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THE TRAINING OF TEACHERS OF MATHEMATICS

FOR

THE SECONDARY SCHOOLS OF THE
COUNTRIES REPRESENTED IN THE
INTERNATIONAL COMMISSION ON
THE TEACHING OF MATHEMATICS

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INTRODUCTION.

During the deliberations of the International Congress of Mathematicians at Rome in 1908 steps were taken to organize an International Commission on the Teaching of Mathematics, the members of which were to prepare or procure reports on the teaching of mathematics in different countries. Most of these reports were ready for the Cambridge congress in 1912, but since then several more have appeared. At this writing 18 countries have published 178 reports, containing over 12,000 pages. Germany has already issued 50 reports, with a total of 5,393 pages. About a fifth of this space is required by the United States for its 14 reports (the present report being the fifteenth), and about a sixth of the same space by each of the following countries: Austria, with 13 reports; Great Britain, with 34 reports; Switzerland, with 9 reports; and Japan, with 2 volumes. The reports of France cover some 700 pages. Of more modest dimensions are, in order of size, the reports from Belgium, Russia (including Finland), Italy, Sweden, Spain, Netherlands, Hungary, Denmark, Australia, and Roumania (1 report of 16 pages).

From this statement it will be observed that much greater detail is given in the case of some countries than in others. Moreover, even in reports of about the same length different subjects are emphasized. As this bulletin is based very largely upon facts drawn from the reports to the International Commission, the treatment of its sections varies with the extent of data at hand, and lack of uniformity is a necessity. No claim is made for originality of presentation.

For the most part only those schools which are under the immediate direction of the Government have been considered. And even here discussion is limited to the best schools for boys and to the teachers in such schools. As a rule the schools for girls are not as completely organized nor of so high a standard.

It has seemed to me desirable to include in this bulletin, when possible, very brief independent sketches of the educational conditions in the various countries, so that the reader may receive here in connected form condensed but definite accounts of the following phases of educational work in the country under discussion, in so far as they bear on the preparation of teachers of secondary mathematics:¹ (1) The general educational scheme; (2) Secondary

¹ For greater detail along this line the reader may be referred to Monroe's *Cyclopedia of Education* (5 volumes, New York, 1911-1913). As to mathematical instruction the *Bibliography of the Teaching of Mathematics, 1900-1912*, by D. E. Smith and K. Goldsamer (Bu. of Educ., Bul., 1912, No. 29), Washington: Government Printing Office, 1912, contains titles supplementary to the bibliographies in the following pages.

schools and their relation to that scheme; (3) the mathematics taught in the secondary schools and the pupils to whom it is taught; (4) the inducements (such as salary, pensions, social position) to young men to take up secondary-school teaching as a profession; (5) the universities of the country, the courses of mathematics and allied subjects they offer, and the diplomas or certificates they confer. With this in mind, one may get an intelligent idea both of the preparation of the secondary-school teacher for his duties and of the type and caliber of men who take up such work. An endeavor has been made to picture conditions of the present day. Only occasional brief historical comments have been introduced.

At a meeting of the International Commission on the Teaching of Mathematics held at Paris in April, 1914, the commission decided to study the theoretical and practical preparation of professors of mathematics in the secondary schools of different countries. It was considered that such a work "would constitute in a certain sense the crown of the labors of the commission." Early in 1915 a questionnaire was published¹ in order to acquaint those who might consent to prepare the special reports for different countries with the questions which the commission wished to have them answer. As far as I have been able to learn, only two reports based upon this questionnaire have been published at this writing; these are the brief report (14 pages) by W. Lietzmann, concerning Germany, and the longer report on Belgium by J. Rose. It is hoped that the general report submitted herewith may be considered a worthy contribution to the commission's special inquiry.

At the present time superintendents, inspectors, and principals in many parts of the United States have been forced by public opinion to consider numerous radical changes in methods of secondary-school education. If a high minimum standard of preparation were required on the part of each teacher, and the position of the teacher were made such as to attract in sufficient numbers the best talent in the country, other difficulties would disappear. Most countries considered in this bulletin have far higher standards than we with respect to teachers of mathematics in secondary schools. The degree of this superiority is exhibited throughout the following pages, and some of the chief points are summarized in the last chapter.

In what follows, statements concerning countries now at war refer, for the most part, to ante bellum days.

R. C. A.

December, 1917.

¹For fuller account of the work in mathematics see *Curricula in Mathematics, a comparison of courses in 127 countries represented in the International Commission on the Teaching of Mathematics*, by J. C. Brown (Bu. of Educ., Bul., 1914, No. 45), Washington, 1915.

²*L'Enseignement Mathématique*, tome 17, January, 1915, pp. 61-65. See also pp. 129-145.

TRAINING OF TEACHERS OF MATHEMATICS FOR THE SECONDARY SCHOOLS.

I. AUSTRALIA.

Australia is politically divided into five States (New South Wales, Victoria, Queensland, South Australia, Western Australia), which with the island of Tasmania form what has been known since 1901 as the Commonwealth of Australia. At each of the six capitals (Sydney, Melbourne, Brisbane, Adelaide, Perth, and Hobart) there exists a university supported in part by public funds¹ and in part by private endowments and fees paid by students. And while the educational conditions vary greatly in the different States and not a little in the same State, the universities, whatever their status, are the crown of the educational system of which they form a part. The conditions in New South Wales and Victoria, each with a population of more than one and one-half millions and with universities founded well over half a century ago, are greatly superior to those in the enormous State of Western Australia, with its scattered population of less than 300,000 people (in 1911) and a university which has been in operation but a short time.

The peculiarity of Australian education is that the State not only controls, but completely and absolutely supports and regulates the system of public education, without support from or interference by the localities in which the schools may lie. Australian education tends therefore to be centralized, systematic, and homogeneous; but since local interest is naturally fitful, the external equipment of the schools is usually of an inferior character, while the qualifications of the teachers are distinctly superior. Primary education throughout Australia is free, but secondary is not. The State secondary schools are fewer and somewhat less important than those of a semipublic endowed or denominational character.

The organization of secondary education in Australia is passing through a period of active development. But until very recently the chief influence upon the work of the secondary schools has been exerted by the universities, not only through their requirements at matriculation, but also through a system of public examinations taken by pupils of the schools whether they proposed to enter the univer-

¹ "The universities, though not State universities in the usual sense of the term, are in most cases largely supported by the State. In some instances the proportion of their revenues derived from the treasury is so large that, except for the freedom of their administration, it would be difficult to distinguish them from State institutions."

sities or not. These examinations are similar to the Cambridge and Oxford local examinations. The machinery in connection with them has differed from State to State. Since 1912 they are gradually being displaced by a much more satisfactory system of State intermediate and leaving certificates. The requirements for these examinations and certificates give a fair idea of the ground the teacher must cover in his instruction. For the States in order these requirements are:

JUNIOR AND SENIOR EXAMINATIONS.

UNIVERSITY OF SYDNEY.

Till very recently the junior examination was held in June of each year. Candidates averaged 15 years of age. The examination was intended to cover the first two or three years' work of the secondary school. Every student matriculated at the university had to pass the mathematical papers of this or an equivalent examination. Almost all of those taking mathematical classes in their university course also passed the senior examination in mathematics before entrance. The senior examination was usually taken from one and a half to two and a half years after the junior; additional papers or separate questions were set for honor candidates.

In New South Wales the above junior and senior examinations are now replaced by the system of State intermediate and leaving certificates. The former is awarded after the successful completion of two years' work in the high school. The courses of study in the mathematical subjects are, in outline, as follows: -

Arithmetic: This forms a part of the first and second years' work for all pupils. It includes mensuration, the plane figures named in the syllabus being the rectangle, triangle, parallelogram, quadrilateral, and circle. The solids are the rectangular box, prism, pyramid, cylinder, cone, and sphere. The simple numerical trigonometry of the right-angled triangle is also introduced. This is not taken in the arithmetic course till after a simple geometric treatment has made possible a satisfactory discussion of the points involved.

Algebra: The work in algebra of these two years goes up to simple case of simultaneous quadratics. The variation and change of sign of the expressions $ax+b$ and ax^2+bx+c are studied graphically and algebraically.

Geometry: This course covers the subject matter of Euclid's Elements, Books I-III, with some freedom from his methods and sequence. Prop. 4, Book I, is accepted as an axiom. Preliminary practical work in geometry will in most cases have been done before entrance upon the course of the secondary school.

The third and fourth years' work in the high schools is divided into pass and honor sections. Practically all pupils have to do some mathematical work in these two years, but only those who have shown special aptitude for this study attempt the full course. Indeed, some take only part of the pass course, but all who desire leaving certificates to count as equivalent to the matriculation examination

have to satisfy the examiners in one of the two pass mathematical papers, and thus have to reach a certain standard in algebra, geometry, and trigonometry. There are three higher papers set in mathematics—one devoted to geometry and trigonometry; another to algebra, coordinate geometry, and the elements of the differential calculus; and the third to mechanics.

The nature of the work in mathematics tested at the leaving certificate examination is as follows:

Algebra: The pass work in algebra distributed over the third and fourth years includes what may be described as "up to the binomial theorem, with a positive integral index." Interest and annuities are introduced after logarithms, and graphical illustrations of maxima and minima, etc., are given. The arithmetic course is supposed to have been completed in the first two years.

The additional work for honors includes: Convergence of series; the binomial theorem with fractional or negative index; the exponential and the logarithmic series.

It comprises a course in coordinate geometry of the straight line and circle, and a short introduction to the differential calculus. Only differentiation of powers of x and simple algebraic expressions need be attempted; the results are applied to the equations of tangents in coordinate geometry, to velocity and acceleration, and to the determination of important areas, surfaces, and volumes.

Geometry: The pass work completes the usual elementary course in geometry, without solid geometry. The additional work for honors is as follows: Modern geometry—including transversals; nine point circle; harmonic ranges and pencils; pole and polar; similitude; and inversion. Solid geometry—including the substance of Euclid's Elements, Book XI, 1-21, together with theorems relating to the surfaces and volumes of the simpler solid bodies. Geometric conics—including the more important properties of the parabola and the ellipse.

Trigonometry: The pass work takes the pupils up to the solution of triangles. The honors work, in addition, includes a fuller treatment of the preceding, with circular measure, De Moivre's theorem, and certain types of series.

Mechanics: This subject is not divided into pass and honors. It comes as one of the higher papers in mathematics. It is intended to be preceded and accompanied by experimental work. Indeed, in many schools the course is under the direction of the science master. It includes the usual elementary work in statics and dynamics, with elementary hydrostatics and atmospheric pressure.

UNIVERSITY OF MELBOURNE.

Here the scheme of public examinations includes the junior and the senior examinations. The usual age for the junior is about 16 and for the senior about 18.

At-entrance to the university a certain amount of mathematics is required for all the faculties, except for those of law and music. In *arts*—arithmetic, algebra and geometry; in *engineering*—arithmetic, algebra, geometry, and trigonometry; in *science*—arithmetic, algebra, and geometry. The scope of the examinations in these subjects is very similar to that indicated above in connection with the intermediate and leaving certificates.

UNIVERSITY OF ADELAIDE.

Among the public examinations in this university are the junior and senior and a higher examination. These examinations cover much the same ground as the junior and senior examinations in New South Wales and Victoria, but the programs (in mathematics at least) do not include the work prescribed for honors. The requirements at entrance to the different faculties are as follows: *Science and medicine*—the senior examination must be passed in arithmetic, algebra, and geometry; *engineering*—the senior examination must be passed in arithmetic and algebra, geometry, and trigonometry.

Up to 1913 some of the schools of Western Australia sent up their pupils to the public examinations of the University of Adelaide, but with the foundation of the University of Western Australia this practice has been discontinued.

UNIVERSITY OF QUEENSLAND.

Until the year 1910 the secondary schools of Queensland usually sent up their pupils for the public examinations of the University of Sydney. The public examination system has now been adopted by the new University of Queensland. The mathematical programs for its junior and senior examinations are practically the same as those in Sydney.

UNIVERSITY OF TASMANIA.

The scheme of public examinations includes a junior and senior examination somewhat like the corresponding examinations in Sydney. The scope of these examinations as regards mathematics has, however, been much narrower. A beginning of better things was made with the foundation of two State secondary schools in 1913. The education department proposes, with the assistance of departmental officers, secondary school teachers, and representatives of the university, to frame a four-year curriculum on lines similar to that adopted in New South Wales, and it hopes to get the university authorities to recognize it.

TRAINING OF TEACHERS.

For teachers in the secondary schools—public or private—a university degree has been in the past, and still remains, an almost indispensable qualification. It is in all cases recognized to be desirable, and in some cases insisted upon by the regulations for registration, that special training in the theory and practice of education be included in addition to the work of the regular university curriculum in arts or science. Further, in at least one State a fair number of traveling scholarships enable the best of the candidates for the teaching profession to enjoy a year or more in Europe or America.

at the chief institutions for the training of teachers, or in special study at the universities.

It remains to describe the work of the teachers' colleges and to give indications of the courses in mathematics at the universities which have been available for the future secondary school teachers. For this purpose it will be convenient to take the different States separately.

NEW SOUTH WALES.

The Teachers' Training College in Sydney was founded in 1906¹ for the training of State school teachers and others who might desire to take advantage of the courses on instruction given therein. The college provides a variety of courses of training, varying in length from six months to four years.² The ordinary college course is the two years' course,³ which qualifies for teaching in the classes of the primary school. For entrance to the regular college course the leaving certificate or its equivalent is soon to be required. For the present it has been found desirable to admit students having received only the intermediate certificate granted by the high school at the end of two years of study.

The regular students of the college who have reached the standard of the matriculation examination in the faculties of arts or science are encouraged to attend the university classes, instead of those at the college, in the subjects of their course. They are admitted to these classes without fees. Those who do satisfactory work at the

¹ The erection of a new building for the college was commenced in 1914 on land provided by the University of Sydney. When completed it was planned that it should provide training for private secondary and primary schools, as well as for State service.

² The minimum age of admission to the college is 17 years. The fees for training courses are as follows: Six months, 7£ 10s.; one, two, three, or four year courses, 15£ per annum for those taking both general and professional subjects; fees are returned to those who enter the service of the Department of Education.

³ Every student of the first year must take the following mathematical classes:

Arithmetic.—There is no set academic work in arithmetic; one hour per week of each term is given to the method of teaching the subject. Amongst others the following topics are discussed: Brief historical review; reasons for teaching arithmetic present tendencies; real applied problem movement; nature and source of problems; use and limitation of objective material; place of oral and written work in the various classes; topical-spiral treatment, various methods of teaching sections of arithmetic, the beginnings of number work; plan and purpose of mechanical work; teaching of processes; the primary school curriculum.

Geometry.—Brief historical sketch of the development of geometry; Egyptian geometry; Greek geometry; the work of Euclid; the reform movement in the teaching of geometry; the present attitude toward the subject; the nature and place of definition, axiom, postulate, proof; experimental and demonstrational geometry; the relation of solid and plane geometry; congruence, similarity, and homology of figures and their applications; extension to trigonometry.

A short course in solid geometry.

Algebra.—Those topics in elementary algebra which require special treatment in class teaching are discussed, including the following: Relation of algebra to arithmetic; literal arithmetic, formulae from arithmetic and mensuration; notion of a negative; operations involving negatives; the equation; solution of problems by means of equations; factors and their applications; indices, logarithms, calculations, with use of logarithm tables and slide rule; irrationals, numerical evaluations, using tables; ratio, proportion, variation, and their application; notion of a function; graphical representation of the variations of a function; discussion of roots of equations, maximum and minimum values from graphs.

Trigonometry.—The purpose of the course is to investigate those cases of the solution of triangles which are used to obtain heights and distances.

Notion of an angle; instruments in common use for measuring angles; sine, cosine, and tangent of acute angles; reading sine and tangent tables; solution of right-angled triangles; use of four-figure tables; extension

university in their first two years have their college scholarships continued, so as to enable them to graduate in arts or science. The staffs of the high schools are chiefly recruited from students of the Teachers' College who have graduated at the university in this way.

In addition to these courses, a one-year course is provided at the college for graduates of the university who have not entered upon the course of training in the regular way. This class is exclusively professional and qualifies for the second-class certificate of the Department of Education.

Further, special provision is made for training of teachers for secondary schools. Graduate students of the college take the courses required for the university diplomas in education or similar courses at the college itself. Candidates for this diploma must have graduated in arts or science before admission to the course. The special work for the diploma can be completed in one year by those who are able to devote their whole time to it. The requirements of the course are as follows:

Lecture work:

- I. A first course in philosophy of education.
- II. A higher course in education.
- III. A course in principles of teaching.
- IV. A course in school hygiene.

Practice work:

- I. Continuous practice—from 8-10 hours per week.
- II. Observation and discussion of lessons—from 2-4 hours per week.

In the course on principles of teaching, some instruction in the methods of teaching mathematics is given by one of the lecturers in mathematics.

Practice schools for primary and secondary work are associated with the college. The principal is also professor of education at the

tion of definitions of sine, cosine, and tangent to angles between 90° and 180° ; relations between sides and angles of a triangle; area of triangle; notion of triangulation and application to simple surveys; heights and distances in more than one plane.

The mathematical work in the second year is optional. The program for the class includes algebra, trigonometry, coordinate geometry, mechanics, infinitesimal calculus, and history and method of elementary and secondary school mathematics.

Throughout the academic work the professional aspect is kept prominently in view.

Algebra.—Ratio, proportion, variation, the progressions (arithmetic, geometric, harmonic), permutations and combinations, mathematical induction, binomial theorem.

Trigonometry.—Continuation of first-year course, including angles of any magnitude, $\sin(A \pm B)$, $\cos(A \pm B)$, $\tan(A \pm B)$, $\sin A \pm \sin B$, $\cos A \pm \cos B$. Relations between sides and angles of a triangle, solution of triangles, heights and distances, circular measure, De Moivre's theorem, simple trigonometric series.

Infinitesimal calculus.—A course on the processes and applications of the differential and integral calculus. Graphical methods are freely used where advantageous. The course is designed particularly to complete the work in geometry, mechanics, and trigonometry.

Mechanics.—The aim of the course is to teach the fundamental mechanical principles. Simple courses of experiments illustrating the following: Composition and resolution of forces; principle of levers; pulleys, inclined plane; friction; motion of falling bodies; circular motion; principle of Archimedes; hydraulic press and pump; atmospheric pressure; easy practical calculations within this range.

Reading.—Students do directed reading of mathematical works in the college library, as well as in the following textbooks: J. W. A. Young, *Teaching of Mathematics in the Elementary and the Secondary Schools*; Fink, *History of Mathematics*, translated by Beman and Smith.

university, and the work of the college is carried on in cooperation with the university, so far as possible.

Let us now take note of the university mathematical courses, which are open to our prospective high-school teachers in connection with his course. There are three classes—mathematics I, II, and III. Each is divided into three sections—class A, class B, and class C. Candidates for the degree of B. A. or B. Sc. with honors attend the honors section (class A) in each year, but it is possible to reach the lowest grade of honors by specially good work in the second section (class B) in the three years. The programs for class A are as follows:

Mathematics I (first year): Algebra, geometry, trigonometry, statics and dynamics, analytic geometry of two dimensions, and the elements of the calculus. Those who enter it are expected to have previously completed the mathematical work required for honors in the senior public examination or an equivalent course. The class is attended by students in arts, science, and engineering.

Mathematics II (second year): The infinitesimal calculus, differential equations, spherical trigonometry, analytic statics, dynamics of a particle, and elementary rigid dynamics. The class is attended by the best students of Mathematics I when they proceed to their second year.

Mathematics III (third year): Analytic geometry of three dimensions, rigid dynamics, higher analysis, and some applied mathematical subject, e. g., hydrodynamic, sound, the theory of electricity and magnetism.

In addition to these courses a two-year course has recently been instituted in the mathematics of insurance, chiefly for actuarial students and others who desire instruction in the mathematics of statistics.

Since 1908 the teacher's promotion depends not only on the classification of his school as determined by average attendance, but also on the improvement of his qualifications. To qualify for a higher grade the teacher must pass a series of examinations as well as show "the requisite degree of proficiency in practical work."

Salaries paid to high-school teachers.

Teachers.	Men.	Women.
Principal of—		
Boys' or girls' school.....	£400-£600	£300-£450
Mixed school.....	250-450
"Master" or "mistress" of a department.....	300-400	200-300
"Assistant".....	200-300	180-250
"Junior staff".....	168-228	144-180

VICTORIA.

Those preparing to become teachers in the secondary schools of Victoria receive almost all of their instruction at the university. Under regulations now in force, every teacher in a secondary school must possess the university diploma in education or an equivalent qualification. All candidates for that diploma must have passed through at least two years of some degree course, and must have a

final year of special work in education. As a matter of fact, many of those working for the diploma take their B. A. or B. Sc. degree before entering upon the special course in education for the diploma.

Of the various groups of students entering the Teachers' College at Melbourne only those taking the three years' course for the secondary certificate are of interest to us. Those accepted for this class must have passed the senior public examination or an equivalent examination. The number admitted to the class is limited to 15 each year. Their course of training is the same as that for the diploma of education at the university, with the addition of some special subjects. The first two years are spent in regular attendance upon university classes in arts or science. Having passed the first and second years' examinations in arts and science, they are then admitted to the special university courses in education for the diploma. These courses include lectures on the theory of education, with special reference to the methods of teaching the various subjects. The course in mathematics contains the following:

- (i) Special methods of teaching arithmetic, algebra, geometry, and trigonometry.
- (ii) A short history of elementary mathematics.
- (iii) General considerations on the teaching of mathematics.

Students are required further to teach 120 hours under supervision; to attend lessons given by members of the college staff; and also to criticize lessons. Part of this practice is obtained in the primary schools, and part in the special secondary practicing school attached to the Teachers' College, and in other schools.

There are about 50 to 60 regular students of the college each year preparing for this diploma, and in addition about the same number of university students are qualifying for it, in order that they may be enabled to take up work in the secondary schools outside the control of the Department of Education.

The authorities of the college are entitled to nominate six of those who distinguish themselves in this course for a further year's work.

At the University of Melbourne courses in pure and applied mathematics are offered for each of the three years:

Pure I. Pass—The course is analytical. It deals with the elementary algebraic, trigonometric, exponential, logarithmic, and hyperbolic functions, with their graphs and derivatives; maxima and minima; elementary processes of integration; the definite integral as the limit of a sum. *Honors*—Algebra, trigonometry, elementary analytic geometry of two dimensions, and elementary calculus.

Applied I. Pass—Elementary statics and dynamics, with hydrostatics. *Honors*—A fuller treatment of the subjects of the pass class with the elements of vector algebra.

Pure II. Pass—In this course further work is done in plane analytic geometry and calculus. *Honors*—This class continues the work of I (honors) in plane analytic geometry and the calculus, and it begins analytic geometry of three dimensions.

Applied II. Pass—Elementary analytic statics and dynamics, with hydrostatics. *Honors*—Analytic statics, dynamics of a particle, elementary rigid dynamics, and hydrostatics.

Pure III. Pass—Solid geometry and the calculus, with differential equations. *Honors*—In addition to the subjects for II (honors): The functions of a complex variable, Fourier's series and integrals, differential equations and calculus of variations. *Applied III. Pass*—Analytic statics and dynamics, the elements of the theories of potential, hydrodynamics, elasticity, and electricity. *Honors*—More advanced work in these subjects.

SOUTH AUSTRALIA.

The following is an outline of the course of training for teachers in the schools of South Australia under the department of education in Adelaide:

1. At the age of 14 the future teachers enter the Adelaide high school. They remain there for three years and receive a general education. At the end of this period the majority will have passed the senior public examination in the University of Adelaide; practically every student passes in arithmetic and algebra; a large number pass also in geometry and in trigonometry.

2. The students in training now become "junior teachers," and spend one to two years in primary schools, devoting practically all their time to their teaching work.

3. Thereafter they enter the Teachers' Training College. Here the students attend university classes in various subjects, a certain amount of freedom of selection being permitted. About 20 per cent take up what is called first-year pure mathematics, an elementary class at about the standard of the higher public examination in algebra, geometry, and trigonometry.

The students also attend several training college lectures, among these being a course in the principles of teaching. This course deals with the methods of teaching the primary school subjects; about 16 out of a total of 80 hours are devoted to arithmetic. They also spend one morning per week at actual teaching in the primary schools.

Those who have shown special ability both as students and teachers are permitted to remain a second year, or even a third year, at the university. These students invariably take up a certain amount of mathematics, usually attending second-year pure mathematics, as they will in most cases have already taken the first class. Some also take the elementary applied mathematics class. It is not uncommon for mathematics, physics, and chemistry to form the bulk of their work.

Such students have practice as before; also weekly discussions on the methods of teaching. Five of these discussions out of a total of twenty-five deal with elementary mathematics. The majority of these students become high-school teachers.

The mathematical courses offered in the University of Adelaide are very similar to those of the universities of Sydney and Melbourne. Horace Lamb and W. H. Bragg were for many years professors in this university.

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QUEENSLAND.

Queensland does not yet possess any special college for the training of teachers. In the "grammar schools" the staff consists principally of university graduates. The high schools also aim at making a university degree an almost indispensable qualification for their teachers.

The University of Queensland, which was founded at Brisbane in 1909 and formally opened in 1911, has an important sphere of influence, and the same general scheme of mathematical instruction as in other States of the Commonwealth is in vogue.

TASMANIA.

Under the scheme for training of teachers which is being introduced in Tasmania candidates pass the senior public examination and matriculate at the university. The next year is spent in teaching in selected schools. Thereafter they enter the training college at Hobart, the minimum age at entrance being 18 years. They attend university classes for two or three years. All receive professional training at the college.

Mathematical courses at the university are similar to those already described.

WESTERN AUSTRALIA.

The organization here being in its infancy, there is nothing of exceptional interest to record.

The universities of Australia are staffed by British professors, and thus the mathematical work of the country is fashioned in conformity with much the same ideals as those held in the motherland.

BIBLIOGRAPHY.

H. S. CARSLAW, *The Teaching of Mathematics in Australia. Report presented to the International Commission on the Teaching of Mathematics.* Sydney, Angus & Robertson, 1914. 79 pp.

In the above sketch I have many times quoted verbatim from Prof. Carshaw's most readable report

J. B. TRIVETT, *The Official Year-Book of New South Wales for 1915.* Sydney, W. A. Gullick, 1917.

"Education," pp. 213-274.

Rept. of U. S. Commis. Educ., 1915-16, Vol. 1, Washington, 1916.

"Education in Australia," pp. 641-659.

Two other recent publications contain considerable information about the universities:

(1) *Universities in the Overseas Dominions*, Board of Education, Special Reports on Educational Subjects, vol. 25, London, 1912, pp. 116-171, 196-198, 238-269;

(2) *Congress of the Universities of the British Empire, 1912, Report of the Proceedings*, London, 1912; also the *Year Book*, 1914.

II. AUSTRIA.

Austria is nearly 116,000 square miles in extent, and its population totals in the vicinity of 29,000,000. Of these, about 10,000,000 are Germans; nearly 6,500,000 are Bohemians, Moravians, and Slovaks; at least 5,000,000 are Poles; and some 3,500,000 are Ruthenians.

The minister of public instruction is at the head of the Austrian educational system. He has inspectors of secondary schools in each of the 14 Provinces of the Empire and certain matters of administration are assigned to local boards. Church and private schools are subject to the same regulations as those of the State.

SECONDARY SCHOOLS.

Before 1908 there were three general types of middle or secondary schools: (1) The Gymnasium, (2) the Realschule, and (3) the four-year Realgymnasium, the courses of study in all of which were based upon four years of work in the primary schools. The first two of these types are still the most prominent. The normal age of the pupil commencing secondary education is about 10 years.

The complete Gymnasium provides for a course of eight years' study, divided into two parts of four years each. The first part is referred to as the Untergymnasium; the latter as the Obergymnasium. The course of study is characterized by the great emphasis laid on instruction in Latin, Greek, and German. In passing from one class to another the pupils undergo very searching examination. This is characteristic of all the Austrian secondary schools.

The seven years' course of the Realschule consists of two cycles of three and four years each, corresponding to programs of the Unterrealschule and Oberrealschule. Here the emphasis is laid on modern languages (Latin and Greek are not taught at all), mathematics, physics, chemistry, etc.; that is, on those subjects which are designed to impart technical knowledge and afford suitable training to those intending to follow industrial pursuits.

As a result of a ministerial decree in 1908, the establishment of Realgymnasien and Reform-realgymnasien, each with an eight-year course, was also authorized. But, in what follows, the Gymnasium and Realschule¹ only will be treated as representative of Austria's best secondary schools.

¹ In 1915-16 there were 336 Gymnasien, of various forms, for boys and 148 Realschulen.

16 TEACHERS OF MATHEMATICS FOR SECONDARY SCHOOLS.

The number of hours per week devoted to class work in mathematics (M.), geometric drawing, and descriptive geometry (D. G.), in these types of schools, is exhibited in the following table:

		I	II	III	IV	V	VI	VII	VIII	Total
Gymnasium	(M.)	3	3	3	3	3	3	3	2	21
	(D. G.)	3	3	2	2	0	0	0	0	10
Realschule	(M.)	3	3	3	4	4	3	5	0	25
	(D. G.)	0	2	2	3	3	3	2	0	15

The total number of hours in these subjects represents about 14.7 per cent of all class periods in the Gymnasium and about 16.5 per cent of those in the Realschule.

In classes I-VIII of the Gymnasium instruction is given, according to the program of 1909, in arithmetic, algebra, plane and solid geometry, plane trigonometry, plane analytic geometry, and elements of calculus.

To give a definite idea of the nature of the contents of these courses a few selections from the program may be made:

Class I: Extension and completion of arithmetic and algebraic studies of the previous class. Continued practice in solution of equations of the first degree arising "from the manifold spheres of thought in which they may be applied." Practice in powers and roots by straightforward examples. *Solid geometry.*—Oblique projections¹ of the simplest bodies (also of crystalline forms). Plan and elevation of simple objects by observation and common sense. Conceptions and laws concerning the mutual relations of straight lines and planes; limited to the fundamental and typical propositions and proofs with constant appeal to observation and intuition. Properties and calculations for surface and volume of prisms (cylinders), pyramids (cones), the sphere, and their sections and portions when in section. Euler's theorem ($S+F=E+2$); regular polyhedra. *Obergymnasium—Class VI: Trigonometry.*—The trigonometric ratios, their geometric representation, especially for the purpose of giving a firm grasp of the characteristics and relations of these functions. Solution of triangles. Repeated comparison of trigonometric propositions and methods with those of plane and solid geometry. Varied applications of trigonometry to problems in surveying, geography, astronomy, etc., the data where possible to be obtained from direct (even though rough) measurements by the pupil. *Class VII: Algebra.*—Arithmetic progressions (of the first order); geometric progressions. Application of the last especially to compound interest. Permutations and combinations in simplest cases. Binomial theorem for positive integral exponent. Theory of probabilities. *Analytic geometry.*—As a continuation of the graphic representation of single functions previously given, further use is made of the analytic method in dealing with lines of the first and second degree with reference, when occasion offers, to the geometric treatment of the same figures and their relations. Representation of direction coefficients (chiefly those of the curves dealt with in class) by means of derivatives. Simple problems of differentiation in connection with problems in mathematics and physics. Approximate solution of algebraic equations (and of the simplest transcendental equations which occur) by graphic methods. In *Class VIII* the two hours a week are

¹ Drawings of solid bodies to show several faces, etc., not in true perspective, but free-hand or constructed according to very simple rules. Cf. A. Höfler, *Didaktik des mathematischen Unterrichts*, p. 207.

² Ordinary arithmetic progressions are said to be of the first order. Series such as $1^2 + 2^2 + 3^2 + \dots$, $1^3 + 2^3 + 3^3 + \dots$, are called A. P. of higher orders.

given up to a comprehensive recapitulation of the whole range of school mathematics, especially equations, series, solid geometry, trigonometry, and analytic geometry. Broader and deeper treatment of particular parts of the subject. Applications to the various subjects of the curriculum and to practical life in place of merely formal exercises. Retrospective and prospective consideration from historical and philosophical points of view.

For the Realschule the work is very similar. There is the same effort to correlate mathematics with other branches of instruction on which it happens to bear and with practical applications in actual life.

Though searching examinations of "orientation" and "classification" have been the portion of students of the Gymnasium and Realschule each year, the final examinations (Maturitätsprüfungen) leading to the certificate of maturity were formerly often much feared.¹ Some amelioration with respect to the extent of detail here required was made imperative by the decree of 1908.²

The main object of these examinations is to determine whether or not the maturity, general efficiency, and development of intelligence on the part of the candidate is sufficient to allow him to take up studies in the universities or higher technical schools. Whilst the examinations for "classification" establish in some measure the rating of a pupil's acquisition with reference to a certain part of the program taught, the "Maturitätsprüfung," on the other hand, embraces the whole range of knowledge acquired by the pupil in the Gymnasium or Realschule.

The examination consists of two parts, a written and an oral. The former includes questions on the required languages, on mathematics, on exercises in descriptive geometry, and tests in facility with free-hand drawing. The oral examination is on geography, history, mathematics, descriptive geometry, physics, chemistry, and natural history.

The examination commission pronounces its verdict after consideration of all the written and oral examinations as well as of the grades received by the candidate in tests during the last year of his course. When a candidate fails he may present himself a second time at the end of a semester or of a year, but he may not repeat the examination more than twice.

UNIVERSITIES AND PREPARATION OF SECONDARY SCHOOL TEACHERS.

There are eight State universities in Austria. The largest is the University of Vienna (German), with over 10,000 students.³ The oldest, dating back to the fourteenth century, is the German-Bohe-

¹ Cf. F. Wallentin, *Maturitätsfragen aus der Mathematik. Zum Gebrauche für die obersten Klassen der Gymnasien und Realschulen zusammengestellt.* 10. Auflage. Wien, C. Geroldssohn, 1912. 8+197 pp. *Auflösungen*, 3. Auflage, 1906. 8+221 pp.

³ In the summer of 1915 this number was reduced to 3,473.

mian University of Prague. There are three other German universities, at Gratz, Innsbruck, and Czernowitz. The Polish universities are at Krakow and Lemberg.

Instruction in the universities is imparted by means of: I, general courses; II, special courses; and III, exercises in proseminary and seminary.

I. The general courses are organized with a double aim: (1) To furnish fundamental theoretical knowledge to those studying mathematics in a purely scientific spirit and proposing to terminate their studies with the doctorate; (2) to prepare students destined for secondary-school teachers. Especially in view of this second aim are the general courses usually organized in cycles of three or four years, and each year a course for beginners is arranged that they may be taught differential and integral calculus as soon as possible, since it is necessary for the study of theoretical physics.

During the five years 1905-6 to 1909-10 professors at (1) the University of Vienna and (2) the German section of the University of Prague gave the following general courses:

(1) Differential and integral calculus, 5 hours weekly during 2 semesters; theory of numbers, 5 hours during 2 semesters; theory of differential equations, 5 hours during 2 semesters; calculus of probabilities, 3 hours; definite integrals and calculus of variations, 5 hours during a semester; theory of linear differential equations, 5 hours; elliptic functions, 5 hours; theory of functions, 5 hours during 2 semesters; algebra, 5 hours during 2 semesters; analytic geometry, 4 hours during 2 semesters; theory of invariants with geometric applications, 2 hours; algebraic curves, 2 hours; curves and surfaces of the third order, 2 hours; synthetic geometry, 4 hours during 2 semesters; differential geometry, 2 hours during 2 semesters; line geometry, 2 hours; continuous groups, 2 hours; noneuclidean geometry, 2 hours; insurance mathematics, 4-6 hours during 2 semesters; mathematical statistics, 3 hours; sickness and accident insurance, 2 hours.

(2) Applications of infinitesimal calculus to geometry, 3-4 hours; theory of invariants, 2 hours; differential and integral calculus, 4-5 hours during 2 semesters; systems of algebraic equations, 1 hour; differential equations, 5 hours; introduction to calculus of variations, 1 hour; fundamental notions of analysis, 2 hours; analytic geometry, 3 hours through 3 semesters; elements of the theory of functions, 3 hours; elements of the theory of numbers, 2 hours; algebraic equations, 4 hours; introduction to descriptive geometry, 2 hours; characteristic features of infinitesimal calculus, 3 hours during 2 semesters; theory of groups and algebraic equations, 2 hours; selected chapters of analytic geometry, 2 hours; vector analysis, 2 hours; differential equations, 3-5 hours; contact transformations, 2 hours; theory of transformations, 5 hours. An equally elaborate

series of courses is offered in the Bohemian section of the University of Prague.

Mathematical instruction in Austria is much handicapped by reason of the fact that the faculties of the universities are so sparsely staffed with professors of pure and applied mathematics. In 1909-10, except at Vienna, no university had more than two such professors and only four had even one Privatdozent. In Italy and Russia the number of these professors per university averages nearly twice as many as in Austria; in France and Germany nearly one and one-half times as many. Indeed, at each of a score of universities in these four countries there are more professors of mathematics than even at the University of Vienna.

In a memorial presented in 1908 to the minister of public instruction and finance on behalf of the faculties of philosophy in Austrian universities the following normal program of mathematical study at a university for a candidate as a teacher in a Gymnasium was outlined:

First year.—Introduction to mathematical analysis and to differential calculus, elements of integral calculus with exercises, 5 hours a week. Introduction to geometry, first principles of analytic and synthetic geometry, with exercises, 3 hours a week. A course of mathematics for students of physics and chemistry, 5 hours a week.

Second year.—Integral calculus and first principles of the theory of functions of complex variables, with exercises, 5 hours a week. Analytic and differential geometry, with exercises, 5 hours a week. Principal properties of algebraic equations and elements of the theory of numbers, with exercises, 3 hours a week.

Third, fourth, and later years.—Higher courses on the theory of differential equations, the calculus of variations, the theory of functions, the theory of groups of transformations, on the principles of arithmetic and of geometry, etc., at least 3 hours a week. Seminary for mathematical analysis, 2 hours a week. Seminary for geometry, 2 hours a week. Seminary for the theory of numbers and higher algebra, 2 hours a week.

On this scheme the memorial comments as follows:

But it should be emphasized that this program indicates only that which is absolutely necessary for future professors in Gymnasien; if one wished to obtain a more extended preparation, which it should be the principal aim of the university to furnish, it would be necessary to augment this program by a whole series of courses in special fields of pure mathematics and on different branches of applied mathematics.

The program indicated corresponds, then, only to a minimum. Now, the execution of such a program requires at least three professors, and further, this number will only suffice if each of two of the three chairs has an assistant at its disposal. The exercises corresponding to elementary courses ought to be directed by assistants under the control of professors. For, in order that exercises be truly profitable, it is necessary that they be individual, as has been the case for a long time in higher technical schools for all practical branches and for descriptive geometry. That is to say, it is not sufficient to work out some examples on the blackboard in the classroom with one or more students; rather is it necessary to lay before the students a selection of examples from which they then choose according to their personal inclinations some problems to solve under the guidance of the professor and assistant. This personal activity, even to such a minor degree, is particularly valuable for the

future professor; for, in his teaching he ought to display certainty and enthusiasm. On the other hand, the proposed reform of the teaching of mathematics in the secondary schools demands a suitable preparation of the professor.

Since of course the whole scheme of instruction in a country depends first of all on instruction in the university—that is to say, since a good organization of university instruction is of capital importance for general instruction—the faculty of philosophy of the Austrian university regard it as urgent to satisfy these exigencies of university instruction.

There ought then to be at least three chairs of mathematics in the faculty of philosophy of every Austrian university, one in each of the following fields: (a) For the theory of numbers and higher algebra; (b) for mathematical analysis; (c) for geometry. At least two of these chairs ought to be held by "ordinary" professors provided with assistants.

As this memorial did not bring about any material change in the situation, we remark that in most Austrian universities the opportunities for scientific preparation of the professor of mathematics in secondary schools are inadequate.

II. Having noted typical general courses which may contribute to this training, let us next consider the special courses. There were no special courses given by Privatdocenten in the German division of the University of Prague during the five-year period referred to above. At the University of Vienna, however, Privatdocenten offered the following courses: Mathematical statistics, 3 hours, during 2 semesters; elliptic functions, 2 hours, during 2 semesters; quaternions and other hypercomplex number systems, 2 hours; theoretical arithmetic, 3 hours; differential geometry, 2 hours; potential theory, 2 hours; Pfaff's problem, 2 hours; elementary theory of functions, 3 hours; during 2 semesters; functions of a single real variable, 2-3 hours, during 2 semesters; hypergeometric series, 2 hours; infinite double series, 2 hours; infinite groups, 2 hours; theory of numbers, 3 hours; foundations of geometry, 2 hours, during 2 semesters; finite discrete groups, 2 hours; review of more recent mathematical works, 2 hours; complex number systems, 2 hours; selected chapters of higher algebra, 3 hours; calculus of variations, 3 hours, during 2 semesters; theory of integral-transcendental functions, 2 hours; determinants, 1 hour; the principle of duality in geometry, 2 hours; integral equations, 3 hours; selected chapters of theory of functions, 2 hours; analysis situs, 2 hours, during 2 semesters; Taylor's series and its analytic extension, 2 hours; synopsis of differential calculus with applications, 3 hours; general theory of groups, 2 hours.

The number of these courses offered at the University of Vienna is nearly twice as large as the total of similar courses offered at all other Austrian universities during the same period. The three courses given at the University of Krakow were: Mathematics in Poland at the end of the seventeenth century, 1 hour; Arabian

mathematics, 1 hour; history of mathematics in antiquity and the Middle Ages, 2 hours.

Special lectures on topics having more direct connection with the mathematical work in secondary schools are not numerous. Occasional courses on selected chapters of elementary mathematics have been given by Prof. Zindler at Innsbruck. Prof. von Sterneck claims that the two-hour year course on elementary mathematics which he first offered in 1909-10 and which was organized so as to be given every third year at the University of Gratz, was the only regular course of this nature in an Austrian university. He tells us that he expounded thoroughly the laws governing positive integers; discussed the introduction of negative integers in the usual way; treated rational numbers in a purely abstract manner as number pairs; discussed in detail Cantor's theory of number sequence and the related theory of irrational numbers, as well as the successive extensions of the idea of power. In order to discuss exactly the convergence of infinite series, especially the geometric, the idea of regular sequence was also introduced. Rules governing operations on complex quantities defined in connection with pairs of real quantities were presented in detail and the connection with the n roots of unity mentioned. The foundations of geometry were not presented with the same completeness as to rigor. There the didactic points of view, first recommended by Borel and already employed by Suppanschitsch in his textbooks on geometry for the secondary schools, were developed, the questions of the independence of the axioms being left entirely to one side.

Spherical trigonometry is not taught in the Gymnasien. At most universities the fundamental formulæ are derived in the lectures on analytic geometry of space; for example, at Krakow, Gratz, and Innsbruck. More extended development occurs in seminary exercises; for example, those of Gmeiner, at Innsbruck, and Mertens, at Vienna; the latter in his proseminary lectures on spherical trigonometry and considers the analogues on the sphere of such results as Feuerbach's theorem and Malfatti's problem. Otherwise, spherical trigonometry is developed in the lectures on astronomy.

Many persons feel strongly that regular lectures on descriptive geometry should be given at all Austrian universities,¹ but so far such courses have been established at Gratz and Innsbruck only.

III. At every Austrian university, in addition to the lectures, there are mathematical seminary exercises which allow of a certain amount of individual activity on the part of the student as opposed to the compulsory receptive attitude in the lecture room. Each "ordinary" professor is supposed to conduct a section of such

¹ Cf. E. Müller, "Anregungen zur Ausgestaltung des darstellend-geometrischen Unterrichts an technischen Hochschulen und Universitäten." *Jahresbericht der deutschen Mathematiker-Vereinigung*, Band 19, 1910, pp. 19-24.

seminary exercises. These exercises are for both beginners and advanced students.

The section for beginners is referred to at the universities of Vienna, Prague (Bohemian), and Czernowitz as proseminary. The number of hours per week devoted to all seminar exercises varies from 2 at Gratz and Innsbruck to 4 (2 hours proseminary) at Czernowitz in the winter semester and 4 at Krakow; of the 3 hours a week at Vienna, one hour is for the proseminary.

Active participation of members of the seminary in its exercises is not generally compulsory. An exception to this is in the section of the seminary conducted at Gratz by Prof. v. Dantscher, who requires three students each semester to work on some topics, usually quite independent of subjects of lectures during that semester, for presentation before fellow students. This gives the students in the seminary opportunity to learn, with the aid of the professors' criticisms, how such matters should be treated, before the preparation of the required "Hausarbeit" in connection with the examination for secondary school teachers (Lehramtsprüfung). Examples of themes of such required papers are as follows: Explanation of the convergence of an additive aggregate of infinitely many rational numbers; examination of the connection between m functions of n variables (Dini); the projective generation of space curves of the third order; discussion of double integrals.

In most cases the exercises are oral. Those for the proseminary have, mainly, direct connection with the current lectures. In the seminary the topics are chosen sometimes on the initiative of the students and sometimes on the recommendation of the professor. As a general thing it has been found that about one-half of those who regularly attend seminary exercises participate in its discussions. The value of such participation is generally recognized, and Prof. v. Dantscher has proposed that it should be required, during two semesters at least, for all candidates as teachers in secondary schools, just as exercises in experimental physics are required.

Examinations.—The present decrees regarding the preparation of secondary school teachers were promulgated in 1911. These require that candidates shall pass two examinations: (1) A preliminary examination on philosophy and pedagogy; (2) the Lehramtsprüfung. While more or less preparation for these examinations is provided at the universities, the examinations are not conducted by the universities, but by State examination commissions whose members are, however, mostly university professors.

(1) In the preliminary examination the candidate must show that he has acquired the knowledge of philosophy and pedagogy essential for every teacher. This knowledge should embrace the main ideas and principles of the theory of education and instruction and their

theoretical foundations in psychology and logic, as well as general survey notions of the principal periods of the history of higher education since the sixteenth century.

The candidate is not admitted to this examination before the end of the fifth semester. In preparation for it he is recommended to take a four-hour course of lectures in pedagogy, a four-hour course in philosophy (especially psychology), and, when possible, courses in school hygiene and in the methodology of his special subjects. He is also strongly recommended to take part in seminary exercises in pedagogy and in his special subjects.

The examination is conducted by the professors in philosophy and pedagogy under the guidance of the director of the examination commission, or of his deputy, and lasts about half an hour. The commission gives the candidate a certificate as to the results of the examination. If a candidate is not admitted to the examination on account of inadequate preparation, he is not allowed to present himself for examination again until another semester has elapsed. A certificate that he has passed the preliminary examination must be presented to the commission with the application for admission to the Lehramtsprüfung.

(2) The time required, as student in a university, in preparation for the Lehramtsprüfung is about three and one-half or four years, and the examination is the same whether the candidate is later to be a teacher in the Gymnasium or in the Realschule.

The examination is on subjects grouped in a certain way. Those groups which involve mathematics are:

- (a) Mathematics and physics as majors.
- (b) Descriptive geometry and mathematics as majors.
- (c) Natural history as major, mathematics and physics as minors.
- (d) Natural history as major, mathematics and geometrical drawing as minors.
- (e) Philosophy and mathematics as majors and physics as minor.
- (f) Chemistry as major with mathematics and physics as minors.

For mathematics as major the official requirements are: Knowledge of "general arithmetic," of the foundation of higher algebra and theory of numbers and their significance for elementary mathematics; elementary geometry, synthetic and analytic geometry, of the plane and of space; foundations of descriptive geometry; differential and integral calculus and its applications to geometry; the elements of the calculus of variations; familiarity with the foundations of modern theory of functions; and acquaintance with the principal results of the investigations concerning the foundations of mathematics.

For mathematics as minor: Knowledge of elementary arithmetic, insight into the structure of the field of real numbers and into operations with them; knowledge of elementary geometry to the extent

of what is taught in secondary schools, and exercises in space perceptions; accuracy and speed in the solution of simple examples applying the idea of a function and the elements of differential and integral calculus to functions coming up in secondary school work.

For (a) *descriptive geometry* and (b) *geometrical drawing*: (a) Thorough grounding in orthogonal, oblique, and central methods of projection including axonometry, knowledge of relief perspective; of the most important map projections, especially of stereographic projection and of cyclography. Familiarity with the constructions relating to curved lines (especially curves of the second order, space curves of the third and fourth order and helices) and curved surfaces (chiefly surfaces of the second order, surfaces of revolutions, helicoidal, ruled and envelope surfaces, particularly in lighting-constructions). Acquaintance with some applications of descriptive geometry (as construction of sundials, roof trusses, sections of stone (stereotomy)). Knowledge of projective and infinitesimal geometry in so far as these subjects are necessary in the applications of descriptive geometry. Exactness and facility in constructive drawing.

(b) Elements of descriptive geometry to the extent of the program of Realschulen. Axonometric representations. Elements of shadows and linear perspective; the geometrical construction of, and by means of, polygons and the most important plane curves, especially the conic sections. Exactness and facility in constructive drawing.

Each examination comprises three parts: The *Hausarbeiten* (theses), the written examinations, and the *oral examinations*.

To prepare each of the theses the candidate has three months. These theses need not necessarily show capacity for discovery on the part of the candidate, but they must indicate thorough familiarity with the literature and content of the subjects. In connection with a very large subject a general presentation of the outstanding results may be permissible. Again, certain illustrations of a theory may be worked out. The character of the topics is shown by the following selection of themes of theses in recent years. (None of these subjects were discussed in the lectures at universities where they were assigned.)

(a) For major:

The theory of Fourier's series.

The isoperimetric problem.

Systematic presentation of the proofs of the fundamental theorem of algebra.

On Abel's equation.

Theta functions and their applications in the theory of surfaces of the fourth order.

Historical presentation of the progress in the theory of algebraic equations (in its leading features).

Triply orthogonal systems of surfaces.

Transcendental numbers, especially e and π .

Jacobi's functional determinants and their most important applications.

Method of derivation of large prime numbers.

(a) For major—Continued.

The series for $\tan x$ and $\sec x$ and the most important properties of their coefficients.

The significance of π in the calculus of probabilities and theory of errors.

Focal properties of surfaces of the second degree.

Algebraic treatment of the 27 lines on a surface of the third order.

Reye's complex.

Rational space curves of the fourth order.

Study of the representation by $w = \int \frac{dz}{\sqrt{(z-a)(z-b)(z-c)}}$

(b) For minor:

Solution of equations of the third and fourth degree.

Properties of the nine-point circle.

Theorems of Pascal and Brianchon and their proof in the case of the circle.

The general term of the Lamé series and the proposition with regard to the greatest common divisor of two expressions.¹

Elementary geometric treatment of the problem of tangencies of Apollonius.

A presentation of pages C₂-G of Girard's *Invention nouvelle en l'algèbre* (1629) in modern mathematical phraseology and notation.

The determination of the 15 Archimedean solids in terms of the radius of the circumscribed sphere.

Weierstrass's theory of irrational numbers.

The theorems of Fermat and Wilson.

In connection with the written and oral examinations, questions are usually asked on the theory of symmetric functions, the algebraic solution of equations of the first to the fourth degrees, and better candidates may also be questioned on the elements of the theory of groups and on the proof of the impossibility of the algebraic solution of equations of degree higher than the fourth. Questions on the calculus of variations are usually omitted. In general, clarity of perception and certainty in handling fundamental theorems are valued more than the extent of minute knowledge.

In the written examination there is also opportunity to indicate ability to apply theoretic knowledge to practical problems. Further, the examinations pay special attention to the subjects of the students' university lectures and seminary exercises. The written examination for a major lasts eight hours, two sessions of four hours on the same day; for a minor four hours are allowed.

In 1908-1910, 241 persons passed the professorship examination, with mathematics and physics as majors; 89 at Vienna, 80 at Prague, at Czernowitz, etc., only 1.

Comparatively few students in Austria proceed to the doctorate in mathematics—less than 50 in the last 45 years. Half of these were at Vienna.

In the examinations of candidates with mathematics as a minor—that is, of those who may expect to become teachers in the lower

¹This title refers to G. Lamé's "Note sur la limite du nombre des divisions dans la recherche du plus grand commun diviseur entre deux nombres entiers", *Comptes Rendus*, 1844, tome 19, pp. 867-870. The series of numbers 1, 1, 2, 3, 5, 8, 13, 21, . . . , which here comes up, is the recurring series often called the Fibonacci Series.

mathematical classes of the middle schools—the ground covered is usually that of the Maturitätsprüfung. Some examiners also demand spherical trigonometry and such things as the solution of an equation of the third degree or lay special emphasis on the development of the power of space perception.

It is felt very strongly by some Austrian educators that the lower standard for teachers in the Untergymnasien and Unterrealschulen is decidedly detrimental to the best interests of these institutions.

After a candidate for a professorship in a secondary school has passed the professorship examination, he has yet to undergo a "trial year" before he can be assigned to a post. At least this is true in theory; in practice the need for teachers has been so great that most of them have not had adequate professional preparation. Of over 200 candidates approved by the Vienna examination commission and supposed to be having a "trial year" in 1908-9, only one actually completed the work of the year.

This trial year is passed in a Mittelschulseminar, which is most successful when it is directly connected with a Gymnasium or Realschule. On entering the Seminar the student is placed under a certain professor who has charge of his development during the trial year. In the first few weeks the candidate visits classes which his professor teaches, and notes the methods employed and the personality of the pupils. The candidate next gives instruction with the aid or under the direction of the professor. At the beginning of the second semester the candidate instructs without aid for at least a month. Furthermore, all candidates have weekly conferences with the guiding professor and the director of the Seminar with reference to various questions regarding instruction, school discipline, pedagogy, school hygiene, and notable publications in pedagogic literature.

After 8 years of service professors in secondary schools are entitled to a small pension in case of need. The rate of pension gradually increases until after 30 years of service it amounts to full salary.

BIBLIOGRAPHY.

- F. BERGMANN, *Der mathematische Unterricht an den Realschulen*. K. KRAUS, *Der mathematische Unterricht an den Volks- und Bürgerschulen*. Berichte über den mathematischen Unterricht in Österreich, veranlasst durch die Internationale mathematische Unterrichtskommission Heft 1. Wien, A. Hölder, 1910, 6+81 pp.
- E. DINTZL, *Der mathematische Unterricht an den Gymnasien*. (I. M. U. K., Heft 3.) Wien, Hölder, 1910, 8+78 pp.
- A. ADLER, *Der Unterricht in der darstellenden Geometrie an den Realschulen und Realgymnasien*. E. MÜLLER, *Der Unterricht in der darstellenden Geometrie an den technischen Hochschulen*. (I. M. U. K., Heft 9.) Wien, Hölder, 1911, 124 pp.

* See also *L'Enseignement Mathématique*, tome 10, 1908, pp. 516-522; tome 12, 1910, pp. 326-341; tome 13, 1911, pp. 159-166, 237-243, 332-33; tome 15, 1913, pp. 79-84; par J. Renard.

- Syllabus of Mathematics for the Austrian Gymnasien.* (Educational Pamphlets, No. 22.) London, Board of Education, Eyre and Spottiswoode, 1910, 10 pp.
- E. CZUBER, *Der mathematische Unterricht an den technischen Hochschulen.* (I. M. U. K., Heft 5.) Wien, Hölder, 1910, 6+39 pp.
- R v STERNECK, *Der mathematische Unterricht an den Universitäten.* (I. M. U. K., Heft 7.) Wien, Hölder, 1911, 6+50 pp.
- T. KONRATH, *Der mathematische Unterricht an den Bildungsanstalten für Lehrer und Lehrerinnen.* (I. M. U. K., Heft 2.) Wien, Hölder, 1910, 27 pp.
- J. LOOS, *Die praktische Vorbildung für das höhere Lehramt in Österreich.* (I. M. U. K., Heft 4.) Wien, Hölder, 1910, 21 pp.
- A. HÖFLER, *Die neuesten Einrichtungen in Österreich für die Vorbildung der Mittelschullehrer in Mathematik, Philosophie und Pädagogik.* (I. M. U. K., Heft 12.) Wien, Hölder, 1912, 103 pp.
- Enzyklopädisches Handbuch der Erziehungskunde unter Mitwirkung von Gelehrten und Schulmännern ...* herausgegeben von J. Loos. 2 Bände. Wien und Leipzig, A. Pickers Witwe, 1906-08.
Articles: "Probefahr," "Hospitieren," and "Pädagogische Seminare."
- W. FRIES, *Die wissenschaftliche und praktische Vorbildung für das höhere Lehramt.* 2. umgearbeitete Auflage. München, Beck, 1910, 6+216 pp.
Band II, Abteilung I von *Handbuch der Erziehungs- und Unterrichtslehre für höhere Schulen ...* herausgegeben von A. Baumelster.
- E. MARTINAK, "Zur pädagogischen Vorbildung für das Lehramt am Mittelschulen." *Zeitschrift für die österreichischen Gymnasien*, 1904, 11. Heft.
- A. STITZ, "Die Ausbildung der Mittelschullehrerungskandidaten." *Mittelschule*, 1910, 4. Heft.

III. BELGIUM.

The area of Belgium is less than that of the States of Maryland and Delaware together, but the population is somewhat greater than that of the Dominion of Canada.

Education is controlled by the minister of sciences and arts, who has under him two general directors, one for primary and one for secondary and higher education. For secondary education the ministry also has an inspector general, nominated by the King, and two ordinary inspectors, one for the humanities, the other for mathematics and science. Authority is exercised over schools by the ministry in effective manner through control of the Government appropriations, appointment of teachers, regulation of programs, and prescription of textbooks.

SECONDARY SCHOOLS.

In Belgium the better secondary schools proper may be roughly divided into two classes, those supported by the Government and those maintained by the communes. The former are of two kinds: (a) the *Athénées Royaux* (royal athenaeums, called also higher middle schools); and (b) the Lower Middle Schools or Middle Schools. The communal secondary schools (*collèges communaux*) are mostly controlled by the church or religious orders. In 1912 they included 15 collèges, which ranked about as high as the athénées.

(a) The athénées royaux, 20 in number, are subject to official control under the direction of the King. In accordance with a decree of 1888 the courses in the athénées were arranged in three parallel divisions: (1) The *humanités grecques-latines*, with seven years of Latin and five years of Greek; (2) the *humanités latines*, with seven years of Latin, no Greek, and a very extensive course in mathematics; (3) the *humanités modernes*, where modern languages served as the basis for teaching during the seven years. The three higher classes of the *humanités modernes* comprise two sections, the scientific section and the commercial section.¹ The classes during the seven years of each of the divisions are numbered VII-I. Pupils entering VII are about 12 years of age² and have had the equivalent of 6 years of training in the primary schools.

¹ Note that the scheme is somewhat similar to that of the French lycées. In Germany these different types of instruction are given in different schools: the *Gymnasium*, the *Realgymnasium*, and the *Realschule*.

² The minimum age of admission to the athénées is 11 years, and an entrance examination must be passed.

The mathematical subjects taught in the athénées are arithmetic, algebra, plane and solid geometry, plane and spherical trigonometry, plane and analytic geometry, descriptive geometry, and surveying.

Number of hours per week devoted to mathematics and its applications in the different divisions.

	VII	VI	V	IV	III	II	I	Total
Greek-Latin humanities	3	3	3	3	3	3	3	21
Latin humanities	3	3	4	4	6	6	8	34
Modern humanities:								
(a) Scientific section	3	3	4	4	6	6	8	34
(b) Commercial section	3	3	4	4	3	3	2	22

The courses in ancient humanities bifurcate at the beginning of the third year, V.

In the first two years the mathematics is the same for all, and the programs for the Latin humanities and the scientific section of the modern humanities are identical. The commercial section differs from the scientific section in Classes III-I only. In Class III of the scientific section the following subjects are taught:

In *arithmetic*: General theory of divisibility of numbers, highest common divisor, least common multiple, theory of prime numbers, Fermat's Theorem, conversion of ordinary fractions into decimal fractions and reciprocally, numerical approximations, weights and measures, operations on complex numbers, cube root. In *algebra*: Discussion of the general equation of the first degree in one and two unknowns, complete discussion of the general equation of the second degree, properties of trinomials of the second degree, questions of maxima and minima, progressions, logarithms, use of tables, compound interest, and annuities. In *geometry*: Regular polygons, measure of the circle, determination of π , problems, notions on the theory of transversals. In *plane trigonometry*: Fundamental formulæ, identities, construction, and usage of trigonometric tables, solution of triangles. *Surveying and leveling*.

In Class I of the Latin humanities two of the eight hours a week are devoted to a thorough review of algebra, geometry, and trigonometry, with new applications of the theories. In the remaining six hours some of the topics taken up are:

Determinants of the second, third, and higher orders; elementary properties, application to the solution of a system of n equations of the first degree; in *spherical trigonometry*: Solution of triangles, spherical excess, radii of inscribed and circumscribed circles of a triangle, distance between two points on the earth's surface, volume of the parallelepiped and tetrahedron in terms of the edges and angles; in *analytic geometry*: Principal properties of points, lines, conics, conics as sections of a cone, intersection and similitude of two conics; in *descriptive geometry*: Introductory notions.

It may be added that the instruction in the athénée is maintained at a very high standard and carried out in such way as to arouse keen competition for honors and prizes. These are distributed as the outcome of three examinations each year for each class, the third being called the *Concours général*. A sample paper in the *Concours général* of 1910 for Class I in Latin humanities and scientific

section will give a further indication of the mathematical standards of the athénée:

I. *Analytic geometry*.¹—Given a rhombus $ABCD$ whose diagonals AC, BD , are, respectively, equal to $2a$ and $2b$, and intersect in O . (a) Form the general equation of the conics S whose conjugate diameters have the directions of the sides BA, BC , and which meet the diagonal AC in two points E and F such that, $OE \times OF = -a^2$. Show that the conics S pass through four fixed points and construct these points. (b) Find the locus of the poles of AC with respect to the conics S . Find also the locus of the points of contact of the tangents drawn to the same conics, parallel to AC . (c) Consider, in each of the conics S , the axes of symmetry l , the polar p of the vertex B , and the perpendicular d dropped from D on p . Find the locus of the points of section of d with l and construct this locus. II. *Descriptive geometry*.—(Given a line c , and the horizontal line a cutting the frontal line b in the point A . (a) Find on the line c the point S , equally distant from the sides of the acute angle formed by a and b . (b) From S drop the perpendiculars SD, SB on a and b , respectively. (c) Construct the quadrangular pyramid $S-ABCD$, which is found by cutting with the plane (a, b) the solid angle whose four edges are SA, SB, c, SD . (d) Give a representation of this pyramid, applying the conventions with respect to the parts of the drawing of projections of parts seen and hidden. III. Demonstrate that the six dihedrals of a tetrahedron satisfy the relation,

$$\begin{vmatrix} -1 & \cos(ab) & \cos(ac) & \cos(ad) \\ \cos(ba) & -1 & \cos(bc) & \cos(bd) \\ \cos(ca) & \cos(cb) & -1 & \cos(cd) \\ \cos(da) & \cos(db) & \cos(dc) & -1 \end{vmatrix} = 0,$$

where the four faces are denoted by a, b, c, d , and (pq) denotes the dihedral angle formed by the faces called p and q .

If the trihedral angle formed by the faces a, b, c is trirectangular, show that the preceding equality reduces to

$$\cos^2(ad) + \cos^2(bd) + \cos^2(cd) = 1.$$

The student who completes any one of the courses of instruction in an athénée and passes the final examination receives a *diplôme de sortie*, which admits him to the goal of his ambition, a university. It will admit him to any faculty. Only in the special schools must an applicant for admission, whether he has a *diplôme* or not, be examined on the program of the Latin or scientific sections.

(b) The State lower middle schools, of which there are about 90 for boys,² "were created by the Government to meet the needs of the higher artisan and commercial classes." Entering pupils for these schools and for the athénées have the same preparation. The courses of study are arranged so as to occupy three years. The obligatory courses are: French, Flemish, history, geography, arithmetic, algebra, geometry, zoology, botany, physics, chemistry, commerce, drawing, and gymnastics. As to mathematics, it corresponds approximately to what is taken up in the first four years at the

¹ For solution of these questions, see *Mathesis*, 1911, vol. 31, pp. 35-38, 61, 67.

² Mr. Rose seems to be incorrect in stating (p. 351) that there are only about 50 of these schools. *Statemen's Year-Book*, 1917, and Reports of the U. S. Commissioner of Education, 1913-14, etc.

athénées.¹ Pupils who have completed the course of a lower middle school are admitted to IV of an athénée without examination or to III after successfully passing an examination.

THE UNIVERSITIES.

There are no higher normal schools in Belgium, and except in very rare instances a candidate for a professorship in an athénée must have received the degree of doctor from a university.

There are four universities—two belonging to the State, at Ghent and Liege; the free university at Brussels; and the largest of all, the Roman Catholic university at Louvain.² Each of these universities has certain special schools or institutes connected with it. Perhaps the most famous of them is the technical school attached to the University of Liege. In each of the universities there are four faculties—philosophy, law, medicine, and sciences. It is in connection with the last-named faculty that the future professors of mathematics and professors of natural science are formed. On entering the faculty of science as students these candidates are required to present a *diplôme de sortie* from an athénée or a *collège*, or else to pass equivalent examinations either (1) before the faculty or (2) before a jury composed of professors of secondary teaching and appointed by the minister of sciences and arts. The students are usually graduates from the scientific section of an athénée.³

In addition to pure mathematics the future professor is required to study general physics and mathematical physics, rational mechanics, chemistry, and crystallography. The program also includes a course in psychology, logic, and moral philosophy, as well as in the history of mathematics.

Didactic preparation takes place at the same time as scientific preparation, each university possessing a special chair of mathematical methodology.

The scientific preparation extends over four years. During the first two years the student prepares to secure the certificate as *candidate in physical sciences and mathematics*. For three years the courses are the same for all the students of mathematics; in the fourth year each takes up, according to his tastes, one or other of these groups: Analysis (including differential geometry), higher geometry, astronomy and geodesy, rational mechanics and celestial mechanics, physics. The thesis for the doctorate is on a question related to the group chosen.

¹ Selected titles from the official list of mathematical texts used in the athénées and lower middle schools are given on pages 234-235 of the report of the subcommission.

² The buildings of this university were completely destroyed by the Germans Aug. 26, 1914.

³ To meritorious and needy students the State awards, on the basis of a *concours*, annual scholarships amounting to 400 francs. These scholarships may be received each year of the course. There is generally one such scholarship for the section of mathematics in each university.

The materials of the program, which are about the same for all universities, have been arranged in the following manner by Mr. Rose:

(a) PURE MATHEMATICS.

1. *Analysis—First year.*—Differential and integral calculus. Three hours a week. Limits; aggregates; derivatives and differentials; Taylor's and Maclaurin's theorems; explicit and implicit functions; change of variable; maximum and minimum; series; geometric applications of differential calculus to curves and to surfaces; integrals—processes of integration; various types of integrals; areas, surfaces, volumes.

Second year.—Definite integrals, differentiation and integration; Eulerian integrals; differential equations; integrable types; simultaneous differential equations; partial differential equations of the first order; total difference equations; calculus of differences and calculus of variations. Three hours a week.

Third year.—Theory of a complex variable; synectic functions; study of works of Abel, Cauchy, Riemann, Weierstrass, and their disciples. Theory of elliptic functions (after Legendre). Three hours a week.

Fourth year.—Six hours a week, and more for students working for their doctorate in analysis. Searching study of a topic in the theory of functions. Elliptic functions according to Jacobi and Weierstrass. Research in differential geometry based upon the work of Darboux¹ and Bianchi².

The masterly work of M. de la Vallée-Poussin³ gives a good idea of the subjects covered in the first two years.

2. *Analytic Geometry—First year.* Three hours a week. Revision of analytic geometry of two dimensions and study of analytic geometry of three dimensions. Particular study of homogeneous, tangential, triangular, and tetrahedral coordinates. Generation of surfaces. Surfaces of the second degree. For such work the notable treatise by Servais of the University of Ghent may be consulted.

Second year. Three hours a week. *Projective geometry:* Study of forms, involution, homography, homology, correlation, duality, polarity, properties and generation of conics, pencils, nets, generation of quadrics, properties. The texts of F. Follie,⁴ F. H. G. Deruyts, and of Servais⁵ illustrate the requirements.

For the pupils who specialize in geometry during their third and fourth years, the professor takes up either the theory of plane and cubic curves and of cubic surfaces, or the theory of forms in higher geometry. The number of hours per week depends on the professor. The works of F. Follie, F. H. G. Deruyts, M. Stuyvaert, Fairon,⁶ and L. Godeaux may be mentioned.

¹ G. Darboux: 1. *Leçons sur la théorie générale des surfaces*. 4 parties. Paris, Gauthier Villars. 1:2e éd., 1914, 7+618 pp. 2:2e éd., 1915, 3+579 pp. 3:1894, 8+512 pp. 4:1896, 8+548 pp. 11. *Leçons sur les systèmes orthogonaux et les coordonnées curvilignes*. 2e éd. Paris, Gauthier Villars, 1910. 3+567 pp.

² L. Bianchi, *Lezioni di geometria differenziale*. 3 vols. Pisa, Spoerri, 1902-1909 (2d edition of volumes 1-2). Second German edition by M. Lukat. Leipzig, Teubner, 1910. 18+721 pp.

³ C. J. de la Vallée-Poussin, *Cours d'analyse infinitésimale*. 2 tomes. Louvain, D'edonne, 1:3e éd. 1914; 2:2e éd. 1912. 9+452+9+464 pp.

⁴ F. J. P. Follie, *Éléments d'une théorie des faisceaux*. Bruxelles, Hayez, 1878. 110 pp. *Optométrie supérieure cartésienne*, Bruxelles, Hayez, 1872.

⁵ C. Servais: 1. *Sur les imaginaires en géométrie, application aux cubiques gauches*. Gand, 1894. 2. *Cours de géométrie analytique*. 2e éd. Gand, 1906.

⁶ J. Fairon, *Sur les involutions du quatrième ordre*. Bruxelles, Hayez, 1909, 68 pp.—M. Stuyvaert, "Recherches relatives aux coniques de l'espace" (1901) and "Étude de quelques surfaces algébriques engendrées par les courbes du deuxième ou du troisième ordre" (1902).—F. Deruyts, "Mémoire sur la théorie de l'involution et de homographie unicursale" (1890).

(a) PURE MATHEMATICS—Continued.

3. *Algebra*—Thorough study of determinants. General theory of equations; resolution and methods of approximation to the roots, study of imaginaries. Two hours a week in the first year. Texts: *Algebra* by J. Neuberg or by P. Mansion.
4. *Calculus of Probabilities*—Third year. One hour a week. Principles and problems; various species of probabilities. Bernoulli's theorem; theory of play; law of large numbers; theory of least squares; application to annuities and life insurance. Text by Boudin.¹

(b) APPLIED MATHEMATICS.

5. *Astronomy*—Second year. Physical astronomy. Three hours a week. Study of the system of the world, systems of coordinates, motions, sun, moon, planets, stars, comets. Applications of spherical trigonometry, elements of geodesy. Text: *Astronomy* (Collection L'auté) by Stroobant.²
- Third year. Three hours a week. Mathematical astronomy, application of analysis to astronomy, refraction, eclipses, calculation of orbits.
- In the fourth year the students make a thorough study of some branch of mathematical astronomy.
6. *Descriptive Geometry*—First year. Four hours a week. Review of the principles of the point, the line, and the plane. Study of trihedral angles, of curves and of surfaces, surfaces of the second degree and ruled surfaces, intersections, *géométrie cotée*. Treatises: By Chomé,³ Breithof,⁴ de Locht, Van Rysselberghe, and Charpis.
7. *Rational Mechanics*—First year. One hour a week. Vector geometry and statics: Forces, equilibrium, virtual velocities, funicular curves, machines.
- Second year. One hour a week. Kinematics: Velocity and acceleration, finite motion, instantaneous motion, continuous motion.
- Third year. Dynamics. Three hours a week. Study of the motion of free point, of a point on a surface or on a curve. Relative motion. D'Alembert's principle and the general equations of dynamics (Lagrange and Hamilton). Motions of a system. Rigid systems. Percussions. Attraction of ellipsoids. Mechanics of fluids; hydrodynamics. Text: The remarkable work by Massau.⁵
- Students who continue the study of mechanics during the fourth year take up equations of mechanics and the principal theories of celestial mechanics.
8. *Mathematical Physics*—Third year. Three hours a week. Study of the principal theories of optics, of magnetism, and of electricity.

(c) HISTORY OF MATHEMATICS.

Third year. Two hours a week. Mathematics in antiquity among the Egyptians, Assyrians, Chaldeans, Greeks, and Romans. Mathematics of the Hindus and Arabs. Middle ages.

Fourth year. Two hours a week. Renaissance, modern times, contemporary history, detailed study of each of the branches: Arithmetic, algebra, geometry, analysis, mechanics, physics.

¹ E. J. Boudin, *Leçons sur le calcul des probabilités*. The first edition was published anonymously in autograph form in 1865; 3e éd. (same as second). Gand, De Witte, 1889. Autographie, 123 pp.

² P. Stroobant, *Précis d'astronomie pratique*. Paris, Masson, 1903. 158+16 pp.

³ F. Chomé, 1. *Cours de géométrie descriptive*. 3e édition entièrement revue, corrigée et augmentée. 3 vols. Bruxelles, 1896-1904. 2. *Éléments de géométrie descriptive*. Bruxelles, Kouwenass, 1896. 12+150 pp.

⁴ N. Breithof, *Cours de géométrie descriptive: surfaces, courbes*. 2 vols. Louvain, 1875.

⁵ J. Massau, *Cours de mécanique*. 2 vols. Gand, Meyer Van Loo, 1892-1896. (Autographie.) 395+84+312 pp. *Leçons de mécanique rationnelle*, 2 tomes. Gand, L'association des ingénieurs sortis des écoles spéciales, 1911-1913. 15+259+17+343 pp.

(c) HISTORY OF MATHEMATICS—Continued.

10. *Elementary Mathematics.*

In the course on methodology (three hours a week, fourth year) the principles and foundations of such matters are considered. Review of the principal theories studied in the athénée with a view to practical lessons. Notions of higher arithmetic, of various kinds of geometry, of transcendental numbers. Text: Methodology, by Dauge.¹

(d) OTHER COURSES.

11. *General Physics—First year.* Four hours a week. Study of the principal theories of experimental physics. Treatises: By Schoentjes and Hben.
12. *Chemistry—Second year.* Four hours a week. General organic and inorganic. Treatises: By Swarts, A. F. Spring, P. Henry, Chivante.
13. *Psychology, Logic, and Moral Philosophy—Second year.* Three hours a week. The principal theories especially of logic.
14. *Crystallography—Second year.* Three hours a week. Systems, properties, representation. Types and properties of minerals. Treatises: By E. A. Stöber, G. Cosaro, Ledoux.

Students in a university have to pass annual examinations on each of the subjects of study during the year² before being admitted to the work of the following year. Having satisfactorily completed the first two years of work they receive diplomas as *candidats*.

The tests at the end of the fourth year include: (1) The presentation and public defense of a *thésis*; and, for those who are to become teachers, (2) the public delivery of two lessons, one on mathematics, the other on experimental physics. The subjects of these lessons are given in advance by the jury and are chosen from the program of the athénées. All tests having been successfully passed, the candidate becomes a doctor of physical sciences and mathematics.

The examinations occur each year in July and in October, and there are several grades of diplomas: With success, with distinction, with great distinction, and with greatest distinction.

PROFESSIONAL PREPARATION.

It is noteworthy that the program for the doctorate includes the elements of the history of mathematics and a course on methodology of the teaching of mathematics and of physics. This latter course deals equally with subjects taught in the athénée and with the methods of mathematics in general. The course lasts one or two years (third and fourth) and averages about three hours a week. The lessons are conducted by a university professor who has generally been a teacher in the secondary schools in his earlier career. They have a bearing on the methods of teaching each of the parts of the program of the athénée, and the professor usually expounds each of these subjects through the medium of the students themselves, aided by his counsel and advice. Each student gives before his fel-

¹ F. Dauge, *Cours de méthodologie mathématique*. 2^e [last] édition. Gand, Hoste, 1890. 10+525 pp.

² The pass mark is 50 per cent.

lows practical lessons in mathematics and physics. These are afterwards passed under critical review by professor and students.

While students are required to take a course in psychology, pedagogy is not taught at all systematically.

It is with such scientific and professional preparation that the future teacher in the better secondary schools enters upon his duties.

TEACHERS IN SECONDARY SCHOOLS.

In the athénées the teaching staff consists of an inspector of studies (*préfet des études*), professors, and masters (*surveillants*). The head of a lower middle school is called a *rector*. The inspectors, rectors, and professors are nominated by the King, and must each have secured the doctor's degree at a university. The masters, who are chosen from candidates, are appointed by the minister of sciences and arts.

In general the newly made doctor enters first either (1) as professor in a free school (*établissement libre*) or communal college; or (2) as temporary or permanent *surveillant*; or (3) as substitute professor in an athénée. After some years have passed in one or another of these capacities, he may be promoted to a chair in an athénée; but in many cases the doctor must proceed to this goal by way of the position as *surveillant*.

The mathematical chairs vary in attractiveness, according to the divisions: (A) Greek-Latin, (B) Latin, (C)-(D) modern humanities, with which they are connected. In establishments of secondary importance (averaging about 200 pupils) there are ordinarily three professors of mathematics, one for division (A) in VII and VI, the course being the same for the divisions (A) and (B); a second for the modern humanities VII, VI, V, and IV, and for (B) V and IV; finally a third for division (C): III, II, I scientific, and division (B): III, II, I. There is only one corresponding professor in each athénée; he is the *professeur de mathématiques supérieures*. So, also, there is always only one professor in division (A). On the other hand, the number of professors in the division of modern humanities may be two or three and sometimes four, according to the number of pupils (400 to 700).

But in any case as there are only 20 athénées, and a smaller number of similar ranking collèges communaux, the number of mathematical chairs is relatively limited.

The number of teaching hours per week varies from 18 to 21, according to the divisions.

The salary is composed of two parts: (1) A variable part, from the *minerval*,¹ which accrues from equal distribution among the professors² of fees paid by the pupils; and (2) a fixed part.

¹ This term is applied to the fee paid by the pupils for scholastic instruction.

² Professors of drawing, gymnastics, and music are excepted.

If the minerva part does not amount to at least 700 francs a year, the State makes up the deficiency. In larger athénées this part of the salary may range from 900 francs to 2,000 francs, or even more.

As to the fixed part of the salary, the initial amount is 2,600 francs. By periodic increments it may reach 5,500 francs in the following manner:

Francs.	Francs.
Initial salary, 2,600	After 15 years, 4,100
After 3 years, 2,900	After 19 years, 4,500
After 6 years, 3,200	After 23 years, 5,000
After 9 years, 3,500	After 27 years, 5,500
After 12 years, 3,800	

Surveillants commence with a salary of 2,200 francs, but have an increase of 200 francs every three years; the years passed as surveillant or as substitute teacher count in fixing the salary of the teacher, who finally becomes a professor.

