight lines and circles are made and numerous angles are measured. No geometric drawing is taught.

Modern language lycée.—Four hours a week are devoted to mathematics and one to geometric drawing.

The course in arithmetic includes the study of fractions, the extraction of the square root, progressions, and the commercial applications of percentage.

Geometry.—The pupils are taught how to divide a line into parts having a given ratio. Similar triangles are studied, and the definitions of sine, cosine, and tangent are taught. The pantograph is extensively used in connection with the study of similar polygons. Fourth proportionals and mean proportionals are constructed. Regular polygons are studied, and the formulas for the area of a rectangle, parallelogram, triangle, trapezoid, and polygon are developed. The areas of similar polygons are compared, and the areas of circles are computed. Some simple curves, such as the cissoid and the conchoid, are constructed. The course in geometric drawing is practically the same as in the sixth school year, with the addition of pen tracing.

GERMANY.—The seventh school year is the fourth year of the Bürgerschule, Gymnasium and Realgymnasium, and Oberrealschule.

The course in the Volksschule includes simple interest, profit and loss, commercial discount, averages, and alligation. In geometry, right-angle triangles, quadrilaterals, circles, cubes, prisms, pyramids, and cones are studied. A good deal of attention is devoted to measurements and to drawing. The drawing board is extensively used.

Gymnasium and Realgymnasium.—One-third of the time is given to arithmetic and almost two-thirds of the time to geometry. Only a small part of the time is given to algebra.

The course in arithmetic includes interest, bankruptcy, and alligation. Addition, subtraction, multiplication, and division of algebraic expressions and the fundamental properties of equations of the first degree are taught. Instruction in geometry is of a propædeutic nature and includes the construction of plane figures, and the study of the congruency of triangles, the properties of the circle, chords, and angles. Some attention is devoted to geometric drawing.

Realschule and Oberrealschule.—Two hours are usually devoted to arithmetic and algebra and two to geometry. The course in arithmetic includes practical business

procedures and abridged computations. In algebra the equation of the first degree is studied. Positive and negative numbers are introduced, and the idea of function is emphasized.

The course in geometry includes the study of triangles, quadrilaterals, and polygons, special attention being given to the study of similarity. The properties of circles are extensively considered, and the pupils are led to discover the rules for areas of plane figures.

HOLLAND.—The seventh school year is the first year of the middle school and of the Gymnasium. The middle school of Amsterdam has a course of five years. Six lessons of 50 minutes each are given weekly in mathematics. The fundamental operations with integers, common and decimal fractions are reviewed. Tests for divisibility, greatest common divisor, and least common multiple are studied, and some attention is given to the subject of circulating decimals. The instruction in algebra is closely related to that in arithmetic. Simple equations in one unknown are studied with short cases of multiplication, and easy types of factoring are presented. Geometry is studied intuitively. The course includes the study of elementary properties of angles formed by parallel and transverse lines; the properties of triangles, quadrilaterals, and polygons.

Gymnasium.—Four hours a week are devoted to the subject of mathematics. Usually one hour is given to arithmetic, two to algebra, and one to geometry. All pupils have the same course in mathematics during the first four years of the Gymnasium. The fundamental operations with integers, common and decimal fractions are reviewed. The four operations with algebraic equations are taught. The course in geometry is of a propædeutic nature.

HUNGARY.—The seventh school year is the third year of the Bürgerschule, Gymnasium, and Realschule. In the Bürgerschule six hours a week are usually devoted to mathematics. The course includes the study of simple cases of proportion, easy computations in percentage, simple exercises in physics, and alligation. Some attention is devoted to geometric and free-hand drawing.

Gymnasium and Realschule.—Three hours a week are devoted to the study of mathematics and two hours to geometric drawing. The course in arithmetic includes the applications of percentage, simple and compound proportion, profit and loss, the making and receipting of bills, and the solution of numerous problems based on commercial and national transactions. A good deal of attention is devoted to the subject of savings banks.

The course in geometric drawing includes the construction of plane figures, the reduction and enlarging of figures, and the making of designs.

ITALY.—The seventh school year is the third year of the Ginnasio and of the modern school.

One hour a week is usually devoted to the study of practical arithmetic. Geometric drawing and intuitive geometry are taught. The course in algebra includes the formal introduction of literal computation and the study of the four fundamental operations. The course in geometry includes the study of the principal definitions and postulates of formal geometry; the relation of position of perpendiculars, parallels, straight lines, and planes; the construction of trihedral angles, parallelograms, polygons, and the regular polyhedra.

JAPAN.—The seventh school year is the first year of the higher primary and of the middle school.

The subject matter of the instruction in the higher elementary school consists of a more intensive study of the work of the ordinary elementary school, with the addition of ratio and proportion.

In the first year of the higher elementary school fractions, percentage, and ratio and proportion are presented.

Middle school.—Four hours a week are devoted to the subject of mathematics. The course includes the study of numeration, notation, integers, common and decimal fractions, compound numbers, tests for divisibility, prime numbers, factors, greatest common divisor, least common multiple, and ratio and proportion.

ROUMANIA.—The seventh school year is the third year of the Gymnasium. The pupils are taught the tests for divisibility, the greatest common divisor, and least common multiple. Common and decimal fractions are comprehensively reviewed; and the subjects of square root, ratio and proportion, interest, and double entry bookkeeping are studied.

The course in geometry includes the study of the similarity of triangles and polygons, regular polygons, and formulas for the areas of the common plane figures. Geometry is studied intuitively.

RUSSIA.—The seventh school year is the fourth year of the Realschule and of the Gymnasium. The course in both types of schools is practically the same. Four hours are devoted to the study of mathematics, two being given to algebra and two to geometry. The pupils study the operations with polynomials, simple fractions, negative numbers, exponents, simple equations with one or more unknown, involution and evolution, and the theory of proportion.

The course in geometry includes the study of the straight line, angles, parallels, triangles, quadrilaterals, polygons, circles, and the fundamental constructions. A large number of numerical exercises are solved.

SWEDEN.—The seventh school year is the fourth year of the Realschule. Five hours a week are devoted to the study of mathematics. Two hours are given to arithmetic, one to algebra, and two to geometry. The instruction is largely of a practical nature. Applications of percentage and simple bookkeeping are taught. The pupils study only enough algebra to enable them to solve simple equations and to factor the simplest types.

The course in geometry includes the simple constructions of triangles and quadrilaterals and the application of geometry to physics. All instruction in geometry is closely correlated with the instruction in drawing.

SWITZERLAND.—The seventh school year is the third year of the middle school and the first year of the Gymnasium.

In the Gymnasium four hours a week are usually devoted to arithmetic and algebra and one to geometry. The work of previous years is reviewed and extended. The study of the operations with common and decimal fractions is completed, and the application of percentage to profit and loss and interest are studied. The course also includes the study of alligation, square root, abridged computations, and foreign money.

The course in geometry includes the use of the ruler and compasses in making simple constructions. The congruency of triangles is established intuitively, and a great deal of attention is devoted to practical measurements and drawings.

UNITED STATES—(New York State course).—It is assumed that pupils who have completed the work of the six previous school years should be able (1) to read reasonably large figures at sight and to write numbers rapidly from dictation; (2) to perform all fundamental processes in arithmetic rapidly and accurately; (3) to reason quickly and explain simple problems; (4) to compute with common and decimal fractions without hesitation; (5) to comprehend the fundamental principles of percentage and their application.

During the seventh school year a good deal of attention is given to oral drill in getting approximate results. The daily activities of the pupils are drawn upon freely to make the arithmetic interesting, informational, and practical.

Interest, including the subjects of commercial paper, bank discount, and compound interest, is studied. Tables are used in the computation of compound interest. The pupils are taught how to deposit money and draw checks. Many problems are

discussed without actual computations being made. Ratio and simple proportion are studied. The equation and the unknown quantity are used in proportion. The pupils are required to memorize the squares of numbers to 25. Square root is taught and is applied to numerous problems. The various tables of denominate numbers are reviewed, and numerous problems are based upon them. The work in mensuration includes the study of the surfaces of the parallelogram, rectangle, triangle, circle, and trapezoid; and the volume of the cube, sphere, cylinder, pyramid, and cone. No formal geometric demonstrations are attempted.

In some schools the topics of commercial discount, commission, and taxes are also studied during the seventh school year. Within recent years a great deal of attention is being devoted to the study of banks and banking, the saving and investment of money, and loans. In many schools it is thought profitable to organize school banks, to elect officers, and to carry on a regular banking business, usually with imitation money.

SUMMARY OF THE SEVENTH YEAR'S WORK.

In all of the European countries the course in mathematics is decidedly more advanced than in the United States.

In practically all of the countries abroad the subject of intuitional or observational geometry is emphasized during the seventh school year. This is the case in a few schools of the United States, but it is the exception rather than the rule. In some schools abroad the most elementary ideas of trigonometry are introduced in connection with the study of similar figures. The pupils abroad receive much training in constructive work, drawing to scale, reducing and enlarging figures. In some of the schools the pantograph and the planimeter are used. Squared paper is extensively used in England and to some extent in a few other countries. In Austria, France, Germany, Hungary, Italy, and Sweden especial attention is devoted to instruction in the elements of geometric drawing.

Geometry and arithmetic are very closely correlated in the instruction. In England more emphasis is put upon the geometric locus than in any other country during this school year.

In the United States more emphasis is probably put upon a few of the applications of percentage, but several topics in arithmetic not given much attention here are introduced abroad. Alligation is taught in few American schools, but the pupils in some of the schools of Germany, Hungary, and Switzerland study the subject. Recurring decimals, cube root, and the check for multiplication and division by casting out the nines, and the functional concept are more extensively taught abroad than in the United States. The function concept receives marked and growing emphasis in Austria, Germany, France, Hungary, and Switzerland.

It is the exception in the United States for any algebra beyond the simple equation and the evaluation of easy formulas to be introduced into the seventh school year. In practically all of the European countries, on the other hand, the pupils learn to solve simple linear equations containing one unknown, to evaluate the formulas of mensuration and percentage, and to perform the four fundamental operations with simple algebraic expressions. In a few countries some easy factoring and graph work are introduced.

IX. THE WORK IN MATHEMATICS IN THE EIGHTH SCHOOL YEAR.

AUSTRIA.—The eighth school year is the last year of the Bürgerschule and the fourth year of the Gymnasium, Realgymnasium, and Realschule.

Bürgerschule.—The course in arithmetic includes powers and roots, ratio and proportion, simple and compound interest, partnership, alligation, and simple bookkeeping.

The four operations with algebraic expressions and simple equations of the first degree with one unknown are taught.

The course in geometry includes the intuitive study of the more important theorems of plane and solid geometry. The fundamental ideas of the ellipse and parabola are also presented. No distinction between plane and solid geometry is made, because much of the study is based on models. Geometry and arithmetic are closely correlated, and the function idea is emphasized.

The course in geometric drawing includes the representation of bodies in outline, transverse and longitudinal lines and triangles. Some attention is given to free-hand drawing and to designing. The course for girls is much the same as for boys, but the geometric drawing includes the construction of various designs for use in handwork.

Realschule.—The course in the Realschule begins with a thorough review of the fundamental operations in arithmetic. The principles underlying these operations are explained, and numerous practical problems are presented. Interest, alligation, proportion, and graphs are studied. Functional thinking is emphasized in the teaching of algebra by showing how the expression varies in value with different values of the unknown. Transformation of algebraic expressions and the solution of simple linear equations are taught. The pupils learn how to factor simple expressions according to type by both factoring and the Euclidean methods. Pure quadratic equations are studied as far as is necessary for the purposes of plane geometry.

In the course in geometry the theorems dealing with congruency and similarity are especially emphasized, and a good deal of attention is devoted to construction problems and to problems involving numerical computations. The time allotted to geometry in this class is generally considered inadequate.

Gymnasium.—The study of intuitional geometry ends with the preceding school year, and a more systematic study of geometry begins during this year. The idea of function is not considered as a separate topic, but it permeates the whole course. Linear functions are considered in the fourth and fifth years of the Gymnasium, and graphs are used to supplement the idea. The course in geometry includes the study of straight lines, angles, parallels, triangles, and quadrilaterals. The congruency theorems and symmetry of plane figures are studied, and the study of regular polygons and circles is an important part of the year's work. Inscribed and circumscribed figures, tangents, chords, central, and inscribed angles are studied. Algebra and geometry are closely correlated, especially in the mensuration of plane figures. The geometry is a mixture of the Euclidean and post-Euclidean geometry. Pencils of rays and other modern concepts are studied.

Realgymnasium and Realschule.—The study of descriptive geometry is introduced through the theory of projections. The pupils learn to represent by drawings the various conic sections and to construct tangents to and from a point outside of a cone. Projectional drawings are made of simple bodies in various positions. Some simple shadow constructions are made. A study is made of the intersection of projecting planes. In the fourth-year class of the Realgymnasium there are no separate hours for geometric drawing.

BELGIUM.—The eighth school year is the second year of the middle school and of the Athénée Royal.

In the middle school the boys devote four hours and the girls three hours a week to mathematics. The course for girls does not include algebra, and the geometry for girls is the same as for boys in the previous school year, with the addition of some elementary constructions.

Course in arithmetic.—The work of the previous year is thoroughly reviewed and the subjects of factoring and the highest common factor and the lowest common multiple by means of factoring are presented. Drill on the reduction of common to decimal fractions is continued. Discount, proportion, averages, mixtures, alligation, and aliquot parts are studied. Numerous oral exercises are given daily.

Algebra.—The formulas of interest and of mixtures are studied, and the pupils learn to evaluate formulas for any letter and to translate formulas into words. Simple linear equations and ratio and proportion are studied.

Geometry.—The chief aim of the work in geometry is to familiarize the pupil with the principal properties of triangles, quadrilaterals, and circles. Numerous problems involving the computation of the various parts of a triangle are given.

Athénée Royal.—In both the classic and modern schools three hours a week are devoted to the study of mathematics, and the course is practically the same in both types of schools. The course includes the rationalization of the rules for the four processes with integers, common and decimal fractions. The tests of divisibility by 2, 3, 4, 5, and 9 are presented. Checks are extensively used. Simple interest, commercial discount, mixtures, and alligation are studied. Problems in proportion are solved by the use of unitary analysis. An intensive study of the metric system is made.

DENMARK.—The eighth school year is the third year of the intermediate school. Six periods a week are devoted to the subject of mathematics. Provision is made for daily drill in oral arithmetic. The subjects of interest, partnership, profit and loss, simple bookkeeping, and proportion are taught. The tests for divisibility by 2, 3, 4, 5, 9, and 11 are presented. Simple areas and volumes are studied.

The subject of the geometry is introduced before algebra. The study is of an intuitive nature, and two hours a week are devoted to it. Accurate drawing is emphasized, and special attention is devoted to the construction and mensuration of circles. The Pythagorean theorem is intuitively established. Simple areas and volumes are computed, and numerous drawings and models are made.

ENGLAND.—The subjects of rates and taxes, insurance, bankruptcy, and dividends are studied. Interest formulas are derived and used. Averages and proportion are given some attention. The subject of mensuration is treated largely from a practical point of view and includes numerous problems of carpeting, papering, and painting. Generalized arithmetic is introduced. Graphs are used to illuminate the subjects of proportion and interest, and squared paper is extensively used in the computation of areas. The investment of savings and the various methods of transmitting money are studied, and pupils become familiar with business practice. Simple lessons on the formation of companies are given, and the terms capital, stock, shares, shareholder, dividends, and investments are given a practical setting. Problems in measuring include areas and easy volumes. The course in geometry includes the study of the properties of triangles, the construction of triangles and ellipses, and the construction of simple geometric figures and designs. Ratio and proportion are studied. The Pythagorean theorem is discovered intuitively.

FINLAND.—The eighth school year is the fourth year of the lycée. The study of algebra is continued, and geometry is introduced.

FRANCE.—The eighth school year in France is the fourth year of the lycée. It is the last year of the first cycle.

In the classical lycée three hours a week are devoted to the study of mathematics. The work in arithmetic involves a study of the metric system and of proportion. In algebra, the pupils study the four operations with monomials and binomials. Negative numbers are introduced. Numerical equations of the first degree in one and two unknowns are solved, and inequalities of the first degree in one unknown are introduced.

Geometry.—The work of the previous year is thoroughly reviewed. The study of proportion is introduced. The theorems involving the similarity of triangles and of polygons and of homogeneity are studied and the terms sine, cosine, tangent and cotangent are introduced. In the study of similarity, the pantograph is extensively used. The relations between the sides of a right triangle are completed. The properties of the circle and of tangents, chords, and secants are studied. The formulas for

the area of the principal plane figures are developed, and the areas of two similar polygons are compared. No course in geometric drawing is offered in the classical lycée.

The modern lycée.—Four hours a week are devoted to the subject of mathematics. The course in algebra includes all that is given in the classical lycée, and in addition to this equations of the second degree involving one and two unknowns, graphs, logarithms, and antilogarithms to four decimal places, and compound interest are studied.

The course in geometry includes the study of planes and of straight lines in space. The terms polyhedral angles, prism, and pyramid are introduced, and the subject of projections is briefly studied. The formulas for the surfaces and volumes of prisms, pyramids, cylinders, and cones are deduced. The formulas for the surface and volume of the sphere are learned without proof. The pupils learn to draw simple designs and the elementary notions of surveying and of leveling are introduced. In geometric drawing, shadow drawings are given a great deal of attention and some of the drawings are colored.

GERMANY.—The eighth school year is the last year of the Volksschule and the fifth year of the Bürgerschule, Gymnasium, Realgymnasium, and Oberrealschule.

Volksschule.—The course in arithmetic includes the study of insurance and of negotiable paper, and a review of the work of previous years, with special emphasis upon business computations. In some of the States, series and the theory of the extraction of roots are taught, but these are being eliminated.

The course in geometry includes the mensuration of circles, triangles, quadrilaterals, and ellipses. The cone and the sphere are studied. A good deal of practice is given in drawing, especially in drawing to scale. In some of the States the pupils make geometric designs and ornaments.

Gymnasium.—The course in algebra includes the four operations with algebraic expressions, the evaluation and transformation of formulas, and the solution of simple linear equations.

The course in geometry includes the study of lines, angles, and parallel lines, the congruency of triangles, quadrilaterals, circles, inscribed and central angles, loci, and the solution of simple problems by analysis.

Realgymnasium.—Four hours a week are devoted to the subject of mathematics. The course in algebra includes the four fundamental operations and the transformation of simple algebraic expressions. The course in geometry includes the study of straight lines, angles, triangles, and polygons.

The course in the Realschule is practically the same as in the Oberrealschule. Equations of the first degree with more than one unknown are studied. The pupils learn to represent graphically the multiplication tables and certain statistical data and to graph functions of the first and second degrees. Involution and evolution are taught and simple quadratics with one unknown are presented. Two periods a week are devoted to algebra.

Geometry.—The course in geometry includes the study of the circle and the regular polygon. Numerous algebraic-geometric problems involving constructions are given. The reciprocal dependence of angles and sides of triangles is emphasized. The study of triangles prepares for the study of trigonometry. The important properties of prisms, cylinders, cones, polygons, and the frustums of polygons and cones are studied. These solids are presented in parallel perspective, and by orthogonal projections. The surfaces and volumes of the solids enumerated above are computed.

HOLLAND.—The eighth school year is the second year of the middle school and of the Gymnasium. In the middle school, six periods of 50 minutes each are devoted to the study of mathematics.

The course in arithmetic includes the study of proportion and its applications, square root, and approximate computations.

In algebra, factoring, the highest common factor and lowest common multiple, fractions, simple simultaneous equations, and radicals are studied. The instruction in arithmetic and algebra is closely correlated.

In geometry, emphasis is put upon the study of proportion and of similar figures. Simple areas are computed.

Gymnasium.—Three hours a week are devoted to the subject of mathematics. One hour is given to arithmetic, one to geometry, and one to algebra. The same course is given in all types of schools.

The course in arithmetic includes the divisibility of numbers, proportion, and the metric system.

The course in algebra includes the study of factoring, special products, and simple linear equations.

The course in geometry is practically the same as in the middle school.

HUNGARY.—The eighth school year is the last year of the Bürgerschule and the fourth year of the Gymnasium and the Realschule.

Bürgerschule.—Two hours a week are devoted to the subject of arithmetic. Proportion, interest, commercial transactions, savings banks, stock companies, taxes and duties, and approximate computations are studied. Some consideration is given to the comparative study of money systems, and to domestic and foreign exchange.

In algebra, negative numbers, the four operations, and simple linear equations are studied. One to two hours a week are devoted to this work.

Two hours a week are devoted to the work in geometry. The course includes the reducing and enlarging of figures, simple designs, and architectural drawings.

Gymnasium.—Three hours a week are devoted to mathematics, and two to geometric drawing. In the Realschule, four hours are devoted to mathematics and two to geometric drawing. The mathematics course in both types of schools is practically the same, but the course is more intensive in the Realschule.

Algebra is introduced by the study of formulas. Addition and subtraction, multiplication and division, involving both positive and negative numbers, are studied. The pupils learn how to square and cube the sum or the difference of two numbers, and to compute the squares and the cubes of decimal expressions. Simple linear equations are solved and the theory of proportion is studied.

The course in geometric drawing includes simple constructions with circles and arcs, parabolas, ellipses, and hyperbolas. Some attention is given to the drawing and coloring of ornaments. Cones, spirals, and cycloids are also drawn.

ITALY.—The eighth school year is the fourth year of the Ginnasio and of the modern school. Three periods a week are devoted to the subject of practical arithmetic.

JAPAN.—The eighth school year is the second year of the higher primary school and the second year of the middle school.

In the higher primary school the computations by proportion, previously learned, are reviewed and compound proportion is taught. Computation in connection with bookkeeping is also presented. A comprehensive review of the subject matter of the previous year is given.

In the middle school two hours a week are devoted to the study of arithmetic and two to algebra. The study of ratio and proportion is continued and alligation is taught. Percentage is introduced and simple interest is studied. Squares and square root, cubes and cube root, are introduced in connection with the work in mensuration.

The course in algebra includes the definition of symbols, the use of simple algebraic expressions, the introduction of negative number, addition, subtraction, multiplication, and division of integral expressions, and simple equations with one unknown quantity.

ROUMANIA.—The eighth school year is the fourth year of the Gymnasium. The course in algebra includes the four operations with algebraic expressions, the solution

of simple equations with one or two unknowns, applied problems, and negative numbers.

The course in geometry includes the study of the parallel and perpendicular planes, the principal properties of the prism, cylinder, cone, and sphere, and the areas and volumes of these solids. The most difficult problems are omitted. An attempt is made to give the pupil some idea of elementary surveying. The final examinations which occur at the end of this year involve problems the solution of which demands some knowledge of arithmetic, algebra, and geometry.

RUSSIA.—The eighth school year is the fifth year of the Realschule and of the Gymnasium.

In the *Gymnasium* four periods a week are devoted to mathematics. Two hours are usually given to algebra and two to geometry.

The course in algebra includes the solution of quadratics with one unknown, a discussion of the properties of the roots of quadratic equations, the factoring of the quadratic trinomial, the solution of simultaneous equations, and radical equations.

The course in geometry includes the measurement of straight lines and angles, the proportionality of segments, the similarity of triangles and polygons, the numerical relations between the sides of a triangle, elementary ideas of limit, the principal properties of the circle, areas of rectilinear figures and of circles, simple problems in construction, dihedral, and polyhedral angles.

In the *Realschule* three periods a week are devoted to algebra and three to geometry. The course in algebra is said to include the study of square root, irrational numbers, simple quadratics, irrational roots, the relation between roots and coefficients in quadratics, construction of squares from given roots, simultaneous quadratics, arithmetical and geometrical progressions, infinite series, and circulating decimals.

The course in geometry includes the measurement of angles, the study of proportional intercepts, the similarity of triangles and polygons. Inscribed and circumscribed triangles and quadrilaterals are studied and elementary ideas of limits are presented. The pupils learn how to compute the area of a circle. Simple constructions are made, and the principle of homogeneity is presented.

SPAIN.—The report does not give details for the work of this year, but the statement is made that algebra and trigonometry are taught, six periods a week being devoted to the subject of mathematics. The instruction in geometry is of an intuitive character.

SWEDEN.—The eighth school year is the fifth year of the Realschule. Equations of the first degree in one and two unknowns are studied. Two hours a week are devoted to arithmetic and two hours to geometry. Simple bookkeeping is usually included in the course in arithmetic. Square-root tables are extensively used. The opinion prevails that algebra should be introduced through the simple equation. No textbook is used in the introductory work in geometry. The regular plane figures and the circle are studied. The principal properties of the ellipse are considered and some attention is given to projections.

SWITZERLAND.—The eighth school year is usually the last year of the middle school. It is the second year of the Gymnasium.

The Gymnasium.—Half an hour a week is usually devoted to the study of arithmetic, two hours to algebra, two to geometry, and one to geometric drawing. In several of the Cantons no special instruction is given in the subject of geometric drawing. The courses in the Gymnasia vary in the different Cantons, but in general, they are somewhat as follows:

In arithmetic, simple and compound proportion, abridged multiplication and division, square root, and the computation of simple areas, partnership, alligation, and business transactions are studied.

The course in algebra includes the four operations and the solution of simple linear equations.

The course in geometry includes the fundamental ideas of the subject up to the equality of surfaces. The applications of the theorems are emphasized, and a good deal of attention is placed upon drawing and the simple constructions.

UNITED STATES.—The business applications of percentage and the mensuration of common surfaces and of solids include all that is usually offered in mathematics during the eighth school year in the United States. In some schools, a comprehensive review of the arithmetic of previous years is given. In the most progressive schools of the country no arithmetic is given during the eighth school year, or at least during the last half of the eighth school year, and the time is devoted to the study of algebra combined with arithmetic.

The New York State course of study outlines the following in arithmetic for the eighth school year:

Daily drill in rapid mental and written computations. This includes certain short methods in multiplication; the computation of simple interest by short methods; and drill in the use of proper business forms, such as invoices, bills, and account sales. The pupils are taught how to keep the simple accounts of daily life, of the home, and the farm. Stocks and bonds are considered. The various types of insurance are studied, and the subject of taxes is closely correlated with the subject of civics and is taught from the standpoint of local conditions. The simple formulas common in various mechanical journals and in trade papers are used to introduce the study of algebra. The study of these formulas is followed by the solution of simple linear equations.

Throughout the eighth school year, especially, an attempt is made to correlate the work in mathematics with the instruction in manual and household arts and agriculture. The problems are largely of a practical nature, and much data is based on local conditions. In some of the schools, a great deal of attention is devoted to the subject of proportion. This is especially true in those schools where the subject of mensuration is emphasized, and the simple equation is not introduced early.

SUMMARY OF THE EIGHTH YEAR'S WORK.

In the United States the eighth school year is the last year of the elementary school. In most of the schools but little work in mathematics is attempted beyond the study of mensuration, proportion, and some of the applications of percentage. In an increasing number of schools some work in algebra is being introduced during the eighth school year. The nature and extent of this work varies, but in general it may be said to consist chiefly of the simple equation and the evaluation of formulas. It is but rarely that a school is found in which any attention is given to the subject of geometry except such as is necessary in the study of mensuration.

In all European countries the course in mathematics is more extensive than in the United States. In no European country is less geometry or algebra offered during the eighth school year than in the United States, and in practically all the countries the work in both of these subjects is much more extensive than in this country. In Austria, Germany, and Switzerland the most emphasis in algebra is put upon the four operations, the solution of simple equations, and proportion. In a few of the countries the subject of factoring is also taught. This is especially true in certain schools of Austria, Roumania, and Holland. The course in algebra reported for Russia is more advanced than in any of the other countries, but it is probably safe to assume that only the most elementary ideas of some of the topics mentioned are presented to the pupils.

In practically all of the countries, except the United States, a good deal of emphasis is put upon the study of intuitive geometry during the eighth school year. Especial attention is devoted to the study of congruent and similar figures and to simple constructions. In a few of the countries the pantograph is used in connection with the

study of similar figures, and the terms sine, cosine, and tangent are introduced. This is notably true in France. In a few of the modern lycées of France logarithms are introduced. The subject of geometric drawing is given a good deal of attention in several of the countries, especially in Austria, Hungary, and in some of the Cantons of Switzerland. In a few of the schools of Austria some instruction is given in the subject of descriptive geometry. In practically all of the schools abroad the subjects of geometry and arithmetic are more closely related than in this country. Continued emphasis is put upon the function concept idea, especially in Austria, Germany, and Switzerland.

In most of the foreign countries less time is devoted to the study of arithmetic during the eighth school year than in the United States. The study of alligation is continued in some of the schools of Austria, Belgium, Switzerland, and Japan; and a good deal of emphasis is put upon the study of tests for divisibility.

It is customary in all of the European countries to teach algebra and geometry simultaneously. During the eighth school year the time is about evenly divided between these two subjects, from two to three hours a week being devoted to each. An attempt is not made to fuse the subjects, but the interrelations between them are kept constantly in mind, and the pupil is not permitted to forget his geometry while studying his algebra, or vice versa. Each subject is considered an instruction unit, but it is used whenever possible as a tool in the study of the other. By the time a European boy has completed his eighth school year, he is at least a full year in advance of the American boy in his knowledge of mathematics.