In the collèges communaux the initial salary varies from 1,800 to 2,400 francs; the increases vary according to the schools, and the minerval is not distributed among the professors. The years spent in a collège communal are always counted toward promotion when a professor is called to an athénée.

At the head of each athénée is a préfet des études who does not teach and who has been chosen from among the professors, at least 40 years of age, in another establishment. Apart from the variable minerval the salary of a préfet ranges from a minimum of 4,400 francs to a maximum of 5,900 francs; he has also free residence, heat, and light.

The chairs at athénées of large cities are most sought after, because of the higher minerval and the attractions which large centers offer. As a general thing professors of mathematics start in division (A) or in division (D), and after some years pass to division (C) if they have acquired distinction by their professional aptitude and their publications. There is no definite rule concerning advancement, though the rule of seniority is ordinarily respected.

Every professor 60 years of age is retired with a pension. This pension may be obtained when he is 55 years old if he has taught for 30 years, or if he has had to give up work on account of disability. The basis of calculating the pension is the average salary, minerval included, for the last five years of service. The pension is $1/55$ of this amount for every year of service, including the four years of study. Thus a professor beginning with any title in secondary-school work at the age of 24, and pensioned at 60 years, counts first 36 years of service, then the 4 years at the university. He has then the right to $40/55$ of his average salary, say $(40/55)(5500 + 700) = (40/55) \times$

6,200, if he has taught in a school of ordinary importance. The pension must not exceed 7,500 francs.

As a rule Belgian professors of mathematics in secondary schools do not find time for scientific research. Those who do promote science by their publications may aspire to university chairs. Such has been the line of advancement of Profs. Neuberg and Fairon to the University of Liege, of Profs. Schoentjes, Servais, and Stuyvaert to the University of Ghent, and of Profs. Donder and Mathy to the University of Brussels.

To prepare *régents* or teachers for the State lower middle schools, the Government has instituted two normal schools at Nivelles and Ghent. Admission is gained (1) by examination, (2) after study in a primary normal school, or (3) after having completed the III or II in an *athénée*. The course of studies lasts three years. The students who prepare themselves for the scientific examination¹ specialize in mathematical studies in the second and third years. The program of such studies strongly resembles that of the scientific divisions of the *athénée*, except that the study of spherical trigonometry is replaced by that of mechanics. Pedagogy and methodology are studied in thorough fashion; during their whole course the future *régents* are required to give practical lessons to pupils of the "école d'application" connected with the normal school, and the final examination calls for two lessons—one in science and one in mathematics. The candidate who has passed all necessary examinations is qualified to become a *professeur agrégé de l'enseignement moyen du degré inférieur* or *régent de l'école moyenne*. Owing to their excessive number, only about one-half of such graduates eventually find a place in a State or communal school. They start ordinarily as *instituteurs* in a primary section connected with the lower middle school, and after a term of years are appointed as professors of lower middle schools. The salary of a *régent* varies from 2,100 francs to about 4,000 francs. Rectors get from 500 to 800 francs more. The *régent* is required to teach about 25 hours a week.

The professors in the normal schools are selected from among: (1) Doctors who have completed their university studies; (2) the best of the *régents*.

¹There are three types of courses organized to develop teachers for different groups of studies in the lower middle schools.

BIBLIOGRAPHY.

- DOCK, L. MONTFORT, et H. PLOUMEN, *Rapports sur l'enseignement des mathématiques du dessin et du travail manuel dans les écoles primaires, les écoles normales primaires, les écoles moyennes, les athénées et les collèges Belges.* (Commission internationale de l'enseignement mathématique, sous-commission Belge.) Bruxelles, J. Goemaere, 1911, 348 pp.

The three authors of the four reports which go to make up this volume are, respectively, inspectors of the normal primary schools, of teaching of drawing and design, and of secondary education.

- J. ROSE, "La préparation théorique et pratique des professeurs de mathématiques de l'enseignement secondaire en Belgique" *L'Enseignement Mathématique*, tome 18, 1916, pp. 335-352.

This interesting article was written in May, 1916, by an exiled teacher from the Athénée Charleroi.

- "Sur l'enseignement des mathématiques en Belgique," *L'Enseignement Mathématique*, tome 18, 1916, pp. 20-36.

IV. DENMARK.

Although Denmark is less than 16,000 square miles in extent, it has a population of close upon 3,000,000 people. Their educational system, which has always been closely associated with the Lutheran Church, ranks high. At present church control is merely nominal, although "both the bishops and clergy serve, as members of school committees ex officio, and aid in the selection of teachers and in the administration of the schools." But apart from this the schools are under civil control.

The minister of ecclesiastical affairs and public instruction is at the head of the whole educational system, including the university. In each of the 18 counties of Denmark he delegates certain duties to the school council or *skoleraad*; and to each of 60 districts is given control, within certain limits, of such things as the appointment of teachers, arrangement of courses of study, and selection of texts.

The system of Danish elementary and secondary schools was newly organized by enactments of 1903, which did not come into complete operation until 1910. As one result much greater coordination between the branches of education was brought about. It is now possible to find a connected course leading from the primary schools to the university. Secondary education proper begins in the Mellemskole or middle school, which the pupil may enter at the age of 11 years. The regular course lasts four years.¹ Those who satisfactorily complete this course may pass on to the Gymnasium. Here, as in Sweden, the pupil must elect to follow one of three parallel lines of study which he will pursue during the three years of the course. These lines are: *Mathematics-science*, with neither Latin nor Greek; *modern languages*, with Latin, but no Greek; *ancient languages*. When he completes the work in any one of these sections the pupil, who is then about 18 years of age, presents himself for the "student's examination." A certificate that he has passed this examination is sufficient to admit the student to the university; indeed, the university grants to every such student a "letter of academic citizenship."

In the mathematics-science section of the Gymnasium mathematics is taught for six periods a week during each of the three years,

¹ An extra year is added for those who wish to prepare for the Realexamen, which is accepted as an entrance standard for middle professional schools.

as compared with two periods a week each year in the other sections. The topics taken up are as follows:

In Arithmetic and Algebra: General equation of second degree; maxima and minima; infinitely great and infinitely small quantities; symmetric equations; equations of higher degree in two unknowns (it is shown by examples in connection with the solution of such equations that roots may be lost while extraneous roots are introduced); theory of exponents; calculation with irrational quantities; theory and practice of logarithms; arithmetic, geometric and harmonic series; permutations and combinations; binomial theorem with positive index; interest and annuities; complex numbers; prime numbers; proof that a number can be broken up into prime factors in only one way; algebraic equations; the expression of the coefficients in terms of the roots; the cyclotomic equation.

In Plane Geometry: Proof of the theorem of proportionality of the sides of two similar triangles; general proof of the theorem on the area of a right triangle; general theory of similitude with applications to simple construction problems; regular polygons; division of circle into 4, 6, 10, and 15 parts and calculation of corresponding chords; length of the circumference of a circle and its arcs; area of a circle and of circular sectors; the trigonometric functions of acute and obtuse angles with simple applications to solution of triangles; application of rectangular coordinates to graphic representation of simple functions (e. g., ax , ax^2 , ax^2+bx+c , $a:x$) for special values of the coefficients; various loci involving proportion; harmonic ranges and pencils; applications to construction problems.

In Trigonometry: Trigonometric functions of any angles; formulæ for the functions of the sum, or the difference of two angles, and for functions of double and half of an angle; limit of $\frac{\sin x}{x}$ for $x \doteq 0$; solution of simple trigonometric equations; logarithms and solution of triangles.

In Solid Geometry: Principal propositions on lines and planes; convex polyhedral angles; the rectangular trihedron and the determination of a point in space by rectangular coordinates; polyhedra with proof that there are not more than five species of regular convex polyhedra and that the tetrahedron, cube, and octahedron exist; cylinder and cone; the fundamental spherical formulæ and their application to the right spherical triangle; congruence, symmetry, and similitude; area of curved surfaces, such as of the cylinder of revolution, cone of revolution, sphere; volumes of prisms, pyramids, truncated pyramids, cones, spheres, and sections of spheres; proof that plane sections of a cone of revolution may be ellipses, hyperbolas, or parabolas. In the instruction especial emphasis is laid on the development of space perception.

In Analytic Geometry: Determination of points and curves by rectangular and polar coordinates; the most important formulæ for the equations of the straight line and circle; parabola, ellipse, hyperbola; tangents, normals, asymptotes, diameters.

In addition to these courses one of the following, A or B, is given:

A. Arithmetic and Algebra: Determinants with applications to linear equations; continued fractions with applications to calculation of irrational square roots and to the solution of indeterminate linear equations. *Analytic Geometry:* Discussion of the general equation of the second degree in two variables. *Solid Geometry:* Icosahedron and dodecahedron; representation of a simple polyhedron by orthogonal projection on two planes at right angles; plane sections of these bodies.

B. Infinitesimal Calculus: Infinitesimals; continuous and discontinuous functions; derivatives of x^n (n rational), of trigonometric functions, of sums, products, and quotients, of a function of a function; Rolle's theorem; maxima and minima; Taylor's series for integral functions; definite and indefinite integrals; integration by parts; simple applications to geometry and to physical problems.

MATHEMATICS AT THE UNIVERSITY.

The university course for the scientific training of the teacher in the secondary schools ends with the "Skoleembedsexamen (teachers' examination). According to regulations of 1906, when a candidate presents himself for this examination with either mathematics (or physics) as a major, he must also present as minors astronomy with applied mathematics, and chemistry with physics (or mathematics). The examination consists of two parts. In the second part the examination is on the major only. Before taking this, indeed at the end of the first year, the candidate must pass a university examination, called the "Filosofikum," in logic, elements of the history of philosophy, and psychology.

In the first part of the examination the mathematical subjects are: Analytic geometry of the plane and of space; algebra; differential and integral calculus, including the theory of infinite series; differential equations with a single independent variable; total and linear partial differential equations in three variables; application of analysis to geometry; statics, kinetics, and hydrostatics; advanced portions of gymnasial mathematics from a higher point of view. For astronomy, in connection with applied mathematics are required: Theory of interpolation; facility in numerical calculation, especially in the use of tables and ephemerides, as well as in the numerical solution of equations.

For the second part of the examination with mathematics as major, candidates must be prepared to answer questions in the following: Function theory and elementary number theory; methods of descriptive geometry; projective geometry in synthetic and analytic presentation; more thorough treatment of kinematics and kinetics; special study of some part of mathematics; the history of mathematics (in connection with which the candidate must make himself familiar with Euclid's Elements and Descartes's Geometry),¹ and either with the complete development of a single branch of mathematics or with the whole field of mathematics in a given period.

Both parts of the examinations are oral and written; the oral are public; the written examination of the first part lasts four hours and the special problem of the second part ten hours. The candidates who have passed the Filosofikum and Skoleembedsexamen are called *candidati philosophiae* and *candidati magisterii*, respectively.

The degree of doctor of philosophy may be won by any *candidatus magisterii* who has received the highest grade and who has prepared

¹ To every mathematician familiar with Greek mathematics or with the history of mathematics the names of Heiberg and Zeuthen, professors at the University of Copenhagen, are well known. Their influence is doubtless observable in these unusual requirements.

a satisfactory thesis which has been printed. Since the degree of doctor gives the *jus docendi*, the thesis may be regarded as a *Habilitationschrift*. Within the past 100 years less than two score of these doctors have been created.

THE TEACHER IN THE SECONDARY SCHOOL.

Since 1908 everyone who has been appointed as teacher in a complete State secondary school is a *candidatus magisterii* and has had two years of professional training. The first year has been normally spent at the Pedagogic Seminar established in 1906 and maintained by the State. Before entering upon the work of this seminary each candidate must, as a rule, hold a degree from the university. During the first semester he receives theoretical instruction which includes: The history and principles of education and methods of teaching; a study of the development and present organization of Danish education; school hygiene, including the physiology and hygiene of adolescence. Professors of university rank are in charge of the instruction. The examinations covering the work of the course are both written and oral. During the second semester the candidate is occupied in practical work under the direction of the inspector of candidates for teaching positions in gymnasia. He is given special training in the teaching of his two special subjects. At first he listens only, then instructs in the presence of the teacher or the school director or the inspector. The day's work closes with critical discussion. The candidate's work of the semester ends with a preliminary examination which consists of two hours of teaching in his major subject and one in his minor, in the presence of his adviser, the headmaster of the school, and the State inspector of complete secondary schools. After a further year of activity as assistant or regular teacher in a State or private secondary school the candidate must take his final examination.

This form of training may be omitted "if a candidate has worked at least two years at a school and has his skill in teaching tested by an *examenskommission* consisting of three experts appointed by the ministry for that purpose." This last method is followed by most candidates for positions in the secondary schools. The private schools in Denmark, as in Norway, have been the training ground for the teachers in the public schools.

The salaries are low, in general, even for Denmark. The maximum salaries range from 2,400 krone (\$646) for assistant teachers to 5,000 krone (\$1,346) for principals, but a residence is also provided for the principal.

This sketch may be appropriately concluded by a sample of the examination problems in the *skoleembedsexamen*:

FIRST PART.

I. (4 hours.)

Give a presentation of the theory of pole and polars for the circle $x^2 + y^2 = r^2$. Determine the geometric locus of the pole of the tangents to an ellipse with respect to a circle whose middle point is a focus of the ellipse.

II. (4 hours.)

(1) Prove that the two infinite series

$$A = \sum_{n=1}^{n=\infty} \frac{1}{n^2(n+1)^2}, \quad B = \sum_{n=1}^{n=\infty} \frac{1}{n^2}$$

satisfy the condition $A + 6B = 10$.

(2) Given that $x \neq 0$ and $a \neq 0$, and $F(x) = x^n + a^n - (x+a)^n$, $f(x) = x^2 + ax + a^2$. Prove that $F(x)$ is divisible by $f(x)$ or $(f(x))^2$ according as $n = 6m - 1$ or $n = 6m + 1$; m is a positive integer.

(3) Let $f(x)$ be a rational integral function of x , let a and b be constants and $a \neq 0$. Prove that the differential equation

$$(ax + b) \frac{dy}{dx} + \frac{a}{2}y = f(x)$$

has one and only one particular integral which is a rational integral function of x ; and give a method for determining this particular integral.

III. (4 hours.)

(1) On the arc of a curve is a fixed point O and any point M . The length of arc $OM = s$ can assume the values from $-a$ to $+a$, and the radius of curvature in M is given by $\rho = \sqrt{a^2 - s^2}$. With rectangular coordinate system having origin at O and x -axis coincident with the tangent to the curve at O , find the coordinates of M expressed in terms of the angle between the x -axis and the tangent at M .

(2) Determine the coordinates of the center of gravity of the arc of the catenary (regarded as a material homogeneous line),

$$y = \frac{a}{2} \left(e^{-\frac{x}{a}} + e^{\frac{x}{a}} \right),$$

which extends from the intersection with the y -axis to a point with the abscissa x . Show that the center of gravity has the same abscissa as the intersection of the tangents at the ends of the arc, and an ordinate which is half of the ordinate of the point of intersection of the normals at the end points of the arc.

SECOND PART.

I. (4 hours.)

(1) If the power series $\sum a_n x^n$ has the radius of convergence $r > 0$, the region of convergence of the series $f(x) = \sum a_n (x^2 + x)^n$ is determined. Seek the nature of convergence of this series and show that the sum of the series for $r > \frac{1}{2}$ satisfies the condition $f(-x-1) = f(x)$, and ascertain the series derived from the identity $(1+2x)^2 = (1+4(x^2+x))^2$ in the above form.

(2) The point $(1, 1, 1)$ in trilinear coordinates is projected from the three angular points a, b , and c of the fundamental triangle on the opposite sides to the points p, q , and r . A conic, K_1 , is tangent to ac in q and ab in r and cuts pr again in u and pq in v . A second conic K_2 is tangent to ba in r and bc in p and cuts qr again in y , pq in z ; z is the point of intersection of the lines uv and yz . What is the geometric locus of z if K_1 and K_2 vary in such a way that v and z are harmonically conjugate to p and q .

ii. (4 hours.)

(1) Given that $|x| \neq 1$, and that the positive integer n increases without limit. Determine the limits of

$$A_n = \frac{1}{n} \log(x^{2n} - 1)$$

and

$$B_n = \frac{1}{n} \sum_{p=1}^{p=n-1} \log(1 - 2x \cos \frac{p\pi}{n} + x^2),$$

and write the last as a definite integral.

(2) Determine the radius of convergence of the power-series

$$1 + \sum_{n=1}^{\infty} \binom{x^n}{n+2n}$$

and discuss the behavior of the series on the periphery of the circle of convergence. If for special values of x some of the binomial coefficients in the above series are zero, the corresponding members of the series are to be left out.

(3) Show that $x=3, y=11$ are the only positive integral values of x and y which satisfy the indeterminate equation

$$1 + x + x^2 + x^3 + x^4 = y^2.$$

iii. (4 hours.)

A rectangular coordinate system (X, Y, Z) in space is turned with an angular velocity ω about the Z -axis which is vertical. A straight, material, homogeneous rod, whose thickness is neglected, of length $2l$ and mass M , is so placed that its end points a and b move without friction along the X -axis and the Y -axis. At a certain instant a is at the origin and has the velocity $2l\omega$ in the direction of the X -axis, while b has a positive Y -coordinate. Determine the angular velocity of the rod in its relative motion with respect to the system (X, Y, Z) at a given instant; also determine the relative, as well as the absolute motion of the end and of the middle point of the rod. If the system (X, Y, Z) has turned through 45° and is suddenly stopped, the rod will move further, since a and b slide without friction on the X -axis and the Y -axis. Determine the angular velocity of the rod immediately after the impulse.

iv. Course problem (10 hours.)

Discuss figured numbers, especially polygonal numbers and their application to the representation of other numbers.

Heegaard gives a list of works used by candidates in preparing for such examinations.

BIBLIOGRAPHY.

- P. HEEGAARD, *Der Mathematikunterricht in Dänemark* (I. M. U. K.). København, Gyldendalske Boghandel, 1912. 107 pp.
- A. ROHRBERG, *Der Mathematische Unterricht in Dänemark* (Berichte und Mitteilungen veranlasst durch die I. M. U. K., Zweite Folge, I), Leipzig u. Berlin, Teubner, 1915. 8+69 pp.
- J. S. THORNTON, *Schools Public and Private in the North of Europe*, London, Board of Education, *Special Reports*, 1907, vol. 17, pp: 66-136.
- M. HEGLAND, *The Danish People's High School, including a general Account of the Educational System of Denmark*. (Bureau of Education, Bulletin, 1915, No. 45.) Washington, 1915. 182 pp.

After an elementary school course of eight years, there follows a four-year continuation school preparing for the two-year course in the "people's high school." This school is not properly classed with secondary schools. The normal age of pupils entering it is 18.

V. ENGLAND.

It has been well remarked that—

few nations show the influence of so many different forces in their educational history as may be recognized in that of England—the church, the state, economic conditions, private enterprise, philanthropic endeavor, educational theories—all have contributed some tradition to what is gradually developing into a well-defined system.

By act of 1899 a central Board of Education was created. This consists of a president and various State representatives, the chancellor of the exchequer, and the secretary of the treasury. The act also provided for a committee of 18 members (15 men and 3 women) to act in an advisory capacity to the board. The number of members in this committee was increased to 21 in 1907.

The board is divided into four departments: (1) Elementary education; (2) higher education; (3) technical education; (4) university education.

SECONDARY SCHOOLS.

The secretary of "higher education" has 3 assistant secretaries and 15 regular district inspectors under him; there are also 27 part-time inspectors.

A secondary school as defined by the board, is a school which—
offers to each of its pupils a progressive course of instruction (with the requisite organization, curriculum, teaching staff, and equipment) in the subjects necessary to a good general education, upon lines suitable for pupils of an age-range at least as wide as from 12 to 17. This definition, which applies both to those schools recognized for grant² and to those which, though not in receipt of grant, are placed by the board on their list of efficient schools,³ determines the minimum requirements upon which the board must insist.

¹ The term "higher education" as employed here refers to the work of the secondary schools and of those schools, such as evening schools, which give instruction "higher" than "elementary." Cf. note 3, page 78.

² To secondary schools that meet certain specified conditions (including the provision for free instruction of a certain number of pupils coming from the elementary schools), annual grants are paid, as follows: For each pupil between 10 and 12 years of age, and who, for two years immediately before entering the secondary school, had attended a public elementary school, £2. For each pupil 12 to 18 years of age, £5. An additional grant of £1 for each pupil 12 to 18 years of age in a school which satisfies the following conditions: (1) Provides for the preliminary education of elementary school teachers as burners or has a pupil-teacher center forming an integral part of the school; (2) has offered not less than 25 per cent of free places.

Extra grants are made on the basis of certain other considerations. (A. T. Smith, in *Cyclopedia of Education*, edited by Monroe.)

In 1909-10 the grants to secondary schools and for the training of secondary school teachers amounted to £791,250.

³ On July 31, 1913, there were 1,008 schools on the board of education list of "efficient" secondary schools for boys and girls. At these schools there were, in 1913, 179,058 pupils (boys, about 54 per cent). Of this total more than one-fourth were on the free basis.

Those cardinal subjects which must be taught in every such school are: English language and literature, at least one language other than English, geography, history, mathematics, science, and drawing. All "efficient" schools must be open for inspection by the board at all times.

While the "list" has a positive value as guaranteeing that the schools included in it have been found by the board to be efficient, no conclusions should be drawn from the absence of the name of any particular school. There are many schools of high, indeed of the highest, efficiency which have not applied for inspection, and which are therefore not included in the "list." Such, for example, are the following nine "great public schools," known throughout the world as the schools patronized by rich and noble families, and all founded well over 300 years ago: Winchester (founded in 1384), Eton (1440), Westminster (1560), Rugby (1567), Harrow (1571), Charterhouse (1611), Shrewsbury (1551), which are boarding schools; and the day schools, St. Paul's School (1509) and Merchant Taylors' School (1561).

The authority of the board of education has steadily increased since its creation in 1899. This authority, however, bears no resemblance to that centralized in the French ministry of public instruction.

It rests upon the voluntary assent of civic or institutional authorities desirous of sharing in the treasury grants or of promoting unity of aim and economy of resources through a national agency. All Government measures are closely scrutinized by local authorities intolerant of any encroachments upon their rights, and are subjects of keen analysis and criticism by the numerous educational associations for which England is noted. Apart from their mastery of professional problems, these associations exercise great influence either by their political affiliations, or by their effective organization of popular opinion.

The educational system, like the national life of England, not only progresses by compromise, but holds to what is enduring by a marvelous system of checks and counter checks.¹

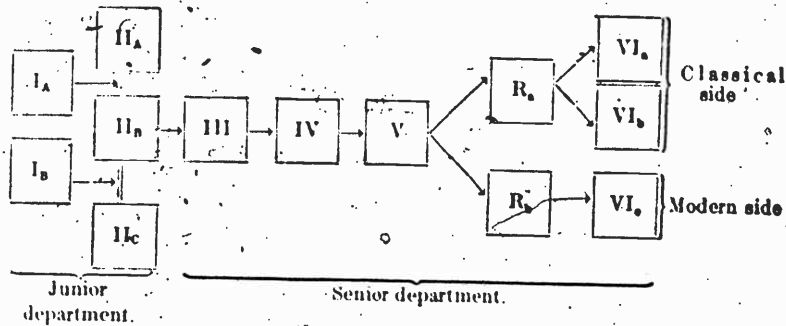
This must suffice to indicate general relations between the board of education and secondary schools. A wide range of designation is used for these schools—for example, Liverpool Institute, Eton College, and Callington County School—and their organization differs greatly.

In the schools with simpler organization there is usually a six-year course. Corresponding to these years are "Forms" numbered I, II, III, IV, V, and VI, but in this there is great variation of usage. I and II are in the "Junior department," III-VI in the "Senior department." In large schools where there is more than one class in the same subject the better pupils are often put in one class and the less advanced in another. Furthermore, there is sometimes bifurcation through election on the part of students of the "classical side," with Latin and Greek; as opposed to the "modern side," with French

¹Anna T. Smith, in Monroe's *Cyclopedia of Education*, vol. 2, 1911.

and German. Occasionally one finds the "science side" and the "Woolwich side" (as at Harrow) with emphasis on mathematics and natural science. But no single scheme can be indicated which would give an approximately definite idea of any large group of schools. Some particulars may, however, be given of two "efficient" schools: (1) The Liverpool Institute, with an annual attendance of about 500 boys, and (2) the Bradford Grammar School; and of a certain ideal school.

At the Institute the forms may be arranged in schematic array as follows:



We find here after V a "Remove" form (R), often called "Shell." Since Ia and Ib are the same, the better pupils can be put in one group and the less advanced in the other. The subjects studied and the distribution of hours is displayed in the annexed table:

Subjects.	Junior department.					Senior department.									
	Ia	Ib	IIa	IIb	IIc	III	IV	V	Ra	Rb	VIa	VIb	VIc		
Scripture.....	1	1													
Latin.....			3	3	3	3	4	5	5	5	4	5	5	5	
Greek.....									3	3	4	5	5	5	
French.....						3	3	3	3	3	4	5	5	5	
German.....										3	4	5	5	5	
Geography.....	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
History.....	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
Literature.....	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
Composition.....	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
Grammar.....	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
Reading.....	2	2	3	3	3	3	3	3	3	3	3	3	3	3	
Writing and spelling.....	3	3	3	3	3	3	3	3	3	3	3	3	3	3	
Mathematics.....	4	4	6	5	5	5	6	6	6	6	4	5	5	6	
Physics.....						3	3	3	3	3	4	4	4	3	
Chemistry.....											3	4	4	3	
Drawing.....	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
Workshop.....															
Singing.....															
Drill.....															
Hours per week.	24	24	27	27	26	27	27	27	27	27	(1)	27	27	27	

¹ Each boy's time table in upper sixth is made up to 27 hours by electives.



Norwood and Hope planned courses in ideal schools preparing for business life and for entry into a university.¹ The following is the scheme of work for the latter:

Ideal school scheme of work leading to university matriculation.

Subjects.	Preparatory.		Lower course.				Classical special-ists.			Other special-ists.		
	Age (about).		Age (about).				Age (about).			Age (about).		
	10	11	12	13	14	15	16	17	18	16	17	18
Religion.....	1	1	1	1	1	1	1	1	1	1	1	1
English.....	6	6	3	3	2	2	3	2	2	3	2	2
Latin.....			6	6	6	6	6	6	6	3	3	3
Greek.....					6	6	6	6	6	3	3	3
French.....	6	6	5	5	3	3	3	3	3	4	4	4
German.....					6	6	6	6	6	3	3	3
History.....	3	3	2	2	2	2	3	3	3	4	4	4
Geography.....	2	2	2	2	2	2	3	3	3	3	3	3
Mathematics.....	6	6	6	6	5	5	5	5	2	14	16	17
Natural science.....					3	3	3	3				
Botany and zoology.....	3	3	3	3								
Writing.....	1	1										
Drawing.....	2	2	2	2	2	2						
Workshop.....	2	2	2	2	2	2						
Class hours per week.	32	32	32	32	32	32	32	32	32	32	32	32

¹ Sunday service is also to be attended.
² Either Greek or German is selected.
³ In the upper classes German rather than French is usually elected.
⁴ These hours may be divided at will between these subjects.

The other "efficient" school, namely, the Bradford Grammar School, with about 200 pupils, has a somewhat different organization, and its course in mathematics is among the best in England. We have here a "classical side" and a "modern side." In the former the course lasts nine years, in the latter eight. "V" is for the seventh year "classical" and sixth year "modern"; "Remove classical" or "Matricu-

¹ While some schools are especially well equipped to prepare pupils for university matriculation, it should be noted that such preparation is not now unusual in any secondary school, even though there be only an occasional pupil to be so prepared. But under new regulations of the bill not yet an act an attempt is to be made to constitute special departments in the best schools of a district, to which promising boys will be transferred from their own schools. In connection with its discussion of the relations of the secondary school to the university, the board of education laid down the following principles, among others, in 1913:

"The specialization proper to the upper part of a secondary school is to be distinguished from the specialization which is natural and proper to a university, and it is the duty of the board to secure that the higher work done in schools, while constituting a proper preparation for university work, does not anticipate it either in the methods of study or in the nature of the curriculum.

"Candidates for degrees in mathematics or science will from the time they enter the university generally devote the whole of their time to the study of mathematics or of one or more branches of science. While at school their work is not (as too often happens) limited in any way. Those who propose to study natural science should continue to give an adequate amount of time to the study of mathematics, and those who wish to be mathematicians should, similarly, continue to do work (including practical work in the laboratory) at some branch or branches of natural science. The board also consider that those who will afterwards be entirely occupied with mathematical and scientific work should, so long as they are at school, continue to give a substantial amount of time to literary work. A thorough proficiency in the use of the English language and a good acquaintance with other languages will be in later years of greater value to them than the small amount of additional specialized knowledge which, by neglecting those subjects, they might acquire at school. It is, moreover, of great importance that by the continued study of selected masterpieces (whether humanistic or scientific in content) they should train their minds to deal with the more general aspects of human thought."

lation" for eighth year "classical"; "Matriculation" or "Remove science and mathematics" for seventh year "modern"; Sixth classical for ninth year "classical" and "Sixth science" or "Sixth mathematical" for eighth year "modern." The following is a synopsis of mathematical work.

Preparatory—Arithmetic: Elements.

I—Arithmetic: Use of decimal numbers and fractions; factoring; highest common factor and least common multiple; use of brackets.

Algebra: Introduction to algebraic symbolism.

Geometry: Fundamental concepts of geometry, such as line, point, direction, area, triangle, solids. Intuitive and practical introduction.

II—Arithmetic: Repeating numbers; coins, weights, and measures; profit and loss; simple interest.

Algebra: Very simple equations to fix the idea of algebraic symbolism.

Geometry: Construction and mensuration of bodies; simple division of bodies.

III—Arithmetic: Ordinary and decimal fractions; rule of three; square root; percentages.

Algebra: The four fundamental operations with applications; equations of the first degree; graphs.

Geometry: Angle, triangle, parallels; the simple bodies; exact drawing and measuring; simple exercises.

IV—Arithmetic: Harder problems in fractions; change of ordinary fractions into decimal numbers and conversely; abbreviated calculations with decimal numbers, and simple interest calculation; rebates and discount; logarithms.

Algebra: Breaking up of sums into factors; simple quadratic equations.

Geometry: Through propositions on the circle (about equivalent to Books 1-3 of Euclid's Elements).

Trigonometry: To the solution of right-angled triangles.

V—Arithmetic: Bank, rebate, and discount calculation; stocks and shares; interest and annuities.

Algebra: Quadratic equations and problems; theory of indices; logarithms; series; permutations and combinations.

Geometry: To similar figures (Euclid's Elements I-V).

Trigonometry: Logarithms; measurement of a calculation of heights and distances; solution of triangles.

Matriculation—Arithmetic: General review.

Algebra: Quadratic equations; arithmetic and geometric series; calculation of roots; proportion.

Geometry: Through theory of similitude (Euclid's Elements, I-VI).

Elementary mechanics and hydrostatics.

Remove science and mathematics.

Algebra: Theory of indices; logarithms; equations; series; graphs.

Geometry: Through theory of similitude.

Trigonometry: Solution of triangles; goniometry.

Mechanics and hydrostatics.

Remove classical.

Algebra: Fractions; highest common factor; least common multiple; square root; equations of first and second degree; logarithms; proportion; series; graphs.

Geometry: Through theory of similitude.

¹ With regard to solid geometry in English secondary school programs, reference may be given to a report in the *Mathematical Gazette*, January, 1914, vol. 7, p. 222.

Sixth mathematics.

Theory of equations; plane trigonometry; statics; dynamics; hydrostatics; synthetic and analytic geometry; modern geometry; differential and integral calculus.¹

Sixth science.

Trigonometry; statics; dynamics; synthetic and analytic geometry; differential and integral calculus.¹

But while such extensive mathematical courses as these may be found in secondary schools, there are many schools where the mathematics includes only arithmetic, algebra to "progressions," and geometry equivalent to Books I-IV of Euclid's Elements. In some schools, also, neither the algebra nor the geometry is even so extensive as this. In other schools permutations and the binomial theorem for a positive integral index and the equivalent of Book VI of Euclid's Elements (proportion and similitude) are added; but the last subject,

interesting and important as it is, is too often wholly omitted because it is not included in the syllabus for the London matriculation examination, and therefore is actually discouraged in those schools (still too numerous) where it is regarded as dangerous to go a hair's breadth beyond the examination syllabus. It is similarly discouraged in those schools which use the examination of the Oxford and Cambridge joint board.²

The programs of studies in secondary schools are largely determined by the universities. This occurs through the influence not only of matriculation examinations, but also of such examinations as those of "joint boards," of the "Oxford local examinations," and of the "Cambridge local examinations."³ These latter (local) examinations are of three main types:

1. Preliminary (for pupils 12-14 years of age);
2. Junior or lower (for pupils 14-16 years of age);
3. Senior or school or leaving examinations (for pupils 16-19 years of age).

This scheme of examinations was established well over half a century ago, but not many years had passed before a standard examination corresponding to university matriculation was demanded, and to meet this demand the Oxford and Cambridge joint board was called into existence.

¹ For an interesting report by C. Godfrey on Calculus in the Public and Secondary Schools of England, see *Mathematical Gazette*, January, 1914, vol. 7, pp. 233-240.

² That is, the board of examiners for matriculation into Oxford and Cambridge Universities. A similar board is organized for the Universities of Leeds and Sheffield, Victoria University, and the University of Liverpool. The mathematical requirements here are (the University of Leeds calendar 1913-14, pp. 154-155): "Arithmetic. The elementary geometry of triangles, parallelograms, and circles, and of similar rectilinear figures. Algebra, including quadratic equations, with the arithmetical and geometrical progressions and an elementary treatment of irrational quantities and of proportion."

³ These examinations under the direction of Oxford and Cambridge Universities are referred to as "local" because they take place at the schools, or other convenient centers, and not at the universities.

This board conducts examinations for three certificates:

1. Lower certificate.
2. School certificate,

and a third certificate to be referred to later.

Corresponding to the matriculation examinations of the University of London¹ are the "responsions" at Oxford and the "previous examination" or "little go" of Cambridge University. Broadly speaking these examinations, the Oxford senior local, the Cambridge senior local, and the Oxford and Cambridge school certificate have the same value. Some characteristics of the mathematical parts of these types of examinations may be noted by studying the papers given in Appendix A. Into these characteristics I shall not go further than to remark that all examinations are written, and successful candidates are rated as "pass" or "honor" (first or second).

Sometimes to attain "honor," questions on "additional mathematics" and of a more difficult nature must be answered.

But there are yet other examinations for which many secondary schools prepare pupils. These are for the entrance scholarship examinations in various universities of the country.

The entrance scholarship examinations at the Oxford and Cambridge colleges are conducted on practically identical systems and differ but slightly in detail. At a rough estimate, the Cambridge colleges award 50 or more scholarships, exhibitions, and sizarships every year for proficiency in mathematics. Their value varies from £30 to £80 per annum. The Oxford colleges annually award for mathematics about 20 scholarships of £80 and 10 exhibitions of from £30 to £60. Most of these are tenable for two years, but they are renewed for one, two, or three more years, subject to satisfactory progress being made. The age limit at the time of examination is 19. The subjects of examination in both universities are: Analytic and synthetic geometry, algebra, trigonometry, differential and integral calculus, and mechanics.

Papers set at Cambridge in 1910 are given in Appendix B of this bulletin.

The preparation for the entrance scholarship examinations is similar to that for the "higher" examinations, the passing of which confers certain rights in university and college. Among such examinations are the Oxford higher local, the Cambridge higher local, and that for the higher certificate of the Oxford and Cambridge schools examination board.²

¹ Examination papers for 1916 are to be found in Appendix A. The following works may be consulted in this connection:

1. *Matriculation mathematics papers*. Being the papers in elementary mathematics set at the matriculation examinations of the University of London for June, 1908, to June, 1915, with full solutions to the papers of June, 1915. (University Tutorial Series.) London, Clive, 1915. 136 pp.

2. *Matriculation model answers: mathematics*. Being the London University matriculation papers in mathematics from January, 1913, to June, 1915. (University Tutorial Series.) London, Clive, 1915. 140 pp.

3. *Matriculation mechanics papers*. Being the London University matriculation papers from September, 1903, to June, 1915, with model solutions to the last paper and additional questions. London, Clive, 1915. 130 pp.

² Papers set at these different examinations in 1900 and 1910 are given in Special Reports on Educational Subjects, Board of Education, London, vol. 26, pp. 473-488, 500-518.

In the preceding pages some of the noteworthy features of the relation of secondary schools to the general educational scheme have been described, their wide divergence in ideal and in work has been illustrated, and the nature of mathematical courses taught has been indicated. The prominent rôle played in the school organization by preparation for examinations by various boards of examiners suggests the thought that much of the energy of the teachers and pupils must be unfortunately diverted to attacking certain types of problems and examinations, rather than to developing a mastery of the subject in question. But this evil has been recognized and is being dealt with.¹

Having now observed what mathematics is taught in secondary schools, we must next consider one of the main features of the preparation of the teacher for his work, namely, his course in one of the universities.

UNIVERSITIES.

The universities of England are situated at Birmingham, Bristol, Cambridge, Durham, Leeds, Liverpool, London, Manchester, Oxford, and Sheffield; there are also university colleges at Newcastle, Nottingham, Reading, and Southampton. All of these institutions, with the exception of those at Oxford, Cambridge, and Durham, receive Government grants. The annual attendance of day students is about 12,000, of students at evening classes about 8,000. In addition to these there are at Oxford in the vicinity of 3,400 students, at Cambridge 3,700.

The organization of such universities as those at Birmingham and Leeds especially reminds one of that at the better American universities. The University of London, long merely an examining and degree-conferring institution, was reconstituted by statutes of 1900 as a teaching university and a federation of 26 colleges and schools giving instruction in arts, law, medicine, theology, science, engineering, economics, and music. Sons of noble and wealthy families who seek a university education usually go to either Oxford or Cambridge. Here one finds many of the most brilliant students of the country, those who have won in competition one of the numerous entrance scholarships. It is especially among graduates of these universities—homes of culture and all that is finest in English life—that teachers for secondary schools are sought. As Cambridge is preeminent in mathematics in England, teachers of this subject are much in demand among "honors" men there.

For definiteness, therefore, I shall confine my brief comment to the Universities of Oxford, Cambridge, and London.

It is well known that Oxford has 21 colleges, each with its own teaching staff of tutors and lecturers; that each student is assigned

¹ Compare Mr. Jolliffe's paper, especially pp. 270/271.

to one of these colleges and has a tutor to whom he looks for guidance, advice, and inspiration in preparing for the various examinations of his university career; and that this preparation demands a high standard of scholarship. Great stress is laid on ease and facility of expression, on the ability to form independent judgments, on originality. No one can get "first" in the class lists on mere hard work and "grinding," or by a display of erudition and an imposing array of facts. The strain of the examinations, especially in the final honors school, is very severe. At Cambridge the scheme is very similar.

In all three universities the courses lead normally to a "pass-degree" or an "honors-degree," as bachelor of arts; the pass B. A. is attained three years after matriculation. The principal examinations may be exhibited in the following table:

University examinations.

Examinations.	Oxford.	Cambridge.	London.
Entrance.....	Responsions.....	Previous (little-go)....	Matriculation (matric.).
Intermediate.....	Pass moderations (pass mods.) [after 3 terms].	Tripos Part I [after 3 years].	Pass intermediate or Pass pre- liminary arts [after 1 year].
Pass B. A.....	Final pass school (groups) [after 3 years].		Pass final [after 3 years].

Examinations for the honors B. A.

Examinations.	Oxford.	Cambridge.	London.
Entrance.....	Responsions.....	Previous (little-go)....	Matriculation (matric.).
Intermediate.....	Honor moderations (honor mods.) [after 5 terms].	Tripos Part I [after 3 years].	Honors intermediate or Honors preliminary arts [after 2 years].
Honors B. A.....	Final honor school [after 4 years].	Tripos Part II [after 4 years].	Honors final [after 3 years].

Let us suppose that our student wishes to pursue mathematical studies in the honor school at Oxford. He will be allowed to specialize almost to his heart's content. According to the regulations of 1913, the following is the examination program for the final honor school of mathematics, which is one of nine schools.

Algebra, including the elements of the algebra of quantities; theory of equations; trigonometry, plane and spherical; infinite series and infinite products.

Geometry, pure and analytic, of two and three dimensions.

Differential and integral calculus; differential equations.

The elements of the theory of functions of a complex variable, with applications to the elementary functions and to elliptic functions.

The elements of the calculus of finite differences.

The elements of the calculus of variations.

Statics and dynamics of particles, rigid bodies, and strings; the elements of analytical dynamics; statics of rods slightly bent. Hydrostatics; the elements of hydrodynamics; waves on liquids.

Attractions; theory of potential.

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Electrostatics; magnetostatics; steady electric currents (flow in linear circuits, laminae, and solid bodies).

Electromagnetism (magnetic force due to currents, induction); electrodynamics (mechanical effects of currents); dielectric currents (propagation of plane waves in a homogeneous dielectric).

Vibrations of strings; propagation of sound; vibrations of air in pipes.

The elements of geometrical optics.

Spherical astronomy.

Half of the 10 examinations in this program are in pure and half in applied mathematics. They occupy about 30 hours, on 6 consecutive days.

Oxford concedes that the most talented of English mathematical students usually go to Cambridge. It is not surprising, therefore, that the program for the tripos examination is much more elaborate than even that for honors at Oxford. On the other hand, the mathematical opportunities for the specialist in the University of London are not as numerous as those at Oxford.¹

But while it is possible that a graduate of one of these universities may have received a very broad training in mathematics, it is also true that he may, at the end of his course, know no more mathematics than are required for responsions, namely: Arithmetic, and either Euclid's Elements, Books I-II, or algebra.

TEACHERS IN SECONDARY SCHOOLS.

The organized training of teachers for English secondary schools is in its infancy and extends at most to a postgraduate year in a university. For nearly a decade the board of education has given financial assistance to institutions training secondary school teachers, but while this feature is being increasingly emphasized, it is nevertheless true that less than 40 per cent of the total number of secondary teachers (of boys and girls) have had some training, and that not more than 15 per cent have been trained for the specific work in which they are now engaged.

According to regulations which went into force in 1913, students may be trained as teachers in secondary schools in (1) training colleges, (2) certain secondary schools.

In order to be recognized as a training college under these regulations—

an institution must be an institution or a department of an institution organized for the purpose of giving instruction in the principles and practice of teaching specially designed for persons who are preparing to become teachers in secondary schools as defined in the regulations.

¹ This program is given on pages 194-95 of *Special Reports on Educational Subjects*, Board of Education, London, vol. 27. The examination papers for Parts I and II of the mathematical tripos are published annually in pamphlet form by the Cambridge University Press.

² Cf. H. Bateman, "The Work of an English Mathematical Student," *Mathematics Teacher*, vol. 5, 1913, pp. 147-153.

2. The training college course must be confined to purely professional instruction.
4. (a) Adequate provision must be made, in secondary schools approved by the board for this purpose, for the instruction and practice of students in teaching and in school organization and management.
- (b) If the training college is a department of a secondary school, this condition may be satisfied, provided that the student has ample practical experience during the year of training in the school of which the college is a part.
6. The course must extend over not less than a full academic year.
7. At least 60 school days must be spent in contact with class work under proper supervision in schools approved for this purpose by the board. Not less than two-thirds of the teaching practice must be taken in a secondary school or schools.

Training of teachers in secondary schools may be carried on under the following conditions:

21. (a) The school must be on the list of secondary schools recognized as efficient by the board.
- (b) Any person proposed for recognition as a teacher in training must be not less than 21 years of age and must have obtained an approved degree conferred by some university of the United Kingdom or some other university of recognized standing. In the case of a woman who is not eligible to receive a degree, a certificate showing that she has fulfilled all the conditions which entitle a man to obtain an approved degree will be accepted for the purpose of this article.
- (c) The course followed by a teacher in training must provide for a systematic course of study both in the practice and in the principles of teaching in accordance with a scheme approved by the board. The scheme must provide in each case for a special study of the methods of teaching a particular subject or group of allied subjects.
- (d) The course must extend over not less than a full school year. The whole year must ordinarily be spent in the school to which the teacher in training is admitted, but arrangements may, in certain cases, be made for the absence of the teacher from the school for part of the year for the purpose of attending a course of instruction in the principles of teaching at a university.
- (e) A school will only be approved for the purpose of this chapter if the head master or head mistress or some other senior member of the staff is specially qualified and has the necessary interest and leisure to supervise the teacher's training.
22. When a teacher in training has been trained for a year in an approved school under conditions which the board can regard as satisfactory, the board will indorse a certificate given by the head master or head mistress of the school stating that the teacher has completed the period of training in a satisfactory manner.

The regulations give also a "List of qualifications other than degrees which will be accepted as qualifying students for admission to training colleges." These are:

- I. A tripos certificate granted by the University of Cambridge to women, provided that the examination taken was one which, if passed by a man after three years' residence, would entitle him to a degree without further examination. Women students who have been allowed the ordinary degree in a tripos examination will be regarded as possessing the necessary qualification.
- II. A diploma or certificate showing to the satisfaction of the board that the applicant, if a woman, has fulfilled all the conditions which, if the University of Oxford granted degrees to women, would entitle her to a degree in that university; so that she has obtained honors in the second public examination or has passed the first and

second public examination¹ at that university or such examinations as are accepted by the university as equivalent thereto.

III. A special honors certificate of the higher local examinations (Oxford and Cambridge) granted under the following conditions:

- (a) That the certificate includes at least a pass in two languages (other than English) and a pass either in mathematics or in logic; and
- (b) That the holder either (i) has passed in four groups or sections, obtaining a first or a second class in at least two of them; or (ii) has passed in three groups or sections, obtaining a first or a second class in at least two of them, and holds in addition (1) an Oxford or a Cambridge senior local certificate in honors, including at least one subject not included in the three higher local groups or sections, or (2) a higher certificate of the Oxford and Cambridge schools examination board, gained in one year, exclusive of drawing and music, and including at least one subject not included in the three higher local groups or sections.

Among the institutions recognized as efficient under regulations of the board of education for the training of teachers for secondary schools are: (1) Departments controlled by, or forming part of, a university or university college; (2) training colleges provided by other bodies. Departments of the first type are to be found at Birmingham University, Durham University—Armstrong College, Leeds University, Liverpool University, London University, Victoria University, Oxford University delegacy for the training of secondary teachers, and Reading University College. An example of a training college of the second type is the Clapham High School, in London.

To illustrate the methods of operation, some details follow with regard to organization in units of each of these types. For the most part the statements have been made by the institutions in question.

(a) *University of Liverpool*.—It is little more than a decade ago that a diploma in education for graduates, along the present lines, was established. The courses of study qualifying for the diploma were placed under the control of a special board, in organic relation to the faculties of arts and science, and including persons representing secondary education in the city.

Candidates for the diploma must be graduates of some university in the United Kingdom, or have obtained such other academic qualifications as shall be approved by the senate of the university. Before admission to the examination candidates present certificates of (1) having fulfilled the conditions as to practical teaching, and (2) having attended for at least one session² subsequent to their final examination for a degree a course of study approved by the diploma board.

The diploma examination consists of two parts, theory and practice. Candidates are required to pass written examinations in the following subjects:

¹ First public examination—moderations after 3 semesters; second public examination—examination for the degree.

² One session consists of 3 terms of about 10 weeks each.

1. Logic, ethics, and psychology, including psychophysiology, in their application to education.
2. Principles of education, with special reference to methods of teaching the usual subjects of the secondary school curriculum; principles of general physiology and school hygiene.
3. A prescribed period of the history of educational theory and practice.

In respect to practice, candidates are judged (1) upon reports by the professor of education, and (2) upon written records of their work in school and upon their teaching before the examiners. Candidates are required to attend at approved practicing schools for a period of at least 250 hours. They are, as a rule, attached to one school throughout the session, and, so far as possible, they undertake work such as would be allotted to a member of the staff. They also attend lessons given by members of the school staff and study the methods of teaching the special subjects in which they are interested. The general supervision of their work is in the hands of the university staff, but each student is also under the direction of one teacher, who reports on his progress. The students are required to keep a record of teaching observed and of courses of lessons given, to be submitted to the examiners.

The fee for the complete course is £10 and the examination fee is £2 additional.

(b) *Clapham High School*.—The department for the training of teachers for secondary schools was opened in 1902. Students desiring to enter must possess a degree, or equivalent qualification, or hold a higher local honor certificate.

Throughout the course students follow lessons given in the school by experienced teachers. They themselves also teach, under supervision, which is relaxed as they gain experience and power. They learn the various duties of a form teacher by being sent as assistants in a form, for not less than half a term at a time. Visits are paid to other schools of different types, and students give criticized lessons in outside schools. Lessons are followed by general discussion, with the specialist in charge of the subject.

The course covers three terms, and the plan of study includes instruction in the theory of education, based on psychology, logic, ethics, and the history and practice of education. Instruction is also given in the use of the blackboard, voice production, and school hygiene.

The fee for the course is £24.

Such organizations give promise of a more efficient group of secondary teachers in the near future. Not so long ago—

of the 9,126 full-time teachers [men and women] in secondary schools which received grants from the board of education, 5,348 were without professional training. Of this

group of untrained teachers, 2,731 were also without degrees from college or university. The total number of secondary teachers without degrees was 3,715.

It appears that very few secondary teachers appointed in the last decade have not had the equivalent of a university course supplemented by practical training. But even so, it is noteworthy that a university graduate may be engaged to teach a subject about which he has not increased his knowledge since he left the secondary school. As Mr. Fletcher comments: It is "a matter of grave concern that half of our mathematical teachers of boys and girls have had no instruction in mathematics beyond what they have had at school."

If we consider only the best grade of secondary schools for boys, the scholastic equipment of the teachers in the subjects taught ranks high, the work that the teachers do is characterized by great thoroughness, and their personal influence is such as to inspire the finest ideals of manhood.

I have been unable to procure any comprehensive statement of salaries paid to teachers in English secondary schools. But even if such a statement has not been published, enough has come from authoritative sources to demonstrate the wide variation in salaries and, in most instances, their great inadequacy. The serious effect of this condition on the personnel of the teaching forces is now generally recognized, and it is hoped that through the authority of the board of education much needed reform in this connection may be brought about.

In August, 1900, Dr. W. H. D. Rouse, headmaster of the Perse School, published some striking facts which would seem to give a true presentment of general conditions at the time.¹ He found that the average salary of assistant masters in 300 schools mentioned in the returns of the Charity Commission was £135.22, and that a similar average for 20 East Anglian schools was £103.6.

In 11 smaller schools the average salary is £52, and these data combined give a sum just below £120 as the average salary of the assistant. Residence, i. e., board and lodging, is included in some cases; but we may leave this out of account, because it is payment for extra work done out of school. The Victorian public schools, such as Clifton, Cheltenham, and Marlborough, and others which though noted in the past are of late growth, such as Tonbridge and Bedford, are not included in the above list, which is meant to illustrate the usual condition of country grammar schools. If these be included, the average will be slightly raised. At Cheltenham there are one or two posts at about £300, one at least of £100 only, and the others range from £200 to £250 as a rule. Clifton and Marlborough do not greatly differ. The state of things in Bedford is thus described by one who knows: "There is no scheme of salaries in either of the two big schools. Each man fights for what he can get; if he makes a good bargain to start with, well for him." A few years ago the average salary at Bedford Grammar School was £174, but many form masters received £150 or less, some under £100, all these being nonresident. At Tonbridge there is in my table only one salary higher than £200. There is usually no automatic increase. If a master wishes to

¹ "Salaries in Secondary Schools," *Contemporary Review*, vol. 78, pp. 275-280.

marry or thinks his increased experience makes his services more valuable, he may have to get another post (if he can) in another school. As to the smaller schools, the account of the career of a Cambridge B. A. of my acquaintance may be of interest. He began in Andover Grammar School at £15, resident, and after several moves from one private school to another, where the pittance was somewhat increased, he attained, after nine years' experience, to the magnificent stipend of £140, nonresident, in the grammar school of a country town which for his sake I forbear to mention. A London B. A., whose life story is also before me, now receives £130, nonresident, after 18 years' experience. The same pitiful story comes from scores of small country schools.

The headmasters' salaries present a pleasing contrast. In the best-paid of the schools mentioned, Tonbridge, the headmaster receives £5,000 and upward, while his assistants have less than £200. The usual average is ten times that of the assistant, falling to five times in the East Anglian schools and even occasionally to less. So far as my knowledge goes (and as regards some of these schools it is not negative knowledge), neither headmaster nor governing body has expressed any dissatisfaction with the state of affairs or has ever considered means whereby the salaries of assistants might be permanently improved. The chance of succeeding to a boarding house keeps hope alive in some schools, and this rather than the earned reward of merit would seem to be the present educational ideal.

The following definite indications of salaries in four types of secondary schools were taken by Norwood and Hope in 1909 (pp. 567-568) from a list issued by the assistant masters' association:

A. LONDON SCHOOLS.

1. *City of London School.*

- (a) Upper scale, £300, rising to £450 by annual increments of £5 15s. 4d.
- (b) Lower scale, £200, rising to £350 by same increments. Five masters are paid on the higher scale and 15 on the lower.

2. *Mercers' School.*

- (a) Upper scale, £160, rising to £300.
- (b) Lower scale, £120, rising to £200.

B. PROVINCIAL SCHOOLS.

1. *Manchester Grammar School.*

Ordinary scale, £180, rising by £20 every two years to £300.

2. *King Edward VII School, Sheffield.*

- (a) Five masters, £220 to £250, rising by £10 annually to £300.
- (b) Five masters, £180 to £200, rising by £10 annually to £250.
- (c) Five masters, £150, rising by £10 annually to £200.

C. MUNICIPAL SECONDARY SCHOOLS.

1. *Bournemouth.*

£120, rising to £150, with three special salaries of £170, rising to £250; £160, rising to £200; £140, rising to £170.

2. *Hartlepool.*

£150, rising to £200.

D. COUNTRY AUTHORITIES.

1. *London County Council.*

£150, rising by annual increments of £10 to £300, on satisfactory reports, with further annual increments of £10 to £350.

2. *Surrey.*

Nongraduates, £100, rising by annual increments of £5 to £150.
Graduates, £130, rising by annual increments of £7 10s. to £250.

Provision for the pensioning of teachers in secondary schools is certainly meager.

A number of wealthy endowed secondary schools have for some-time had individual pension systems of their own, operating under provisions of their respective schemes of government of charters, most of which were received under the endowed schools acts for England beginning in 1869. * * * Also in a few cases secondary teachers in schools under public management have been pensioned by local authorities. With these exceptions there has been nothing remotely approaching any general provision of pensions for secondary teachers.

BIBLIOGRAPHY.

G. WOLFF, *Der mathematische Unterricht der höheren Knabenschulen Englands*. (Berichte und Mitteilungen veranlasst durch die Internationale Mathematische Unterrichtskommission.) Leipzig, Teubner, 1915. 6+207 pp.

To anyone outside of England this work has met a great need because of the inadequacy of the reports published by members of the International Commission from the United Kingdom.

Memoranda on Teaching and Organization in Secondary Schools. Curricula of Secondary Schools. Board of Education, London, 1913.

Regulations for the Training of Teachers for Secondary Schools. (In force August 1, 1913.) Board of Education, London, 1913.

Special Reports on Educational Subjects. *The Teaching of Mathematics in the United Kingdom*, being a series of papers prepared for the International Commission on the Teaching of Mathematics. Board of Education, London, 1912. Vols. 26, 27. 588+339 pp.

Among others these reports contain papers: By W. C. Fletcher, "The position of mathematics in secondary schools in England"; C. S. Jackson, "The calculus as a school subject"; W. Newbold, "Higher mathematics for the classical sixth form"; A. L. Dixon, "Mathematical examinations at Oxford"; A. Berry, "Recent changes in the mathematical tripos at Cambridge"; F. S. Macaulay and W. J. Greenstreet, "Examinations for mathematical scholarships"; A. E. Jolliffe, "Examinations for mathematical scholarships at Oxford"; G. H. Hardy, "Examinations for mathematical scholarships at Cambridge"; T. P. Nunn, "The training of teachers of mathematics"; G. H. Bryan, "Research and advanced study as a training for mathematical teachers."

List of Secondary Schools in England Recognized by the Board of Education as Efficient. 1912-1913. Board of Education, London, 1914.

C. NORWOOD and A. HOPE, *The Higher Education of Boys in England*. London, Murray, 1909. 14+568 pp.

C. H. JUDN, *The Training of Teachers in England, Scotland, and Germany*. (Bureau of Education, Bulletin, 1914, No. 35.) Washington, 1914. "England," pp. 1-45.

The Training of Women Teachers for Secondary Schools. A Series of Statements from Institutions Concerned. (Educational Pamphlets No. 23.) Board of Education, London, 1912. 90 pp.

R. W. SIES, *Teachers' Pension Systems in Great Britain*. (Bureau of Education, Bulletin, 1913, No. 34.) Washington, 1913. 88 pp.

VI. FINLAND.

The area of Finland is somewhat less than half that of Texas, and its population is but slightly in excess of 3,000,000; yet it publishes many important periodicals and is the home of not a few learned societies. Of the inhabitants, about 80 per cent are Finns and about 11 per cent Swedes.

By ordinances of 1869 and 1872 the general administration of the national system of primary education, as well as that of the secondary schools which heretofore had been under ecclesiastical direction, was vested in a central board or council of education. The members of this council are appointed by the senate (constituted by imperial appointment), upon the nomination of the Diet (legislative assembly). The executive chief of the system, the director general, is also appointed by the senate. An assistant director has charge of primary education, and there are Government inspectors for both secondary and primary schools.

Those State secondary schools which have interest for us are the lyceer (classical and real), each with an eight years' course. In the real, as compared with classical lyceum, one additional hour per week is spent on mathematics, and greater emphasis is laid on physics. Such lyceer prepare directly for the university.

There is no examination in passing from one class to another in the lyceum. But after finishing the work of the VIII class, the pupils, who are about 17 years of age, are required to submit to both written and oral tests. The written tests are the same throughout the country and occur on the same days. They consist of (1) a composition written in the mother tongue; (2) an examination in the other language, Finnish in a Swedish school, Swedish in a Finnish school; (3) a Latin translation in the classical lyceer, a French, German, or Russian in the real; (4) a test in mathematics. In connection with (4), questions are also given on physics. The test lasts for six hours, and the pupil who answers at least three questions, out of about ten, in a manner satisfactory to the professors in the school, is declared admissible. The paper is then submitted to the examination commission of the university, who pass upon it finally. The successful pupils who have also passed the oral test, given by the professors of the university, on all the subjects of the course, receive a "leaving certificate." This certificate now admits the student to the university.

THE ALEXANDER UNIVERSITY.

As experience has shown that the course of mathematics in many lyceer of the country has not been thorough, on account of the great amount of time taken up with languages, the university undertakes to review and complete certain parts, especially in trigonometry. Then follows a course in analytic geometry, which deals with the conic sections and surfaces of the second degree. Only the elements of projective geometry are taught. The text is the manual of analytic geometry, by L. Lindelöf.

Spherical trigonometry is taught in connection with analytic geometry of space. It is taken up more in detail in the courses on spherical astronomy.

Differential and integral calculus is begun in the first year at the same time as analytic geometry, and its study is carried on for two years. The texts of L. Kiepert and E. Czuber are the ones most in demand.

An elementary course in differential equations is frequently given. It contains special geometric applications of the theory. Sometimes developments lead to such problems as the conduction of heat and other questions in the domain of mathematical physics.

Algebra and the theory of numbers are given in the second year. The pupils use G. Bauer's *Vorlesungen über Algebra* as a manual.

Then, further, a course on analytic functions, which takes account of the methods of exposition of Cauchy, Riemann, and Weierstrass, is given every year. H. Burkhardt's *Algebraische Analysis und Einführung in die Theorie der analytischen Funktionen* is recommended for the students.

Besides the above-mentioned regular courses, there are frequently given special courses in certain domains of mathematics, such as minimal surfaces, application of the theory of groups to the resolution of algebraic equations, elliptic functions.

The teaching in the sections "historico-philologique" and "physico-mathématique" of the faculty of philosophy is organized in such a way that the students can, in general, after four or five years of study, pass the necessary examination to obtain the degree of "candidate of philosophy."

This examination comprises, apart from a written test in a modern language, interrogation bearing on at least four of the sciences in the domain of the faculty in question, the combination of subjects being submitted by the candidate and approved by the section. In each subject the successful candidate obtains a predicate—*approbatur*, *approbatur cum laude*, or *laudatur*—these predicates depending on the extent of the program studied. To pass, the candidate must receive either the predicate *laudatur* in two subjects, or

laudatur in one and approbatur cum laude in two others. In addition to this, an essential part of the examination is a study pro gradu. In this paper, which requires a long time to prepare, the candidate treats, in a personal way, some scientific question given or approved by a professor.

To obtain the highest predicate in mathematics in this examination, it is necessary to know thoroughly the courses in analytic geometry, differential and integral calculus with its applications to geometry, differential equations, algebra and theory of numbers, and the theory of functions. For the predicate approbatur cum laude, the theory of functions disappears from the program, and only the first principles of algebra and the theory of differential equations are required. The program for the predicate approbatur is confined to plane trigonometry, plane analytic geometry, and the fundamental notions of the differential and integral calculus.

The candidate who has obtained the degree of "candidate of philosophy" can obtain the degree of licentiate in the faculty of philosophy by continuing his studies, publishing and publicly sustaining a thesis, and passing an oral examination, in which he shows thorough knowledge in three subjects within the domain of the faculty.

The candidate of philosophy receives at the annual promotion the title of master in arts; under the same conditions the licencié may receive the title doctor in philosophy.

In order to be competent to fill a post in secondary teaching, it is necessary to have passed the examination for a "candidate of philosophy" or the examination for the "certificate of aptitude in teaching." This latter examination is passed at the university. It differs from the first only in that it is not absolutely necessary to obtain the highest predicate in one of the subjects. Nearly all candidates become "candidates of philosophy."

NORMAL LYCEER.

The professional education of the professors in secondary schools is concentrated in two normal lyceer, one for professors in Finnish schools, the other for the Swedish. These two lyceer are under the special control of the professor of pedagogy in the university. In order to be inscribed as "stagiaire ordinaire" in a normal lyceum it is necessary to have a "certificate of aptitude in teaching" or to be a "candidate of philosophy." As a general rule, the mathematical candidates have passed the second of these examinations and have obtained the highest or the next to the highest grade in this subject. They are placed under the immediate direction of the senior professor of mathematics.

The extent of training at the normal lyceum is two semesters. The "stagiaire" is thoroughly drilled in the theory and practice of pedagogy. Under the surveillance of the senior professor he teaches in a State lyceum. When his major is in mathematics, he also gives instruction in physics and chemistry.

In addition to work of this kind, candidates are assembled in conferences of three types, monthly, general, and weekly. The monthly conferences are presided over by the professor of pedagogy; ordinarily, the class examines some one pedagogic work each semester. Each of the "stagiaires" in turn makes a report on a part of this work, which at the hour of the conference is the basis for discussion. The portions of the work which concern mathematics are assigned to mathematical candidates. In the general conferences questions of general pedagogy are discussed. The weekly conferences are either the conferences of the candidates in a special subject or those related to it, or special conferences arranged by the senior professors for the individuals. In the mathematical section a mathematical work is generally examined in a manner similar to that obtaining in the monthly conferences.

The "condition of competence" required to obtain a post as professor is that the candidates possess the degree of "candidate of philosophy" with the highest grade in the major subjects required by the post. For the position as "chargé de cours," the same degree or the "certificate of aptitude," with at least the second highest grade in the major subjects required by the position, is necessary. For both classes of teachers an examination in pedagogy and a test in practical pedagogy are required. These tests are conducted by a council of the senior professors of the lyceum, Finnish or Swedish, as the case may be. This council consists of five members; the rector of the normal lyceum, the senior professors of the subject in question from the two normal lyceum, and two members of the council of the normal lyceum where the candidate is being tested. The candidate is invited to teach in assigned classes for the purpose of showing his aptitude for the career which he wishes to embrace. On the basis of this test is given the grade *approbatur*, *approbatur cum laude* or *laudatur*, according to the work done.

CONCLUDING REMARKS.

About one-half of the students in the faculty of philosophy at the university look forward to a career in connection with secondary education. Nevertheless, the professional education of such teachers has been conducted almost wholly by those outside the university. Some slight departure from this policy with regard to mathematics was made in 1912, when an associate professor of mathematics in the

university was appointed to a recently created chair. The special task of this professor is "to give courses and direct the practical exercises for the future professors of mathematics in the secondary schools."

Among topics which this professor takes up are:

In *geometry*: The axioms of Euclidean geometry; the principles of projective geometry; a sketch of the various systems of geometry; a systematic exposition of the elementary methods of resolution of geometric problems; the history of elementary geometry.

In *trigonometry*: The historical development of the science.

In *arithmetic*: Methods of numerical calculation; the historical development of elementary arithmetic; the extension of the idea of number.

In *algebra* and the *theory of numbers*: The notion of divisibility in the theory of numbers and in algebra; the historical development of algebra and abridged notation; the application of algebra to the resolution of problems in geometric constructions with the aid of various instruments.

BIBLIOGRAPHY.

- Rapport sur l'Enseignement des Mathématiques dans les écoles de Finlande. Rédigé par une commission instituée par le Sénat Impérial de Finlande (Commission internationale de l'Enseignement mathématique. Sous-Commission Russe). Helsingfors, Imprimerie de la société de littérature Finnoise, 1910. 52 pp.*
- J. S. THORNTON. *Schools, public and private, in the north of Europe. Special Reports on Educational Subjects, vol. 17, London Board of Education, 1907. Finland: pp. 25-35; p. 123f.*

VII. FRANCE.

Few countries in the world can compare with France in the high standards maintained with regard to the appointment of teachers of mathematics for the secondary schools. One may almost say that the educational scheme in France is largely organized with reference to the preparation and selection of secondary school teachers. It is important therefore to have this general scheme in mind.

For educational purposes France is divided geographically into *arrondissements*. The assemblage of Government schools (primary, secondary, and superior) in each *arrondissement* forms an *académie* over which a *recteur* presides. There are thus the 16 *académies* of Aix-Marseille, Besançon, Bordeaux, Caen, Chambéry, Clermont, Dijon, Grenoble, Lille, Lyon, Montpellier, Nancy, Paris, Poitiers, Rennes, Toulouse, as well as a seventeenth at Algiers. With the exception of Chambéry, these names correspond to the seats of the French universities.

The assemblage of *académies* constitutes the *Université de France*, at the head of which is the minister of public instruction, who is ex officio the "*Recteur de l'Académie de Paris et Grand Maître de l'Université de Paris*." For the *Académie de Paris* there is a vice *recteur* whose duties are the same as those of the *recteurs* of other *académies*. Although nominally lower in rank than the heads of *académies* in the Provinces, he is in reality the most powerful official in the educational system. Since the position of the minister of public instruction is so insecure by reason of changing governments, continuity of scheme is assured by three lieutenants who have charge respectively of the primary, secondary, and superior education. They in turn have an army of inspectors who report on the work and capabilities of the *recteurs* and their *académies* as far as primary and secondary instruction are concerned.

The present system of secondary education in France dates from the great reform of 1902 (important modifications were introduced in 1905, 1909, and 1913¹) and is carried on for the most part in *lycées* and *collèges communaux*, which are to be found in nearly all cities. Because of their preeminence we shall consider the teachers in the former only, which are under control of the State. Here the boys who come from families in comfortable circumstances may enter at the age of 5 or 6 years and be led along in their studies till they

¹ Rept. of Comms. Educ., 1911-12, vol. 7, Washington, 1913, pp. 528-532.

receive the baccalauréat at the age of 16 or 17. Many lycées have still more advanced courses to prepare for entrance into such schools as the École Normale Supérieure, École Polytechnique, École Centrale, École Navale, or École de Saint Cyr. For our purpose it is desirable to consider these advanced courses and the students who, having taken them, enter the École Normale Supérieure.

Instruction in fully equipped lycées may be divided into four sections: (1) Primary, five years, for children from 5 to 10 years of age; followed by (2) first cycle, four years, the first section of the secondary education, properly speaking; which leads to (3) second cycle, three years, on the completion of which the student receives the State degree known as the baccalauréat. In about one-third of the 120 lycées in France there are also (4) the classes de mathématiques spéciales. In considering the training of teachers for the lycées certain general facts with regard to (2) and (3) should be borne in mind, while detailed information as to (4) is essential.

Since the reforms of a decade or so ago the pupils of the first cycle are divided into two groups. In the one are those who learn Latin, with or without Greek, and in the other are those who have nothing to do with a dead language. Pupils of the latter group go into the science-modern languages section of the second cycle. Into this section may also enter certain pupils who give up the study of Latin to specialize in modern languages and science. But in the second cycle there are three other sections entitled, "Latin-science," "Latin-modern languages," and "Latin-Greek." The work in mathematics is decidedly heavier in the science sections, and as this work must be taken by all prospective teachers of mathematics, we may confine our attention to these sections. The classes of these sections in the successive years are named as follows:

Classes of the sections.

	First year.	Second year.	Third year.
Latin-science.....	Seconde C.....	Première C.....	Mathématiques A.
Science-langues.....	Seconde D.....	Première D.....	Mathématiques B.

The work in mathematics is the same throughout in both sections. At the end of the second year the first of the examinations for the baccalauréat are taken. They are both written and oral and of an elaborate and exacting nature. Among the examiners, six in number, in a given arrondissement, three are professors from the university. Under similar conditions the second and final tests are applied at the end of the third year and the successful pupil becomes a

¹ E. LORMEAU ET P. MERLET, *Problèmes de géométrie donnés aux examens du baccalauréat Latin-Science, Sciences-langues, Mathématiques A (1907-1908) énoncés dans la géométrie André et Lormeau. Énoncés et Solutions.* Paris, André-Guédon, [1909].

bachelier. Little, if any, exaggeration is made by regarding the bachelier as upon a plane of scholastic equality with the student who has finished the sophomore year at one of the best American universities. Several of France's most brilliant mathematicians and teachers of our day became bacheliers when only 16 or 17 years of age.

If the bachelier who is proficient in mathematics be not turned aside by circumstances or inclination to seek immediately a career in civil or Government employment, he is likely to proceed to prepare himself for the highly special and exacting examination necessary for entrance into one of the great schools of the Government. The method of this preparation exhibits a very peculiar feature of the French system. Whereas with us, or with the Germans, the boy who has finished his regular course in the secondary school goes directly to some department of a university for his next instruction, the bachelier, who has a perfect right to follow the same course, returns to this old lycée (or enrolls himself at one of the great Paris lycées, such as St. Louis, Louis le Grand, or Henri IV), to enter the classe de mathématiques spéciales préparatoires which leads up to the classe de mathématiques spéciales. The latter is exactly adapted to prepare students for the École Normale Supérieure, the École Polytechnique, and the bourses de licence. As mentioned above, only a small proportion of the lycées have this classe, but with the exception of Aix they are to be found in all university towns. On the other hand, still other lycées have classes which prepare specially for the less exacting mathematical entrance examinations of the École Centrale, École de St. Cyr, École Navale, etc. But the number of pupils who on first starting out deliberately try to pass examinations for these schools is small in proportion to the number who eventually reach them after repeated but vain effort to get into the École Polytechnique or the École Normale Supérieure. Just what makes these two schools famous and peculiarly attractive will appear later. When the pupil has won his baccalauréat he may immediately matriculate into a university, and although it might be possible for him to keep pace with the courses, it would be, in mathematics at least, a matter of excessive difficulty. There is then in reality, between the baccalauréat and the first courses of the universities, a distinct break, bridged only by the classes de mathématiques spéciales.¹

The élèves who enter the preparatory section of this class are, generally, bacheliers leaving the classes de mathématiques. Natural science, history and geography, philosophy—indeed practically every study except those necessary for the end in view—have been dropped, and from this time on to the agrégation and doctorat, all energies

¹ It is only for mathematical or scientific students that such a break occurs, as no special classes are provided in other subjects except in the case of half a dozen Paris lycées which have classes in "letters" preparatory for entry into the École Normale Supérieure.

are bent in the direction of intense specialization. This is the most pronounced characteristic of French education to-day. In mathematics, instruction now occupies 12 instead of 8 hours per week. New points of view, new topics and broader general principles, are developed in algebra and analysis, trigonometry, analytic geometry, and mechanics. Physics and chemistry are taught during 6 hours instead of 5. Add to these, German, 2 hours; French literature, 1 hour; descriptive geometry, 4 hours; drawing, 4 hours. After one year of this preparatory training the élève passes into the remarkable *classe de mathématiques spéciales*.

Eight years of strenuous training have made this class possible for the young man of 17 or 18 years of age, who is confronted with no less than 34 hours of class and laboratory work per week and no limit as to the number of hours expected in preparing for the classes. The program seems a well-nigh impossible performance for one year. Surely no other country can show anything to compare with it.

It would be interesting to consider fully the mathematical program as given at the end of the plan d'étude, but I shall hastily refer to only a few of the subjects treated. In *algebra and analysis* we find developed the fundamental ideas concerning irrational numbers, convergency and divergency of series, the elements of the theory of functions of a real variable, power series, their multiplication and division, their differentiation and integration term by term, Taylor's formula, the theory of algebraic equations, including symmetric functions, but omitting the discussion of infinite roots. The latter part of the course treats of differentiation of functions of several variables, elementary ideas concerning definite integrals, integration of such functions as are considered in a first calculus course of the best American colleges, rectification of curves, calculation of volumes, plane areas, moments of inertia, centers of gravity, differential equations of the first order, solutions of simpler differential equations of the second order which occur in connection with problems of mechanics and physics. Whenever possible in the discussion of these topics the power to work numerical examples is emphasized.

Plane trigonometry and the discussion of spherical trigonometry through the law of cosines are treated in class, and five-place tables are used.

In the course on *analytic geometry* there is given a thorough discussion of equations of the second degree, of homography and anharmonic ratios as they enter into the discussion of curves and surfaces of the second degree, of points at infinity, asymptotes, foci, trilinear coordinates, curvature, concavity and convexity, envelopes, evolutes. The professor also discusses thoroughly the various questions connected with the treatment of quadric surfaces and less completely, the theory of surfaces in general, of space curves, osculating planes,

curvature of surfaces. The elements of the theory of unicursal curves and surfaces and of anallagmatic curves and surfaces are also taken up.

So also, we find broadly arranged programs in mechanics and descriptive geometry. The whole number of class hours per week is broken up as follows:

Mathematics, 15; physics, 7 (2 in laboratory); chemistry, 2; descriptive geometry, 4; drawing, 4; German, 2; French, 1. The scope of the mathematical work may be judged from some books which were prepared with the needs of such a class especially in view: B. Niewenglowski, *Cours d'algèbre*, I, 382 pp.; II, 508 pp.; supplement—G. Papelier *Précis de géométrie analytique*, 696 pp.—Girod, *Trigonométrie*, 495 pp.—P. Appell, *Cours de mécanique*, 650 pp.—X. Antomari, *Cours de géométrie descriptive*, 619 pp.

If anything, this list underestimates the work actually covered¹ by those who finally go out from the class. Tannery's *Leçons d'algèbre et d'analyse* (I, 423 pp.; II, 636 pp.), might well replace Niewenglowski's work; while Niewenglowski's *Cours de géométrie analytique* (I, 483 pp.; II, 292 pp.; III, 569 pp.) represents the standard almost as nearly as Papelier's volume. Another treatise on mechanics widely used is that of Humbert and Antomari.

When we further realize that the main parts of the books in this list, which represents the work for only one of a half dozen courses, are covered by the professor in about 15 months—the last three months of the second year are given over to drill in review and detail—we begin to get some conception of what the *classe de mathématiques spéciales* really stands for. In his instruction the professor is officially "recommended"—

not to overload the courses, to make considerable use of books, not to abuse general theories, to expound no theory without numerous applications dealt with in detail, to commence invariably with the more simple cases, those most easy to understand, leading up finally to the general theorems. Among the applications of mathematical theory, those which present themselves in mathematical physics should be given the preference, those which the young people will meet later in the course of their studies either theoretical or practical. Thus, in the construction of curves, choose as examples those curves which present themselves in physics and mechanics, as the curves of Van der Waals, the cycloid, the catenary, etc.—in the theory of envelopes choose those examples of envelopes which are met in the theory of cylindrical gearing—and so on. The pupils should be trained to reason directly on the particular cases and not to apply the formulæ. To sum up, the aim should be to develop the pupil's judgment and initiative, not his memory.

The success of a class is, by happy arrangement, not left to depend wholly upon a single man. Take, for example, lycée St. Louis in Paris. It is the greatest preparatory school for the *École Normale Supérieure* and the *École Polytechnique*. There are four *classes de mathématiques spéciales*, and for all the members of these classes, conferences, interrogations, and individual examination are organ-

¹That is, much more than what is called for by examination questions is studied. The students find truth in the adage: *Qui peut le plus peut le moins.*

ized. These exercises, which are complementary to the daily instruction, are conducted by one of the professors in the lycée, or by one of another lycée, or by one of those from the Collège de France, the Sorbonne, the École Polytechnique, or the École Normale. Incapable students are thus speedily weeded out, a marked solidity of the training is attained and, perhaps most valuable of all, the interest of the pupil is sustained at a high pitch.

With the end of the year the pupil has his first experience of a concours or competitive examination. Previously he has found that it was necessary only to make a certain percentage in order to mount to the next stage in his scholastic career; but now it is quite different. In 1908, 1,078 pupils tried for admission to the École Polytechnique, but only 200, or 19.5 per cent, were received; for the department of science in the École Normale Supérieure, 22 out of 274, or 8 per cent, succeeded. In each case the number was fixed in advance by the Government according to the capacity of the school; the fortunate ones were those who stood highest in the examinations, written and oral. In the case of the École Polytechnique, the written examinations were held in all the lycées which had a classe de mathématiques spéciales. The 387 candidates declared "admissible" were then examined orally at Paris, and from them the 200 were chosen. Similarly for the École Normale, the written examinations are conducted at the seats of the various académies and the oral at Paris. Since 1904 the concours passed by students wishing to enter the École Normale has been that for the stipends known as bourses de licence,¹ open to candidates of at least 18 years of age and not more than 24. Certain dispensations in the matter of age are sometimes granted. The value of the bourse, for the section of science, is from 600 to 1,200 francs a year and is intended to help the students to prepare for the licence and other examinations required of prospective professors in the lycées and universities. The candidates leading the list in the concours are sent to the École Normale Supérieure for from three to four years. It is necessary for the six or seven other boursiers to prepare for future examinations at the various universities of the Provinces. Their bourses last regularly for two, and exceptionally for three, years.