X. THE WORK IN MATHEMATICS IN THE NINTH SCHOOL YEAR.

AUSTRIA.—The ninth school year is the fifth year of the Gymnasium and of the Realschule.

In the Realschule four hours a week are devoted to the study of mathematics. This time is about evenly distributed between arithmetic and geometry. These two subjects complement each other and form one instruction unit. Accurate observation and concise description are emphasized. Modeling is extensively used. There is a close correlation between plane and solid geometry, because of the extensive use of models. The theory of powers and roots is studied. Quadratics with one unknown are solved, and the pupils learn to graph the various types of quadratic equations. Only the simplest types of simultaneous quadratics are considered. Irrational, imaginary, and complex numbers are studied as far as is necessary for the solution of quadratic equations. The subject of logarithms is thoroughly studied. In geometry the work of the fourth class is continued and completed during the first semester. The course includes the study of proportional lines, similarity, and the computation of areas. During the second semester the subject of stereometry is introduced. The instruction begins with the study of solid angles. Tetrahedrons are studied, and surfaces and volumes are computed. A systematic study of descriptive geometry is made during this year. The course includes the study of straight lines and planes, vertical and horizontal projections. Oblique projections are used occasionally. Constructions are applied in the study of the regular pyramid and prism and of their shadows.

In the Realgymnasium two hours a week are devoted to descriptive geometry. No special hours are assigned for the subject of geometric drawing. With this exception, the course is practically the same in mathematics as in the Gymnasium.

In the Gymnasium neither descriptive geometry nor geometric drawing is obligatory. The arithmetic of previous years is extended and supplemented. Special

attention is given to the subject of powers and roots. Stereometry is introduced and emphasis is put upon functional thinking. Models are very extensively used.

BELGIUM.—The ninth school year is the last year of the middle school and the third year of the Athénée Royal.

Middle school.—A comprehensive review of the arithmetic of previous years is given. Powers and roots are studied, special attention being devoted to approximate roots. Compound interest is computed by the use of tables. Bonds, shares, savings banks, annuities, and insurance are studied. Tables are extensively used in all computations.

Algebra.—The course includes the study of the fundamental operations, the square and cube of a binomial, fractions, equations of the first degree involving two and more unknowns, negative numbers, and indeterminate equations.

Geometry.—The work of previous years is reviewed, and proportionality and similarity are introduced. The relations between the sides of a right triangle are computed. The regular polygon, circle, and the sector are studied. Some elementary exercises are given in surveying and in leveling. The surfaces and volumes of polyhedra and of cones, cylinders, and spheres are computed and applied in solving practical problems.

During this year, girls are not required to study algebra. The course in arithmetic is the same for boys and for girls. The course in geometry is somewhat less extensive for girls than it is for boys. A study is made of parallels and of elementary theorems. The areas of rectangles, parallelograms, triangles, trapezoids, and circles are computed, and the formulas for the surface and volume of prisms, cylinders, pyramids, cones, and spheres are used without proof. From three to five hours a week are devoted to the subject of mathematics.

Athénée Royal.—In the classical schools three hours are devoted to mathematics, and in other types of schools four hours are devoted to the subject.

The course in the classical school includes a study of the changes which a qoutient and remainder undergo when the dividend and divisor, or one of them, are increased or decreased in a certain ratio. Tests for divisibility are studied. Checks by the casting out of nines and the elevens are applied in multiplication and division. Simple interest, bank discount, annuities, stocks and bonds, mixtures, alligation, negotiable paper, partnership, arithmetical and geometric progression, simple and compound proportion, and the computation of easy surfaces and volumes are studied during the year.

The course in the modern school is much the same as in the classical school, except that no attention is given to the study of arithmetical and geometrical progression, and more emphasis is put upon the study of congruency and simple constructions in geometry.

DENMARK.—The ninth school year is the last year of the intermediate school. At the close of the year a comprehensive examination is given, and the student must pass this satisfactorily before he is entitled to enter a higher grade. Seven hours a week are devoted to the subject of mathematics. Two hours are usually given to arithmetic, three to algebra, and two to geometry. After the work of the previous year has been thoroughly reviewed, the subjects of proportion, powers and roots, simple equations and quadratic equations with numerical coefficients are studied.

The course in geometry includes a review of the work of the previous year and a study of the similarity and congruency of triangles and polygons. Simple constructions are made. Provision is made for daily drill in mathematics.

ENGLAND.—The lack of uniformity in the English school system, especially in the upper years, makes any exact statement in regard to courses quite difficult.

In some of the schools a course similar to the following is offered for boys who do not wish to specialize in mathematics: Extraction of the square root by rule, the Pythagorean theorem, the study of circles, chords, arcs, tangents, and angles; the construction of circles from simple data, the construction of regular polygons, the

solution of quadratic equations with numerical coefficients, the simplifying of fractions, and the solution of applied problems.

The boys who pursue this course are not expected to continue the study of mathematics after leaving the school. The majority of them do not enter the university. The boys who expect to enter the universities and who are preparing at any of the great schools, such as Eton, Rugby, Harrow, and Winchester, spend from three to seven hours a week in the study of mathematics. By the time a boy is 15 or 16 years old he has usually completed the study of the five books of Euclid and of algebra to the progressions.

FINLAND.—Boys who expect to pursue their education beyond the elementary school usually enter the lycée or the secondary school. The lycée prepares directly for the university. The classical and modern lycée are somewhat similar to the German Gymnasium and Realschule. Both algebra and geometry are taught. Emphasis is put upon the solution of simple and quadratic equations. Solid geometry is briefly studied. The development of the intuition receives a good deal of attention. Further details of the course are not available.

FRANCE.—The ninth school year is the fifth year of the lycée. It is the first year of the second cycle. The second cycle lasts two years and has four divisions:

A. Latin-Greek (classical).
B. Latin-Modern languages.
C. Latin-Science.
D. Science-Modern languages.

Course in A and B.—Two hours a week are devoted to the subject of mathematics. The work in algebra includes a review of the work of the previous year. The four operations are performed with positive and negative numbers. Problems involving uniform movement are solved. Inequalities of the first degree are studied. Variations of the expression $ax+b$ are considered. Graphs are introduced, and the pupils represent the variations of x^2 and of $\frac{1}{x}$.

Solid geometry.—Dihedral angles and perpendicular and parallel planes are studied. Polyhedral angles are considered; and formulas for the surface and volume of prisms, pyramids, cylinders, cones, and spheres are developed. The course does not include geometric drawing.

Course in C and D.—Five hours a week are devoted to the subject of mathematics. The work is more intensive and more extensive than in courses A and B. The four operations with positive and negative numbers, the solution of equations of the first degree involving one and more unknowns, and inequalities of the first degree are studied. The variation of the expression $ax+b$ is considered and represented graphically. Equations of the second degree in one unknown are solved, but no equations involving imaginaries are considered. The relations between roots and coefficients are studied. The quadratic trinomial and inequalities of the second degree are introduced. A good deal of emphasis is put upon the subject of graphs. The variations of the expression $\frac{ax+b}{a'x+b}$ are considered. The notion of derivatives is applied to simple numerical problems and to functions previously studied. Arithmetical and geometric progressions are introduced. Four-place logarithms are used, and the subject of compound interest is studied.

The course in geometry includes a systematic study of lines, angles, parallels, perpendiculars, triangles, quadrilaterals, polygons, and circles. The theorems for the congruency and similarity of triangles are especially considered, and the terms sine, cosine, tangent, and cotangent, for angles from zero to 180° are introduced. The construction of mean and fourth proportionals and the harmonic division of a line are considered. The areas of triangles, polygons, and circles are computed. Ele-

mentary notions of surveying are introduced. The pupils learn to use the surveyor's chain and the square.

In the classes in drawing, the ruler, compasses, square, and protractor are used. The simple geometric constructions are made.

The course in geometric drawing includes the making of designs particularly for tiles, parquets, and mosaics. Many of the drawings are colored.

GERMANY.—The ninth school year is the sixth year of the Bürgerschule, Gymnasium, Realgymnasium, Realschule, and Oberrealschule.

In the Gymnasium two hours a week are usually devoted to the study of algebra and three to geometry. The courses vary somewhat in the different States, but the work in algebra usually includes the solution of simple equations with one or more unknowns, graphs, the study of simple functions, and the solution of simple quadratic equations. In some of the States the course includes also the use of five-place logarithms and the slide rule, and the study of arithmetical and geometric progressions, compound interest, and annuities. The course in geometry usually includes the study of similar figures, regular polygons, and cyclometry.

In most of the Realgymnasia and Realschulen, three hours a week are devoted to algebra and three to geometry. The work is more intensive than in the Gymnasium, especially in the subject of logarithms and quadratic equations. Elementary trigonometry and solid geometry are sometimes introduced, and oblique and parallel projections are considered.

The course in the Oberrealschule is more advanced than in the other types of schools. In general it may be said to be an intensive study of the work that is offered in the Gymnasium, with the addition of the following: Trigonometric functions, and goniometry are frequently introduced with the study of similar figures. Solid geometry is studied. Euler's theorem is considered. The study of regular polyhedra is introduced, and numerous exercises involving numerical computations are solved.

HOLLAND.—The ninth school year is the third year of the middle school and of the Gymnasium.

Seven periods a week are devoted to the study of mathematics in the middle schools. The previous work in radicals is reviewed and extended. Fractional negative exponents are studied. Logarithms, arithmetical and geometrical progression, and compound interest are introduced. Equations of the first and higher degrees involving one or more unknowns are solved. The course in geometry includes the study of circles, regular polygons, and goniometry.

The Gymnasium.—All types of Gymnasia offer the same course in mathematics. One hour a week is usually devoted to the subject of algebra and two to geometry. The course in algebra includes the study of equations of the first and second degrees with one or more unknowns and the subject of radicals. Details of the course in geometry are not available.

HUNGARY.—The ninth school year is the fifth year of the Gymnasium, Bürgerschule, and Realschule.

Three hours a week are devoted to mathematics in the Gymnasium. Systematic instruction in geometry begins during this year, and a good deal of attention is devoted to the study of models. The pupils are required to make numerous indoor and out-of-door measurements and estimates. The axioms, postulates, and the principal theorems of plane geometry are studied. The course in algebra includes the solution of equations of the first degree with one or more unknowns, square root, cube root, radicals, and quadratic equations of the types used in geometry.

Realschule.—Five hours are devoted to the study of mathematics. No geometric drawing is given in either Gymnasium or Realschule. The course is practically the same as in the Gymnasium, but the work is more intensive. Some of the fundament theorems of solid geometry are introduced.

Bürgerschule.—Four hours a week are devoted to the study of mathematics. The course includes the four fundamental operations with positive and negative numbers, fractions, simple linear equations in one unknown, the chief theorems of planimetry, the study of congruent and similar plane figures, the construction of plane figures, and the study of perimeters and areas.

ITALY.—The ninth school year is the last year of the Ginnasio and the third year of the modern school. Three hours a week are devoted to the study of practical arithmetic and two hours to geometry. The first book of Euclid is studied. Some attention is devoted to the study of the simple solids and to the proportionality of magnitudes. Equations of the first degree are solved. Four periods a week are usually devoted to mathematics.

JAPAN.—The ninth school year is the last year of the higher primary and the third year of the middle school.

The higher primary.—Four hours a week are devoted to the study of mathematics. The lessons of the previous years are reviewed and supplemented. There is a repetition of the mensuration previously learned, to which are added the mensuration of the pyramid, circular cone, frustum of a pyramid, frustum of a circular cone, expression of square root and cube root and a review of the four rules, proportion and percentage, together with arithmetical and geometric progression.

Rules for finding the volume of pyramids and circular cones are given dogmatically and verified by actual measurement. Simple bookkeeping is sometimes taught.

The middle school.—Four hours a week are devoted to the study of mathematics. The study of equations is continued, and linear simultaneous equations are solved. Formulas relating to the distributive law are considered, factoring, the greatest common factor, and lowest common multiple are studied. The reduction of fractional expressions and the four fundamental operations with fractions are included in the course. Quadratic equations with one unknown quantity, equations with one unknown quantity reducible to quadratics, and simultaneous equations containing quadratic equations are presented.

The course in geometry includes the study of straight lines, triangles, parallelograms, circles, chords, tangents, and segments. Two hours a week are devoted to this work.

ROUMANIA.—The ninth school year is the first year of the lycée. Six hours a week are devoted to the study of mathematics. The four operations with common and decimal fractions are explained. The use of tests for divisibility is continued. Prime numbers are studied. The idea of the limit is introduced. Square and cube root are studied. The aim is to cultivate mathematical reasoning. The work is more intensive than in previous years.

Algebra.—Positive and negative numbers, algebraic fractions and radicals are introduced. Equations of the first degree are solved, and some attention is devoted to the study of reciprocal equations. The quadratic trinomial is considered. Arithmetical and geometrical progression, logarithms, compound interest, and annuities are taken up.

The course in geometry includes the study of angles and triangles, quadrilaterals, circles, and proportion. Numerous exercises are solved.

RUSSIA.—The ninth school year is the sixth year of the Realschule and of the Gymnasium.

In the Gymnasium four hours a week are devoted to the subject of mathematics. The work in algebra includes the study of progressions, logarithms, and compound interest. The course in geometry includes the study of regular polyhedra, the formulas for the area and volume of the prism, cylinder, pyramid, cone, and sphere. A good deal of attention is devoted to estimates.

In the Realschule two periods are devoted to algebra, two to geometry, and two to trigonometry. Fractional, negative, and zero exponents are introduced. Irra-

tional expressions are considered. Logarithms, exponential equations, combinations, the binomial theorem, continued fractions and their application to square root and to logarithms, compound interest, and annuities are studied.

The course in geometry includes the study of the relative positions of straight lines and planes in space, the chief characteristics of plane and solid angles, the regular polyhedra, and the study of the formulas for the surfaces and volumes of the regular solids.

In trigonometry, the sine, cosine, tangent, and cotangent for both acute and obtuse angles are studied. The limiting values of these functions are considered and the logarithmic and trigonometric functions are used. The elementary formulas of plane trigonometry and the formulas for the solution of right and oblique triangles are developed. The areas of triangles are computed by the use of the trigonometric formula, and trigonometry is applied to the solution of geometric problems and exercises.

SWEDEN.—The ninth school year is the last year of the Realschule and the first year of the Gymnasium.

In the Realschule square root and its application to geometric problems, the computation of compound interest by means of tables, graphs, and simple bookkeeping are taught. Two hours a week are devoted to the study of arithmetic.

The work in geometry includes the drawing and making of models, simple projections, the use of the compasses, ruler, and protractor, and the making of simple constructions. One hour a week is devoted to geometry.

Gymnasium.—Five hours a week are devoted to mathematics in the classical gymnasium, and seven hours a week in the realgymnasium. The algebra of the Realschule is reviewed, and square root and quadratic equations are studied. Simple functions are emphasized. The geometry of the Realschule is reviewed, and numerous applications are made. Linear drawing is given considerable attention, and the principles of the theory of projection are studied. Special attention is directed to the study of the intersection of plane and solid figures.

SWITZERLAND.—The ninth school year is the third year of the Gymnasium and the first year of the Realschule. The course is the same in the Classical and the Realgymnasium. Two hours a week are devoted to algebra, two to geometry, and in some Cantons two hours a week are devoted to geometric drawing in the Realgymnasia.

The course in algebra includes the study of logarithms, arithmetical and geometrical progression, compound interest and annuities, the four operations with fractional exponents, and the study of functions.

The course in geometry includes the study of transversals, triangles, quadrilaterals, circles, construction problems, the theorems of congruency and similarity, and the study of surfaces and volumes.

Realschule.—One hour a week is devoted to the study of arithmetic, two to algebra, three to geometry, and two to geometric drawing. The course is practically the same as above except that a more intensive study is made. More attention is given to abridged multiplication and division than is the case in the Gymnasium.

UNITED STATES.—The course in mathematics varies somewhat for the ninth school year, but in general it may be stated as follows:

The four fundamental operations with rational integral expressions, factoring, highest common factor and lowest common multiple, fractions, reduction, the fundamental operations with fractions, linear equations with one unknown, applied problems, simple and simultaneous equations with one and more variables, graphic solutions, involution, the method of extracting square and in some cases cube root, fractional and negative exponents with proofs of fundamental laws, radicals, irrational expressions and equations, problems, the solution of quadratic equations of the type $ax^2+bx+c=0$ by factoring.

SUMMARY OF THE NINTH YEAR'S WORK.

The ninth school year is usually the first year of the secondary school in the United States. In the majority of secondary schools the subject of algebra is completed up to quadratic equations. In a few of the schools less work is attempted in algebra and more attention is devoted to the subject of intuitional geometry. In such schools, emphasis is put upon the use of the simple geometrical instruments, the making of easy constructions, measurements, and simple designs.

In all European schools both algebra and geometry are taught during the ninth school year. In most of the countries the time is divided evenly between these two subjects, from two to three hours a week being devoted to each. Here, as in the preceding year, the subjects supplement each other, and no attempt is made to fuse them. The relations between the two subjects are emphasized much more extensively abroad than in the United States. In most of the schools of Europe the distinction between plane and solid geometry is less marked than in the schools of the United States. This is, in part at least, due to the fact that models and drawings are very extensively used abroad. During the ninth school year the work in algebra in most of the European schools is but slightly more advanced than in the best schools of the United States. In some of the schools of Austria, France, Denmark, Holland, and Hungary the course is somewhat more advanced than in the United States. In these schools logarithms, proportion, and quadratics are studied. The course in the Realschulen of Russia is said to be very advanced.

In practically all of the European countries geometry is studied during the ninth school year. Especial emphasis is put upon the study of proportional lines, congruent and similar figures, areas, and volumes. In all of the schools models are extensively used. In Belgium, France, Germany, and Holland the study of similar figures is supplemented by elementary exercises in surveying, and in some of the French schools the terms sine, cosine, tangent, and cotangent are introduced and used.

Descriptive geometry is introduced in some of the Realschulen and Realgymnasia of Austria. In France, Sweden, and in the Realgymnasia of Switzerland some attention is devoted to the subject of geometric drawing. In most of the countries continued emphasis is put upon the idea of functions. This is especially true in Austria, France, Germany, Sweden, and Switzerland. The subject of logarithms is introduced in most of the countries, and in some of the schools of Germany the pupils become familiar with the slide rule.

In a few of the countries the study of arithmetic is continued during the ninth school year. Especial emphasis is put upon the use of tables for finding interest, and upon abridged processes.

Arithmetic, algebra, and geometry are more closely related in the schools abroad than in the United States. Each subject is considered as an instruction unit, but it is closely correlated with the others. When a European boy has completed his ninth school year he has a mastery of all the topics in algebra that the American boy has studied, and in addition to these he is familiar with practically all of the more important theorems of plane and solid geometry. In some of the schools of Finland, Germany, and Russia the elements of trigonometry are introduced.

XI. THE WORK IN MATHEMATICS IN THE TENTH SCHOOL YEAR.

AUSTRIA.—The tenth school year is the sixth year of the Gymnasium, Realschule, and Realgymnasium.

Realschule.—The course in algebra includes the study of logarithms and exponential equations of the simplest kind, arithmetical and geometrical progressions, annuities, and compound interest.

Goniometry and plane and spherical trigonometry have an important place in the curriculum, on account of their practical value. The course in trigonometry may be said to dominate the work of this school year in the Realschule. The functions are defined and developed for acute and obtuse angles and are graphically represented in the unit circle. All the fundamental formulas are developed and used in the solution of acute and oblique triangles. Logarithmic tables and the natural functions are used. The spherical triangle and the chief formulas for its surface are studied. These formulas are applied to the solution of certain problems in stereometry.

The instruction in geometric drawing in the Realschule and in the Realgymnasium includes an extensive study of vertical projections of plane figures, the shadow of the circle projected on a plane by parallel and by oblique rays, the derivation of the properties of the ellipse conceived as the projection of the circle, the representation of cylinders and cones of revolution, tangential planes, cones and cylinders, shadow constructions, and the study of the sphere. Three hours a week are devoted to the subject of geometric drawing.

Gymnasium.—Equations of the second and of higher degrees that can be reduced to quadratic equations are solved. Surds, imaginaries, and complex numbers, graphs, and the nature of roots of quadratics are considered.

Geometry.—A full year is devoted to the study of solid geometry, a great deal of emphasis being put upon drawing. The pupils draw vertical and horizontal projections of all the simple bodies.

Goniometry and trigonometry.—The course is not so extensive as in the Realschule, and the slide rule is not much used in the instruction. Five-place logarithmic tables are almost always used.

BELGIUM.—The tenth school year is the fourth year of the Athénée Royal. In the classical course three to four hours a week, and in the modern course four hours a week, are devoted to the study of mathematics.

Classical course.—The arithmetic of previous years is reviewed. The greatest common divisor by successive division is studied. Factoring and greatest common divisor and lowest common multiple by factoring receive a good deal of attention.

The course in algebra includes the transformation of equations, the study of proportion, linear equations with one unknown, applied problems, the evaluation of formulas, and alligation.

The work in geometry includes the study of the first book of Euclid. Numerous constructions are made.

Modern course.—The work in arithmetic is the same as in the classical course, with the addition of the study of tests for divisibility.

The course in algebra includes all that is given in the classical course, with the addition of negative expressions, the square and cube of binomials, the four fundamental operations, the remainder theorem, the four operations with fractions, the reduction of fractions, squares and the square roots of numbers, and of algebraic expressions, radicals, and simple quadratics.

The course in geometry includes a complete review of all the work of preceding years and a study of the circle, the mensuration of quadrilaterals, the study of proportional lines and similar figures, and the calculation of the elements of the triangle.

DENMARK.—The tenth school year is the first and only year of the Realklasse and the first year of the Gymnasium.

Realklasse.—This class is for those who wish an additional year, but are not going to the university.

The work in arithmetic includes the study of business forms, the use of interest, and logarithmic tables.

The course in algebra includes the general solution of quadratic equations. The rules for the sum and the product of the roots, the solution of easy equations reducible

to quadratics, symmetrical equations, simultaneous quadratics, powers and roots, and annuities are also studied. The subject of geometry is elective, and the course is practically the same as in the corresponding year of the Gymnasium.

Gymnasium.—In the classical Gymnasium the course includes the study of general quadratic equations, the rules for the sum and the product of the roots of quadratic equations, inequalities of the second degree, maxima and minima, limits, simultaneous quadratics, imaginaries, four-place logarithms, simple exponential equations, arithmetical and geometrical progression, infinite series, compound interest, and annuities.

The course in geometry includes the study of similar figures, regular polygons, the study of the circle, the trigonometric functions of acute and obtuse angles with simple applications, and the graphic representation of simple functions.

The course in the scientific Gymnasium includes all that is offered in the classical Gymnasium, with the addition of the study of approximate computations, harmonic progressions, complex numbers, permutations and combinations, indeterminates with two unknowns, and complex roots.

ENGLAND.—The tenth school year is the last year of the five-year preparatory course. The work in arithmetic includes the mensuration of plane and solid figures and the study of simple business arithmetic.

The course in geometry includes simple constructions, loci problems, and propositions concerning parallels, the congruency and similarity of triangles, areas, and proportional parts.

The course in algebra includes the theory of exponents, logarithms, and variations.

The course in trigonometry includes the numerical solution of triangles by means of the sine and cosine formulas, the use of tables, and simple problems from surveying.

The course in girls' schools for the corresponding year is more elementary. The course in arithmetic includes the study of general arithmetic, compound interest, stocks and bonds. The course in algebra includes quadratic equations, the theory of indices, surds, progression and proportion. In geometry, Books I to IV of Euclid are usually completed.

FINLAND.—The report makes no division of the work by years. Algebra is begun in the third year of the lycée, geometry in the fourth, and trigonometry in the fifth. The subject of solid geometry is somewhat abridged. The tenth school year is the sixth year of the lycée.

FRANCE.—The tenth school year is the sixth year of the lycée. It is the second and last year of the second cycle. In the classical lycée two hours a week, and in the scientific lycée five hours are devoted to the study of mathematics.

Classical lycée.—The course in algebra includes exercises involving equations of the first degree and of the second degree in one unknown, variations of trinomials of the second degree.

The course in geometry includes the mensuration of angles, the study of plane figures, the definition of sine, cosine, and tangent for angles from zero to 180 degrees, and the formulas for surfaces and volumes.

Scientific lycée.—The course in algebra includes the study of equations of the second degree, applications of geometry to trigonometry, equations of derivatives of simple functions, the study of variations and of graphic representations, the study of motion in connection with the theory of derivatives.

The course in geometry includes the study of polyhedra, cones, spheres, surfaces, and volumes. The ideas of translation and rotation are extensively used and the subject of symmetry is given a good deal of emphasis. Rather an extensive course in descriptive geometry is offered in many schools. Geometric drawing is also taught in most of the scientific schools. The trigonometric functions are studied, and the principal formulas for the solution of right and oblique triangles are derived.

GERMANY.—The tenth school year is the seventh year of the Bürgerschule, the Gymnasium, Realgymnasium, Realschule, and Oberrealschule.

Gymnasium.—Equations of the second degree, including simultaneous quadratics and equations in the quadratic form, are studied. Four and five place logarithmic tables are also used. Emphasis is put upon the application of algebra in the solution of geometric problems.

Geometry.—The course in geometry includes the study of congruent and similar figures, inscribed and circumscribed polygons, cyclometry, and harmonic functions. The plane trigonometry of the right and oblique triangle and goniometry are introduced. In some schools harmonic points and rays are studied.

Realgymnasium.—Six hours are devoted to mathematics, three to algebra, and three to geometry. Arithmetical and geometrical progression, compound interest and annuities, are studied. Equations of higher degree, difficult quadratics, reciprocal and binomial equations are introduced.

The course in geometry includes the study of straight lines, planes, triangles, polygons, and circles. The principal properties of the pyramid, cylinder, prism, cone, sphere, and logarithms are studied, and the study of trigonometry is continued. In a few Realgymnasia the elements of analytic geometry are begun. The straight line, poles and polars, the ellipse, parabola, and hyperbola are considered.

Realschule and Oberrealschule.—Two hours a week are devoted to the subject of algebra, and three to geometry and trigonometry.

The course in algebra includes the study of imaginary and complex numbers, De Moivre's theorem and its application, the graphic solution of equations, the binomial theorem, and reciprocal equations.

The course in geometry includes the study of similar figures; the functions for any angle are taught and are represented by line values; and the applications of spherical trigonometry to solid geometry and to mathematical geography are presented. In some schools map projection is studied. In most of the schools a brief course in descriptive geometry is offered. This includes a systematic review of the descriptive geometry of previous years, and in addition the study of the intersection of planes and solids by other planes and solids.

HOLLAND.—The tenth school year is the fourth year of the Gymnasium and of the middle school.

Middle school.—Four hours a week are devoted to the study of mathematics. The algebra of previous years is reviewed. Indeterminate equations of the first degree are introduced; Permutations and combinations and the binomial theorem are studied. Trigonometry is introduced. The work in solid geometry includes a study of the relative position of straight lines and planes, dihedral and polyhedral angles, the properties and volumes of prisms, pyramids, cylinders, cones, and spheres. A course in the elements of descriptive geometry is usually offered. Two hours a week are usually devoted to the study of mechanics and one to cosmography.

Gymnasium.—Three hours a week are devoted to the study of mathematics, one hour being given to algebra and two to geometry. Irrational and complex expressions, fractional and negative exponents are studied. All of plane geometry is completed.

HUNGARY.—The tenth school year is the sixth year of the Gymnasium and of the Realschule.

Gymnasium.—Involution, involving fractional and negative exponents, is taught. Brigg's logarithms are used, and arithmetical and geometrical progression are studied.

Most of the course in geometry is devoted to the study of the circle.

The functions of acute angles are taught and used in the solution of triangles. Formulas for the sine and cosine are developed and applied. Functions of positive and negative angles are considered. Some simple surveying is done.

Realschule.—The courses in algebra and in geometry are the same as in the Gymnasium, but the work is somewhat more intensive.

Three hours a week are devoted to the study of descriptive geometry. The course includes the study of orthogonal projections, the projections of circles in simple positions, the study of points, straight lines and planes, angles, and shadow problems.

Bürgerschule.—Three hours a week are devoted to the study of mathematics. Linear equations, square and cube root, quadratic equations, and involutions with fractional and negative exponents are taught. Some attention is given to irrational and imaginary numbers. Logarithms, the theory of combinations, and annuities are also presented.

The course in geometry includes the study of the Pythagorean theorem and its applications, and the study of the circle. In some classes one hour a week is devoted to simple bookkeeping.

ITALY.—The tenth school year is the first year of the liceo. Six hours are devoted to the subject of mathematics, considerable attention being devoted to arithmetic. The study of geometry includes a review of the first book of Euclid, which was studied in the preceding year. Books II and III are also completed. The study of algebra is completed up to radicals and fractional exponents. The study of arithmetic is completed with the theory of square root and incommensurable numbers.