But to return to the students of the classes de mathématiques spéciales. At the end of the first year, when 18 or 19 years old, they usually present themselves for the concours of both the bourse de licence and the École Polytechnique, the examinations in the former being more strenuous and searching. Only from 2 to 5 per cent succeed on the first trial. The others then go back to the lycée and take another year in the classes de mathématiques spéciales. Many points not fully understood before are now clear and at the end of

¹ The mathematical questions at the concours of 1913 are given in Appendix C.

the second year from 25 to 28 per cent are successful. The persevering of those who fail again return to their classe and try still a third time (the last permitted for the bourse de licence); but it is a matter of record that less than one-half of those who enter the classe de mathématiques spéciales succeed even with three trials. This is usually the last trial possible for entry into the École Polytechnique, as the young man who has passed the age of 21 on the 1st of January preceding the concours may not present himself. The remainder of the students either seek for entrance into Government schools with less severe admission requirements, and thus give up their aspirations to become mathematicians, or else continue their studies at the Sorbonne or other university. The candidate who heads the list in each of these concours has his name widely published. In the case of the bourse de licence he is called the *cacique*, and he very frequently tops also the École Polytechnique list.

If the work in the classe de mathématiques spéciales is so enormously difficult that only 2 to 5 per cent of its members can, at the end of one year, meet the standard of requirements of the examinations for which it prepares, why is not the instruction spread over two? Since nearly all the mathematical savants who now shed luster on France's fair fame have passed from this remarkable class on the first trial, there can be no doubt that the answer to this question may be found in the fact that the French Government ever seeks her servants among the élite of the nation's intellectuals.

Those who pass from the classe de mathématiques spéciales at the early age of 18 years are not numerous, but Borel and Picard are such men, while Goursat entered the École Normale Supérieure at 17 years of age. For the average boy the lycée course is heavy, and more than once he may have to halt in order to repeat a year. The system of training is largely formulated to develop to the full the powers of the brilliant boy and to promote his rapid advancement. For such youths poverty is no deterrent. Every lycée has a number of bursaries (covering all expenses) which it distributes to just such boys coming with distinguished records from the primary schools. If the boy's record is sustained, renewal of his bursary from year to year is assured.

There has been occasion to point out the strong influence which the École Normale Supérieure and École Polytechnique exert on the careers of the flower of the French youth; how that instead of entering the university on passing the baccalauréat, as in America or in Germany—

they seek to enter these schools. The reason for this is not difficult to find. The École Polytechnique which prepares its pupils as military and naval engineers,

Cy. P. Appell, "Sur la classe de mathématiques spéciales," *L'Enseignement mathématique*, vol. 2, 1900, pp. 340-346.

artillery officers, civil engineers in Government employ, telegraphists and officials of the Government tobacco manufactories, offers all of its graduates a career which is at once rapid, brilliant, and certain. The *École Normale* practically assures its graduates at least a professorship in a lycée and prepares its students for this or for a university career, better and more rapidly than the university can do it.

Let us suppose that our future mathematical professor in the lycée is one of the 11 mathematical students who is successful in getting into the *École Normale* in a given year. He studies there for three years and receives special drill in pedagogy; simultaneously he also hears courses of lectures at the Sorbonne¹ and at the Collège de France.² Almost the whole purpose of the drill and instruction is to prepare for two examinations, the licence and the agrégation.

The diploma *Licence ès Sciences*, which is necessary for all those who take up secondary teaching, is granted to those who have three "certificats" in any one of three groups of subjects. Our mathematician is examined in the following subjects: (1) Differential and integral calculus; (2) rational mechanics; (3) general physics or some advanced topic in mathematics. The examinations may be taken singly in July or in November; each examination successfully passed entitles the student to a certificat for that subject. The examination consists of three parts, *épreuve écrite*, *épreuve pratique*, *épreuve orale*. The first two are written examinations of about four hours each. Theoretic considerations abound in the écrite, while numerical calculation is characteristic of the pratique. The oral lasts for 15 to 20 minutes and is held before a jury of those professors who have the whole examination in charge. The pass mark is 50 per cent.

To understand just what is implied in the possession of the diploma, *Licence ès Science*, let us consider the value of such a certificat as that in differential and integral calculus. To prepare for the examination in this subject at the Sorbonne (and the methods at other universities in France are little different) the candidate attends three courses during the preceding year. The first of these consists of about 60 lectures of 70 minutes each on differential and integral calculus by Goursat; in the second, 30 lectures of one hour each on applications of analysis to geometry are given by another professor; the third course is made up of 60 lectures of about 70 minutes each on problems illustrative of the above 100 lectures. In this way practically all of the first volume and a portion of the second volume of Goursat's *Cours d'analyse mathématique* is discussed. Upon this work the examination for the first certificat is based. In a

¹ The Sorbonne is the name of a building in the University of Paris where the instruction in science and letters is given.

² The Collège de France, founded in the sixteenth century, is independent of the general educational scheme but ranks higher than any of the universities. Two or three courses in pure and applied mathematics are offered each year. In them the lecturers usually set forth original developments of the subjects.

similar way there are three courses each in preparation for the examinations to secure the certificates in rational mechanics and general physics. There is probably no graduate school in America where the student attains such a comprehensive grasp and mastery of these subjects.

Unlike the baccalauréat and the licence, the agrégation is a competitive examination and is conducted by the State. The number who become agrégés each year is fixed in advance by the minister of public instruction according to the needs of the lycées in the country. This number in recent years has been about 14; the number of candidates is usually about 80. Our candidate for this examination must have four certificates: (1) Differential and integral calculus; (2) rational mechanics; (3) general physics; and (4) a subject chosen at pleasure in the advanced mathematical fields in which courses are offered.

To pass the agrégation our future professor disposes of his three years as follows: During each of the first two years he passes the examinations for two of the four certificates. With these successfully taken he turns his whole attention to preparing for the agrégation proper. This examination is unique in its difficulty and exactions. As it is organized for selecting the most efficient young men in the country to take charge of the mathematical classes in the lycées, the examination turns largely on the subjects there taught. It consists of *épreuves préparatoires* and *épreuves définitives*. The former are four written examinations, each of seven consecutive hours in length (7 a. m. to 2. p. m.). The first two of these are on subjects chosen from the program of the lycée in *mathématiques élémentaires* and *mathématiques spéciales*. The last two, based on the work of the candidates in the universities, are a composition on analysis and its geometric applications and a composition on rational mechanics.¹ The *épreuves* are held at the seats of the various académies of France. Those who have reached a sufficiently high standard are declared "admissible." Their number is usually a little less than twice the possible number to be finally received. They must present themselves at Paris for the *épreuves définitives*. These consists of two written examinations and two *leçons* or specimens of class-room instruction. The written tests are an *épreuve de géométrie descriptive*, and a *calcul numérique*. Their duration is fixed by the jury, but it is usually four hours for each. A *leçon* is supposed to be the treatment of some topic such as a professor might give (during three-fourths to one hour) in a lycée. The subjects are drawn by lot and are taken from the programs of the following classes: (a) *Mathématiques spéciales*; (b) *Seconde, Première C, D, and Mathé-*

¹ Copies of examination questions and of the program of a concours may be consulted in Appendix D.

matiques A. B. For each lesson the candidate has four hours to think over what he is going to say. No help from book or other source is permitted. The unfortunate who has little to say is speedily "adjourned."

The agrégés are those specially prepared by the State for the positions of professeurs titulaires in the lycées. Although this title is not conferred regularly till the agrégé has completed his twenty-fifth year, those who are younger receive temporary appointment, for every agrégé may demand a position as his right. The salaries vary according to the "classe" of the professor. At Paris the lowest salary is 6,000 francs per year, and the highest, 9,500. In this range seven classes are represented; six, each differing from the one before by 500 francs, and the hors classe, for which the salary is 9,500 francs. Promotion from one class to another takes place by selection and by seniority. From the sixth (the lowest classe) to the third, the number of those who can be advanced each year by selection is equal to the number which can be advanced by seniority. In the second and first classes two advancements may be made by selection to one by seniority. Those named for the hors classe are advanced by selection alone. The promotions are made at the end of each calendar year, and take place so that there are always 20 per cent of them in the sixth classe, 18 in the fifth, 18 in the fourth, 16 in the third, 14 in the second, and 14 in the first. This arrangement is obviously a happy one, both by way of recognition of the merits of the unusually successful teacher, as well as those of one whose service is rather characterized by faithfulness.

In addition to the professeurs titulaires there are professeurs chargé de cours, who are usually selected from those who have been students at the École Normale and those admissible to the agrégation, who fail to become agrégés. After 20 years of service they may become professeurs titulaires and receive the salaries we have indicated above. The Government has, however, recently passed a law which gives a higher reward to the agrégé. It is to the effect that 500 francs per year shall be added to the regular salary of every agrégé. The real range of salaries mentioned above is, then, 6,500 to 10,000 francs; in the provinces this reduces to 4,700 to 6,700. For the professeurs chargés de cours the salaries at Paris vary from 4,500 to 6,000 francs; in the provinces, from 3,200 to 5,200.¹ In the premier cycle the professors have 12 hours of teaching per week, in the second cycle and classe de mathématiques spéciales, 14 to 15 hours. Except for correcting exercises and filling out reports, the professors have absolutely no obligations outside of class hours. They do not live in the lycées. The superintendence of the study of the pupils is

¹ The tendency of recent legislation is to exclude from the lycées all professors who are not agrégés.

carried on by repetiteurs, the more advanced of whom receive at Paris 2,600 to 4,600 francs for 36 hours' service per week.

Attractions connected with a professorship in a lycée¹ are that the remuneration is ample to live on comfortably, that the work is not onerous and often quite decidedly inspiring, that colleagues are brilliant specialists in the same or other lines of study, that the professorships are positions of honor and prominence in the community, and that the incumbents are in demand in many ways which frequently materially increase their regular income. In general the charge of the classe de mathématiques spéciale, enrolling in its membership the pick of the French youth of mathematical bent, is confided only to a professor in the first or hors classe, but to this position all may aspire.

With inducements such as these it causes no surprise to learn that France draws to the development of her system of secondary education much of the best mathematical talent in the country.

BIBLIOGRAPHY.

Of first importance are the five volumes issued (Paris, Hachette, 1911) by the Commission Internationale de l'Enseignement Mathématique. These volumes are entitled: 1. *L'Enseignement primaire*; 2. *L'Enseignement secondaire* publié sous la direction de M. C. Bioche; 3. *L'Enseignement supérieur* publié sous la direction de M. A. de Saint-Germain; 4. *L'Enseignement technique*; 5. *L'Enseignement des jeunes filles*. 675 pp. Each volume contains reports by several different specialists and the whole is very interesting reading.

For information concerning the scheme of secondary education, and of mathematical work in particular, the inquirer should turn to *Plan d'études et programmes d'enseignement dans les lycées et collèges de garçons* (Paris, Delalain, 1910).

A very valuable general work by several different authors is *Secondary and University Education in France*. London, Board of Education, Special Reports, vol. 24, 1911. 554 pp.

Other references are given in my papers: (1) "Mathematical Instruction in France" (*Transactions and Proceedings of the Royal Society of Canada* for 1910, IV, 89-152); (2) "Mathematical Instruction and the Professors of Mathematics in the French Lycées for Boys" (*School Science and Mathematics*, 1913, XIII, 43-56; 105-117).

¹ In normal times men only are allowed to teach in lycées for boys, but 104 women were employed in 1915 in such lycées and 208 in the collèges. (Rept. of U. S. Commis. of Educ., 1916, vol. 1, Washington, 1916, p. 691.)

VIII. GERMANY.

In 1913 Germany or the German Empire was third among European countries in area and second in population, being next after Russia and Austria-Hungary in area, and second only to Russia in population. Politically the Empire is composed of 26 States and divisions: The four Kingdoms—Prussia, Bavaria, Saxony, and Württemberg; six Grand Duchies, the largest of which are Baden and Mecklenburg-Schwerin; five Duchies, of which the largest is that of Brunswick; seven Principalities; the three free towns of Lubeck, Bremen, and Hamburg; and the Imperial Territory, Alsace-Lorraine, which is about the same size as the Grand Duchy of Baden. The Kingdoms embrace more than four-fifths the area of the Empire, which is upward of 208,000 square miles. Prussia alone exceeds three-fifths, and its population of approximately 40,000,000 people is but little less than two-thirds that of the Empire.

There is no such centralization of education in Germany as in France. The only point of direct contact between the Empire and education lies in—

the mutual undertaking of the federated States to bring the law of compulsory attendance to bear upon all subjects of the Empire resident within their respective borders. Of far greater moment is the moral influence exerted upon the other States by the Prussian hegemony, in virtue of which the Prussian educational system comes to be in all essential characteristics typical and representative of Germany as a whole.

In Prussia the highest educational authority is the minister of spiritual and educational affairs,¹ who is appointed by the King. He is usually a jurist by profession and represents his department in the Prussian Parliament.

His individual obligations are numerous. He has charge of the financial affairs of his own department; appoints, with the approval of the Crown, counselors and other officials; confers titles upon teachers; ratifies their appointments, and makes promotions, except when this right has been granted to other authorities; and he is the court of final appeal in all matters connected with this branch of government.²

The ministry of spiritual and educational affairs is organized into departments of ecclesiastical affairs and education. The direction of the department of education devolves upon under-secretaries, who are the minister's deputies, associated with 30 to 35 special counselors; 10 or 15 assistants are also attached to the department. It

¹ Formerly this officer was minister of religious, educational, and medicinal affairs, but in 1911 the department of public health was organized as a separate body. The demand for a separate ministry of education is insistent.

² J. E. Russell, *German Higher Schools*, new edition, New York, 1907, p. 112. I have been more than once indebted to this very interesting and valuable work.

consists of two main divisions: The one which "has charge of common schools, normal schools, high schools for girls, and institutions for the education of defective children"; the other which exercises supervision of higher education, including universities and secondary schools.

The immediate administration and supervision of secondary education in each of the 13 Provinces of Prussia is delegated to a provincial school board,² which is composed of four or five counselors selected by the minister from among directors of training colleges and Gymnasien. The provincial Ober-Präsident is chairman of the board by virtue of his office.

Among other duties prescribed for the board are:

The establishment of regulations for conducting final examinations (Maturitätsprüfungen or Reifeprüfungen) and revision of reports of the same.

Inspection, revision, and direction of the reports of those higher schools³ which admit to the university.

Appointment, dismissal, suspension, and discipline of higher-school teachers (not directors).

SECONDARY SCHOOLS.

The official classification of secondary schools in Prussia is in accordance with the curricula, and the following types are recognized: I. Classical schools: (a) Gymnasium, with nine years' course; (b) Progymnasium, with six years' course. II. Modern schools: (a) With Latin but without Greek (semiclassical)—(i) Realgymnasium (nine years' course), (ii) Realprogymnasium (six years' course); (b) without Latin or Greek (nonclassical)—(i) Oberrealschule (nine years' course), (ii) Realschule (six years' course).

The three leading types of secondary schools are: Gymnasium, Realgymnasium, and Oberrealschule. The following are the names of the classes and their abbreviated forms:

Lower stage:		Upper stage:
Sexta..... VI		Obersekunda..... O II or II A.
Quinta..... V		Unterprima..... U I or I B.
Quarta..... IV		Oberprima..... O I or I A.
Intermediate stage:		
Untertertia..... U III or III B.		
Obertertia..... O III or III A.		
Untersekunda..... U II or II B.		

¹In Prussia there are 10 universities, namely, those at Berlin, Bonn, Breslau, Göttingen, Greifswald, Halle, Kiel, Königsberg, Marburg, and Münster; in the whole Empire there are 21. The others are situated in the Kingdom of Bavaria, at Erlangen, Munich, and Würzburg; in Saxony, at Leipzig; in Württemberg, at Tübingen; in the Grand Duchy of Baden, at Freiburg and Heidelberg; in the Grand Duchy of Hesse, at Giessen; in the Grand Duchy of Mecklenburg-Schwerin, at Rostock; in the Grand Duchy of Saxe-Weimar, at Jena; and at Strassburg, in Alsace. The university at Frankfurt, Hesse, was opened in the autumn of 1914.

² Provinzial-Schulkollegium.

³In Germany secondary schools are called "higher schools" (higher than the elementary schools), and institutions of university rank are called "high" schools.

The schools with six-year courses lack the "upper stage" and are usually found in cities unable to support the full course.¹

Pupils usually enter all of these schools at the minimum age of 9 and spend one year in each class, although some, requiring more time to complete the work, remain longer. To enter a Gymnasium the pupil must have had a three years' preparatory course in reading, writing, arithmetic, and religion. This course may be obtained in the elementary public or private schools, or in the special Vorschulen connected with the Gymnasium. These are preferred by the wealthier classes.²

The Gymnasium is a classical stronghold; French is the required modern language, English being an optional subject. About one-half of the 304 lesson hours in a week³ of the course in the Gymnasium are devoted to linguistic study. Apart from cultural development, the course is intended to prepare students for university specialization in either arts or sciences.

The aim of the Realgymnasium is "to give the youth a liberal education founded especially on instruction in the modern languages, mathematics, and the natural sciences." English takes the place of Greek in the Gymnasium program, and more time is devoted to French and natural sciences.

In the Realschule and Oberrealschule the pupils receive broad preparation for business careers. With this end in view the study of modern languages and of the natural sciences is emphasized.

To formulate more definitely our conceptions of Prussian ideals in standard secondary schools, it will be well to consider the program of the official Lehrplan. While certain optional studies are offered, only exceptional students attempt them, as the regular work is extremely heavy. From IV on, in all of the schemes, music is demanded only of pupils with real ability. The bracketed numbers in two of the tables indicate when hours may be differently distributed.

¹ One reason for the success of the six-year schools has been that boys who graduate from them are granted the much prized "privilege of serving only one year in the army and of entering upon the career of an officer in case they choose the army as a profession." Another reason is that large business houses and banks frequently recruit their apprentices from such graduates.

² In the public elementary schools (öffentliche Volksschulen) the course is usually laid out for eight years, and attendance of all children is compulsory from the ages of 6 to 14, unless a child attends some other school or receives private instruction. The Volksschulen are designed to train good citizens, while the Vorschulen and Gymnasien impart instruction with a view to preparing men of education and culture. The cleavage between Volksschulen and "higher schools" has been much criticised, as it is "founded wholly upon a basis of wealth. The child who passes his tenth year in a Volksschule where tuition is free, instead of a Gymnasium or Realschule where a tuition fee of from \$30 to \$50 is charged, finds himself, as a rule, permanently entered upon a career in which university attendance, preparation for a liberal profession, or qualification for any of the higher governmental positions is definitely prohibited." (See W. S. Learned, "An American Teacher's Year in a Prussian Gymnasium." A Report to the Carnegie Foundation for the Advancement of Teaching. *Educational Review*, 1911, p. 389.)

³ A lesson hour contains about 47 minutes. Classes occur on 6 days of the week, and the school year lasts about 40 weeks.

30. TEACHERS OF MATHEMATICS FOR SECONDARY SCHOOLS.

Gymnasium.

Subjects.	VI	V	IV	U III	O III	U II	O II	U I	O I	Total.
REQUIRED.										
Religion.....	3	2	2		2	2	2		2	19
German.....	4	3	3	2	2	3	3	2	2	26
Latin.....	8	8	8	8	8	8	8	8	8	68
Greek.....				6	6	7	7	7	7	36
French.....				2	2	3	3	3	3	20
History.....	2	2	2	2	2	2	2	2	2	17
Geography.....				1	1	1	1			9
Arithmetic, algebra, geometry, and trigonometry.....	4	4	4	3	3	4	4	4	4	34
Natural science.....	2	2	2	2	2	2	2	2	2	19
Writing.....	2	2	2	2	2	2	2	2	2	19
Drawing.....	3	3	3	3	3	3	3	3	3	27
Gymnastics.....	2	2	2	2	2	2	2	2	2	18
Music.....										
Total.....	30	30	34	35	35	35	35	35	35	304
OPTIONAL.										
Drawing.....						2	2	2	2	
Hebrew.....							2	2	2	
English.....							2	2	2	

Realgymnasium.

Subjects.	VI	V	IV	U III	O III	U II	O II	U I	O I	Total.
REQUIRED.										
Religion.....	3	2	2	2	2	2	2	2	2	19
German.....	4	3	3	3	3	3	3	3	3	28
Latin.....	8	8	7	5	5	4	4	4	4	49
French.....			5	4	4	4	4	4	4	29
English.....				3	3	3	3	3	3	18
History.....	2	2	2	2	2	2	2	2	2	17
Geography.....				2	2	1				11
Arithmetic, algebra, geometry, and trigonometry.....	4	4	3	5	5	5	5	5	5	42
Natural science.....	2	2	2	2	2	4	5	5	5	29
Writing.....	2	2	2	2	2	2	2	2	2	16
Drawing.....	3	3	3	3	3	3	3	3	3	27
Gymnastics.....	2	2	2	2	2	2	2	2	2	18
Singing.....										
Total.....	30	30	34	35	35	35	36	36	36	307
OPTIONAL.										
Geometrical drawing.....						2	2	2	2	

Oberrealschule.

Subjects.	VI	V	IV	U III	O III	U II	O II	U I	O I	Total.
REQUIRED.										
Religion.....	3	2	2	2	2	2	2	2	2	19
German.....	5	4	4	3	3	3	3	3	3	34
French.....	6	6	6	6	6	6	6	6	6	47
English.....				4	4	4	4	4	4	25
History.....	2	2	2	2	2	2	2	2	2	18
Geography.....				2	2	1	1	1	1	14
Arithmetic, algebra, geometry, and trigonometry.....	5	5	6	6	6	5	5	5	5	47
Natural science.....	2	2	2	2	2	4	6	6	6	36
Writing.....	2	2	2	2	2	2	2	2	2	16
Freehand drawing.....	3	3	3	3	3	3	3	3	3	27
Gymnastics.....	2	2	2	2	2	2	2	2	2	18
Singing.....										
Total.....	30	30	34	35	35	35	36	36	36	307
OPTIONAL.										
Geometrical drawing.....						2	2	2	2	

Percentage of time spent on different subjects in the typical schools.

Subjects.	Gymnasium.	Realgymnasium.	Oberrealschule.	Subjects.	Gymnasium.	Realgymnasium.	Oberrealschule.
Religion.....	6.2	6.2	6.2	Mathematics.....	11.2	12.7	14.3
German.....	8.5	9.1	11.1	Science.....	6.0	9.4	11.5
Ancient languages.....	34.2	16.0		Writing.....	1.3	1.2	2.9
Modern languages.....	6.6	15.3	23.5	Drawing.....	2.6	5.2	5.2
History.....	5.0	5.5	5.8	Gymnastics.....	8.9	8.3	5.5
Geography.....	3.0	3.0	4.5	Singing.....	6.9	6.9	6.8

The extent to which these different type schools appeal to the Prussians appears from the following statistics:

Number of schools and of pupils.

Classes of schools.	Number of schools.		Number of pupils.	
	Feb. 1, 1909.	Feb. 1, 1912.	Feb. 1, 1909.	Feb. 1, 1912.
Gymnasien.....	336	342	102,297	108,314
Realgymnasien.....	133	166	41,202	80,319
Oberrealschulen.....	85	102	24,735	41,986
Progymnasien.....	25	30	4,497	3,787
Realprogymnasien.....	45	44	4,878	4,346
Realschulen.....	169	177	33,350	32,421

These figures show that for the three-year period 1909-1912 there was little change in the number of Gymnasien and of pupils attending them, while the increase in this respect in connection with Realgymnasien and Oberrealschulen was very striking. There seems little reason to doubt that these schools will outnumber the Gymnasien in the not very distant future. For more than a century the Gymnasien have been the "center and strength of the German school system; and while schools of different nature have been established with a view to modern economic needs, popular prejudice is so strong that only graduates of the Gymnasien are regarded as cultured."

In 1901 it was enacted that those who are "ripe," that is, those who have passed the Reifeprüfung of the Gymnasien, Realgymnasien, and Oberrealschulen should be admitted to practically equal privileges, with respect to both civil-service preferment and university matriculation. So far as our future teacher of mathematics is concerned, he may have entered the university from any one of the higher schools, but if he had any teachers of Latin and Greek in the secondary schools they must have experienced their fundamental training in Gymnasien. The Reifeprüfung also gives the right of admission to the technical Hochschulen, to the careers of officer in the army and navy, and, ultimately, to State examinations at the end of a university or technical course. Indeed, it is almost the only entrance "to all higher positions of honor and trust in the service of the State, and social recognition in which it is held is correspondingly high."

To obtain a clearer idea of the early mathematical training the future teacher has received, as well as better to appreciate the relation between the teacher's work and his professional training, let us briefly consider the mathematical content of Prussia's secondary-school courses as well as the nature of the emphasis in their presentation.

On entering Sexta at nine years of age the boy is supposed to have the ability to add, subtract, multiply, and divide simple whole numbers. The general scheme of the programs of the Realgymnasien and Oberrealschulen is as follows:

- VI (four or five hours, weekly): *Arithmetic*, fundamental operations with whole numbers; weights, measures, and currency; simple tasks in decimal fractions.
- V (four or five hours): *Arithmetic*, common and decimal fractions; rule of three. *Geometry*, propaedeutic, "enough to get the pupils looking at things from a geometric point of view."
- IV (four or six hours): *Arithmetic*, rule of three problems in calculation of profit and loss, interest, discount. *Geometry*, lines and angles; theory of triangles and quadrilaterals. *Algebra*, propaedeutic.
- U III (five or six hours): *Arithmetic*, commercial. *Algebra*, reckoning with abstract quantities; proportion; equations of the first degree with one unknown quantity. *Geometry*, theory of quadrilaterals; circle and area; constructions.
- O III (five hours): *Algebra*, theory of powers and roots; equations of the first degree in several unknowns; simple equations of the second degree in one unknown. *Geometry*, proportion; theory of similitude; the length of a circumference.
- U II (five hours): *Algebra*, logarithms; equations of the second degree; graphs. Elements of *trigonometry*. Elements of *solid geometry* of plane and line; mensuration; oblique parallel projection.
- O II (five hours): *Algebra*, arithmetic and geometric progressions; compound interest and annuities; imaginary quantities; reciprocal and binomial equations, harder quadratic equations. *Geometry*, harmonic ranges and pencils; radical axes; centers of similitude; power, poles, and polars of circles. Systematic treatment of *solid geometry*. Conclusion of plane *trigonometry*.
- O I (five hours) and U I (five hours): *Algebra*, permutations and combinations, with applications to the theory of probabilities; binomial theorem for any exponent; the most important series; exponential, logarithmic sine, cosine, and arithmetic series of second order; cubic equations; maxima and minima. *Descriptive geometry*. *Geometrical conics*. *Analytic geometry*, conic sections. *Spherical trigonometry*, with applications to mathematical geography and spherical astronomy.

It is only in a general way that such a synopsis as the above conveys a definite idea; this can be more adequately derived from consulting some such texts as those of Heis, and Koppe-Diekmann.¹ The

¹ E. Heis: *Sammlung von Beispielen und Aufgaben aus der allgemeinen Arithmetik und Algebra*, Köln, 1837. Over 300,000 copies of this famous book have been sold; the one hundred and eleventh edition was published in 1912. Koppe-Diekmann's *Geometrie zum Gebrauche an höheren Unterrichtsanstalten* (21. Auflage) 3 Teile. *Lehrbuch für Reallehranstalten* 5. Auflage der neuen Bearbeitung von J. Diekmann. Essen, 1903. There is probably a later edition, but this is the last I happen to have. Plane and solid geometry, plane and spherical trigonometry, geometrical conics, and analytic geometry are all treated in this single work.

See also W. Lietzmann, *Stoff und Methode des Raumlehreunterrichts in Deutschland* (IMUK, Band V, Heft 2), Leipzig, 1912. 84-88 pp.; B. Hoffmann, *Mathematische Himmelskunde und niedere Geodäsie an den*

following official "Notes on method" are also instructive:²

1. In the higher schools the chief object of the mathematical instruction is to provide such a training of the intellect as will enable the pupil to apply correctly in independent work the ideas and knowledge which he has gained. Therefore in all the various divisions of this branch the aim must be to secure a clear comprehension of the propositions to be developed and of their proof, as well as practice and skill in their application. Accordingly, the teacher of mathematics, equally with the teacher of any other branch, is urged to use every opportunity to foster a proper use of the mother tongue, and this point is most particularly to be remembered in the correction of written work, especially of more independent work done at home, which in the upper classes is to be done, as a rule, every four weeks, in addition to the usual class exercises.

2. The instruction in geometry begins with a preparatory course, which, starting with the consideration of simple bodies, develops the power of observation and at the same time gives the pupil opportunities for gaining practice in the use of compasses and ruler.

3. In all schools the most careful attention is to be given to practice in problems of construction, and this practice must accompany the instruction up to the highest classes; but any problem must be excluded which demands for its solution a knowledge of remote principles or special skill in execution. By means of a reasonable selection of exercises, which may be solved by common methods and out of the materials already at the pupil's command, and through the clearness of his instruction, the teacher must awaken in his pupils a feeling of independent capacity and use to the full the formative force of such exercises.

4. The omission of a preparatory course in trigonometry and solid geometry in IIb of the Gymnasium does not preclude that, when these branches are taken up at the later stages, the first instruction should not be of such an introductory character. Trigonometry is to be treated at first by means of figures, that is, geometrically; and, in order to reach as soon as possible the solution of triangles, only such formulæ are to be practiced as are absolutely required for this purpose. Solid geometry is to begin with the consideration of simple bodies (e. g., cubes and prisms), and a more strictly systematic instruction is not to be given till later. In modern schools such introductory courses in trigonometry and solid geometry must be exclusively and under all circumstances followed. Here, as in the earlier stages, models and mathematical wall charts will prove of great assistance in securing proper visualization and thoroughness of instruction.

5. For the highest class of the Gymnasium the program prescribes the introduction of the pupils to the important theory of coordinates and the simplest presentation possible of the fundamental properties of conic sections, which can also be given synthetically. But it is not intended that systematic instruction in analytical or the so-called new geometry should be given. Equally little do the formulæ necessary to understand mathematical geography and astronomy call for any extended treatment of spherical trigonometry. They may be simply deduced from a consideration of the properties of solid angles. Here, too, as elsewhere, care must be taken that, side by side with such knowledge, skill in its application is acquired, and this point of view must determine the selection and the extent of the material of instruction.

6. The defect of the mathematical instruction in the upper stage—viz, that it is too exclusively of the nature of calculation—will be best remedied by practice in geometrical observation and construction. In solid geometry (quite apart from the

¹ *Höheren Schulen*. (IMUK, Band III, Heft 4), Leipzig, 1912. 6+68 pp.; and P. Zühlke, *Der Unterricht im Linearzeichnen und in der darstellenden Geometrie an den deutschen Realschulen* (IMUK, Band III, Heft 3), Leipzig, 1911. 2+92 pp.

² *Centralblatt für die gesamte Unterrichts-Verwaltung in Preussen*, June-July, 1901. See also *Special Reports on Educational Subjects*, vol. 9 (London, 1902), p. 201f.

question of descriptive geometry) it is specially important to prepare and assist the pupil to understand the principles of projective drawing.

7. In the highest class, side by side with the solution of problems in the different branches of mathematics, there must be a comprehensive review of the theories and processes already learned. An opportunity will thereby present itself of giving to the pupils a fuller understanding of the theory of functions with which they have become acquainted at a previous stage.

8. The independent position which mathematics occupies in the curriculum of the higher schools does not prevent—least of all in the upper stage—the instruction from gaining in value, if the pupil learns how the results of this science may be applied to other branches of knowledge, whether they be those of everyday life or physical science, and if opportunity be given for the development of his mathematical sense by practice in application in these directions. Accordingly, it is permissible to make a more extended use of those parts of physics, which admit of mathematical treatment, not only in the physics lesson, but also in the mathematical instruction. In the Gymnasium, it is true, owing to the small number of hours allotted to the branch, such practice can be really fruitful only if the instruction in physics and mathematics be intrusted to one and the same teacher, as has been suggested by the brackets of the time-table. In the modern schools, owing to the greater number of hours there is not an equal measure of necessity for this combination.

To give an indication of the relations between teacher and pupil I quote once more from W. S. Learned's most interesting report (pp. 354-355, 365):¹

The fundamental conception of the teacher is that of one who inculcates a given body of knowledge in the mind of a preferably passive pupil. The ideal teacher is a master of his subject and an expert in its presentation; the ideal pupil is one who is completely receptive. The teacher imparts and drills; the pupil receives and repeats. With this in view nothing is omitted that will assist in focusing the attention directly on the teacher. "Textbooks" such as are used everywhere in America, with introduction, explanatory notes, tables, charts, diagrams, etc., do not exist. Textbooks in Germany are literally confined to the "text" or, in history, to the barest outline of the ground to be covered, while all supplementary material, such as biographical and explanatory notes, suggestions on hard places, the flesh and blood of historical narrative, etc., are provided by the teacher, who sets all forth in the best style at his command in the form of an uninterrupted lecture from which the pupils make what notes they wish. For all of this they are held directly responsible on the following day in an exhaustive oral examination which forms the counterpart to the lecture, the two exercises dividing the time about equally. * * * Voluntary questioning on the part of the pupil was in my experience wholly lacking. The whole spirit of the instruction is against it, and I think it would be regarded in most classes as irrelevant if not impertinent. * * * What we know as school loyalty simply doesn't exist. It is a matter of keen regret on the part of many a warm-hearted director and master, but the fact remains that the German student in both university and school regards his alma mater with the total indifference and apathy which he naturally feels toward, perhaps, the postal system put at his disposal. This is varied only by the sense of freedom and relief with which he leaves the institution which in his mind stands for strain, struggle, and restriction, without the counterbalancing notions of opportunity and sympathetic inspiration.

The most important examinations in connection with the secondary schools are the intermediate examination (Abschlussprüfung) at the end of the sixth school year, and the leaving examination (Ab-

¹ Some aspects of a Gymnasium pupil's activities 40 years ago were given by Hugo Münsterberg in "School Reform," *Atlantic Monthly*, May, 1900, vol. 85, p. 650.

gangsprüfung, Abiturientenprüfung, Maturitätsprüfung, or Reifeprüfung) at the end of the ninth year. This latter is the first real test of the pupil's ability, and we have already seen that those who meet its requirements have essentially advanced in their careers. All secondary schools approved by the minister of education have the right of holding this examination.

The examination committee consists of a commissioner, who is usually the inspector (Schulrat) of the school (appointed by the Provinzialschulkollegium), the director of the Gymnasium, Realgymnasium, or Oberrealschule, as the case may be, and the teachers of Oberprima.

The examinations are both written and oral, and the questions must be such as would be familiar to those in Prima. For the Oberrealschule the written examinations are in (1) a German essay; (2) a French or English essay; (3) a translation from German into French or English; (4) four problems in mathematics—one each from algebra, plane geometry, solid geometry, trigonometry, and analytic geometry; and (5) one problem in physics or chemistry. The subjects of oral examination are religion, French, English, history, mathematics, and either chemistry or physics.

In the written examinations the time allowed for the German essay is five hours, for mathematics five hours, for French or English three hours. The papers in each subject are read by the teachers concerned, and the grades "very good," "good," "satisfactory," "barely satisfactory," or "unsatisfactory," assigned according to the merits of the papers.

After all papers are examined and the errors noted, the committee discusses the attainments of each candidate and decides who should or should not, in its opinion, be admitted to the oral examination. A detailed report of the decisions in respect to the standing of all pupils is then made to the Schulkollegium. To it also are submitted copies of the examination papers and of the answers handed in by the students. These are finally examined by the Schulrat, who approves the decisions or makes such changes as he may deem wise. Some weeks later he sets a date for the oral examination.

At the oral examination pupils are brought in groups of 10 before a disconcerting array of 30 or 40 critical witnesses, many of them garbed in frock coats and white gloves. The Schulrat presides, and others present include a representative of the city authorities and the whole teaching staff of the institution. The teachers of the subjects of the examination take the lead in the quizzing, but at any point the Schulrat may also interject questions.

¹ In this connection the following work is of interest: A. Schaeffer, 1400 mathematische Abiturienten-Aufgaben mit 700 Lösungen und Anleitungen zur Lösung als Übungsbuch für Sekunda und Prima sowie zum Selbstunterricht. Zabern, A. Fuchs, 1910. 16+240 pp.

A candidate fails to pass if he is "unsatisfactory" in German or in both modern languages. He may pass, however, even though "unsatisfactory" in one modern language, if he is at least "good" in the other or in German or in mathematics. If "unsatisfactory" in mathematics, he must be at least "good" in physics or chemistry. In case of failure the last year's or half year's work must be repeated.

The final grades are based not only upon the results of the written and oral examinations, but also upon an inspection of the complete records of the candidate's work during Prima. As compared with the French system two features are especially striking: (1) The large part which the teacher plays in the Abiturient examination; (2) the small percentage of failures—in 1902 only 15 per cent in the Abiturient examination as against 55 per cent in the bachelier examination in France. That the difference in the method of examinations contributes largely to the disparity in the percentage of candidates who pass is usually conceded. A closer study of the comparative questions involved would be of interest. Of course the young Abiturienten are on an average a year older than the bacheliers. Their scholastic training is therefore about equivalent to that of a student who has completed his junior year at one of the best American universities. The average age of such a student is about 22. The following table gives the percentage of Abiturienten of different ages in Prussia in 1907-8.

	Percentage of students aged—				
	17	18	19	20	21 and older.
Gymnasialisten.....	4.9	26.6	30.2	20.0	18.4
Realgymnasialisten.....	4.1	30.1	34.8	18.6	12.4
Oberrealschüler.....	3.6	26.4	34.4	23.9	11.8

Examples of mathematical questions set in Reifeprüfungen in Württemberg and Hamburg are given in Appendix E.

TRAINING OF SECONDARY-SCHOOL TEACHERS.

The regulations with regard to the scientific training of teachers for secondary schools in Prussia have been adopted by 11 other States,¹ and there is no great divergence in the methods of still other States. It should not be inferred, however, that the same method of preparation implies that equal standards are maintained. It is nevertheless true that a candidate who has satisfied the requirements of Leipzig, or Karlsruhe, or Giessen, or Rostock, or Jena, or

¹ Alsace-Lorraine, Mecklenburg-Schwerin, Oldenburg, Brunswick, Mecklenburg-Strelitz, Anhalt, Saxony-Coburg-Gotha, Waldeck, Lubeck, Bremen, and Hamburg.

Brunswick, or of Strassburg has equal privileges in Prussia with the candidate who has passed the corresponding examinations there, just as the latter has equal privileges with the former in Saxony, or Baden, or Hesse, or Mecklenburg, or one of the Thuringian States¹ or Brunswick, or Alsace-Lorraine, or in States which have no examination commission.²

Because of its predominating influence, a description in some detail³ is first given of the Prussian system of training secondary school teachers; then follow brief sketches of the corresponding organization and, when possible, mathematical examination questions for each of the seven other States with a population of not less than 1,000,000, namely: Bavaria, Saxony, Wurttemberg, Baden, Alsace-Lorraine, Hesse, and Hamburg.

1. PRUSSIA.

A candidate for appointment as teacher of mathematics in one of the higher schools must first of all have completed the full course of a Gymnasium or Realgymnasium or Oberrealschule. He must also have studied for not less than 6 semesters (the average is nearer 8 or 10) at one or more of the German universities,⁴ directing his attention specially to those subjects in which he is later to be examined.

The examination is known as the examination pro facultate docendi or the Staatsexamen and is conducted by a royal examination commission (Königliche Wissenschaftliche Prüfungskommission) which is composed for the most part of university professors and schoolmen; the chairmanship is given to a schoolman. There are 10 of these examination boards in Prussia, one in each of the university centers mentioned above.⁵

¹ The Thuringian States are: Schwarzburg-Rudolstadt, Schwarzburg-Sondershausen, Reuss & L., Reuss j. L., Saxe-Weimar, Saxe-Meiningen, Saxe-Coburg-Gotha and Saxe-Altenburg.

² See W. Lorey, *Staatsprüfung und praktische Ausbildung der Mathematiker*, page 90.

³ In this connection I have frequently drawn upon the official regulations of J. F. Brown's work.

⁴ It is only in exceptional cases that a candidate may spend less than three semesters in a Prussian university. For candidates who expect to teach mathematics, physics, and chemistry, regular study in a German technical school (Technische Hochschule) is reckoned equal to study in a German university to the extent of three semesters. A candidate who seeks a teaching certificate in French or English and has devoted himself to linguistic studies along with scientific pursuits in countries where these languages are spoken may, by special permission, have this period of study count for as much as two semesters toward the prescribed period of university work.

⁵ In 1910 the number of members in the examining boards varied from 28 at Königsberg to 45 at Berlin. In all 10 commissions there were 350 members. Of these, 37 were examiners in mathematics, 21 in physics. In pure mathematics the examiners were as follows: *Königsberg*—Meyer and Schoenflies; *Berlin*—Lampe, Knoblauch, and Färber; *Greifswald*—Engel and Vahlen; *Strassburg*—Rosanes, Sturm, Krieger, and Vogt; *Halle a. S.*—Cantor, Wangerin, Gutzmer, and Eberhard; *Kiel*—Pochhammer, Heffter, and Landsberg; *Göttingen*—Klein, Hilbert, and Landau; *Münster*—Killing, v. Lillenthal, and Blankenburg; *Märburg*—Hansel and Neumann; *Bonn*—Study, London, and Hausdorff. All of these are professors in universities except Lampe, of the Technische Hochschule; Färber, professor in an Oberrealschule; and Vogt and Blankenburg, professors in Gymnasien.

A written application for examination must be sent by the candidate to the chairman of the commission in the district where the candidate has spent the last and at least one earlier semester of his university course. The application must specify (1) the subjects and the grades or classes¹ for which the candidate expects to qualify to teach; (2) the field in which the candidate wishes to receive subjects for the two home essays or theses, one in the "general examination" and one in the "special-subject examination."

The application must be accompanied by—

(a) A biographical sketch in the candidate's own handwriting, including a detailed account of his school training and academic studies.

(b) The originals of the certificates showing that the candidate has done the required work at higher school and university.

(c) A statement of military status.

(d) In case the candidate has already received the degree of doctor of philosophy, a copy of the doctor's dissertation and of the doctor's diploma.

The examination consists of two parts:² The general examination and the special-subject examination. Both of these are divided into written and oral parts.

The general examination is for the purpose of testing whether or not the candidate possesses the general culture demanded of all teachers in higher schools. The subjects for examination are philosophy, pedagogy, German literature, and, for the candidate who belongs to the Christian church, religion. The home thesis may be on any one of these subjects, and in it the candidate must not only manifest adequate information and intelligent judgment concerning the subject treated, but also show himself capable of a grammatically correct, logically arranged, clear, and sufficiently skillful presentation. For the oral examination it is required that the candidate (a) shall be acquainted with the most important facts in the history of philosophy and with the principles of logic and psychology, and shall have intelligently read an important philosophical treatise; (b) shall be familiar with pedagogy in respect to its philosophical foundations as well as with the most important phenomena in their development since the sixteenth century, and shall have already attained to some understanding of the problems of his future calling; (c) shall demonstrate that he is familiar with the general course of development of German literature from the beginning of the blooming period in the eighteenth century, and that since leaving school

¹ Certification in individual subjects is of two grades: (a) The second grade for lower and intermediate stages of secondary schools—that is, including Untersekunda; (b) the first grade for the upper stage—that is, including also the classes Obersekunda, Unterprima, and Oberprima.

² Exclusive of the stamp to be placed on the certificate, the fees amount to 80 marks (\$14.28) for the entire examination.

he has also read with understanding for his own further development the important works of the present day; (d) shall show himself acquainted with the content and connection of the Holy Scriptures, with a general survey of the history of the Christian church, and with the principal doctrines of his denomination. The general examination is a formidable ordeal, and as each examiner is usually a university professor and a specialist in the subject the candidate frequently anticipates this examination with greater apprehension than that in the special subjects which he is preparing to teach.

The special-subject examination is to test the acquaintance of the candidate with the subjects which he expects to teach. The State regulations list 17 subjects,¹ in 3 of which,² a major and two minors, the candidate must qualify. Choice is limited in several respects. For example, when pure mathematics is a major, physics must be a minor; and applied mathematics as a major can be chosen only in connection with pure mathematics as a minor. Pure and applied mathematics and physics are frequently chosen together; chemistry with mineralogy, or botany and zoology, or geography are also elected with pure mathematics and physics, but such a third subject as French or English is also possible.

To pass the special-subject examination the candidate must qualify for the first grade in at least one of the subjects and in two more subjects for the second grade; he may be certified to instruct in the first grade, even though in his application he intended to qualify only for the second grade.

The official regulations make the following statements with regard to the requirements of the special-subject examination in pure mathematics, applied mathematics, and physics:

Of candidates who wish to qualify to teach *pure mathematics* there is required: (a) For the second grade, a thorough knowledge of elementary mathematics, familiarity with plane analytic geometry, especially the principal properties of conics, as well as with the fundamental principles of differential and integral calculus; (b) For the first grade, in addition, an acquaintance with the principles of higher geometry, arithmetic, and algebra, of higher analysis and *statics* sufficient to enable the candidate to work out a fairly difficult problem in this field without assistance.

Of candidates who wish to qualify to teach *applied mathematics* there is required, in addition to qualification to teach pure mathematics: Knowledge of descriptive geometry, including the principles of central projection and corresponding facility in drawing; acquaintance with the mathematical methods of technical mechanics, especially of graphic statics with elementary geodesy and the elements of higher geodesy, in addition to the theory of adjustment of errors in observation.

Of candidates who wish to qualify to teach *physics* there is required: (a) For the second grade, knowledge of the more important phenomena and laws of the whole field of

¹ (1) Christian religion; (2) introduction to philosophy; (3) German; (4) Latin; (5) Greek; (6) Hebrew; (7) French; (8) English; (9) history; (10) geography; (11) pure mathematics; (12) applied mathematics; (13) physics; (14) chemistry with mineralogy; (15) botany with zoology; (16) Polish; (17) Danish.

² A candidate is not prohibited from choosing a larger number of subjects.

this science, as well as the ability to prove these laws mathematically as far as this is possible without the application of higher mathematics; acquaintance with the most important apparatus required for school instruction and practice in its use; (b) for the first grade, in addition, a more exact knowledge of experimental physics and its applications; familiarity with the fundamental investigations in one of the more important fields of theoretical physics, and a general survey of the whole field of science.

The second thesis is on a subject for which the candidate seeks a first grade. To finish this thesis as well as that of the general examination a limit of 16 weeks, beginning with the day of delivery of the themes, is granted. Under certain circumstances the examination commission may extend the time not to exceed 16 weeks more. For one of the two theses a dissertation regarded by a Prussian faculty of philosophy as satisfactory for the degree of doctor of philosophy may be accepted as equivalent. The examination commission may require a candidate to stand a written examination on any subject of the special-subject examination. The time allowed for this does not exceed three hours.

The oral examinations occur after the written examinations, and after the theses have been handed in. Upon special request, of the candidate the chairman of the commission can separate the oral examinations of the general and special subject examinations by a period not to exceed three months.

After the oral examinations the commission determines whether or not according to the results of all tests the certificate is to be marked "satisfactory," "good," or "excellent." For the predicate "good" or "excellent" the candidate must have qualified for the first grade in at least two subjects. The certificate is now much less elaborate than formerly. The following is a sample of a certificate at Munster in 1906:

Herr, son of, born the of 1882, at, and of confession, graduated Easter, 1902, at the Gymnasium in and studied mathematics and natural science from Easter, 1902, to Easter, 1903, at Munster, in the summer of 1903 at Gottingen; and from the spring of 1903 to the spring of 1905 at Munster.

On the basis of authorization, dated October 25, 1905, to be admitted to the examination for the position of teacher in the higher schools, he received the following themes for theses:

In mathematics: The geodetic representation of a surface on a plane. In philosophy: The different interpretations of psychophysical law.

The oral examinations took place in the period from November 20 to December 4, 1906:

Herr took the examination for the position of teacher in the higher schools, and according to the entire result of written and oral examination, the certificate is marked "Good"; he is qualified to teach mathematics as well as botany and zoology in the first grade; physics in the second grade.

For information concerning the work of the Seminarjahr, you are referred to the ordinance, dated March 15, 1890, for the practical education of candidates for the position as teacher in higher schools.

ROYAL SCIENTIFIC EXAMINATION COMMISSION.

Munster, December 4, 1906.

(Signed)

CAUER.	SPICKER.
KILLING.	KONEN.
BUSSE.	ZOPP.
FELL.	BELLOWITZ.
GEYSER.	BUSZ.

Under certain conditions examinations covering deficiencies in the first examination are allowed. The percentages of failures in the Staats-Examen in all subjects (there are no special statistics for mathematics) are considerable. For the year 1908, these varied from 20 per cent at Gottingen to 47 per cent at Munster. To most of the mathematical candidates who passed the predicate "Satisfactory" was assigned. The following exact statistics may be of interest:

Results of the Staats-Examen.

Universities.	Satisfac- tory.	Good.	Excel- lent.
Berlin.....	28	6	5
Bonn.....	17	8	0
Breslau.....	3	2	0
Gottingen.....	18	14	2
Greifswald.....	9	0	0
Halle.....	16	5	1
Kiel.....	4	2	0
Konigsberg.....	7	4	2
Marburg.....	23	0	0
Munster.....	13	6	0
Total.....	138	47	10
Per cent.....	70.7	24.1	5.2

Of all dissertations presented in 1908, 34.9 per cent were not accepted. Less than one-half of the Prussian teachers have taken the trouble to secure the doctor's degree, which they look upon as a somewhat unnecessary luxury. For many years no written examination was required of candidates with a university degree.

Anyone who has the above-mentioned certificate can, within the six following years, take an additional examination, either to qualify for instruction in new subjects or to extend the certification already granted, in order to raise the final standard of the certificate, provided he is recommended for admission to such examination by the Provinzialschulkollegium under which he has been working or is appointed to work.

The examination pro facultate docendi was first established in 1810, and by 1826 Prussia had introduced a plan requiring all candidates who had completed this examination to spend a probationary year (Probejahr) as trial teacher (candidatus probandus) in some approved school. In time it became apparent that the Probejahr did

not adequately meet desired requirements, that something should be done to shut out undesirable candidates and to elevate the standard of professional attainments. These considerations and pedagogical experiments made by Ziller at Leipzig, Stoy at Jena, and especially by Frick at Halle, led to a decree in 1890, establishing a Seminarjahr in which the candidates are to be "made familiar with the theory of education and instruction in its application to the secondary schools and with the methods of the individual subjects of instruction; they are also to be introduced to the practical work of the teacher and educator." The Probejahr now serves primarily as a test of the teaching skill won during the Seminarjahr. The following descriptions of the two years of practical training of candidates in Seminarjahr and Probejahr are based upon the revised regulations of 1908:

Seminarjahr.—Application is made to the Provinzialschulkollegium in whose district the candidate wishes to pass the Seminarjahr. The application must be accompanied by: (a) The certificate of having passed the Staatsexamen; (b) a certificate from an official physician attesting that the candidate has the necessary health and the physical qualifications for the profession of a teacher; (c) a statement setting forth the ability of the candidate to support himself during the period of practical training; (d) a statement of military status.

Seminarien are organized in connection with various higher schools, and the Provinzialschulkollegium assigns the candidates to schools at Easter and in the autumn. Not more than six candidates may be assigned in a given year to any one seminar and in their distribution the teaching subjects of the candidates, as well as the specialties of the different Seminarien, are taken into consideration.

The director and the teachers of a Seminar are named by the Provinzialschulkollegium from among the staff of the higher school where the Seminar is located. The systematic training and practice of the candidates is carried out in accord with the following regulations:

For the instruction of the candidates weekly sessions of at least two hours each are to be held during the whole year (vacation time excepted) under leadership of the director or one of the teachers named; to these sessions the other teachers are to be admitted also. In these meetings the academic form of lectures is to be omitted as far as possible; on the other hand, emphasis should be laid on discussion and instruction concerning the requirements of practical school life. The following subjects in particular must be treated:

Principles of training and instruction, together with their applications to the general problems of the secondary schools, and with special reference to the conduct of the subject in which the candidate is preparing to teach.

Historical survey of the evolution of secondary schools, including an account of the leaders in pedagogy and also including comments on the most important present-day situations in the educational world.

The constitution and organization of secondary schools, the official course of study and regulations regarding examinations, the rules regarding student credentials and promotion.

Principles of school discipline illustrated so far as possible by current examples, and by discussions of examples of a concrete type which have come up in the school at

other times, rules of conduct for students, discussions of the relation between school and home.

School hygiene with special reference to equipment of the classroom and to the conduct of the class work.

Supervisory authorities, rules, and regulations governing the official relations of the teacher and class masters (Ordinarien¹), including the forms of official communications and reports.

Provision is also made for (1) visits to classes by the candidate; (2) supervision of the preparations for instruction made in the subject which the candidate is himself to teach, and of the methods of correcting and handing back the work of pupils; and (3) discussion of the personal or other aspects of the trial teaching of candidates.

In the same section of the regulations we find that—

According to the requirements of the chairman, the candidate must deliver short reports on subjects lying within his field; he must also give lectures in which special emphasis should be laid on the training of the candidate in fluent speaking.

Minutes of each Seminar meeting must be prepared by one of the candidates. These are kept as formal record of the work done at the conference. Parts of these minutes, as well as of the topics given to the candidates for their reports and their theses, to be referred to presently, are exchanged between Seminaristen for the mutual stimulation of teachers and directors.

Practice teaching begins as soon as the candidate feels somewhat at home in the institution. The regulations state:

In these lessons the teaching topics, which at first are limited in time and importance, have to be gradually extended according to the ability of the candidates, so that they may have the opportunity to test their own power and to be trained in independent instruction. For these lessons the candidates must outline the subject matter, and, as long as the supervising teacher thinks it necessary, they must prepare a lesson plan.

About once a month the candidates must teach lessons at which, as a rule, in addition to the director, the regular teacher of that subject and the other candidates must be present. These lessons are to be discussed in the general meetings with reference to their plan and development (as noted above); in this discussion attention must be drawn to mistakes which the candidates have made in their preparation, in the pedagogical treatment of the pupils, and in their own attitude before the class.

The candidates must also take part in the direction of play hours, and, when necessary, also of work hours, as well as in the physical exercises of the pupils and in school excursions.

As far as the local school conditions permit, candidates should sometimes be given opportunity to attend lessons in the Seminaristen of elementary teachers and in different kinds of elementary schools.

¹ Each of the nine classes of a higher school has a class master or Ordinarius, who is a regular teacher chosen by the director to act as his representative. "The class master is expected to be a teacher, guide, and friend of his class. All other teachers report to him, and the class record is his special care. Pupils must first seek his advice before going to the director; he becomes personally acquainted with the parents of his boys; he studies the condition of their home life and their conduct out of school; he has charge of the trial teachers who may be assigned to the class, and sees to it that no harm comes from their teaching; and in all faculty conferences he acts as spokesman for his class. From the reports of other teachers he knows just what each boy is doing, and is prepared to talk intelligently with teacher or boy, as the case may be. Thus, he is responsible for the industry, progress, and morals of his charges."

About two months before the end of the Seminarjahr every candidate must hand in a thesis on a topic assigned to him by the director. These theses, in the choice of topics for which the reasonable wishes of the candidates should be considered, are, as a rule, so constructed that they include theoretical considerations and practical applications. They should not involve the treatment of an elaborate literary subject, but they should give the candidate the opportunity, to work out a literary subject within his field and to connect it with his own observations and experiences. Even if the candidate has had a very extensive teaching experience, exemption from this final thesis is not granted.

The titles of some recent theses at the Seminar connected with the Realgymnasium at Reimscheid are as follows:

On instruction in plane geometry for beginners.

Introduction to algebra based upon simple equations in *Untertia*.

The discussion of trigonometry in *Unterssekunda*.

Differential and integral calculus in the mathematical instruction of the *Realschulen*.

At the Seminar connected with the Klinger-Oberrealschule in Frankfurt on the Main the themes assigned included the following:

1901. Three lessons in *Unterprima* of the *Oberrealschule* forming an introduction to synthetic geometry of conics.

1902. The area of the circle. Three lessons in *Obertertia*.

1903. The reflection of light, discussed in four lessons in *Unterssekunda* of an *Oberrealschule*.

1905. The circle and proportion. Four lessons in *Obertertia*—pole and polar with respect to a circle and other conics. Four lessons in *Unterprima* of an *Oberrealschule*.

1907. The triangle in geometry for beginners (*Quinta*). The complete quadrilateral and quadrangle in *Unterprima*. The principle of duality, three lessons. Three lessons on maxima and minima.

1910. Discussion of the *regula falsi* and Newton's method for approximate solutions of an equation. Three lessons.

At least three weeks before the end of the Seminarjahr the director must present to the Provinzialschulkollegium a detailed characterization of each candidate on the basis of personal observation made during his training. This characterization treats of—

the conduct and activity of the candidate during the Seminarjahr, of his ambition, his capacity for scientific work, his ability to teach and the stage reached in his practical training; it further covers his state of health, his financial position, his social attitude, and his behavior toward his colleagues, so that the supervising authorities may be acquainted with special talents as well as with striking shortcomings in the candidate's conduct, ambitions, and attainments. The thesis with the criticism of the director or the teacher in charge¹ and the application for admission to the Probejahr, must accompany the characterization.

If it is the unanimous opinion of the Provinzialschulkollegium that the candidate appears unfit for the teaching profession, the candidate is informed that he can not be admitted to the Probejahr.

In 1909 there were 125 Seminarieren throughout Germany. Of these the 49 in Prussia with a capacity of approximately 300 candi-

¹ The predicates "satisfactory," "good," and "excellent" are employed in this connection.

dates were of two types: (1) The 12 State institutions under the direction of the Provinzialschulkollegien; (2) the much more numerous class of institutions connected with leading Gymnasien, Realgymnasien, and Oberrealschulen, and each under the control of the director and some associates in the school.*

There are many who consider that the Seminar system of Saxe-Weimar is founded on sounder basis than is that of Prussia. In this Duchy the candidate is required to attend lectures on general pedagogy at the University of Jena, to teach at least two hours per week in the university practice school, and to carry on the work of the Seminarjahr and Probejahr in the city Gymnasium where the higher-school practice teaching is done.† In this way prospective secondary-school teachers can avail themselves of more scientific and critical treatment of training than is possible on the part of a busy director and his staff.

In the summer semester of 1909 and the winter semester of 1909-10 the following courses were given at the University of Jena: Elements of empirical psychology (2 hours a week during a semester); applied psychology (2 hours); attention (1 hour); general didactics (2 hours); pedagogic seminary (3 hours a week during a year); psychology (2 hours a week during a semester); neurological diagnosis with practical exercises (1 hour); discovery and treatment of mental weakness in youth, for physicians and teachers; principal problems of ethics and jurisprudence (2 hours); Herbart (2 hours); special didactics (3 hours).

There are other university Seminarien at Leipzig and Giessen, but while most of the universities give courses which are especially valuable for the professional training of teachers, some universities are hostile to closer relations. The Universities of Munich and Wurzburg are of the opinion that "pedagogy as an isolated science is able to produce no creative, scientific work;" that "the university has for its purpose the advancement of the scientific and purely cultural training of students;" that "the union of practice schools with the universities should be opposed not only by considerations of principle, but by great practical difficulties as regards the personnel of teachers, the number of pupils, and local relations.‡"

The following courses were offered at the University of Berlin§ in the summer semester of 1909:

General psychology (2 semester hours); theoretical and experimental exercises in psychological institute (3 hours); the soul of primeval races (3 hours); exercises in race psychology (1½ hours); pedagogic theories from Plato to Rousseau (2 hours);

* The best book on the subject of the gymnasial Seminar seems to be *Das pädagogische Seminar*, by Dr. Karl Neff, of Munich (Beck, 1908). It is the result of 11 years' experience since Bavaria adopted the Prussian system in 1897.

† In his work on *Training of Teacher*, J. F. Brown gives (pp. 92-107), "Regulations for the pedagogical seminar of the university and its practice school at Jena."

‡ Cf. Rep. U. S. Comm. of Educ., 1912-13, vol. 1, Washington, 1914, pp. 823-24.

scientific pedagogic exercises (1 hour). In the winter semester: Psychology with demonstrations (4 hours); lectures, and experimental and theoretical exercises, in psychological institute (10 hours); general psychology (2 hours); the psychological foundations of education (1 hour); ethics (2 hours); Kant's ethics (1 hour); problems of modern culture (2 hours); the theory of instruction (2 hours); scientific pedagogic exercises (2 hours); discussion of experimental pedagogy in connection with lectures (1 hour).

Probjahr.—The Probjahr gives opportunity for consecutive teaching under continual inspection and criticism in an institution which is usually different from that in which the Seminarjahr is spent. The candidate is intrusted, according to his qualifications, with larger connected teaching problems and 8 to 10 hours a week of actual class work. The candidate is also obliged to exercise supervision, to attend teachers' conferences, to assist the director in making out reports and checking lists, and otherwise to identify himself with the life of the school. For this heavy burden of work no compensation is given. In exceptional cases the Provinzial-schulkollegium may employ the probandus as a scientific assistant, for substitute teaching or for additional work.¹ In this case the candidate receives remuneration; he also receives the right to vote in teachers' conferences on all questions which concern the classes he conducts.

The director must watch the conduct and activity of the candidate, visit him from time to time in his lessons, and draw his attention to accidental mistakes and, if necessary, warn him seriously, by indicating the consequences of disregarding this advice.

The teacher in charge of the training of the candidate is obliged to attend the candidate's lesson very often in the beginning, later at least twice a month; to examine the papers which he has corrected; and to advise him concerning matters outside of instruction.

The candidate should learn to use critically Reidt's *Anleitung zum mathematischen Unterricht*, Simon's *Didaktik*, and Höfler's recent *Handbuch des mathematischen Unterrichts*; similarly with the handbooks and encyclopedias of Killing-Hovestadt, Thieme-Färber, Schwering, and Weber-Wellstein.² The candidate should also work through F. Klein's *Vorlesungen über Elementarmathematik vom höheren Standpunkte aus*.³ The study of the history of mathematics is also specially recommended to the candidate if he has already carried on some historical study in mathematics.

¹ In such cases the amount of class teaching may be as much as 20 to 24 hours a week.

² W. Killing und H. Hovestadt, *Handbuch des mathematischen Unterrichts*. Leipzig, Teubner, 1910-1913. 4 vols., 456+482 pp.

³ H. Thieme und C. Färber, *Grundlehren der Mathematik*. Leipzig, Teubner, 1909-1912. 1 vols.

K. Schwering, *Handbuch der Elementarmathematik für Lehrer*. Leipzig, Teubner, 1907. 416 pp.

H. Weber und J. Wellstein, *Enzyklopädie der Elementar-Mathematik*. Leipzig, Teubner, 1909-1912. 4 vols.; 550+608+548+681 pp.

⁴ A sketch of the teaching, in German universities, of elementary mathematics from higher standpoints, is given in Lorey's *Das Studium der Mathematik an den deutschen Universitäten*, pp. 263-271.

As evidence of the amount of pedagogical insight obtained, the candidate must present to the director toward the end of the Probejahr a report concerning his own work as teacher.

At least three weeks before the close of the year the director reports concerning the candidate to the Provinzialschulkollegium. The nature of the report is similar to that described above in connection with the duties of the director of a Seminar. On the basis of these reports, of the observations of the district advisers (Schulräte) and of the result of the two years of practical training, the Kollegium soon decides whether or not the candidate possesses *Anstellungsfähigkeit* or qualification to have his name inscribed on the list of teachers eligible to appointment in the higher schools.

If there is doubt as to the candidate's qualifications, the Probejahr is prolonged six months. In any case a certificate concerning the teaching ability of a candidate can never be granted if—

in the meantime it has been proved that the candidate either through physical defects or through unavoidable pedagogic defects is unable to fulfill his duties as a teacher or educator of youth, or if the candidate on account of his conduct within or without the school appears unfit for the teaching profession.

It rarely happens that a candidate fails of certification at the end of the Probejahr, since his defects are almost sure to be brought to light in the Seminarjahr which is extended for a year or half year at another seminar when any doubt exists as to his qualifications for the Probejahr.

Teachers in all secondary schools, whether under State, city, or other patronage must be selected from the provincial list. But while State institutions are required to select their teachers with the desired qualifications in order of seniority on the list, other schools are free to select, regardless of order, from the six highest on the list. In recent years the appointment of teachers to positions has followed very soon after their names have been placed on the official list. For interesting details regarding appointment and promotion of teachers, the reader may consult J. E. Russell's work (pp. 370-381), written about 18 years ago.¹

To make more apparent the opportunities of the future teacher for studying mathematics at the university it may be well to consider details concerning specific cases. As even in Germany the very large number of courses in pure and applied mathematics which are offered each year at the University of Göttingen are quite exceptional,² the following illustrations are cited: (a) The courses offered

¹ An interesting discussion of the preparation of teachers in Prussia is contained in W. Killing's "Bemerkungen über die Ausbildung der Gymnasiallehrer," *Jahresbericht der deutschen Mathematiker-Vereinigung*, Band 22, 1913, pp. 20-34.

² For an account of mathematics at Göttingen, see Lorey's *Das Studium der Mathematik an den deutschen Universitäten*, pp. 347-356.

at the University of Berlin during the summer semester of 1914; (b) the actual list of courses of study (with the exception of those in botany, zoology, and philosophy) which an Oberlehrer took at the University of Bonn in preparation for his present position. He spent the fourth semester of university study at Berlin, where he heard lectures on the theory of numbers by Frobenius and on algebraic equations by J. Schur.

(a) Derivation of the most important properties of conic sections by means of elementary geometry (2 hours a week) by Schwarz; analytic geometry (4 hours), Schottky; differential calculus (4 hours), Knopp; integral calculus (4 hours), Knoblauch; theory of determinants (4 hours), Frobenius; theory of infinite series, infinite products, and continued fractions (4 hours), Knopp; introduction to the theory of ordinary differential equations (2 hours), Hettner; space curves and curved surfaces (4 hours), Schwarz; analytic mechanics (4 hours), Lehmann-Filhés; applications of elliptic functions (4 hours), Knoblauch; calculus of variations (4 hours), Schwarz; theory of theta functions (4 hours), Schottky; exercises in differential calculus (2 hours fortnightly), Knopp; exercises in differential and integral calculus (1 hour), Knoblauch; exercises in mechanics (1 hour), Lehmann-Filhés; mathematical colloquium for advanced students (2 hours fortnightly), Schwarz; mathematical seminary (2 hours), Schwarz, Frobenius, Schottky (each professor directed the seminary once in three weeks).

(b) 1. Semester (1906): Differential calculus with exercises, Kowalewski; descriptive geometry with exercises in drawing, London; experimental physics (electricity and optics), Kayser. 2. Semester (1906-7): Laboratory physics for beginners, Kayser; experimental physics (mechanics, heat, acoustics), Kayser; integral calculus with exercises, Kowalewski; elements of analytic geometry with exercises, London; introduction to algebra, Schmidt; theory of determinants, Schmidt. 3. Semester (1907): Laboratory physics for beginners, Kayser; experimental physics (repeated), Kayser; physical units and constants, Kaufmann. 4. Semester (1908): Selected chapters in analytic geometry, London; foundations and history of higher analysis, Kowalewski; readings and reviews of selected writings of Leibnitz and Newton, Kowalewski; mathematical seminary. 5. Semester (1908-9): Fourier series and applications, Kowalewski; foundations of the theory of aggregates, Kowalewski; theory of electricity with exercises, Pflüger; mathematical seminary. 6. Semester (1909): Differential geometry, study; differential equations, Kowalewski; theory of light, with exercises, Pflüger; elements of the theory of elections, Bucherer; mathematical seminary. 7. Semester (1909-10): Synthetic geometry with exercises, London; introduction

¹W. Lorey, *Das Studium*, etc., pp. 342-43.

to theory of functions, Kowalewski; mechanics, Pflüger; physics colloquium, Kayser; applied theory of physics, Eversheim; mathematical seminary. 9. Semester (1910): Analytic geometry, second part (repetition), London; foundations of geometry, Hessenberg; mathematical seminary.

In the tenth semester this candidate passed the Oberlehrerprüfung and was certified as having scientific knowledge requisite for teaching mathematics, physics, botany, and zoology in the first grade. The combination of botany and zoology with mathematics and physics appears to be very common at Bonn as well as at other universities.

2. BAVARIA.

The method of training teachers for higher schools in Bavaria differs considerably from that in Prussia. In what follows there is given a description of (I) the training based upon regulations in force from 1895 to 1912; (II) the training required by decree of September 4, 1912.

I. The training consists of (a) an examination (Lehramtsprüfung) and (b) a so-called Seminarjahr. The Lehramtsprüfung is divided into two sections, the first at the end of four semesters of study in a university or technical school, and the second after eight semesters at a university or technical school.

In the first section of the examination the candidate is liable for examination in the following subjects: (1) Algebraic analysis and algebra, including equations of the third and fourth degree; (2) plane and solid geometry; (3) plane and spherical trigonometry; (4) elements of differential and integral calculus; (5) analytic and synthetic geometry of conics; (6) elements of descriptive geometry. About four hours are allowed for the examination in each of these subjects. Then five hours are set aside for (7) a German essay on a theme the treatment of which displays the general culture of the candidate. For each candidate the oral examination, lasting an hour, covers the subjects of the written examination and the foundations of physics. The following questions were set in 1908:

Algebraic analysis (2 hours).—Prove that the series

$$1 + \frac{\cos \phi}{1!} + \frac{\cos 2\phi}{2!} + \frac{\cos 3\phi}{3!} + \dots$$

converges for all values of ϕ ; calculate the sum of this series and express the sum in real form.

Algebra (1½ hours).—For what values of λ does the equation $x^3 - 5x^2 + 2x = \lambda$ have the property that the difference of two roots is equal to 3? Determine the corresponding system of solutions of the equation and express the linear relation between λ and x .

¹ The current questions are to be found in *Bay. Zeitschr. f. Realschulwesen* or *Blätter f. d. Gymnasial-schulwesen*. Questions set for many years have been collected in *Prüfungsaufgaben f. d. Lehramt der Math. und Phys. a. d. Kgl. bayr. hum. u. techn. Unterrichtsanstalten (1873-1893)*. Ansbach, C. Brügel u. Sohn, 1894.

Plane geometry (2 hours).—Any three points A', B', C' are taken on the sides BC, CA, AB , respectively, of a triangle ABC . Prove that—

- (1) the circles circumscribed about the triangles $A'B'C, B'C'A, C'A'B$ meet in a point;
- (2) the centers of these circumscribed circles are the vertices of a triangle similar to the triangle ABC .

Solid geometry (1½ hours).—On an equilateral triangle the radius of whose circumscribed circle, r , is given, a right pyramid of height h is constructed. From the middle point of the circumscribed circle of the base perpendiculars are dropped on the faces of the pyramid and through each pair of perpendiculars planes are drawn. Find the common area and volume of any one of the resulting pyramids and the original pyramid.

Plane trigonometry (2 hours).—In a triangle ABC there is given $AB=c=9$, the length of the line segment $CC'=1=50$ (where C' is the middle of the side AB), and the difference of the angles A and B for $a-b=8=9^\circ 49' 8''$. Find the other two sides, first in general terms and then numerically.

Physical trigonometry (2 hours).—At a place whose latitude is $\phi=48^\circ 8'$, and at 3 o'clock in the afternoon on the longest day (declination of the sun $\delta=23^\circ 27'$) it is desired to find the approximate cardinal points of the compass from the position of the sun, and accordingly it is supposed that the sun is in the southwest. How great is the error of this orientation?

At what hour of the day would this orientation error, due to interchange of azimuth and hour angle, be a maximum on that day and at that place?

Differential calculus (1½ hours).—If $u=f(x)$ and $v=\phi(x,y,z)$, find the derivative

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial x} \frac{\partial v}{\partial y} + \frac{\partial u}{\partial x} \frac{\partial v}{\partial z}$$

and express them as derivatives of f and ϕ .

Integral calculus (2 hours).—Prove by partial differentiation that

$$\int_0^{\infty} \frac{\sin x}{x} dx + \int_0^{\infty} \left(\frac{\sin x}{x}\right)^2 dx = 1, \quad \int_0^{\infty} \left(\frac{\sin x}{x}\right)^3 dx = 3, \quad \int_0^{\infty} \left(\frac{\sin x}{x}\right)^4 dx.$$

Analytic geometry of conics (2 hours).—A fixed tangent a of a parabola meets the tangent at a variable point P in T . The line segment PT is divided at Q so that $\frac{PQ}{QT} = \lambda$. Show that—

- (1) the locus of Q is a parabola tangent to the given parabola at the point A where a is tangent to the curve;
- (2) the foci of Q corresponding to different positions of a are congruent.

Synthetic geometry of conics (1½ hours).—In a plane two lines g_1, g_2 and two points P and Q are given. Determine that line through P such that the part cut off by g_1 and g_2 is seen from Q under a right angle.

Elements of descriptive geometry (4 hours).—A plane (s_1, t_2) is given by its traces, also a point A by its projections A_1, A_2 . A circle through A which touches the first trace s_1 and has at A a tangent parallel to s_1 is projected orthogonally on the plane (s_1, t_2) . Draw—

- (a) the projection plan;
- (b) the true form of this projection;
- (c) the cylinder employed in the projection; and
- (d) its development (Abwicklung) so far as it is bounded by the plane (s_1, t_2) and the circle-plane.

The conditions to be satisfied before admission of a candidate to the second section of the Lehramtsprüfung are: (a) He shall have studied at a university or technical school for at least four years, of

which seven semesters have been devoted to the study of mathematics and physics; (b) he shall have taken courses in laboratory physics and theory or history of pedagogy, and two courses in the second section of the philosophical faculty, especially one in inorganic chemistry; (c) he shall have completed a scientific thesis in the field of pure or applied mathematics or physics.

The titles of some theses in recent years are as follows:

- On the radii of curvature¹ of the C_n , their envelope and pedal curve with respect to the center.
- Envelope of the planes which cut off from a trirectangular trihedral angle a tetrahedron of constant area.
- To exhibit three coplanar concurrent rays as the central projection of three rays concurrent in space.
- Discussion of Rosenhain's threefold periodic functions with the aid of Weierstrass' "p" and "σ" functions.
- Chissettes of points of curves tangent to a given line in a fixed point.
- On space W-curves and W-surfaces.
- On the geometric foundations of function theory based upon hyperbolic measurement.
- On Laguerre's contact transformations.
- Groups of substitutions which may be analytically exhibited as functions of the form $x^2 + y^2 + z^2 = 1$.
- The employment of elliptic functions for the theory of the point systems of a general curve of the third degree.
- Presentation of the methods of solution of the equation of the fifth degree in their historical development.

The oral examination consists of questions on the thesis, and on analysis, geometry, analytic mechanics, and physics. In particular such subjects as foundations of geometry, theory of functions, differential equations, differential geometry of plane and space may be discussed. In conducting the examination the special line of study of the candidate is taken into account. Furthermore, emphasis is laid on maturity in connection with subjects of the first examination. In mathematics, especially careful consideration is bestowed on those topics whose mastery tends to a truer understanding of subjects taught in secondary schools. In physics the candidate's facility in experimentation is tested. Theory and history of pedagogy are also subjects for examination. The whole examination lasts two hours.

Both parts of the Lehramtsprüfung are held in Munich before an examination commission of the royal State ministry.

The examination regulations contain yet another requirement. The candidate has to give a trial lesson, (Lehrprobe) on an assigned theme in one of the middle schools of Munich before the assembled examination board. This test of teaching ability, for which a special predicato is assigned, is very peculiar in that the candidate is tested

¹ I have so translated *Krümmungsradien* (p. 73 of Wieleitner's report on Bavaria).

in something concerning which he has never been taught. At best it can only tend to indicate his natural ability. On the other hand a naturally good instructor may be confused by the large audience and the fact that he is not acquainted with any of his pupils. Nevertheless, every candidate who satisfactorily passes the Lehramtsprüfung and the Lehrprobe test is declared competent to teach all subjects in secondary school mathematics.

Following the example set by Prussia, certain Seminarien for the training of teachers in ancient classics were established in 1893. It was not, however, till the spring of 1904 that two mathematical Seminarien came into being at the Theresiengymnasium and at the Realgymnasium in Munich. By 1906 two others had been established: One at the Realgymnasium in Augsburg and the other at the Realgymnasium in Würzburg, but up to June, 1912, these four institutions were the only establishments of their kind in Bavaria. Since none of these Seminarien could train more than 6 or 8 candidates annually, while 40 to 50 were coming from the universities, training in a Seminar was not required for the candidate in mathematics. With the new regulations of 1912 the Seminarjahr became compulsory.

In the mathematical Seminar there are two special teachers (Seminarlehrer, one mainly in mathematics, the other in physics) in addition to two members of the regular staff who are Seminarleiter, or teachers who conduct the training. Of the four teachers only two are specialists in mathematics and physics.

Candidates enter the Seminar at the end of November; in the first weeks they observe the instruction of their Seminarlehrer and of the other teachers in the institution and after that give trial lessons (Probekationen) at first for half an hour and later for the full hour. On these occasions the other candidates and the Seminarlehrer are always present. At the special Seminar meetings, which usually take place twice a week, the candidates must report concerning all classes which they have attended either as instructor or observer. At these meetings the "Praktikanten" hear discussions of a pedagogic-didactic nature and give lectures on general pedagogic questions or on methods of treating certain parts of mathematics and physics. The trial lessons of the Praktikanten are here criticised and every lecture is followed by general discussion. At the meetings the Seminarleiter and Seminarlehrer also give lectures on general questions, on duties as a teacher and to society, and on special topics of didactics and method. The number of lectures which each candidate gives varies from two to six. Themes employed for such lectures are as follows: Herbart's views on mathematical instruction; Klein's endeavors for reform; quadrature of the circle in instruction; mathematical instruction in France; hygiene in the schoolroom; the handling of lying in the school; the best texts in different sub-

jects; how is rest in connection with instruction best taken? the slide rule; suicide of pupils;¹ review of new books; methods of construction according to Adler, Petersert, and Alexandroff; and similar themes in physics.

Minutes of all Seminar meetings must be kept by the candidates in turn. In this way each one has to prepare eight to ten sets of minutes. Before May 1 each one must also hand in a larger pedagogic thesis of which the theme was assigned the preceding January in consultation with the Seminarleiter and Seminarlehrer. The theses are discussed in Seminar meetings and written judgments are passed upon them. A selection of subjects (in so far as they are not in pure physics) assigned at the four Seminarerien is as follows:

Of what significance for instruction in mathematics and physics are the six "interests" of Herbart? Appreciation of the attack of Ellen Key, Forel, and other moderns leveled against the traditional methods of instruction. Comparison of the light and dark sides of the system of specialist teachers and of class teachers. Self-criticism as one of the most necessary conditions of progress in successful instruction. Relation between pupil and teacher both within and without the school. The question of overworking and the humanistic Gymnasium of the day. What place should art occupy in a scheme of instruction? Comparative consideration of programs in mathematics and physics at various German institutions. Mental arithmetic. Teaching of fractions by question and answer. Graphic methods in the instruction of arithmetic and algebra as well as of trigonometry. How is the idea of a function to be developed according to the program of the Bavarian Oberrealschule? In the interests of concentration in mathematical instruction in the upper classes, how can the study of physics or natural science be promoted? (To be discussed on the basis of the program of Bavarian secondary schools.) How can the interest of a pupil be aroused to recognize mathematical manifestations in daily life? The German Museum² and the humanistic Gymnasium. Introduction to the similarity of figures. Theory of equality of areas and transformation of areas. Weights and measures in arithmetic instruction.

At the close of the Seminar course the authorities notify the ministry of approved candidates who in due course receive Befähigungszeugnisse as teachers. There is no Probejahr as in Prussia; and it is also to be observed that the Seminarerien of Bavaria, organized for the most part to train in a special subject, are, in this respect, different from those in Prussia.

At the University of Munich in the summer semester of 1909 the following courses were offered: Psychology, with special reference to pedagogic questions involved (4 hours a week); psychology (4 hours); psychological exercises (2½ hours); art and race psychology (2 hours); anthropological psychology (4 hours); introduction to ethical problems (2 hours); introduction to problems of sociology (3 hours); his-

¹ This is a serious matter in Germany. That it is largely due to overwork seems to be established by the appalling condition of recruits for the army and navy from secondary schools of Prussia. More than one-third of the students are found physically unfit (see Rep. U. S. Commis. of Educ., 1909-10, vol. 1, Washington, 1910, pp. 482-3).

² Presumably the deutsches Museum von Meisterwerken der Naturwissenschaft und Technik, in Munich, of which the corner stone was laid in 1906.

tory of pedagogic theories (4 hours). In the winter semester, 1909-10: General psychology (5 hours); psychology seminary (1½ hours); psychological exercises (1½ hours); psychology of Aristotle (1 hour); fundamental problems of ethics (4 hours).

In Bavaria, university "vacation courses" for those who are teaching in secondary schools are often largely attended. Such lectures were given for example at: (a) Würzburg, April 21-27, 1908—Dr. Rost, 6 lectures on the productiveness of the employment of graphical aid in carrying out geometrical constructions, and Prof. Prym, 6 lectures on the derivation of fundamental functions in passing from lower to higher analysis; (b) Erlangen, April 13-19, 1909—Prof. Gordan gave 2 lectures on corresponding conics, Prof. Noether 2 lectures on the variation principle in mechanics, and Dr. Hilb 2 lectures on relations between higher and elementary mathematics.

Courses in the history of mathematics are not often given in Germany, but at the University of Munich Dingler is a privat-docent for the didactics, philosophy and history of mathematical sciences. His appointment in 1912 was the first of the kind.

II. Regulations of 1912. The candidate is still required to pass two examinations, namely, the two sections of the Lehramtsprüfung, but the first section does not come till after at least four years of study in a university or technical high school and the second section after the completion of a Seminarjahr. There is also a voluntary special examination.

The candidate's choice of subjects is more circumscribed in Bavaria than in Prussia. He is required to make selection from one of the following groups:

1. Classical languages, German language, and history.
2. German language, history, French or English language.
3. Modern languages.
4. Mathematics and physics.
5. Chemistry, biology, and geography.
6. Commercial science and geography.
7. Drawing.

Mathematics and physics must be chosen together, and no other subject may be elected with them.

For admission to the first division of the Lehramtsprüfung in mathematics and physics the following are required:

- (a) The Reifezeugnis of a Gymnasium, Realgymnasium, or Oberrealschule.
- (b) The proof of study at a German university or technical high school for at least four years, at least three of which have been devoted to attending lectures on mathematics and physics, to laboratory practice, and to drill in exercises.
- (c) A certificate of successful completion of courses of exercises in mathematics and physics.

- (d) Proof of attending two courses of philosophic, historic, or geographic content.
- (e) The presentation of two "Zulassungsarbeiten," which must be authenticated by directors of exercise drill or laboratory practice; one "Arbeit" of moderate scope, on some topic in mathematics or theoretical physics, should have been prepared in seminary or similar exercises, the other Arbeit should be in the nature of a report of work done in laboratory physics. The Zulassungsarbeiten must each display familiarity with the scientific subject and its literature, a methodical development, and well-ordered presentation.

In these conditions it is notable that practical exercises in both mathematics and physics are required and that certificates of the successful pursuit of these exercises are to be presented.

The first division of the examination is both written and oral. The written examinations all take place within a few days (again in contrast to Prussian arrangements, where they extend over months), and are arranged in three groups:

GROUP 1:

- (a) Elementary plane and solid geometry (2 hours).
- (b) Plane and spherical trigonometry with simple applications to mathematical geography (2 hours).
- (c) Descriptive geometry, including the elements of astronomy and perspective with applications, e. g., to construction of shadows (drawing very carefully done) (2-3 hours).
- (d) Algebra (2 hours).

GROUP 2:

- (a) Analytic and synthetic plane and solid geometry (2-3 hours).
- (b) Differential and integral calculus with applications (2-3 hours).
- (c) Elements of the theory of ordinary and partial differential equations and the elements of differential geometry (2-3 hours).
- (d) Theory of series and elements of the theory of functions (2-3 hours).

GROUP 3:

- (a) Experimental physics, Division I (mechanics, acoustics, theory of heat) (2 hours).
- (b) Experimental physics, Division II (electricity, magnetism, optics) (2 hours).
- (c) Analytic mechanics of rigid and deformable bodies (2-3 hours).
- (d) Elements of theoretical physics (2-3 hours).

In addition to these written examinations, the candidate is given four hours to write a German essay to test his general culture. The subject of the essay must be selected from one of three fields stated to him.

The elaborate home essay, which is the central feature of the written examination in other States and which is regarded as of special value in determining cultural development, is entirely lacking in Bavaria.

In the oral examination, which is not public, there is more searching questioning in certain fields of the written examination; these fields include the foundations of the theory of space (Group 1), the theory of numbers (Group 2), and chemistry (Group 3). The candidate is also given opportunity to prove his familiarity with some special field of mathematics or physics which he has himself chosen.

The oral examination in each of the groups 1 and 2 lasts for 40 minutes; in group 3, 50 minutes.

Between the first and second divisions of the Lehramtsprüfung the candidate receives his practical or professional training in a Seminar, to which he is assigned by the ministry. The training here has been described already.

In the second division of the examination the candidate—

(a) Presents his home thesis.

(b) Gives lessons of one hour each in some topics of mathematics or physics (in connection with which opportunity is given to indicate his facility in sketching on the blackboard and in experimentation), and conducts an exercise in physics for pupils. The topics are assigned one day in advance.

(c) Is examined orally on the theory of education and instruction, especially of secondary school pedagogy and its history, as well as on related fundamental questions in philosophy and psychology. In addition to the general knowledge, the candidate must show a more thorough acquaintance with the pedagogic theories and cultural development in some modern epoch selected by himself. Connected with this are questions on the newer development of methods of instruction in topics of mathematics and physics, whereby the candidate is given an opportunity to demonstrate his acquaintance with separate phases of development in mathematics and physics.

A candidate who fails in the second section of the examination must repeat the Seminarjahr before he is permitted to try the examination again. A second repetition of the examination is not permissible. The examination is conducted almost entirely by schoolmen, including the Seminar director and Seminarlehrer.

The optional "special examination" was a feature of regulations in force from 1873 to 1895. In somewhat modified form it reappeared in 1912. This examination must be taken within 10 years after the second division of the Lehramtsprüfung; it may, however, be taken at the same time or even before, but in this latter case a certificate of having passed the examination will not be awarded until the Lehramtsprüfung has been completed.

For admission to the special examination, there is required either the presentation of a scientific thesis¹ in the field of pure or applied mathematics or of physics, or the presentation of a practical work, with proof of thorough scientific study in one of the following fields: Geodesy, astronomy, technical mechanics, technical physics, electro-technical science.

The first part of the oral examination is on the field from which the subject of the scientific thesis is taken and lasts about an hour; the second part covers the whole range of the special subject chosen and

¹ Or prize essay or doctor's dissertation or other printed paper.

lasts at least an hour and a half. No second trial of this examination is permissible.

Note that a scientific thesis and a piece of practical work are put on a plane of equality in this examination, of which there is no analogue elsewhere in Germany, although it is somewhat similar to the examination for doctorate. In the earlier regulations it was ordered that those who passed the special examination, even though it involved nothing of a pedagogic character, were to get the better positions in the secondary schools. This is not necessarily the case in the newer regulations, and just what the effect of the examination will be, remains to be seen.

In illustration of the excellence of the pedagogically arranged lists of courses offered at the University of Munich, that for the summer semester 1914 is given: Introduction to higher mathematics for students of philosophy (3 hours, including one for exercises), Dingler; differential calculus (5 hours, including one for exercises), Böhm; exercises in plane analytic geometry (2 hours), Dingler; descriptive geometry II (5 hours), Hartogs; exercises in descriptive geometry (3 hours), Hartogs; analytic geometry of space (4 hours), Voss; integral calculus (5 hours), Lindemann; definite integrals and Fourier series (4 hours), Pringsheim; synthetic geometry II (4 hours), Rosenthal; exercises in synthetic geometry (1 hour), Rosenthal; in seminary, exercises for middle semesters (higher analysis with applications) (2 hours), Rosenthal; theory of algebraic forms (invariants) (4 hours), Lindemann; introduction to the theory of partial differential equations (4 hours), Voss; seminary, theory of ordinary differential equations (2 hours), Voss; applications of elliptic functions (3 hours), Pringsheim; on the problem of squaring the circle (2 hours), Lindemann; seminary (2 hours), Lindemann; geometric morphology (3 hours), Brunn; elementary theory of life insurance for natural scientists, political economists, and mathematicians (4 hours, including one for exercises), Böhm; lectures and reviews of mathematical-statistical questions, for advanced students (2 hours), Böhm. Mechanics of the continua (hydrodynamics, acoustics, elasticity) (4 hours), Sommerfeld; seminary exercises on hydrodynamics, etc. (2 hours), Sommerfeld; physics of the science of earthquakes (4 hours), Bidlingmaier; mechanics of the heavens II (4 hours), Seeligor.

3. SAXONY.

The regulations formulated in 1908-9 for the preparation of teachers of mathematics for the secondary schools of Saxony differ very little from those promulgated in Prussia in 1898. The examination consists of two parts, the general and the special subject. The home theses are to be finished before the oral examination. The general examination is on philosophy, pedagogy, and German literature, but

not on religion. The special subject examination, more limited than in Prussia, is confined to—(1) pure mathematics; (2) applied mathematics; (3) physics; (4) chemistry; (5) mineralogy and geology.¹

The candidate must choose at least three subjects; it is desirable that he be also prepared in a fourth; and he is not prohibited from taking all five. The following combinations are permissible:

- (a) Pure mathematics, applied mathematics, physics.
- (b) Pure mathematics, physics, chemistry.
- (c) Chemistry, mineralogy with geology, physics.
- (d) Chemistry, mineralogy with geology, pure mathematics.

For home theses the candidate is given two problems, one for the general examination, the other for the special subject examination in that subject in which he wishes to prove himself capable of first grade. In case this subject is pure mathematics he receives, in addition to the problem for the general examination, two problems for the special subject examination to work at home, of which one must be in mathematics; in this case the candidate has no written examination, but otherwise written examinations in all subjects of the special subject examination, to the extent of four hours, may be required. For the preparation of each home thesis eight weeks are allowed. The whole period of examination in Saxony as well as in Prussia thus extends over several months instead of a few days as in Bavaria.

The regulations with regard to pure mathematics and physics in the special subject examination are practically identical with those in Prussia, but in applied mathematics, the requirements are somewhat different by reason of the final clause. The whole paragraph is as follows:

Of candidates who wish to qualify for instruction in applied mathematics there is required, in addition to a certificate in pure mathematics, knowledge of descriptive geometry, including the principles of central projection, and corresponding facility in drawing; familiarity with the mathematical methods of technical mechanics, especially of graphical statics, with lower geodesy and the elements of higher geodesy and with the theory of adjustment of errors in observation. The candidate is free to indicate in which one of the three parts he would rather be examined.

The preparation for the special subject examination usually occupies not less than four years and this may be made either at the University of Leipzig or at the Technische Hochschule in Dresden. For the student whose special subjects are pure and applied mathematics and physics, the following scheme of courses is recommended at the Technische Hochschule:

¹ There seems to be a discrepancy in A. Witting's report unless different regulations govern candidates studying at the University of Leipzig from those at the Technische Hochschule. The regulations quoted above went into effect for the Hochschule January 25, 1909 (p. 57). On p. 47, however, in connection with regulations in force May 1, 1908, for the university, German is not mentioned as a general examination subject, while nine instead of five subjects are mentioned as possible special subjects: the other four are history, botany, zoology, and geography. Geography might be elected instead of chemistry by the mathematician. Cf. Lorey's report on Prussia, p. 66.

1. Semester (summer).
 - Higher Mathematics I (analytic geometry and higher analysis), with exercises.
 - Descriptive Geometry I, with exercises.
 - Inorganic chemistry.
 - Inorganic chemistry in laboratory.
2. Semester (winter).
 - Higher Mathematics II, with exercises.
 - Descriptive Geometry II and perspective, with exercises.
 - Technical Mechanics I, with exercises.
 - Experimental Physics I.
 - Geodesy I.
 - Laboratory physics.
3. Semester (summer).
 - Higher Mathematics III, with exercises.
 - Analytic geometry of conics.
 - Technical Mechanics II, with exercises.
 - Experimental Physics II.
 - Praktikum in geodesy.
 - Method of least squares.
 - Laboratory physics.
4. Semester (winter).
 - Higher Mathematics IV, with exercises.
 - Analytic geometry of surfaces of the second degree.
 - Technical Mechanics III, with exercises.
 - Geodesy II.

For the first, third, and especially the fourth semester it is recommended that the candidate take yet other courses in the fields of mathematics and physics. The above-mentioned courses are simply those which are fundamental.

Later semesters should serve for deeper study in fields of interest. For this purpose there are each semester mathematical seminars, praktika in physics, and special lectures in mathematics and physics, especially in spherical trigonometry; theory of real and complex functions, elliptic functions, higher algebra, differential equations, theory of space curves and surfaces, theory of geometric transformations, theory of algebraic curves and surfaces, geometry of motion, analytic mechanics, potential theory and other parts of mathematical physics, spherical astronomy, and insurance.

The following courses in mathematics and astronomy were offered at the University of Leipzig in the summer semester of 1914: Descriptive geometry (2 hours), by Rohn; plane analytic geometry (4 hours), Herglotz; algebraic analysis (introduction to the differential and integral calculus), Herglotz; theory of numbers (2 hours), Koebe; algebraic curves (4 hours), Rohn; differential equations, with exercises (4 hours), Koebe; general theory of functions of a complex variable (4 hours), Hölder; calculus of variations (2 hours), Hölder; theory of collective measurement (4 hours), Bruns. Mathematical seminary: (1) Exercises in theory of functions (1 hour), Hölder and ———; (2) exercises in descriptive geometry (2 hours),

Rohn and ———; (3) exercises in algebraic-curves (1 hour), Rohn; (4) introduction to the literature of modern theory of functions (conformal representation and uniformization), lectures by the members (fortnightly), Herglotz and Koebe. Exercises in analytical geometry (1 hour), Herglotz; practical work in the observatory, Bruns.

Further, all candidates have opportunity to hear, at regular intervals, lectures on history of philosophy, logic, psychology, systematic pedagogy, and history of pedagogy.

In spite of the statement of P. Ziertman in Monroe's *Cyclopedia* (p. 91), I am in doubt as to whether the Probejahr is obligatory in Saxony, since even the Seminarjahr does not seem to be required but only "recommended." (A. Witting, p. 62.) The single Saxony Seminar for the mathematician is the practical-pedagogic Seminar which was organized in 1895 and stands in official connection with the University of Leipzig.¹ It is in charge of the rector of the Thomas Gymnasium, and Dr. Ernst Lehmann directs the work of mathematics and natural sciences. The weekly sessions of two hours each in a Gymnasium embrace visiting the classes of experienced teachers, practice teaching by students, and criticism of the work done. The number of students is large.

¹ It is not even mentioned by A. Witting, in connection with the required training of the teacher. True, one does find the following in section 8 of the regulations concerning the practical Seminar mentioned below: "Upon his departure from the Seminar every member can demand a certificate, which is to be given by the leader of the subject group and countersigned by the director. These certificates must be affixed to the petition to the royal ministry of public instruction when the candidates are admitted to the Probejahr." (J. F. Brown, p. 112.)

Upon appealing to the late Miss Anna T. Smith, specialist in foreign educational systems at the Bureau of Education, Washington, for references leading to a more conclusive statement, I received the following information:

"I regret to say that I am unable to refer you to any sources of information which positively settle your inquiry, although to my mind it may be inferentially determined. A little volume by Prof. Dr. Jacob Wychgram, entitled *Das höhere und mittlere Unterrichtswesen in Deutschland*, and published at Berlin and Leipzig in 1913, purposes to give the requirements existing at that time with reference to the qualifications of teachers of the secondary schools in Germany. In the introductory chapter, which is a general survey for the entire Empire, p. 25, it is stated that after completing the *Seminar and Probejahr* the candidates receive the certificate of capacity for appointment as professors and may be admitted into the public, State, or city school service unless special difficulties interfere. This statement is in exact accordance with the requirements published for Prussia in 1898, i. e., *Ordnung der Prüfung für das Lehramt an höheren Schulen* von 12. September, 1898.

"By a decree (Verfügung) dated Dec. 29, 1892, a mutual arrangement was entered into between different parts of the German Empire which provided for the mutual recognition of candidates in the States of the Empire named. The Kingdom of Saxony does not appear in the number. (Compare pp. 86-87.) It is known that, while the general features of the Saxon scheme of qualification are the same as those of Prussia, there are some slight modifications. This distinction is indicated by regulations pertaining to the examination of candidates for the higher school service issued Jan. 25, 1900. Section 29 of this publication simply states that the candidate for appointment, before receiving the certificate of capacity, must have sustained the State examination and have passed successfully the Probejahr. No mention is made of the year in the Seminar. From this it might be inferred that the latter is not essential although it is well known that it is customary. (See *Bekanntmachung, über die Ordnung der Prüfung für Kandidaten des höheren Schulamtes der mathematisch-physikalischen und chemischen Richtung an der Königlich-Technischen Hochschule zu Dresden*, sec. 29, pp. 14-15.)"

² "Über das Studium der Pädagogen an der Universität Leipzig," in Klein's *Aktuelle Probleme der Lehrerbildung*, pp. 16 ff.

4. WURTEMBERG.

The present regulations for the training of teachers for secondary schools date from 1898. Broadly speaking, they require that a candidate shall successfully pass (1) two service examinations (*Dienstprüfungen*), that is, examinations preparatory to serving as teachers; and (2) a "Vorbereitungsjahr" or Probejahr.

The examinations are held at Stuttgart, usually in the spring, before a commission composed of university professors and schoolmen appointed by the minister of church and school affairs. The commission is under the direction of a member of the department of the ministry for higher schools.

Application for admission to the first service examination must be accompanied by: (a) A scientific thesis; (b) proof of attendance at lectures and seminary exercises in a German university or technical high school during four years; (c) certificates of participation in the exercises of a high-school institute in physics or chemistry, and a testimonial descriptive of the success attending this participation.

Of the required attendance at university and technical high school at least two of the semesters must be at a university, and during not less than six semesters the candidate must specialize in the subjects of examination, in class and seminary.

Two advanced lecture courses in philosophy and one in pedagogy are also required of all candidates.

The first service examination of the mathematics-natural science course is arranged in two divisions: (a) Mathematics-physics; (b) chemistry-biology, between which the candidate has to make choice. The subjects in each division are grouped as majors and minors.

Except in cases to be noted presently the examinations are both written and oral in all subjects; the commission can nevertheless waive oral examination of a candidate in a special subject, when he has done exceptionally well in the written examination.

The majors in the mathematics-physics division are:

1. Mathematics.
 - (a) Higher algebra, including the theory of elimination.
 - (b) Differential and integral calculus, including partial differential equations and elements of the theory of functions.
 - (c) Analytic geometry, including the elements of the theory of higher curves as well as the theory of curvature.
 - (d) Trigonometry, with mathematical geography.
 - (e) Synthetic geometry, including surfaces of the second degree.
 - (f) Descriptive geometry.
2. Mechanics, especially mechanics of a rigid system.
3. Physics.
 - (a) Experimental physics.
 - (b) Theoretical physics in the four fields: Mechanics in physics, optics, theory of heat, electricity and magnetism (written examination only).
 - (c) Exercise in manipulation of apparatus for physics.

In the written examination the candidate is expected to solve one or more of the "not too difficult exercises" of the subject in question, while in the oral examination the candidate's comprehension of the theory is ascertained.

In mathematics, the candidate has a choice in the written examination between (a) and (b); in theoretical physics exact acquaintance with only two of the four fields mentioned is required.

The exercise in manipulation of apparatus consists in carrying out certain experiments; the commission may waive this exercise if an official certificate from the authorities of the Institute of Physics or of the State University, or of the Technical High School in Stuttgart indicates sufficiently long drill and adequate dexterity in such exercise.

The only minor of the first part is chemistry, in which a knowledge of the most important theories of general chemistry and acquaintance with the presentation and the properties of the most important compounds is required. Furthermore, demonstrations suitable for school instruction may be called for.

The majors of the chemistry-biology division are chemistry, mineralogy with geology, botany, zoology.

The minors of this part are:

1. Algebra and lower analysis.
2. Elements of differential and integral calculus.
3. Elementary geometry, including trigonometry and the elements of modern geometry.
4. Elements of plane analytic geometry.
5. Elements of descriptive geometry.
6. Experimental physics, in which the demonstration of acquaintance with the most important apparatus for instruction in physics may be required.

Mathematical questions which have been set in Wurttemberg examinations in both the mathematics-physics and the chemistry-biology divisions are given in Appendix E.

For upper classes the teacher is allowed to instruct only in the subjects of his first service examination; in the two upper classes he must usually confine himself to the major subjects of this same examination.

Concerning the second service examination the regulations state: After the first service examination the candidate is assigned to a Realschule for a year in order to acquire a methodical introduction into the theory and practice of teaching (Vorbereitungsjahr). The second service examination consists of (a) a German essay; (b) an examination in free-hand drawing; (c) a supplementary examination (Ergänzungsprüfung), in which the extent of the requirements is determined by the needs of instruction in middle *real* classes; and (d) the delivery of three trial lessons.

The peculiar supplementary examination of the mathematics-natural science group includes (1) French and (2) English, and is a

survival of other days. In practice, however, the young teacher equipped to instruct in mathematics has to start out in a country school and possibly instruct in just such subjects.

The Vorbereitungsjaar is never properly practical because of the great scarcity of teachers. Hence the professional training of the teacher in Wurttemberg really amounts simply to a year of teaching.

At the University of Tübingen in the summer semester of 1909 the following lectures were given: General psychology (4 hours a week); church and school in the nineteenth century. In the winter semester, 1908-10: Psychology (4 hours a week); philosophical ethics and jurisprudence (4 hours); exercises in ethical questions (1 hour).

6. BADEN.

The examination ordinance of 1903 for teachers in secondary schools in Baden is copied in its essentials from the Prussian of 1898.

Before admission to the examination the candidate must have pursued special studies at a German university for not less than eight semesters (instead of six). If mathematics or natural science is to be the major of the examination, regular study at a German school of technology may be counted equivalent to study at a German university up to four semesters (ordinance of 1909). During at least four semesters of the period of academic study the candidate must have successfully participated in scientific or practical exercises in school of technology or university seminaries, or laboratories, or institutes.

The examination, which occurs every spring in Karlsruhe before a special commission, consists of two parts, the general and special subject examinations. Both are written and oral; the written usually precedes the oral.

The subjects for examination are:

- A. In the general examination for each candidate, (1) philosophy, (2) German literature.
- B. In the special subject examination, according to the choice of the candidate:
In the mathematics-natural science field of instruction: (1) Mathematics,¹ (2) physics, (3) chemistry and mineralogy, (4) botany and zoology.

Geography may be chosen in combination with mathematics, zoology, and botany, or as another subject.

The candidate must choose at least three examination subjects—two as majors and a third as minor.

Candidates in the mathematics-natural science division must always choose mathematics. The candidate with mathematics as a major must present a certificate indicating his participation in the exercises of a university seminar or other school of higher education during at least four semesters.

¹ Applied mathematics does not occur as a special examination subject.

In the general examination all candidates must demonstrate—(a) Their general knowledge of the most important facts in the history of philosophy, the principles of logic, and the chief facts of empiric psychology; (b) their acquaintance with the general development of German literature from the beginning of its "blooming period" in the eighteenth century; (c) their thorough knowledge of a not too limited field in philosophy and literature.

For the examination in mathematics there is required—

(A) As minor: Knowledge of the elements of higher analysis, plane analytic and synthetic geometry; spherical trigonometry, elements of astronomy.

(B) For mathematics as major in addition to the knowledge required in A: Acquaintance with the foundations of arithmetic and geometry, of analysis and algebra, including the theory of functions, of analytic and synthetic geometry of space, and of analytic mechanics, as well as with the elements of descriptive geometry and with the chief facts of the history of mathematics.

For the home theses every candidate receives two problems, one for the general examination, the other for the special subject examination in one of the subjects chosen by the candidate as major. The examination commission may, however, so choose a theme that the latter home thesis involves both majors.

For the preparation of the two home theses a period of 20 weeks is allowed.

The extent of the requirements in this connection may be judged by some of the themes assigned in the years 1908 and 1909:

Derive the necessary and sufficient conditions for the validity of the theorem concerning the independence of the value of the integral $w = \int_{z_0}^z f(z) dz$ taken over the path of integration, and apply the theorem to some examples (Cauchy) for finding the value of definite integrals.

Symmetric functions.

The proofs of the transcendence of π exhibited in their characteristic features. Connection between the theory of functions and hydrodynamics, especially explanations of Kirchhoff's theory of water spouts.

A presentation of the most important theorems in the theory of cylindrical functions.

The addition theorems of elliptic integrals of the first, second, and third species according to Euler's development, and their principal geometric applications.

The derivation of Lagrange's differential equations of the second kind in mechanics after the presentation of Boltzmann and the researches of Von Hölder on the Hamiltonian principle.

Development of the most important properties of spherical harmonics according to the lectures of Helmholtz, due regard being paid to the works by Heine, F. Neumann, and C. Neumann.

On the numerical solution of algebraic equations.

On the transformation of Laplace's differential equation of the potential by Jacobi, among others, together with applications to individual problems.

On those plane curves of the fourth order which have the imaginary circular points at infinity as double points.

(On singular asymptotic curves and surfaces (Inaugural dissertation).
Cyclides.

A prize memoir crowned by the faculty of philosophy or natural science in a State high school or university, or a dissertation accepted by such a faculty may be considered by the examination commission as equivalent to the home thesis in the special subject examination.

In determining the result of the whole examination slight lack of knowledge in one part may be compensated for by good performance in another. The candidate has passed if he has met the requirements of the general examination, is satisfactory in at least one of his chosen majors, and has shown a knowledge of two more subjects at least equivalent to that required for minors. If the examination is passed, one of the predicates "satisfactory," "good," or "excellent" is assigned, according to the character of the written and oral examinations and home theses. In this connection the majors are to be considered as first in importance, the minors and general examination as secondary. If the candidate has failed in his examination, he may try again within two years, at the latest. If he fails a second time, permission for further trial may be granted only by the ministry.

Having passed the examination the candidate must next become a *Praktikant* in the training of a "Probejahr," which is more in the nature of a Prussian Seminarjahr.

The activities of the *Praktikant* in the institution consist—(a) In visiting classes which will aid in his development; (b) in trial teaching, first for part of a lesson, then for whole lessons, and finally for continuous instruction; (c) in study of notable works in general and special pedagogy and didactics, which are to be found in the library of the institution. This study is directed by the teacher in charge of the candidate.

From this teacher at the beginning of the second half of the year the *Praktikant* receives a theme on which he has to prepare a thesis of not more than 20 written quarto pages in extent. This theme relates to the activities of the *Praktikant* in the institution, but its treatment also serves to indicate his knowledge of the related pedagogic literature of ancient and modern times. The thesis, accompanied by the critical judgment of it on the part of the director of the institution, is laid before the inspector.

Furthermore, the *Praktikant* has to give a trial lesson to a class, which he has taught consecutively for at least two weeks, before the director of the institution and the guiding teacher. These each make independent reports of the trial lesson to the inspector. The report of the director is also accompanied by a statement concerning the candidate's industry and his general bearing both inside and outside of the institution.

On the basis of the various reports the inspector decides on the candidate's qualifications as a teacher. If he decides that the requirements of the "Probeyahr" have not been met, the probationary period may be extended for a year.

At the University of Freiburg in the summer semester of 1909, the following courses were offered: Psychology (4 hours a week); physiological psychology (1 hour); history of pedagogy (2 hours). In the winter semester: Physiological psychology (1 hour); the higher school systems of the present (2 hours); school hygiene (1 hour).

In corresponding semesters at the University of Freiburg the following courses were offered: Ethics (5 hours a week for a semester); pedagogical classics, Rousseau and Pestalozzi (1 hour a week):

As in Bavaria, university "vacation courses" are given from time to time. For example, at Freiburg, Easter, 1906: Löwy lectured on mathematics of life insurance, and Lürth on some fields of infinitesimal calculus which are suitable for introduction into instruction in secondary schools. At Easter, 1908, Lürth gave three lectures, of an hour each, on spherical trigonometry.

6. ALSACE-LORRAINE.

Since 1899 the regulations for the scientific training of the teacher for the secondary school have been the same as those of Prussia.

The examination is conducted at Strassburg by an imperial scientific examination commission.

The professional training of the teacher in Alsace-Lorraine has not changed in form since 1871. The young expectant is sent as "probationary candidate" to some higher school where he is required to give instruction for six or eight hours weekly. The pedagogic and didactic direction, the fixing of class hours when the candidate listens to the instruction of other teachers, the giving of advice in the choice of works which serve to advance pedagogic and didactic development, are either undertaken by the director, if he is a specialist in mathematics, or (as happens more frequently in larger institutions) by a capable teacher who exercises a more or less satisfactory guidance. At the end of the Seminarjahr the candidate must present a thesis on a theme given him in the first half of the year. As usual in such cases the theme gives the candidate an opportunity to treat some practical pedagogic question on the basis of his own teaching experience and of his study of related literature. The reports to the inspector are of the same nature as in Baden, and he decides as to the candidate's "Anstellungsfähigkeit."

Wirz well remarks: In this organization for the training of candidates, it is evident that chance plays altogether too prominent a rôle.

When Reye and Weber were at the university of Strassburg the courses of lectures were arranged according to a plan which considered the needs of those preparing to be teachers in secondary schools. In the list of courses for the summer semester of 1914, however, there is little evidence of any such plan, as there is only one course, on the theory of numbers, which can be possibly considered as a course for beginners. The list is as follows: By Faber: Integral calculus II (4 hours); theory of numbers (2 hours); mathematical seminary (the distribution of prime numbers) (2 hours). By Schur: Theoretical mechanics (4 hours); theory of ordinary differential equations (2 hours); mathematical seminary (2 hours). By Simon: History of mathematics in the Middle Ages (2 hours). By Wellstein: Elliptic and hyperelliptic functions (4 hours). By v. Mises: Technical mechanics II (hydraulics and aerodynamics) (4 hours); integral equations and their applications (4 hours); seminary exercises in applied mathematics (technical mechanics) (2 hours). By Epstein: Calculus of variations (2 hours). By Speiser: Theory of aggregates (2 hours); the equation of the fifth degree with exercises (1 hour).

Arrangements for the scientific development of the teacher of mathematics are wholly lacking and there are no "vacation courses" as at several universities in Prussia, Baden, and elsewhere.

In the summer semester of 1909 and the winter semester of 1909-10 the following courses were given at the university of Strassburg: Psychological exercises (2 hours a week through a semester); history of philosophy (2 hours); psychology (4 hours); introduction to experimental psychology (1 hour); ethics (2 hours); history of Greek ethics (2 hours).

7. HESSE.

According to an ordinance of 1908 the scientific capabilities of a candidate are tested by an examination commission at the University of Giessen. The commission is composed only of professors in the faculties of philosophy and theology, the idea being that after candidates have studied at the university their teachers are best qualified to examine them. In addition to the examination, a professional training of two years is required.

Candidates for admission to the examination in mathematics and natural science—(a) must have a Reifezeugnis from a Gymnasium, Realgymnasium, or an Oberrealschule; (b) must have attended a German university for at least eight semesters, three semesters at a technical high school being counted as equivalent to those at a university.

The examination consists of a general examination and a special subject examination. The general examination is on philosophy, pedagogy, and German literature. In philosophy a home thesis is

required, six weeks being assigned for completing it; otherwise the examination is oral. It is expected that the candidate have a general knowledge of the history of philosophy, acquaintance with the most important laws of psychology and of the facts of empiric psychology, and also familiarity with the history of pedagogy, the essentials of method (Methodik), and with the general development of German literature.

The regulations with regard to the special subject examination are almost word for word the same as those in Prussia.

The first of the two years of professional training must be obtained at one of the six pedagogic Seminarien connected with and conducted by the officials of some of the secondary schools. Candidates are here familiarized with the pedagogic theories required in their special scientific development for the purposes of instruction and education, and are guided in the practical application of knowledge thus won. This training takes place (1) at meetings which occur once or several times each week for introductory instruction in the theoretical pedagogy and method of individual subjects; (2) in the classroom, where the candidate first observes the instruction of a special subject-teacher and then after sufficient preparation and assistance gives some trial lessons. At the end of the Seminarjahr each candidate must present a thesis on a pedagogic subject.

The second year of professional training is at one of the secondary schools, where the candidate gives consecutive instruction under the direction of one of the teachers or of the director. After successfully meeting the conditions of professional training the candidate, who has up to that time borne the title of *Lehramtsreferendar*, is named *Lehramtsassessor*.

At the University of Giessen in the summer semester of 1909 and the winter semester of 1909-10 the following courses were offered: Exercises in experimental psychology; history of education and pedagogy since the age of humanism (3 hours per week); the nature, origin, and development of speech (2 hours per week); Pestalozzi in philosophic seminary (1 hour); psychology of will (1 hour); introduction to scientific works in the sphere of psychology and pedagogy; ethics; outlines of pedagogy (2 hours per week).

8. HAMBURG.

For the training of their teachers for secondary schools the Hanseatic cities of Hamburg, Lubeck, and Bremen are governed by the regulations in Prussia. The scientific training of the prospective teacher for these States, as well as the professional training in the case of Bremen, takes place in other States, such as Prussia or Hesse. I shall, therefore, confine my comment to the professional training of the candidate in Hamburg.

The candidate in mathematics and natural science is assigned, as a rule, to an Oberrealschule or a Realgymnasium. The number of Seminar candidates at one institution varies from 4 to 12. Each candidate is assigned to an experienced teacher in his special subject. This Oberlehrer decides what lessons the candidate must attend in order to prepare him most quickly to give instruction himself. The period of simple listening rarely exceeds two weeks; then follow the first attempts at teaching in the presence of the Oberlehrer. A candidate showing some aptitude gives one or more lessons alone, even during the first quarter of the year, that he may display his powers of preserving discipline. Not till this chrysalis stage has passed does the director listen to a candidate's lesson and discuss it with the Oberlehrer. Such conferences are frequently repeated. Interruption of a lesson by Oberlehrer or director rarely occurs, as it is recognized that the loss suffered by the candidate's authority is greater than the benefit which would accrue. Moreover, at the end of the lesson there is ample opportunity to point out any error. In case of need a written preparation for the next lesson may be required. It is arranged that the candidate gives at least four such inspected lessons, so-called "Anleitungsunterricht," weekly.

At the same time, in accordance with the advice of the Oberlehrer, he visits classes, first, in all grades of his own special subject, then also in other school subjects to a moderate extent. He has to render an account of his activities to the director every week. In some institutions a written report of lessons heard and given is required. This proceeds by way of the Oberlehrer, who suppresses indiscretions or evidences of poor judgment. Since in a large faculty frequent substitutes (Vertretungen) are necessary, the candidate is early and frequently called upon to act as deputy. In these cases he is quite alone and must report to the Oberlehrer what he has done. The weekly amount of Anleitungsstunden, hours of visiting classes, and Vertretungsstunden should not exceed 18 and usually varies between 12 and 15 hours.

In the second quarter the candidate gives frequent instruction in one or two classes of different grades. He must direct practical class exercises in physics unassisted. If a class is divided into two sections for carrying out exercises, it is customary for the Oberlehrer to take the first section, while the candidate assists in the lesson. The candidate then conducts the second section himself. When the candidate has the necessary independence, especially with respect to discipline, the attendance of the Oberlehrer during his teaching hours is confined more to regular visits.

In the second semester regular teaching hours are frequently assigned to the candidate, and for these he receives remuneration. In addition to this he retains Anleitungsunterricht in one subject.

In the Seminarjahr the candidate must become thoroughly acquainted with the school equipment, and the laboratory exercises give the necessary incentive. In addition all Seminar candidates of physics are required to attend the weekly instruction (2-3 hours) in practical physics in the Oberrealschule at the Uhlenhorst.

For theoretical instruction a conference of one hour a week is appointed. This is led by the director. So far as possible, general questions of instruction and education are dealt with by discussion. Experienced specialists lecture on methods of instruction in individual subjects; discussion follows immediately or a week later in the presence of the lecturer. Every candidate must present a review of some valuable work; for example, by Locke, Basedow, Pestalozzi, Herbart, Jäger, Förster, Wernicke, Lietz, Wetekamp, Reidt, Simon, Schwering, Klein, Wellstein, Killing, or publications of the International Commission on the Teaching of Mathematics. The zealous use of the Seminar library is promoted by the director. In some institutions the minutes of the Seminar conferences are taken by the candidates in turn. At others each candidate has to prepare minutes of all general or special conferences.

The candidate has to take part in all conferences and class examinations in so far as he is not prevented by his own instruction. When the reports of these examinations are decided upon by class teacher and director, the candidate has the right to offer a dissenting opinion; indeed, he is expressly summoned to declare himself, if the higher courts disagree.

At the end of the Seminarjahr the candidate must present a scientific pedagogic thesis, and a more practical report on his total teaching experience is due at the end of the Probejahr. In the Probejahr the candidate is usually busy with a large amount of remunerative teaching. If he remains at the same institution as that in which he spent the Seminarjahr, the directing Oberlehrer is frequently changed in order that different methods may be learned. Monthly revisions by the Oberlehrer and a thorough examination of the classes by the director at the end of each semester are the rule.

The inspector follows the development of the candidates with special attention. At each of his visits to the institution, or when the director appears before the school authorities, all candidates are discussed in detail. Special occurrences are immediately communicated either orally or in writing. The inspector hears the candidate at least once in each semester, and considerably oftener if doubt arises as to the desirability of admitting him to the Probejahr or of awarding him an Anstellungsfähigkeitszeugnis. On the basis of his own observations, of repeated oral reports by the director, and of written final reports by the Oberlehrer and director, the inspector passes upon the candidate's fitness for the Probejahr or for a teaching

certificate. The refusal of the first as well as the second occurs repeatedly, either by postponement (when the work is usually continued at another institution) or with finality.

In conclusion, some titles of seminar theses may be mentioned. The related literature is collected by the candidate from Rethwisch's *Jahresberichten über das höhere Schulwesen*, and the director then furnishes the more important original works for thorough study. The problems are so chosen that the candidate has opportunity to test theory in practice or to modify it. Since he receives the theme in the first month, and does not render the thesis till the ninth, ample time is allowed to assemble personal experiences. The titles are as follows:

What limitations can be imposed on arithmetic-algebraic instruction in the secondary school classes?

On what grounds can the constant use of oblique parallel projection be recommended in commencing the instruction of descriptive geometry?

How far can and how far should arithmetic in the lower classes of the higher schools prepare for the instruction in algebra?

What part of the theory of circles may be placed at the beginning of instruction in plane geometry?

Geometrical instruction in Quarta, with special reference to M. Schuster's *Geometrische Aufgaben*.¹

How may instruction in spherical trigonometry be built up on the theory of right-angled dihedral angles as found in solid geometry?

The treatment of the integral calculus in Oberprima, starting from definite integrals.

Logic in the program of the Oberrealschule.

How formulate in Sekunda the treatment of trigonometry that has the closest possible connection with plane geometry?

To what extent is history of philosophy connected with instruction in physics and chemistry?

The instructional value of drawing.

Induction and deduction in geometric instruction.

When a candidate for a teaching position in a secondary school takes up the work of his preparation in the German university, he is not without suggestions for his guidance. Two illustrations of weighty pronouncements of this nature may be indicated: I. The plan recommended in 1907, after three years of labor,² by the commission on instruction of the Gesellschaft deutscher Naturforscher und Ärzte (resulting in the formation of the Deutschen Ausschusses

¹ *Geometrische Aufgaben und Lehrbuch der Geometrie*, Nach konstruktiv-analytischer Methode bearbeitet. Nach dem Tode des Verfassers herausgegeben von W. Lietzmann. Ausgabe A: für Vollanstalten, 3 Teile, Leipzig, Teubner. I. Teil: *Planimetrie*, 3. Auflage, 1908, 168 pp.; II. Teil: *Trigonometrie*, 2. Auflage, 1911, 126 pp.; III. Teil: *Stereometrie*, 2. Auflage, 1908, 110 pp.

² Cf. A. Gutzmer, *Die Tätigkeit der Unterrichtskommission der Gesellschaft deutscher Naturforscher und Ärzte*. Leipzig, Teubner, 1908. The plan and recommendations were also published in *Universität und Schule, Vorträge auf der Versammlung deutscher Philologen und Schulmänner zu Basel, 1907*. Leipzig, Teubner, 1907. See also W. Lorey, *Das Studium der Mathematik an den deutschen Universitäten*, pp 294-298.

für den mathematischen und naturwissenschaftlichen Unterricht);
II. Advice (Vorschläge) issued in 1913,¹ at the University of Göttingen, to students of mathematics and physics.

I. In this plan it is recommended that the general studies in mathematics and physics should be taken during the first six semesters according to the following scheme:

Semester.	Specialty studies.			General studies.
1	Differential and integral calculus, I. Exercises, Praktika, Seminars.	Analytic geometry. Exercises, Praktika, Seminars.	Experimental physics, I.	
2	Differential and integral calculus, II.	Descriptive geometry (with projective geometry). Exercises, Praktika, Seminars.	Experimental physics, II.	
3	Differential equations.	Elementary mechanics (with graphical and numerical methods). Exercises, Praktika, Seminars.	Introduction to chemistry.	
4	Algebra with theory of numbers.	Curves and surfaces. Exercises, Praktika, Seminars.	Higher mechanics.	History of philosophy and pedagogy.
5	Theory of functions.	Curves and surfaces. Surveying with calculus of probabilities. Exercises, Praktika, Seminars.	Theoretical physics, I.	Logic.
6	A comprehensive lecture.	Astronomy with geophysics. Exercises, Praktika, Seminars.	Theoretical physics, II.	Psychology.

¹ In the *Praktikum* of a German university or technische Hochschule instruction is not confined to informative lectures, but includes guidance in practical application of the knowledge gained, as in laboratory work for physics and chemistry, case procedure for law, and so on.

In addition to this there will be added in most cases two to four semesters during which—

the industrious student finds time to complete these studies in some direction which appeals to him. Above all it is of importance that he deepen his study in some special field, in which it may well happen that he is led to the doctorate. Candidates with other gifts will strive for a suitable extension, not too far-reaching, of the "specialty studies" in the scheme outlined above.

As the most essential point of this scheme one may well put forward the most impelling effort to make the mathematical training of the average candidate as far as possible independent of the special fields of investigation standing for the time being in the foreground of interest.

B II. The Göttingen Vorschläge.

The employment of years of study for the acquisition of a general training, by virtue of which the candidate may later understandingly become a part of the whole organiza-

¹ *Ratschläge für die Studierenden der Mathematik und Physik an der Universität Göttingen*, Leipzig, Teubner, 1913, pp. 30-31. See also *Jahresbericht der Deutschen Mathematiker-Vereinigung*, Band 22, 1913, pp. 99-100, and W. Lorey's *Das Studium der Mathematik an den deutschen Universitäten*, pp. 291-292.

tion of the school in which he has to work, and may have the ability to make himself potently felt in his department, is indeed a very important matter. Important, too, is appreciation, for the time being, of the pedagogic problems which stand in the foreground in the later activity of his calling.

In connection with these matters, however, there are two contrary conceptions.

The one, sustained more by historical tradition, understands general training as knowledge of an encyclopedic nature, as it were, of just those fields which are far removed from the special studies in mathematics and natural science, such as historical, literary, or even artistic phases of human development. According to this conception the pedagogic instruction puts in the foreground the general ideas which have been suggested in the course of history with regard to the problems of education.

The other more modern conception demands that the candidate in mathematics and natural science shall be in a position to establish the significance of his mastery of a particular field of instruction. In this sense we must refer to the importance which mathematics and natural science possess for the logical and theory of knowledge side of philosophy, for psychology, for history of civilization and technology, for geography, finally for the general field of civic interests. The biological side of natural science is, besides this, the basis of the rational theory of health, and it has to that extent also a quite particular importance for school management. Corresponding to this conception it would be desirable that the candidate at the university should soon acquire a certain understanding as to the significance which his scientific studies possess for the didactics of his subsequent field of instruction. On the pedagogic side, however, the great and important field of child study opens up.

At the present time examination procedure always follows the first of the conceptions. We would not, however, omit to mention the other which is ever gaining currency. We do not recommend definite lectures or Praktika in one or the other direction; rather do we believe that very much which is worthy of attainment there is better obtained by private study or, in some cases, by association with those who have a similar purpose. Whichever course the candidate may follow, we urgently recommend that during the years of his university study he shall not lose sight of the importance of the formation of his own style and delivery, and in any case shall attain as great knowledge as is requisite to the successful pursuit of his studies. We must also think of and observe the necessity for physical exercises so that on the side of the management of instruction special weight be laid upon the acquisition of ability to give instruction in gymnastics.

On account of conflict of statements in authoritative sources, it seems well here to formulate definitely the observation that even the *general* requirements for the training of teachers of mathematics in secondary schools are not the same for all the eight States we have specially considered. Prussia, Alsace-Lorraine, and Hamburg each require not less than three years of study in a university or school of technology, but all the other States require not less than four years.¹ All States except Württemberg, apparently, now require a Seminarjahr; this has been true of Bavaria only since 1912, and there is some doubt

¹ According to a new Prussian project, which had been formulated when the war broke out, the following changes in the training of secondary-school teachers were contemplated: (1) A minimum of four instead of three years of study at a university or school of technology; (2) instead of a major and two minors in the special-subject examination, two majors and one minor; (3) following the lead of Bavaria, an examination at the close of the professional training. (See *L'Enseignement mathématique*, tome 18, 1916, pp. 355, 356, 368; Lorey's *Das Studium der Mathematik an den deutschen Universitäten*, pp. 254-256, 298; and *Berichte und Mitteilungen, veranlasst durch die Internationale Mathematische Unterrichts-kommission*, No. X, Leipzig, Teubner, 1915, p. 66.)

regarding Saxony. The Probejahr is not required in at least three of the eight States. In Wurttemberg the so-called Probejahr is simply a year of teaching. Statements made to the effect that two years of professional training are required of all German teachers in secondary schools must therefore be accepted with considerable reserve; at least four States do not have two years. The facts may be exhibited in tabular form. Note that the requirements in Hesse are more extensive than those in the other States.

States.	Least number of years required at university or technical school.	Seminarjahr required?	Probejahr required?
1. Prussia.....	3	Yes.....	Yes.
2. Bavaria.....	4	(1)	No.
3. Saxony.....	4	?	?
4. Wurttemberg.....	4	No.....	Yes (1)
5. Baden.....	4	Yes.....	No.
6. Alsace-Lorraine.....	3	Yes.....	No.
7. Hesse.....	4	Yes.....	Yes.
8. Hamburg.....	3	Yes.....	Yes.

¹ No. before 1912, yes, since 1912.

It seems a little curious if the man who has just completed his Seminarjahr in Alsace-Lorraine, where there is no Probejahr, should be immediately qualified to teach in Prussia (because of the interchangeability of certificates) when a Prussian must submit to another year of training.

Salaries in Prussia.—The following scheme of salaries went into effect in 1909:

- Salaries of Provinzialschulräte: 6,300-8,100 marks (\$1,499.20-\$1,727.80).² The increase of 600 marks comes at the end of three, six, and nine year periods of service.
- Salaries of directors of nine-year schools: (a) In Berlin, 6,600-7,800 marks (\$1,590.80-\$1,856.40);² (b) in other cities, 6,000-7,800 marks (\$1,428-\$1,856.40).² In addition to these salaries directors receive a rent indemnity of 1,800 marks (\$428.40) in Berlin and 900-1,500 marks (\$214.20-\$347) elsewhere. Salaries in both (a) and (b) increase by 600 marks every three years till the maximum is reached.
- Salaries of directors of six-year schools: 5,200-7,600 marks (\$1,237.60-\$1,808.80).² The rate of salary increase and rent indemnity is the same as in 2.
- Salaries of the Oberlehrer, teachers of science: 2,700-7,200 marks (\$622.60-\$1,713.60). The salary increases occur after every three years of service. The first three increases amount to 700 marks each; the next four to 600 marks each. The Oberlehrer's rent indemnity in Berlin is 1,200 marks (\$285.60); in other places 760-880 marks (\$133.28-\$209.44).
- Salaries of scientific assistants from whom the Oberlehrer are chosen: 2,100 marks (\$499.80) for the first year, 2,400 marks (\$571.20) for the second, 2,700 marks

¹ *Statistisches Jahrbuch der höheren Schulen*, XXXIII. Jahrgang, 1912-13, Leipzig.

² This computation is made on the basis of 23.8 cents to the mark. These salaries each include 600 marks "pensionsfähige Zulage."

³ These salaries each include 400 marks "pensionsfähige Zulage."

(\$642.60) for the third, and 3,000 marks (\$714) for the fourth year. The rent indemnity for Berlin is 720 marks (\$171.36); for other places, 290-580 marks (\$69.02-\$138.04).

In other parts of Germany salaries are sometimes not quite so high (for example in Hesse), but in Hamburg directors receive as much as 13,000 marks without residence or 11,000 marks with residence, and salaries of the Oberlehrer range from 4,000 to 11,000 marks without rent indemnity.¹

Pensions.—In practically all of the States of the German Empire the teachers of higher schools make no contribution to the State pension funds. Such pensions may therefore be considered as adding to the attractions of the teachers' positions.

If a Prussian teacher is retired after 10 years of service, he receives 33½ per cent of his salary. The amount of pension increases with each additional year of service; after 25 years it is 58½ per cent of his last salary, and after 40 years of service the maximum of 75 per cent is reached. The period of service includes time (1) spent on leave of absence; (2) in the Seminarjahr and Probejahr; (3) in military service if performed after the age of 20.

Sixteen of the German States give larger pensions after 40 years of service than Prussia does, and several States grant 40 per cent of the salary to teachers forced to retire after 10 years of service. Three States (Hamburg, Hessen, Anhalt) give the full salary in pension after 50 years of service. (*Statistisches Jahrbuch*, 1913-14.)

Prof. J. W. A. Young has calculated that a very rough approximation to the annual cash value of a Prussian teacher's assurance of a pension after 10 years of service is \$222.² If this be added to the salaries and rent indemnities, we find that the range of income of directors is from \$1,673.80 to \$2,506.80; of Oberlehrer from \$977.88 to \$2,145.04, this last salary being the reward after 21 years of service.

Concluding comment.—Characteristic features have been outlined to indicate a fair uniformity of procedure throughout the Empire in the selection of teachers of mathematics for the secondary schools, and it has appeared that in very few of the 26 States does the standard fall below the remarkably high one set by Prussia. The possibility of this uniformity is largely caused by the standardization of all secondary schools "for the purpose of administering military privilege which is under the control of the Central Government."

In Prussia the secondary teachers as a class now form an important group in the social system of the country as the result of struggle during more than half a century for economic equality with the

¹ For further details with regard to different States the current *Statistisches Jahrbuch* may be consulted.
² *The Teaching of Mathematics in the Higher Schools of Prussia*, London, 1900, p. 29. Prof. Young's statement of \$224 is derived on the basis of 1 mark as the equivalent of 24 cents instead of 23.8 cents. It should also be remarked that Prof. Young's estimate was based upon a lower scale of salaries than is now in force.

Richter or judges of the lower courts. In his delightful volume on the *Oberlehrer*, Mr. Learned writes as follows:

Owing to the peculiar structure of the German State, the classes of public servants are numerous, and their ranks, rights, titles, and privileges are elaborated with great nicety. The nine grades in the civil service make a fairly accurate basis for measurement of public importance, and class differences are minutely scrutinized. Now it happens that of all State officials, the judges in courts of the first instance are most closely comparable to the *Oberlehrer*. They are members of a court organized on the collegiate principle; they have approximately the same training as the *Oberlehrer*, and are not far from the latter in numbers.

It was claimed that—

the professional qualifications, the conditions of service, and the social obligations of the *Oberlehrer* were fully equal to those of the *Richter*; that the service of the *Gymnasien* to society was not inferior to that of the law court; and that, primarily, for the sake of these schools, an equal rating was necessary to secure the ablest brains for the profession and to give the prestige necessary for the *Oberlehrer* in the discharge of his duties.

The Kaiser, Bismarck, and Paulsen were among the *Oberlehrer*'s champions in the fight which was finally won by parliamentary decree in 1909.

The instructors in a secondary school of Prussia consist of (1) the *Probandus*, or candidate on probation; (2) the *Hilfslehrer*, or part-time instructor, who is qualified for a higher position but awaits an opening; (3) the *Oberlehrer*; (4) the professor; (5) the director. The title of professor is purely honorary and was granted as an offshoot of the struggle referred to above for improving the social standing of the teacher. The ministry may appoint one-third of the regular staff of the higher school to a rank equivalent to that of university professor, and all those so nominated are authorized to use the title. Although it was intended that the honor should be a reward of merit, in practice it is awarded irrespective of merit to the upper and older section of the faculty.

As for the number of hours of service per week, German teachers compare well with American and English, but badly with their French colleagues. Special teachers give 26 hours of instruction per week; *Hilfslehrer*, 24; *Oberlehrer*, 22; and professors, usually 20; but all are liable to extra work without, as in France, extra remuneration. The teachers are expected to watch over their pupils' health, to prepare their lessons carefully and mark their exercises regularly; to consult the director before undertaking any private work and the *Provinzialschulkollegium* before venturing to marry, and above all to refrain from political controversy and pamphlet writing. It will be noticed that French masters, though equally civil servants, are bound by none of these petty regulations.

In addition to an almost overwhelming burden of official duties, the director must usually also teach 12 to 15 hours a week. But in spite of his great responsibilities, he is warned that even in official relations with teachers he should not emphasize the fact of his precedence. As Mr. Learned remarks, "He is but *primus inter pares*,

and the members of the Kollegium do not allow him to ignore or forget the fact.

Although the school obligations of the Oberlehrer are arduous, it is common practice for him to supplement his small salary by private tutoring and by boarding students at his home. He can by frugal management comfortably rear and educate a large family, while constantly enjoying a sense of security in the tenure of his office and in provisions made for disability and advancing age. Moreover, his social status in the community is of importance commensurate with that of one whom Bismarck declared to be "the most important factor in patriotic education of the rising generation."

To such training, career, and rewards the candidate who presents himself for preparation as a teacher in a secondary school in Germany may look forward.

The scientific publications of professors in secondary schools consist mainly of textbooks and of discussions of methods in teaching. Among mathematicians who have produced purely scientific work during professional activity, the notable case of Grassmann may be cited.

It was formerly normal for a university professor to start his career by teaching in a secondary school. For example: Weierstrass, Clebsch, Fuchs, Kummer; and among those living, Sturm, Killing, Lampe, and Wangerin. But since about 1870 this has happened rarely. At the Technische Hochschule in Berlin certain courses are given by secondary school professors; some of whom have been called to universities; for instance, Jahnke and Salkowski. At Strassburg, Simon has been both professor in a secondary school and honorary professor in the university since 1903.

BIBLIOGRAPHY.

In addition to references already given in this chapter, the titles of the following publications which have a bearing on the subjects under discussion may be mentioned:

- W. LOREY, *Staatsprüfung und praktische Ausbildung der Mathematiker an den höheren Schulen in Preussen und einigen Norddeutschen Staaten.* (Internationale Mathematische Unterrichtskommission, Band I, Heft 3.) Leipzig, Teubner, 1911. 6+118 pp.
- W. LIETZMANN, *Die Organisation des mathematischen Unterrichts an den höheren Knabenschulen in Preussen.* (I. M. U. K., Band I, Heft 2.) Leipzig, Teubner, 1910. 8+204 pp.
- K. KÖRNER, *Die Mathematik in Preussischen Lehrerbildungswesen.* (I. M. U. K., Band V, Heft 7.) Leipzig, Teubner, 1916. 8+136 pp.
This work was unprocurable in America when this report was written.
- J. WIRZ, *Der mathematische Unterricht an den höheren Knabenschulen sowie die Ausbildung der Lehramtskandidaten in Elsass-Lothringen.* (I. M. U. K., Band II, Heft 7.) Leipzig, Teubner, 1911. 6+58 pp.
- A. WITTING, *Der mathematische Unterricht an den Gymnasien und Realanstalten nach Organisation, Lehrstoff und Lehrverfahren und die Ausbildung der Lehramtskandidaten im Königreich Sachsen.* (I. M. U. K., Band II, Heft 2.) Leipzig, Teubner, 1910. 12+78 pp.

- A. THAER, N. GEUTHER, A. BÖTTGER, *Der mathematische Unterricht in den Gynnasien und Realanstalten der Hansestädte Mecklenburgs und Oldenburgs.* (I. M. U. K., Band I, Heft 4.) Leipzig, Teubner, 1911. 6+93 pp.
- H. SCHNELL, *Der mathematische Unterricht an den höheren Schulen nach Organisation, Lehrstoff und Lehrverfahren und die Ausbildung der Lehramtskandidaten im Grossherzogtum Hessen.* (I. M. U. K., Band II, Heft 5.) Leipzig, Teubner, 1910. 6+51 pp.
- H. CRAMER, *Der mathematische Unterricht an den höheren Schulen nach Organisation, Lehrstoff und Lehrverfahren und die Ausbildung der Lehramtskandidaten im Grossherzogtum Baden.* (I. M. U. K., Band II, Heft 4.) Leipzig, Teubner, 1910. 2+48 pp.
- E. GECK, *Der mathematische Unterricht an den höheren Schulen nach Organisation, Lehrstoff und Lehrverfahren und die Ausbildung der Lehramtskandidaten im Königreich Württemberg.* (I. M. U. K., Band II, Heft 3.) Leipzig, Teubner, 1910. 2+101 pp.
- H. WIELEITNER, *Der mathematische Unterricht an den höheren Lehranstalten sowie Ausbildung und Fortbildung der Lehrkräfte im Königreich Bayern.* (I. M. U. K., Band II, Heft 1.) Leipzig, Teubner, 1910. 12+85 pp.
- W. LIETZMANN, E. GECK, H. CRAMER, *Neue Erlasse in Bayern, Württemberg und Baden.* (I. M. U. K., Band II, Heft 8.) Leipzig, Teubner, 1913. 6+49 pp.
- C. HOSSFELD, *Der mathematische Unterricht an den höheren Schulen in den Thüringischen Staaten.* (I. M. U. K., Band II, Heft 6.) Leipzig, Teubner, 1912. 2+18 pp.
- W. LOREY, *Das Studium der Mathematik an den deutschen Universitäten seit Anfang des 19. Jahrhunderts.* (I. M. U. K., Band III, Heft 2.) Leipzig, Teubner, 1916. 12+431 pp.
- W. LIETZMANN, *Die Ausbildung der Mathematiklehrer an den höheren Schulen Deutschlands.* (Berichte und Mitteilungen veranlasst durch die Internationale Mathematische Unterrichtskommission. Erste Folge, No. XI.) Leipzig, Teubner, 1915. 14 pp.
- This work was unrepresentative in America when this report was written. Cf. J. P. Dumas, *L'Enseignement Mathématique*, tome 18, 1916, pp. 250-301.
- R. SCHIMMACK, *Die Entwicklung der mathematischen Unterrichtsreform in Deutschland.* (I. M. U. K., Band III, Heft 1.) Leipzig, Teubner, 1911. 6+116 pp.
- W. LIETZMANN, *Stoff und Methode im mathematischen Unterricht der norddeutschen höheren Schulen auf Grund der vorhandenen Lehrbücher.* (I. M. U. K., Band I, Heft 1.) Leipzig, Teubner, 1909. 12+102 pp.
- D. KATZ, *Psychologie und mathematisches Unterrichts.* (I. M. U. K., Band III, Heft 8.) Leipzig, 1913. 2+120 pp.
- F. KLEIN, *Aktuelle Probleme der Lehrerbildung.* Vortrag . . . am 6 Juni 1911 . . . gehalten. (Schriften des deutschen Ausschusses für den mathematischen und naturwissenschaftlichen Unterricht, Heft 10.) Leipzig, Teubner, 1911. 4+32 pp.
- G. LEIBUSCHER, *Über Notwendigkeit der Ausbildung der Lehrer in Gesundheitspflege.* (Schriften des deutschen Ausschusses für den mathematischen und naturwissenschaftlichen Unterricht, Heft 7.) Leipzig, Teubner, 1911. 14 pp.
- F. KLEIN, *Vorträge über den mathematischen Unterricht an den höheren Schulen.* Bearbeitet von R. Schimmack. Teil 1: Von der Organisation des mathematischen Unterrichts. Leipzig, Teubner, 1907. 9+236 pp.
- The conclusion of this work is *Elementarmathematik vom höheren Standpunkte aus*. 2 Teile Leipzig, 1908-9; 2. Auflage, 1911-1914.
- A. BEIER, *Die höheren Schulen in Preussen (für männliche Jugend) und ihre Lehrer.* 3. Auflage. Halle a. d. S., Verlag der Buchhandlung des Waisenhauses, 1909. 24+1218 pp.

- F. MAROTTE, *L'Enseignement des sciences mathématiques et physiques dans l'enseignement secondaire des garçons en Allemagne*. (Publications du musée pédagogique, nouvelle série, no. 7) Paris, Imprimerie nationale, 1905. 121 pp.
- Encyklopädisches Handbuch der Pädagogik*. 2. Auflage mit 3 Ergänzungsbände. 10 Bände. Herausgegeben von W. Rein. Langensalza, Beyer, 1903-1912.
- Encyklopädie des gesamten Erziehungs- und Unterrichtswesens*. 2. Auflage. 10 Bände. Herausgegeben von K. A. Schmid u. a. Leipzig, Fues, 1876-1887.
- Handbuch der Erziehungs- und Unterrichtslehre für höhere Schulen*. 2. Auflage, 4 Bände. Herausgegeben von A. Baumeister. Munich, Beck, 1902-1910.

In particular, the work contains Simon's didactic questions of mathematics, which is also published separately.

- J. F. BROWN, *The Training of Teachers for Secondary Schools in Germany and the United States*. New York, Macmillan, 1911. 10 + 335 pp.

The chapter on Certification of Teachers in Prussia contains a useful complete re-daction of the regulations.

- W. S. LEARNED, *The Oberlehrer. A Study of the Social and Professional Evolution of the German Schoolmaster*. (Harvard Studies in Education, volume 1.) Cambridge, Harvard University Press, 1914. 14 + 150 pp.

- E. E. BOLTON, *The Secondary School System of Germany*. (International Education Series.) New York, Appleton, 1900. 19 + 308.

- C. H. JUDG, *The Training of Teachers in England, Scotland, and Germany*. (Bureau of Education, Bulletin, 1914, No. 35.) Washington, 1914.

"The Training of Teachers in Germany," pp. 54-84, secondary teachers, p. 741.

- C. NORWOOD and A. H. HOPE, *The Higher Education of Boys in England*. London, Murray, 1909.

"Second Education in Germany," pp. 102-127. "Professional Training in Germany," pp. 251-256.

- D. E. SMITH, "German versus American Conditions," *Teachers College Record*, Columbia University, New York, March, 1912.

Pages 5-11 of *The Present Teaching of Mathematics in Germany*.

- M. E. SADLER, "The Unrest in Secondary Education in Germany and Elsewhere" (pp. 1-192); A. E. TWENTYMAN, "Note on the revised Curricula and Programmes Work for Higher Schools for Boys in Prussia, 1901" (pp. 193-206). *Education in Germany* (Special Reports on Educational Subjects, volume 9). London, Board of Education, 1902.

- M. E. SADLER, "Problems in Prussian Secondary Education for Boys, with special reference to similar questions in England" (pp. 83-252); H. E. D. HAMMOND, "The Higher Schools of the Grand Duchy of Baden; their Development and Organization" (pp. 322-414). (Special Reports on Educational Subjects, volume 3.) London, Board of Education, 1908.

- P. ZIERTMANN, "Education in Germany," *Monroe's Cyclopedia of Education*, vol. 3, New York, 1912. Pp. 68-93.

- G. B. M. COORE, "Education: National Systems," *Encyclopædia Britannica*, eleventh edition, 1910.

Germany has two periodicals which are specially devoted to promoting the teaching of mathematics; (1) *Zeitschrift für mathematischen und naturwissenschaftlichen Unterricht*, (Leipzig, Teubner), founded in 1870; and (2) *Unterrichtsblätter für Mathematik und Naturwissenschaften*, (Berlin, Salle), established in 1895 as the organ of the Verein zur Förderung des mathematischen und naturwissenschaftlichen Unterricht.

While current volumes of the *Universitäts-Kalender* indicate all lectures delivered at German universities, from semester to semester, titles of most of those on mathematical topics are probably more readily accessible in the *Bulletin of American Mathematical Society* or in *L'Enseignement Mathématique*.

IX. HUNGARY.

Exclusive of Croatia and Slavonia, Hungary is about 109,000 square miles in extent and has a population of over 18,265,000, of which more than one-half are Hungarians (Magyars).

Hungarian education is administered, for the most part, by the ministry of worship and public instruction, which is organized in 10 departments, including the departments of elementary, secondary, and higher education. Although not all schools are controlled by the State, all are subject to State inspection.

SECONDARY SCHOOLS.

The right of inspection over all secondary schools was given by the secondary school act of 1883. Another significant feature of this law was that it gave to the national speech, the Hungarian language, its proper place in instruction by directing that it should receive attention in all school courses.

Secondary schools are divided into Gymnasias and "Real Schools." The aim of these schools is to provide boys with a higher general training, and to prepare them for the universities. The Gymnasias attempt to carry out this aim by means of a humanistic training, especially classical; the Real schools by means of modern languages, mathematics, and natural science.

The secondary school course of eight years (Classes I-VIII) is based upon the first four years of the primary school course.¹ In the Real schools, the pupil studies 9-11 subjects each year and receives 28-30 hours of instruction per week. In the whole course 30 hours per week of class work have been devoted to mathematics and 22 hours to drawing and geometry, or two-ninths of the total number of hours in the course has been assigned to these two subjects. In the Gymnasias this proportion is reduced to two-thirteenths. Here arithmetic is taught in Classes I-III; algebra, plane geometry, and plane trigonometry in Classes IV-VI; algebra, solid geometry, spherical trigonometry, and graphs in Classes VII-VIII. In the Real schools these subjects are developed at greater length, especially the section on

¹The six-year courses of the "burgher" schools (Polgári iskolák) for boys are also based upon this course but the form and manner of instruction is that of higher elementary schools.
In Croatia and Slavonia the eight-year course of secondary education is in the Realgymnasias.
The Hungarian scheme of secondary education is almost identical, even to the programs, with that of Austria.

graphs, which becomes a course in plane analytic geometry. Eight hours of descriptive geometry also come into the program of Classes VI-VIII of the Real schools.

A public examination takes place at the close of each school year. At the end of his eighth school year the pupil must take the final examination. This examination is partly written and partly oral. The oral part is public. The student who passes this examination successfully receives a "certificate of maturity." He is then usually about 18 years of age.

The certificate of maturity from a Gymnasium entitles the pupil to entrance into any Hungarian university. The certificate of maturity from a Real school entitles a pupil to entrance into a Polytechnikum, or the mathematical or science department of a university.

THE UNIVERSITIES AND POLYTECHNIKUM, AND SECONDARY SCHOOL TEACHERS.

In Hungary there are five universities maintained by the State: The University of Budapest, with over 7,000 students and the chief center of classical and scientific study and research in the Kingdom; the University of Kolozsvár, with over 2,000 students; and the universities at Zagráb (Agram), Pressburg (Pozsony), and Debreczen, the last two of which were founded in 1912.¹ The Royal Josefs-Polytechnikum at Budapest has about 170 professors and about 1,800 students.

The secondary school education act of 1883 gives definite instructions concerning the training of secondary school professors. The candidate must meet the following requirements:

(a) He must have graduated from a Gymnasium or Real school. In the latter case he must be able to read and understand easy Latin, for example Caesar's writings, Ovid's poetry, and Cicero's letters.

(b) He must have spent four years in a university or Polytechnikum devoted to the study of the special subjects he wishes to be qualified to teach. He must choose at least two subjects, and he is strongly recommended to take three. He must also show (1) proficiency in the Hungarian language, literature, history, and general Hungarian culture; (2) a thorough acquaintance with pedagogy, history, principles and methods of education; (3) a knowledge of the special methods of teaching his special subjects; and (4) that he has studied logic, psychology, and the history of philosophy.

(c) A year of practical teaching in a secondary school after the completion of the four-year university course, or five years at the university and at least a complete year of secondary school teaching accomplished during that time.

The candidates are examined by a special examination commission organized by the universities. The members of the commission are

¹ Neither of these universities has a faculty of mathematics and science. The University of Pressburg was developed from the Royal Academy of Pressburg, and the University of Debreczen from a high school which had been in existence for over 300 years. Cf. Rept. of U. S. Commis. Educ., 1916, vol. 1, Washington, 1916, pp. 684-5.

mostly professors from the universities and Polytechnikum, but some are also Privatdocenten or eminent secondary school professors.

According to an ordinance of 1888 the candidate must choose at least two special subjects which he wishes to teach, v. g., mathematics with physics or descriptive geometry. The ordinance recommends that the candidate should also make a special study of a third scientific subject¹ or else of philosophy.

The examination is threefold: I. The general examination at the end of the fourth semester; II. the examination on special subjects at the end of the eighth semester; III. the pedagogic examination at the end of the next year, which the candidate usually employs in professional practice.

Those who fail in an examination may come up again six months later. Those who fail a second time may not again present themselves for examination till a year later, during which period they must have attended lectures at a university or at the Polytechnikum.

I. The *General Examination* is composed of two parts, an oral and a written examination. The former is public. In the latter and in other written tests 10 hours are allowed. The candidate must write a theme in the Hungarian language on some subject of interest in Hungarian literature.

In the oral examination the mathematical candidate is examined in—

(1) Plane and spherical trigonometry; (2) analytic geometry; (3) analysis—complex numbers, infinite series, elements of the theory of equations and of differential and integral calculus; (4) descriptive geometry—(a) orthogonal parallel projection, axonometry, oblique parallel projection, central projection and collineation; (b) elements of the constructive theory of lines and surfaces; (c) shades and shadows; (d) perspective; (5)² physics—(a) experimental physics; (b) elements of analytic mechanics; (6) Hungarian language and literature; and (7) one modern language (German, French, English, or Italian), with the literature of the same.

At this examination the candidate must be provided with his certificate of birth, a medical certificate, his certificate of graduation from a secondary school, and a certificate to prove that he has studied the requisite time in a university.

It may be well to give indications of what knowledge is expected for the above examinations in analytic geometry and in analysis:

In analytic geometry: Points, lines, and complete discussion of conic sections; generation of conics by projective pencils, parabolas through four points; conics through five points, common properties of two conics, pencils of conics and circles, radical axis; line coordinates, trilinear coordinates, principle of duality, conics in line coordinates, generation of a conic by projective ranges, theorems of Pascal and Brianchon; point coordinates in space, lines and planes in space, coordinate trans-

¹ It is not uncommon for a candidate to offer as three subjects mathematics, physics, and descriptive geometry.

² If a candidate has not elected to specialize in both (4) and (5) he is examined in only one of them.

formation; facility in determining geometric loci.—In analysis: Thorough discussion of irrational and complex numbers, De Moivre's theorem, general theory of root extraction (binomial equations), properties of roots of unity; infinite series, conditional and unconditional convergence, series with complex members; convergency of power series, circle of convergence, continuity of power series; definition of elementary functions by power series; binomial, exponential, and logarithmic series; connection between exponential and trigonometric functions; Taylor's and MacLaurin's expansions; maxima and minima of one and of several variables; multiple integrals, surface, volume; fundamental theorem of algebra that every equation has a root; symmetric functions of the roots; cubic and biquadratic equations; approximation methods of Horner and Lagrange; elimination, methods of Sylvester and Bézout.

II. After two more years of university study the candidate may present himself for examination in his *special subjects*. Five months before he presents himself he is given a theme in each of his subjects, upon which he has to write a dissertation. In this dissertation the candidate must show familiarity with the literature of the subject as well as originality and knowledge in its presentation. All authorities must be carefully indicated. These themes are passed upon by the professors of the examination commission. If a dissertation is regarded as "insufficient," another one has to be prepared.

In addition to these dissertations an oral examination is required. In this the candidate must show decided proficiency in each of his subjects. All aids, except a table of logarithms, are forbidden. In case of failure at the oral examination, another trial may be made in six months. The oral examination in *mathematics* is on: (1) The mathematics of the secondary schools; (2) certain parts of geometry, algebra, and analysis common for all candidates; (3) the following five subjects, one of which the candidate must know thoroughly, the others in a general way—(a) Modern geometry and the theory of algebraic forms; (b) number theory and higher algebra; (c) general theory of skew curves and surfaces; (d) general theory of analytic and elliptic functions; (e) advanced portions of integral calculus.

In connection with (1) the candidate must not only display such acquaintance with the subjects as is requisite for graduation, but also a deeper scientific insight into elementary mathematics and a thorough appreciation of rigorous analysis. Facility in geometric drawing is also required. As to (2) the general examination is supplemented by consideration of the following topics:

Elements of synthetic geometry, ranges and pencils, harmonic and anharmonic ratio, involution, projective properties of conics; point coordinates in space, different forms of the equation of a plane, tetrahedrons, classification of surfaces of the second degree, general properties, particular surfaces, pencils of quadrics; adjoint and compound determinants, volume of tetrahedron and other geometric applications, functional determinants; continued fractions and indeterminate equations; infinite products, Wallis's formula; total differential equations, existence of a solution, general and particular solutions, simplest differential equations of the first and second

order, integrating factor, linear differential equations, variation of the constant, application of differential equations to geometric questions.

Under (3) the candidate is examined in the subjects indicated in the following synopses:

(a) Homogeneous coordinates in the plane and in space; conics and quadrics in these coordinates; symbolic notation; line coordinates in space; line geometry; projective properties of lines and surfaces; singularities; algebraic forms; binary and ternary forms; the most important invariants and covariants; general theory of quadratic forms; orthogonal substitutions.

(b) Elements of number theory; congruences; general theory of quadratic residues and quadratic forms; general theory of algebraic numbers; substitutions and groups; rational functions of roots of an equation; resolvents; cyclotomy; Abel's equations; conditions for the solution of algebraic equations; theorems of Sturm, Sylvester, and Hermite on the separation of roots of an equation; theory of elimination; chief proofs of the fundamental theorem of algebra (Gauss, Cauchy, etc.).

(c) Skew curves: Tangent and normal planes, first and second curvature, evolutes; curved surfaces; tangent and normal planes, lines on surfaces, surface elements, tangency of surfaces; theory of curvature, lines of curvature, orthogonal surfaces, confocal quadrics, Dupin's theorem, elliptic coordinates, conformal representation of surfaces; geodesic lines; partial differential equations of surfaces; theory of ruled surfaces.

(d) Functions of a complex variable: Single and multiple-valued functions, Riemann's surfaces, integration of functions of a complex variable, analytic expressions for single and multiple valued functions, Dirichlet's principle; algebraic functions, logarithms, trigonometric integrals; the three kinds of elliptic integrals and reduction of general elliptic integrals, the modulus of periodicity of integrals; periodic functions in general, fundamental properties of elliptic functions, theta functions, development into series; Abel's theorem, the problem of transformation.

(e) The more important definite integrals, multiple integrals, Fourier series and integrals, gamma functions, spherical harmonics; fundamental ideas of the calculus of variations, variation of functions, simple and multiple integrals, maxima and minima, second variation; simultaneous differential equations (general, complete, and singular solutions), partial differential equations of the first order; partial differential equations of the second order, integration of partial differential equations of mathematical physics by means of infinite series, methods of Monge and Ampère.

In *descriptive geometry* and *physics*; elaborate requirements are also made. To this oral examination the candidate must bring his certificate to show he has passed the general examination, and a certificate to show that he has studied two years further in the university or Polytechnikum.

III. A year after the special examination, the candidate may present himself for his *examination in pedagogy*. This consists of a philosophic or pedagogic dissertation, and an oral examination in history of philosophy, logic, psychology, pedagogy, history of pedagogy, and special methods for teaching mathematics. The examiners have constantly in mind the candidate's qualifications with regard to teaching all classes of the secondary schools. There is no such thing in Hungary as a teacher's being qualified to teach only in the lower classes of the State secondary schools. We have seen that this is not unusual in Austria.

In pursuing the courses leading to these examinations brilliant students may receive an annual stipend of 1,000 crowns (\$200). The plan of study requires not less than 20 hours a week of lectures. In the first two years are:

History of Hungarian literature (2 semesters, 2 hours weekly); Hungarian language (1 s., 2 h.); Hungarian culture (1 s., 1 h.). In the third and fourth years are: Logic (1 s., 2 h.); psychology (1 s., 2 h.); theoretical pedagogy (1 s., 2 h.); history of philosophy (1 s., 2 h.); history of pedagogy (1 s., 2 h.).