JAPAN.—The tenth school year is the fourth year of the middle school. Two hours a week are devoted to algebra and two to geometry. The course in algebra includes the study of irrational expressions, ratio and proportion, the progressions, permutations and combinations, logarithms, the use of tables, and the binomial theorem with positive integral exponents.

The course in geometry includes the study of inscribed and circumscribed figures, areas, congruency, proportion, and the applications of proportion to similar figures.

ROUMANIA.—The tenth school year is the second year of the lycée.

Algebra.—The functions of independent variables, limits, continuity, simple derivatives, graphs, maxima and minima, the theory of permutations and combinations and the binomial theorem are studied. In some schools, the extraction of the square root of polynomials by indeterminate coefficients is taught. In a few schools, determinants of the third order, and their use in the solution of homogeneous and nonhomogeneous equations are taken up. Graphs are extensively used in all courses in algebra.

Geometry.—The course in geometry includes the study of regular polygons, circles, transversals, areas, congruency, and similarity. Some attention is devoted to the study of harmonic functions. Many applications are presented. Dihedral and trihedral angles, parallel and perpendicular planes, and the formulas for surfaces and volumes are studied.

Trigonometry.—The trigonometric functions for acute and obtuse angles are taught and represented graphically. The principal formulas of plane trigonometry are developed and applied, and the tables of logarithms are used. The idea of periodic functions is emphasized.

RUSSIA.—The tenth school year is the seventh year of the Realschule and of the Gymnasium.

Gymnasium.—Three to four hours a week are devoted to the study of mathematics in the Gymnasium. The course in algebra includes simple and quadratic equations involving one or more unknown quantities, indeterminant equations of the first and second degrees, the progressions, logarithms, the binomial theorem, and continued fractions.

Trigonometry.—One and a half hours to two hours are devoted each week to the study of trigonometry. Plane trigonometry is completed during this year.

Realschule.—The course in algebra in the Realschule is practically the same as in the Gymnasium, but the work in the Gymnasium is somewhat less intensive. In the

Realschule more emphasis is put upon the subject of complex numbers, and upon integral functions and their roots. The course in trigonometry is the same as in the Gymnasium.

Plane analytics and the elements of infinitesimal calculus are also introduced during the tenth school year in the Realschule. The course in analytic geometry includes the derivation of the principal theorems by means of rectangular coordinates. The circle is studied and its equation is derived by the use of rectangular and by polar coordinates. The spiral of Archimedes is studied. The equation for the ellipse, the parabola, and the hyperbola are derived both in rectangular and polar coordinates. The ellipse, regarded as the projection of the circle, is also taken up.

Infinitesimal calculus.—The principles of the theory of limits and their application to the mensuration of the circle and of the surface and volume of the cylinder, cone, and sphere are studied. The limiting value of $\frac{\sin z}{z}$ as z tends to zero and that of $\left(1+\frac{1}{n}\right)^n$, when n increases without limit, are discussed. The system of natural logarithms is presented. Continuity of functions is studied. The geometric representation of functions and derivatives receive attention. Formulas for the derivatives of the sum, difference, product, and quotient of functions are taught, and the derivatives of inverse functions are also presented. The subject of maxima and minima is taken up. Equations for tangents and normals at a given point of a curve are derived. Indefinite and definite integals are taken up.

Those pupils who elect this course in mathematics are usually preparing for the school of technology.

SPAIN.—No pure mathematics is offered during the tenth school year. Both algebra and trigonometry are completed during the preceding year and physics is studied during the tenth year.

SWEDEN.—The tenth school year is the second year of the Gymnasium. In the classical Gymnasium four hours a week, and in the Realgymnasium six hours a week are devoted to the study of mathematics.

Classical Gymnasium.—Involution and evolution and the use of tables of logarithms are taught. The pupils graph simple functions. The geometry of previous years is reviewed and numerous applied problems are solved.

Realgymnasium.—Simple trigonometric computations for right and oblique triangles are made. The function concept is emphasized throughout the year. The work in trigonometry is a continuation of the theory of projections and the intersection of planes by planes and of solids by planes.

SWITZERLAND.—The tenth school year is the fourth year of the Gymnasium and the second year of the Realschule.

Gymnasium.—One and a half hours a week are devoted to the study of algebra. The course includes powers and roots, equations of the second degree involving one unknown and equations of higher degree reducible to equations with one unknown, imaginary and complex numbers.

One and a half hours are usually devoted to the study of geometry. The course includes the study of similarity of polygons, cyclometry, applications of algebra to geometry, harmonic division and transversals.

Plane trigonometry is introduced and the function concept is given a great deal of emphasis.

Realgymnasium.—Two hours a week are devoted to the study of algebra, and three and a half hours to geometry and trigonometry. The course includes all that is given in the Gymnasium, with the addition of dihedral and polyhedral angles, and a more intensive study of trigonometry and goniometry.

Realschule.—The course is practically the same as in the Realgymnasium. The work in algebra is somewhat more intensive and includes some equations of the third degree, logarithms, arithmetical and geometrical progressions.

UNITED STATES.—There are two general plans for organizing the course in mathematics for the tenth school year.

The first is the older plan and is still in more general use, but the second is growing in popularity.

First plan: Half of the year is devoted to the study of algebra. The course includes the theory of exponents, radicals, the theory of quadratics, simultaneous quadratics, the binomial theorem for positive integral exponents. This is followed by a half year of plane geometry. The first two books are completed.

Second plan: The entire year is devoted to the subject of plane geometry, and the five books are completed.

SUMMARY OF THE TENTH YEAR'S WORK.

The tenth school year is the second year of the secondary school in the United States. In most of the schools the entire year is devoted to the study of plane geometry. In some of the schools half of the year is devoted to the study of algebra and the other half to the study of plane geometry.

In the schools of Europe the mathematics of the tenth school year is distinctly in advance of that in the United States. The difference in the mathematical courses in the two countries is most marked in the subjects of geometry, trigonometry, and drawing.

In most of the European schools the course in algebra is practically the same as in the first half of the third year in the secondary schools in the United States, that is, the course begins with the study of quadratic equations and continues through the progressions and the binomial theorem. In many of the European countries the subjects of logarithms, compound interest, and annuities are given a good deal of attention.

The courses in geometry abroad usually include the study of congruent and similar figures, cyclometry, regular polyhedra, and the formulas for the surfaces and volumes of the common solids.

The subject of trigonometry is taught during the tenth school year in almost all of European countries. The extent of the course varies, but usually the entire subject of plane trigonometry is completed during this school year. In a few of the countries spherical trigonometry is also studied, and the knowledge thus gained is applied in the study of solid geometry and of mathematical geography.

Descriptive geometry is introduced in the French scientific lycée, in a few Realschulen and Oberrealschulen of Germany, and in some of the schools of Holland and Hungary. The study of geometric drawing is given more emphasis in France than in other countries. The elements of analytic geometry are introduced in a few of the Realgymnasia and Oberrealschulen of Germany and in some of the Realschulen of Russia. In practically all of the European countries, by the time a boy has completed the tenth school year, if he has pursued the scientific course, he has studied all the mathematics that is offered during the entire 12 school years in most of the schools of the United States.

XII. THE WORK IN MATHEMATICS IN THE ELEVENTH SCHOOL YEAR.

AUSTRIA.—The eleventh school year is the last year of the Realschule and the seventh year of the Gymnasium and Realgymnasium.

Gymnasium.—Plane geometry is completed during the tenth school year. A good deal of emphasis is placed upon the idea of function. Spherical trigonometry is not taught in the Gymnasium. Surveying and mechanics receive a great deal of attention. Analytic geometry is introduced. The study is at first confined to rectangu-

lar coordinates. The principal formulas for distances and areas are developed and applied. Since the idea of function has been developed in the previous years, a good deal of time is saved here. The equation of the straight line is presented in four ways, $y=mx+b$, $ax+by+c=o$, $\frac{x}{a}+\frac{x}{b}=1$, $x\cos\alpha+y\sin\alpha-p=o$. The conic sections and the tangents for each of the conics are studied.

The course in algebra includes the study of arithmetical and geometric progression, compound interest, the theory of permutations and combinations, variations, the binomial theorem for positive integral exponents, and the fundamental ideas of the theory of probability.

Realschule and Realgymnasium.—Two hours a week are devoted to the subject of descriptive geometry. Orthogonal and oblique projections are studied, and a good deal of emphasis is put upon perspective drawings. Tangential planes and plane sections are studied. The principal aim of the course in descriptive geometry is to develop spatial concepts and skill in constructions as a basis for the work in more advanced technical institutions.

One hour a week is usually devoted to the subject of geometric drawing.

In the Realschule, more attention is devoted to the study of analytic geometry than in the Gymnasium. The subject is treated as a continuation and extension of the theory of functions, which has been presented in previous years. The straight line is exhaustively studied as a basis for other figures. Practically all computations are made by the use of rectangular coordinates. A good deal of attention is devoted to the study of conic sections. Special attention is given to problems in which conic sections appear as geometric loci. Easy differential and integral calculus are applied to problems in physics. Only those computations are introduced which simplify or make more intensive the knowledge of physics. There is a decided tendency to restrict the theoretic matter to the smallest possible compass.

BELGIUM.—The eleventh school year is the fifth year of the Athénée Royal. In the Greek-Latin and in the commercial Athénée three hours a week are devoted to the study of mathematics. In the Latin and in the scientific sections six hours a week are devoted to the subject.

Greek-Latin section.—The algebra and geometry of previous years are thoroughly reviewed. Equations of the first degree with two or more unknowns and simple applied problems are solved. The interpretation of negative, indeterminate, and infinite values is emphasized. Results are generalized whenever possible. The remainder theorem is introduced and algebraic fractions are studied.

The course in geometry includes the study of the properties of circles, chords, secants, and tangents, the congruency and similarity of triangles and polygons, proportional lines and problems in construction. A good deal of attention is devoted to drawing to scale and to out-of-door measurements.

The course in the scientific and in the Latin Athénée is practically the same. The work in arithmetic includes a thorough review of the work of previous years, tests for divisibility, greatest common divisor and lowest common multiple, prime numbers, the theorem of Fermat, common and decimal fractions, approximate computations, complex numbers, and cube root.

The course in algebra includes the solution of equations of the first degree with one or two unknowns, and of simple quadratic equations with one unknown, the square root of binomials, the progressions, logarithms, compound interest and annuities, maxima and minima.

In geometry the study of regular polygons, circles, and transversals is emphasized.

The trigonometric functions are defined, and the fundamental formulas are developed and applied. Quite a good deal of attention is devoted to surveying, especially leveling. Numerous figures are drawn to scale.

DENMARK.—The eleventh school year is the second year of the Gymnasium. Two periods a week are devoted to the study of mathematics, in the classical and in the modern language gymnasium, and six periods a week are devoted to the subject in the mathematical-scientific Gymnasuim.

The trigonometric functions are defined and represented graphically, and the principal formulas of plane trigonometry are developed and applied. Some attention is devoted to the subject of goniometry.

The formulas for the surfaces and volumes of regular solids are developed and the fundamental ideas of conic sections are presented. The principal theorems of solid geometry are studied. A good deal of emphasis is put upon the study of spherical triangles and of symmetry and similarity.

ENGLAND.—The eleventh school year is the last year of the central school. By the time a boy has completed the work of this year, he should be able to solve any ordinary problem in arithmetic, especially any problem of a practical nature. Workshop methods are emphasized. Constructions and approximations are given a good deal of attention. Many of the problems involve weights, volumes, and density. In some schools simple surveying is given and graphs are taught.

The course in algebra includes the study of quadratics, graphs, fractional coefficients, and indices, formulas, radicals, logarithms, arithmetical and geometric progressions.

The geometry of previous years is reviewed and extended, and the work is closely correlated with mechanical drawing and with science. The pupils study the circle, the ellipse, easy vectors, and areas, and make simple constructions.

The work in trigonometry includes the study of logarithms and problems involving heights and distances. Angles with given sine, cosine, or tangent are constructed.

FINLAND.—The eleventh school year is usually the last year of the course. No details are given, but when the pupil has completed the course, he has finished the study of algebra up to permutations and combinations. He has studied plane and solid geometry and the elements of trigonometry. In the classical lycees, only the study of the right triangle is included in trigonometry.

FRANCE.—The eleventh school year is a one-year course to prepare for the second part of the bachelor degree. It is called the special mathematics course. Eight hours a week are devoted to the study of mathematics.

The theoretical parts of arithmetic are presented. Special emphasis is put upon the theory of prime numbers and of circulating decimals.

The course in algebra includes a review and an extension of the work of the preceding year. Equations of the first and second degrees involving two or more unknowns are solved. Inequalities of the first and second degrees are studied. Arithmetical and geometrical progression, logarithms, compound interest, and annuities are studied. The idea of coordinates is introduced, and some simple equations are derived. The functions $y=ax+b$ $y=\frac{ax+b}{a'x+b'}$ $y=ax^2+bx+c$ $y=ax^4+bx^2+c$ are represented graphically. The maximum and minimum values of certain functions having numerical coefficients are found. The areas of curves regarded as functions of the abscissa are computed.

The course in trigonometry includes a review and the application of trigonometry to surveying.

The course in geometry includes a thorough review of the work of preceding years and in addition to this, radical axes, polars, inversions, the theory of vectors, and central projections are studied. The ellipse, hyperbola, and parabola are considered, and their equations are derived. Problems involving tangents to these three curves are solved.

The course in descriptive geometry includes the study of the straight line, plane, circle, cone, cylinder, and sphere. Plane sections of solids are considered. Shadows are studied and the ideas underlying the construction of topographical maps are pre-

sented. A good deal of emphasis is put upon the subject of mechanics and cosmography during this year.

GERMANY.—The eleventh school year is the eighth year of the Bürgerschule, Gymnasium, Realgymnasium, and Oberrealschule.

The course in the gymnasium includes the study of arithmetical and geometrical progression, compound interest, and annuities. Solid geometry in completed and the elements of plane analytic geometry are introduced.

Realgymnasium.—Five hours a week are devoted to the subject of mathematics. Three hours are devoted to algebra and trigonometry, and two to descriptive geometry. The theory of permutations and combinations, the binomial theorem, determinants, and the general theory of equations of higher degree are studied. Plane and spherical trigonometry are completed and applied to the study of mathematical geography.

The course in descriptive geometry includes the graphic determination of points, lines, and planes, and problems involving these. Numerous problems in which the distances from given points, lines, and planes are to be determined are solved.

Oberrealschule.—Two hours a week are devoted to the study of analysis and to the elements of infinitesimal calculus. The algebra of previous years is reviewed. Permutations, combinations, and the binomial theorem are studied. The ideas of limits and of derivatives are introduced. The derivatives for powers, sines, and cosines are formed. The study is closely correlated with physics and mechanics.

Three hours a week are devoted to the study of analytic and synthetic geometry. Coordinate geometry is introduced. The straight line, circle, pencils of rays, tangents, poles, and polars are studied. Some attention is given to the theory of projections.

The course in descriptive geometry includes the study of the cylinder, cone, and sphere, and the introduction of perspective drawing.

HOLLAND.—The eleventh school year is the fifth year of the middle school and of the Gymnasium. Four hours a week are devoted to the study of mathematics.

Middle school.—The algebra of previous years is reviewed and extended. The study of trigonometry is continued, and circular functions are introduced. Plane geometry is reviewed; and the cone, cylinder, and sphere are studied. The course in descriptive geometry is completed up to the study of curved surfaces. Two hours are devoted to mechanics and one to cosmography.

Gymnasium.—Two hours a week are devoted to algebra and two to geometry. The course in algebra includes the study of quadratics, radicals, fractional and negative exponents, arithmetical and geometric progressions, logarithms and indeterminate equations of the first degree. The study of solids is completed.

In some Gymnasia the pupils who specialize in mathematics and physics take a more advanced course in mathematics. This course includes the above with the addition of maxima and minima, the theory of limits, and some trigonometry.

HUNGARY.—The eleventh school year is the seventh year of the Gymnasium and of the Realschule.

In the Realschule from three to five hours are devoted to the study of mathematics. The course includes the study of arithmetical and geometric progression, circulating decimals, compound interest, and annuities, Government loans, indeterminate equations of the first degree, the binomial theroem, simple formulas involving surfaces and volumes, the study of trigonometry and its applications to surveying and geography.

From two to three periods a week are devoted to the study of descriptive geometry. The course includes orthogonal projections, angles of inclination, the intersection of simple solids by straight lines and planes, shadow constructions, the revolution of plane and simple solid figures about vertical axes, and the introduction of new planes of reference.

Gymnasium.—From two to three hours a week are devoted to the study of mathematics. The course in algebra includes the study of compound interest and annuities,

Government loans, the progressions, circulating decimals, and the theory of quadratics.

The course in geometry includes the study of the coordinates of a point, the distance between two points, the graphs, and the important theorems of solid geometry, especially those involving surfaces and volumes.

ITALY.—The eleventh school year is the second year of the licée. Six hours a week are devoted to the study of mathematics. Books IV, V, VI, XI, and XII of plane and solid geometry are completed. The theory of proportion, power, equations of the first and second degrees with one and more unknowns, radicals, and progressions are studied. The elements of trigonometry are introduced.

JAPAN.—The eleventh school year is the fifth and, in some schools, the last year of the middle school.

Four hours a week are devoted to the study of mathematics, two hours being devoted to geometry and two to trigonometry.

The course in geometry includes the applications of proportion, the study of areas and loci, straight lines and planes, solid angles, polyhedra, the prism, the pyramid, the sphere, the cylinder, and the cone.

The course in trigonometry includes the functions and graphic representation of the functions for acute and obtuse angles, the development and use of the principal formulas, the solution of right and oblique triangles, the use of logarithmic tables, and the applications of trigonometry to surveying.

ROUMANIA.—The eleventh school year is the third year of the lycée. Four hours a week are devoted to the study of mathematics. A good deal of emphasis is put upon mechanics.

The course in algebra includes the study of sines, convergence, the number e, exponential functions, logarithms, derivatives of exponential, logarithmic and circular functions, homogeneous functions, and complex numbers.

In descriptive geometry, estimates of projections, the representation of a point by the aid of projections of two perpendicular planes, and the projection of a straight line are considered.

The pupil's knowledge of algebra and of geometry is utilized to a great extent in the study of mechanics.

RUSSIA.—The eleventh school year is the eighth and last year of the Gymnasium. From three to four hours a week are devoted to the study of mathematics. A complete review of the previous year's work in mathematics is given, one hour a week being devoted to the review in arithmetic. The factor theorem and the transformation of equations with the unknowns in the denominator are presented.

SWEDEN.—The eleventh school year is the third year of the Gymnasium. In the Latin Gymnasium, three hours, and in the Realgymnasium, four hours are devoted to the study of mathematics.

In the Gymnasium, trigonometry and solid geometry are completed. Arithmetical and geometrical progression and compound interest are studied. Graphs of simple functions, such as $y=\sin x$, are introduced.

In the Realgymnasium, in addition to the above course, analytic geometry is introduced and a good deal of attention is devoted to linear drawing. A thorough study is made of the theory of shadows.

SWITZERLAND.—The eleventh school year is the fifth year of the Gymnasium and the third year of the Realschule. In the literary Gymnasium two hours a week are devoted to algebra and one and a half hours to geometry. In the Realgymnasium two hours are devoted to algebra, one to descriptive geometry, and one to bookkeeping. In the Realschule two and a half hours to algebra, three to geometry, and three and a half to descriptive geometry.

Literary Gymnasium.—Logarithms, exponential equations, the progressions, compound interest, annuities, indeterminate equations of the first degree, and plane trigonometry are studied.

Realgymnasium.—In addition to the above course, continued fractions, equations of the second degree in two unknowns, and solid geometry are studied. Goniometry and trigonometry with its applications are introduced.

Realschule.—The course in the Realschule is the same as in the Realgymnasium, with the addition of the approximate solutions of equations of higher degree. The elements of spherical trigonometry applied to geography and the elements of plane analytic geometry are also introduced.

In both the Realgymnasium and Realschule descriptive geometry is studied. The course is more intensive in the Realschule. The relation of points, straight lines, planes and simple solids in vertical and horizontal projections are considered. The fundamental problems in construction are presented. The projections of the circle are studied, and triangles, prisms, pyramids, and regular solids are given special attention.

UNITED STATES.—The eleventh school year is the third year of the secondary school. The courses in mathematics vary somewhat, but in most of the schools a second course in algebra is given during the first semester, and solid geometry is studied during the second semester.

The course in algebra includes the study of radicals and exponents, quadratic equations with one and two unknowns, the theory of quadratic equations, equations in the quadratic form, the progressions, and the binomial theorem.

The course in solid geometry includes the study of Books VI, VII, VIII, and IX.

SUMMARY OF THE ELEVENTH YEAR'S WORK.

The eleventh school year is the third year of the secondary school of the United States. In most of the schools the course in mathematics includes a half year of algebra and a half year of solid geometry. In a few of the schools, the first half year is devoted to the study of solid geometry and the last half to the study of trigonometry.

In the schools of Europe the work in algebra during the eleventh school year includes the study of arithmetical and geometrical progressions, permutations and combinations, the theory of probability, the binomial theorem, and determinates. The preceding topics are included in the most advanced courses in Austria, France, Germany, Holland, Hungary, Sweden, Switzerland, and Roumania. The subject of trigonometry, which was completed during the tenth school year, is applied to surveying. Much more emphasis is put upon the subjects of cosmography and mechanics than in the United States. Descriptive geometry is studied in Austria, Germany, France, Holland, Hungary, and Switzerland, and the study of geometric drawing is continued in Austria. Analytic geometry is introduced in certain of the schools of Austria, France, Germany, Hungary, and Sweden, and the elements of differential calculus are introduced in a few of the Realschulen of Austria and in the Oberrealschulen of Germany. The calculus is applied in the study of physics. Spherical trigonometry is taught in several of the countries, and it is usually applied in the study of mathematical geography.

When a European boy has completed the eleventh school year, if he has elected the scientific course, he has studied more mathematics than is offered in any except a very few of the most progressive secondary schools in the United States.

XIII. THE WORK IN MATHEMATICS IN THE TWELFTH SCHOOL YEAR.

AUSTRIA.—The twelfth school year is the eighth and last year of the Gymnasium and of the Realgymnasium.

Gymnasium.—The mathematics of previous years is thoroughly reviewed, and various ideas are studied more intensively. Practical applications are emphasized, and a good deal of attention is devoted to the history of the subject and to the introduction of general ideas of means in advanced mathematics. In both the Gymnasium and Realgymnasium the elements of differential and integral calculus are presented. The subject matter is treated in close relation with the theory of functions as presented in previous years. Special attention is given to the following functions: $ax+b$, ax^2+bx+c, ax^3, $\frac{c}{ax+b}$, $\frac{a}{x^2}$, $\sqrt{x}$, $\sin x$, $\cos x$, a^x and $\frac{a}{\log x}$. Problems are given to determine the tangents of curves, and velocities; maxima and minima are also studied.

BELGIUM.—The twelfth school year is the sixth school year of the Athénée Royal. In the Greek-Latin and in the commercial courses three hours a week are devoted to mathematics. In the Latin and the scientific courses six hours are devoted to mathematics.

Greek-Latin course.—The square and square root, radicals, quadratic equations, equations reducible to quadratics, special quadratics of higher degree, arithmetical and geometric progression, proportion, logarithms, interest, and annuities are studied.

The geometry of the previous year is reviewed, and solid geometry is completed. Trigonometry is introduced. Tables are used, and the formulas for the right triangle are developed and applied in numerous problems.

Scientific and Latin courses.—The algebra and plane geometry of previous years are reviewed, and trigonometry and its applications are studied. The different systems of numeration are considered, and computations involving various number scales are made. Short methods for multiplication and division are emphasized.

The application of indeterminant coefficients in functions, and the relation between the coefficients of algebraic equations in order to satisfy certain given conditions are studied. Certain problems in maxima and minima are given. Continued fractions are presented. Indeterminate equations of the first degree, permutations and combinations, the binomial theorem, the summation of series, powers and roots, radical equations, exponential equations, and Naperian logarithms are studied.

The course in plane geometry includes the study of functions, harmonic pencils, poles and polars, and the theorems of Pascal and Brianchon concerning the circle.

The course in solid geometry includes the general subject matter of Books VI, VII, VIII, and IX.

DENMARK.—The twelfth school year is the third and last year of the Gymnasium. In the classical and in the modern language courses two hours a week are devoted to the study of mathematics, and in the mathematics and scientific courses six hours are devoted to the subject.

The course includes a comprehensive review of the mathematics of previous years. The elements of analytic geometry are introduced. The pupils are taught how to determine points and curves by means of rectilinear and polar coordinates. The formulas for the equation of the straight line and circle, tangents, parabola, ellipse, and hyperbola are developed; and the principal theorems for tangents, normals, and asymptotes are studied.

In addition to this, the pupils elect one of the following courses:

(*A*) Determinants with their applications to linear equations, continued fractions and their applications, the general equation of the second degree treated analytically, the icosahedron and dodecahedron and the representation of similar polyhedra by

means of vertical projections on perpendicular planes, and the plane sections of these solids.

(*B*) Infinitesimal calculus. The course includes the computation with infinitesimal magnitudes as an introduction to differential and integral calculus, the continuity of fractions, the derived function of x^n, (n being rational) and of the trigonometric functions, of the sum, product, and quotient, and of functions of a function, the theorem of Rolle, and maxima and minima. Taylor's theorem for integral functions is presented. Definite and indefinite integrals are considered, and the simplest functions are integrated. Simple applications are made to geometry and physics.

ENGLAND.—The instruction in the central schools usually closes with the eleventh year. In the great private schools, such as Eton, Harrow, Rugby, and Winchester, there are boys from 13½ to 19 years of age. Many of these boys go to the universities or enter the army. In most of the great private schools, there are three courses:

(*a*) Classical course, which corresponds roughly to the German Gymnasium.

(*b*) The modern course, which corresponds roughly to the German Realgymnasium.

(*c*) The army course.

Courses (*a*) and (*b*) are usually about equal in number of students. Course (*c*) usually has a smaller enrollment.

In the modern course from four to six periods a week are devoted to the study of mathematics. Details of this course are not available in the English reports. Great emphasis is put upon the entrance requirements in mathematics by Oxford and Cambridge, and more time is devoted to the subject in the classical courses in England than in corresponding courses in France or Germany.

FRANCE.—The twelfth school year is usually called Classe de Philosophie.

The relations between algebra and geometry are emphasized, and a good deal of attention is devoted to the subject of graphs. The pupils are taught how to construct a rectangle which has a given side and is equivalent to a given square, and numerous other similar constructions. The notion of coordinates is extended, and the study of functions is continued. Squared paper is used in the determination of areas. The theory of limits is discussed. The formula for the area of a parabola is derived. The application of the methods of infinitesimal calculus to the evaluation of surfaces and volumes of figures considered in elementary geometry is made.

GERMANY.—The twelfth school year is the ninth and last year of the Bürgerschule, Gymnasium, Realgymnasium, Realschule, and Oberrealschule.

Gymnasium.—The formulas of elementary trigonometry are applied to mathematical geography and to elementary astronomy. A comprehensive review of the mathematics of previous years is given, and applications to physics are especially emphasized. The concept of coordinates is introduced and applied to the study of conics.

Realgymnasium and Realschule.—Five hours a week are devoted to the subject of mathematics and two to physics. The subjects are very closely correlated. Three hours are usually devoted to the study of plane analytic geometry and a comprehensive review of the mathematics of previous school years. The study of analytics includes the point, straight line, and circle. Two hours are usually devoted to the study of descriptive geometry. Pyramids, prisms, cones, cylinders, and spheres, and sections of these bodies, are presented. The ellipse and parabola are usually not considered. In some schools cubic equations and maxima and minima are studied.

Oberrealschule.—Five hours a week are devoted to the study of mathematics. The course includes both analytic geometry and infinitesimal calculus. Logarithms and exponential functions with their derivatives are studied. Indefinite and definite integrals with simple exercises are introduced. The principles of calculus are applied to exercises in mechanics.

The course in geometry includes the study of conics treated both analytically and synthetically. These curves are regarded as sections of right circular cones.

The theorem of Quetlet-Dandelin is introduced. The relation between algebra and geometry is especially emphasized.

The course in descriptive geometry is a continuation and extension of the work of the eleventh school year.

A comprehensive review of the mathematics of previous years is given, and the historical development of the subject is especially considered.

HOLLAND.—The twelfth school year is the sixth year of the Gymnasium. It is sometimes called the preparatory year. Plane and spherical trigonometry are studied, and the elements of analytic geometry are introduced. A comprehensive review of the mathematics of previous school years is given. Especial emphasis is put upon the study of permutations and combinations, the theory of probability, determinants, continued fractions, higher series, and functions. The correlation between mathematics and physics is emphasized.

HUNGARY.—The twelfth school year is the eighth and last year of the Bürgerschule, Gymnasium, and Realgymnasium.