Lectures on subjects of the group *mathematics and physics* are distributed as follows (all subjects run through the year):

First year: Geometry and analysis (9-12 hours); algebra and number theory (3 hours); experimental physics (7 hours).

Second year: Analysis, continued (3 hours); geometry, continued (2 hours); algebra or number theory (3 hours); exercises with reference to the program of the secondary schools (2 hours); mechanics (6 hours); numerical exercises in physics (2 hours).

Third year: Special lectures in mathematics (5 hours); mathematical exercises (2 hours); scientific treatment of the subjects in the secondary schools (2 hours); theoretical physics (4 hours); exercises in experimental physics (5 hours).

Fourth year: Special lectures in mathematics (5 hours); mathematical exercises (2 hours); scientific treatment of the subjects in the secondary schools (2 hours); theoretic physics (4 hours); chemistry (2 hours).

For the group *mathematics and descriptive geometry*, the schedule is as follows:

First year: Geometry and analysis (9-12 hours); algebra and number theory (3 hours); descriptive geometry (5-6 hours); constructive drawing (7 hours).

Second year: Continuation of analysis (3 hours); continuation of geometry (2 hours); algebra and number theory (3 hours); exercises with reference to the program of the secondary schools (2 hours); projective geometry (2 hours); special lectures on descriptive geometry (2 hours); review and questions of descriptive geometry (3 hours).

Third year: Special lectures in mathematics (5 hours); mathematical exercises (2 hours); scientific treatment of secondary school mathematics (2 hours); special lectures and exercises in descriptive geometry (4 hours).

Fourth year: Special lectures in mathematics (5 hours); scientific treatment of secondary school mathematics (2 hours); mathematical exercises (2 hours); special lectures and exercises in descriptive geometry (4 hours).

In Budapest the student may attend such courses in both the university and the Polytechnikum.

The Baron Eötvös College in Budapest is an important institution—the object of which is to give deserving students of the Budapest University who intend to enter the teaching profession an opportunity for holding social intercourse with their fellows and for acquiring the theoretical and practical knowledge necessary to qualify them for their work. The college is directly subordinate to the minister of public instruction, who delegates his authority to the curator. For expert guidance of the resident students four tutors (chosen by the curator from among the teachers in the service of the State) are appointed by the minister (for periods of three years in rotation) for special duties. They are present in the college during the hours devoted to private study to give individual or combined instruction to the candidates, and, as occasion arises, to hold special courses of lectures.

We have yet to consider the institutions where our future secondary school professor, who has completed his four years in the university, receives a year of professional training.

Under the direct control of the minister of education is a State Training College (founded in 1895) for Secondary Teachers. It is established at Budapest, and its aim is to train university students for the profession of teaching. The students all live together in the training college, and continue their university studies simultaneously with their pedagogic training. At the head of the college is a curator, and under him are four professors who are appointed by the minister. It is the duty of these professors to teach the students the principles underlying education, and to show them how they can be applied. The students teach in a Gymnasium connected with the training college for that purpose. The methods adopted are Herbartian as worked out by Ziller and Stoy. At the end of each school year the curator and professors formally confer together in regard to the students' progress, and weed out the unsatisfactory ones. Students may not remain in the college longer than four years.

There is also the famous Pedagogic Seminar at Budapest, founded in 1872 by Dr. Kármán.¹ This institution was modeled on Ziller's "Seminar" at Leipzig. It has three distinct aims: (1) To be a gymnasium; (2) to be a practice school for secondary teachers; (3) to be an institution for promoting special methods of teaching in Hungarian secondary schools.

The students in the Budapest Pedagogic Seminar are young men who have spent eight years in a secondary school and four years in a university and have passed the general and special examinations given by the examination commission.

They make a thorough study of the theory of special secondary school teaching in connection with practical experience of the same. Their philosophic and theoretic studies are already completed. The staff consists of leading professors of special subjects, a professor of pedagogy, and a director.

After each candidate announces the special subject he wishes to teach, he is placed under the direction of the professor of that subject. At first he is not himself permitted to teach, but he must attend all the lessons the directing professor gives to the various classes; he must then work out series of lessons under the direction of the professor; and after about three months he may begin to teach. The professor is always present at his lessons to offer him advice and criticism.

Every week there is a *Praktikum* or criticism lesson. In the early part of the year these lessons are given by the professors, with the

¹ For a description of this Seminar the following articles on "Das Pädagogische Universitäts-Seminar in Budapest" in *Pädagogische Studien* (Neue Folge herausgegeben von W. Rein at Dresden) may be consulted in Jahrgang 8 (1887), pp. 219-222, by E. Leonhardt; in Jahrgang 9 (1888), pp. 99-103. It will also be of interest to turn to M. KÁRMÁN *Beispiel eines rationalen Lehrplans für Gymnasien*, Halle a. S., 1890.

whole Seminar present. In the latter part of the year the candidates give lessons, and conferences are held two days after each lesson, in which all points raised by the Praktika are fully discussed. Twice a week a *Theoretikum* is held in which general principles and special methods of secondary school teaching are discussed.

Teachers of secondary schools attain to full service after three years' probationary service. At this stage they receive an annual salary of 2,000 crowns (\$400) in the capital, and 1,600 crowns (\$320) in the provinces. The teachers under full appointment are divided into two classes. Salaries in the lower class begin at 2,600 crowns and rise by periodical increase, to 3,200 crowns; in the higher classes the salaries increase by successive additions, from 3,600 to 4,400 crowns. Directors receive from 4,800 to 6,000 crowns.

After 10 years' service, according to the regulations of 1894, a teacher is entitled to a pension of 40 per cent of his salary, and for each year of further service the pension increases by 3 per cent, so that at the end of 30 years' service the teacher may retire on full salary.

Teachers contribute to the pension fund one-third of the excess of a year's salary above 690 crowns, and one-third of every increase in salary. These sums are paid only once.

BIBLIOGRAPHY:

- J. VOLENSZKY, *Der mathematische Unterricht an den Bürgerschulen*. (Internationale Mathematische Unterrichtscommission). Budapest, Franklin-Verein, 1912, 18 pp.
- E. BEKE, *Der mathematische Unterricht an den Mittelschulen*. (Gymnasien und Realschulen). (I. M. U. K.). Budapest, Franklin-Verein, 1912, 20 pp.
- K. GOLDZIEHER, *Der mathematische Unterricht an den Lehrerbildungsanstalten*. (I. M. U. K.). Budapest, Hungaria Buchdruckerei, 1912, 13 pp.
- G. RADOS, *Der heutige Stand des mathematischen Unterrichts am königlich ungarischen Jos. Polytechnikum*. (Technische Hochschule in Budapest.) (I. M. U. K.). Budapest, Hungaria Buchdruckerei, 1911, 14 pp.
- P. von SZABÓ, *Der Unterricht der Mathematik am Übungsgymnasium*. (I. M. U. K.) Budapest, Hungaria Buchdruckerei, 1912, 17 pp.
- J. KÜRSCHÁK, *Die Ausbildung der Mittelschulprofessoren*. (I. M. U. K.) Budapest, Hungaria Buchdruckerei, 1911, 20 pp.
- Cf. J. Renard, in *L'Enseignement Mathématique*, t. 14, 1912, pp. 407-416.
- Abhandlungen über die Reform des mathematischen Unterrichts in Ungarn im Auftrage der mathematischen Reformkommission des Landesvereins der Mittelschulprofessoren nach dem Ungarischen Original unter Mitwirkung der Herren M. Balog und J. Rados deutsch herausgegeben von E. Beke und S. Mikola*. Leipzig und Berlin, Teubner, 1911, 160 pp.
- This volume contains "Abhandlungen" by 11 different authors.
- C. I. DODD, "Hungarian Education." *Special Reports on Educational Subjects*, Vol. 8, London, Board of Education, 1902, vol. 8, pp. 481-536.
- A. T. SMITH, "Education in Hungary" in *Cyclopædia of Education*, edited by Monroe, vol. 3, New York, 1912.

X. ITALY.

Italy is about 110,600 square miles in extent, and its population of upward of 36,000,000 is in general perfectly homogeneous in language, except for about 250,000 people of French, Teutonic, Slavonic, Albanian, Greek, and Spanish origin.

The educational system of the country is under the direction of a minister, who is a member of the cabinet and either a deputy or a senator. He is assisted and in some respects controlled by the *consiglio superiore* or higher council which, in accord with legislation of 1909, has 36 members. These members consist of 6 senators elected by the Senate, 6 deputies (not university professors) elected by the Chamber of Deputies, 12 members nominated by the minister, and 12 designated by the ordinary and extraordinary university professors. The consiglio, on request of the minister, prepares and examines all bills and general provisions relating to the organization of schools, appointment of professors, etc. The power of control on the part of the consiglio is illustrated by the fact that the minister can neither dismiss nor suspend a professor without its consent.

Three departments of the ministry of public instruction are concerned, respectively, with elementary and normal education, secondary education, and higher education.

SECONDARY SCHOOLS.

The secondary-school training of Italy is based upon four years' preparation in the elementary schools (pupil's age 6 to 10 years). Apart from certain normal schools there are two broad types of secondary schools: (1) Classical schools and modern schools with Latin; (2) modern schools without Latin, and technical schools. Of the first type there is the *ginnasio*, with a five years' course leading up to the *liceo*, whose three years' course prepares for entry into the universities. The second type includes the *scuola tecnica* and *scuola complementare*, each with a three years' course. They are preparatory for such schools as the *istituto tecnico*, with a four years' course, and the *istituto nautico*, with a course of three years.

¹ As an exception the course for the industrial section of the *Istituto di Bergamo* lasts five years. Concerning mathematical instruction in technical schools and institutes, see G. Scorza, *L'insegnamento della matematica nelle scuole e negli istituti tecnici*. (Commissione internazionale l'insegnamento matematico.) Roma, 1911. 34 pp.

In 1912-13 the relative numbers of these schools for boys and girls were as follows:

	Government.	Private.
Normal schools.....	144	124
Ginnasi.....	291	262
Licei.....	162	77
Technical schools.....	344	207
Technical institutes.....	79	32
Mercantile marine institutes.....	20	0

No student is admitted to a secondary school unless he has passed an examination called *maturità*.

The course of study in the classical ginnasio includes the Italian language and literature, Latin, Greek, French, mathematics, and a little drawing and natural history. In the corresponding liceo, philosophy, physics, and chemistry are taught in addition to all the subjects (except French) of the ginnasio. In the "modern" ginnasi and licei, established by an act of 1911, Greek is no longer taught; instruction in Italian, Latin, French, German, or English, geography and history, mathematics, natural history, drawing, and physical culture is given at these ginnasi. Besides these the course at the corresponding licei includes, in addition, political economy, philosophy, civics, physics, chemistry, astronomy, and physical geography. In cities having a ginnasio and a liceo, the two schools are combined in a *liceo-ginnasio*. The "modern" schools with Latin are being established in those cities which have more than one liceo-ginnasio. The *licenza ginnasiale*, or diploma, of a ginnasio represents the standard of training for minor posts in the civil service and for the licei. The diploma *licenza liceale*, awarded after the satisfactory completion of the eight-year period of secondary education, admits to the universities.

At this time the student who has taken the possible mathematical courses of the liceo has been taught—

In *algebra*: Equations of the first and second degree, progressions, theory of indices, theory of logarithms, binomial theorem, irrational numbers, prime numbers and divisibility, indeterminate equations of the first degree. In *geometry*: Relations of positions, equality of solids; proportion and similitude in plane geometry; measurement; theory and application to plane geometry; practical rules for the measure of curved surfaces and of solids; equivalence and similitude of solids; theory of the measure of curved surfaces and of solids; applications of algebra to geometry. In *trigonometry*: Through the solution of plane and spherical triangles.

This work is covered by four hours a week of work in the first year, three hours in the second, and two in the third.

From these indications we observe that in the liceo-ginnasio mathematics does not occupy the central position, as in corresponding schools of other countries such as France.

In the physico-mathematical section of an istituto tecnico six hours a week are devoted to arithmetic, algebra, and geometry in

the first year; five hours to the same subjects in the second year; and five hours in the third year to algebra, geometry, plane and spherical trigonometry, and the elements of descriptive geometry. All of the mathematical subjects of the liceo-ginnasio are here treated much more fully. The following topics are sometimes introduced: Geometry of the triangle, geometrography, derivatives and their applications to maxima and minima, equations of the third and fourth degree, probabilities, determinants, history of elementary mathematics.

Thorough grounding in geometry is characteristic of Italian secondary schools. It is only necessary to recall in this connection the works of Sannia and D'Ovidio, Faifofer, Lazzeri and Bassani, and de Paolis.

Having thus gathered impressions as to subjects in which a teacher of mathematics in leading secondary schools has to give instruction, let us next consider those schools where the mathematical teachers are trained.

THE UNIVERSITIES AND TRAINING OF SECONDARY SCHOOL TEACHERS.

The term "university" is applied in Italy only to those schools of the highest grade in which students are instructed in special branches for the professions of their choice. There are in Italy 21 universities,¹ 17 bearing the description "royal" and receiving State subsidies and 4 independent of Government control. The free universities are at Perugia, Ferrara, Urbino, and Camerino. The oldest university was founded at Bologna about 1200 and the youngest at Palermo in the early nineteenth century. The other universities are at Padua, Macerata, Naples, Genoa, Pavia, Rome, Pisa, Siena, Turin, Catania, Parma, Messina, Sassari, Cagliari, and Modena. The largest university is at Naples, where there were more than 4,000 students in 1913-14; in point of size the university at Rome, with over 3,000 students, ranks next.

Repeated efforts have been made to reduce the number of royal universities, which exceed both the needs and the resources of the nation. As a consequence a few vigorous universities are found in the same class with a number of struggling institutions that are quite unable to maintain standards and prestige. The condition has been the subject of much discussion in the legislature, but so far no modification has resulted. Meanwhile complaints have arisen that the standards of secondary education are depressed as a consequence of the competition for students on the part of the universities.²

¹ There are about a score of schools known as "superior institutes," which have the scholastic program of universities. Though private they are more or less under the control of the Government. Probably the oldest now in existence is that founded at Florence in the fourteenth century. An account of mathematics taught in the five-year course of the Reale Istituto Tecnico Superiore at Milan is given by V. Snyder in "Mathematics at an Italian Technical School," *Bulletin of the American Mathematical Society*, December, 1916, vol. 23, pp. 149-151.

² Rep. Commis. of Educ., 1913-14. Washington, 1915, vol. 1, pp. 757-758.

The faculty of sciences in the university has a section of mathematics in which the object is twofold: (1) To give to future engineers the preparation which will fit them for the Scuole di Applicazione (schools of applications); (2) to prepare candidates for the degree of doctor in mathematics.¹ This degree is a prerequisite for teachers of mathematics in secondary schools.

Preparation for the degree requires four years of study, the first two leading to the licenza. The courses followed during this period are in physics, inorganic and organic chemistry, algebraic analysis, infinitesimal analysis, analytic geometry, projective geometry, descriptive geometry, and drawing.

The *licenziati dell'universita* then attend lectures at a school for preparation of secondary school teachers (*scuola di magistero*) and follow courses at the university in five or six of the following subjects: Higher analysis, higher geometry, higher mechanics, theoretical geodesy, astronomy, mathematical physics. The *scuole di magistero* are connected with the faculties of sciences, and in them one or two professors give lectures on methods of teaching, etc. The diploma di *magistero* obtained from these scuole is a distinct advantage to its holder when he is seeking a position in secondary teaching.

At the end of a year the *licenziato* generally offers himself for examination in at least three of his five courses; his second year is largely occupied with his dissertation and the preparation of his minor theses. The final examination, which is oral, is held before a commission of 11, consisting of 7 professors of the faculty and 4 *privati docti*. If the examiners award the candidate a mark of not less than 60 per cent, he is proclaimed *dottore in matematiche pure* by the dean of the faculty.

It very often happens that candidates add to the required four years of study a fifth, for the preparation of the dissertation and for attendance at special courses such as those of the R. Scuola Normale Superiore of Pisa and of the "Istituto consorziale" of Pavia, or of the recently established mathematical seminaries of the faculties of science at Rome and Naples.

While the annual number of mathematical *dottori* in Italian universities has not appreciably increased in the last 30 years, the number of secondary schools has greatly augmented, and the problem of procuring properly qualified teachers has been a very difficult one to deal with. Furthermore, the problem is not altogether one of numbers. It is strongly felt by many that while the larger universities furnish an admirable scientific training, far too little attention has been paid to the professional and practical training.

The first appointment of a teacher in a secondary school is for three years, after which the appointment may be made permanent if the inspector's report is favorable;

¹A doctor's degree is required for admission not only to the professions (physician, lawyer, teacher in secondary school, etc.), but also to the highest grade clerkships in the Government service in all branches.

if not, the appointment is extended for another year, when the teacher is definitely appointed or dismissed. During this probationary period the teacher (*professore*) is called extraordinary (*straordinario*); after the definite appointment, ordinary (*ordinario*).¹ As a rule a teacher is appointed for a single study or for two related subjects.

Teachers in the first three classes of the ginnasio receive as salary about \$347 (1,800 lire)² a year (if extraordinary) and from \$386 to \$926 (2,000 to 4,800 lire) if ordinary. In the case of the teachers in the licei and in the fourth and fifth classes of the ginnasi, these salaries are increased to \$414 if extraordinary and \$482 to \$1,042 (2,500 to 5,400 lire) if ordinary.³

All ordinary teachers receive four quinquennial increases in salary of 500 lire each. An increase of one-tenth of this salary is made at the end of two six-year periods. "In this way they pass from the minimum to the maximum salaries as given above. Two of the four fixed increases, but not two consecutive ones, may be anticipated one year in the case of exceptionally able teachers."

Headmasters in schools are appointed from teachers who have taught for thirteen years, and from a list compiled by the consiglio superiore—

in accordance with results of inspections. They are appointed for a first period of five years, during which they teach and receive extra compensation varying from 750 to 1,000 lire a year. After the probationary period they are permanently appointed and receive salaries not higher than 5,750 lire for such schools as ginnasi and 6,500 lire for schools of the licei standard. Schools equivalent to Government schools must pay the same salaries.

The teacher is entitled to a pension proportional to his salary and the length of his service. A pension equal to the full salary at the time of retirement is due to the teacher who has served continuously for 25 years or more. In case of his death two-thirds of the pension awarded to the husband or father is paid to the widow or to the children if orphans.

NORMAL SCHOOLS.

The normal schools form a third type of school in secondary education. All that has been stated above with reference to the appointment, salaries, and pensions of licei teachers applies to teachers in the normal schools. Their courses are designed to qualify teachers for the primary schools.

¹ In certain cases when a teacher would have only a few hours a week a temporary appointment is made. Such appointments are also made if a regular appointee can not be found for the place, and in schools with a large number of pupils when it is necessary to divide classes and the regular teacher can not take charge of all.

² The lira equals 19.3 cents.

³ A comparison of these salaries with those of university professors may be of interest. In accordance with a recent law, the salary of the ordinary professor of State universities is fixed at 1,000 lire minimum; that of the extraordinary professor at 4,500 lire. The salary of the ordinary professor may be increased until a maximum of 10,000 lire is reached through quinquennial increases of 750 lire each. The salary of the extraordinary or special professor has a one-tenth quinquennial increase, but must never exceed the initial salary of an ordinary professor.

A licenziato of a scuola tecnica is prepared to enter a scuola normale with its course of three years. To students with the licenza ginnasiale many ginnasi offer a two-year corso magistrale.

The plan of studies for a scuola normale includes pedagogy, ethics, Italian, history, geography, the elements of mathematics,¹ accounting, physics, chemistry, natural history, and hygiene. Drawing, penmanship, singing, agriculture, gymnastics, practice in elementary schools, and manual training.

In England and the United States it is important that a teacher should be able to maintain discipline in the classroom. But in Italy matters are ordered differently.

Youngsters in their early teens aim to shape municipal policies; they get up demonstrations and indignation meetings, and go on strikes when their instructors fail to comport themselves to their liking.²

In the university—

there are unannounced holidays. At periods of political excitement the students have a patriotic custom of marching on the professors for speeches. Should the doors be locked they will applaud enthusiastically thereon with hands and feet. As there is no discipline to deal with these cases, and it is not etiquette to introduce the police, the authorities frequently adopt the prudent course of closing the university until word arrives that the students are ready to return to ordinary academic ways.

BIBLIOGRAPHY.

- F. SCARFIS, *L'insegnamento della matematica nelle scuole classiche*. I. I successivi programmi dal 1867 al 1910. (Commissione Internazionale dell'insegnamento matematico. Atti della sotto-commissione italiana). Roma, cooperativa tipografica Manuzio, 1911, 11 pp.
- G. FAZZARI, *L'insegnamento delle matematiche nelle scuole classiche*. II. Critiche e proposte. (C. I. i. m.) Roma, Manuzio, 1911, 16 pp.
- A. CONTI, *L'insegnamento della matematica nelle scuole normali*. (C. I. i. m.) Roma, Manuzio, 1912, 71 pp.
- S. PINO HERLE, *Sugli studi per la laurea in matematica e sulla sezione di matematica delle scuole di magistero*. (C. I. i. m.) Roma, Manuzio, 1911, 16 pp.
- A. PADOA, *Osservazioni e proposte circa l'insegnamento della matematica nelle scuole elementari, medie e di magistero*. (C. I. i. m.) Roma, Manuzio, 1912, 22 pp.
- C. SOMIGLIANA, *Intorno all'ordinamento degli studi matematici nel primo biennio universitario in Italia*. (C. I. i. m.) Roma, Manuzio, 1911, 11 pp.
- G. SCORZA, "Sui libri di testo di geometria per le scuole secondarie superiori" *Bollettino della "mathesis,"* Roma, Manuzio, Anno IV, 1912, 15 pp.
- Cl. E. Chatelain, *L'Enseignement Mathématique*, tome 14, 1912, pp. 156-161, 249-253, 416-424; tome 15, 1913, pp. 170-176, 427-430.
- J. L. COOLIDGE, "The Opportunities for Mathematical Study in Italy," *Bulletin of the American Mathematical Society*, vol. XI, 1904, pp. 9-17.
- T. de SUZZAZA-VERDI, and A. OLDRINI, "Education in Italy," Report of the Commissioner of Education for 1901-2, Washington, Government Printing Office, 1903, pp. 741-787.
- A. FANTI, "Education in Italy," in *Cyclopedia of Education*, edited by Monroe, vol. 3, New York, 1912.
- R. BETTAZZI, "Un essai de reforme des études moyennes classiques en Italie," *L'Enseignement Mathématique*, tome 7, 1905, pp. 400-406.

¹ Two hours a week: Arithmetic, plane and solid geometry, accounting.

² Rep. of U. S. Commis. Educ., 1906, vol. 1, Washington, 1907, p. 82.

XI. JAPAN.

The area of Japan proper is less than 150,000 square miles, and its total population is about 56,000,000.

One of the most prominent features of its educational system is the almost absolute centralization of authority. Educational affairs are administered by a department of education, at the head of which is the minister of education, who is a member of the imperial cabinet and is directly or indirectly in charge of everything pertaining to education. He is assisted by a vice minister, several directors of bureaus, and a number of councilors and secretaries. The department of education was first definitely established in 1871, and in the following year there was issued a code which stipulated that the whole country should be divided into 8 *large educational* districts, each with a university. Each of these districts was to be subdivided into 32 *intermediate educational* districts, each with a secondary school, every one of which was in turn to be further subdivided into 210 *small educational* districts, each with an elementary school. This plan, which bore striking resemblance to the French scheme, was found too ambitious to carry out in its entirety. Elementary schools were indeed established largely as planned. The standard for secondary schools is still unsettled although, since the promulgation of the code, reforms in secondary education have been of a sweeping character. As recently as April, 1912, certain new reforms went into effect. Of these I will take no account in what follows. As to the universities, not more than one of the four in existence is 20 years old, so that this branch of the educational system falls far short of the ideal of the code.

Immediately above the elementary course of six years is the *middle school* course of five years, "socially obligatory" for membership in the educated class. A one-year supplementary course is sometimes offered. Pupils begin in the middle school at the age of 12 years. Next in order to the middle schools comes the *higher middle school*, whose course extends over three years. Much of the discussion as to secondary education is centered upon the question whether this school, intermediate between the middle school and the university, is to be affiliated with the former or with the latter. On the whole the tendency seems to be in the direction of giving greater prominence to the higher general education. And the aim of the higher middle school is preparation for the different faculties of the university. There are

four imperial universities at present, namely, those at Tokyo, at Kyoto (the ancient capital), at Kiushiu (the southern island), and at Tôhoku ("northeastern"). The normal age of a student prepared to enter a university is about 20 or 21. In the higher middle school he has covered the work of about the first two years in an American college of the better class.

Baron Kikuchi writes:

In all cases graduation from one school is a qualification for entrance into the next higher stage, but in recent years the demand for higher education has increased at such a rate that, notwithstanding the very great and rapid expansion of educational resources, a competitive examination for admission has to be held at almost every stage, the number of candidates for admission being from twice or three times to even in some cases as large as 10 times the number of those that can be admitted.

This is particularly true of the higher middle schools of which there are only eight. The students who arrive at the university are thus likely to be the élite of the youthful intellectuals.

The only branch of mathematics taught in the *elementary schools* is arithmetic. There is much practice in *soroban* calculations.¹

In the *middle schools*: Arithmetic (conclusion); algebra through such subjects as ratio and proportion, progressions; permutations and combinations, binomial theorem with positive index; logarithms; elements of plane and solid synthetic geometry, as in Baron Kikuchi's book which is based upon the syllabi of the Association for the Improvement of Geometrical Teaching; plane trigonometry, through the solution of triangles.

The competitive examination for entrance into the higher middle schools is excessively severe and of a nature that is far from winning general approval. The examinees are picked boys from over 300 middle schools scattered all over the country. Among them are those who have failed one, two, three, or even more times. The percentage of those who succeed on first trial is small.

In the *higher middle schools* there are three sections; the first is for those who wish to enter the faculty of law or literature in the university; the second for those who wish to enter the faculty of science, technology, or agriculture, or a certain part of the faculty of medicine; the subjects in the third section are religion, the Japanese language, foreign languages (German and either English or French), Latin, mathematics, physics, chemistry, zoology, botany, and gymnastics. The chief mathematical training is given in the second section. The subjects of study include trigonometry, algebra, analytic geometry, and calculus.

In *trigonometry*, Todhunter's "Plane Trigonometry for the use of Colleges and Schools" is used as a textbook.

¹ The *soroban* is a variety of abacus.

In *algebra*, besides R. Fujisawa's "Sequel to Elementary Algebra" (written in Japanese), Todhunter's "Algebra for the Use of Schools and Colleges" and C. Smith's "Treatise on Algebra" are used as texts. Some of the topics discussed are: Inequalities; theory of determinants; theory of probability; theory of numbers; continued fractions; indeterminate equations; convergency and divergency of infinite series; symmetric functions of roots; cubic and biquadratic equations; methods of elimination.

In *analytic geometry*, besides Kikuchi's "Plane and Solid Analytic Geometry," Puckle's "Conic Sections and Algebraic Geometry" and N. S. Aldis's "Treatise on Solid Geometry" are used as texts.

In *differential and integral calculus* the textbooks used are those of Todhunter and Williamson. The chief books of reference in current use, though not necessarily confined to higher middle schools, are the calculus treatises by Kiepert, Czuber, J. A. Serret, Byerly, Lamb, Greenhill, as well as Appell's "Éléments d'Analyse Mathématique" and J. W. Mellor's "Higher Mathematics for Students of Chemistry and Physics."

THE UNIVERSITIES.

The higher middle schools are preparatory institutions for the imperial universities, where the greater part of the students enter the college of engineering. The first article of the imperial ordinance for the founding of imperial universities reads: "Imperial universities shall have for their objects the teaching of such arts and sciences as are required for the purpose of the State, and the prosecution of original research in such arts and sciences." The second article of the same ordinance runs: "Each imperial university shall consist of a university hall and faculties, the university hall being established for the purpose of original research, and the faculties for instruction theoretical and practical." At present only three of the universities have faculties of science. The length of a course is either three or four years. At the end of every year there is a thorough examination. A graduate is entitled to call himself *gakushi* or "graduate of a university faculty."

Owing to the insufficiency of schools, boys are often required to pass competitive examinations in going from an elementary to a middle school as well as from a middle to a higher middle school. Moreover, as is only natural, some of them may not be able to finish a school course in the prescribed number of years. Thus it happens, not infrequently, that a student reaches the age of 30 or thereabout by the time he has finished his university course.

The newer universities are modeled upon the University of Tokyo, which is far more completely organized. At this university the following mathematical courses are offered in the faculty of science:

FIRST YEAR.

Differential and integral calculus, four hours a week through the three terms of the year. This includes discussion of: Volume and area of surfaces; moments of inertia; integration of ordinary differential equations of the first and higher orders and their applications; linear and simultaneous differential equations; calculus of variations.

Solid analytical geometry, four hours for one term. Lines; planes; surfaces of the second degree; elements of the general theory of surfaces.

Projective geometry, two hours a week in second and third terms. Synthetic and analytic discussion of projective forms of the first order; curves of the second degree and second class.

Select chapters in elementary mathematics, three hours a week in second and third terms. Elementary algebra; domain of rationality and divisibility of integers, of rational integral functions of one variable and of many variables; algebraic solutions of the general equations of the first, the second, the third, and the fourth degrees. Sturm's theorem, etc.

Astronomy and least squares, three hours a week through the year. Some of the topics are: Spherical trigonometry; fixed stars; sun; calendar; planets; moon; eclipses; rotation of planets; precession and nutation; theory of probabilities; law of errors; method of least squares.

General physics, three hours a week through the year.

SECOND YEAR.

General theory of functions and theory of elliptic functions, three hours a week through the year.

Differential and integral calculus and differential equations, two hours a week through the year. Topics here discussed are: Fourier's series and integrals; Legendre's and Bessel's functions and hypergeometric series; total and partial differential equations, etc.

Theory of numbers and algebra, five hours a week through the year.

Higher geometry, three hours a week through the year. Differential, noneuclidean, or descriptive.

General dynamics, three hours a week through the year.

THIRD YEAR.

The titles of the courses in this year are practically the same as those in the second; the student is lead on to deal with advanced topics.

Other courses in astronomy and theoretical physics are also given, but further details are not germane to our subject.

PREPARATION OF MIDDLE SCHOOL TEACHERS.

To become a regular teacher in a middle school it is necessary to have a license granted by the minister of education. This is given (1) without examination to graduates of Government schools and institutes especially established for training teachers and to others specified later, and (2) after examinations to still others. The examinations are conducted at least once a year by an examining board appointed from among the professors of the imperial universities, of higher normal schools, of higher technical schools, and of similar institutions. The examinations for teachers' licenses in mathematics are of four grades: The first consisting of arithmetic, algebra, and geometry;

the second of trigonometry; the third of analytic geometry; and the fourth of differential and integral calculus.¹ The examinations are held separately for each grade, and at the same time there is a test as to sufficient knowledge of the outlines of pedagogy and of pedagogic methods. Teachers of mathematics in the middle schools are not regarded as fully equipped for their work unless they have been successful in passing the examination for all four grades. The majority, however, simply have licenses in the first grade. Graduates of the regular and special courses in mathematics in the higher normal schools, as well as certain others, are regarded as having successfully passed the examinations in all four grades and are entitled to licenses in them all.

HIGHER NORMAL SCHOOLS.

The schools specially established for the training of teachers consist of higher normal schools, of which there are two at present, one at Tokyo and the other at Hiroshima. The single temporary institute engaged in training teachers in mathematics is located at Sondai.

Each of the higher normal schools has a four years' course, one year preparatory, and three years' regular course. The preparatory course is common to all students. The regular course, in which only special subjects are taught, is divided into five departments, that having special interest for us being called the "Department of mathematics, physics, and chemistry." This in its turn is divided into two minor divisions, one for students of mathematics and physics, and the other for students of physics and chemistry.

Students entering the preparatory course of the higher normal schools are mostly graduates of middle schools or of normal schools, or are students recognized to have scholarship equal to that of the graduates of either of these schools. In the Tokyo Higher Normal School, out of applicants from all parts of the Empire only those are admitted who have the highest grade in the entrance examination. At present, in the department of mathematics, physics, and chemistry, about 50 are admitted as preparatory students out of three or four hundred applicants. In Hiroshima the method of selection is different, but the number in this department is less than at Tokyo.

In both schools students enter at the average age of 21; so that the average age of graduates is 25.

The students in the preparatory course of the Tokyo Higher Normal School are required to take 30 hours' classroom work a week, of which 4 hours are devoted to mathematics—making a total of 160 hours for the year. Of these, 20 to 25 hours are given to arithmetic, 40 to 45 hours to algebra, some 50 hours to geometry, and about

¹Copies of these examination papers for 1911 are given in Appendix F.

30 hours to plane trigonometry. The work is based upon that of the middle school rather than that of the normal school, which is fuller and more advanced. Nearly the same instruction is given in the preparatory course at Hiroshima.

In the Tokyo Higher Normal School the first-year students of the department of mathematics, physics, and chemistry are required to take a total of 26 hours of classroom work a week throughout the year, in addition to hours for laboratory experiments in physics and chemistry. Of the 26 hours, 2 are devoted to algebra, 2 to trigonometry, and 2 to geometry. The topics taken up are:

In algebra—Inequalities; theory of quadratic equations; maxima and minima; equations of higher degree; simultaneous quadratics; ratio, proportion, and variation; series; progressions; permutations and combinations; binomial and multinomial theorems; limiting values; exponential theorem; logarithmic series; determinants.

In geometry—Foundations of geometry; definitions and axioms; noneuclidean geometry; demonstrations of theorems; maxima and minima; geometric loci; problems of geometric constructions; geometry of the triangle; modern geometry; history of elementary geometry.

In trigonometry—Identities; relations between the sides and angles of a triangle and a quadrilateral; measurements of heights and distances; inverse trigonometric functions; trigonometric equations; trigonometric inequalities; maxima and minima; trigonometric solution of algebraic equations; use of tables of trigonometric functions, and tables of logarithms; construction of these tables; rules of proportional parts.

The courses in algebra and trigonometry are required for all students in this department, but students specializing in physics and chemistry may substitute for the course in geometry described above a course in cartesian geometry.

The second and third year students are divided into two sections, the one consisting of those who take mathematics and physics as their specialties and the other of those who take physics and chemistry. The students of both sections have 23 hours of classroom work per week in all subjects. To students of the first section experiments in physics and exercises in mathematics are assigned outside of these hours. In the second year, 6 of the 23 hours are devoted to higher algebra, modern geometry, and analytic geometry. In addition 4 hours a week are allotted to mathematical exercises or to lectures on mathematical subjects.

Higher algebra, two hours a week. Some of the subjects treated are: Infinite products; continued fractions; indeterminate equations; properties of rational integral functions; algebraic equations; symmetric functions of roots of equations and their applications; solutions of numerical equations; elementary theory of numbers; divisibility; congruences.

Analytic geometry, four hours. Some of the topics are: Straight lines; circles; parabolas; ellipses; hyperbolas; general equation of the second degree; systems of conics; homothetic and similar conics; coordinates in space; lines and planes; surfaces of the second degree.

Of the four hours for mathematical exercises two are devoted to supplementing the above-mentioned course of algebra, with assigned exercises in algebra. The remaining two hours are devoted to lectures, some subjects of which are as follows:

Complex numbers; De Moivre's Theorem; roots of a number; binomial equations, relations of surds; trigonometric factorizations; powers of a complex number the exponent of which is a complex number; logarithm of a complex number; expansions in trigonometric series; demonstrations of theorems and solutions of problems in Euclidean space; problems impossible of geometric construction—trisection of an angle, duplication of the cube, quadrature of the circle; transcendence of e and π ; constructible regular polygons.

In the third year the students are engaged in practical teaching during the whole of the last term. During the first two terms 23 hours a week are devoted to lectures, of which 10 hours are in mathematics, as follows:

Differential and integral calculus, six hours. Some of the topics are: Higher differential coefficients and differentials; curvature of plane curves; evolutes and involutes; functions of two variables, partial differentials; total differentials; surfaces and osculating paraboloids; indicatrix; principal curvatures; curves in space; tangent, normal, binomial, principal normal, rectifying line and cone, curvatures and spherical curvatures; Gregory's series, evaluation of π ; existence theorem of integral calculus; differentiation and integration under the integral sign; quadrature of areas; center of gravity; moments of inertia; Lagrange's interpolation formulae; mechanical quadrature; nonequidistant ordinates of Gauss; Eulerian integrals, Stirling's formulae; linear differential equations with constant coefficients, characteristic equations; particular and general solutions; differential equations of the first order, singular solutions; existence proof according to Lipschitz; uniqueness of solutions; linear equations of the second order; Fourier's functions, trigonometric series; equations of potential functions, of small vibrations, and of heat diffusion.

Two hours a week are given to discussion of additional work in *Analytic geometry*: General theory of curves; construction of curves in rectangular and polar coordinates; singular points; limaçon, epicycloid; spiral of Archimedes; logarithmic spiral; strophoid; etc.

The remaining two hours are allotted to problems in calculus and dynamics: Velocities and accelerations; straight motions; circular motions; parabolic motions; elliptic motions; constrained motions; elements of rigid dynamics and hydrodynamics.

The mathematical requirements in the Hiroshima Higher Normal School are very similar to those in the Tokyo school. The aim in teaching mathematics in both of these schools is to develop in the student a well-disciplined logical faculty, to lead him to acquire a practical knowledge, and to heighten his æsthetic sense of scientific beauty.

Among the graduates of the higher normal schools there are a very limited number who, instead of immediately engaging in teaching, take the graduate course. This course, still looking toward the teaching of mathematics in middle schools, prescribes theories of mathematics, considerably in advance of those discussed in the regular course. The aim is to strengthen the student's grasp on the fundamentals of

mathematics prescribed by the curriculum of middle schools. An account of the condition of mathematical instruction throughout the world is generally given. The theory of numbers, higher algebra, theory of curves and surfaces, hydrodynamics, etc., are sometimes offered. The time allotted is from 6 to 12 hours per week.

Very few of the graduates of the higher normal schools enter the imperial universities for further study.

The mathematical requirements in the Sendai Institute are almost identical with those in the higher normal schools.

Besides the three institutions mentioned above whose graduates are given teacher's licenses, there is a private school in Tokyo, called the Tokyo Butsuri Gakko (Physics School at Tokyo), most of whose graduates go out to teach in the middle schools. The graduates of this school, however, are required, in order to obtain teachers' licenses, to pass the examination conducted every year by the educational department. Similar conditions obtain in connection with the recently established Waseda University, a private institution which has established a higher normal course containing a branch for training teachers of mathematics.

Teachers' licenses in mathematics are also given without examination to the following:

1. Graduates of the schools named below or of their special courses, having the approval of the minister of education: (a) Mathematical course, astronomical course, theoretical physics course, experimental physics course of the Science College, Tokyo Imperial University; (b) mathematical course, physics course, of the Science and Engineering College, Kyoto Imperial University; (c) mathematical course, physics course of the Science College, Tōhoku Imperial University.

2. Graduates of a normal or a middle school who have studied three or more years at any of the schools recognized by the minister of education as qualifying students for teachers' positions.

3. Graduates of a normal or a middle school who have studied abroad at a university or at a school of similar grade and have degrees or diplomas from such schools.

4. Graduates of schools abroad similar in grade to a normal or middle school in Japan, who have studied at a university or at a school of similar grade abroad and have degrees or diplomas from such schools.

5. Persons who have teachers' licenses for schools equal in grade to, or higher than, the one in which they wish to teach.

Teachers with a license have the title *Kyoyu*; otherwise, however learned or proficient they may be, they are known as *Kyoyu Kokoro* (temporary or substitute *Kyoyu*), assistant *Kyoyu*, or assistant

Kyoyu Kokoroe. These licenses are granted either to those who pass the examination or to those who are considered able to pass it. The supply of teachers in mathematics is entirely inadequate to meet the demand. It has also been felt by some that the lack of general education on the part of the teachers is a serious defect in the training, however admirable may be the extensive special knowledge which the teacher has in the subject in which he instructs.

I find no information in the reports with reference to the preparation of teachers for the higher middle schools of Japan. Prof. Hayashi, of the Tōhoku Imperial University, has, however, very courteously supplied me with the following facts: In the eight higher middle schools during 1915-16 there were 256 professors, and 27 assistant professors, and about 70 lecturers. Among these instructors were 27 mathematical teachers, almost all of whom were *gakushi*. But some of them were not *gakushi*, and only one was a *hakushi* (doctor) as well as a *gakushi*. There are no special regulations with regard to the training of teachers of mathematics for higher middle schools.

Although competition is keen for the various teaching positions, there is little monetary inducement to take up teaching as a profession. As Baron Kikuchi writes:

The salaries of teachers, from the university professors to primary-school teachers, are very inadequate; this, no doubt, is one of the reasons why the supply of teachers is not sufficient to meet the demand. More than half of the 140,000 primary-school teachers have salaries ranging between 15 and 24 yen (\$7.50 to \$12) a month, which even allowing for the low rate of living in Japan is very inadequate; the highest salary for a university professor is about 4,000 yen (\$2,000). All teachers in Government or public schools and colleges are entitled to a pension equal to one-fourth the amount of their salary at the time of retirement if they retire after 15 years of service, and to an additional one two-hundred-and-fortieth of the amount of their salary for every year exceeding 15.¹

BIBLIOGRAPHY.

Report on the Teaching of Mathematics in Japan prepared by the Japanese Subcommission of the International Commission on the Teaching of Mathematics. Tokio, 1912. 549 pp.

This beautifully bound volume contains 15 monographs by various authors. Numerous quotations have been made from this volume.

R. FUJISAWA. *Summary Report on the Teaching of Mathematics in Japan.* Tokio, 1912. 16+239 pp.

Another bound volume giving a summary of the above-mentioned monographs and additional information.

D. KIKUCHI. "Education in Japan," *Monroe's Cyclopædia of Education*, New York, Macmillan, vol. 3, 1912.

"System of Public Instruction in Japan." *Report of the Commissioner of Education for 1914-15*, Washington, 1915, vol. 1, pp. 749-771.

¹Prof. Hayashi informed me that the pension increment, one two-hundred-and-fortieth of the salary, is not allowed for the years of service in excess of 40.

XII. THE NETHERLANDS.

The Kingdom of the Netherlands has an area of about 12,600 square miles, and its population at the end of 1914 was about 6,340,000.

Every grade of education in the Netherlands is under the control and supervision of the State, the work of administration being handled by a special department under the ministry for the interior. So-called secondary schools (burgher schools, higher burgher schools, agricultural schools, and industrial, trade, and technical schools) were established by a law of 1863. This law requires that every commune with a population of 10,000 provide a burgher school. All of these are now organized as evening schools, and they, as well as the higher burgher schools (*Hoogere Burger Scholen*), are mainly intended for those engaged in industrial or agricultural pursuits.

The higher burgher schools are divided into two classes, those with a three-year course and those (about two-thirds of the whole number) with a five-year course. Pupils commence this five-year course at the age of 12 or 13 years, after having had six years of training in the primary schools.¹ The instruction in mathematics (which includes arithmetic, algebra, plane geometry, solid geometry, elements of plane trigonometry, descriptive geometry, geometric drawing, mechanics, and cosmography) occupies about one-quarter of the 30 to 32 recitation periods (50 minutes each) per week. Algebra is taught in Classes I-IV; geometry in Classes II-V. In algebra such topics as theory of indices, logarithms, progressions, compound interest, and equations of the first and higher degree in one or several unknowns are taken up.

The diploma of a higher burgher school entitles the pupil to go to the polytechnic school at Delft or to one of the universities to study medicine or natural science.² The examination for the diploma is decidedly severe. Within a period of nine days the pupils have to write 42 hours in the examination room: Algebra (3 hours), Dutch (3 hours), geometry (3 hours), French (3 hours), nature study (4 hours), bookkeeping (2 hours), trigonometry (3 hours), free-hand drawing (6 hours), mechanics (3 hours). This is followed by an oral examination, which includes certain subjects not mentioned in

¹ All pupils entering the higher burgher schools and the gymnasia, to which I shall presently refer, must pass an entrance examination. Usually about 25 per cent of those who try the examination fail.

² For other departments of the universities, examinations in Greek and Latin must be passed.

the above list. The examination is held at some convenient center, before a committee appointed by the provincial authorities. In some cases two or three Provinces are combined for this purpose. The examination lasts for three days, not more than three hours a day.

Higher education, as defined by the law of 1876, is that given in the gymnasia (public and private), at the atheneum—later the communal university—at Amsterdam, and at the State universities. The pupils entering the gymnásium and higher burgher school are of the same age, and the courses of these schools are to a great extent parallel, although the duration of the course in the gymnásium is six instead of five years.¹ It is, accordingly, somewhat remarkable to find gymnasia classed with institutions of higher education. This classification of gymnasia is found only in the Netherlands. Every town with a population of 20,000 (and there are at least 30 such in the Netherlands) must provide a gymnásium unless specially relieved from the obligation. The affairs of each gymnásium are administered by a college of curators nominated by the municipal council. This council also appoints the teachers, upon the recommendation of the curators and the advice of the inspector, dismisses them when necessary, and fixes the rate of their salaries, subject to the approval of the minister of the interior.

The gymnasia prepare students to enter directly the various departments of the universities. During the first four years of the course this preparation is the same, but in both the fifth and the sixth year there are, along with hours common to all, certain hours for (a) the "humanist" alone, and (b) the "realist" alone. The common mathematical program in the fifth and sixth years for (a) and (b) includes discussion of quadratic equations, theory of indices, and solid geometry. But students of section (b) are also taught mathematics in three extra hours per week during the two years. They have such subjects as the progressions, logarithms, indeterminate equations of the first degree, permutations and combinations, determinants, probabilities, plane and spherical trigonometry, and elements of the theory of plane analytic geometry. They also review their arithmetic, algebra, and geometry, and are taught applications. The (b) courses in geometry include at times such topics as the theorems of Menelaus and Ceva, radical axes, centers of similitude, Euler's line, nine-point circle, prismatoid, Euler's theorem, harmonic ranges and pencils, poles and polars, and "Guldin's theorem." The mathematics of section (b) occupies the student for about one-sixth of his time.

¹ Graduates of a higher burgher school with a five-year course are usually better prepared to take up the study of physics and mathematics in a university than are the graduates from a gymnásium.

The final examinations are conducted by the staff of each school, under the supervision of Government delegates, usually three in number. As a rule, the inspector is one of them. The examination is similar to that for graduates from the higher burgher schools, and covers practically all the studies of the course. These include Greek, Latin, Dutch literature, French, German, history, geography, mathematics, physics, chemistry, and natural history. The Dutch would, therefore, seem to have taken as a motto Lessing's saying: "Aus einem tüchtigen Philologen lässt sich alles machen."

In the Netherlands there are five universities. Besides the State universities at Leyden, Utrecht, and Groningen, there are the Communal and the Free Universities of Amsterdam. The Free University has no faculty of mathematical science and physics.

The mathematical instruction in the universities of the Netherlands varies to a certain extent, but in all it includes mathematical physics and theoretical mechanics. During the first two years (courses are reckoned by years, not semesters) the programs of study contain *higher algebra* (determinants, irrational and complex numbers, theory and methods of solution of equations of higher degree, linear substitutions, theory of invariants); *differential calculus* and an introduction to the *integral calculus*; *analytic geometry* of the plane and of space (lines, curves of the second degree, planes, surfaces of the second degree); *descriptive geometry* (methods of projection, applications to the theory of curves and surfaces).

The subjects of study in later years are very varied, but some of the principal ones are: Integral calculus, differential equations, theory of functions, general theory of curves and algebraic surfaces, differential geometry, calculus of probabilities, calculus of variations, theoretical mechanics, and mathematical physics.

The title of doctor is obtained after two academic examinations and the submission of a dissertation. The student generally presents himself for the first examination, called "the examination for candidature," after three years of study. For the candidate in mathematics and physics this examination is on higher algebra, analytic geometry, descriptive geometry, differential calculus; astronomy; and physics. In the second examination, the examination for the doctorate, the candidate is examined on integral calculus, theory of functions, calculus of probabilities in its applications to physics, theoretical mechanics, and physics. It is especially notable that even for the mathematicians, higher geometry is not a subject for examination.

For students in mathematics and astronomy the course is much the same as in mathematics and physics.

Candidates generally present themselves for the examination for the doctorate three years after the examination for candidature, but

the total period of the study for this degree is often more than six years, since the time ordinarily spent on the dissertation is one or two years. Occasionally a student accomplishes the work for this degree in less than six years. It occurs more frequently, however, that a student passes the examination for the doctorate and then presents his dissertation one or several years after having left the university.

According to a law of 1905 the student who has successfully passed the examination for the doctorate has the right to teach in a gymnasium or other secondary school, in one of the subjects in which he was examined. It thus happens that those who have arrived at professorships in this way have received an admirable scientific preparation from a theoretical point of view, but have had absolutely no practical preparation for the chairs which they have been appointed to fill. In most cases the future teacher is transferred directly from the university to the school, without ever having taught and without any knowledge of theoretical and practical pedagogy. His only experience is what may be gained by conducting from time to time, in the presence of a professor, a lecture on some mathematical subject, either on a paper which has appeared in a periodical or on a subject chosen by the student or proposed by the professor. Through these lectures and the criticisms to which they lead the student becomes somewhat accustomed to the method of formulating personal scientific research; but for development as professor these lectures have slight value, since the subject of a conference is almost invariably taken from some part of higher mathematics, and but rarely is a scientific treatment of elementary questions given.

Those who have passed the examination for the doctorate are not, however, the only ones who have a right to teach in secondary schools. The law of 1863 established a series of State examinations leading to a Certificate of Capacity for secondary teaching. For mathematics and mechanics there are two examinations, of which the first, examination *A*, accords the right to teach the branches of this examination in a secondary school with a course of three years, while the second, examination *B*, accords this right for a secondary school with a course of five years. Examination *A* includes arithmetic, algebra, plane geometry, solid geometry, plane and spherical trigonometry, principles of descriptive and analytic geometry, principles of theoretic and applied mechanics, machinery and technology, physics, chemistry, cosmography, geology, mineralogy, botany, and zoology. In the 50 years that the law has been in force a very small number of persons have passed this exceedingly elaborate examination. But the law also permits special certificates of capacity for mathematics alone, for theoretical mechanics, machinery, technology, etc. Thus examination *A* is decomposed into four less onerous examinations, and

the candidate who has passed any one of them may teach the branch named in a secondary school with a three-year course. Examination *B* includes descriptive geometry, analytic geometry, differential and integral calculus, theoretic and applied mechanics. Those who have passed the two examinations *A* and *B* completely may teach mathematics and mechanics in a secondary school with a course of five years.

Those who present themselves at these examinations have rarely if ever attended any course in mathematics or theoretic mechanics. The majority of the candidates have been prepared by private tutors or by personal effort. The percentage of failures is consequently very high.

The salaries of directors of higher burgher schools range from 2,250 florins (with home) to 4,000 florins (\$904.80 to \$1,608) per annum; those of professors from 1,000 florins to 3,050 florins (\$402 to \$1,225).

BIBLIOGRAPHY.

Rapport sur l'enseignement mathématique dans les Pays-Bas publié par la Sous-Commission nationale (Commission internationale de l'enseignement mathématique). Delft, J. Waltman, 1911. 151 pp.

This volume contains several papers by 10 different authors.

J. CARDINAAL, "L'enseignement mathématique en Holland," *L'Enseignement Mathématique*, tome 2, 1900, pp. 317-339.

Pages 317-327 of this article are mainly historical.

J. C. MEDD, *A Short Account of Education in the Netherlands*. (Special Reports on Educational Subjects, vol. 8, suppl.) London, Board of Education, 1902, 72 pp.

XIII. ROUMANIA.

Roumania contains about 7,500,000 inhabitants.

The general control of its education is vested in the minister of public instruction, but other ministers have charge of schools pertaining to their special provinces. The ministry of public instruction is organized in three departments: (1) Primary instruction, including primary normal schools; (2) commercial, technical, and private schools; (3) secondary and higher education.

The real organization of secondary education dates from the law of 1898, which was modified in 1909 and again in 1910 in the direction of increasing the modern character of the curriculum. In accordance with these measures secondary education is arranged in two cycles of four years each: The lower course (first to fourth, I-IV) and the higher course (fifth to eighth, V-VIII). Those secondary schools which have only the lower course are the gymnasia. Those with both cycles are the lycées. The higher course is organized in three parallel sections after the model of the French lycée; namely, science-modern languages, Latin-Greek, and Latin-modern languages. In each section the material forms a unit of study. At the end of the eighth class the pupils who successfully pass the final examinations obtain the "certificate of secondary study." In general, the pupils leave the lycée at the age of about 19 years. They are then prepared to enter various higher schools, including the universities, of which there are two, one at Bucharest, the other at Jassy.

In the first cycle the subjects taken up in mathematics are: Practical and rational arithmetic, elementary geometry (plane and solid), elementary algebra, and notions of bookkeeping and surveying. With regard to the second cycle, reference is made only to the science-modern language section from which the future mathematicians come. The special object of the mathematical work in the second cycle is to develop the reasoning powers. Because of this some of the more subtle questions of arithmetic are introduced. The study of algebra is developed at length in courses extending through all four years. This study includes convergence and divergence of series, the number e , derivatives of exponential, logarithmic, and circular functions, and Horner's method of solution of equations with numerical coefficients. In plane and solid geometry such questions as transversals, harmonic division, poles and polars with respect to a circle, areas of polygons and of a circle, and area and volume of a sphere are taken up with many problems and applications. Then there are also plane trigonometry in VI, descriptive geometry and mechanics in VII, analytic geometry and cosmography in VIII. Mathematics occupies 6 hours out of 28 a week (21.4 per cent) in V and VI, 5 out of 28 (17.9 per cent) in VII and VIII.

For the pupils in the science-modern languages section who specialize in mathematics there is a special publication *Gazeta matematica*, which has appeared since 1895. It is somewhat similar to the *Revue de mathématiques spéciales* prominent in the French lycées. Quite apart from the obligations connected with their courses of study, the mathematical pupils of the section not only solve the problems in pure and applied mathematics in the *Gazeta*, but also propose others and contribute notes and articles. The progress of mathematical science in Roumania stands in intimate relation to this review:

Such then is the nature of the work which the mathematical professor has to conduct. There remains the consideration of his preparation for the work.

The directors and professors in the secondary schools are university men and receive their appointments from the Government. Since the law of 1898 the mathematical professors are appointed from among those who have passed the "examination of capacity," which is held every three years. To register for this examination it is necessary to have (1) the diploma of *licence ès sciences mathématiques* of a university; (2) the certificate of a pedagogic seminar.

"The students who have passed the examination of *licence ès sciences mathématiques* have sufficient theoretical preparation to become good professors in secondary education," as it is modestly expressed by Prof. Tzitzéica. The examinations for the *licence* are in higher algebra, analytic geometry, descriptive geometry, differential and integral calculus, theory of functions, mechanics, and astronomy. These examinations require at least three years of preparation.

During his university studies and afterwards the future mathematical teacher takes a course in pedagogy at the university. He also commences at the same time his practical training at a pedagogic seminar. The pedagogic seminar in connection with the universities were created by the law of 1898.

The examination of capacity consists of (1) three written examinations on the elementary material of secondary education and on the more advanced material of the *licence*; (2) two oral examinations, one on mathematical questions proposed by the jury, the other on pedagogy; and (3) two practical examinations—that is to say, two lessons such as the candidate might deliver to pupils in a lycée.

In order to claim a teaching position as a right, one must have passed an examination of capacity for a secondary specialty. For example, physics or geography may be offered with mathematics.

BIBLIOGRAPHY.

G. TZITZÉICA, *L'Enseignement Mathématique en Roumanie. Enseignement secondaire*. Bucarest, Imprimerie, "Gutenberg" Joseph Gobl & Co., 1912. 16 pp.

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XIV. RUSSIA.

It has been estimated that the Empire of Russia contains more than 182,000,000 people. Since the treaty of Portsmouth its area, exclusive of inland waters, is about 8,400,000 square miles.

Finland (*q. v.*) has a separate system of public schools much more highly developed than that of the remainder of Russia, where the general education of the people has been of a very low order. Except in certain parishes of the Baltic provinces, education is not compulsory. The numerous central authorities in connection with Russian education include the ministry of public instruction and the Holy Synod. The former controls the universities and the great majority of the secondary schools of all classes.

The secondary or middle schools include: ¹ (1) Seminaries for teachers, of which the ministry established 63 ² in 1891 for the preparation of teachers for certain primary schools; (2) institutes for teachers (33 in number in 1914) which prepare teachers for another class of primary schools; and (3) establishments such as the progymnasia (of which there were 29 in 1914), gymnasia (441), "real schools" (284), and technical schools (68 in 1910),³ under the control of the ministry of public instruction and providing general instruction for the youth.

The progymnasia are incomplete gymnasia with four or six classes instead of eight classes (I-VIII), one for each year of the course. Among secondary schools, only the gymnasia and real schools are considered here. Bobynin has given the mathematical programs of the seminaries and institutes for teachers.

GYMNASIA.

Pupils normally enter the gymnasia at the age of 10 years. In the explanatory remarks prefatory to the program of the course of mathematics in the gymnasia for boys, the ministry of public instruction sets forth the object of the course:

Mathematics as an exact and abstract science furnishing a simple, and consequently appropriate, means of assuring suitable development of the mind constitutes one of the foundations of general instruction. The essential object of study at the gymnasium being the intellectual development of the pupil, the mathematical instruction should be characterized, before all else, by the thoroughness and systematic rigor of a theo-

¹ Bobynin, 1903 (see bibliography); and A. T. Smith in *Cyclopedia of Education*, edited by Monroe, in article on Russia.

² Since increased, according to a report of Jan. 1, 1914, to 122 (*Statesman's Year-Book*, 1917).

³ In 1912 there were 30 technical schools of secondary school grade.

retic course; practical applications should be introduced, first, as illustrations of the theory, and then, to habituate in computations.

The gymnasia of the ministry of public instruction are of two types: (1) Those in which only one ancient language, Latin, is taught; and (2) those in which two ancient languages are taught. There were only five of these latter gymnasia in 1910.

While the whole number of hours devoted to mathematics in these two types of gymnasia is not the same, the programs are identical. The subjects taught are arithmetic, algebra, geometry, trigonometry, physics, mathematical geography (cosmography), and notions of mechanics.

Arithmetic is taught in Classes I-III and reviewed in Class VIII, where certain parts, omitted earlier on account of their difficulty, are taken up. In the third year topics of discussion are: Ratio and proportion, problems relating to the so-called rule of three; interest and proportional parts.

Algebra (III-VIII).—Some of the matters taken up in Classes VI-VIII are: Progressions and logarithms; calculation of compound interest; indeterminate equations of the first degree in two unknowns; continued fractions; binomial theorem; resolution of a system of equations by the method of Bézout.

Geometry (IV-VI), with review in VIII.—In V-VI the student is instructed in the measurement of lines and angles; proportionality of segments; similitude of triangles and polygons; regular polygons; notion of a limit; length of the circumference; notions on the calculation of π ; area of rectilinear figures, of the circle and of its parts; simple problems of instruction, and numerical applications for each article of the program; regular polyhedra; evaluations of area and volumes of prisms and pyramids; cylinder, cone, and sphere, evaluation of their areas and volumes.

Trigonometry (VII-VIII).—Elements of plane trigonometry, including the use of tables, solutions of triangles, calculation of areas and applications to problems in surveying. Trigonometric equations and inverse trigonometric functions are not discussed.

Cosmography (VIII).—Rotary motion of the celestial sphere; rotary motion of the earth; true shape and size of the earth; apparent annual movement of the sun; annual movement of the earth round the sun; measurement of time; constitution and dimensions of the sun; moon; eclipses; planets; comets; law of gravitation; tides.

Mechanics (VI, VIII).—Motion and force—laws of motion, law of inertia, law of relative motion, equality of action and reaction; force as a cause of motion and of pressure; resistance of motion (friction); equilibrium of forces; composition and decomposition of forces; levers. Theory of gravitation. Theory of motion—uniform motion; velocity; acceleration; uniformly accelerated and retarded motion; motion of a projectile; notions regarding curvilinear motion and centrifugal force; pendulum. Theory of energy—work; lever; pulley; inclined plane; toothed wheels; kinetic and potential energy; transformation of mechanical work into heat and inversely; principle of conservation of energy.

In most gymnasia the pupils have the same teacher of mathematics in all of their courses. This arrangement seems to cause the pupil to think of the bonds uniting one course to another. As to the connection between mathematics and physics and mechanics, the courses in physics and mechanics (as well as in cosmography) are conceived.

in such a way as constantly to give the pupil occasion to apply his knowledge and his experience of mathematics.

About one-third of the pupil's time in the ordinary gymnasium is devoted to the study of Latin and Greek, and about one-fifth to mathematics and physics.

At various stages in his course the pupil is required to pass written and oral examinations. In the final examination the written portion in mathematics lasts five hours and requires the solution, with detailed explanations, of two problems, one in algebra and the other in trigonometry applied to geometry. The mathematical portion of the oral examination is in arithmetic, algebra, solid geometry, and trigonometry. The student who has successfully passed this examination receives a certificate of maturity (*attestat zrolosti*). The bond between secondary and higher education is formed by the requirement of this certificate for admission to the universities and that of either the gymnasium or the real school for admission to the higher technical colleges.

"REAL SCHOOLS."

The normal age of entrance into these schools is 10 years. Most schools now possess, in addition to the former regular six classes (I-VI), a seventh class (VII). Each class lasts for a full year.

The subjects common to all real schools are: Russian, with Slavonic; modern languages—German, French; geography; history; mathematics and geometric drawing; natural history; physics; drawing; calligraphy. In a comparison of Real schools with the gymnasia, it is to be noted that logic and the classical languages have disappeared to make room for more of science and of modern languages.

The students of the real school cover in six years about the same amount of mathematics as those in the gymnasia cover in eight. Special reference may be made, however, to the five-hour course of mathematics given at many real schools in the supplementary seventh year. The subjects taught are arithmetic, algebra, trigonometry, elements of analytic geometry, and infinitesimal calculus. The leading topics taken up are:

Arithmetic: Principal propositions on factoring of numbers; the highest common divisor of two numbers; solution in positive integers of indeterminate equations of the first degree in two unknowns.

Algebra: Complex numbers; fundamental properties of an integral function and its roots; special cases, the functions $x^n - a^n$ and $ax^{2p} + bx^p + c$; discussion of equations of the first degree with one unknown and of a system of two equations of the first degree with two unknowns—indeterminate case, contradictory equations.

Geometry: Relative positions of lines and planes in space; principal properties of dihedral and polyhedral angles; regular polyhedra; measurement of the surface and volume of the right cylinder, the right cone, and the sphere and its parts; examples leading to computation and construction problems.

Trigonometry: Inverse circular functions; trigonometric equations; etc.

Analytic geometry: Rectangular and polar coordinates; transformation of coordinates; circle, parabola, ellipse, hyperbola; equation of ellipse and hyperbola in bipolar coordinates; loci; diameters.

Calculus: Fundamental theorems, in limits; application to the measurement of circumference and area of circles, of the surface and volume of cylinders, cones, and spheres; limit $\frac{\sin z}{z}$; limit $\left(1 + \frac{1}{n}\right)^n$; differentials of algebraic and transcendental functions; geometric explanation of Rolle's theorem; Lagrange's theorem; increasing and decreasing functions; equations of tangents and normals of the conic sections; definite integrals.

Since 1911, students who have satisfactorily completed the seven years' course of the real school are admitted to the universities without examination. Teachers in the real schools are, for the most part, graduates of a university in the subject which they teach.

To form a true conception of secondary education in Russia, one should bear in mind that, taken as a whole, the boys attending the secondary day schools are drawn from a lower social stratum than are those attending such schools in England or France. As a rule neither the children of the aristocracy nor those of the higher officials attend the gymnasia; the former are educated for the most part by private tutors, the latter in special schools open only to the nobility.¹

THE UNIVERSITIES.

In Russia, exclusive of Finland, there are 10 universities. The largest and oldest is at Moscow; The others, in order of foundation, are at Yuriev (formerly Dorpat), Kazan, Kharkof, Petrograd, Kief, Odessa, Warsaw, Tomsk, and Saratov.² The last was founded in 1909, and neither there nor at the University of Tomsk has a faculty of physics and mathematics been established.

While secondary education in Russia still leaves much to be desired, the standard of teaching in the universities is, on the whole, very high, and may be compared to that of the German universities.³

The scholastic year consists of about 27 weeks of lectures, and the course of study at the university covers four years.

The Russian faculty of physics and mathematics is composed of two sections: The section of mathematical sciences and the section of

¹ This paragraph is equivalent to what Darlington states in this connection on pp. 347-348 of his report (which sought to represent actual conditions at the end of 1904). The Bureau of Education at Washington has, however, kindly volunteered the information that "about 50 per cent of Russian gymnasium students are sons of nobility and merchants. 'Aristocracy' is represented by at least 20 per cent of the students. The special schools for aristocracy have long ago become insufficient and now are discarded." The bureau states that the authority it quotes is: Russia, Ministry of Public Instruction, Report for 1912, [in Russian], Petrograd, 1915, pp. 50 and 51.

² A Popular University bearing the name of Gen. Alphonse Shanilavsky, who has given the funds necessary for its creation, has existed at Moscow since autumn, 1908. In 1916 a Woman's University was created at Petrograd with the power of conferring the degree of doctor.—*Statesman's Yearbook, 1917.*

³ *Encyclopaedia Britannica*, 11th edition, 1911, Article "Russia." W. S. Jesien has recently reported great improvement in Russian secondary education during the past decade.

natural sciences. With the exception of general courses in physics and chemistry given to students in both sections, all subjects in the two courses are different.

Since 1906, several universities, including those at Moscow and Petrograd, have divided the studies in the mathematical section into three groups: (1) Mathematics and analytic mechanics; (2) astronomy; and (3) physics, subdivided into (a) physics and (b) physical geography and meteorology. The student may elect to take up the studies of any one group.

The courses for the mathematics and analytic mechanics group at the University of Petrograd in 1909-10 were as follows:

Semester 1.—Spherical trigonometry; introduction to analysis; analytic geometry; exercises on analytic geometry; higher geometry or descriptive geometry; physics of molecular forces; descriptive astronomy. Recommended: Course in general chemistry.

Semester 2.—Analytic geometry and exercises; differential calculus and exercises; higher geometry or descriptive geometry; descriptive astronomy; physics; heat. Recommended: Course in general chemistry.

Semester 3.—Higher algebra; application of differential calculus to geometry; integration; spherical astronomy; optics; acoustics and electricity.

Semester 4.—Higher algebra; integration; geometric applications of integral calculus; exercises on the integral calculus; statics, optics, acoustics, electricity; spherical astronomy.

Semester 5.—Definite integrals; integration of ordinary differential equations; finite differences; kinematics.

Semester 6.—Definite integrals; integration of ordinary differential equations and the calculus of variations; theory of numbers; dynamics of a particle and exercises.

Semester 7.—Partial differential equations; finite differences; calculus of probabilities; elliptic functions; mechanics of systems and exercises. Recommended: Theoretical astronomy.

Semester 8.—Partial differential equations; calculus of probabilities; theory of attraction, hydrostatics and hydrodynamics. Recommended: Celestial mechanics.

In addition to the general and recommended courses there are certain special courses not given every year, for example:

1. Theory of Abelian integrals (1906-7).
2. Theory of surfaces (1907-1909).
3. Analytic theory of differential equations (1907-1910).
4. Vector analysis (1908-1910).
5. Historical and critical review of the principles of rational mechanics (1908-1910).

These courses are given, for the most part, by the Privatdozent. The final examination for the students in the faculty of science is conducted by a commission appointed by the State. The examinations are both written and oral. The programs of the State examinations divide university courses into two classes: (1) Fundamental or principal courses, (2) complementary courses. The programs of the principal courses are the same for all universities and contain only the most essential and elementary parts of the four sciences.

(mathematics, mechanics, physics, astronomy), which are taught in the mathematical section of the faculty. In these subjects there are two written and five oral examinations. The programs of the complementary courses are not fixed. The choice of two complementary courses is made by the student himself. In mathematics he may choose theory of numbers, theory of elliptic functions, higher geometry, or some other course of three semester hours per week. The examination is oral.

A student who passes all the examinations receives a diploma of the first or second grade, which gives certain civil rights, such as the right to teach in the secondary schools without any special examination. The diploma conferred by the State examining commission is not technically a degree, though it is fully equivalent in value to the B. A. degree conferred by British or American universities. With this diploma the great majority of students rest satisfied. Only a select few remain at the university to prepare for a degree, and most of these do so with the view of ultimately obtaining a professorial chair. There are two degrees, magister and doctor; but discussion of these is beyond the scope of this sketch.

TEACHERS IN SECONDARY SCHOOLS.

Every teacher of mathematics in a gymnasium under the control of the ministry of public instruction must be a graduate of the mathematical section of the faculty of sciences in a Russian university. Until very recently such a graduate had no training in pedagogy and no practice in teaching when he first presented himself at a gymnasium as a professor. In 1909, however, a provisional set of courses for the preparation of teachers of mathematics and other subjects was organized in the educational district of Petrograd. This plan has been followed by other educational districts.

At Petrograd the courses are under the direction of a council composed of the directors of establishments of secondary education and presided over by the inspector of the district. Information issued to pupil-teachers by the council includes the following:

1. The courses of one year in duration are especially designed to give practical preparation to teachers of mathematics in secondary schools.
2. The courses are open to those students who have a diploma from the mathematical section of a faculty of sciences and are bearers of a certificate of good conduct.
3. The more necessitous of the candidates for the course receive a stipend of 300 rubles (\$156). In addition to this assistance in money, student teachers may, under certain circumstances, earn money as tutors, or as substitute teachers during the absence of the incumbent of a chair.

4. Pupil teachers of the same specialty are sent as probationers, in groups of two or three, into gymnasia and real schools.

5. The immediate direction of the work and courses of the pupil teachers is confided to chiefs of probationers chosen from the teaching personnel of the establishment by the administration of the district.

6. The obligations of the pupil teachers are the following:

(i) To study carefully, in works and books indicated by the chiefs of the probationers, the parts of their specialty which figure in secondary education, as well as the methodology of the subject.

(ii) To attend lectures by the chiefs of the probationers or by other professors in logic, psychology, history of pedagogy, pedagogy, and to make report of these lectures to the chief.

(iii) To deliver lessons, as often as possible, before the chiefs of the probationers, other professors, and other probationers in the specialty, in accordance with a plan arranged in advance and approved by the chief.

(iv) To attend the trial lessons of other probationers of their group.

(v) To take part in the discussion of the trial lessons of probationers of their group.

(vi) To prepare papers on questions of general pedagogy, studies of methodology in the specialty, reviews of textbooks.

(vii) To seek to understand the child's mind, by contact with and observation of pupils in general, or of a particular group, while engaged in recreations, games, excursions, and walks.

(viii) To attend the pedagogic council.

(ix) To attend the conferences and pedagogic discussions (three times a week) and make report.

7. In case these requirements have been met, the probationers are "éliminés du contingent" and are qualified to teach.

At the end of the school year, in a general conference of the committee and the various chiefs of probationers, the results of the work of the year and the characteristics of the probationers are discussed.

The preparation of teachers in the real schools is practically the same as of those in the gymnasia.

Apart from the question of supply, the two main difficulties which remain to be solved in connection with the teaching staff of the Russian secondary schools are (1) the question of remuneration and (2) the question of professional training. An attempt to dispose of this latter difficulty has been described above. With regard to the first point there seems to be general agreement, Mr. Darlington writes, that the present rate of remuneration, which was fixed 30 years ago, is not sufficiently high either to attract the best talent into the teaching profession or to enable teachers to give their undivided attention to their school duties. An assistant teacher in a State secondary school, teaching 30 hours a week, which is accepted as the

normal limit, receives 1,830 rubles (about \$915) a year for the first five years of his service, after which his salary is increased by 150 rubles, thus reaching 1,980 rubles (about \$993) a year in all. Extra payments (e. g., for correcting exercises and acting as "class tutor") may raise the remuneration to 2,230 rubles a year, but in any ordinary case, so long as he continues to be an assistant teacher, his salary remains as above stated from the sixth to the twenty-sixth year of his service, when he receives his pension. An income of less than 2,000 rubles a year may be sufficient to enable an assistant teacher to live in a manner suitable to his position, so long as he remains a bachelor; it is wholly inadequate to the needs of a married man with a family. With the most rigid economy a moderate-sized middle-class household can not be maintained, at any rate in the larger towns of Russia, on less than 3,000 rubles a year. A married teacher is therefore under the absolute necessity of earning on an average an extra thousand rubles a year in addition to his official salary in order somehow to keep soul and body together and provide proper education for his children.

The salary of an ordinary professor in a Russian university is 3,000 rubles a year; of an extraordinary professor 2,000 rubles.

The pension rights attached to the teaching profession in the higher and secondary branches are very considerable in Russia. Twenty years of work gives to such a teacher a right to a pension equal to half his salary; 25 years' service entitles him to a pension equal to his full salary. Moreover, if the teacher is prevented by shattered health or some incurable disease from continuing in the exercise of his profession, he is still more generously treated.

BIBLIOGRAPHY.

- V. V. BOBYNIN. 1. "L'Enseignement mathématique en Russie, aperçu historique." *L'Enseignement mathématique*. Paris, I, 1899, pp. 77-100.
 2. "L'Enseignement mathématique en Russie. État actuel.—Enseignement primaire." *L'Enseignement mathématique*, I, 1899, 420-446.
 3. "L'Enseignement mathématique en Russie. État actuel.—Enseignement secondaire." *L'Enseignement mathématique*, V, 1903, pp. 237-261.
- Commission internationale de l'enseignement mathématique, Sous-Commission Russe:
1. B. S. *L'Enseignement mathématique dans les écoles primaires et les écoles normales*.
 - M. KONDRATIEV, *L'Enseignement mathématique dans les gymnases de garçons du Ministère de l'Instruction Publique*. St. Pétersbourg, 1911, 29 pp.
 2. K. W. VOGT, *Bericht über den mathematischen Unterricht an den Russischen Realschulen*. St. Petersburg, 1911, 16 pp.
 3. O. FOSSÉ, *Rapport sur l'enseignement mathématique dans les universités... en Russie*. St. Petersburg, 1910, 100 pp.
- T. DARLINGTON, *Education in Russia*. London, Board of Education, Special Reports, vol. 23, 1909, 569 pp.
- W. S. JESIEN, "Education in Russia," Rept. of U. S. Commis. Educ., 1916, vol. 1, Washington, 1916, pp. 597-615.

XV. SPAIN.

At the close of 1913 continental Spain had an area of about 194,000 square miles and a population of over 19,600,000.

The whole system of public instruction is controlled by the minister of education, an advisory council, and a corps of inspectors, one for each of the 49 Provinces or administrative divisions of the country. The secondary schools and universities are intimately associated, by reason of the fact that Spain is divided into 11 university districts, the secondary and higher education in each of which is under the control of a rector. He is assisted by a university council which acts in an advisory capacity.

The State secondary schools, of which there must be one, at least, in every Province are styled institutos de segunda enseñanza. The colegios de segunda enseñanza are boarding schools in charge of local authorities and are feeders for the institutos. Pupils who enter the institutos are about 10 years of age. The general course is divided into two parts, one part covering two years and the other four. In the first part the only subjects of a mathematical nature are arithmetic and drawing. Algebra, geometry, and trigonometry are studied in addition to arithmetic in the second part. Physics and chemistry are also taught in this part. The student who has successfully completed the six years of study and passed the corresponding State examination is called a bachiller en artes, and is entitled to proceed to a university.

The University of Madrid has the largest attendance; the University at Salamanca is the most ancient. Other universities are those established at Granada, Sevilla, Barcelona, Valencia, Santiago, Zaragoza, Valladolid, Oviedo, and Murcia.¹

The bachiller who wishes to prepare himself to teach mathematics in the secondary schools must study in the faculty of sciences of a university and secure the title of *licenciado* (licentiate). At Madrid candidates may then proceed to the doctorate; Barcelona and Zaragoza are the only other universities which have a faculty of science preparing completely for the licenciatura.

The mathematical studies leading to the licenciatura are as follows:

1. *Mathematical analysis*.—First and second courses, supplementary to the work in the institutos. The first course comprises theories of arithmetic not explained in

¹ This university was established in 1915 or 1916. Cf. *The Statesman's Yearbook*, 1915-1917.

secondary education, the whole field of elementary algebra, plane and spherical trigonometry with analysis of circular functions. The second course deals with higher algebra; and the general theory of equations, as well as an introduction to modern higher algebra; is presented. During one year at the University of Madrid, for example, this introduction dealt with the theory of binary forms (discriminants; Jacobians, Hessians, and Wronskians; linear substitutions; invariants and covariants; canonical forms).

2. *Metric geometry*.—This geometry is simply the ordinary Euclidean geometry, modified by the use of modern synthetic methods. The course in the faculty of sciences is divided into two parts: The first deals with fundamental theories, those on which the theorems of all previous questions are based; the second consists in studies of each group of figures in detail, without losing sight of the groupings which facilitate the employment of the principles of duality, projectivity, etc.

(a) *Fundamental theories*: First notions; angles; perpendiculars and parallels; relation between the elements of triangles and of trihedral angles, isosceles or scalene; distances and inclinations large and small; elementary cases of symmetry and of equality of triangles, trihedrals, and tetrahedrons; equality and orthogonal symmetry in general; geometric loci; circle, cone, and sphere; proportional lines; similar figures; products of segments in triangles; concept of length of curves in general and of the areas of conics; plane and spherical trigonometry sufficient for the solution of right triangles; orthogonal projections.

(b) *Studies*: More complete consideration of each of the following figures: Ranges of points, pencils of rays or of planes; cross ratios and projection; segments and angles; triangles and trihedral angles; quadrilaterals, tetrahedral angles, and tetrahedrons; ordinary polygons, polyhedral angles, and polyhedrons in general, and in particular those which are regular and semiregular; prisms and cylinders; pyramids, prismatoids, cones, and spherical figures; systems of circles; of cones, and of spheres; particular determination of length, of areas, and of volumes; their comparison and proportionality; homology, homothety, involution, and symmetry in general; polarity of a circle, of a cone, or of a sphere; inverse figures; stereographic projection.

3. *Analytic geometry*.—This course is based on projective methods and is especially developed at Madrid.