In the Gymnasium and Bürgerschule two hours a week are devoted to mathematics, and in the Realschule three hours are devoted to the subject of mathematics and two to descriptive geometry.

Gymnasium and Bürgerschule.—Permutations and combinations, the theory of probability, the binominal theorem, and Pascal's triangle are studied, and a comprehensive review of algebra is made. The formulas of spherical trigonometry are applied in the study of geography. The elementary ideas of coordinates are introduced. The study of free-hand drawing is continued and a thorough review of geometry is made.

Realschule.—A comprehensive review of the algebra of previous years is made. The idea of coordinates is introduced, and the principal equations of points, distances, straight lines, and circles are developed and applied. The ellipse, parabola, and hyperbola are studied as geometric loci, and the entire subject of geometry is thoroughly reviewed.

The course in descriptive geometry includes the study of orthogonal projections, the representation of cones, cylinders, spheres, and the intersection of these bodies by lines and planes, tangent planes, shadow constructions, and a thorough review of the work of previous years.

ITALY.—The twelfth school year is the third and last year of the liceo. No mathematics is offered during this year.

JAPAN.—In most of the schools the middle school closes with the eleventh school year. When an additional year is offered, the time is usually devoted to a comprehensive review of the mathematics of previous years, and to a more intensive study of the subject.

ROUMANIA.—The twelfth school year is the fourth and last year of the lycée. Five hours a week are devoted to the study of mathematics.

The course in algebra includes the theory of roots, theorem of Rolle, Descartes's theorem, the solution of equations of higher degree, the study of integral, fractional and irrational roots, the methods of approximation, and other elementary ideas of the theory of equations.

The course in analytic geometry includes the study of rectangular coordinates, and problems concerning the straight line, circle, ellipse, hyperbola, and parabola, and their properties. A good deal of emphasis is put upon the study of cosmography, and mathematics is closely correlated with this subject.

RUSSIA.—The course in Russia closes with the eleventh school year.

SWEDEN.—The twelfth school year is the fourth and last year of the Gymnasium. In the Latin Gymnasium five hours a week and in the Realgymnasium six hours are devoted to the study of mathematics.

Latin Gymnasium.—The course includes a comprehensive review of the mathematics of previous years and the introduction of the elementary ideas of analytic geometry. Linear drawing is taught, and the principles of perspective are emphasized.

Realgymnasium.—The concept of derivatives is introduced and applied to finding maximum and minimum points and to the determination of equations for perpendiculars. Only simple expressions, such as $y=x^2$, $y=x^3$, $y=\sqrt{x}$, $y=\sqrt[3]{x}$, and $y=sine\ x$ are developed.

SWITZERLAND.—The twelfth school year is the sixth year of the Gymnasium and the fourth of the Realschule.

In the classical Gymnasium four hours a week, and in the Realgymnasium four and a half hours, are usually devoted to the study of mathematics. In the Realschule nine hours a week are frequently given to the subject.

In the Realgymnasium two hours are usually devoted to algebra, one and a half to geometry, and one to descriptive geometry. In the Realschule three hours are devoted to algebra, two and a half hours to geometry, and three and a half hours to descriptive geometry.

Gymnasium.—The course includes the solution of difficult quadratics and the study of permutations and combinations, the theory of probability, and the binomial theorem. The correlation of mathematics with mechanics and physics is emphasized. In some Gymnasia the elementary ideas of analytic geometry and of infinitesimal calculus are introduced.

Realschule.—The course includes the study of series, compound interest and annuities, permutations and combinations, and the binomial theorem. The study of analytic geometry is introduced and the fundamental formulas for a point, straight line, ellipse, parabola, and hyperbola are developed and applied. In most of the Realschulen the elements of infinitesimal calculus are introduced. In some of the Realgymnasia and Realschulen de Moivre's theorem and operations with complex numbers are introduced. Approximate methods for the solution of numerical and indeterminate equations, continued fractions, maxima and minima, and spherical trigonometry are also taught in a few of the Realgymnasia and Realschulen. In many of the Realschulen the elements of solid analytic geometry are also taught. About one-third of the Realschulen take up the general equation of the second degree with two variables. The study of conic sections is quite thorough in most of the Realschulen. In the courses in differential and integral calculus the transition from differences to derivatives is carried out by objective geometric methods in every case. Exponential and logarithmic functions are developed, and the rules for the derivative of the sum, product, and quotient, and of a function of a function are taught. The equations of tangents of the conic sections and of other curves are studied. In a few schools differential, but not integral, calculus is taught. In some schools differential calculus is taught, but only the merest elements of integral calculus are studied. In most of the schools the elements of infinitesimal calculus have been taught for many years.

The course in descriptive geometry includes the construction of plane sections, of the regular solids and their intersections; also the representation of right cylinders and cones and tangent planes.

THE UNITED STATES.—The twelfth school year is the last year of the secondary school. The courses in mathematics vary somewhat, but in most of the schools plane trigonometry is studied during the first half of the year and college algebra during the second half. In many of the schools the application of trigonometry to surveying is emphasized, and some practice in actual surveying is given.

Spherical trigonometry is usually not included, but it is not unusual for the course to include the study of the right spherical triangle.

The course in college algebra usually includes a thorough review of quadratic equations with two or more unknowns, graphs, proportion, the progressions, the binomial theorem for positive integral exponents, determinants, inequalities, permutations and combinations, the theory of probability, and methods for the approximation of roots. The mathematics of the twelfth school year is always elective.

SUMMARY OF THE TWELFTH YEAR'S WORK.

The twelfth school year is the last year of the secondary school in the United States. In many of the schools the first half of the year is devoted to the study of solid geometry and the last half to trigonometry or to business arithmetic. In some of the schools the first half of the year is devoted to the study of plane trigonometry and the last half to the study of college algebra. It is rare that courses in spherical trigonometry, beyond the study of the right spherical triangle, are offered in the United States.

In practically all of the European countries the twelfth school year begins or ends with a comprehensive review of the mathematics of the preceding years. Special emphasis is put upon such reviews in Austria, Germany, Denmark, Holland, and Hungary. The practical applications of mathematics are emphasized in most of the countries, and in Austria and Germany some attention is devoted to the history of mathematics. Spherical trigonometry is offered in the schools of Holland, Germany, Hungary, and Switzerland; and analytic geometry is studied in Denmark, Germany, Austria, Hungary, Sweden, Switzerland, and Roumania. In a few of the Cantons of Switzerland solid analytic geometry is studied. Descriptive geometry is offered in Germany, Austria, Hungary, and Switzerland, and advanced algebra is studied in most of the countries. Differential and integral calculus are offered in the schools of Austria, Belgium, Denmark, France, Sweden, Switzerland, Russia, Germany, and Roumania.

The relations between algebra and geometry are especially emphasized in France, and the relations between mathematics and physics receive special emphasis in Germany, Holland, and Switzerland.

When a European boy has completed his twelfth school year he has had the opportunity of studying more mathematics than is offered in any of the secondary schools of the United States. He has had more practice in applying his mathematics in physics, cosmography, and mathematical geography than is the case with the American boy. The simultaneous study of several mathematical subjects results in a more complete mastery of each. He sees the unity of mathematics in a way that is seldom true with the American boy. He can use his arithmetic and algebra in the solution of geometrical problems and his arithmetic and geometry in the solution of algebraic problems much better than the average American boy. He has some knowledge of analytic geometry and of the infinitesimal calculus. The frequent drills and reviews so common in European schools have furnished him with a large number of mathematical facts and formulas that he can use more readily than his American brother. Mathematics to him is an interesting and a fruitful subject, because he has learned to appreciate something of its deeper significance.

XIV. THE WORK IN MATHEMATICS IN THE THIRTEENTH SCHOOL YEAR.

In Belgium and in Switzerland the course in the secondary schools extends over a part of the thirteenth school year.

In Belgium the thirteenth school year is the seventh and last year of the Athénée Royal. In the commercial course two hours a week, in the Greek-Latin three hours, and in the Latin and in the scientific courses eight hours a week, are devoted to the study of mathematics.

Greek-Latin course.—The subjects of geometry and algebra are thoroughly reviewed, and the progressions, logarithms, annuities, and loans are especially emphasized. The theorems for the surface and volume of the prism, pyramid, cone, and sphere are applied in numerous problems. Spherical geometry is given a good deal of attention.

The fundamental formulas of trigonometry are studied and applied in the solution of triangles. The application of trigonometry to surveying is emphasized, and the pupils learn to use the surveyor's instruments.

Scientific and Latin courses.—Two hours a week are devoted to review and to new applications of the mathematics of previous years. Determinants are studied and applied in the solution of equations. The principal theorems of spherical trigonometry are developed and applied. The study of analytic geometry is continued and the principal formulas for the straight line and circle are developed and applied. Both rectilinear and polar coordinates are used, and the tables are taught for the transformation of coordinates. Poles and polars are extensively studied, and the equations of conic sections are developed.

The course in descriptive geometry includes the study of the principal theorems relating to points, straight lines, and planes; simple rotations are introduced.

In most of the Cantons of Switzerland the final year of the course lasts but one semester. A thorough review of the mathematics of previous years is given, and the study of plane analytic geometry and calculus is extended.

Tables 2 to 6 indicate the time at which each subject is introduced into the various types of schools and the school years during which the subject is taught.

The nomenclature of some of the mathematical subjects varies to such an extent in the various countries that it is not possible to be exact in all details in such tables. These tables are intended to indicate usual practices, and not exceptional cases.

The term geometry, as used in Table 4, includes all that is usually thought of in this country under the headings of observational, demonstrative (plane and solid), and descriptive geometry and geometric drawing. In many cases the reports of the countries do not indicate the divisions between two types of geometry sufficiently to justify the construction of a separate graph for each.

XV. GRAPHIC REPRESENTATION OF WORK IN MATHEMATICS.

TABLE 2.—*Years of study of arithmetic.*

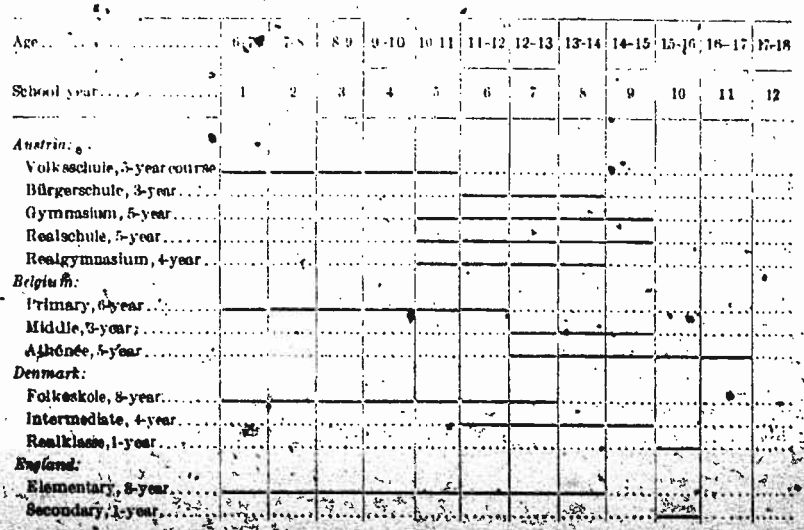

TABLE 2.— *Years of study of arithmetic*—Continued.

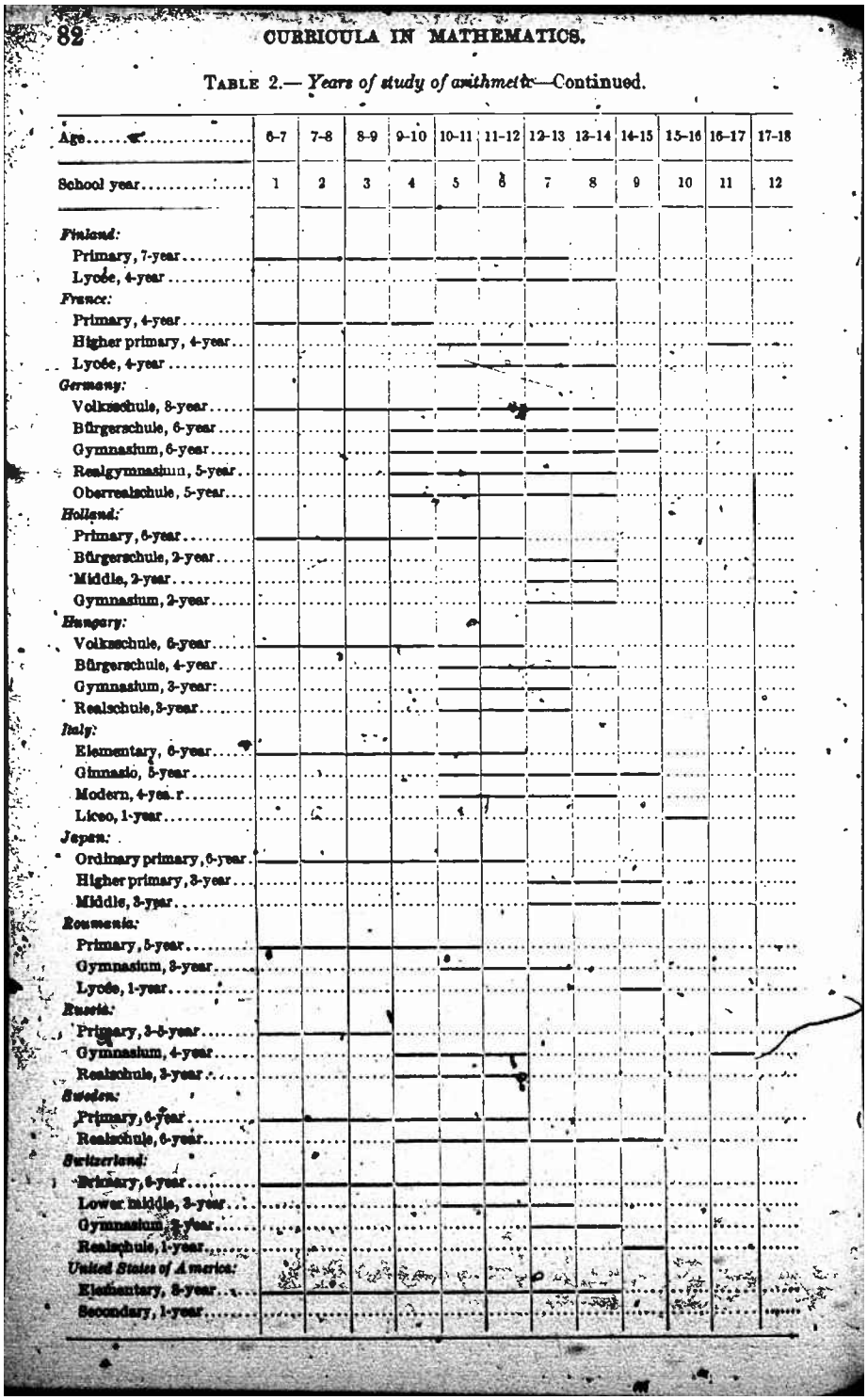

TABLE 3.— *Years of study of algebra.*

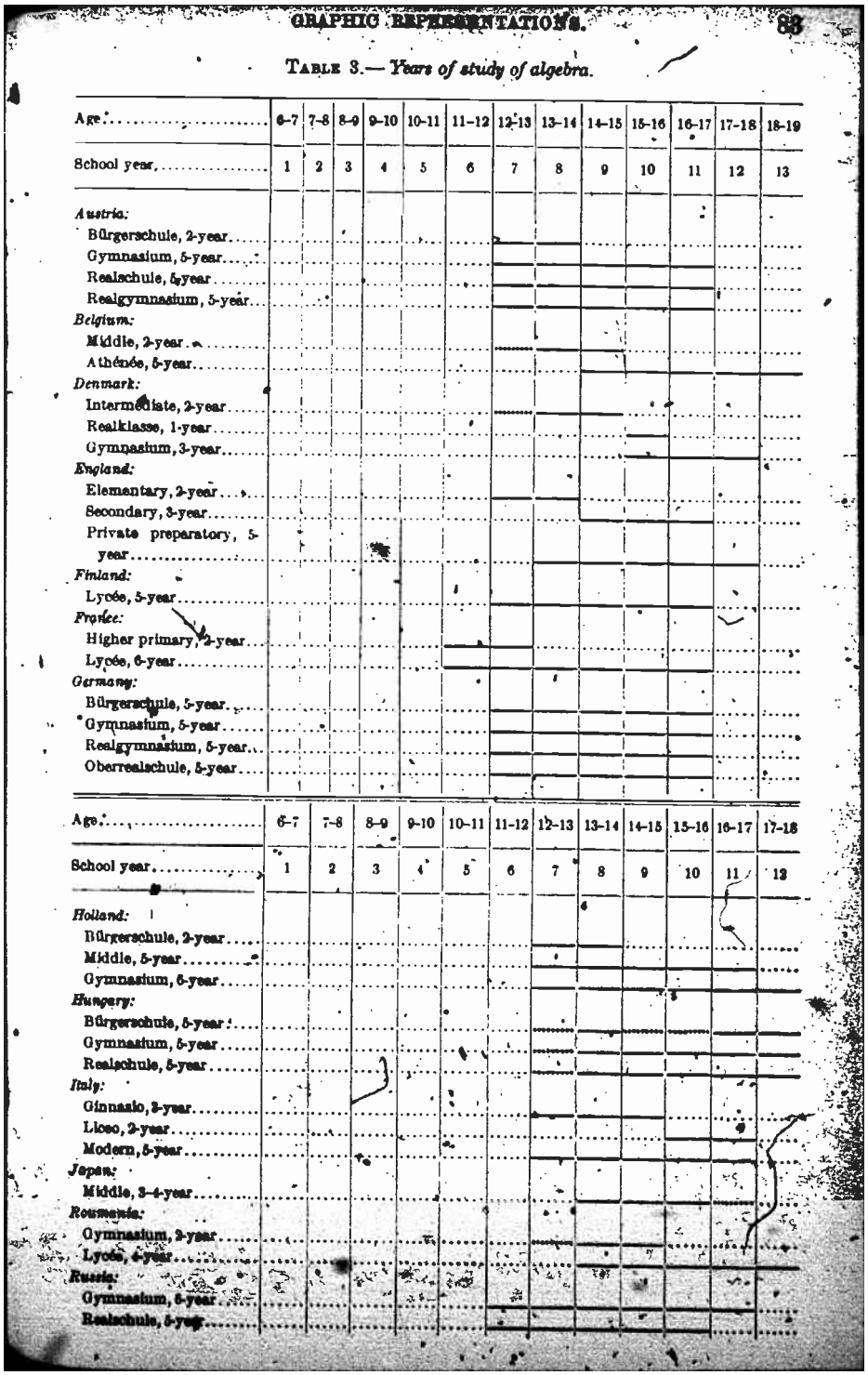

TABLE 3.— *Years of study of algebra*—Continued.

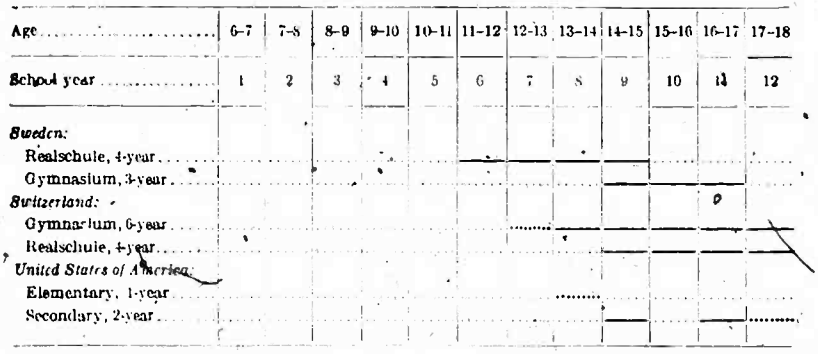

TABLE 4.— *Years of study of geometry*

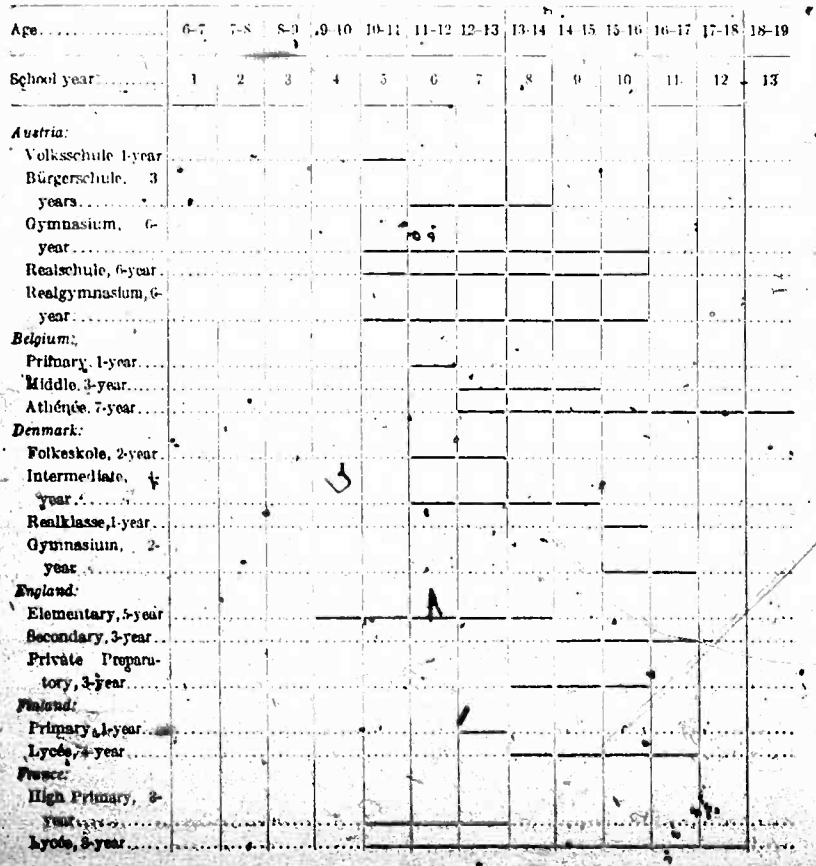

TABLE 4.— *Years of study of geometry*—Continued.

Age	6–7	7–8	8–9	9–10	10–11	11–12	12–13	13–14	14–15	15–16	16–17	17–18	18–19
School year	1	2	3	4	5	6	7	8	9	10	11	12	13
Germany:													
Volksschule, 7-year													
Bürgerschule, 7-year													
Gymnasium, 7-year													
Realgymnasium, 7-year													
Oberrealschule, 7-year													

Age	6–7	7–8	8–9	9–10	10–11	11–12	12–13	13–14	14–15	15–16	16–17	17–18
School year	1	2	3	4	5	6	7	8	9	10	11	12
Holland:												
Bürgerschule, 2-year												
Middle, 5-year												
Gymnasium, 6-year												
Hungary:												
Volksschule, 2-year												
Bürgerschule, 8-year												
Gymnasium, 8-year												
Realschule, 8-year												
Italy:												
Elementary, 1-year												
Gymnasium, 5-year												
Liceo, 2-year												
Modern, 6-year												
Japan:												
Middle, 3–4 year												
Roumania:												
Gymnasium, 4-year												
Lycée, 3-year												
Russia:												
Gymnasium, 3-year												
Realschule, 3-year												
Sweden:												
Realschule, 4-year												
Gymnasium, 3-year												
Switzerland:												
Primary, 2-year												
Lower Middle, 3-year												
Gymnasium, 6-year												
Realschule, 3-year												
United States of America:												
Elementary, 1-year												
Secondary, 1½-year												

TABLE 5.—*Years of study of trigonometry.*

Age	6-7	7-8	8-9	9-10	10-11	11-12	12-13	13-14	14-15	15-16	16-17	17-18	18-19
School year	1	2	3	4	5	6	7	8	9	10	11	12	13
Austria:													
Gymnasium, 2-year										—	—		
Realschule, 2-year										—	—		
Realgymnasium, 2-year										—	—		
Belgium:													
Athénée, 2-year											—	—	—
Denmark:													
Gymnasium, 1-year											—		
England:													
Secondary, 2-year										—	—		
Private preparatory, 2-year										—	—		
Finland:													
Lycée, 3-year									—	—	—		
France:													
Lycée, 2-year										—	—		
Germany:													
Bürgerschule, 2-year										—	—		
Gymnasium, 3-year										—	—	—	
Realgymnasium, 3-year									—	—	—		
Oberrealschule, 3-year									—	—	—		
Holland:													
Middle, 2-year										—	—		
Gymnasium, 2-year											—	—	
Hungary:													
Gymnasium, 2-year										—	—		
Realschule, 2-year										—	—		

Age	6-7	7-8	8-9	9-10	10-11	11-12	12-13	13-14	14-15	15-16	16-17	17-18
School year	1	2	3	4	5	6	7	8	9	10	11	12
Italy:												
Liceo, 1-year											—	
Japan:												
Middle, 2-year											—	—
Roumania:												
Lycée, 2-year											—	—
Russia:												
Gymnasium, 3-year											—	—
Realschule, 2-year										—	—	
Sweden:												
Gymnasium, 2-year											—	—
Realgymnasium, 2-year											—	—
Switzerland:												
Gymnasium, 3-year											—	—
Realschule, 2-year											—	—
United States of America:												
Secondary, 1-year												—

TABLE 6.— *Years of study of analytic geometry and the calculus.*

	Analytic geometry.					The calculus.					
Age	14–15	15–16	16–17	17–18	18–19		14–15	15–16	16–17	17–18	18–19
School year	9	10	11	12	13		9	10	11	12	13
Austria:											
Gymnasium, 2-year						1 year					
Realschule, 2-year						2 yrs.					
Realgymnasium, 2-year						1 yr.					
Belgium:											
Athénée, 2-year											
Denmark:											
Gymnasium, 1-year						1 yr.					
England:											
Private preparatory, 1-year						1 yr.					
France:											
Lycée, 2-year						1 yr.					
Germany:											
Gymnasium, 2-year											
Realgymnasium, 2-year						1 yr.					
Oberrealschule, 3-year						2 yrs.					
Holland:											
Gymnasium, 1-year											
Hungary:											
Realschule, 1-year											
Gymnasium, 2-year											
Bürgerschule, 1-year											
Roumania:											
Lycée, 1-year											
Russia:											
Realschule, 1-year						1 yr.					
Sweden:											
Gymnasium, 1-year						1 yr.					
Realgymnasium, 1-year											
Switzerland:											
Gymnasium											
Realschule, 2-year						2 yrs.					
United States of America: [Analytic Geometry is seldom taught in secondary schools.]											

XVI. CERTAIN IMPORTANT POINTS OF DIFFERENCE BETWEEN THE WORK IN MATHEMATICS ABROAD AND IN THE UNITED STATES.

The European schools are doing certain kinds of work that we are not doing, some that we can not hope to do under present conditions, and some that we might not wish to do if we could. They are also doing some work that we wish we could do, and some that we shall probably do before many years have elapsed.

In every country of Europe the secondary school period extends over at least six years. In most of the countries the majority of the teachers above the primary school have had the advantage of college or university training. The teachers have a margin of scholarship that is not common among teachers in the United States.

Abundant provision is made for daily drill in mathematics. The educator of Europe realizes that this daily drill is absolutely necessary in order to give the pupil a real mastery of number facts and relations. A little smattering of the subject will not suffice. The pupil is expected to know thoroughly certain facts and principles, and to this end daily drill is provided. No small part of the thoroughness in detail, which is so characteristic of most of the schools of Europe, may be traced to this drill. The American pupil has some information on a great variety of topics, but much of his knowledge is vague and indefinite, rather than clear-cut notions about definite things.

Everywhere algebra is introduced earlier than in the United States. In certain of the German schools some work in algebra is introduced during the sixth school year, and in no country, except the United States, is this introductory work postponed later than the seventh school year.

Some instruction in constructional, observational, or intuitive geometry is always offered during the sixth, seventh, and eighth school years. This instruction is always of a propædeutic nature. Much emphasis is placed upon estimates and constructions.

In all of the schools of Europe algebra and geometry are studied simultaneously during a considerable number of years. The various mathematical subjects are more closely correlated than in this country. A pupil who is studying geometry can use his arithmetic and his algebra more readily than is the case with the average American boy. The introduction of the trigonometric functions while the pupil is studying similar figures in geometry has the sanction of most of the best teachers abroad. The distinction between plane and solid geometry is much less marked than in this country. This is due, in part at least, to the fact that models are very extensively used in the study of geometry.

Everywhere the attempt is being made to find genuine applications of mathematics that are really within the experience of the pupil and to link the subject of mathematics as closely as possible with the activities of real life. Drawing and physics are frequently taught by the same teacher, and the correlation between these subjects is found to be to the advantage of each.

European school men believe that a course in mathematics should be planned by those who know some mathematics rather than by educators who are practically ignorant of the subject. The reports do not indicate that the schools of Europe are hearing a demand for weak algebra and anemic geometry, or even for no work in these subjects. If any pressure of this sort exists, it has hitherto produced no modification of the course of study.

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