4. *Elements of infinitesimal calculus*.—This course includes a discussion of theory of limits; continuity, orders of infinitesimals, derivatives and differentials of functions of one or of several variables; change of variables; hyperbolic functions; Legendre's polynomials and developments into series by formulae of Taylor, Maclaurin and Lagrange; maxima and minima; elements of integral calculus; differentiation and integration under the integral sign with application to the calculation of definite integrals and of the Eulerian integrals. In some programs this first part of the calculus course concludes with a discussion of curvilinear and surface integrals and of integrals in the formulae of Green, of Stokes, and of Dirichlet, and their applications in mechanics and physics.

5. *Cosmography and physical geography*.

6. *Geometry of position*.

7. *Descriptive geometry*.

8. *Rational mechanics*.

9. *Spherical astronomy and geodesy*.

The title of licenciado in mathematical science is obtained by a candidate after three examinations are successfully passed: (a) A written examination on two questions drawn, by lot, from those proposed by the tribunal; (b) an oral interrogation by the three

J. REY PASTOR, *Fundamentos de la geometría proyectiva superior*. Tomo I. Madrid, 1916. 22+444 pp.
Cf. *Bulletin des sciences mathématiques* tome 41. 1917 p. 229.

judges, for half an hour each; (c) a practical exercise consisting in the solution of a problem in descriptive geometry or in rational mechanics drawn, by lot, from the problems of the questionnaire, and response to observations of the tribunal.

The examinations for secondary school professorships are open only to licentiates and comprise: (1) A written examination four hours in length, consisting of the development of two themes of a questionnaire; (2) an oral examination consisting of response to five questions drawn by lot; (3) a practical exercise; and, after the exclusion of those who have not met with success, (4) delivery of a lesson after eight hours of preparation to be criticized by one or two opponents; and (5) exposition of a given topic and replies to criticisms of the tribunal.

The best mathematical positions in the secondary schools are obtained by the doctors of mathematical sciences, who are also eligible for positions on the faculty of sciences of a university. To prepare for the doctorate it is necessary to follow courses at the university in higher analysis, advanced parts of geometry, astronomy of the planetary system, and mathematical physics, and to present a memoir on a subject selected by the candidate and satisfactorily sustained against objections on the part of the tribunal.

The titles of some advanced courses offered to those preparing for the doctorate may be given: Ordinary differential equations; calculus of variations; integral equations; quaternions; functions of a complex variable; elliptic functions; Galois's theories.

It usually takes four years in the university to pass the licenciatura, and one extra year, for those who are apt scientific investigators, to make the doctorate.

The professors of secondary and higher education are appointed by the King; in the case of the institutos one of the professors is appointed as director.

BIBLIOGRAPHY.

Z. G. DE GALDEANO, *Nueva Contribución á la Enseñanza de la Matemática con indicaciones de sistematización matemática*. Zaragoza, E. Casañal, 1910. 64 pp.

— *L'enseignement mathématique en Espagne*. (Commission internationale de l'enseignement mathématique. Sous-Commission espagnole.) Zaragoza, E. Casañal, 1910. 8 pp. 2^{me} rapport, Zaragoza, E. Casañal, 1911. 18 pp.

C. J. RUEDA *L'enseignement des mathématiques en Espagne*. Mémoires présentées au congrès de Cambridge par M. C. J. R. délégué en Espagne de la commission internationale de l'enseignement mathématique. Tome I^{er} Madrid, Tip. de la "Rev. de Arch., Bibl. y Museos," 1912. 143 pp.

Among the memoirs in this volume are: "M. Torroja et l'évolution de la géométrie en Espagne" by M. Vegas; "Enseignement de la géométrie métrique à la Faculté des Sciences" by C. J. Rueda; "Les cours d'analyse mathématique aux Facultés des Sciences espagnoles" by L. O. de Toledo. "L'enseignement du calcul infinitésimal aux Facultés des Sciences espagnoles" by P. Peñalver; "Enseignement des mathématiques aux Écoles Normales" by L. Ferreras.

XVI. SWEDEN.

The area of Sweden is a little less (excluding the lakes) than 170,000 square miles, and the population was estimated to be on December 31, 1916, about 5,757,000. Of these, all but about 100,000 belong to the established Lutheran Church. It is, therefore, not surprising to find church and school both placed under the administration of the *Ecklesiastikdepartementet*, or the ecclesiastical department. The trend of circumstances recently has been toward their separation. It was not so long ago that the chapters (*domkapiteln*), composed of ministers and laymen, were still the local boards of administration, not merely of ecclesiastical affairs, but also of affairs relating to the secondary schools and to the elementary school system. Since 1905, however, the central government of the State secondary schools and equivalent educational establishments receiving State aid has been in the hands of a board called the Royal Board of Secondary Schools; the result has been that the powers of the chapters as regards these educational institutions have been very considerably curtailed. In 1913, by the establishment of a central board for the elementary schools, these also were placed under the administration of expert laymen.

The royal board of secondary schools deals with such matters as curriculum, discipline, training of teachers, appointment of teachers, etc. It is also the duty of its members to inspect the schools personally and to give instruction and advice in the course of their inspection.

Since 1905 the State secondary schools for boys have been classified into two groups: *Realskolor* or modern schools (independent), and *högre allmänna läroverk*, each comprising a *realskola* and a *gymnasium*.

There are 77 secondary schools for boys (*allmänna läroverk*), 38 of these are *högre allmänna läroverk* and 39 are independent *realskolor*. Among the latter, 18 are coeducational schools. There is no independent State *gymnasium*.

Into the *realskola*, which has six one-year classes (one, the lowest, to six), the boy may enter at 9 years of age.¹ The course at the *gymnasium* is based upon the work of the five lower classes of the

¹Note that the secondary education is built on the third year of primary education, instead of the fifth, as in Denmark. The primary schools give a six-year course, while in some cases continuation courses are offered for three years longer.

realskola and is divided into two "sides"; the Latingymnasium and the realgymnasium, each consisting of four one-year classes called "rings" (I to IV). The boys in the sixth class of the realskola are thus of the same age as those in the first ring of the gymnasium.

REALSKOLOR.

Mathematical instruction, which occupies about one-sixth of the pupil's time, is here given during five hours weekly in each of the classes except in the first and fifth, where it occupies four hours. The subjects taught are arithmetic, algebra, and geometry. In the sixth class the pupils are instructed in: (1) *Algebra*—evolution and involution, proportion, equations of the first and second degree with one unknown, graphs, and problems; (2) *geometry*—geometric exercises and amplification of preceding course, which has dealt with circles and polygons and simple problems. Drawing exercises of the Realskola include, in class 4, geometric construction of parallel lines, triangles, parallelograms, and polygons; in class 5, drawing of regular figures, such as the ellipse and limacon; elements of descriptive geometry; in class 6 further exercises in descriptive geometry.

In 1904-5 about 60 out of 75 schools used Euclid's Elements as textbook in geometry. Four years later, with the development of the more practical or modernized scheme of secondary education, about 60 out of the 75 schools had adopted texts similar to the school geometries now so common in England.

General scheme of studies in a realskola.

Subjects.	Classes.					
	1	2	3	4	5	6
Religion.....	3	3	3	2	2	2
Swedish.....	5	5	6	4	3	3
German.....	6	6	6	4	4	3
English.....				5	5	4
History.....	2	3	3	3	3	4
Geography.....	2	2	2	2	2	2
Mathematics.....	4	5	5	5	4	5
Biology.....	2	2	2	1	2	2
Physics.....				2	1	2
Chemistry.....					2	1
Writing.....	2	2				1
Drawing.....	1	1	2	2	2	2
Total number of hours a week.....	27	29	30	30	30	30

The final goal of the realskola is a State examination (Realskol-examen), which gives admittance to various technical schools and to schools of forestry, agriculture, and mining, and qualifies for various appointments in the post office, railway or telegraph service, etc. The examination consists of two parts, the one written and the other oral. The questions of the first part (in Swedish, German, English,

and mathematics) are the same for the whole country and the requirements are moderate.

GYMNASIA.

In 1913 about 57 per cent of the pupils in the gymnasia were in attendance at the realgymnasium.

The number of class periods per week in the realgymnasium (including gymnastics, fencing, singing, and religious instruction) is 38 to 41, of 45 minutes' duration. There must be a pause of 10 minutes between two periods. About one-quarter of the total time, apart from instruction in gymnastics, fencing, etc., is given to mathematics and drawing. In order to avoid overpressure and to permit of a pupil's devoting himself to some special study for which he displays marked aptitude, some options (*valfrihet*) are allowed in the last two years of the gymnasium course.

The extent of requirements in the different rings of the gymnasium may be seen in the following table:

Subjects.	The realgymnasium.				The Latin gymnasium.			
	Ring I.	Ring II.	Ring III.	Ring IV.	Ring I.	Ring II.	Ring III.	Ring IV.
Religion.....	2	2	2	2	2	1	2	2
Swedish.....	3	2	3	3	3	2	3	3
Latin.....					6	6	6	6
German.....	2	2	2	2	2	2	2	2
English.....	3	3	2	4	2	2	2	2
French.....		4	4	4		4	4	4
History.....	3	3	2	3	3	3	3	3
Geography.....	2	1			2	1		
Elements of philosophy.....			1	1			1	1
Mathematics.....	7	6	6	6	5	4	4	5
Biology.....	1	2	2	1	1	2	2	1
Physics.....	3	2	4	3	2	1	2	2
Chemistry.....	2	2	2	2				
Drawing.....	2	2	2	2	2	2	2	2
Total number of hours a week..	30	31	33	33	30	31	33	33

¹ Those pupils who have elected Greek (7 hours a week) in Ring III drop mathematics, drawing, and one hour of English; in Ring IV Greek (7 hours) is then substituted for mathematics and drawing.

As to mathematics, instruction in a realgymnasium includes: (a) *Algebra*—theory of indices, logarithms, arithmetical and geometrical series, compound interest; (b) *geometry*—proportion applied to geometry, problems (especially in plane mensuration), solid geometry; (c) *plane trigonometry*—simple computations in connection with right and oblique triangles; (d) *analytic geometry*—curves of the second degree; the notion of a derivative is made clear and much emphasis is laid on graphic representation of functions.

The course in a gymnasium is concluded with a final examination (studentexamen, or afgangsexamen, or maturitetsexamen) which, in either "side," entitles those who have passed it to matriculation at the universities. Examination commissioners called "censors" are appointed by the Government to superintend each examination.

They are chosen for the most part from professors of the universities, and usually number about 18. The censors are assisted by "examiners," who are the teachers of the schools; the examiners always act under the direction of a censor and in his presence. Questions for the written part of the examination are drafted by the censors but must be approved by the royal board of secondary schools. This examination takes place simultaneously in all the schools some weeks before the oral examinations and lasts four or five days.

The written examinations in both "sides" include Swedish, modern language, and mathematics, and in the realgymnasium physics, in the Latingymnasium Latin. French may be substituted for either of the "modern languages," which are German and English. The single examination in mathematics lasts six and one-half hours. The paper contains eight or nine questions. An adequate discussion of three at least is necessary for passing.¹ The answers are first looked over by the teacher himself and one colleague, and are graded with one of six predicates, the fifth of which in descending order is "satisfactory," and the sixth "unsatisfactory."

The candidate is not admitted to the oral part of the examination unless in each of his papers he gains at least the mark "satisfactory." When the teachers have thus arrived at their decisions, the papers are stitched together into books and sent to the royal board, which distributes them among the censors for their inspection, much of which they make as they travel in the train from school to school, from one end of Sweden to the other. The final decision is not reached until censors and teachers sit together in conclave somewhat later.

The oral examination is wider in range than the written examination and embraces all the subjects taught in the later years of school life. The rektor or headmaster in each school arranges the candidates in groups of five or six in a room; and in these rooms they are kept throughout the examination, while the censor is sometimes in one room and sometimes in another, according to the subjects of the examination. Various arrangements emphasize the importance of the occasion. At least three persons of position, nominated by the inspecting local authority, represent the public. Evening dress is *de rigueur* for censors, examiners, and candidates; and the censors and the teachers wear their robes and decorations. The teacher of the candidates is generally the examiner; and, as a rule, puts all the questions under the direction and guidance of the censor. The examination, though apparently of the pupil alone, is really quite as much a test of the teacher, enabling the censor to judge of the efficiency of his work. It is also deemed useful for university professors to be brought for a few days in each year into such close touch

¹ I quote freely from Mr. Thornton in what follows.

with the pupils graduating from the public schools in the country, and for school and university thus to be brought into relationship. As in the case of the written, so also in the oral examination, each candidate is given one of six grades, of which the sixth is unsatisfactory.¹ It is so difficult for an ill-prepared candidate to be promoted to the highest class that the number of rejections at the final examination² is exceedingly small.

The aim of the realskola is to provide a common citizens' education of wider scope than that of the elementary school. The aim of the gymnasium is to add to general education imparted in the realskola a preparation for the universities or equivalent educational institutions. In 1912 the number of graduates from the realskolas was 675, and from the Latin gymnasias 510. Of these 1,185 pupils, 160 continued their careers at business or commercial colleges, 144 entered on a military career, 111 went to higher technical schools, and 535 to universities, etc.

THE UNIVERSITIES.

There are two State universities in Sweden; one at Upsala, founded in 1477 and the oldest in Scandinavia, and the other at Lund, founded in 1668. Both of these are in provincial cities. There are, however, two privately organized universities, the Högskola at Stockholm and the Högskola at Goteborg (the second largest city in Sweden). Furthermore, the State supports a medical college, the Karolinska Institutet, at Stockholm, which it established in 1810.³

According to statutes of 1908 the direction of the State universities and medical college is exercised by a chancellor appointed by the Crown, without salary,⁴ on the recommendation of certain authorities in these institutions. It is the duty of the chancellor to see that the statutes are observed, to issue instructions regarding the administration of the university finances and estates, to pronounce "finally and officially in questions of nomination, and, in general, in all university matters which are submitted to the decision of the Government. * * * The immediate management and supervision of all matters relating to the university is in the hands of its rektor, who is elected for a term of three years" (but is eligible for reelection) by certain university professors.

¹ These grades are usually named A (mark of distinction), a (exceedingly satisfactory), AB, B, b (satisfactory), and C (unsatisfactory).² The words in parentheses are doubtless Mr. Thornton's translations of the corresponding Swedish expressions given on p. 177.

³ As a rule only about one-fourth of the pupils entering the gymnasia ever pass this examination.

⁴ There are also two foreign universities founded in the seventeenth century under Swedish rule, namely, the universities at Helsingfors in Finland and at Dorpat in Russia.

⁵ The chancellor is entitled to appoint a salaried secretary.

In 1913 there were about 5,200 students at the Swedish universities and medical college: 2,461 at Upsåla, 1,421 at Lund, 395 at the Karolinska Institutet, 741 at the Stockholm Högskola, and 235 at the Göteborg Högskola.

The Högskola of Stockholm is under the superintendence of the chancellor of the universities. The governing body is a board of nine members, the president of which is nominated by the Government. The rektor of the university is an ex officio member of the board. Six other members are elected by the Swedish Academy, the Academy of Sciences, the Stockholm city council, and the faculty. The ninth member of the board is elected by the board itself.

It is under the direction of professors at the Stockholm Högskola that *Acta Mathematica* has been published.² This has been made possible by the annual grant of 3,000 kronor.

At each of the universities (except that at Göteborg) mathematics is taught in the faculty of philosophy; and in this faculty, till lately, three degrees were conferred—those of candidate, licentiate, and doctor. According to a statute of 1907, however, the examination for the first of these degrees was replaced, for those expecting to go into secondary school teaching, by a secondary school teachers' examination (*filosofisk ämbetsexamen*). All candidates for teaching positions in a gymnasium must have passed this examination. Should the candidate aspire to be a professor (*lektor*), he must also pass the second university examination (*filosofie licentiateexamen*), and defend a thesis for the degree of doctor (*filosofie doktor*). The first of these examinations requires about seven or eight semesters (that is, about four years) of study; the second, about four years more.

Filosofisk ämbetsexamen.—The faculty of philosophy is divided into two sections: The section of the humanities, and the mathematics-natural sciences section. To the latter section belong mathematics, astronomy, physics, mechanics, chemistry, geology and mineralogy, botany, and zoology. Since 1909 political economy may, under certain circumstances, be included in this section. Geography belongs to both sections. The filosofisk ämbetsexamen is based upon the following groups of subjects:

¹ In the State universities "every student is obliged to belong to one of the unions called *nationsföreningar*, or simply *nation*, or *landskap* (literally, province), into which the students are grouped, according to the part of the country from which they come, the object of these unions being to encourage hard study and pure morals, as well as the rendering of mutual aid. At Upsåla there are 13 and at Lund 12 of these nation societies, each under control of an *inspector* chosen by the society from among the professors of the university. At Upsåla these unions have as a rule their own club premises * * *; at Lund there is one clubhouse common to all the 'nations,' that of the *Akademiska Föreningen*. These unions have played an important rôle in the life of the students, both in its serious and its more convivial aspects. The distinctive feature of the students' dress is the white cap (*alla mössan*), known abroad from the tours of students' choral unions."

² The first number of its forty-first volume appeared recently.

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|---------------------------------------|---------------------------------------|
| Group 1. Northern languages, Latin. | Group 8. German, romance languages. |
| Group 2. Northern languages, German. | Group 9. English, romance languages. |
| Group 3. Northern languages, English. | Group 10. History, geography. |
| Group 4. Northern languages, history. | Group 11. Geography, botany, zoology. |
| Group 5. Latin, Greek. | Group 12. Mathematics, physics. |
| Group 6. Latin, history. | Group 13. Physics, chemistry. |
| Group 7. German, English. | Group 14. Chemistry, botany, zoology. |

Application for the *ämhetsexamen* must be accompanied by—(1) a statement of the choice of subjects; (2) a certificate that the applicant has been a member of a nation society during the term in which the examination is to take place; (3) the certificate, showing that the candidate has passed the studentexamen; (4) the candidate's record book of university work (*tentamensbok*); (5) a fee of 15 kronor (about \$4.09), plus 3 kronor for making out reports, plus 1 krona for janitor service.

As we shall be frequently referring to the grades which the candidate obtains in university examinations, it may be stated at once, that, as in the case of secondary school marking, they are six in number. If, for definiteness, we employ the letters that have been already mentioned, these grades are: *A*, with honor (*berömt*); *a*, with exceptional praise (*med utmärkt beröm godkänd*); *AB*, with praise (*med beröm godkänd*); *Ba*, not without praise (*icke utan beröm godkänd*); *B*, satisfactory (*godkänd*); *C*, not satisfactory (*icke godkänd*).

To pass the examination we are considering, the candidate must obtain seven points. He has three points to his credit for each *A* which he receives in the subjects of the examination, two points for each *a* or *AB*; and one point for each *Ba* or *B*.

From this we see that mathematics and physics, the subjects of group 12, could contribute at most six points. But the candidate must also pass an examination in some third subject, as well as in psychology and the history and theory of pedagogy. In the case of group 12 the third subject may be any other subject in the mathematics-natural science section. Mathematics may be taken as a third subject with any two-subject group.

Whatever selection of group is made for the major, if it is a two-subject group, the candidate must obtain at least a mark *AB* in each; if it is a three-subject group at least a mark *AB* in two subjects and *B* in the third. Moreover, in the mathematics-natural science section six of the seven points must be obtained in subjects selected from the following list: Theoretical philosophy, literary history and poetics, German, English, romance languages, pedagogy, geography, mathematics, astronomy, physics, mechanics, chemistry, geology and mineralogy, botany, and zoology.

In the *ämhetsexamen* at the University of Lund, the requirements in mathematics for several grades assigned are as follows: (1) For *B*

the candidate must show that he has a good general knowledge of analytic geometry, elements of the differential calculus and of the most important parts of the integral calculus; (2) for *AB* the candidate must be familiar with the subject matter in (1), with differential calculus and its geometric applications, with an extensive and thorough course in algebraic analysis, with a good course in integral calculus, with the elementary theory of equations, with the foundations of theory of numbers and theory of probability, and with a short course in modern geometry, particularly the application of the theory of projection and reciprocation to conic sections; (3) for *A* the candidate must show remarkable facility in handling the material of (1) and (2), and also of some important course the content of which may be optional with the examinee to a considerable extent, but each particular choice must be approved by the examiner.

The examination in every subject is oral. The professor reports to the examiner his personal opinion of each candidate whom he has taught. He not only indicates the extent of work covered, but also expresses an opinion as to the thoroughness of the student's knowledge and as to the independence and maturity of judgment which he has manifested.

The titles of some courses offered at the university will be given later, and it will be seen that certain of these are formulated to prepare the student for the searching tests. Among the great number of books which are found useful by the mathematician in preparing for the *filosofisk ämbstsexamen* the following are officially listed at Lund:

- K. R. COLLIN, *Lärobok i plan analytisk geometri*. 2 upplagan. Stockholm, 1898. (Rather brief.)¹
- BRIOT et BOUQUET, *Leçons de géométrie analytique*. 17e édition. Paris, 1900.
- C. F. E. BJÖRLING, *Lärobok i differentialkalkyl och algebraisk analys*.² 3. upplagan. Lund, 1909.
- C. F. E. BJÖRLING, *Lärobok i integralkalkyl*. Upsala, 1877.
- SERRET-HARNACK, *Lehrbuch der Differential- und Integralrechnung*. Bearbeitet von Scheffers. 4. und 5. Auflage. Band I: Differentialrechnung. Leipzig, 1908. Band II: Integralrechnung. Leipzig, 1911.
- W. NERNST und A. SCHÖNFLIES, *Einführung in die mathematische Behandlung der Naturwissenschaften. Kurzgefasstes Lehrbuch der Differential- und Integralrechnung, mit besonderer Berücksichtigung der Chemie*. 7. Auflage. München und Berlin, 1913.
- TODHUNTER, *Theory of Equations*. 4th edition. London, 1880.
- WEBER und WELLSTEIN, *Encyklopädie der Elementar-Mathematik*. Band 1: Algebra und Analysis. 3. Auflage, Leipzig, 1909; Band 2: Elemente der Geometrie. 2. Auflage. Leipzig, 1907; Band 3: Angewandte Elementar Mathematik. 2. Auflage. 2. Teilen. Leipzig, 1910-1912.

¹ This comment and others of a similar nature which follow occur in the original list.

² In this work, applications to plane geometry are considerably more extensive than is usual in books of a similar nature.

TODHUNTER, *Algebra for the use of schools and colleges with numerous examples*. 5th edition. London, 1871. (For theory of numbers and theory of probabilities.)
 C. F. E. BJÖRLING, *Lärobok i nyare plan geometri*.¹ Lund, 1896.

The requirements in mechanics and mathematical physics,² in the filosofisk ämbetsexamen at the University of Lund, for various marks are: (1) For the mark *B*, the candidate must know the elements of the kinematics and statics of rigid bodies, including computation of stability, the elements of dynamics of a particle (also relative motion), the elements of the dynamics of solid bodies moving in a plane, and central impulse with friction, and must have the ability to solve simpler problems within the limits of the course in a *reliable* manner; (2) for the mark *AB* the candidate must be familiar with the subject matter in (1), with discussion of statics of a flexible string, with more extended development of rigid dynamics, including Lagrange's equations of motion and the elements of the theory of the motion of a rigid body about a fixed point, with hydrostatics, with differential equations of hydrodynamics and Bernoulli's theorem, with the theory of small vibrations applied to organ pipes and strings, and with attraction between spheres, and he must also have facility in the solution of problems within the range of the course; (3) for the mark *A* the candidate must complete the course in (2) with respect to general dynamics and by means of a detailed study of some field in mathematical physics or in mechanics of continuous media, chosen by the candidate in consultation with the examiner. He must also have skill in applying practically the theoretical knowledge appertaining to the courses.

Prerequisites in mathematics.—For a *B* in mechanics it is necessary to know what corresponds to the requirement for *B* in mathematics. For an *AB*, in addition to the requirements in analysis for the same predicate in mathematics, the candidate must have some notion of ordinary differential equations and the most elementary ideas of partial differential equations. For the mark *A* there may be required in addition to ordinary and partial differential equations important branches in one part or another of mathematical analysis.

Studies and textbooks.—The studies are appropriately begun in the free preliminary course (for *B*) as prescribed every spring term in connection with practical exercises. In order to obtain any advantage from this it is, however, necessary at the same time to work at home, partly by carefully reviewing the notes taken at every lecture

¹ The principal parts of this work include a treatment of the theory of conics from projective and dualistic points of view, in addition to a presentation of the theory of invariants in linear substitutions for binary and ternary forms, and, finally, after a section on "Curves of higher order" where among other results the Plückerian formulæ are derived, a treatment of the theory of cubic curves. Other works, not mentioned above but widely used in Sweden, are: L. Kiepert, *Grundriss der Differential- und Integralrechnung*, umgearbeitet von M. Stegemann. 2 Bände. Band I, 10. Auflage. Hannover, 1906; Band II, 9. Auflage, 1908. De la Vallée-Poussin, *Cours d'analyse infinitésimale*. 2 tomes. 3. et 2. éditions. Louvain, 1912 and 1914. Goursat, *Cours d'analyse mathématique*. 3 tomes, tomes 1-2. 2. édition. Paris, 1910-15.

² Studiehåndbok, p. 168-173; astronomy and physics are treated on pp. 166-167.

and carrying on collateral reading in appropriate textbooks, and partly by diligently solving problems in addition to the one which has been taken up in the practical exercises.

At the conclusion of the preliminary course similar regularly recurring lectures and exercises for the next higher certificate can be taken up. (To profit by these the home work should be somewhat of the same nature as that in connection with the preliminary course.)

Really satisfactory textbooks for beginners are not readily found at present. For the time being, however, the most suitable work which can be recommended is Lindskog's *Lärobok i mekanik*, Stockholm, 1894, which also contains practical problems. Chapters 13 and 17 may be omitted, and for the certificate *B* also paragraphs 140-142, 155-159, 167-168.

For the certificate *AB* a more thorough knowledge of Lindskog's mechanics is required. It is not expedient to recommend a definite textbook for this certificate, because a few of this kind which are worthy of notice are just now being published or revised. The following may be mentioned first:

- P. APPELL et S. DATHÉVILLE, *Précis de mécanique rationnelle*. Paris, 1910.
 A. G. WEBSTER, *The Dynamics of Particles, and of Rigid, Elastic, and Fluid Bodies*. Leipzig, 1904. (Out of print, but a German as well as a new English edition is being prepared.)

The first-named book, which contains also a collection of practical problems, corresponds most nearly to the prescribed course. Still, parts of chapters 14, 16, and 18 can be omitted. The latter book contains comparatively more of the mechanics of fluids and of elastic bodies and is specially to be recommended to those who intend to devote themselves to physics.

Among problem collections we may mention:

- H. L. DE ST. GERMAIN, *Recueil d'exercices sur la mécanique rationnelle*. 2. édition. Paris, 1889.
 A. FUHRMANN, *Aufgaben aus der analytischen Mechanik*. 2 Bände. Leipzig Band I, 3. Auflage, 1904; Band II, 2. Auflage, 1882.
 W. WALTON, *A collection of problems in theoretical mechanics*. 3. edition. Cambridge, 1876.
 —, *A collection of problems in hydrostatics and hydrodynamics*. Cambridge, 1847.
 M. JULLIEN, *Problèmes de mécanique rationnelle disposés pour servir d'applications aux principes enseignés dans les cours*, 2. édition. 2 vols. Paris, 1866-67.
 K. W. F. KRAFT, *Sammlung von Problemen der analytischen Mechanik*. 2 Bände. Stuttgart, 1884-85.

Filosofie licentiatexamen.—Those who have passed the filosofisk ämbetsexamen are eligible for the filosofie licentiatexamen. In each examination subject the candidate must make *AB* at least. An acceptable scientific thesis must also be written.

¹ The second English edition was published in 1912, shortly after this was written.

Application for the examination must be accompanied by—(1) a statement of the choice of subjects; (2) a certificate that the applicant has been a member of a nation society during the term in which the examination is to take place; (3) the certificate showing that the candidate has passed the filosofisk ämbetsexamen; (4) the candidate's record book of university work (tentamensbok); (5) a fee of 15 kronor (about \$4.09), plus 3 kronor for making out reports, plus 1 krona for janitor service.

The present regulations governing the licentiatexamen went into effect in 1911. They provide that every student who has passed either that examination or the filosofisk ämbetsexamen has the right to ask for an examination in supplementary subjects, for the sake of obtaining higher reports. No one may take more than one supplementary examination. The standing in supplementary examination carries with it the same rights as in other subjects. The fee for this examination is 5 kronor for each subject, plus 3 kronor for making out reports, plus 1 krona for janitor service.

The licentiatexamen calls for insight into the more fundamental parts of modern higher mathematics. The higher certificates require an exceptional scientific thesis based upon special studies appropriately chosen. There is considerable latitude in making the choice. But "however the subjects are arranged," the announcement concerning the examination reads, "one must neither thrust to one side modern insistence on logical exactness nor neglect geometrical intuitive methods."

In preparation for the filosofie licentiatexamen in mathematics, the lectures in higher mathematics delivered by professors or assistants are intended either to illuminate such parts of broad fields as are little available on account of gaps in texts, etc., or else to orient the student in some special topic. The seminary exercises have the same end in view.

Among the texts which are useful in preparing for this examination in the most fundamental fields the following are mentioned in the Studiehåndbok of the University of Lund:

É. PICARD, *Traité d'analyse*. 2. édition. 3 tomes. Paris, 1901-1909.

W. F. OSGOOD, *Lehrbuch der Funktionentheorie*. Band I. 2. Auflage. Leipzig, 1912.

(More complete in some respects than Picard.)

N. NIELSEN, *Laerebog i elementær Funktionsteori*. København, 1909.

H. DURÉGE, *Elemente der Theorie der Funktionen einer komplexen veränderlichen Grösse*. 5. Auflage bearbeitet von Maurer. Leipzig, 1906.

Can be used with advantage for an introductory course.

C. JORDAN, *Cours d'analyse de l'École Polytechnique*. 3. édition. 3 tomes. Paris, 1909-1915.

Separate sections on elliptic functions and also introductory chapters on partial differential equations.

O. A. A. BRIOT et J. C. BOUQUET, *Théorie des fonctions elliptiques*. 2. édition. Paris, 1875.

In many respects now antiquated, but can still be used with profit.

L. SCHLESINGER, *Einführung in die Theorie der Differentialgleichungen*. 2. Auflage. (Sammlung Schubert.) Leipzig, 1904.

K. HENSEL und G. LANDSBERG, *Theorie der algebraischen Funktionen einer Variablen und ihre Anwendung auf algebraische Kurven und Abelsche Integrale*. Leipzig, 1902.

C. F. E. BJÖRLING, *Lärobok i nyare plan geometri (och algebra)*. Lund, 1896.

G. SALMON, *Traité de géométrie analytique: courbes plans*. 3. édition par O. Chemin. Paris, 1903.

H. WEBER, *Lehrbuch der Algebra*. 2. Auflage. 3 Bände. (Band III: Elliptische Funktionen und algebraische Zahlen.) Braunschweig, 1898-1908.

E. NETTO, *Substitutionentheorie und ihre Anwendung auf die Algebra*. Leipzig, 1882.

J. P. C. PETERSEN, *De algebraiske Ligninger's Theori*. København, 1877. (For an introduction to the subject.)

J. TANNERY, *Introduction a l'étude de la théorie des nombres et de l'algèbre supérieure*. Conférences rédigées par E. Borel et J. Drach. Paris, 1895.

For an introduction to the subject.

F. KLEIN, *Vorlesungen über das Ikosaeder und die Auflösung der Gleichungen vom fünften Grade*. Leipzig, 1884.

SCHNEIBNER, *Beiträge zur Theorie der linearen Transformationen als Einleitung in die algebraische Invariantentheorie*. Leipzig, 1908.

G. SCHEFFERS, *Anwendung der Differential- und Integralrechnung auf Geometrie*. Band II: Einführung in die Theorie der Flächen. 2. Auflage. Leipzig, 1913.

P. BACHMANN, *Grundlehren der neuern Zahlentheorie* (Sammlung Schubert). Leipzig, 1907.

A. R. FORSYTH, *Treatise on differential equations*. 4. edition. London, 1914.

Contains a great number of useful examples, but is not satisfactory from a theoretical point of view.

SERRET-HARNACK, *Lehrbuch der Differential- und Integralrechnung*: Band III: Differentialgleichungen und Variationsrechnung. 3. Auflage. Bearbeitet von G. Scheffers. Leipzig, 1909.

G. Bohlmann and E. Zermelo were the editors of the second German edition in 1904.

The statements of the University of Lund concerning the filosofie licentiatexamen in mechanics are as follows:

To obtain the mark *B*, the requirements are the same as for the highest certificate in the filosofisk ämbetsexamen in mechanics.

To obtain the mark *AB*, the candidate must (a) have prepared himself in a complete course in general dynamics and mechanics of a rigid body in conformity with the requirements for the highest certificate in the filosofisk ämbetsexamen; (b) have some acquaintance, with the most important branches of mathematical physics and of the mechanics of continuous media; (c) have a thorough grasp of at least one of the sciences (for example, hydrodynamics, or theory of elasticity or thermodynamics, etc.), together with a

* The second edition by W. Jacobsthal of H. Maser's German translation (with the solutions of the problems) of the third English edition of Forsyth's work was published at Leipzig in 1912.

detailed study of the modern literature connected with the science in question.

To obtain the mark *A*, the candidate must not only meet the requirements just indicated, but must also prepare a piece of research work of actual scientific value.

The person who intends to devote himself specially to mechanics or mathematical physics is advised to direct his studies for the *ämhetsexamen* in such a way as to enable him to obtain two certificates in physics and the highest certificate in mathematics. In the latter subject the main emphasis should be on analysis, such as ordinary and partial differential equations, calculus of variations, general theory of functions, elliptic functions, together with spherical harmonics and Bessel's functions.

Literature list.—The literature list given below lays no claim to completeness, and the entries are chiefly of works suitable as textbooks. Among books in one and the same subject those stand last which on account of size or character are better suited to profound study in the special subject. Short or popular texts come first.

H. WEBER, *Die partiellen Differentialgleichungen der mathematischen Physik*. 2 Bände. 5. Auflage. Braunschweig, 1910-1912.

A. WANGERIN, *Theorie des Potentials und der Kugelfunktionen*. (Sammlung Schubert.) Band I. Leipzig, 1909.

A. GRAY and G. B. MATHEWS, *A Treatise on Bessel functions*. London, 1895.

CHRISTIANSEN-MÜLLER, *Elemente der theoretischen Physik*. 3. Auflage. Leipzig, 1910.

P. APPELL, *Traité de mécanique rationnelle*. Tomes 1-2, 3. édition. Paris, 1909-1911; tome 3, 2. édition, Paris, 1908.

The first volume may be dispensed with, since it has been already read for the certificate in the *ämhetsexamen*. The third volume contains the theory of potential, hydromechanics, and theory of elasticity.

E. MACH, *Die Mechanik in ihrer Entwicklung historisch-kritisch dargestellt*. 7. Auflage. Leipzig, 1912.

Critical presentation of the foundations of mechanics. Recommended even for the highest certificates in the *ämhetsexamen*.

G. R. KIRCHOFF, *Vorlesungen über mathematische Physik*. Band 1: Mechanik. 4. Auflage von W. Wien. Leipzig, 1897.

Succinct presentation—especially from the mathematical standpoint—of the whole of mechanics.

W. THOMSON and P. G. TAIT, *Treatise on Natural Philosophy*. New edition, 2 volumes. Cambridge, 1903.

A masterly presentation of mechanics from the physicist's standpoint, but unsystematic and for that reason unsuitable as a text. In part perhaps somewhat out of date, but none the less readable. Complements Appell and Kirchhoff in valuable manner.

E. J. ROUTH, *Treatise on the Dynamics of a System of Rigid Bodies*. 2 volumes. Volume 1: The elementary part, seventh edition, London, 1905. Volume 2: The advanced part, sixth edition, London, 1905.

¹ English translation of the first edition by W. F. Magie. London, 1867.

² English translation from the second German edition by T. J. McCormack, Chicago, 1893. 2d edition, revised, 1902. Supplement by P. E. B. Jourdain, Chicago, 1915.

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H. R. HERTZ, *Gesammelte Werke*. Band 3: *Die Prinzipien der Mechanik in neuem Zusammenhange dargestellt*. Herausgegeben von P. Lenard. 2. Auflage. Leipzig, 1910.

Short courses in hydromechanics and theory of elasticity were written by Webster, Appell, and Kirchhoff (see above).

W. WIEN, *Lehrbuch der Hydrodynamik*, Leipzig, 1900.

H. LAMB, *Hydrodynamics*. 3. edition, Cambridge, 1906. (Also in German by Friedel, 1907). 4. English edition, 1916.

A. E. H. LOVE, *A Treatise on the Mathematical Theory of Elasticity*. Second edition, Cambridge, 1906.

Also translated into German by Timpe, 1907.

H. LAMB, *The Dynamical Theory of Sound*. London, 1910.

J. W. S. RAYLEIGH, *Theory of Sound*. New edition. London, 1894-1896.

M. PLANCK, *Vorlesungen über Thermodynamik*. 4. Auflage. Leipzig, 1913.

W. VOIGT, *Thermodynamik*. 2 Bände. (Sammlung Schubert.) Leipzig, 1903-4.

E. MACH, *Die Prinzipien der Wärmelehre historisch-kritisch dargestellt*. 2. Auflage. Leipzig, 1900.

G. JÄGER, *Die Fortschritte der kinetischen Gastheorie*, Braunschweig, 1906.

O. E. MEYER, *Die Kinetische Theorie der Gase*. 2. Auflage. Leipzig, 1899.

JEANS, *The Dynamical Theory of Gases*. 2. edition. Cambridge, 1916.

L. BOLTZMANN, *Vorlesungen über Gastheorie*. 2 Bände, 2. Auflage. Leipzig, 1912.

J. W. GIBBS, *Elementary Principles in Statistical Mechanics. Developed with special reference to the rational foundation of thermodynamics*. New Haven, 1902. (Also in German by Zermelo, 1905.)

P. DRUDE, *Die Physik des Aethers auf elektromagnetischen Grundlage*. Stuttgart, 1894.

A. FÖPPL-M. ABRAHAM, *Theorie der Electricität*. Band 1. 3. Auflage. Leipzig, 1907.

P. DRUDE, *Lehrbuch der Optik*. 3. Auflage von E. Gehrcke. Leipzig, 1912.

H. A. LORENTZ, *The Theory of Electrons and its Applications to the Phenomena of Light and Radiant Heat*. Leipzig, 1909.

M. PLANCK, *Vorlesungen über die Theorie der Wärmestrahlung*. Leipzig, 1906.

Doctorate.—In order to procure the degree of doctor of philosophy the candidate must have passed the filosofie licentiatexamen, and have published and publicly defended an original scientific thesis on a subject connected with the studies in which he has received commendation in the licentiatexamen. The thesis and defense or disputation must be satisfactory to the faculty.

The members of the mathematics-natural science faculty are supposed to be present at the disputation, in which a representative of the faculty and two supporters of the candidate take part. The regulations seem to imply that such functions are not always of an amiable character: "The dean, or some other professor whom he may appoint, shall see to it that order and decorum are preserved. In case disorder or disturbance arises and can not be quelled by pleasant means by him or his deputy, he has the right to dismiss the assembly. Whatever he requires must be immediately observed by all present." The paragraph then concludes, "After the disputation has lasted four

¹ English translation of the second revised edition, by R. N. Boyes. London, 1899.

hours the dean, or his deputy, may conclude the same if he is so inclined."

Courses.—In 1891 regulations as to the nature and method of instruction in Swedish universities were formulated. They provided that all instruction given with a view to preparing for examinations must be so arranged that "scientific lectures in each examination subject shall be given each year, and that exercise courses, partly introductory courses, and partly seminary courses shall be furnished." They provided also that "one course within the purview of the faculty instruction should conclude with an oral or other examination suitable for those electing the course."

Consequently, as far as the number of instructors will allow, an endeavor is made to provide annually the essential requirements for the predicate *AB* in the filosofisk ämbetsexamen, and in addition at least the especially fundamental parts of analytic geometry and calculus, which properly belong to a propaedeutic course. In special exercises, on which great stress is laid, an attempt is made to have the students acquire the habit of solving with facility harder problems, such as are generally treated in the courses. In Lund these courses and exercises are compulsory to a certain extent; in Upsala, requirements of this kind have not yet been introduced in connection with mathematics.

The public lectures aim to give, on the one hand, broad general presentations of fields which properly belong to examination courses; on the other hand a thorough discussion of special questions. In the seminary exercises analysis of memoirs and report and discussion of original papers encourage the development of independent scientific work.

The following courses in astronomy, pure mathematics, and mechanics were offered at the University of Lund during the four semesters (which are now referred to as first, second, third, and fourth), from the autumn semester, 1914, to the spring semester, 1916, inclusive:

During the first semester Prof. C. V. L. Charlier lectured on "Motions of star clusters"; during the second on "Multiple correlation" and "Use of the kinetic theory of gas in the motion of star clusters," three hours a week; during the third on "Selected parts of the theory of enumerating the stars"; and during the fourth for three hours a week on "Selected parts of star statistics." In each of the semesters Charlier and an assistant conducted a seminary in astronomy for those aspiring to the marks *B*, *AB*, or higher in the filosofisk ämbetsexamen.

T. Brodén, professor of mathematics, conducted: (a) In the first semester—(i) a preliminary course with the aid of an assistant; (ii) a course of about 20 lectures on the theory of functions; (iii) an elementary course of about 10 lectures on infinite series; (iv) an elementary seminary course, two hours every fortnight. (b) Second semester—(i) a preliminary course; (ii) a course on selected topics of the ämbetsexamen; (iii) a course on theory of functions (about 30 lectures); (iv) a course on some phase of the modern theory of aggregates; (v) an elementary seminary course. (c)

Third semester—(i) a course in theory of equations, a preliminary course in calculus, selected topics in theory of conics and certain arithmetic applications of higher analysis, three hours a week; (ii) an advanced seminary course on the connection between analysis and the theory of numbers, one hour. (d) Fourth semester—(i) preliminary course; (ii) elementary course in algebraic analysis and arithmetic applications of higher analysis, two hours; (iii) advanced seminary exercises, one hour.

V. W. Ekman, professor of physics: First semester—(i) mechanics, two hours a week for two certificates in the ämbetsexamen; (ii) hydrodynamics, one hour; (iii) exercises in mechanics, one hour. Second semester—(i) mechanics (first course), three hours; (ii) motion of liquids and theory of friction, one hour; (iii) exercises in mechanics, one hour. Third semester—(i) mechanics (continuation course and course for two reports in ämbetsexamen); (ii) theory of elasticity, one hour; (iii) exercises in mechanics, one hour. Fourth semester—(i) preliminary course in mechanics, two hours; (ii) theory of elasticity, one hour; (iii) exercises in mechanics, one hour.

N. E. Nörlund, professor of mathematics: First semester—(i), elliptic and automorphic functions, three hours; (ii) advanced mathematical seminary, one hour. Second semester—(i) elliptic and automorphic functions, one hour; (ii) theory of numbers, one hour; (iii) advanced mathematical seminary, one hour. Third semester—automorphic functions and higher algebra, three hours; (ii) elementary mathematical seminary, one hour. Fourth semester—(i) higher algebra, three hours; (ii), elementary mathematical seminary, one hour.

F. A. Engström, astronomical observer: All four semesters—(i) use of simpler astronomical instruments for determination of the time and of latitude, one hour; (ii) exercises in astronomical observations.

H. G. Block, docent in astronomy: First, second, and third semesters—in each, celestial mechanics, 15 lectures.

N. G. O. Kuylenstierna, docent in mathematics: First semester—on Laplace's equation, 15 lectures. Second semester—integral equations, 20 hours. Third semester—selected chapters in the theory of linear differential equations, 15 lectures.

K. A. W. Gyllenberg, docent in astronomy: Third semester—(i) spherical astronomy, for meeting requirements for *AB* in filosofisk ämbetsexamen; (ii) general astronomy and the stars (requirements for *B* in same examination). Fourth semester—(i) determination of orbits of comets, 3 hours; (ii) spherical astronomy (for *AB* in ämbetsexamen), 20 hours.

S. D. Wicksell, docent in mathematical statistics: Third semester—elementary principles of mathematical statistics, 25 hours. Fourth semester—theory of correlation, 25 hours.

At the University of Upsala in the spring term of 1916 courses in the following subjects were offered: Linear differential equations, theory of functions, differential equations, trigonometric series, theory of electricity, rational mechanics, integral equations, and theoretical astronomy.

SECONDARY-SCHOOL TEACHERS.

There has been given above an outline of the work that a mathematical teacher must do in the State secondary schools, and of work in school and university which he must have completed by way of preparation. But Sweden requires still more, namely, a year of probation (*provår*). This year of professional training is taken at one of the seven schools officially prescribed for this purpose, three of

which are located at Stockholm, one at Upsala, one at Lund, and two at Goteborg. The number of "candidates" is about 70 a year.

Having finished his university course in the spring,¹ and having decided to become a teacher in secondary schools, the candidate sends to the royal board of secondary schools copies of his final examination certificate and of his various university certificates, naming at the same time the three subjects he wishes to be trained in, which are, say, mathematics, physics, and chemistry. In the autumn he is assigned to some one of the schools along with perhaps nine others. After he has had an interview with the head master or rektor, there are assigned to him from the members of the staff, various supervisors or directors, under whose particular direction he remains during his training. A man may have one supervisor given him for mathematics in the lower classes, another for mathematics in the higher, and a third for physics and chemistry. The first two or three weeks may be spent in watching the work of the classes (the lowest for the most part) and listening to the instruction given. Then comes a little teaching, first in the presence of the teacher and then in his absence. After that comes the first "criticism lesson"² given, perhaps, to the lowest class, in the presence of the rektor and the teacher of the class and the rest of the students in training. As soon as possible after the termination of the lesson all those who thus witnessed it meet with the candidate and discuss its matter and manner. And so in the second, third, and fourth classes there might be listening and teaching and a second "criticism lesson." In the fifth class there is possibly continuous teaching for a month, both with and without the teacher; then "criticism lessons" follow; and the first term closes with the class leaving the realskola for the gymnasium.

In the spring term the candidate watches the teaching of algebra and geometry in Ring I, going through his first "criticism lesson" in those subjects in Ring II. He may have a criticism lesson in Ring III in trigonometry and solid geometry, and in Rings II-IV, he may have criticism lessons in physics and chemistry, or he may give a month of continuous instruction in one of these subjects followed by a criticism lesson. In addition, he is expected to take class teaching if one of his supervisors is kept away on account of illness.

Besides all this, an elaborate series of lectures is given to the candidates on the theory and history of education and pedagogy. Lectures are also given on method in connection with special subjects.

The rektor, having heard the criticism lessons and received periodical reports both from the candidate and his supervisors, holds a *scrutinium*, attended by those of his colleagues who have taken part

¹ From Mr. Thornton's report.

² A "criticism lesson" is a lesson specially prepared for criticism on the part of his fellow candidates and of his teachers.

in the training; and the precise character of the certificate to be given the candidate is then settled by discussion and by full interchange of opinion and evidence. This certificate becomes an important asset in the teacher's stock in trade, one upon which the character of his first employment largely depends. A sample follows:

TRAINING CERTIFICATE.

X Y Z, *Licentiate in Philosophy,*

who in autumn term 1893 and spring term 1894 according to the regulations in force has undergone a training course at the *Elementarläroverk* in this place, has displayed

in the theoretical course:

an exceedingly satisfactory¹ knowledge of the theory and history of education;

in the practical course:

In Mathematics (Class I, III, V, VI 2, VII, 1)² a distinguished³ capacity for teaching.

In Physics (Class VI and VII 1) an exceedingly satisfactory capacity for teaching.

In Chemistry (Class VI and VII 1) an exceedingly satisfactory capacity for teaching.

In (Class).

and there has therefore been awarded to him, as a public testimony to his capacity for teaching, the certificate "exceedingly satisfactory." He has besides shown distinguished industry and exceptional aptitude for the teacher's calling.

Upsala, the 2d of June, 1894.

(Signed) A. B.,
Principal in the
practical course.

(Signed) C. D.,
Principal in the
theoretical course.

The teaching staff of a gymnasium consists of the rektor or head master and three ranks of permanent teachers: (a) Professors (*lektor*) who teach chiefly in the upper classes of the higher schools and who must be doctors; (b) assistant professors (*adjunkter*), who teach in *realskolor*, in lower classes of large gymnasia, and in the smaller gymnasia, and who, as far as the university is concerned, need only to have passed the filosofisk ämbetsexamen;⁴ and (c) instructors (*övningslärare*) teaching drawing, music, gymnastics, and the use of arms.

According to the official regulations the applicant for a position as a lektor or adjunkt on the staff of a State secondary school must—

- (1) Be 23 years old, at least.
- (2) Be in good health and not crippled in such a way as to be unable to get about.

¹ That is, a.

² These Classes VI 1, 2, and VII 1, 2, are now known as Rings I, II, and Rings III, IV.

³ That is, A.

⁴ The prospective adjunkt of mathematics who has passed this examination may have taken mathematics as a third subject with one of the groups in the humanities.

- (3) Be known for honor and probity, and in case the chair includes instruction in religious matters, must confess belief in the true evangelical faith.
- (4) Be conspicuous for sincerity and strength of character, and for the good temper necessary in the guidance of youth.
- (5) Be endowed with ability easily and clearly to impart instruction.
- (6) Be thoroughly conversant with the subjects connected with the chair he seeks.
- (7) Be prepared by a year of professional training.
- (8) Have performed, with industry and skill, for a period of not less than two years, the duties which devolve upon a lektor or adjunkt in (a) a boys' high school; or (b) some institution of learning under the jurisdiction of the royal board; or (c) a training seminary for elementary schools; or (d) a technical elementary school; or (e) the royal naval academy; or (f) a high school supported by the State. Or have served as *docent* in a university for not less than two years, in such a manner as would entitle him to an honorarium.

The prospective lektor must also (a) have obtained *AB* or higher in all those subjects of the *ambetsexamen* which correspond to subjects in which the chair has been announced as vacant; (b) have passed the *licentiatexamen* with the grade *AB* or higher in one of those subjects; (c) have satisfied the requirements for the doctorate. The lektor who teaches mathematics must also be conversant with the mechanics and astronomy in the course for the licentiate.

In the larger gymnasia the rektor are required to teach from 12 to 16 hours a week, in the smaller from 20 to 24; the professors from 18 to 22; and the assistant professors from 20 to 28 hours a week. A rektor at a *realskola* is obliged to teach from 18 to 20 hours weekly.

An adjunkt begins with a salary of 3,000 kronor (about \$818) which may be increased 500 kronor every five years until the maximum salary of 5,000 kronor is reached. In Stockholm, Göteborg, and Norrköping schools the municipality also makes a grant toward rent of apartments. The salaries of lektor range from 4,000 to 6,000 kronor. There are two scales of salaries for the rektor; namely, 6,000 kronor in the higher schools, with an increase of 500 kronor after 10 years, and 5,000 kronor in the *realskolor* with like increase after 10 years. In addition to this, rektor have free lodging or an equivalent allowance.

All teachers are obliged to contribute an annual quota (maximum, 200 kronor) to their future pension. On attaining the age of 65 and after 35 years of service, teachers on the permanent staff are entitled to a pension—4,000 kronor (about \$1,090.80) for lektor and 3,400

for adjunker. The rektor receives his pension (5,000 kronor in the gymnasia, 4,000 kronor in the realskolor), provided he has held the post of rektor for at least 15 years. The widow and children of a teacher on the permanent staff receive a considerable pension from a State-aided and State-controlled Widows' and Orphans' Pension Fund.

BIBLIOGRAPHY.

Der mathematische Unterricht in Schweden. Berichte der Schwedischen Abteilung der internationalen mathematischen Unterrichtskommission. Herausgegeben von H. von Koch und E. Göransson. Stockholm, Stockholms-Tryckeriet, 1911. 215 pp.

This volume contains eight reports by E. Göransson, E. Hallgren, H. von Koch, A. Wiman and others. Cf. *L'Enseignement mathématique*, 1911, tome 13, pp. 166-172, 339-344.

J. S. THORNTON, *Schools public and private in the north of Europe.* Special Reports on Educational Subjects, Board of Education, London, 1907, vol. 17. Sweden: pp. 1-24; 99 ff.

A. NORDFELDT and P. E. LINDSTRÖM, *Sweden Historical and Statistical Handbook.* By order of the Swedish Government, edited by J. Guinchard. Second edition, English issue. First part. Stockholm, Norstedt, 1914.

Pages 385-390: "State secondary schools for boys," by A. Nordfeldt.

Pages 408-417: "State and private Universities," by P. E. Lindström.

Studiehandbok för afläggandet af examina inom filosofiska fakulteten i Lund. Utgifven af fakulteten. Lund, Håkan Ohlssons Boktryckeri, 1911. 197 pp.

Föreläsningar och öfningar vid Kungl. Universitetet i Lund. (a) Höst-terminen,¹ 1914; (b) Vår-terminen,² 1915; (c) Höst-terminen, 1915; (d) Vår-terminen, 1916. Lund, Håkan Ohlssons Boktryckeri, 1915-16.

J. G. LEGGE and M. E. SADLER, "Education in Sweden." Special Reports on Educational Subjects, vol. 8, London, Board of Education, 1902. pp. 95-158.

Summarized translation of "Enseignement et culture intellectuelle en Suède," issued in connection with Paris Exhibition, 1900, by the Swedish Government.

O. GÅLLANDER, "The Secondary Schools of Sweden." Special Reports on Educational Subjects, vols. 3, London, Board of Education, 1898. pp. 649-671.

"The Educational System in Sweden." Report of the Commissioner of Education for 1903-4. Washington, 1906. vol. 1, pp. 767-797.

Abridged from a pamphlet, *Education in Sweden*, published in connection with the Swedish educational exhibit at the Louisiana Purchase Exposition.

N. G. W. LAGERSTET, "Current Educational Movements in Sweden." Report of the Commissioner of Education for 1912-13. Washington, 1914. vol. 1, pp. 763-772.

¹ That is, autumn term—from Sept. 1 to Dec. 16.

² That is, spring term—from Jan. 15 to June 1.

XVII. SWITZERLAND.

Switzerland is a confederation of 25 Cantons. It has a total area of less than 16,000 square miles and a population of about 3,900,000. The Cantons vary greatly in size; for example, Bern (Berne), Graubünden (Grisons), Valais (Wallis) are each over 2,000 square miles in extent, while the area of the Canton of Zug (Zoug) is less than 100 square miles.¹ German is spoken by the majority of the inhabitants in 19 of the Cantons, French in 3, and Italian, in 1. About two-thirds of the total population speak German; about 800,000, French; and about 300,000, Italian.

There is no centralization of control in educational matters. Each Canton is almost completely autonomous in the arrangement of its school system, and while there are many similar features in the organization, the ideals are often widely divergent. But it is certainly true that in no country is the importance of training for the primary school teacher more clearly recognized than in Switzerland. There are no fewer than 4 State or private training colleges for such teachers in this little Republic.

SECONDARY SCHOOLS.

The secondary schools leading to higher studies have different names in the different Cantons: *Collège*, *gymnase*, *Oberrealschule*, *Kantonschule*. This last is the most common in the German Cantons.

Examined from the point of view of their external organization, these establishments present many notable differences. We generally find two *cycles*. The first cycle is of three or four years and forms the *collège* or *gymnase inférieur*; the second cycle comprises from four to four and one-half years. Pupils enter this latter cycle at the age of 14 or 15 years.

A gymnasium may be divided into two, three, or even four sections, according to the foreign languages taught. The two main sections common to all the gymnasia are: (a) The *classical* section, which leads to all the university faculties (and to the Federal Polytechnic School after special preparation in mathematics), the special studies being Latin, Greek, and philosophy; (b) the *technical or industrial* section, which leads more particularly to scientific, technical, or industrial careers. The students who go out from this section are admitted directly to the faculties of science and of letters, to the technical

¹ Although Bern is not the largest Canton, its population of about 665,000 is the greatest. The Canton of Zurich, with an area of some 670 square miles, ranks next, with a population of about 540,000.

faculty at Lausanne, and to the Federal Polytechnic School. The gymnasium at Geneva possesses two other sections: (c) The *Real* section, in which Latin is taught in addition to modern languages. (It corresponds almost exactly to the section Latin-sciences in France or to the Realgymnasium in Germany, and its certificates of graduation admit to university faculties.) (d) The *pedagogic* section, which prepares candidates for teaching in the primary school and which also leads to the faculties of science and letters.

Although secondary schools are under cantonal or municipal direction, there is in their programs a certain necessary common minimum brought about by Federal influence in connection with requirement for *examens fédéraux de maturité pour les candidats aux professions médicales*. The requirements in mathematics for this examination are as follows: (a) *Algebra*.—Equations of the first and the second degree in one and several unknowns; logarithms; arithmetic and geometric progressions; compound interest and annuities; permutations and combinations; probabilities; binomial theorem with integral exponent. (b) *Geometry*.—Plane and solid geometry; plane trigonometry; facility in construction of geometric figures; analytic geometry of point, line, circle, and conic; application of the theory of graphic representation to simple analytic functions and elementary functions of physics and mechanics.

The mathematical requirements of secondary schools often exceed these. In the Oberrealschule of Basel, which is open to suitably prepared students who are 14 years of age, the scheme of courses is as follows:

Class I. Arithmetic and algebra to equations of the first degree in several unknowns (3 hours). Plane geometry and the beginning of solid geometry (3 hours). Geometric drawing (2 hours).

Class II. Algebra: Theory of indices, logarithms; equations of the second degree (3 hours). Solid geometry (2 hours). Geometric drawing (2 hours).

Class III. Algebra: Progressions; compound interest, annuities and applications to insurance; determinants (3 hours). Plane and spherical trigonometry (3 hours). Geometric drawing: with practical exercises (2 hours).

Class IV. Algebra: Binomial theorem; series; complex numbers; solution of equations of higher degree; transcendental equations (2 hours). Analytic geometry (2 hours). Descriptive geometry and geometric drawing (4 hours).

Class V (1 semester). Elements of differential calculus with simple applications to geometry and physics (3 hours). Analytic geometry of three dimensions (3 hours). Descriptive geometry and drafting (4 hours).

At the end of the gymnasium course a certificate of maturity is awarded to those who pass written and oral examinations on the studies of the previous year or two years. The examiner is the teacher, but the examination is conducted under the supervision of one of the various forms of "commissions" to be found in the Cantons.

In nearly all the examination rules, it is noticeable that greater emphasis is laid on the display of intellectual maturity than on range of knowledge. The "maturity rules" of the gymnasium and Real

school in Zurich state: "In mathematics and descriptive geometry the pupil shall indicate any new problems or applications which suggest themselves to him." At the Realschule in Zurich the oral examination is conducted in a manner similar to an ordinary class. The inspector makes a choice of three subjects proposed to him by the teacher. Some of the examination themes during the years 1907-1910 are as follows: Maclaurin's series; the equation $x = \tan \bar{x}$; maxima and minima problems in optics; sketch of the applications of the idea of the derivative; Huygens's approximate construction for the rectification of a circle; problems on geographic and geocentric latitude; analytic treatment of problems from Steiner's paper "Ueber das Maximum and Minimum bei den Figuren;" curvature of conics; perspective of circular cones; construction of sun dials.

All schools unite in the opinion that the grade in the certificate of maturity shall not, in general, depend upon the examination but rather on the performance of the pupil in regular classroom work.

HOCHSCHULEN.

Pupils with appropriate certificates of maturity may be regularly admitted to: (1) The *Eidgenössische Technische Hochschule* (Federal Polytechnic School) in Zurich; (2) a cantonal *École d'Ingénieurs* in Lausanne; (3) seven universities. In all of these training in advanced mathematical work is given.

(1) The *Polytechnic School*, which is the only Swiss scholastic institution under the direct supervision of the Federal authorities, is one of the greatest institutions of its kind in the world. It is organized into 11 sections, each with a course of 4 to 8 semesters. Most of the students turn to the engineering and chemistry sections. But somewhat over a score are in the section of mathematics and physics. The work of this section is supplemented by the general section, one part of which is known as the "Mathematisch-naturwissenschaftlich-technische Sektion."

Scheme of lectures in the mathematics and physics section during the first three semesters.

	Lectures.	Exercises.	Colloquium.
	Hours.	Hours.	Hours.
1. Semester (24 hours):			
Higher mathematics I.....	5	2	1
Analytic geometry.....	4		1
Descriptive geometry.....	4	4	1
Mathematical exercises.....		2	
2. Semester (25 hours):			
Higher mathematics II.....	5	2	1
Applications of descriptive geometry.....	2	3	
Mathematical exercises.....		2	
Mechanics I.....	6	2	1
3. Semester (22 hours):			
Higher mathematics III.....	3	1	
Geometry of position.....	2	1	
Mathematical exercises.....		2	
Mechanics II.....	4	2	1
Physics (heat).....	4		1

Cf. note on p. 78.

Some of the topics in *Higher Mathematics II-III* are the following: Space curves, tangent, principal normal, and binormal, osculating plane; ordinary differential equations; equations of the first and higher order; linear equations of any order; systems of simultaneous linear differential equations; curvilinear integrals; Green's formula and harmonic functions; conformal representation; Fourier's series; elements of calculus of variations. In *analytic geometry*: Analytic treatment of projectivity, cylinders, and cones of revolution, general surface of second degree, focal conics, generation of ruled surfaces by projective pencils. In *geometry of position*: Projective forms of the first, second, third, and fourth species.

From the fourth to the eighth semester there is no fixed program of mathematical work, but the following courses are given periodically: Function theory, elliptic functions, algebraic equations, number theory, differential geometry, plane curves, algebraic surfaces. During the last two years, such instruction is supplemented by discussions in the mathematical seminary, and lectures on special topics such as: Quadratic forms, theory of definite integrals, theory of transformation groups, axioms of arithmetic and of geometry, line geometry. In applied mathematics the titles of some of the courses given in recent years are: Partial differential equations of physics, cylindrical and spherical harmonics and their applications in physics, electro-mechanics, thermodynamics, celestial mechanics, astrophysics, map making, theory of probabilities, mathematics of insurance.

To able students who have assiduously applied themselves to the work in a section a diploma is given after successful examination. The examination comprises three parts: (a) The *Vordiplomprüfung*, on subjects of the courses of the first three semesters; (b) the *Schlussdiplomprüfung*, on function theory, synthetic and analytic treatment of geometry, higher arithmetic and algebra, theoretic physics and astronomy; and (c) the *Diplomarbeit*, or thesis.

At the polytechnic school the mathematical student may proceed to the doctor's degree in the usual way. The standard for the thesis is high.

(2) *L'École d'Ingénieurs*, although part of the faculty of sciences of the University of Lausanne, has a certain measure of autonomy. The classwork (39-43 periods a week) of the first four semesters includes about the same amount of mathematics as in the first three semesters of the above-mentioned section in the Polytechnic. *L'École* gives diplomas in civil, mechanical, electrical, and chemical engineering.

(3) The *Swiss Universities* are situated at Basel, Zurich, Bern, Geneva, Lausanne, Fribourg, and Neuchatel. In the winter semester of 1913-14 Zurich and Bern had the largest teaching staffs and the largest number of students; Geneva came next.

At the university of *Basel* three types of lectures are given: (a) Introductory lectures, (b) course lectures, (c) special lectures. The introductory lectures consist of analytic geometry of the plane and of space (4 hours, 1 semester), and differential and integral calculus (5 hours, 2 semesters). These, and some supplementary lectures on algebraic analysis and descriptive geometry, are to prepare suitably for taking up course lectures¹ students who, though they may have the certificate of maturity, are not adequately prepared in mathematics. The course lectures treat of such subjects as algebra, function theory, elliptic functions, ordinary differential equations, partial differential equations, linear differential equations of mathematical physics, projective geometry, and theory of surfaces. The program of the special lectures is not fixed, but it includes lectures on elementary geometry and number theory on account of their application in school instruction, and on calculus of variations because of its significance in mathematical physics.

Attendance on course lectures, special lectures, participation in seminary exercises and the preparation of a suitable thesis lead to the doctorate in about eight semesters.

At the *University of Bern* there are also three classes of mathematical lectures: (a) Eight or nine hours a week during four semesters in preparation of candidates for positions as teachers in the higher primary schools, the so-called "Sekundar-schulen"; (b) 12 or 14 hours weekly during six semesters for future secondary school teachers and those who are specializing in mathematics; (c) "free lectures" on such topics as theory of hypergeometric series, selected chapters from the theory of differential equations, non-Euclidean geometry, calculus of variations, and method of least squares.

In the courses for teachers in the higher primary schools, practical geometry, plane and spherical trigonometry, descriptive geometry, analytic geometry, and calculus (2 hours, 2 semesters) are taken up.

The subjects for the mathematical specialist and secondary school teacher include: Definite integrals (6 semester hours), differential equations (4 semester hours), function theory (4 semester hours), theory of determinants (1 semester hour), analytic geometry of space (3 semester hours), theory of higher-plane curves (3 semester hours), gamma functions (3 semester hours), and Fourier's series and integrals (three semester hours).

Examinations for secondary school teachers will be referred to later. The requirements leading to the doctorate are similar to those in the other universities.

At the *University of Geneva* the first two years may be spent in preparing for the baccalauréat ès sciences mathématiques, which is a sort of licence ès sciences. The examination for this consists of

¹ Course lectures are often taken simultaneously with introductory lectures.

two parts, an oral and a written. The oral examination is on functions of a real variable, series, theory of equations, plane and solid analytic geometry, descriptive geometry and the elements of projective geometry, differential and integral calculus, mechanics, astronomy, physics, physical geography and meteorology, inorganic chemistry, and mineralogy. The written examinations cover all of the above-mentioned subjects except the last four.

In 1908 the university established the *certificat d'aptitude à l'enseignement des sciences dans les établissements secondaires supérieurs*. A candidate for this certificate must (1) have the diploma of the "baccalauréat" from the university; (2) spend four semesters in further study at the university; and (3) finally show himself well versed in six academic studies. The examination consists of two parts, both of which may be taken at once. The first part of the examination is a thesis on a theme selected from the field of the major subject, and two trial lessons related to the minors. The second part consists of an oral examination in the three subjects chosen. The major and one minor are selected from the following: Infinitesimal analysis, algebra and higher geometry, mechanics, astronomy, physics, chemistry. The third subject, or second minor, is "the science of education," unless the candidate has the certificate of maturity from the pedagogic section of the gymnasium in Geneva, or an equivalent certificate, when philosophy or experimental psychology is substituted for the science of education.

In the oral examination some choice of subjects is possible. In *Infinitesimal Analysis* two of the following subjects may be selected: (a) Theory of analytic functions; (b) theory of elliptic functions; (c) differential equations; (d) partial differential equations of the first order; (e) calculus of variations; (f) introduction to mathematical physics (trigonometric series, Green's formula, etc.). In *Algebra and Higher Geometry* the examination covers: (a) Theory of equations, equations solved by square roots, applications in geometry; transcendence of e and π ; (b) methods in geometry; (c) infinitesimal geometry, curves traced on surfaces, applicable surfaces, and (d) one of the following: (i) Theory of algebraic forms; linear transformations; invariants; geometrical applications; (ii) notions on the theory of substitution groups and its applications to the theory of equations; (iii) projective geometry; principal properties of conics and quadrics; study of some transformations; (iv) line geometry; ruled surfaces; congruences and complexes of lines. In *Mechanics* two of the following subjects may be selected: (a) General methods for integration of equations in mechanics; (b) kinematics and machinery; (c) hydrostatics and hydrodynamics; and (d) theory of elasticity. In *Astronomy* the examination is on practical astronomy and geodesy, and either spherical or theoretical astronomy.

Upon satisfaction of the usual requirements, the university also confers the degree *doctorat ès sciences mathématiques*.

At the *University of Lausanne* a student who has pursued certain courses for four semesters may present himself as a candidate for the *Licence ès sciences mathématiques pures*. The examinations for this degree are oral, written, and practical. The oral covers differential and integral calculus; theory of analytic functions; elliptic functions; analytic geometry; descriptive geometry; geometry of position; rational mechanics; applied mechanics; astronomy; mathematical physics; and selected chapters from analysis, geometry, and analytic mechanics.

The written examination consists of three tests in analysis, geometry, and mechanics. The practical examination is a geometric drawing (*épure*).

The *Licence ès sciences physiques et mathématiques* is given under similar conditions. At the University of Lausanne the degree of *doctorat ès sciences* is also conferred.

I have already referred to the "école d'ingénieurs" connected with this university.

According to the organization of the *University of Fribourg*, the diploma *Licence ès sciences mathématiques* is awarded to students who have attended and have taken part in the practical exercises of a Hochschule, for not less than eight semesters (two at least at Fribourg), and have passed successful examinations in four subjects: (a) Differential and integral calculus, including the theory of definite integrals, of differential equations, and of differential geometry; and (b) analytic and synthetic geometry. The other two subjects can be chosen from c₁) function theory and d₁) higher algebra, or c₂) experimental physics and analytic mechanics, or c₃) experimental physics and mathematical physics.

The degree *doctor philosophiae naturalis* is also conferred at Fribourg.

SECONDARY-SCHOOL TEACHERS.

For secondary school teachers in Switzerland there are no fixed examinations similar to those for teachers in primary schools. Nor are there colleges for training such teachers, although, as we have seen, the University of Geneva has a pedagogic department. For most of the teaching positions in the secondary schools (*Mittelschulen*) the majority of the cantons demand that the candidate shall have completed a Hochschule course of four or five years. Such are the demands, for example, in the Canton of Fribourg where the *Licentiatdiplôme* is necessary, and in the Canton of Geneva with its *certificat d'aptitude à l'enseignement des sciences dans les établissements secondaires supérieurs*. Those candidates with a record of but two years in a Hochschule can, usually, only be appointed as teachers in the

lower classes of the better secondary schools. As an exception, however, according to a law of 1908 for the Canton Vaud, the licence certificate, which a candidate may secure after two years at the University of Lausanne, is the only necessary preparation for a teacher in the secondary schools. Students with a diploma from the École d'Ingénieurs with its two-year mathematical course are also in demand.

Several Cantons prefer that prospective teachers who have completed their studies shall have been engaged for several semesters as assistants in the Technische Hochschule; others prefer candidates who either before or after university studies have taught in one of the lower school grades. And still others have recently made some appointments to mathematical positions in Realschulen, of candidates holding engineering diplomas; not so much because of the lack of well-trained mathematical teachers as on account of the desire to emphasize, in the instruction, the applications of mathematics. But whatever the basis may be upon which the selection of secondary teachers is made, the standard is sure to be of a high order of its kind.

At this place further comment should be made with regard to work at the University of Bern. We have noted the six semester course for the oral and written *Patentprüfungen von Kandidaten des höheren Lehramts*. According to regulations of 1907-1911, the corresponding diploma entitles men to give instruction in the higher classes of the gymnasia. This diploma implies that the possessor has had, in addition to class instruction in mathematics and pedagogy, at least four weeks of practical experience in listening, or giving instruction to higher classes in the gymnasia.

For more than a decade past the *Verein Schweizerischer Mathematiklehrer* (*Société suisse des professeurs des mathématiques*) has been a source of inspiration to teachers through the opportunities which it offers both for social intercourse and exchange of ideas, and for methodic organization leading to advances in instruction of the science. The feeling is very general among members of this organization that the preparation of secondary school teachers should always include adequate professional training by means of courses in psychology and pedagogy at the university, for example, and by means of actual teaching in schools.

As to the remuneration of secondary school teachers, in the Canton of Zurich in 1900 the salaries ranged from about \$780 to about \$1,460. The head (raktor) of the secondary school is chosen from the staff. He serves for a period of three years and may be reelected. In Bern the ordinary teacher (25-31 hours a week) would receive about \$750. Increases in salary, amounting to about \$60 each are made at the end of 4, 8, and 12 years of service.

The range of pensions for secondary schools in Zurich, in 1893, was from \$200 to \$600 annually. More recent information is not available.

BIBLIOGRAPHY.

- H. FEHR, *L'Enseignement mathématique en Suisse*. Commission internationale de l'Enseignement Mathématique. Rapports de la Sous-Commission Suisse publiés sous la direction de H. Fehr avec la collaboration de MM. Badertscher, K. Brandenberger, L. Crelier, J. H. Graf, M. Grossmann, E. Gubler, M. Lacombe, K. Matter, L. Morf, F. Scherrer, J. Stöcklin. Bale et Genève, Georg & C^o, 1912. 704 pp.
- H. FEHR, "Les mathématiques dans l'enseignement secondaire en Suisse." *L'Enseignement mathématique*, tome 10, 1908, pp. 285-296.
- A. MORGAN, "The Training and Status of Primary and Secondary Teachers in Switzerland." Special Reports on Educational Subjects, vol. 8, London, Board of Education, 1902, pp. 177-261.
- R. L. MORANT, "The National Organization of Education of all grades as practiced in Switzerland." Special Reports on Educational Subjects, vol. 3, London, 1898, pp. 1-82.

XVIII. THE UNITED STATES.¹

Like Germany and other federated States, the United States has no national system of education. It is, however, a notable fact that the educational systems in the 48 States and Commonwealths of the country are all constructed on the same general lines, and that the differences, although of local importance, are of a minor character when the systems are viewed as a whole.

In each State one finds that the pupil normally passes in order through: (1) The kindergarten, where the pupils are from 3 to 5 years of age;² (2) the elementary school, which has an eight-year course for pupils 6 to 13 years of age; (3) the secondary school or high school, with a four-year course for pupils of 14 to 17 years; (4) the college or institution of collegiate rank, for students from 18 to 21; and then on to (5) the university or institution of university rank.

The American secondary school, unlike similar schools in Europe, takes the pupils at 14, on the completion of an elementary course covering 8 years. This is due in part to the earlier establishment of elementary schools which aimed to give a general education extending in some cases to 9 or even 10 years. High schools are now very important establishments in the educational system of the country, and the increase in their number in recent years is enormous. In 1910-11 the number of "accredited secondary schools" in the United States was 12,213. In 1913-14 this number had increased to 13,714.³ By an "accredited secondary school" is meant one—

which is equipped to prepare students for colleges requiring at least 14 units⁴ for unconditional admission and which has been investigated and approved for this purpose by one of the following agencies: A State officer of education, a university or college inspector or committee on admissions, or an officer or committee of an accrediting association.⁵

This standard imposes slight restrictions on the latitude of the program of study, so that it is quite impossible to indicate a program in any wise representative for even a large proportion of high schools in the country. But it may be worth while to give (1) a single pro-

¹ As different phases of mathematical instruction and its problems in the United States have already received detailed treatment in reports of the International Commission on the Teaching of Mathematics published by the Bureau of Education, this sketch is added mainly for the sake of completeness. It is intended merely to emphasize some outstanding features and to give supplementary information.

² Here and in what follows I shall assume that the normal lower limits of age at the beginning of each year are considered.

³ S. P. Capen, *Accredited Secondary Schools* (Bu. of Educ., Bul., 1916, No. 20), Washington, 1916, p. 7.

⁴ "A unit represents a year's study in any subject in a secondary school, constituting approximately a quarter of a full year's work." This statement takes—(1) the four-year high-school course as a basis and assumes that (2) the length of the school year is from 36 to 40 weeks; that (3) a period is from 40 to 60 minutes in length; and that (4) the study is pursued four or five periods a week.

⁵ S. P. Capen, *Accredited Secondary Schools* (Bu. of Educ., Bul., 1916, No. 20), Washington, 1916, p. 7.

gram, and this of a well-known school founded over 270 years ago; and (2) illustrations of the variations of accrediting agencies.¹ These illustrations roughly indicate the relative importance in which subjects are regarded by large groups of schools.

(1) The program selected is that (without Greek) of the Roxbury Latin school in 1915-16. The course of study there is designed to afford training for boys between the ages of about 12 and 18 and to qualify those who complete it to enter Harvard College, or other colleges or scientific schools, like the Institute of Technology.² The entering students are in the sixth class, those graduating in the first. The work covered in the fourth, third, second, and first classes is that which corresponds to the typical high-school course. The numbers in the different columns are class periods of 40 to 45 minutes.

Class periods in the Roxbury Latin School, 1915-16.

Subjects.	Sixth class.	Fifth class.	Fourth class.	Third class.	Second class.	First class.	Total.	Percentage.
English.....	190	152	152	114	76	114	798	13.7
Latin.....	190	190	190	228	190	190	1,178	20.3
French.....		190	190	152	152	190	874	15.0
German.....				190	190	190	570	9.8
History.....			114	114	114		342	5.9
Mythology.....			38				38	.7
Geography.....	76						76	1.3
Physics.....						228	228	3.9
Chemistry.....						228	228	3.9
Elementary science.....		76					76	1.3
Trigonometry and solid geometry.....						152	152	2.6
Plane geometry.....		76			190		266	4.6
Algebra.....			190	152			342	5.9
Arithmetic.....	114	114					228	3.9
Writing.....	76						76	1.3
Drawing.....	76	76					152	2.6
Gymnastics.....	76	38	38				152	2.6
Music.....	38						38	.7
Total.....	836	912	912	950	912	1,292	5,814	100.0

(2) The range of work accepted by accrediting agencies is shown by the following illustrations:

(a) The College Entrance Examination Board recognizes the following subjects as permissible in a standard high-school course:³

Subjects.....	Units. ⁴	Subjects.....	Units. ⁵
English, up to.....	3	Spanish.....	2
Mathematics ⁶	4½	History.....	4
Latin.....	4	Science.....	6
Greek.....	3	Drawing.....	1
French.....	4	Music.....	2
German.....	4		

¹ For a discussion of entrance to college by examination see "Examinations in mathematics other than those set by the teacher for his own classes." International Commission on the Teaching of Mathematics. (Bu. of Educ., Bul., 1911, No. 8.) Washington, 1911.

² Quinquennial catalogue Roxbury Latin School, 1915-16.

³ College Entrance Examination Board. *Document 68*, New York, Dec. 1, 1914, pp. 11-12.

⁴ In each case the number of units given is the maximum number which can be taken in the corresponding subject. In mathematics credit may be obtained for any number of half units up to nine.

⁵ Three quinquennial volumes of "Examination Questions in Mathematics" of this board have been published in Boston. One volume is for the period 1901-1905, another for 1906-1910, the third for 1911-1915.

(b) In contrast to this schedule there are State universities and privately endowed institutions, like Leland Stanford Junior University, which permit a wide range of electives. The University of Minnesota, for example,¹ accepts the following:

	Units. ²		Units. ³
English.....	3 -4	Vocational subjects (made up of the following).....	4-4
Mathematics.....	2 -3½	Business arithmetic.....	½
Latin.....	2 -4	Business law.....	½
Greek.....	2	Bookkeeping.....	1 -2
French.....	1 -4	Stenography and typewriting... 1 -2	
German.....	1 -4	Freehand drawing.....	2
Spanish.....	1 -4	Mechanical drawing.....	2
Scandinavian.....	1 -4	Shopwork.....	2
History and social science.....	½-7	Modeling and wood carving.....	1
Natural science.....	½-6	Domestic art and science, up to..	4
Agriculture.....	1 -4		
Normal training subjects.....	1 -3		

(c) Finally, it may be of interest to give a composite picture of class work in the 15 States of the North Central Association (Colorado, Illinois, Indiana, Iowa, Kansas, Michigan, Minnesota, Missouri, Montana, Nebraska, North Dakota, Ohio, Oklahoma, South Dakota, and Wisconsin). The percentage of units given in the different subjects of the high schools of these States in 1913-14 was as follows:⁴ English (13.1), Latin (11.5), history (9.5), commercial (9.2), German (8.5), algebra (5.2), geometry (4.8), manual training (4.7), physics (3.3), domestic science (3), chemistry (2.6), cooking (2.5), drawing (2.5), sewing (2.4), normal subjects (2.4), agriculture (2.0), botany (2.0), French (1.7), physical geography (1.7), music (1.6), civics (1.5), physiology (1.2), zoology (0.9), education (0.8), other subjects (1.4).

After completing his high-school course our future teacher in one of the secondary schools must of necessity take a college course. There are upward of 800 so-called "colleges" and "universities" in the United States, but many of these are really secondary schools. In 1908-9 there were only 261 colleges which had 100 collegiate students enrolled in the four regular college classes, or which had an endowment to the amount of \$100,000.⁵ There are about 40 endowed and 40 State universities⁶ in the United States. The student who wishes to go to one of the chief university mathematical centers would

¹ Illustration given by Capen (*l. c.*).

² In each case the number of units given is the maximum number which can be taken in the corresponding subject.

³ G. B. Counts, *A Study of the Colleges and High Schools in the North Central Association*. (Bu. of Educ., Bul., 1915, No. 6.) Washington, 1915, p. 115. See also the association's "Investigation into the status of the teaching of almost all subjects appearing in the secondary schools, programs of study," by L. V. Koos: *The Administration of Secondary-School Units*, Supplementary Educational Monographs, University of Chicago, vol. 1, No. 3, July, 1917.

⁴ Monroe's *Cyclopedia of Education*, vol. 2, p. 74.

⁵ There are also about 50 State colleges. See *Statistics of State Universities and State Colleges* (Bu. of Educ., Bul., 1917, No. 65), Washington, 1918.

probably select one of the following: Harvard, Chicago, Illinois, Princeton, or Columbia.

The college course leads to the degree of A. B., Sc. B., Litt. B., Ph. B., or some other degree of similar rank, but the title alone does not convey a very definite idea of achievement.¹

As to work leading to the degree of A. B., some colleges do not require any courses in mathematics; others require higher algebra, solid geometry, plane and spherical trigonometry, plane and solid analytic geometry, calculus and differential equations. But the normal requirement is a three-hour semester course in each of the subjects: (1) Higher algebra, or (2) solid geometry, and (3) plane trigonometry.² If our undergraduate student wishes to proceed in mathematics beyond the required work, the opportunities for doing so vary widely with different colleges. In one none but required courses in mathematics are offered; in another so many courses in mathematics are offered that under the "elective system" there in vogue it would be possible for the student to elect the major part of the courses he takes for his degree from those given by the department of mathematics.

THE TRAINING OF TEACHERS FOR SECONDARY SCHOOLS.

But before going into this question more particularly let us consider how a student who is taking a college course may definitely prepare himself to be a high-school teacher. To get a broad view of the question, we should recall the "joint recommendations of the committee of seventeen on the professional preparation of high-school teachers," adopted by the National Education Association.

The committee on the preparation of high-school teachers recommend:

- I. That the academic preparation include the following elements:
 - A. A detailed and specialized study of the subjects to be taught. The program of studies selected by each student should include work in subjects outside of those in which he is making special preparation, sufficient to give some insight into different fields of knowledge and to avoid the dangers of overspecialization.
 - B. One or more subjects from a group including history, economics, and sociology, which will give the teacher a proper outlook upon the social aspects of education.
 - C. A course in general psychology and at least one from a group of subjects including history of philosophy, logic, and ethics, which will give the teacher a proper outlook upon education as the development of the individual.

¹ In 1911 the Bureau of Education published an interesting *Classification of Universities and Colleges with Reference to Bachelor's Degrees*. The author, Dr. K. C. Babcock, a specialist in the bureau, divided the institutions into four classes. The first and highest class he defined as "Institutions whose graduates would ordinarily be able to take master's degree at any of the larger graduate schools in one year after receiving their bachelor's degree, without necessarily doing more than the amount of work regularly prescribed for such higher degree." In the first class were only 15 State and 44 endowed and private institutions. See also Rep. of U. S. Commis. of Educ. for 1914, vol. 1, Washington, 1915, p. 168.

² Compare *Undergraduate Work in Mathematics in Colleges of Liberal Arts and Universities*. International Commission on the Teaching of Mathematics. (Bu. of Educ., Bul., 1911, No. 7.) Washington, 1911.

II. That definite study be given to each of the following subjects, either in separate courses or in such combinations as convenience or necessity demands:

- A. History of education.
 1. History of general education.
 2. History of secondary education.
- B. Educational psychology with emphasis on adolescence.
- C. The principles of education, including the study of educational aims, values, and processes. Courses in general method are included under this heading.
- D. Special methods in the secondary-school subjects that the students expect to teach.
- E. Organization and management of schools and school systems.
- F. School hygiene.

III. That opportunity for observation and practice teaching with secondary pupils be given.

The committee recognizes the difficulties involved in this recommendation, but believes that they are not insurmountable. Each of the following plans has proved successful in some instances:

- A. The maintenance of a school of secondary-school grade that may be used for observation and practice.
- B. Affiliation with public or private high schools so situated geographically that practice teaching can be done without interfering with other work of the college course.

In addition to the above, the committee suggests that where competent critical supervision is possible, cadet teaching, in schools more remotely situated, may be attempted. In such cases, a teacher's diploma might be granted after a year's successful work as a cadet teacher.

IV. That the minimum requirement for a secondary-school teacher be graduation from a college maintaining a four-year course and requiring four years of high-school work for admission, or from an institution having equivalent requirements for admission and giving equivalent academic scholarship.

A year of graduate work divided between academic and professional subjects is desirable. Discussions of the relative value of college and normal schools for secondary-school teachers are to be found in the references below.¹

V. That the study of subjects mentioned under II be distributed through the last two years of the college course.

The proportional amount of time given to these subjects will vary with local conditions, but an irreducible minimum is one-eighth of the college course. They should be preceded or accompanied by the subjects mentioned in I B, C. Recommendations as to the amount of time given to particular courses will be found in several of the accompanying papers.²

It will now be illuminating to consider in detail a definite scheme involving practice teaching in a manner which has won high praise from prominent authorities in recent writings.³ I refer to the pioneer system at Brown University, in Providence, R. I., where it has been in operation for over 20 years. The fundamental principles of the

¹ Given on p. 538, Proc. Nat. Ed. Assoc., 1907.

² See the report in Proc. Nat. Ed. Assoc., 1907, pp. 521-668.

³ J. F. Brown, *The training of teachers for secondary schools in Germany and the United States*, New York 1911, p. 242f. W. S. Learned, *The Oberlehrer. A study of the social and professional evolution of the German schoolmaster*. (Harvard Studies in Education, Vol. 1). Cambridge, Mass., 1914, p. 131f.

scheme which has been advocated as worthy of wide acceptance have been summarized by Prof. Jacobs as follows:¹

1. Practice teaching should be open only to graduate students; that is, students who hold a bachelor's degree. This rule is inflexible and has never been broken. Brown University looks askance upon the custom which has developed of including practice teaching and extended professional preparation as a part of the four years' work for the first college degree and regards it as a lowering of standards. It holds that a fifth year of college work is necessary to the proper preparation of secondary-school teachers.

2. Practice teaching should be under actual schoolroom conditions. Hence, Brown University uses for its practice teaching the public and private secondary schools of the city of Providence and neighboring cities and towns.

3. Practice teaching should include the continuous instruction and control of a class for a long period. At Brown University the amount of practice teaching required varies from a minimum of 5 periods a week to 15 periods a week for one year. Student teachers teach very few classes, but they teach them continuously for a semester or a year. After many trials it has been found best to limit the student teachers to one or two subjects. To give a few sporadic lessons before a class is one thing; to teach a subject continuously is quite another matter.

4. Practice teaching must be under the continuous direct or indirect supervision of an experienced teacher who knows the school, the class, and the detailed progress of the subject taught. Hence, all supervising teachers at Brown University are selected from the experienced teachers of the schools. Each student teacher has one supervising teacher, and each supervising teacher one student teacher. The work is strictly individual.

5. Practice teaching must be closely correlated with the university work. At Brown University supervising teachers are selected by the university and paid a small remuneration. Each student teacher is visited once a week by the professor in charge of the practice teaching, and private conferences are held. He also meets once a week in a general conference all the student teachers. Plan-books for the past week are presented and discussed and later returned to the student teachers. Student teachers are at the same time pursuing other courses—(1) in education, a course in secondary education and a seminary in current educational problems, and (2) in departments allied with the subjects they are teaching.

Student teachers who teach more than five periods a week usually receive some remuneration for their work from the school. In the case of the city of Providence this is provided for by an agreement between the university and the city of Providence. In other cases it is arranged as the cases arise. Student teachers who have shown themselves efficient are assigned other classes under supervision. For this work they receive remuneration. The work then becomes closely similar to what is known as "part-time work" in vocational education. The work at Brown University, however, long antedates the vocational "part-time work."

Graduate students who are admitted to the practice teaching at Brown University usually have taken as undergraduates four semester courses in education. These courses are: History of education, principles of education, educational psychology, and general method. In the last course there is some systematic work in observation and some teaching of the class by members of the class. For this last purpose the class is divided into sections of about 10 each. The student, then, who is admitted to the graduate practice teaching, is not a mere novice, but one who has already had some experience.

6. The last principle is one which is fundamental and appears in all of the work. Practice teaching must not be an injury to the school or to the pupil, but rather a

¹ *School and Society*, Apr. 8, 1916, vol. 3, p. 534.

benefit. Hence the student teacher is called upon freely to assist the supervising teacher or the principal of the school in doing a limited amount of clerical work, work with individual pupils, or other work which can be assigned with profit to the student teacher and to the school alike. The student teacher becomes, to all intents and purposes, a part of the school staff, subject to regulations as other teachers and working as the other teachers are, in harmony with the general purposes and spirit of the school. The work of the student teacher is frequently superior to that of many of the regular teachers. He has more time for preparation and individual work with pupils and frequently more enthusiasm.

But, quite apart from these general requirements for all teachers, the department of mathematics at Brown University lays down a minimum course of study which it requires students who are prospective teachers of mathematics to take if they wish the backing of the department in starting on their careers. In outline the course is as follows: Plane trigonometry (3 semester hours), higher algebra (3), solid geometry (3), plane analytic geometry (4), differential and integral calculus (8), teachers' course in algebra (6), and teachers' course in geometry (6). The teachers' course in algebra constitutes an introduction to some of the concepts of modern analysis. Among the topics treated are: The number system with special reference to irrational numbers, limits, infinite series, the fundamental operations, and determinants.

In the teachers' course in elementary geometry the student is taught: Methods for attacking Euclidean problems; discussions of famous problems; the existence of transcendental numbers and the proofs of the transcendence of e and π ; means of rigorous discussion of the more delicate and difficult parts of the subject, such as the systems of axioms; something about (a) the history and literature of elementary geometry; (b) the most important French, German, and Italian texts; and (c) non-Euclidean geometry.

The students are also urged to take a year each (three hours a week) in physics and chemistry, in addition to the courses in education and psychology referred to above. For the A. B. degree they are required to take, also, 12 hours in rhetoric, composition, and English literature, 6 hours in European history; 6 hours in economics and either social or political science. Other courses are elective.

Students preparing to teach in secondary schools constantly elect further mathematical work, such as (1) the two-hour course in differential equations; (2) the six-hour course in theory of functions of a real variable (text, first part of Goursat-Hedrick's *Mathematical Analysis*); (3) the six-hour course in functions of a complex variable (text, Pierpont's or Burkhardt-Bischoff's work); (4) six-hour course in differential geometry (text, latter part of Goursat-Hedrick's work and Gauss's memoir); (5) six-hour course in projective geometry; (6) three-hour course in solid analytic geometry; (7) three-hour course

in tangential coordinates (French text by Papelier); (8) three-hour course in algebra (German text by Weber, Band 1).

In the United States, as in Germany, the general courses in colleges and universities have exerted more potent influences in molding teachers of secondary schools than any courses in professional training. All the better secondary schools now require that newly appointed teachers shall be college graduates. Moreover, the ideals embodied in the report of the committee of seventeen are rapidly becoming generally accepted and have led to yet higher standards on the part of many schools of the best type.

From the Atlantic to the Pacific there is constant development of schools of education, and the greatest in the country are associated with, or integral parts of, universities. Perhaps the School of Pedagogy of New York University, Teachers College of Columbia University,¹ and the School of Education at the University of Chicago are the most prominent—the last two being the only schools of the type organized for research work in the teaching and history of mathematics leading to the degrees of A. M. and Ph. D.

Another great source of benefit and training to secondary-school teachers is the opportunity offered by the recent unparalleled development of summer schools. In 1913, in a group of 29 university summer schools, 282 courses of the character of professional courses in education were offered.² Such courses are especially valuable in inspiring those teachers who desire to continue their studies during vacations.

The enormous increase in the number of high schools during the past 20 years makes the problem of training teachers for them a serious one. While the State normal schools³ throughout the country deal effectively with the problem of preparing teachers for elementary schools, the training of high-school teachers is a small part of the work of a few of the schools. In no case can this training be considered adequate from the point of view of the standard set by the "Committee of Seventeen." But apart from normal schools, although in practice this standard is fully maintained in such a State as Rhode Island, for example, there and in every other State except California the *legal* requirements for certification of high-school teachers are not commensurate with that standard.⁴

¹ The Teachers College, Columbia University, was changed into a graduate school in 1914.

² Rep. of U. S. Commis. of Educ. for 1913. Washington, 1914, vol. 1, pp. 545-551.

³ Statements made in this connection are based upon Ch. XII in *Problems involved in standardizing State Normal Schools*, by C. H. Judd and S. C. Parker. (Bu. of Educ., Bul., 1916, No. 12.) Washington, 1916.

⁴ For full details on "The Certification of American Teachers," see pp. 207-235 of *The Training of Teachers for Secondary Schools in Germany and the United States*, by J. F. Brown. New York, 1911.

208 TEACHERS OF MATHEMATICS FOR SECONDARY SCHOOLS.

As to the "tangible rewards" which form an attraction to the high-school teacher we are able to give some statistics but recently accumulated and published.¹

The salaries for men and women in city high schools range about as follows:

1. In cities having more than 250,000 inhabitants.

	Minimum	Maximum	Average
Principal.....			
Vice principal.....	\$1,700	\$5,000	\$3,565
Head of department.....	1,000	2,850	2,164
Teacher.....	1,340	4,000	1,951
	450	3,150	1,746

2. In cities having 5,000 and fewer than 10,000 inhabitants.

	Minimum	Maximum	Average
Principal.....			
Head of department.....	\$360	\$2,500	\$1,314
Teacher.....	618	1,800	947
	100	1,800	795

Cities with a population of more than 250,000.

San Francisco:		New York:	
Principal.....	\$3,000- \$3,300	Principal.....	\$4,150- \$5,000
Vice principal.....	2,100	Teacher.....	900- 3,150
Head.....	2,010	Boston:	
Teacher.....	1,500- 1,680	Principal.....	2,844- 3,924
Chicago:		Head.....	2,340- 3,204
Principal.....	3,000- 3,600	Teacher.....	1,360- 3,060
Vice principal.....	2,100- 2,600		
Teacher.....	1,000- 2,600		

Cities with a population less than 250,000.

Albany:		Oakland:	
Principal.....	\$1,000	Principal.....	\$3,000- \$3,300
Head.....	\$1,900- 2,500	Vice principal.....	2,000- 2,300
Teacher.....	900- 1,900	Teacher.....	1,380- 2,100
Providence:			
Principal.....	3,000		
Vice principal.....	2,500		
Teacher.....	550- 2,200		

Cities with a population of less than 100,000 but greater than 50,000.

Springfield:		Troy:	
Principal.....	\$2,800	Principal.....	\$2,500
Head.....	1,300	Vice principal.....	1,600
Teacher.....	\$800- 1,400	Head.....	1,600
		Teacher.....	1,500

Cities with a population of less than 50,000 but greater than 25,000.

San Diego:		Salem:	
Principal.....	\$2,700	Principal.....	\$2,700
Vice principal.....	2,200	Teacher.....	\$400- 1,600
Teacher.....	\$1,416- 2,001		

¹ *The Tangible Rewards of Teaching. A detailed Statement of Salaries paid to the several Classes of Teachers and School Officers.* (Bu. of Educ., Bul., 1911, No. 16.) Washington, 1914.

In the 15 States of the North Central Association already referred to, G. S. Counts gives the following table of median salaries for cities of different sizes:¹

Median salaries in 15 cities.

	Under 2,500.	2,501- 5,000.	5,001- 7,500.	7,501- 10,000.	10,001- 15,000.	15,001- 30,000.	50,001 and over.	Total.
Median salary for superintendents.....	\$1,628	\$1,750	\$1,950	\$2,000	\$2,290	\$2,700	\$2,700	\$1,821
Median salary for principals.....	1,058	1,140	1,292	1,445	1,587	2,005	3,014	1,358
Median salary for teachers.....	723	965	793	861	906	970	1,381	919

The first system of teachers' pensions to be established in the United States appears to be that of Chicago, which was inaugurated in 1893. By the year 1900 nine other systems had been founded. By 1910 there were 25 more. Nearly half (31) of all of the systems, however, have come into existence since 1910. * * * There are now State-wide pension systems for teachers in 22 States. There are local systems in 10 more. Thus more than half of the States are represented in the movement.²

The 65 pension systems are generally administered by special boards, in which the teachers constitute a majority. Provision is as a rule made for retirement on the basis of service and disability, but usually only for teachers entering the service after the establishment of the system. Funds are in most cases provided by teachers' contributions and by public appropriation in approximately equal amounts, but the funds arranged for are frequently insufficient to pay the pensions that have been promised.

In all the systems retirement is on the basis of from 20 to 40 years' service, most frequently 30 years. Teachers contribute to the funds in four-fifths of the systems, most frequently 1 or 1½ or 2 per cent of their salaries. The representative salary (the median, averages ranging from \$307 to \$1,197) of the teachers in 61 systems reporting is \$665 a year. The representative pension in these systems (60 reporting) is \$500 a year (the median of averages ranging from \$181 to \$1,050).

For full information regarding systems of teachers' pensions the reader should consult the seventh, eighth, ninth, tenth, eleventh, and twelfth reports of the Carnegie Foundation, and the bulletin by Ryan and King.³

¹A Study of the Colleges and High Schools in the North Central Association (Bu. of Educ., Bul., 1915, No. 6), Washington, 1915, p. 67.

²This paragraph and the facts which follow are taken from the admirable "Summary of Teachers' Pensions Systems" in the *Truth Annual Report* of the Carnegie Foundation for the Advancement of Teaching, New York, Oct., 1915, pp. 109-162. In organizing the pension systems no distinction has been made between elementary and secondary teachers.

³W. C. Ryan and R. King, *State Pension Systems for Public-School Teachers*. Prepared for the committee on teachers' salaries, pensions, and tenure, of the National Education Association. (Bu. of Educ., Bul., 1916, No. 14). Washington, 1916. 46 pp.

Bibliography, pp. 23-26. Typical pension and retirement laws, pp. 27-39.

No consideration of the various forces which influence and tend to raise the standards for the training of teachers in the United States would be complete without reference to the exchange of teachers between Prussia and the United States. In response to a request from the Prussian minister of education the trustees of the Carnegie Foundation for the Advancement of Teaching voted in 1907 to authorize the president of the foundation to act as the agency in America for an exchange of teachers of English between the United States and Prussia. Since that time about 40 of our college and high-school teachers drawn from more than a score of States have been cordially received in Prussian secondary schools, and a like number of Prussian Oberlehrer sent to us were located in college, university, academy, and high school. Several of our teachers sent over were teachers of mathematics, one of them being a college professor.

The advantages, direct and indirect, accruing from the exchange have been very great, and more general acquaintance with the efficiency and worth of Germany's system of secondary education will surely be a source of weighty influence for progressive reform in this country.

BIBLIOGRAPHY.

In addition to references already given in this chapter, the titles of the following publications which have a bearing on the subjects under discussion may be mentioned: International Commission on the Teaching of Mathematics. American Reports, published by the Bureau of Education, Washington:

1. *Report of the American Commissioners.* (Bulletin, 1912, No. 14), 1912.
 2. *Training of Teachers of Elementary and Secondary Mathematics.* (Bulletin, 1911, No. 12), 1911.
 3. *Mathematics in the Public and Private Secondary Schools of the United States.* (Bulletin, 1911, No. 16), 1911.
 4. *Mathematics in the Technical Secondary Schools in the United States.* (Bulletin, 1912, No. 4), 1912.
 5. *Undergraduate Work in Mathematics in Colleges of Liberal Arts and Universities.* (Bulletin, 1911, No. 7), 1911.
 6. *Graduate Work in Mathematics in Universities and in Other Institutions of Like Grade in the United States.* (Bulletin, 1911, No. 8), 1911.
 7. *Influences Tending to Improve the Work of the Teacher of Mathematics.* (Bulletin, 1912, No. 13), 1912.
- A Comparative Study of the Salaries of Teachers and School Officers.* (Bur. of Educ., Bul., 1915, No. 31), Washington, 1915.
- R. C. BROOKS, *National Education Association: Report of the Committee on Teachers' Salaries and Cost of Living.* Ann Arbor, 1913: 20+328 pp.

¹ For details concerning the exchange of teachers between Prussia and the United States, the following publications of the foundation should be consulted: (1) *A Plan for an Exchange of Teachers between Prussia and the United States*, 1908; (2) Annual Reports, 3 to 8, inclusive.

- D. E. SMITH, "The Teaching of Mathematics in the Secondary Schools of the United States," *Atti del IV congresso internazionale dei matematici*, Roma, Loescher, 1909, tomo 3, pp. 465-477. (French translation by J. P. Dumur in *L'Enseignement mathématique*, 1908, tome 10, pp. 269-284.)
- A. W. STAMPER, "The Training of Teachers of Mathematics in the Secondary Schools of the United States," *School Science and Mathematics*, April 1910, vol. 10, pp. 283-289.
- G. A. MILLER, "The Preparation of Mathematics Teachers in the United States of America," *L'Enseignement mathématique*, 1915, tome 17, pp. 325-335.
- C. W. ELIOT and E. NELSON, *Needed Changes in Secondary Education*. Two papers presented at the Pan American Scientific Congress at Washington, D. C., December 27, 1915, to January, 1916. (Bu. of Educ. Bul., 1916, No. 10.) Washington, 1916.

XIX. SUMMARY AND COMPARATIVE REMARKS.

A summary of some of the principal facts in connection with the training of teachers for secondary schools as well as comment, mainly of a comparative nature, is given in the following paragraphs. All statements should be considered in the light of the fuller discussion in earlier pages.

I. *Australia*.—The standards for the training of secondary teachers in Victoria and New South Wales are those to which all States of the Commonwealth are approaching. In the Teachers' College and University at Melbourne, Victoria, the usual course of training lasts four years—three years to obtain the B. A. or B. Sc. degree and one year of special professional training. The professional training includes (1) attendance at lectures in the university on theory of education with special reference to the method of teaching the various subjects; and (2) 120 hours of teaching under supervision in primary and secondary schools. In New South Wales similar requirements of at least four years of training after leaving the high school are maintained. Professional work is taken up after graduation from the university. This consists in part (1) of study in philosophy, education, principles of teaching, and school hygiene; (2) of continuous practice teaching (8–10 hours a week). In neither Victoria nor New South Wales is it necessary that the teacher shall have had special courses in mathematics in the university. On the other hand the better schools prefer graduates with "honors" in mathematics.

II. *Austria*.—To a certain extent the provisions for the training of secondary school teachers and the conditions under which they work are similar to those prevailing in Germany:

The training required for candidates as teachers in the Austrian Gymnasium and Realschule is practically the same. There must be not less than three and one-half years of scientific and other preparation (not always good) at a university. After five semesters the preliminary examination in philosophy and pedagogy may be taken. When this is passed application may be made not earlier than the end of the seventh semester for the Lehramtsprüfung, with its Hausarbeiten and oral examinations. After this, in theory only one year of practical training at a Mittelschulseminar is required, while a second year is permitted. But in practice very few candidates have had

more than a minor part of the training of even a single Probejahr before taking up actual teaching.

The written and oral examinations for the Lehramtsprüfung may include mathematics as (1) a major or (2) a minor. In the former case (1) the candidate must be familiar with "general arithmetic," the foundations of higher algebra and theory of numbers and their significance for elementary mathematics, elementary geometry, synthetic and analytic geometry, differential and integral calculus and its applications to geometry, the elements of the calculus of variations and foundations of the modern theory of functions, and the principal results of investigations concerning the foundations of mathematics. (2) When mathematics is a minor it is demanded that the candidate shall have knowledge of elementary arithmetic, insight into the structure of the field of real numbers and into operations with them; knowledge of elementary geometry to the extent of what is taught in secondary schools, and exercises in space perception; accuracy and speed in the solution of simple examples applying the idea of a function and the elements of differential and integral calculus to functions coming up in secondary school work.

The candidate is given three months to prepare the Hausarbeit on some subject not discussed in the lectures at the university. With mathematics as major recent Hausarbeiten have dealt with such topics as: "The theory of Fourier series"; "Theta functions and their applications in theory of surfaces of the fourth order"; "Method of derivation of large prime numbers"; and "Algebraic treatment of the 27 lines on a cubic surface." With mathematics as a minor: "Weierstrass's theory of irrational numbers," "Properties of the nine-point circle," and "The theorems of Fermat and Wilson."

The Probejahr includes observation of teaching methods, practice teaching under directions, the preparation of reports and weekly conferences with the staff of the Seminar on such matters as teaching, school discipline, pedagogy, school hygiene, and new publications of interest to teachers.

It is deplored that teachers of mathematics in the Untergymnasien and Unterrealschulen may have had only the scientific training required when mathematics has been taken as a minor.

III. *Belgium.*—The diplôme de sortie of an athénée royal, or of certain of the collèges entitles a student to enter one of the universities at which all teachers of the athénées are prepared. The inspectors of studies and professors in an athénée and the rector (or head) of a lower middle school must have secured the doctor's degree, and the masters (surveillants) in an athénée must have the university diploma of "candidate." (The four universities include two State establishments, the free university at Brussels, and the Roman Catholic University at Louvain.)

During the first two years the student prepares for examinations leading to the certificate as "Candidate in physical sciences and mathematics."¹ The examinations of the first year are on: Analytic geometry, plane and solid; descriptive geometry; higher algebra and the elements of the theory of determinants; differential and integral calculus (first part); analytic statics; experimental and laboratory physics. For the second year: Logic; psychology; moral philosophy; projective geometry; integral calculus (second part); elements of the calculus of variations and calculus of differences; pure kinematics; crystallography; and laboratory exercises.

The first examinations for the doctorate occur at the end of the third year. They are on: Higher analysis; dynamics; general mathematical physics; spherical astronomy and elements of mathematical astronomy; elements of the calculus of probabilities with the theory of least squares.

The subjects of the second set of examinations for the doctorate (in the fourth year) include: Mathematical methodology; elements of the history of the physical sciences and mathematics; and one of the five following groups chosen at pleasure by the candidate: (a) Higher analysis; (b) higher geometry; (c) analytic mechanics and celestial mechanics; (d) mathematical astronomy and geodesy; and (e) experimental physics and mathematical physics. The examination in the "group" chosen is somewhat searching, as the thesis has to do with the development of some part of it. The thesis must be publicly defended. Prospective teachers must also give two public "lessons," the subjects for which are designated in advance by the jury and chosen from the program of the athénées.

The young doctor is qualified to teach in an athénée at once without any professional training. In general, it is, however, only after several years in some such minor position as surveillant that he may reach the status of a professor in an athénée. In the large cities, the salaries, corresponding pensions, and social position connected with such a chair are attractive.

IV. *Denmark*.—The university course for the scientific training of a teacher for a secondary school usually ends in about six years, with the Skoleembedsexamen. This examination consists of two parts, covering the major and two minors; if the major is mathematics, the minors are astronomy with applied mathematics, and chemistry with physics. But in addition to these at the end of the first year the candidate must pass the "Filosofikum," an examination in logic, psychology, and the elements of the history of philosophy. This entitles him to the degree *candidatus philosophiae*. Having passed the Skoleembedsexamen he is *candidatus magisterii*. After

¹ This certificate very nearly corresponds to the bachelor's degree in this country.

two years of professional training (a year in the State Pedagogic Seminary and a year as assistant, or regular teacher, in a State or private secondary school), or its equivalent, the candidate is finally examined as to eligibility to teach. In view of the eight years of such training, the salaries are pitifully small.

In the first part of the Skoleembedsexamen the mathematical subjects include: Plane and solid analytic geometry, theory of infinite series, differential equations with a single independent variable, total and partial linear differential equations in three variables, differential geometry, statics, kinematics, hydrostatics, and advanced portions of gymnasial mathematics from a higher point of view. The second part of the examination with mathematics as a major includes questions on such subjects as function theory, elementary number theory, methods of descriptive geometry, synthetic and analytic projective geometry, and mathematical history.

V. *England*.—The amount of mathematics taught in English secondary schools varies from such a program as the elements of arithmetic, algebra, and geometry required for a London University matriculation examination, to programs which include trigonometry, synthetic and analytic geometry, theory of equations, differential and integral calculus; statics and hydrostatics—indeed all subjects necessary to prepare for an entrance scholarship examination in mathematics at Cambridge University, for example. In the mathematical training of the teacher there is corresponding variation. In practically all cases it is necessary for a candidate for a position to be a college graduate. But on the one hand he may have received a "pass" degree, in three years, without any college work in mathematics; and on the other hand he may have spent four years in obtaining a degree with first-class honors in mathematics. As to the extent of the professional training, this does not exceed a year in the training college of what we call the graduate school of a university, but even this is not yet always insisted upon. Athletic abilities are a valuable asset for a man seeking a position. The salaries are often very low.

That first-class scientific and professional training is not generally demanded of candidates for positions as teachers of mathematics in secondary schools indicates a marked similarity between the standards in England and the United States.

VI. *Finland*.—The steps required in the preparation of the secondary school teacher of mathematics after leaving the lyceum are: (1) To pass the examinations for a "candidate of philosophy," or for the "certificate of aptitude in teaching" after four or five years of preparation in the university; then (2) to spend a year of professional training in one of the normal lycées; and (3) to pass an examination

in theoretical and practical pedagogy. In each of the examinations, grades *approbatur*, *approbatur cum laude*, and *laudatur* are assigned.

A professor in a secondary school must have the degree of "candidate of philosophy" with highest grade in the major subjects required by the post. For the assistant professor, the same degree, or "certificate of aptitude," with at least the second highest grade in the major subjects required by the position, is necessary.

The "candidate of philosophy" has had courses in spherical trigonometry, analytic geometry, differential and integral calculus, differential equations, theory of numbers, theory of functions of a complex variable, and occasional courses in such subjects as minimal surfaces, theory of groups, and elliptic functions.

VII. *France*.—The highest type of secondary school in France is the *lycée*. The following remarks are with reference to preparation of teachers for this institution. The tendency of recent legislation is to demand that all teachers shall have passed the examination of the *agrégation*. The requirements of this examination in respect to knowledge of mathematics and dextrous handling of materials are unparalleled by any other country. Professional training, in comparison with the amount required by Denmark or Prussia, is comparatively neglected, and this has been judged by some as a weakness in the French preparation. It should be borne in mind, however, that it is difficult for an intelligent Frenchman (and the *agrégés* are the élite of the country's intellectuals) to be otherwise than interesting, effective, and elegant in his exposition of any subject.

The bachelor who wishes adequately to prepare himself as a mathematical teacher in a *lycée* usually undergoes the strenuous training of the *classes de mathématiques spéciales* in mathematics, physics, chemistry, descriptive geometry, drawing, German, and French. This preparation, covering at least two years, equips him for the university, where the three-year course continues to be of a highly specialized character. The best students pass the examination for the *licence* at the end of the first year, other tests at the end of the second, and present themselves for the terrific *concours* of the *agrégation* at the end of the third year. Pedagogics and practice teaching form a very meager part of the training of the teacher.

The annual number of *agrégés* has never exceeded 14. For the years 1885-1909 there were only 300, of whom 178 were trained at the famous *École Normale*. In 1909, of the 300 *agrégés* 51 had also become *docteurs ès science mathématiques*. Such a doctorate ranks far higher than any German doctorate, the acquisition of which has been made part of the preparation of many *Oberlehrer*.

As to the mathematical attainments of the *agrégé*, they are equaled by those of only a very small percentage of professors of mathematics in American colleges.

VIII. *Germany*.—Among the nations of the world, Denmark, France, Sweden, and Germany certainly maintain the highest standards in the selection of teachers for their better secondary schools. And no small part of the remarkable training which these teachers get is from master minds and master teachers in the schools themselves. Germany insists much less than France on brilliancy and breadth of mathematical attainment, but, like Denmark and Sweden, decidedly more on theoretical and practical professional training.

The candidate for teaching in a secondary school in Prussia is examined by a commission composed partly of professors and partly of State officials. Before the commission he must place—(1) his Maturitätszeugnis from a Gymnasium, Realschule, or Oberrealschule; (2) documents to show that he has spent at least three years in study at one or more German universities (in point of fact candidates usually spend four or five. For three of the six semesters required the mathematician may have attended a technical high school); (3) a statement of the subjects (for example, pure mathematics and physics and applied mathematics) and classes (Sexta to Unterschunda, inclusive, or Obersekunda to Oberprima, inclusive) for which the candidate hopes to prove his qualifications; and (4) a statement of the field in which he wishes to receive subjects for required essays. Not more than three months, ordinarily six weeks, are allowed to prepare the essays. For one of them some publication of note, such as the candidate's doctor's dissertation, may sometimes be substituted.

The written and oral examinations are in philosophy, pedagogy, religion, German literature, and the candidate's special subjects. For first-grade rank in pure mathematics the candidate must show that he is thoroughly acquainted with the principles of higher geometry, arithmetic, algebra, higher analysis, and analytical mechanics and can solve fairly difficult problems without assistance.

Having passed these examinations *pro facultate docendi*, the *Schulamtskandidat* must still undergo two years of severe professional training, (i) the *Seminarjahr* and (ii) the *Probejahr*.

When favorable report has been made on the work of the *Probejahr* the candidate's name is put on a list of teachers eligible to appointment in the higher schools of the province in which the examination was held.

Several German States follow Prussia's method of training teachers for secondary schools. Others differ in various ways: Bavaria, Saxony, Wurtemberg, Baden, and Hesse each require four years at a university; in Wurtemberg the *Seminarjahr* is unnecessary; and Bavaria, Baden, and Alsace-Lorraine demand no *Probejahr*. There are also differences in details, even when general requirements are the same. But throughout the Empire the attractions of the teach-

er's position are sufficiently strong to draw some of the best mathematical talent of the country.

IX. Hungary.—The method of training secondary school teachers in Hungary is more adequate than that in Austria. Recruits for such training are drawn from among the graduates of the Gymnasias and Real Schools. The elaborate and searching training is carried on during five years at a university, or in the Polytechnikum, and at a training college. In starting out, the candidate must elect two subjects which he wishes to teach; the selection of a third is recommended. Mathematics, physics, and descriptive geometry are not uncommonly chosen together.

The examinations in the course are threefold: (1) The general examination, at the end of the fourth semester; (2) the examination on special subjects, at the end of the eighth semester; and (3) the pedagogic examination at the end of the next year, which the candidate usually employs in professional practice.

In the general examination, the oral part contains questions in plane and spherical trigonometry, analytic geometry, analysis, descriptive geometry, physics, the Hungarian language, and one modern language (German, French, English, or Italian) with the literature of the same.

Five months before the examination in special subjects the candidate is given a theme in each of his subjects, upon which he has to write a dissertation. These dissertations must indicate originality, familiarity with literature, and broad knowledge of the themes. In the oral examinations on special subjects the candidate must show decided proficiency. The oral examination in mathematics is on—(1) The mathematics of secondary schools; (2) certain parts of geometry, algebra, and analysis common for all candidates; and (3) the following five subjects, one of which must be known thoroughly, the others, in a general way: (a) Modern geometry and the theory of algebraic forms; (b) number theory and higher algebra; (c) general theory of curves and surfaces; (d) general theory of analytic and elliptic functions; and (e) advanced portions of integral calculus.

The examination in pedagogy consists of a dissertation and an oral examination in history of philosophy, logic, psychology, pedagogy, history of pedagogy, and special methods for teaching mathematics. While this examination occurs at the end of the fifth year, most of the courses of study have been taken during the first four years at the university. The fifth year is largely devoted to training and practice teaching in a Seminar, such as Kármán's, or the State training college. Teachers do not attain to full service until after three years of probationary activity.

X. Italy.—A candidate for a position as teacher of mathematics in a secondary school must possess the degree of Dottore in matematiche pure from one of the universities. After the candidate has graduated

from a liceo or an istituto four, sometimes five, years are devoted to securing this degree. With an exception to be referred to presently, the courses of study are wholly in pure and applied mathematics and allied fields. An idea of the breadth of some of these courses may be obtained by recalling that the following works consist mainly of university lectures: F. Severi's *Lezione di geometria algebrica* (1908); E. Ciani's *Lezione di geometria proiettiva ed analitica* (1912); and G. Castelnuovo's *Lezione di geometria analitica* (1913).

The first two years of study lead to the certificate *licenza dell'università*. During the last two (or three) years the student (1) carries five advanced courses (theoretic geodesy being put on a plane with higher analysis), (2) writes a dissertation, and, if he wishes to occupy an advantageous position with respect to appointments, (3) attends lectures on methods of teaching leading to the diploma di *magistero* (these lectures are given in the *scuola di magistero* of the university). The final examinations of the university lead to the laureate, of which there are many grades.

The student who has read nothing but what was suggested in lectures is not likely to pass out with "full wishes and praise," as the highest grade is called, nor will the most favorable opportunities probably be waiting for him in future; still he has the highest degree which the university confers. The dissertations, as is natural, present great variation in value and originality. The candidate is required to publish a full résumé of his results, but not the thesis itself; whereby a long-suffering mathematical public is spared much, for the feebler dissertations never see the light. The better ones find ready acceptance by the mathematical journals or are laid before learned societies.

The first appointment of a teacher in a secondary school is for three years, after which the appointment may be made permanent if the inspector's report is favorable; if not, the appointment is extended for another year, when the teacher is definitely appointed or dismissed.

In the preparation of the teacher the lack of professional training is regarded as serious by many; his scholastic equipment in his special subject is probably equal to that of the *Oberlehrer*.

XI. *Japan*.—The usual method of preparation as a teacher in a middle school is (1) to take the course of four years in one of the higher normal schools whose graduates are granted teachers' licenses; or (2) to attend some similar institution and pass the four grades of examinations set by the department of education for the license. A *gakushi* of the faculties of science in the universities of Tokyo and Kyoto, who has no professional training, is entitled to receive a license without examination. Similar privileges are accorded to graduates of foreign universities.

The courses in the higher normal school which prepare teachers of mathematics and physics include professional training, teachers' courses in algebra and geometry, higher algebra, plane and solid analytic geometry, and differential and integral calculus. The four

grades of examination in mathematics set by the department of education are in (1) arithmetic, algebra, and geometry; (2) trigonometry; (3) analytic geometry; and (4) differential and integral calculus.

Teachers in the eight higher middle schools which are closely allied to the universities in their organization are nearly all *gakushi* in mathematics. At the university the *gakushi* have passed examinations in calculus and differential equations, solid analytic geometry, projective geometry, astronomy and least squares, general physics, general dynamics, and theory of numbers.

XII. *The Netherlands*.—Gymnasia prepare students to enter directly the various departments of the universities, but graduates of higher burgher schools must pass university examinations in Latin and Greek. The first examination after entering the university is the examination for candidature, at the end of about three years; the second examination, that for the doctorate, occurs at the end of about six years of study. If, in addition to passing the latter examination, the candidate prepares for the faculty an appropriate thesis, he is entitled to the degree of doctor. It is from those who have passed the examination for the doctorate that teachers for gymnasia and higher burgher schools are almost wholly recruited. They have had absolutely no professional training.

A candidate for appointment as teacher in a three-year or five-year higher burgher school may also prepare himself for the position by passing a series of exceedingly elaborate examinations, the preparation for which is made entirely outside of the universities. For the five-year school, examinations on the following subjects, among others, must be passed: Plane and spherical trigonometry, descriptive geometry, analytic geometry, calculus, theoretical and applied mechanics, physics, chemistry, cosmography, geology, and mineralogy. During the past 50 years exceedingly few have qualified themselves as teachers in this way.

Some of the subjects leading to the examination for the doctorate are higher algebra, calculus, plane and solid analytic geometry, descriptive geometry, differential equations, theory of functions, differential geometry, calculus of variations, and mathematical physics.

XIII. *Roumania*.—Graduating from the *lycée* when about 19 years of age, the student prepares himself to be a secondary teacher by study for (1) the diploma, *licence ès sciences mathématiques* of a university; and for (2) the certificate of a pedagogic seminar. The examinations for the license are in higher algebra, analytic geometry, descriptive geometry, differential and integral calculus, theory of functions, mechanics, and astronomy. It takes at least three years to prepare for these examinations. Professional training is obtained at a pedagogic seminar.

A candidate for the position of secondary teacher should have passed the examination of capacity. To be admitted to this examination the student must possess the diploma and certificate mentioned above.

The organization of the secondary schools and the training of teachers for them is very similar to that in France. In Roumania perhaps more emphasis is put on professional training.

XIV. *Russia*.—Graduates of the gymnasium and of the seven-year "real school" are entitled to register in faculties of the universities. The course for teachers in the gymnasias, as well as for most teachers in the "real schools," occupies four years and leads to a diploma of the first or second grade.¹ The courses for the mathematics-analytic mechanics group at the University of Petrograd include spherical trigonometry, analytic geometry, calculus, partial differential equations, theory of numbers, theoretical mechanics, hydrostatics and hydrodynamics, and astronomy and celestial mechanics.

It was not till recently that any provision was made to give professional training to candidates for teaching positions in State secondary schools. This beginning was made in 1909, when a year course including lectures in logic, psychology, pedagogy and its history, as well as practice teaching, was organized for the arrondissement of Petrograd.

XV. *Spain*.—The bachiller en artes of an instituto may prepare himself to teach mathematics in an instituto at any one of the three universities: Barcelona, Madrid, or Zaragoza. The courses in the faculties of science lead to the licenciatura certificate in about four years; the apt scientific investigator may procure the doctorate in another year. Every candidate for a professorship in the instituto must have a licenciatura certificate; the best positions are obtained by the doctors in mathematical sciences.

The mathematical courses leading to examinations for the licenciatura are as follows: Mathematical analysis, metric geometry, analytic geometry, infinitesimal calculus, cosmography and physical geography, projective geometry, descriptive geometry, rational mechanics, and spherical astronomy and geodesy. (This list of subjects strongly reminds one of those required for the licenza in Italy.) The examinations take on different forms and imply a certain amount of professional training.

To prepare for the doctorate the student must (1) attend courses in higher analysis, higher geometry, astronomy of the planetary system, and mathematical physics; (2) present a memoir on a subject selected by the candidate; and (3) successfully defend the memoir before a tribunal.

¹ This is not a degree; the degrees of magister and doctor require longer preparation.

XVI. *Sweden*.—The very thorough course of the gymnasium concludes with the studentexamen, the sole means by which a student may enter one of the universities; mathematics is there taught in the faculty of philosophy. The courses for secondary-school teachers are very elaborate and lead to (1) the filosofisk ämbetsexamen, at the end of about four years of study; and (2) the filosofie licentiatexamen, after about eight years of preparation. Candidates for positions as teachers in the Realskolor are required to pass the first of these examinations; those seeking higher posts in the gymnasia, the second; but in this latter course the candidate must also defend a thesis for the degree of doctor.

In each case the candidate has to spend a probationary year in connection with some one of five special schools which organize excellent professional training. Moreover, before regular appointment as professor or assistant professor in the Government secondary schools, the candidate must have spent at least two years in successfully performing the duties attached to the position. Sweden probably leads the world in the extent of scientific preparation required for her professors (Lektorer) in the gymnasia.

The subjects of the ämbetsexamen include projective geometry, theory of equations, calculus, analytic geometry, theory of numbers, theory of probabilities, ordinary differential equations with constant coefficients; and generally for the highest predicate elements of the theory of (a) differential equations, (b) analytic functions, and (c) differential geometry. In the preparation for the licentiatexamen the candidate is introduced into various important fields of modern mathematics, including ordinary and partial differential equations of mathematical physics.

XVII. *Switzerland*.—Considering the country as a whole, there are no fixed examinations for secondary-school teachers; nor are there any collèges for training them, although there is a department of pedagogy in the University of Geneva. For most of the teaching positions in the secondary schools the majority of the Cantons demand that the candidate shall have completed a university course, or its equivalent, of four or five years. Typical of such Cantons are Fribourg and Geneva. Candidates with two years' training in a university, or its equivalent, can usually only be appointed as teachers in the lower classes of the better schools; an exception requiring the qualifying word is Canton Vaud. There is a general tendency to favor candidates who have had technical training such as a student derives from an assistantship in the Polytechnic School.

On the whole, high scientific standards of training are demanded on the part of secondary teachers, but their professional training is almost entirely neglected.

Subjects included in examination requirements for secondary-school teachers in courses of the Polytechnic School and the University of Bern are of the following types: Descriptive geometry, calculus, analytic geometry of three dimensions, projective geometry, theory of functions, Fourier's series and integrals, differential geometry, and theory of transformation groups.

XVIII. *United States*.—Practically all of the best secondary schools require, as a general thing, that a candidate for a teaching position shall be a college graduate. It is usually unnecessary to have had any courses in the theory of education, or any special work in the subjects for which the candidate applies to teach, or any professional training. Other things being equal, candidates equipped in these respects are, of course, more readily employed. Some schools demand such preparation, and colleges and training schools in the country have made provision for meeting such demands.

The standards throughout the United States for the training of secondary-school teachers appear to be very similar to those in England, although the professional training is now, possibly, more insisted upon in the latter country. As a general thing, salaries constitute perhaps the chief attraction to college-trained men to take up secondary-school work in the United States.

CONCLUSION.

In the preceding pages an attempt has been made to bring together from a very limited number of sources facts concerning the development of the teacher of mathematics in the better secondary schools of different countries. A presentation which would be at all adequate and complete in its different aspects would require, for each country a volume based upon a far wider range of sources of information. Here simply the descriptions of the few features which happen to be dealt with in available authorities are epitomized; there has been no opportunity for uniform treatment. It is therefore only in a very general way that definite statements may be made in comparing the methods of different countries.

Few will deny that if the secondary education in a country is of a high order, and is extended over a period of seven or eight years, it must materially contribute to the training of the pupil who, after further development in university or professional school, returns to the secondary school as teacher. It is therefore pertinent to inquire, What period is devoted to secondary education in different countries? To what extent is this based upon work of primary schools? When is primary instruction given in secondary schools? When does the pupil enter the university? Bearing in mind the types of schools which have been excluded from consideration in this report, and the fact that, in general, only the better secondary schools (under Govern-

ment or State control) leading directly or indirectly to a university have been considered, fairly definite answers to all of these questions may be found in the accompanying synoptic table.

The age there given is the normal lower limit, and in any one of the countries it is possible for a pupil or student of the given age to be at the stage of scholastic advancement indicated. It should not be inferred, however, that this rate of progress is necessarily characteristic of the country; for example, in Japan the pupil just graduating from the middle school is usually nearer 20 than 17 years of age; and in Germany, while a small percentage of students are ready for a university career at 17 years of age, more than one-half are at least 19, and more than one-third not less than 20 years of age. In the following discussion I shall, however, use the normal age given in the table.

We remark that in 15 of the 18 countries the normal age of entering the university is 18 or 19. These countries are: Austria, Belgium, Denmark, England, Finland, France, Germany, Hungary, Italy, Netherlands, Roumania, Russia, Sweden, Switzerland, and the United States. When the extent of secondary school preparation for the university is considered, we find wide variation of custom. Australia and the United States are at the foot of the list with only 4 years; but 7 countries (Austria, Finland, Hungary, Italy, Japan, Roumania, and Russia) devote 8 years, and 3 countries (France, Germany, Sweden) 9 years to secondary education.

Since the period of school education leading to a university in 14 of the countries is 12 or 13 years, the wide difference of views which these countries hold with respect to the portion of this time which should be assigned to secondary education proper is interesting. On the one hand, the United States holds that during 8 of the years the methods of elementary education should be employed; while, on the other hand, France and Germany consider that best results are obtained when even the 3 or 4 years allotted to primary instruction are given in connection with secondary schools. It is, then, not surprising to find that there is great difference between the scholastic equipment of students coming from these two types of school. The graduates of the *classe de mathématiques spéciales* or of the German *Gymnasium* are about on a par with the youth who has finished his junior year in one of the better American colleges. And in other countries also, like Denmark, Japan, and Sweden, the graduate of a higher secondary school has done more or less work whose equivalent is done in colleges in the United States. It is only in the light of such considerations that the full force of, say, Sweden's requirement of 10 or 11 years of preparation before a graduate of a *gymnasium* may return as a regularly appointed professor can be adequately appreciated.

Again, it should be borne in mind that even with courses of secondary schools covering the same number of years the content may be vastly different. Contrast the 9 years of the German gymnasium, where 19.8 per cent of the time is devoted to mathematics, sciences, and drawing, and 34.2 per cent to ancient languages, with the 9 years of the French lycée in the science-modern language section and classes de mathématique spéciales with about 36.8 per cent of its course in mathematics, science, and drawing, and no time spent on ancient languages. France offers a much more extensive mathematical course in her secondary schools than does any other country in the world.

From such different types of schools come the future teachers of mathematics. Let us next consider in a general way how these teachers were prepared after leaving the secondary schools. Broadly speaking, the training is derived from: I. Courses in a university or similar institution; II. Professional training.

I. All the countries require some university training on the part of candidates for appointment as secondary school teachers.¹ The maximum requirements are in Denmark and Netherlands, each 6 years, and in Sweden about 8 years. On the other hand, for minor positions in the athénées of Belgium and regular positions in Canton Vaud, Switzerland (where most Cantons require 4 or 5 years), only 2 or 2½ years of attendance at a university are compulsory. The complete record is as follows: Australia (Victoria and New South Wales), 3 years; Austria, 3½ to 4; Belgium, 4 to 5, 2 for minor positions in athénées; Denmark, 6; England, 3 to 4; Finland, 4 to 5; France, not less than 3, in addition to 2 years in classes de mathématiques spéciales; Germany, 3 to 4, but rarely 3 and often 5 are taken; Hungary, 4; Italy, 4 or 5; Japan, 3; The Netherlands, 6; Roumania, not less than 3; Russia, 4; Spain, 5, for lower positions, 4; Sweden, usually 8; Switzerland, 4 to 5 years for the most part, in one or two cases 2 to 3; United States, 3 to 4.

Let us consider a single example, to bring out more clearly the implications of these statements. Since the future mathematical teacher entering a German university is about on a scholastic par with the student who has finished the junior year at a college in the United States, we may state, roughly, that the German teacher has generally had at least three years more of scientific training than the American teacher in a secondary school has had.

II. In addition to attendance at universities, some countries require professional training. Australia (Victoria and New South Wales), England (generally), Finland, Roumania, and some States of Germany each require one year (it is only in theory that Aus-

¹ As far as this statement concerns Japan, reference is made to the higher middle schools.

tria requires a year); other States of Germany and Denmark require two years each; in addition to a year in a Seminar, Sweden requires two years of probation as teacher before regular appointment and in a similar way Hungary requires three; and in Italy after four years of trial a teacher may be dropped. In France the professional training may possibly be estimated at half a year. In seven countries no professional training is made compulsory. These countries are: Belgium, Japan (in higher middle schools), the Netherlands, Russia, Spain, Switzerland, and the United States.¹ The question of the far greater efficiency of the training in some countries than in others is not taken up here.

When the courses required for the candidates in different countries are considered, the unenviable conclusion is reached that Australia, England, and the United States are largely in a class by themselves. For in these countries mathematical teachers know practically nothing of their subjects, as they have had no special mathematical training in the universities. Perhaps England is less of an offender on account of the number of trained specialists necessary at schools preparing for Scholarship Examinations. All other countries require of their professors a more or less broad scientific training, and the minimum mathematical requirement is a knowledge of the differential and integral calculus. Most countries include also among their requirements differential equations, analytic geometry of three dimensions, descriptive geometry, projective geometry, mechanics, and physics. A doctor's degree is required of higher teachers in Belgium, Italy, Netherlands, Spain, and Sweden; and the standard for teachers in France and Germany is certainly not below that for the degree of doctor in those countries (indeed much of the training for teacher and doctor is identical).

In the American Report to the International Commission on the Teaching of Mathematics on *Training of Teachers of Elementary and Secondary Mathematics*, the following lines occur (p. 13):

Yet with us, where the public is now beginning to recognize that teaching is a profession, a feeling which will certainly increase as the years go by, the time will undoubtedly come when secondary teaching will be sufficiently attractive financially to enable us to demand from the prospective teacher some such preparation as the following: On the side of pure mathematics we may expect the calculus, differential equations, solid analytic geometry, projective geometry, theory of equations, theory of functions, theory of curves and surfaces, theory of numbers, and some group theory. On the applied side we should demand a strong course in mechanics, theoretical and practical astronomy, descriptive geometry, and some mathematical physics with a thorough course in experimental physics.

We have seen that this ideal for the scientific training of teachers is *now* a matter of course in a number of countries.

¹ The occasional requirements in Russia, Switzerland, and the United States are neglected. For most of the teachers in the middle schools of Japan a certain amount of training is demanded.

On the whole, the salary, pension, social position, and scholastic status of the secondary-school teacher in France and Germany seem to combine to give to the profession an attractiveness not to be found in other countries.

TABLE.

In this table it should be noted that: (1) The separation lines between the primary and secondary schools do not always indicate that regular primary instruction ends there, but only that portion of it preparatory to the secondary school in question; (2) five only, of the six years of the course in the realskola of Sweden, suffice as preparation for a gymnasium; and (3) some university courses extend beyond the limits of age in the table, e. g., in Belgium and in Japan.

Ages of pupils and classification of schools in different countries.

Ages.	Australia. (Victoria, N. S. Wales.)	Austria.	Belgium.	Denmark.
6-7	Primary school..... I	Primäre Schule..... I	École primaire..... I	Forskole..... I
7-8 II II II II
8-9 III III III III
9-10 IV IV IV IV
10-11 V V V V
11-12 VI VI VI VI
12-13 VII VII VII VII
13-14 VIII VIII VIII VIII
14-15 IX IX IX IX
15-16 X X X X
16-17 XI XI XI XI
17-18 XII XII XII XII
18-19 XIII XIII XIII XIII
19-20 XIV XIV XIV XIV
20-21 XV XV XV XV
21-22 XVI XVI XVI XVI

Ages.	England.	Finland.	France.	Germany.
6-7	Primary school..... I	Folkskola..... I	Lycée: Primary..... I	Vorschule..... I
7-8 II II II II
8-9 III III III III
9-10 IV IV IV IV
10-11	Great public schools: Preparatory..... I	Lycéum..... I I I
11-12	(Grammar school, institute, etc.) II II II II
12-13	Junior..... Form I III III III
13-14 Form II IV IV IV
14-15	Senior department..... Form III V V V
15-16 Form IV VI VI VI
16-17 Form V VII VII VII
17-18 Form VI VIII VIII VIII
18-19 Remove IX IX IX
19-20 Form VI X X X
20-21 Form VII XI XI XI
21-22	University..... I	University..... I	University..... I	University..... I

SUMMARY AND COMPARATIVE REMARKS.

Ages.	Hungary.	Italy.	Japan.	The Netherlands.
6-7	Volkschule..... I	Scuola elementare..... I	Elementary school..... I	Primary school..... I
7-8 II II II II
8-9 III III III III
9-10 IV IV IV IV
10-11	Gymnasium..... I	Ginnasio..... I	Middle school..... V	Higher burgher school..... VI
11-12	("Real school")..... II II II II
12-13 III III III III
13-14 IV IV IV IV
14-15 V V V V
15-16 VI VI VI VI
16-17 VII	Istituto tecnico..... III	Higher middle school..... I VII
17-18 VIII VIII VIII VIII
18-19 IX	University..... I	University..... I	University..... I
19-20 X X X X
20-21 XI XI XI XI
21-22 XII XII XII XII

Ages.	Roumania.	Russia.	Spain.	Sweden.
6-7	Primary school..... I	Primary school..... I	Colegio..... I	Elementary school..... I
7-8 II II II II
8-9 III III III III
9-10 IV IV IV IV
10-11	Lyce: First cycle..... I	"Real school" (prep.)..... I	Instituto..... I	Realskola..... I
11-12 II II II II
12-13 III III III III
13-14 IV IV IV IV
14-15 V V V V
15-16	Second cycle..... VI VI VI	Gymnasiums..... VI
16-17 VII VII VII VII
17-18 VIII VIII VIII VIII
18-19 IX	University..... I	University..... I	University..... I
19-20 X X X X
20-21 XI XI XI XI
21-22 XII XII XII XII

For footnotes see p. 240.

Ages.	Switzerland.	United States.
6-7	Primary education..... I	Elementary school: Primary..... I
8 II II
9 III III
10-11 IV IV
11-12	Gymnasium..... I	Grammar..... V
12-13 II VI
13-14 III VII
14-15 IV VIII
15-16 V	High school..... II
16-17 VI III
17-18 VII IV
18-19 VIII	University..... V
20-21 IX VI
21-22 X VII
	University..... I VIII
 II IX
 III X

¹ In Victoria most of the pupils entering the State high schools are 12 years of age; in New South Wales, 13.

² In England typical school progressions do not exist, and the outline given here is simply that in use at some large schools. Norwood and Hope group the forms differently (*Higher Education of Boys in England*, pp. 247-288) - I-V containing a lower course, V-VI a "specialist course," somewhat like the first and second cycles of the French lycées. It is generally believed that the proper time for a student to enter university is when he is 18 or 19 years of age.

³ According to the Report of the International Commission on the Teaching of Mathematics in Finland (pp. 5, 6, 21) there appears to be a gap between the primary and secondary courses. We are told that the normal age of a boy entering the lycée is 10, and that the average age of the boy entering upon the two-year course of the preparatory school (École primaire élémentaire or Ecole préparatoire) is 7.

⁴ In Wurtemberg the nine classes of the secondary school are numbered I (the first) to IX (the last).

⁵ In Russia there is no correlation between elementary and secondary curricula, and while there is nothing to prevent a clever boy whose parents can afford the necessary expense from passing from a primary school at the age of 10 or 11 into a gymnasium or "real school," it is unusual and often inadvisable. Hence many "real schools" introduce this preparatory year.

⁶ The regular course of the "real school" was formerly six years, but this seventh year is now added in most cases.

⁷ With the great variation of systems in the Cantons no very definite statement can be made with reference to the period covered by primary and secondary instruction. In the case of Gymnasien the period usually is 13 years, but some Cantons have 11-year and 14-year periods. In Realschulen the average is 12½ years, though there are also those with 10½ and 11½ year ranges. Primary instruction for boys begins at 6 years of age in most of the Cantons, and the usual range of duration is 8 to 9 years. The term "Realschule" is employed to designate many types. For example, Gymnasium Biel, in Bern; Real section of Kantonschule in Schaffhausen; Collège de Genèbre; Gimnasio e Liceo in Tessen; and Collège et Gymnase in Vaud.

More details in this connection may be found in K. Brandenberger's report, pp. 14-25.

APPENDICES.

APPENDIX A.

ENGLAND.

CAMBRIDGE LOCAL EXAMINATIONS, SENIOR STUDENTS. December, 1915.

GEOMETRY.

(Two hours.)

*The answers to questions marked A and B are to be arranged and sent up to the examiner in separate bundles.
N. B.—Attention is called to the alternative questions B ix, B x, B xi at the end of the paper.*

A.

A 1. In the triangles ABC , DEF . $\angle B = \angle E$, $\angle C = \angle F$, and $BC = EF$. Prove that the triangles are congruent.

Show that the diagonals of a parallelogram bisect each other.

A 2. In a triangle ABC , AD is drawn perpendicular to BC ; show that, when the angle C is acute,

$$AB^2 = AC^2 + BC^2 - 2BC \cdot DC.$$

Prove that the sum of the squares on the four sides of a parallelogram is equal to the sum of the squares on the diagonals.

A 3. From a point O outside a circle two straight lines OPQ , ORS are drawn, the first cutting the circle at P and Q , the second cutting the circle at R and S . Prove that $OP \cdot OQ = OR \cdot OS$.

In the triangle ABC , the angle A is a right angle. From a point D on BC a line DEF is drawn perpendicular to BC , meeting AC at E , and BA produced at F . Show that

$$DE^2 = BD \cdot DC - AE \cdot EC.$$

A 4. Inscribe a regular octagon in a circle of radius 2 inches. Produce alternate sides of the octagon, so as to form a square.

Measure the side of the square.

Show clearly all the construction lines in your figure.

B.

B 5. Show that if a straight line touch a circle, and from the point of contact a chord be drawn, the angles which this chord makes with the tangent are equal to the angles in the alternate segments.

Two circles intersect at A and B . At A , tangents to the circles are drawn, meeting the other circles at X and Y . Show that BA bisects the angle XY .

B 6. Two triangles ABC , DEF are similar, AB and DE being corresponding sides. Show that their areas are in the ratio $AB^2 : DE^2$.

Through each of two opposite corners of a rectangle perpendiculars are drawn to the diagonal which joins the other two corners. Show that if the three parts into which the diagonal is thus divided are equal, the squares on the sides of the rectangle are in the ratio 2 : 1.

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B 7. The tangents to a circle at P and Q meet in T , and C is the center of the circle. TC meets the circle at A and B , and meets PQ at X .

Show that $CX \cdot CT = CA^2$ and that $AX \cdot XB = TA \cdot TB$.

B 8. Two circles (not in the same plane) intersect in two points. Show how to obtain the center of the sphere on which both circles lie.

A, B, C, D, E, F are six distinct points in space, such that A, B, C, D lie on a circle, C, D, E, F lie on a circle, E, F, A, B lie on a circle, no two of the circles lying in the same plane. Show that the three circles lie on a sphere.

N. B.—One or more of the following questions B ix, B x; B xi may be taken instead of an equal number of the questions B 5, B 6, B 7, B 8, but lower marks will be assigned to them.

B ix. Show that the sum of the angles of a triangle is equal to two right angles.

ABC is an acute-angled triangle, and O is the center of the circle which passes through A, B, C . Show that the angles OBC and BAC are complementary.

B x. Illustrate and explain by means of a figure the geometrical theorem corresponding to the algebraical identity

$$(a + b)^2 = a^2 + 2ab + b^2.$$

$ABCD$ is a square; points E, F, G, H are taken on AB, BC, CD, DA such that $AE = BF = CG = DH = \frac{1}{4}$ of the side of the square. Show that the area of the square $EFGH$ is $\frac{3}{4}$ of the area of the square $ABCD$.

B xi. Draw two perpendicular lines AB, BC . Make $AB = 1\frac{1}{2}$ inches, $BC = 2\frac{1}{2}$ inches. Construct a circle to pass through C and to touch AB at A . Measure the radius of this circle.

Show clearly all the construction lines in your figure.

ALGEBRA.

(Two hours and a half.)

Squared paper and tables of logarithms, etc., can be obtained from the presiding examiner.

N. B.—Attention is called to the alternative questions A, B, C at the end of the paper.

1. Show that $\left\{ \frac{4x^2}{(x+1)^2} - 1 \right\} \left\{ \frac{4x^2}{(x-1)^2} - 1 \right\} = \frac{8x^2}{x^2-1} + 1$.

2. Find the factors of

(i) $a^2b - b^2 + abc - b^2c$,

(ii) $x^4 - a^2x^2 + 2abx - b^2$;

and show that if $a + b + c = 0$,

$$a^3 + b^3 + c^3 = 3abc.$$

3. Solve the equations:

(i) $\begin{cases} \frac{p}{x} + \frac{q}{y} = 1, \\ \frac{q}{x} + \frac{p}{y} = 1, \end{cases}$

(ii) $\begin{cases} (x-2)(y-3) = 3, \\ (x-3)(y-2) = 4. \end{cases}$

4. A room is such that its length is 10 feet more, and its height 13 feet less, than its breadth. If it had been 1 foot less each way, it would have contained 1,524 cubic feet less. Find its dimensions.

5. Find by logarithms the value of $1.058 \times (.0247)^{\frac{1}{3}}$

$$(0.892)^{\frac{1}{3}}$$

In what year must £1 have been put out at compound interest at 5 per cent per annum to amount to £100 at the end of 1915?

6. Show how to obtain the formula for the summation of n terms of a geometric progression.

If the first term is 9 and the ratio of the sum of eight terms to the sum of four terms is 97 : 81, find the series.

7. If A is the arithmetic, G the geometric, H the harmonic mean, between two numbers, show that $AH = G^2$ and that $A > G > H$.

$$\text{If } \left. \begin{array}{l} A - G = \frac{1}{4} \\ G - H = \frac{1}{4} \end{array} \right\} \text{ find the numbers.}$$

find the numbers.

8. Find the condition that $ax^2 + 2bx + c$ has always the same sign for all real values of x .

$$\text{Show that, if } \begin{array}{l} x^2 + px + q = 0 \\ \text{and } x^2 + qx^2 + p = 0 \end{array}$$

have a common root, either

$$p + q = -1 \text{ or } (p - q)^2 = -q^2(p^2 - pq + q^2).$$

9. Find the number of different pairs (irrespective of order) that can be formed with $2n$ things.

If, of 14 numbered cards, 5 are red, 6 white, and 3 blue, find the number of groups of three, one of each color, that may be formed.

How many more groups can be formed if any two, but only two, may be of the same color?

10. Enunciate the binomial theorem, and write down the middle term in the

expansion of $\left(1 - \frac{x}{2}\right)^{12}$.

Find the term independent of x in the expansion of

$$\left(x + 1 + \frac{1}{x} + \frac{1}{x^2}\right)^6.$$

11. Plot the graph

$$y = \frac{x^3}{8} - \frac{3x^2}{2} + \frac{11x}{2} - 4 \text{ from } x=1 \text{ to } x=7,$$

and find its gradients when $x=2$, when $x=4$, and when $x=6$.

Draw the tangent at the point $(4, 2)$.

12. Plot the graph $4y = 8 + 5x - 3x^2$ from $x = -2$ to $x = 3$, and show by integration that the area of the portion on the positive side of the axis of x is $\left(\frac{1}{2}\right)^3$.

N. B.—Any of the following questions may be taken instead of an equal number of the questions 8, 9, 10, 11, 12, but considerably lower marks will be assigned to them.

A. Solve the equation:

$$\frac{a(x+a)}{b} - \frac{b(x-b)}{a} = a + b.$$

B. A cask is filled with wine and water in the proportion of 3 : 1; 4 gallons are drawn off, and the cask filled up with water. If the proportion of wine to water is now 3 : 2, find how many gallons the cask can hold.

C. Divide $a^2(b+c^{-1}) + b^2(c^{-1}+a) - c^{-2}(a+b) + abc^{-1}$ by $a + b - c^{-1}$.

TRIGONOMETRY.

(One hour and a half.)

Squared paper and tables of logarithms, etc., can be obtained from the presiding examiner.

N. B.—Attention is called to the alternative questions A, B at the end of the paper.

1. (a) Find to the nearest minute the angle which an arc of length 4 inches subtends at the center of a circle whose radius is 5 inches.

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- (b) Find by drawing and measurement the least positive angle whose tangent is -1.2 ; and calculate the cosine of this angle, correct to three significant figures.
- (c) The length of the slant side of a cone is 4.3 inches, and the angle at the vertex is 115° . Calculate the height of the cone correct to a tenth of an inch.
2. Determine, by drawing the graph of $\sin x + 2\cos x$ for values of x between 0° and 45° , the value of x between these limits for which $\sin x + 2\cos x$ is a maximum.
3. Prove that

$$(a) \cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$(b) \cos(A-B) [1 + \sin(A+B)] = (\sin A + \cos B) (\cos A + \sin B)$$

4. Find the length of the side AB of the triangle ABC , given that $BC=23.4$ inches, $\angle ABC=42^\circ$, $\angle ACB=67^\circ$.
5. An airship is sighted at the same time from two points A and B at the same level 1 mile apart. It is due south of A at an elevation of 34° and due east of B at an elevation of 23° . Calculate its height in feet.
6. (a) Write down all the cube roots of $\cos \theta + \sqrt{-1} \sin \theta$.
- (b) Separate into real and imaginary parts the expression

$$\frac{1 + \sqrt{-3}}{(1 + \sqrt{-1})^3}$$

7. (a) Express $\tan(\theta_1 + \theta_2 + \theta_3 + \theta_4 + \theta_5)$ in terms of $\tan \theta_1, \tan \theta_2, \tan \theta_3, \tan \theta_4, \tan \theta_5$.
- (b) Prove that, if $\tan \alpha, \tan \beta, \tan \gamma$ are the roots of the equation $x^3 + ax^2 + x + b = 0$, where a, b are unequal, then $\alpha + \beta + \gamma$ is an odd number of right angles.

N. B. — Either of the following questions may be taken instead of either of the questions 6, 7, or both may be taken instead of the two questions 6, 7, but considerably lower marks will be assigned to them.

A. A is the foot of a flagstaff AB , and C is a point on it such that $AC = \frac{1}{2}AB$. If the flagstaff subtends an angle of 48° at a point X on the ground, calculate the value of $\tan BXC$.

B. (a) Find all the solutions of the equation $\sin 3\theta + \cos \theta = 0$.

(b) Prove that $\cot \frac{\theta}{2} - \cot \theta = \operatorname{cosec} \theta$.

ANALYTIC GEOMETRY AND CALCULUS.

(Two hours.)

ANALYTIC GEOMETRY.

Tables of logarithms, etc., may be obtained from the presiding examiner.

1. The corners of a triangle are the points $(2, 3), (3, 8), (-1, 5)$. Find the tangent of the angle at $(2, 3)$.
2. Find the equation of the two bisectors of the angles formed by the two straight lines $ax^2 + 2hxy + by^2 = 0$.

The straight line $3x + 2y = 1$ meets the circle $x^2 + y^2 = 8$ in two points P and Q . Find the equation of the straight lines joining P and Q to the origin.

3. Obtain an expression for the length of the tangent from the point (x', y') to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$.

A point P moves so that the tangents from it to the two circles

$$x^2 + y^2 = 4$$

and

$$x^2 + y^2 + 6x - 8y + 3 = 0$$

are in the ratio 3:2. Find the equation of the locus of P .

4. Find the equation of the polar of the point (x', y') with respect to the parabola $y^2 = 4ax$.

P is a variable point on any fixed line at right angles to the axis of the parabola. From P a perpendicular is drawn to its polar, meeting it in Q . Prove that the locus of Q is a circle with its center at the focus.

5. Find the condition that the line $y = mx + c$ may be (i) a tangent, (ii) a normal to the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

If the tangent and normal at any point P of the ellipse meet the major axis in T and T' so that $TT' = a$, prove that the eccentric angle, ϕ , of P , is given by the equation

$$e^2 \cos^2 \phi + \cos \phi - 1 = 0,$$

where e is the eccentricity.

CALCULUS.

6. Obtain the values of

$$\frac{d\sqrt{x}}{dx}, \quad \frac{d}{dx} \sin x, \quad \int \frac{dx}{\sqrt{a^2 - x^2}}, \quad \int_{-1}^1 (x^2 + 2x^3) dx.$$

Prove that $y = A \cos 2x + B \sin 2x + Cx$ satisfies the differential equation

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} + 4y = 0.$$

7. A hollow cone, of height 20 inches and radius at the base 10 inches, is inverted, and water is poured into it at the rate of 12 cubic inches per second. At what rate is the area of the water-surface increasing when the depth of the water in the cone is 15 inches?

8. A circle of radius 6 inches is divided into two segments by a chord at a distance of 1 inch from the center. Prove that the rectangle of greatest area which can be inscribed in the smaller segment has an area of 27.8 square inches approximately.

9. The curve $y = x^3$ meets the line $y = 4x$ in three points. Find the coordinates of these points and obtain the equation of the normal to the curve at each of them.

If P is the one whose abscissa is greatest, find the area contained by the y -axis, the normal at P , and the curve from the origin to P .

APPLIED MATHEMATICS.

(Two hours.)

Squared paper and tables of logarithms, etc., can be obtained from the presiding examiner. The acceleration due to gravity may be taken as 32 foot-second units.

1. Three forces of 30 pounds, 50 pounds, 90 pounds act at a point, the angle between the directions of any two forces being 120° . Find the magnitude of their resultant, and the angle which its direction makes with the greatest force.

2. Prove that the algebraic sum of the moments of two concurrent forces about any point in their plane is equal to the moment of their resultant about the same point.

Forces $2P$, P , P act along BC , CA , AB , the sides of an equilateral triangle ABC . Show that the resultant is a force P parallel to BC and at a distance from it, on the side remote from A , equal to the height of the triangle.

3. Two light rods AC (2 feet long), BC (3 feet long) are hinged to one another at C and to fixed points at A and B , the plane of the rods being vertical. The horizontal distance between A and B is 4 feet, and A is 1 foot higher than B , and C is above AB . From C is suspended a weight of 10 pounds. Find by graphical construction or otherwise the thrusts in the rods.

4. A body is placed on a rough plane inclined to the horizontal at an angle α . Show that it can remain in equilibrium if the coefficient of friction is greater than $\tan \alpha$.

A circular hoop of weight W hangs in a vertical plane over a rough peg. Prove that the greatest weight which can be suspended tangentially from the rim of the hoop without causing the latter to slip is $W \frac{\sin \epsilon}{1 - \sin \epsilon}$, where ϵ is the angle of friction.

5. A steamer is moving along a narrow canal when a man steps ashore after walking across the deck in a line making an angle of 60° with the line from stern to bow. He finds himself 25 feet farther along the canal than if he had stepped off on the other side before walking across. If the vessel is moving at 2 miles per hour and the deck is 25 feet wide, how fast did the man walk?

6. Prove the formula $s = ut + \frac{1}{2}ft^2$.

A mass of 35 pounds is pulled up a rough plane of inclination 1 in 16 by a rope parallel to the plane whose tension is 150 poundals. If the resistance due to friction is 12 poundals, find the velocity acquired by the mass in 10 seconds.

7. A body is projected with velocity V in a direction making an angle α with the horizontal. Find the greatest height that it reaches and its horizontal range.

Find, in foot-pounds, the least energy of projection which must be given to a ball of 3 ounces in order that it may have a horizontal range of 300 yards.

8. A particle moves in a straight line with an acceleration directed toward a fixed point in the line and equal to μ times the distance of the particle from the point. Show that the motion is periodic, with period $2\pi/\sqrt{\mu}$.

OXFORD AND CAMBRIDGE SCHOOLS EXAMINATION BOARD.

EXAMINATION FOR SCHOOL CERTIFICATES.

July, 1910.

ARITHMETIC.

(Two hours.)

- Find the number nearest to 9,999 that can be divided exactly by 3, 4, 5, 7.
- Simplify $\frac{1}{2}(1\frac{1}{2} - \frac{1}{2} + \frac{1}{2}) + \frac{1}{3}(\frac{1}{2} - \frac{1}{3} \text{ of } 1\frac{1}{2})$.
Divide .003125 by .02048, obtaining the answer correct to three decimal places.
- Find the cost of 3 tons 7 cwt. 2 qrs. 11 lb. at £4 8s. 8d. per cwt.
- If it costs 6 dollars 30 cents to carry $4\frac{1}{2}$ cwt. a distance of 500 miles, find, to the nearest cent, the cost of carrying 8 cwt. 360 miles.
- A bankrupt owes £6,000. He pays 12s. 6½d. in the £, and defrauds his creditors by concealing $\frac{1}{5}$ of his assets. Find the value of his estate.
- Find to the nearest penny, the compound interest on £2,525 at 2½ per cent per annum for four years.
- A square field contains 2 acres 3 roods, and is to be fenced with hurdles 6 feet in length. What is the smallest number of hurdles that can be used for each side of the field?
- A rectangular tank is 35 cms. deep, 62 cms. long, and 27 cms. wide. If the tank is filled with water, how many times could a bucket holding 8.5 liters be filled from it, and what would remain over?
- A man buys oranges at 5s. a hundred and sells them at 8d. a dozen. What is his gain or loss per cent?
- A man wishes to obtain £1,000 by selling 2½ per cent consols at 82½. What amount of stock must he sell, and what would be the quarterly income derived from it? (Answers correct to the nearest penny.)

ALGEBRA.

(Two hours.)

- NOTE.—(1) In order to pass in elementary mathematics candidates must satisfy the examiners in Part I.
(2) In order to pass in additional mathematics candidates must satisfy the examiners in Parts I and II taken together.
(3) Candidates for exemption from the army qualifying examination must satisfy the examiners in Parts I and II, taken together, and should NOT attempt the questions A, B, C, D.

Part I.

1. If $a = 2$, $b = -3$, $c = 0$, $d = \frac{1}{2}$, find the value of

$$\frac{(2a + b)(c + 4d)}{a + b} + cd.$$

2. Find the continued product of $b - c$, $c - a$, $a - b$.
Divide $4x^4 - 9x^2 + 6x - 1$ by $2x^2 + 3x - 1$.

3. Simplify:

$$(1) \frac{x^3 - 1}{x - 1} - \frac{x^2 - 2x + 3}{x + 2} - \frac{x + 3}{x^2 + x - 2}$$

$$(2) \frac{\frac{a}{b} - \frac{b}{a}}{a^2 - \frac{b^2}{a}} + \frac{a + b}{b + a}$$

4. Prove that the L. C. M. of two quantities is equal to their product divided by their H. C. F.

Find the H. C. F. of

$$x^3 + x^2 + x^2 + x^2 + x + 1 \text{ and } x^3 + x^2 - x + 2.$$

5. Solve the equations:

$$(1) \frac{1}{2}(x - 2) - \frac{1}{3}(x + 2) = \frac{1}{4}(x - 6) + \frac{1}{6}x;$$

$$(2) \frac{x^2 + 1}{x} = \frac{x + 5}{3}.$$

6. The difference of two fractions, one (the greater) formed by adding a certain number to both the numerator and the denominator of $\frac{1}{3}$, the other (the lesser) formed by subtracting this number from both the numerator and the denominator of $\frac{1}{3}$, is $\frac{1}{6}$. Find the number.

The following questions, A, B, C, D are to be attempted ONLY by candidates who do NOT attempt PART II.

- A. Factorize:

$$(1) 6x^2 + x - 12;$$

$$(2) a^2 + b^2 - c^2 - 2ab.$$

- B. Solve the equations:

$$ax - by = a^2 + b^2, x + y = 2a.$$

- C. Show that the sum of the squares of three consecutive integers is greater by 5 than 3 times the product of the greatest and the least.

- D. Solve *graphically* the equations:

$$\frac{1}{2}x + \frac{1}{3}y = 1.2, \quad 4x = 5y.$$

[Take 1 inch as your unit.]

Part II.

7. Solve the equations:

$$y^2 = 5x + 1, \quad 3x = 2y + 1.$$

8. If $\frac{a + b}{a - b} = \frac{c + d}{c - d}$, prove that $\frac{a}{b} = \frac{c}{d}$.

If a, b, c are in continued proportion, prove that

$$\frac{a - b}{b - c} = \left(\frac{a}{c}\right)^{\frac{1}{2}}.$$

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9. Find the value of

$$(34.97)^2 + (1.08)^2.$$

Obtain a formula for the sum of 27 terms of the series

$$1 + .9 + .81 + .730 + .6561 + \dots$$

and show that the sum is very nearly 9.42.

10. Draw the graphs of $2y - x = 3$, $y = 3 - x^2$ for values of x lying between -1 and 4 .

Find their gradients when $x = 1$, and show that the lines cut at right angles at the point $(1, 2)$.

GEOMETRY.

(Two hours.)

NOTE.—(1) In order to pass in elementary mathematics, candidates must satisfy the examiners in *a*.
 (2) In order to pass in addition: mathematics or to obtain exemption from the army qualifying examination, candidates must satisfy the examiners in *a* and *B* taken together.
 (3) Figures should be drawn accurately with a hard pencil, and all "constructions" clearly shown.

a
 1. Draw a triangle ABC having $BC=3$ in., $ABC=32^\circ$, and $ACB=105^\circ$. Construct the bisector of the angle BAC meeting BC in D . Measure BD .

2. Draw a circle of radius 1 inch and take a point P 2.5 inches from its center. Construct a circle of radius 1.3 inches to touch the former circle and also pass through P .

3. If two triangles have two sides of the one equal to two sides of the other, and also the angles contained by those sides equal, prove that the triangles are congruent.

If the sides AB, AC of a triangle are equal, and equal lengths AE, AD are cut off on AB, AC , prove that CE, BD are equal.

4. State an axiom relating to parallel straight lines, and use it to prove that when a straight line crosses two parallel straight lines it makes alternate angles equal.

Prove that the opposite angles of a parallelogram are equal to one another.

5. Prove that the square on a side of a triangle opposite to an obtuse angle is greater than the sum of the squares on the other two sides by twice the rectangle contained by one of these two sides and the projection on it of the other.

6. Prove that the opposite angles of any quadrilateral inscribed in a circle are supplementary.

Two circles intersect in A, B . C, D are any two points on one circle and CA, DB cut the other circle in P, Q , respectively. Prove that PQ is parallel to CD .

B

7. Prove that the tangent at any point of a circle and the radius through the point are perpendicular to one another.

Two circles whose centers are A and B touch externally at C . The common tangent at C meets another common tangent in D . Prove that ADB is a right angle.

8. Construct a mean proportional to two given straight lines, and prove the construction.

$ABCD$ is a rectangle such that the square on AD is twice the square on AB . BE is drawn at right angles to AC to meet AC in E . Prove that AE is one-third of AC .

9. If two triangles have one angle of the one equal to one angle of the other and the sides about these equal angles proportional, prove that the triangles are similar.

10. Draw a straight line 5 inches long and divide it into three parts proportional to $37 : 46 : 53$.

ELEMENTARY TRIGONOMETRY.

(Two hours.)

1. Define the tangent of an angle.
Construct an acute angle whose sine is .6, and find its cosine and its cotangent.
2. Prove that $\cos(180^\circ - a) = -\cos a$.

Arrange the angles a , β , γ in order of magnitude, if

$$\sin a = .8211, \cos \beta = .7738, \tan \gamma = -0.6104,$$

the angles being positive and each less than 180° .

3. What is the length of the shadow of a man 5 ft. 8 in. high cast by the sun when its altitude is $55^\circ 30'$?
4. Draw the graph of $10 + 10 \cos 2x$ for values of x between 0° and 60° . Find a value of x to satisfy the equation

$$x = 10 + 10 \cos 2x.$$

[Take one-tenth of an inch as unit along both axes.]

5. Prove that in any triangle

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

If $A = 63^\circ$, $B = 49^\circ$, $a = 50$ in., find b to the nearest tenth of an inch.

6. Prove that:

$$(1) \quad \frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} + \frac{\cos \theta - \sin \theta}{\cos \theta + \sin \theta} = \frac{2}{1 - 2 \sin^2 \theta}$$

$$(2) \quad (\sec \theta + \tan \theta)(\operatorname{cosec} \theta - \cot \theta) = (\operatorname{cosec} \theta + 1)(\sec \theta - 1).$$

7. If $2 \sin \theta + 5 \cos \theta = 5$, prove that

$$\tan \theta = 0 \text{ or } \frac{1}{4}.$$

8. Prove that $\sin(A - B) = \sin A \cos B - \cos A \sin B$, where A and B are both acute angles and A is greater than B .

Prove that $\frac{\sin 5.4 - \sin A}{\sin 3.4 - \sin A} = 1 + 2 \cos 2.4$.

9. Show that in any triangle ABC

$$\frac{b+c}{a} = \frac{\cos \frac{1}{2}(B-C)}{\sin \frac{1}{2}A}$$

If $b+c=24.8$ cm., $a=11.89$ cm., $A=39^\circ$, find B and C .

10. A lighthouse is observed from a ship which is steaming due N. to bear 62° W. of N., after the ship has sailed 10 miles, the lighthouse is observed to bear 40° W. of S. Calculate the distance of the ship from the lighthouse when it was nearest to it.

STATICS AND DYNAMICS.

(Two and a half hours.)

[Assume $g=32$ for foot-second units.]

1. State and prove the triangle of forces.
Find graphically, or otherwise, the resultant of the following forces acting at a point viz, 5 lb. due N., 10 lb. 30° S. of W., 15 lb. 60° N. of E., 12 lb. SE. [Scale 1 cm. to 1 lb.]
2. A form 10 ft. long weighs 2 stone; its legs are 1 ft. from each end. A boy weighing 8 stone sits 2 ft. from one end, another weighing 7 stone sits 3 ft. from the other end. Find the pressures of the legs of the form on the ground.

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3. Prove that, if three coplanar forces which are not parallel are in equilibrium, their lines of action meet in a point.

A stiff light rod ABC , bent so that ABC is a right angle and pivoted to a fixed point at A , has a weight of 12 lb. attached to it at C and is maintained with AB vertical and BC horizontal by a horizontal string attached at C . If the tension of the string is equal to the weight of 8 lb. and BC is 4 in., prove that AB is 6 in.

4. O is the center of a rectangular piece of cardboard $ABCD$; the piece AOB is cut out and then fitted to and gummed on to DC so that AB lies along DC . Find the center of gravity of the cardboard in its new form.

5. A mass of 14 lb. rests on a rough plane inclined at 30° to the horizon. If the coefficient of friction is .8, find the least force which, acting directly down the plane, will just move the mass.

6. Prove the formula $v^2 = 2/s$ for uniformly accelerated motion.

A point moving in a straight line with uniform retardation describes 7 ft. in the fifth second of its motion and 5 ft. in the seventh second. Prove that it will be at rest at the end of $11\frac{1}{2}$ seconds.

7. Define *force* and show how it is measured.

Find (1) in poundals, (2) in pounds' weight the force which will bring to rest in 8 seconds a mass of 3 lb. moving at the rate of a mile a minute.

8. Explain the meaning of the terms *work*, *energy*, and *power*.

Find the number of foot-pounds of work done by a man who picks up a stone weighing half a pound and throws it through a window 20 ft. above the ground so that it passes through the window with a velocity of 8 ft. per second.

9. A bullet is fired with a velocity whose horizontal and vertical components are 200 ft. per second and 40 ft. per second, respectively. Find the greatest height to which it will rise and its distance from the firing point when it reaches the ground again.

TRIGONOMETRY, STATICS AND DYNAMICS.

(MATHEMATICS I.)

(Three hours.)

[Assume $g=32$ for foot-second units.]

1. Draw the graph of $5 \cos x$ for values of x from 0° to 60° ; and employ it to find the angle whose cosine is .8.

2. Prove that $\sin^2 A = 1 - \cos^2 A$.

Prove that

$$(\sin A + \sin B)(\cos B - \cos A) = \sin(A - B)[1 - \cos(A + B)].$$

3. Show that in any triangle:

$$(1) b = c \cos A + a \cos C;$$

$$(2) \tan \frac{1}{2} A = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}.$$

If $b=60$ ft., $c=72$ ft., $B=52^\circ$, find the possible values of C .

4. Two men at A and B observe a balloon situated at a point vertically above the line AB (which is horizontal) and between A and B ; the angles of elevation of the balloon are observed to be $68^\circ 12'$ and $78^\circ 41'$, and the distance AB is 240 ft. Show that the height of the balloon is nearly 400 ft.

5. State and prove the triangle of forces.

Find graphically, or otherwise, the resultant of the following forces acting at a point, viz, 5 lb. due N., 10 lb. 30° S. of W., 15 lb. 60° N. of E., 12 lb. SE. (Scale 1 cm. to 1 lb.)

6. A form, 10 ft. long, weighs 2 stone; its legs are 1 ft. from each end; a boy weighing 8 stone sits 2 ft. from one end, another weighing 7 stone sits 3 ft. from the other end. Find the pressures of the legs of the form on the ground.

APPENDIX A.

7. Prove that if three coplanar forces, which are not parallel, are in equilibrium their lines of action meet in a point.

A stiff light rod ABC , bent so that ABC is a right angle and pivoted to a fixed point at A , has a weight of 12 lb. attached to it at C and is maintained with AB vertical and BC horizontal by a horizontal string attached at C . If the tension of the string is equal to the weight of 8 lb. and BC is 4 in., prove that AB is 6 in.

8. A mass of 14 lb. rests on a rough plane inclined at 30° to the horizon. If the coefficient of friction is .8, find the least force which, acting directly down the plane, will just move the mass.

9. Prove the formula $v^2=2fs$ for uniformly accelerated motion.

A point moving in a straight line with uniform retardation describes 7 ft. in the fifth second of its motion and 5 ft. in the seventh second. Prove that it will be at rest at the end of $11\frac{1}{2}$ seconds.

10. Define force and show how it is measured.

Find (1) in poundals, (2) in pounds' weight the force which will bring to rest in 8 seconds a mass of 3 lb. moving at the rate of a mile a minute.

11. A body is attached to a fixed point by a string. If the body is let go when the string is taut and horizontal, prove that when the body is passing through its lowest position the tension of the string is three times the weight of the body.

ARITHMETIC, ALGEBRA, GEOMETRY, AND GEOMETRICAL DRAWING.

(MATHEMATICS I.)

(Three hours.)

1. Three bells begin tolling together at the rates of 50, 55, and 65 times per minute, respectively. How soon will they next toll together?

2. Having given that 1 cubic centimeter of water weighs 1 gram, that

1 kilogram = 2.2046 lbs.

and 1 foot = 30.48 cms.,

find the weight of 1 cubic foot of water to the nearest ounce.

3. Solve the equations:

$$(1) (x^2 - 1)(x + 2) = x^3;$$

$$(2) x - y = a,$$

$$x^2 - ax + by = 0.$$

4. Interpret the expressions a^3 , a^2 , and justify your interpretation:

$$\text{Simplify } \sqrt[3]{a^2b^2c^3} + (a^4b^3c^4)^{\frac{1}{4}}$$

5. Find the sum of n terms of an arithmetic progression, whose first term and common difference are given.

Prove that, if n is an odd number, the sum of n terms of the progression is n times the middle term.

Find the fifth term and the sum of 5 terms of the progression

$$\sqrt{2} + 1, 1, \sqrt{2} - 1 \dots$$

6. Draw the graphs of the equations

$$x + y = 1, y(1 - x) = 1$$

for the values of x between -2 and 3 .

Show that the second curve has equal gradients at the two points where it is cut by the first.

7. Prove that the opposite angles of any quadrilateral inscribed in a circle are supplementary.

Two circles intersect in A, B . C, D are any two points on one circle and CA, DB cut the other circle in P, Q , respectively. Prove that PQ is parallel to CD .

8. Construct a mean proportional to two given straight lines and prove the construction.

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$ABCD$ is a rectangle such that the square on AD is twice the square on AB . BE is drawn at right angles to AC to meet AC in E . Prove that AE is one-third of AC .

9. If two triangles have one angle of the one equal to one angle of the other and the sides about these equal angles proportional, prove that the triangles are similar.

10. Draw a straight line 5 inches long and divide it into three parts proportional to 37:46:53.

UNIVERSITY OF LONDON.

MATRICULATION EXAMINATIONS, JANUARY, 1916.

ARITHMETIC AND ALGEBRA.

Tuesday, January 11—Morning, 10 to 1.

1. Find x from the equation

$$x = 635 - 1540 \times (.642)^2.$$

If .642 denotes a number lying between .6415 and .6425, what are the extreme values of x ? Give all the required values of x to three significant figures.

2. A consumer receives notice at the end of a quarter that the charge per unit of electricity supplied is to be raised p per cent. In the next quarter he succeeds in reducing the number of units consumed by q per cent. Give a formula for the percentage increase in the bill for the quarter.

If $p=10$, and the bill is decreased by $6\frac{1}{2}$ per cent, find q .

3. There are two kinds of floorcloth of the same pattern, one 27 inches wide costing 2s. 6d. per yard length, the other 45 inches wide costing 7s. 6d. per yard length. A lady requires 21 square yards of floorcloth altogether and buys as much as she can of the wider kind, but is determined not to spend more than £5. What length of the wider kind will she be able to buy?

4. Resolve into factors

(i) $x^2 + 2x - y^2 + 2y.$

(ii) $a^2 - ab - 3a + 2b.$

and simplify the expression

$$a^4 - 3(a-1)^4 + 3(a-2)^4 - (a-3)^4.$$

5. If $x = \frac{ay+1}{a+y}$, $y = \frac{ab+1}{a+b}$, and $b = \frac{ac+1}{a+c}$, express x in terms of a and c , and check the result by putting c equal to unity.

6. Solve the equations

(i) $\frac{1}{3}\left(x - \frac{1}{2}\right) + \frac{1}{2}\left(x + \frac{1-x}{4}\right) = \frac{1}{6}\left(1 + \frac{x+1}{8}\right).$

(ii) $(x-3)(x-4) = (y-7)(y-6)$
 $x+y=9$

7. (i) If $3x^2 + 12xy + 8y^2 + 12x - 10y - 63 = 0$, find y when $x=7$.

(ii) Find the values of $\frac{1}{63 \pm \sqrt{3957}}$ each correct to three significant figures.

8. Draw graphs of the equations

(i) $y = 27 - 10x + x^2,$

(ii) $y = 12x - 29 - x^2.$

For what values of x is $27 - 10x + x^2$ less than $12x - 29 - x^2$?

9. A sum of £1,000 is set aside at the end of each year and invested at 5 per cent compound interest. Find approximately the total sum accumulated at the end of 19 years, including the £1,000 set aside at the end of the nineteenth year.

[Use which you require of the following:

$$(1.05)^{18}=2.406619 \quad (1.05)^{19}=2.526950 \quad (1.05)^{20}=2.653298.]$$

10. *A* and *B* could between them type 4,500 pages of manuscript in 100 working hours, and undertook to do so. After 36 working hours *B* was replaced by *C*, and the first half of the task was completed in 54 working hours from the start. After 80 working hours from the start *B* returned, and all three just finished the task in time. Find the average number of pages typed in an hour by *A*, by *B*, and by *C*.

GEOMETRY.

Tuesday, January 11—Afternoon, 2.30 to 5.30.

1. Prove that, if two sides of a triangle are unequal, the angle opposite the greater side is greater than the angle opposite the less. Also, state and prove the converse of this theorem.

In an acute-angled triangle *ABC* in which *AB* is greater than *AC*, the lines drawn from *B*, *C* perpendicular to the opposite sides intersect in *O*. Prove that *OB* is greater than *OC*.

2. Prove that the diagonals of a rhombus bisect each other at right angles.

The side *BC* of a rhombus *ABCD* is produced through *C* to a point *E* so that *CE* is greater than *BC*. The line *ED* is drawn and produced to cut *CA* produced in *F*, and *F* is joined to *B*. Prove that the angles *BFA*, *EFA* are equal.

3. Prove that parallelograms on the same base and between the same parallels are equal in area.

ABCD, *A'EFG* are two parallelograms having a common point at *A*, and having the vertex *E* on *BC*, and the vertex *D* on *FG*. Prove that the parallelograms are equal in area.

4. In a right-angled triangle prove that the square on the hypotenuse is equal to the sum of the squares on the other two sides.

If two unequal right-angled triangles *ABC*, *ADC* are drawn on opposite sides of their common hypotenuse *AC*, and if *AM*, *CN* are drawn perpendicular to *BD*, cutting it in *M*, *N*, prove that $BM^2 + BN^2 = DM^2 + DN^2$.

5. If a straight line *AB* is bisected at *C* and produced to any point *D*, prove that $CD^2 = AD \cdot BD + AC^2$.

Show how to find the position of *D* by a geometrical construction so that the rectangle *AD* · *BD* shall equal the square on *AB*.

6. Show, with proof, how to construct a square equal in area to a given triangle, illustrating your method by a well-drawn figure.

7. Prove that the angle at the center of a circle standing on a given arc is double any angle at the circumference standing on the same arc.

Prove that, if a square be described externally on the hypotenuse of a right-angled triangle, and the right angle be joined to the center of the square, the joining line will bisect the right angle.

8. The angles *A*, *B* of a cyclic quadrilateral *ABCD* are 110° , 85° , respectively, and *AB* subtends an angle of 65° at the point of intersection of the diagonals. Find the angles which the sides subtend at the center of the circle.

9. Prove that in equal circles (or the same circle) equal angles at the centers stand on chords which are equal.

Given the base *BC* and the vertical angle *A* of a triangle *ABC*, the angle *A* being acute; prove that if *BM* is drawn perpendicular to *AC*, cutting it in *M*, and if *CN* is drawn perpendicular to *AB*, cutting it in *N*, the line *MN* is of constant length.

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10. If two chords AB , CD of a circle, on being produced, meet in a point P , prove that the rectangles $PA \cdot PB$ and $PC \cdot PD$ are equal.

Two circles having a common chord AB cut a third circle, the chords of intersection with it being CD , EF , respectively. Prove that AB , CD , EF , produced if necessary, intersect in a common point.

MECHANICS.

Wednesday, January 19—Afternoon, 2.30 to 5.30.

1. The gravitational acceleration at the surface of the moon is approximately 5.4 feet per second. Calculate (a) the time taken by a projectile, starting vertically upward from the surface of the moon with a velocity of 120 feet per second, to return to its starting point, and (b) the maximum height reached.

How do these results compare with those upon the earth's surface?

2. Explain the variations of the force between the floor of a lift and the feet of a man standing in it during the upward and downward journeys from rest to rest.

3. Give exact definitions of the terms *force*, *momentum*, *work*, and *power*, and explain the connections between (a) force and momentum, (b) force and work, and (c) work and power.

4. Explain the principle by which the resultant of two forces not in the same straight line is determined.

The bob of a simple pendulum is deflected so that the string makes an angle of 30° with the vertical. It is then released. Calculate the direction and magnitude of the acceleration with which the bob begins to move.

5. Show how to calculate the magnitude and position of the resultant of a number of parallel forces acting in a plane.

Equal weights are situated at five of the angular points of a horizontally placed regular hexagon of side a . Find the line of action of the single force which would be in equilibrium with the weights.

6. Apply the principle of work to determine the mechanical advantage of a smooth plane inclined at an angle α to the horizontal, the load being raised by a horizontally applied force.

Show what would be the effect of friction on the mechanical advantage; and explain what is meant by the efficiency of a machine.

7. Define the *density* and the *relative density* of a substance.

Describe carefully how you would carry out—using a suitable bottle—a determination of the density of (a) a liquid, (b) a powder soluble in water, but insoluble in the liquid.

8. Explain how the pressure of the atmosphere can be accurately measured.

At the top of a mountain a mercury barometer reads 67.2 cm. What is the pressure in kilograms weight per sq. cm., given that mercury has 13.6 times the density of water.

APPENDIX B.

ENGLAND.

ENTRANCE SCHOLARSHIPS EXAMINATION PAPERS.

CAMBRIDGE UNIVERSITY.

EXAMINATION FOR SCHOLARSHIPS, EXHIBITIONS, AND SIZARSHIPS.

Trinity, Clare, Trinity Hall, Peterhouse, and Sidney Sussex.

December 7, 1910. 9-12.

1. The lines AB , BE , CF are drawn perpendicular to the sides BC , CA , AB of the triangle ABC , and EF , FD , DE are drawn to cut BC , CA , AB , respectively, in X , Y , Z . The tangents at A , B , C to the circle ABC cut BC , CA , AB , respectively, in P , Q , R , and L , M , N are the middle points of AP , BQ , CR , respectively. Prove that the six points X , Y , Z , L , M , N are on the radical axis of the circles ABC and DEF .

2. A , B are the points of contact of a common tangent of two given circles, and any line parallel to AB cuts one circle in P and the other in Q . Prove AP and BQ intersect on a fixed circle coaxial with the given circles.

3. Find three points X , Y , Z on the sides BC , CA , AB of the triangle ABC , such that YZ , ZX , XY will pass, respectively, through three given collinear points L , M , N .

Hence, or otherwise, find the points of contact of three given tangents to a conic having also given the pole of a given straight line.

4. Prove that, if n is a positive integer,

$$1 - \frac{n}{1^2}x + \frac{n(n-1)}{1^2 \cdot 2^2}x^2 - \frac{n(n-1)(n-2)}{1^2 \cdot 2^2 \cdot 3^2}x^3 + \dots$$

$$= e^x \left\{ 1 - \frac{(n+1)}{1^2}x + \frac{(n+1)(n+2)}{1^2 \cdot 2^2}x^2 - \frac{(n+1)(n+2)(n+3)}{1^2 \cdot 2^2 \cdot 3^2}x^3 + \dots \right\}.$$

5. Prove that, if α , β , γ are the three roots of the equation

$$x^3 - 21x + 35 = 0;$$

then will

$$\alpha^2 + 2\alpha - 14 \text{ be equal to } \beta \text{ or to } \gamma.$$

6. Prove that

$$\cos 2\alpha \sin(\beta - \gamma) + \cos 2\beta \sin(\gamma - \alpha) + \cos 2\gamma \sin(\alpha - \beta)$$

$$= 4 \sin \frac{1}{2}(\beta - \gamma) \sin \frac{1}{2}(\gamma - \alpha) \sin \frac{1}{2}(\alpha - \beta) \{ \cos(\beta + \gamma) + \cos(\gamma + \alpha) + \cos(\alpha + \beta) \}.$$

7. The diameter AB of a circle is produced to C so that BC is equal to the radius OB of the circle; CD is drawn perpendicular to OBC and $CD = BC$. A point P is taken on the circle on the same side of AB as the point D , and such that $\angle AOP$ is half a right angle. Prove that, if Q is the point where PD cuts the circle again, $\angle BOQ$ is 1.001 radians, very nearly.

8. Prove that chords of the ellipse $x^2/a^2 + y^2/b^2 - 1 = 0$ which subtend a right angle at a given point P of the ellipse intersect the normal at P in a point P' , such that PP' is equal to $2abd/(a^2 + b^2)$, where d is the semidiameter conjugate to CP .

Prove that as P varies the locus of P' is a similar ellipse, and that the normals to the ellipses at P and P' , respectively, intersect either ellipse in four concyclic points.

9. Show that the locus of the intersection of normals at the extremities of chords of a parabola which pass through a fixed point is a parabola, and find the direction its axis.

10. Find the equation of the tangent at the point $(p^2, 1, p)$ on the conic $\alpha\beta - \lambda^2 = 0$; and prove that, if the sides of a triangle touch the conic and two of its vertices are on the lines $\alpha - \lambda_1\beta = 0$ and $\alpha - \lambda_2\beta = 0$, respectively, the locus of the third vertex is the conic

$$(\lambda_1 + \lambda_2)^2 \alpha\beta - 4\lambda_1\lambda_2\gamma^2 = 0.$$

11. Show that if θ and ϕ lie between 0 and π , a^2 is less than 1, and $(1 - 2a \cos \theta + a^2)(1 + 2a \cos \phi + a^2) = (1 - a^2)^2$,

then

$$\frac{d\theta}{d\phi} = \frac{\sin \theta}{\sin \phi} = \frac{1 - 2a \cos \theta + a^2}{1 - a^2}.$$

12. Prove that if the chord of curvature through the origin is $2r^n/a^{n-1}$ ($n \neq 1$), for any point of a curve at distance r from the origin, then the radius of curvature is proportional to

$$\frac{a^{n-1}}{r^{(n+1)}e^{(n-1)r^{(n-1)}}}$$

December 7, 1910. 1-4.

BOOKWORK.

Candidates are requested to attempt at least ONE question from each section of the paper, and not more than THREE in all.

1. Write a short account of the method of projection in geometry. Include in this account the properties of a projected figure corresponding to (a) circles, (b) right angles, (c) a pair of equal angles, (d) middle points of lines, and (e) foci of conics, in the original figure, and illustrate your theory by stating in a form true for all conics the property that the angle at the center of a circle is double that at the circumference.

2. Prove that if m is prime to a the least positive remainders of the series of integers

$$k, k+a, \dots, k+(m-1)a$$

with respect to m are a permutation of the numbers of the series

$$0, 1, 2, \dots, (m-1),$$

and that

$$a^{\phi(m)} - 1 \equiv 0 \pmod{m},$$

where $\phi(m)$ is the number of integers less than m and prime to it.

Prove, also, that $(m-1)! + 1$ is divisible by m if, and only if, m is a prime.

Show that if m is a prime and $p < m$,

$$(p-1)!(m-p)! + (-1)^{p-1} \equiv 0 \pmod{m}.$$

3. State and prove the leading propositions in the theory of determinants and indicate some applications of the theory.

4. Starting from the definition of a differential coefficient, develop methods and results which will enable you to differentiate any function obtained by combining exponential functions, circular functions, powers, and the inverses of these functions.

5. Establish formulæ for the curvature at any point of a plane curve, including the cases when the curve is defined (a) by a Cartesian equation, (b) by equations of the type $x = \phi(t)$, $y = \psi(t)$, (c) by an intrinsic equation, (d) by a p, r equation, and (e)

as the envelope of a line whose equation contains one variable parameter. Apply such of these formulæ as are suitable to the cases of the ellipse and the parabola.

6. Starting with any definition of $\exp x$ or $\log x$, where x is a real variable, develop the principal properties of these functions, including among your results the expansions for $\exp x$ and $\log(1+x)$, the equation.

$$\frac{d}{dx}(\log x) = \frac{1}{x},$$

and the relation

$$\lim_{n \rightarrow \infty} \left(1 + \frac{x}{n}\right)^n = \exp x.$$

Explain, also, the connection between $\exp x$ and e^x .

A. Prove the theorems on which graphic methods as applied to statical problems depend.

B. Investigate the various theorems concerning the conservation of energy and the conservation of momentum under appropriate conditions for a system of two particles.

Apply your theorems to the solution of the following problem: Two particles of masses m and m' are connected by an elastic string, of natural length l and without mass, and are initially at rest at a distance $\frac{1}{2}l$ from each other at points A and B , respectively. Blows P and Q are applied to the particles in directions perpendicular to AB and toward A , respectively. Discuss the subsequent motion.

C. Investigate the various problems arising from the collision of smooth elastic spheres. In particular consider the loss of kinetic energy.

A mass m of water issues per unit time from a pipe with uniform velocity u and strikes a pail which retains it, there being no elasticity. Initially the pail is at rest, and at a subsequent instant is moving in the direction of the stream with velocity V . Prove that

$$\frac{dV}{dt} = \frac{m(u-V)^2}{Mu^2},$$

and that the loss of energy up to this instant is

$$\frac{1}{2}MuV,$$

where M is the mass of the pail, and gravity is omitted from consideration.

D. Investigate the theory of the isochronism of oscillations, considering harmonic motion in general, motion on a cycloid, and the small oscillations of a pendulum.

December 8, 1910. 9-12.

FOR CANDIDATES IN MATHEMATICS, IN MECHANICAL SCIENCES, AND IN MATHEMATICS APPLICABLE TO PHYSICS.

Mathematical tables and squared paper are provided, and mathematical instruments may be employed for calculation.

Candidates for mathematical scholarships are to omit questions A-C.

A. Prove the formula

$$\cos(A+B) = \cos A \cos B - \sin A \sin B.$$

Show that

$$\sin A \sin(B-C) + \sin B \sin(C-A) + \sin C \sin(A-B) = 0.$$

B. Find the length of the perpendicular from the origin to the line

$$3x + 4y - 1 = 0.$$

Find also the equations of the two lines which are parallel to this line and at twice its distance from the origin.

C. Trace the curve $y^2 - 16x$, and find the points where it is cut by the line $y = 2x - 6$. Prove also that the line cuts the curve at angles ϕ and ϕ' given by $\tan \phi = \frac{1}{2}$, $\tan \phi' = \frac{1}{2}$.

1. The bisector of the angle A of a triangle ABC passes through the center of the square described externally on BC . Show that A is a right angle, or that $AB = AC$.

2. $ABCD$ is a quadrilateral not inscribable in a circle. Show that $AC \cdot BD < AB \cdot CD + BC \cdot AD$.

3. The roots of the equation $x^4 - 12x^3 + 51x^2 - 92x + 60 = 0$

are all integral; find them.

4. Prove that if x is the n th root of a , the error involved in taking x for the $(n+q)$ th root of a is

$$\frac{qx \log_e x}{n+q} \left\{ 1 - \frac{q \log_e x}{2(n+q)} \right\} \text{approximately,}$$

where

$$\frac{q \log_e x}{n+q}$$

is small.

Estimate roughly the extreme magnitude of this error when x and q are not greater than 1,000 and 1, respectively, and n is not less than 200.

5. Solve (graphically or otherwise) to two significant figures the equation

$$x^3 - 10x - 11 = 0.$$

6. Find the general solution of the equation

$$\operatorname{cosec} 4a - \operatorname{cosec} 4\theta = \cot 4a - \cot 4\theta.$$

7. Solve completely the triangles in which

$$a = 113, \quad b = 152, \quad A = 27^\circ.$$

8. In an acute-angled triangle ABC , if LMN is the pedal triangle and ρ its in-radius, show that

$$LM \cdot MN \cdot NL = 2\rho\Delta,$$

where Δ is the area of ABC .

9. Interpret the equations obtained by eliminating x and y in succession between the straight line $x + 2y = 3$ and the conic $x^2 - 3y^2 = 22$, and hence (or otherwise) prove that the circle described on this chord as diameter has for its equation

$$x^2 + y^2 + 18x - 12y - 128 = 0.$$

10. Find the relation between the eccentric angles at the extremities of conjugate diameters of an ellipse.

If the ellipse be

$$x^2/a^2 + y^2/b^2 = 1,$$

prove that the tangents at the ends of conjugate diameters intersect on the ellipse

$$x^2/a^2 + y^2/b^2 = 2,$$

and that the corresponding normals meet on the curve

$$2(a^2x^2 + b^2y^2)^2 = (a^2 - b^2)^2(a^2x^2 - b^2y^2)^2.$$

11. Prove the rule for differentiating the quotient of two functions. Find the differential coefficients of

$$\frac{x-2}{\sqrt{2x^2-2x+5}}, \quad \sin\left\{\frac{1+x^2}{1+x^3}\right\}, \quad \tan(x - \sin^{-1}x).$$

12. A bell tent consists of a conical portion above and a cylindrical portion near the ground. For a given volume and a circular base of given radius, prove that the amount of canvas used is a minimum when the semivertical angle of the cone is $\cos^{-1}(\frac{2}{3})$.

13. Integrate the following expressions:

$$\frac{x}{(x^2+3x+2)}, \tan^2 x, x^2 \log x, \frac{1}{\sqrt{2-x} + \sqrt{1+x}}.$$

14. Apply the integral calculus to evaluate

- (i) the area of a parabola cut off by a line perpendicular to the axis;
- (ii) the volume of a sphere.

15. Trace the curve $xy^2 = 4a^2(2a-x)$ and prove

- (i) that the area between the curve and the axis of y is $4\pi a^2$;
- (ii) that the volume generated by the revolution of the curve about this line is $2\pi^2 a^3$.

December 8, 1910. 1.30-4.30.

FOR CANDIDATES IN MATHEMATICS, IN MECHANICAL SCIENCES, AND IN MATHEMATICS APPLICABLE TO PHYSICS.

Mathematical tables and squared paper are provided, and mathematical instruments may be employed for calculation.

1. A ship leaves a certain port and steams N.W. at 15 knots; 10 hours later another ship leaves the same port, and steams W.S.W. at 12 knots. Their wireless instruments are capable of communications up to 500 nautical miles, how long may the ships expect to remain in touch with one another?

[A knot = a nautical mile an hour.]

2. A train passes station A at 40 miles per hour and maintains this speed for 7 miles, and is then uniformly retarded, stopping at B , which is 8 miles from A . A second train starts from A the instant the first train passes and, being uniformly accelerated for part of the journey and uniformly retarded for the rest, stops at B at the same time as the first train. What is its greatest speed on the journey?

3. A tramcar starts from rest and its velocities at intervals of 5 seconds, are given in the following table:

Time.....	0	5	10	15	20	25	30
Velocity in miles per hour.....	0	8.1	11.8	14.6	16.3	17.7	19

Calculate the distance in yards traveled in the above time. Also, if the car weighs 8 tons, estimate the effective pull exerted on the car at the end of 20 seconds.

4. A particle is projected from a given point with a velocity whose vertical component is given. Prove that the initial angular velocity about the focus of the path is greatest when the angle of projection is 45° .

5. A smooth sphere is tied to a fixed point by an inelastic string, and another sphere impinges directly on it in a direction making an acute angle with the string. Show that, if the second sphere is reduced to rest by the impact, the ratio of the total kinetic energy after the impact to that before is equal to the coefficient of restitution.

6. A light spring is such that m lbs. weight compresses it a feet. It is compressed c feet ($c > a$). If m' lbs. is placed on the top and the spring is released, find the condition that the weight will leave the spring, and its velocity when this happens.

7. Four equal masses are attached at equal distances A, B, C, D at points on a light string, and so placed that $\angle ABC = \angle BCD = 120^\circ$, and the various parts of the string are straight; an impulse I is given to the mass at A in the direction BA , show that the impulsive tension in AB is $\frac{1}{3} I$.

8. A heavy uniform beam, 8 feet long, weighs 20 lbs., and at one end it carries a load of 20 lbs. and at the other end a load of 10 lbs. The beam thus loaded is to be carried on a certain support. Find by a graphical construction the position of the support.

If the beam is to rest on two supports which are to divide the load in a given ratio, show how to find on your diagram any number of suitable pairs of points, and find the position of the supports when they are at a distance 4 feet apart, the pressures on them being in the ratio of 3 to 2.

9. A tripod consists of three equal rods, each of weight w , smoothly jointed at the upper ends. It is placed symmetrically on a rough horizontal table, for which the angle of friction is ϕ , and a weight W is put on the top of the tripod. Show that the rods can not make an angle with the vertical greater than

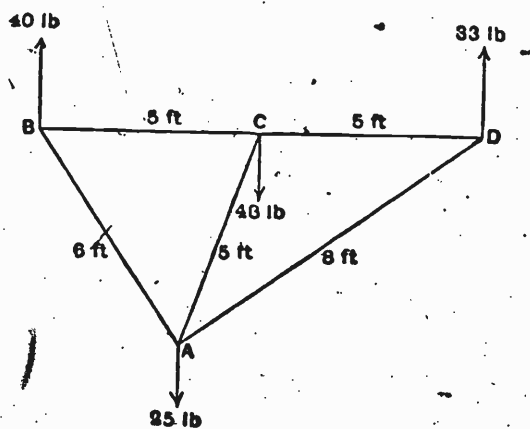
$$\tan^{-1} \left(2 \frac{W+3w}{2W+3w} \tan \phi \right).$$

10. Two equal uniform rods are fastened together so as to bisect one another at right angles. They rest in a plane at right angles to a rough wall, the one rod resting on the edge of the top of the wall, and an end of the other against the vertical side. Prove that the limiting inclination θ of the second rod to the vertical is given by the equation

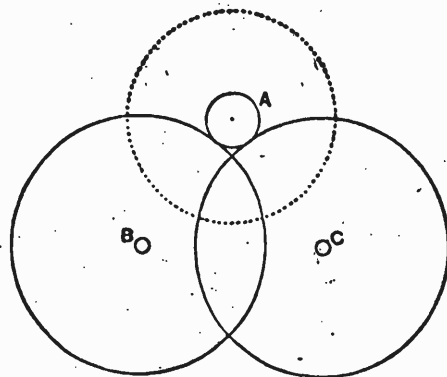
$$\tan \theta \cos^2 \lambda = \cos(\lambda + \theta) \sin(\lambda - \theta);$$

where λ is the angle of friction.

11. The figure $ABCD$ represents a freely-jointed light plane framework, with forces acting in the plane at right angles to BCD . Show that it is in equilibrium, and prove, graphically or otherwise, that the stresses in AB, AC are equal and of opposite sign.



12. Two equal friction-wheels of radii R inches turn on axles B, C of radii r inches; the coefficient of friction between wheel and axle is $\tan \phi$. A wheel of weight W is attached to an axle of radius a inches which rests on the circumferences of the first two wheels as in the figure. Neglecting the weights of the wheels B and C , show that the least couple which will rotate the wheel A is $Wa \frac{\sin 2\phi}{\sqrt{3}}$, where $\sin \theta = \frac{r}{R} \sin \phi$; the centers



of the three wheels forming an equilateral triangle.

(ALTERNATIVE QUESTIONS IN PHYSICS.)

13. A diving bell has the form of a paraboloid of revolution cut off by a plane perpendicular to its axis at a distance h from the vertex, where h is the height of the water barometer. If the bell be lowered so that its vertex is at a depth $5h/2$ below the surface of the water, find how high the water will rise in the bell.

14. A battery is connected to a galvanometer of 40 ohms resistance, and a certain current is observed. The galvanometer is now shunted with a resistance of 10 ohms, and the current in the galvanometer falls to one-half of the former value. Find the internal resistance of the battery.

15. The coefficients of cubical expansion of mercury and brass are .00018 and .00006 per degree centigrade. A mercury barometer with a brass scale reads correctly at 15°C ., standing at 30 inches; what will be the error at 35°C .?

16. One cubic foot of air at a temperature of 500°C . absolute is expanded isothermally from a pressure of 120 lbs. per sq. in. to twice the initial volume; it is then expanded adiabatically to three times the initial volume. Find the pressure and temperature at the end of each stage and calculate, graphically or otherwise, the work done and the heat units taken in during each stage, taking the mechanical equivalent of one thermal unit as 1,400 ft. lbs., and the equation for adiabatic expansion of air as $pV^{1.4} = \text{const}$.

APPENDIX C.

FRANCE.

CONCOURS FOR ADMISSION TO THE ÉCOLE NORMALE SUPÉRIEURE
AND FOR THE BOURSES DE LICENCE IN 1913.

MATHEMATICS.¹

GROUP I.

I.

(Time: 6 hours.)

Being given three axes of rectangular coordinates Ox , Oy , Oz , consider the surface (S) defined by the equation $z = xy + x^2$ and the line (D) defined by the equations $y = b$, $z = c$, where b and c are two given constants, the second not being zero. In all that follows this line (D) remains fixed.

1. Show that the surface (S) is ruled and find its generators.
2. To each rectilinear generator (G) of the surface (S) establish a correspondence of the plane (P) drawn through the line (D) and parallel to the line symmetric to (G) with respect to the plane xOy . Determine the locus of the point of intersection of (G) and of (P), when the line (G) describes the surface (S).
Show that this locus is a curve (C) situated on a quadric (Q), and determine this quadric.
3. Form the equation of the fourth degree, giving the abscissas of the points of intersection of the curve (C) with a plane given by its equation $ux + vy + wz + t = 0$. Calculate the elementary symmetric functions of the roots as a function of u, v, w, t . From this deduce the relation which the abscissas x_1, x_2, x_3, x_4 , of four points of the curve (C) must satisfy in order that these four points should be in the same plane.

This relation will be useful in most of the questions which follow.

4. Deduce from the preceding relation the conditions which the abscissas x_1, x_2, x_3 , of three points of the curve (C) must satisfy in order that these three points shall be collinear.

Form the general equation of the third degree of which the roots are the abscissas of three collinear points of the curve (C). Show that the lines which cut (C) in three points generate one of the families of rectilinear generators of the quadric (Q).

5. Show that the necessary and sufficient condition that the osculating planes to the curve (C) in three given points cut on the curve (C) is that the three points are collinear.

6. Through any point M of the curve (C) there pass two planes enjoying the property of being tangent to the curve (C) at the point M and in another point (that is to say of being bitangent to the curve). Suppose M' and M'' are the second points of contact of these two planes. Show that there exists a plane bitangent to the curve (C) in M' and M'' .

What conditions must be satisfied by the abscissas of the three points M, M', M'' , of the curve (C) in order that any two of them are points of contact of a plane bitangent to the curve (C)?

¹ The solutions of the following problems are to be found in *Nouvelles Annales de Mathématiques*, tome 73, Oct.-Nov., 1914, pp. 467-482.

7. Form the general equation of the third degree whose roots are the abscissas of the points M, M', M'' , of the curve (C) subject to the preceding conditions. Express the coefficients of this equation by means of the abscissa ϵ of the fourth point of intersection μ of the curve (C) with the plane (π) determined by the points M, M', M'' .

Calculate, in terms of ϵ the coefficients of the equation of the plane (π) and the coordinates of the point of concurrence, A , of the tangents to the curve (C) at the points M, M', M'' . This point A is said to be the point associated with the point μ of the curve (C) .

8. Show that there exists an infinity of quadrics, depending only on b and c , with respect to which the point A is the pole of the plane (π) ; determine these quadrics and show that one of them is the quadric (Q) already considered.

Determine the locus (Γ) of the point A , also the envelope of the plane (π) , when the point μ describes the curve (C) .

9. With any three collinear points μ_1, μ_2, μ_3 , on the curve (C) are associated the three vertices A_1, A_2, A_3 , of a triangle inscribed in the curve (Γ) . Determine by supposing $b=0$, the envelope of the sides of this triangle when the line $\mu_1 \mu_2 \mu_3$ varies. Show that in the same hypothesis $b=0$, the circle circumscribed about the triangle $A_1 A_2 A_3$, passes through two fixed points.

II.

(Time: 4 hours.)

Given two rectangular axes, and the differential equation $y - 2xy' + y^2y'^2 = 0$:

1. Show that this equation admits of an infinity of solutions, the curves C , of which the equation is of the form $y^2 = f(x)$, $f(x)$ denoting a polynomial in x . Write the general equation of the curves C ; show that through every point of the plane there passes either one or three curves C , and determine the region of the plane where the point ought to be found in order that the number of the curves which pass through it shall equal three, determine the locus of the points such that two of the curves C which pass through one of them are orthogonal.

2. Given the point $A (x=0.5, y=0)$. Let P be that one of the curves C which passes through A and is concave toward the positive part of the axis Ox ; let B be the point of the curve P which has for ordinate $\sqrt{6}$. Suppose Q is that one of the curves C passing through B , and concave toward the negative part of the x -axis; suppose finally that A' is the point where this curve cuts the axis Ox . Calculate the area bounded by the arcs of curves AB, BA' , and the axis Ox .

3. A moving point, starting from A , traverses successively the arc AB of P , then the arc BA' of Q . Its tangential acceleration is constantly equal to its velocity, and its initial velocity is equal to 1; at the point B suppose that the velocity does not change in magnitude, but only in direction. Calculate to the nearest tenth the time taken for the point to traverse the arc ABA' .

4. At the point B , the acceleration of the moving point suffers a discontinuity. Calculate, by its projections on the two axes of coordinates, the geometric variation of the vector-acceleration at the point B .

Group II.

I.

Consider in a plane two rectangular coordinate axes Ox, Oy . A material point M , of mass equal to unity, is movable in a plane under the action of a force (F) of which the projections X and Y on the axes are $X=x, Y=y-4x$, x and y denoting the coordinates of the point M :

1. Form and integrate the differential equations of the motion of the point M .

2. Determine the motion of M in supposing that at the beginning of the time its coordinates are $(\alpha, 0)$ and that its velocity has $-\alpha$ and 2α for projections on the axes. Construct the trajectory (T) corresponding to this motion.

3. Calculate the time taken by the moving point in going from any point M of its trajectory (T) to the point M' , where the tangent to the trajectory is parallel to the radius vector OM .
4. Prove that the hodograph of the motion is a homothetic curve of the trajectory (T) and calculate to the nearest tenth the ratio of homothety.
5. The trajectory (T) passes through the point O . Evaluate, in terms of the abscissa of the point M , the area bounded by the arc of the curve OM and the chord OM , also the volume generated by this area turning about Oy .

II.

Evaluate to the nearest hundredth the integrals:

$$\int_0^{\pi} \frac{dx}{2\cos x + 3} \qquad \int_0^{\pi} \frac{dx}{(2\cos x + 3)^2}$$

NOTE.—For interesting comment concerning the emphasis on analytical geometry in the above examination, compare E. Blutel's report, page 21 (Commission Internationale de l'Enseignement Mathématique. Sous-Commission Française, Rapports, vol. 2).

APPENDIX D.

FRANCE.

AGRÉGATION DES SCIENCES MATHÉMATIQUES.

As there are no mathematical examinations for teachers in any other country to compare in difficulty with those to which the candidate for a French agrégation is required to submit, it seems worth while to give fuller details. I therefore subjoin:

- I. Concours programs (announced 9-11 months in advance). The examinations were on topics selected from the program.
- II. The corresponding examination papers. The four written examinations occurred within five consecutive days. The first paper may seem short for the time allowed (seven hours), but when the enormously high standard in presentation and detail is taken into consideration, this is not found to be the case.

I. PROGRAM FOR THE CONCOURS.

I.—GENERAL PROGRAM IN ANALYSIS AND MECHANICS, 1914.¹

Since the programs for the certificats d'études supérieures vary among the different universities, the jury indicate in the program below the minimum of general knowledge which the candidates for certificats in analysis (differential and integral calculus) and mechanics are supposed to have acquired.

The subjects of the "compositions" in differential calculus, integral calculus, and mechanics will be chosen from Nos. 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, and 15 of this program; their scope will not exceed the standard set by the subjects of problems proposed for the corresponding certificats for the licence.

DIFFERENTIAL CALCULUS AND INTEGRAL CALCULUS.

1. *Fundamental operations of differential and integral calculus:* Derivatives and differentials; simple integrals, curvilinear integrals, integrals of total differentials, double and triple integrals.

2. *Applications of the differential calculus:* Study of functions of a real variable (Taylor's formula, maxima and minima, functional determinants, implicit functions); calculation of derivatives and differentials; change of variables. Order of connection and class of an area.

3. *Applications of the integral calculus:* Processes of integration. Length of an arc of a curve (plane and gauche), areas, volumes. Differentiation and change of variables under the sign \iint Study of the integral $\int_a^b f(x) dx$ when one of the limits or the function becomes infinite. Green's formula. Study of functions represented by certain series. Properties of power series.

4. *Elements of infinitesimal geometry:* "Infinitesimal properties" of plane and gauche curves (curve envelopes, curvature, torsion). Infinitesimal properties of surfaces; surface envelopes, summary of the results on contact transformations; developable surfaces, ruled surfaces; Meusnier's theorem; principal sections. Conjugate lines, lines of curvature, asymptotic lines in any curvilinear coordinates.

¹ *Bulletin administratif du ministère de l'instruction publique*, année 1913, 26 Juillet, pp. 172-174. Although the program for 1915 was published, no examination has been held since 1914.

5. *Elementary functions of a complex variable*: Simple algebraic functions; circular and logarithmic functions.

6. *Theory of analytic functions*: Properties of the integral $\int f(z) dz$. Series of Taylor and of Laurent. Poles, essentially singular points, residues. Reduction of the hyperelliptic integrals.

7. *Differential equations of the first order*: General solutions, particular solutions, singular solutions. Simple types of integrable equations. Integrating factor. Theorem of Briot and Bouquet on the existence of the solutions in the cases where the known functions are analytic.

8. *Differential equations and systems of equations of any order*: General solution, particular solutions, first solutions. Simple types of integrable equations. Linear equations.

9. *Integration of linear partial differential equations of the first order*.

10. *Integration of differential equations (partial or total) of the first order*.

MECHANICS.

11. *Statics*: Composition of forces applied at a point. Attraction of a spherical homogeneous shell at an exterior or interior point. Elementary properties of potential. Reduction of forces applied to a solid body. Conditions of equilibrium of a solid body. Applications to simple machines. Funicular polygon. Suspension bridges. Catenary. Principal of virtual work.

12. *Kinematics*: Velocity, acceleration. Movement of a plane figure in its plane. Representation of the movement by the rolling of a moving curve on a fixed curve. Movement of a solid body about a fixed point. Representation of the movement by the rolling of a moving cone on a fixed cone. Movement of a solid body in space. Helicoidal movement. Relative movements. Theorem of Coriolis.

13. *Dynamics of a particle*: Work. General theorems. First integrals of the equations of motion. Application to the motion of the planets. Movement of a point on a curve or on a surface. Pendulum in a vacuum and in a resisting medium. Conical pendulum. Geodetic lines.

14. *Geometry of masses*: Centers of gravity. Moments of inertia.

15. *Dynamics of systems*: General theorems. First integrals. Energy, stability of equilibrium. Movement of a solid body about a fixed axis. Pressure supported by the axis. Compound pendulum. Movement of a solid body about a fixed point. General movement of a solid body. Law of friction and slipping. Application of the principle of *vis viva* to machines. D'Alembert's principle. Lagrange's equations. Relative motion. Percussions.

16. *Canonical equations*; Jacobi's theorem.

17. *Hydrostatics*: Equilibrium of a fluid mass. Level surfaces. Pressure on a plane wall. Archimedes's principle. Equilibrium of floating bodies.

18. *Hydrodynamics*: General equations of motion of a fluid mass. Bernoulli's theorem. Torricelli's theorem.

LESSONS.¹

Parts of the programs from which are drawn the subjects of the lessons.

¹ Although the program of "Lessons" for 1914 is given on pp. 175-177 of the *Bulletin Administratif* mentioned above, the examinations on these lessons took place after the war had commenced, and I have been unable to find any published list of lesson-subjects selected at that time. As the program of lessons for 1910 and the corresponding list of lesson-subjects were available, they are given in this appendix. There is considerable variation in the program of "Lessons" from year to year.

MATHÉMATIQUES SPÉCIALES.

Series: Series of positive terms; character of convergence or divergence drawn from the study of the expressions: $\frac{u_{n-1}}{u_n}$, $\sqrt[n]{u_n}$, nu_n

Absolutely converging series. Convergence of series, with terms alternately positive and negative, of which the general term decreases constantly in absolute value and tends toward zero. Numerical examples.

General properties of algebraic equations: Number of roots of an equation. Relations between the coefficients and the roots. Every rational and symmetric function of the roots may be expressed rationally as a function of the coefficients. Elimination of one unknown between two equations by means of symmetric functions. Condition that an equation has equal roots. Study of the commensurable roots. Descartes's theorem. Complex numbers. De Moivre's theorem. Trigonometric resolution of the binomial equation.

Functions: Function of a real variable, graphic representation, continuity. Definition and continuity of the exponential function and of the logarithmic function.

Limit of $\left(1 + \frac{1}{m}\right)^m$ when m increases indefinitely in absolute value. Derivative of a function; slope of the curve represented. Derivative of a sum, of a product, of a quotient, of an integral power, of a function of a function. Derivative of a^x and of $\log x$. Use of logarithm tables and of the slide rule. Rolle's theorem, law of finite increments, graphic representation. Functions of several independent variables, partial derivatives. Law of finite increments. Derivative of a compound function. Derivative of an implicit function (admitting the existence of this derivative). Employment of the derivative for the study of the variation of a function; maxima and minima. Primitive functions of a given function, their representation by the area of a curve.

Functions defined by a power series with real coefficients. Interval of convergence: Addition and multiplication. In the interior of the interval of convergence one obtains the derivative or the primitive functions of the function, on taking the series of derivatives or of the primitive functions (functions which extend to the extremities of the interval are not considered). Examples: Developments in series of $\frac{1}{1-x}$,

$\frac{1}{1+x^2}$, $\arctan x$, $\log(1-x)$, $\log \frac{1-x}{1+x}$. Exponential series. Binomial series. The equations $y' = y$, and $y'(1+x) = my$ serve to determine the sum of the last two series. Development into series of a^x , of arcs in x .

Curves whose equation is resolved or resolvable with regard to one of the coordinates: Tracing. Equation of the tangent at a point; subtangent. Normal, subnormal. Concavity, convexity, points of inflexion. Asymptotes. Application to simple examples and in particular to the conics and to those curves of which the equation is of the second degree with respect to one of its coordinates.

Curves defined by the expression of the coordinates of one of their points as functions of a parameter: Tracing. Numerical examples. Curves of the second order and those of the third order with a double point are unicursal.

Curves defined by an implicit equation: Equation of the tangent and of the normal at a point. Tangents at the origin in the case where the origin is a simple point or a double point. Discussion of the asymptotes in the case of numerical examples of curves of the second and of the third order.

Curvature. Envelopes. Developables.

Polar coordinates: The transformation into rectangular coordinates. Equation of a right line. Construction of curves; tangents, asymptotes. Applications (confined

to the case when the equation is solved with respect to a radius vector). Case of the conics.

Gauché curves: Tangent. Osculating plane. Curvature. Applications to the circular helix.

Study of surfaces of the second degree with reduced equation: Condition of the contact of a plane with a surface. Simple problems relative to tangent planes. Normals. Properties of conjugate diameters. Theorems of Apollonius for the ellipsoid and the hyperboloids. Circular sections. Rectilinear generatrices. The surfaces of the second order are unicursal.

DYNAMICS.

1. *Free material point:* Principle of inertia. Definition of force and mass.¹ Relation between the mass and the weight. Invariability of the mass. Fundamental units. Derived units. Movement of a point under the action of a force, constant in magnitude and direction, or under the action of a force issuing from a fixed center: (1) Proportional to the distance; (2) in the ratio inversely as the square of the distance. Composition of forces applied at a material point.² Work of a force, work of the resultant of several forces, work of a force for a resulting displacement. Theory of living force. Level surfaces. Fields and lines of force. Kinetic energy and potential energy of a particle placed in a field of force.

2. *Material point, not free:* Movement of a heavy particle on an inclined plane, with and without friction, the initial velocity acting along the line of greatest inclination. Total pressure on the plane; reaction of the plane. Small oscillations of a simple pendulum without friction; isochronism.

DESCRIPTIVE GEOMETRY.

Intersection of surfaces: Two cones or cylinders, cone or cylinder and surface of revolution, two surfaces of revolution of which the axes are in the same plane.

II.—LESSONS ON THE SUBJECTS OF THE PROGRAMME OF THE SECONDE AND PREMIÈRE (C AND D) AND MATHÉMATIQUES A.

Seconde (C and D).

Algebra: Resolution of equations of the first degree in one unknown. Inequalities of the first degree. Resolution and discussion of two equations of the first degree in two unknowns. Problems; substitution in equation. Discussion of the results. Variation of the expression $ax+b$; graphic representation. Equations of the second degree in one unknown (theory of imaginaries not discussed). Relations between the coefficients and the roots. Existence and signs of the roots. Study of the trinomial of the second degree. Inequalities of the second degree. Problems of the second degree. Variation of the trinomial of the second degree. Graphic representation.

Variation of the expression $\frac{ax+b}{a^2x+b^2}$; graphic representation. Notion of derivative; geometrical significance of the derivative. The sign of the derivative indicates the direction of the variation; applications to very simple numerical examples, and in particular to the functions studied before.

Geometry: Simple notions of homothetic figures. Similar polygons; Sine, cosine, tangent, and cotangent of positive angles less than two right angles. Metrical relations in a right triangle and in any triangle. Proportional lines in the circle. Fourth proportional; mean proportional. Regular polygons. Inscription in a circle of a

¹ It is admitted that a force applied at a material point is geometrically equal to the product of the mass of the point by the acceleration that it impresses on the point

² It is admitted that, if several forces act at a point, the acceleration that they impress on the point is the geometric sum of the accelerations that each of them impresses on it, if acting alone.

square, of a hexagon; of an equilateral triangle, of a decagon, of a quindecagon. Two regular polygons of the same number of sides are similar. Ratio of their perimeters. Length of an arc of a circle. Ratio of the circumference to the diameter. Calculation of π (confined to the method of the perimeters). Area of polygons; area of a circle. Measure of the area of a rectangle, of a parallelogram, of a triangle, of a trapezium, of any polygon. Ratio of the areas of two similar polygons. Area of a regular convex polygon. Area of a circle, of a sector, and of a segment of a circle. Ratio of the areas of two circles.

Première (C and D).

GEOMETRY.

Translation: Rotation about an axis. Symmetry with respect to a line. Symmetry with respect to a point. Symmetry with respect to a plane. This second kind of symmetry is equivalent to the first.

Trihedral angles: Disposition of the elements. Trihedral symmetry. Each face angle of a trihedral is less than the sum of the other two. Limits of the sum of the face angles of a trihedral. Supplementary trihedrals. Applications. Inequalities of the trihedrals.

Homology: Parallel plane sections of polyhedral angles. Area.

Polyhedra: Homothetic polyhedra, similar polyhedra. Prisms, pyramids. Summary of notions on the symmetry of the cube and of the regular octahedron. Volumes of parallelepipeds and of prisms. Volume of a pyramid. Volume of a pyramid truncated by parallel sections. Volume of a truncated triangular prism. Ratio of the volumes of two similar polyhedra. Two symmetrical polyhedra are equivalent. Sphere: Planè section, poles, tangent plane. Circumscribed cone and cylinder. Area and volume.

Mathématiques A.

Arithmetic: Common fractions. Reduction of a fraction to its simplest terms. Reduction of several fractions to a common denominator. Least common denominator. Operations with common fractions. Decimal numbers. Operations (considering the decimal fractions as particular cases of ordinary fractions). Calculation of a quotient to a given decimal approximation. Reduction of an ordinary fraction to a decimal fraction; condition of possibility. When the reduction is impossible, the ordinary fraction can be regarded as the limit of an unlimited periodic decimal fraction. Square of a whole number or of a fractional number; nature of the square of the sum of two numbers. The square of a fraction is never equal to a whole number. Definition and extraction of the square root of a whole number or of a fraction to a given decimal approximation. Definition of absolute error and of relative error. Determination of the upper limit of an error made in a sum, a difference, a product, a quotient, knowing the upper limits of the errors by which the given quantities are affected. Metric system.

Algebra: Monomials, polynomials; addition, subtraction, multiplication, and division of monomials and of polynomials. Equations of the second degree in one unknown. Simple equations which are equivalent. (The theory of imaginaries is not developed.) Problems of the first and second degree. Arithmetic progressions. Geometric progressions. Common logarithms. Compound interest, annuities.

Trigonometry: Circular functions. Addition and subtraction of arcs. Multiplication and division by 2. Resolution of triangles. Applications of trigonometry to various questions relative to the elevation of planes. (The construction of the trigonometric tables is not to be considered.)

Geometry: Inversion. Applications. Peaucellier's cell. Polar of a point with respect to a circle. Polar plane of a point with respect to a sphere. Hyperbola:

Trace, tangent, asymptotes; simple problems on tangents. Equation of a hyperbola with respect to its axes. Plane sections of a cone and of a cylinder of revolution.

Vectors: Projection of a vector on an axis; linear moment with respect to a point; moment with respect to an axis. Geometric sum of a system of vectors; resultant moment with respect to a point. Sum of the moments with respect to an axis. Application to a couple of vectors.

Descriptive geometry: Rabatting. Change of plane of projection; rotation about an axis perpendicular to a plane of projection. Application to distances and angles; distance between two points, between a point and a line, between a point and a plane; the shortest distance between two lines of which one is vertical or at right angles to the plane, or of two lines parallel to the same plane of projection; common perpendicular to these lines. Angle between two lines; angle between a line and a plane; angle between two planes.

Kinematics: Units of length and of time. Motion. Relative motion. Trajectory of a point. Examples of motion. Rectilinear motion; uniform motion; velocity, its representation by a vector. Varied motion, mean velocity; velocity at a given instant, its representation by a vector; mean acceleration; acceleration at a given instant; its representation by a vector. Uniformly varied motion. Curvilinear motion. Mean velocity, velocity at a given instant defined as vectors. Algebraic value of velocity. Hodograph. Acceleration. Uniform circular motion, angular velocity; projection on a diameter. Simple oscillation in a line. Change of the system of comparison. Resultant of velocities. Examples and applications. (Purely geometrical applications are not to be insisted upon.) Geometrical study of the helix. Helicoidal motion of a body. Screw and nut.

Dynamics: Work of a force applied to a material point. Unit of work. Work of a constant force, of a variable force. Elementary work, total work. Graphical evaluation. Work of the resultant of several forces. Theorem of forces acting on a material point. Simple examples.

Cosmography: Moon. Apparent proper motion on the celestial sphere. Phases. Rotation. Variation of the apparent diameter. Eclipses of the moon and of the sun.

II. EXAMINATIONS IN THE CONCOURS.

(1) *Written, 1914.*

MATHÉMATIQUES ÉLÉMENTAIRES.

(June 30. Time, 7 hours: 7 a. m. to 2 p. m.)

Let A_1, A_2, A_3, A_4 be the four vertices of a tetrahedron T . Let a_{ij} represent the length of an edge A_iA_j and O_{ij} the middle point of this edge. Denote also by (A, B) the sphere described on any segment AB as diameter.

1. Calculate one of the geometric products¹ of two opposite edges of T as a function of the edges of the tetrahedron. Find the relation which exists between three of these products with respect to the three pairs of opposite edges.

2. Find the relation which must subsist between the lengths of the edges of T in order that the two lines $O_{1,2}O_{3,4}$ and $O_{1,3}O_{2,4}$ should be at right angles.

3. Suppose d is the distance between the radical planes of each of the spheres (A_1, A_2) , (A_2, A_4) and of the sphere $(O_{1,2}, O_{3,4})$. Let d' and d'' be the analogous corresponding distances in connection with the two other pairs of opposite edges of the tetrahedron. It is required to find the relations which must subsist between the edges of the tetrahedron in order that $d = d' = d''$.

Three vertices of such a tetrahedron being given, find the geometric locus of the fourth vertex. Discuss.

¹ It is recalled that the geometric product of two vectors AB, CD is the product of the lengths of these vectors and of the cosine of the angle between them.

4. Being given any point M , draw the two bisectors of the angle A_1MA_2 . Let $w_{1,1}$ be the middle of the segment cut by these bisectors on the line A_1A_2 ; there are six points $w_{1,j}$. Demonstrate that these six points are in the same plane π which may be associated with the point M . Conversely, any plane π being given, one may establish a correspondence of this kind with two points M and M' which are themselves associated. What position must be given to the plane π in order that the corresponding points M and M' fall together? How must the plane π be displaced when a corresponding point M describes a circle the plane of which passes through the center of the sphere circumscribed about the tetrahedron?

5. Given a tetrahedron T , construct a point M_0 coincident with its associate and situated at the same distance from the three vertices A_1, A_2, A_3 .

Find the surface S , locus of the vertex A_4 when T is deformed, A_1, A_2, A_3 remaining fixed and the ratio $\frac{M_0A_4}{M_0A_1}$ preserving a constant value k .

Let D be any line passing through A_1 . Construct the points of intersection of D and of S , and find the locus of the lines D tangent to S and the locus of the point of contact.¹

MATHÉMATIQUES SPÉCIALES.

(July 1. Time, 7 hours: 7^h. m. to 2 p. m.)

Given a hyperboloid of one sheet whose equation, with respect to its axes, is $\frac{x^2}{\lambda^2} + \frac{y^2}{\mu^2} - \frac{z^2}{\rho^2} - 1 = 0$.

There exist two families of such hyperboloids susceptible of being generated by the intersection of planes at right angles to one another passing respectively through two fixed lines; we may pass from a hyperboloid H of the first family to a hyperboloid H' of the second family by rotation through a right angle about Oz .

Let D, Δ be the fixed lines with respect to H ; D', Δ' the fixed lines with respect to H' ; find the surface loci of D, Δ and of D', Δ' , when λ, μ, ρ vary, ρ remaining fixed.

There exist planes P parallel to the plane xOy cutting these surfaces in two curves which have a common point A situated on Oz , and a common real point B situated in the trihedral $Oxyz$; evaluate the area bounded by the arcs AB of the two curves, also the volume generated by this area when the plane P has one coordinate (\cos) varying from z_1 to z_2 .

II. With a hyperboloid H_1 of the first family, a correspondence may be established with an infinite number of hyperboloids of the second family such that the fixed lines with respect to H_1 and the fixed lines with respect to one of these latter form a gauche quadrilateral; let H_2 be such a hyperboloid; D_1, Δ_1 , and D_2, Δ_2 the fixed lines with reference to H_1 and H_2 respectively; $ABCD$ the quadrilateral formed by these lines; show that the hyperboloid H_2 generated by the intersection of two planes cutting at right angles and passing respectively through the diagonals of the quadrilateral $ABCD$ appertains to the linear point pencil defined by H_1 and H_2 , that the feet a, b, c, d of the altitudes Aa, Bb, Cc, Dd of the tetrahedron $ABCD$ are on the curve of the pencil and that the lines, other than the altitudes, which join the points A, B, C, D to the points a, b, c, d are on a hyperboloid H_3, H_2 , or H_1 .

III. The hyperboloid H_1 being given through a point A in space one may pass two hyperboloids H_2 of the second family defined as has been indicated (II); on what surface S must A be found in order that these hyperboloids H_2 coincide? So also through the point A one may pass two hyperboloids H_3 defined as in (II); on what surface S' must A be found in order that these hyperboloids H_3 coalesce?

Show that H_1 cuts S and S' along the same curve C , and that the intersection of S and S' is composed of the curve C and an imaginary curve.

¹ These questions are solved in *Nouvelles Annales de Mathématiques*, tome 73, oct.-nov., 1914, pp. 491-505.

IV. Construct the projection Γ on the plane xOy of the curve C ; show that Γ is the envelope of circles orthogonal to a fixed circle and find the locus of the centers of these circles.

Find the locus of the middle points of the chords of the curve Γ which passes through the origin.¹

COMPOSITION ON A SUBJECT OF ANALYSIS.

(July 3. Time, 7 hours: 7 a. m. to 2 p. m.)

Being given three rectangular axes of coordinates $Oxyz$, consider the total differential equation (1) $a^2y^2dx + (z - a^2y^2)dy - ydz = 0$ where a denotes a given constant.

I. Determine the surfaces that are solutions of the equation (1). These surfaces S are ruled; study their lines of striction and their asymptotic lines.

For each asymptotic line of a surface S , express the torsion as a function of the angle which the binormal makes with the axis Oz .

II. The surfaces S all satisfy a partial differential equation of the first order (E) independent of the numerical value of the constant a . Indicate how one may, by means of the surfaces S , generate all the surfaces Σ that are solutions of this equation.

The surfaces Σ contain in general the axis Ox ; determine the exceptional surfaces which do not contain it and indicate their nature.

III. Demonstrate that the characteristic curves of the equation (E) are the asymptotic curves of the different surfaces S , and that they form one of the families of asymptotics of the surfaces Σ . Prove that they can be obtained as contact curves of the surfaces Σ with the right conoids having for axes the parallels to Oz meeting Ox .

IV. Determine the second family of asymptotic lines of the surfaces Σ . Prove that the curves of this second family can be obtained as curves of contact of the surfaces Σ with the right conoids having for axes the parallels to Ox meeting Oz .

Determine the ruled surfaces Σ which are distinct from the surfaces S . Indicate their nature.

V. Suppose T is a surface enjoying the property that one of its family of asymptotic lines is formed from the curves of contact of T with the right conoids having for axes the parallels to Ox meeting Oz . Prove that the surface T either is a ruled surface with director plane parallel to the plane yOz , or else satisfies a partial differential equation of the form (E') $F(z - qy, py) = 0$, $\left(p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}\right)$.

Show that in the second case the asymptotic lines of the other family are characteristics of the equation (E') satisfied by the surface T , and that these lines can be obtained as curves of contact of the surface T with the right conoids having for axes parallels to Oz meeting Ox .

If two surfaces, of which each satisfies an equation of the form (E'), are tangent in a point, they have at this point the same total curvature.²

MECHANICS.

(July 4. Time, 7 hours: 7 a. m. to 2 p. m.)

Motion of a marble in a basin.

A homogeneous spherical marble is let go without initial velocity on the interior hemispherical surface of a fixed basin of which the axis of symmetry is vertical and which is concave upward.

Required to study the motion of the marble on the basin.

In particular solve the following questions concerning this motion:

¹ These questions are solved in *Nouvelles Annales de Mathématiques*, tome 74, jan. 1915, pp. 15-29.

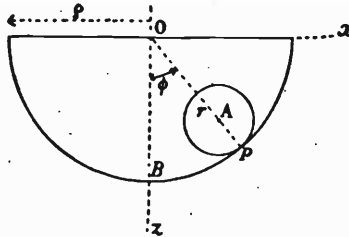
² Idem, tome 73, déc. 1914, pp. 539-547.

I. Suppose first that the marble can be concentrated at a material point A .

1. Indicate the nature of its motion when the bodies in contact are perfectly smooth.

2. Consider then the case where the coefficient of friction f of the marble on the basin is not negligible. Determine first the condition of equilibrium for an initial determinate position A_0 of A . (Denote by ϕ_0 the initial value, comprised between 0 and $\frac{\pi}{2}$, of the angle ϕ of OA with the downward vertical OZ). When this condition is not satisfied, the point A is changed for a time in a determinate sense on a circle Γ . For the moment confine the calculation to that of the expression for the velocity v of A as a function of the angle ϕ during this interval of time T .

II. Then solve the same questions in the case where the radius r of the marble is not negligible.



But when the slipping friction is appreciable, confine the attention (neglecting the rolling friction) to the case where the marble rolls without slipping from the initial instant. Show that for a given initial position, A_0 , of the center A of the marble this condition is necessarily produced if the coefficient of friction f is sufficiently large. (In this case call w the velocity of A , and Ω the instantaneous velocity of rotation of the marble.)

III. Find an approximate value of $\frac{w^2}{v^2}$, w and v being calculated for the same position of OA , very near to the common initial position OA_0 and for the same value of the coefficient f , when the radius r which enters in w^2 is very small.

IV. In a general manner, compare the results obtained in Paragraph I with those of Paragraph II when the radius r of the marble tends toward zero, and the values of ϕ_0 and of f (zero or not) remain the same. It is noted that these results are different; indicate in a few words the origin of this paradox.

V. Return to the case where the marble is reduced to a material point. Show that during the interval of time T the velocity v of the marble supposed rough is less than that, V , which it possesses if it were smooth when it passes through the same point, starting from the same initial position A_0 .

VI. Then continue the study of the motion of the marble reduced to a material point and indicate the various circumstances which can present themselves according as the value of f and the position of A_0 (without now limiting ourselves to the case where the interval of time T when the velocity of A does not change the sense).

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VII. In particular, indicate what happens when the value ϕ_0 is taken equal to 32° , then to 60° , the coefficient of friction having in the two cases the value 0.75, then the value $\frac{1}{2}$.

Notation: Call ρ the interior radius of the basin, O its center and mg the weight of the marble. It will possibly be found useful to set $\tan \theta = f$ and $\tan \theta' = 2f$.

Of the candidates who took the previous examinations, 29 were declared by the jury to be *admissibles*. Further test of each of these by: (1) A numerical calculation; (2) a problem in descriptive geometry (*épure*); and (3) an oral "lesson" occurred on August 21, 1914. This resulted in the final selection of 15 *agrégés*.

The members of the jury for the entire examination were:

Niewenglowski, inspector general of public instruction, president.

Blutel, inspector general of public instruction, vice president.

Cartan, professor at the University of Paris.

Fréchet, professor at the University of Poitiers.

Grévy, professor at Lycée St. Louis.

In 1910 the numerical calculation and *épure* were as follows:

NUMERICAL CALCULATION.

Calculate the integral

$$I = \int_0^{2\pi} \frac{(100 - 10\cos\alpha)^2}{101 - 20\cos\alpha} d\alpha$$

The direct method of primitive functions should be employed and it should be compared with the results obtained by the methods of approximate integration. Indicate, in each of these methods, the number of significant figures that are determined with certainty.

DESCRIPTIVE GEOMETRY (*ÉPURE*).

A parabola P situated in a horizontal plane has for horizontal projection a parabola of which the focus is 96 mm. to the left of the major axis, and 60 mm. below the minor axis of the sheet; its vertex is 102 mm. to the left of the major axis and 54 mm. below the minor axis.

A frontal line D is inclined at 45° to the horizontal plane and it rises from right to left: it passes through the point of the horizontal plane of projection situated at 130 mm. to the right of the major axis and 96 mm. below the minor axis of the sheet. This line is met by the parabola P in a point situated to the right of the profile plane which contains the major axis of the sheet.

Consider the parabolic segment bounded by the arc of the parabola P which contains its vertex and by the parallel to the ground line distant 144 mm.; required to represent in vertical projection only the solid generated by the rotation of this segment about the line D .

Take the minor axis of the sheet for ground line.

SUBJECTS OF THE "LESSONS" IN 1910.

(1) In *Mathématiques Élémentaires*.

Solution of triangles.

Symmetry with respect to a point.

Symmetry with respect to a line. (As in the classe de première.)

Symmetry with respect to a plane.

Variation of $\frac{ax+b}{dx+b}$; graphic representation (program of seconde).

Supplementary trihedral angles. Applications.
 Rabattments. Applications.—Angle between two lines, angle between a line and a plane, angle between two planes.
 Conversion of an ordinary fraction into a decimal fraction. Repeating fractions. (Mathématiques A.)
 Volume of parallelepipeds and prisms. (Do not consider the truncated pyramid or the truncated prism.) (Première.)
 Plane sections of a cone of revolution. (Dandelin's method.)
 Homothetic polyhedra. Similar polyhedra. (Première.)
 Relation between the coefficients and the roots of the equation of the second degree. Applications.
 Summary of notions on the symmetries of the cube and of the regular octahedron.
 Tangents to a hyperbola. Asymptotes. Simple problems on tangents.
 Notion of the derivative, geometric significance of the derivative.
 Application to the variation of simple functions. (Program of seconde.)
 Homothety in plane geometry.
 Inversion (in a plane and in space). Applications.
 Motion of the moon. Phases.
 Problems of the second degree. (Mathématiques A.)

(2) *In Mathématiques Spéciales.*

Small oscillations of a pendulum without friction: Isochronism.
 Asymptotes in polar coordinates: position of the curve with respect to its asymptotes.
 Theory of envelopes in plane geometry.
 Normals to the ellipsoid.
 Curvature of plane curves. Evolutes. Examples (rectilinear coordinates).
 Motion of a point under the action of a force issuing from a fixed center and proportional to the distance.
 Motion of a heavy particle on an inclined plane with or without friction, the initial velocity being zero or directed along the line of greatest slope.
 Number e . Limit of $(1 + \frac{1}{m})^m$.
 Series of positive terms. Criteria of convergence and divergence derived from study of the expressions:

$$\frac{u_{n+1}}{u_n}, \sqrt[n]{u_n}, nu_n^p; \text{ numerical examples.}$$

Complex numbers. $a+bi$. Addition, subtraction, multiplication, division. Geometric representation.
 Use of the derivative for the study of the variations of a function; maxima and minima. Numerical examples.
 Developments into series. Applications to the series of the binomial and of arcs in x .
 Motion of a point attracted by a fixed center in a ratio inversely as the square of the distance.
 Construction of a curve ($\rho=f(\omega)$) in polar coordinates. (It should be assumed that the lessons on tangents and asymptotes have been given.)
 Discussion of the commensurable roots of an equation with integral coefficients. Examples.
 Multiplication of series. Applications.
 Power series. Intervals of convergence. Differentiation. Integration.
 Intersection of a surface of revolution and a cone.
 Trigonometric solution of the binomial equation.
 Infinite branches in the intersection of cones and cylinders. (Descriptive geometry)

Intersection of two surfaces of revolution of which the axes are in the same plane.
Symmetric and rational functions of the roots of an algebraic equation.

The jury for the concours of the agrégation in 1910 consisted of:

Niewenglowski, inspector general of public instruction, president.

Hadamard, professor at the Collège de France, vice president.

Combette, inspector general of public instruction.

Grévy, professor at Lycée St. Louis.

Husson, professor at the university of Chen.

BIBLIOGRAPHY.

A mimeographed pamphlet (16 pp.) published annually from 1901 to 1913, by the Librairie Croville-Morant, Paris, contained the épreuves écrites and subjects of the "Lessons" for the Agrégation des Sciences Mathématiques of the year.

Published solutions of some of the questions of the agrégation during the four years 1910-1913 may be consulted in the following places:

Agrégation 1910.—Composition in Math. Élém.: *Journal de Math. Elem.*, année 35, mai, 1911, pp. 121-124. Composition in Math. Spéc.: (1) *Rev. de Math. Spéc.*, année 21, juin, 1911, pp. 214-221; (2) *Nour. Annales de Math.*, tome 70, janvier, 1911, pp. 72-80; Composition in Analyse: *Nour. Annales de Math.*, tome 70, janvier, 1911, pp. 21-39.

Agrégation 1911.—Composition in Math. Élém.: (1) *Journal de Math. Elem.*, année 36, juin, 1912, pp. 137-140; (2) *Nour. Annales de Math.*, tome 70, août, 1911, pp. 358-373; Composition in Math. Spéc.: *Revue de Math. Spéc.*, année 22, juin, 1912, pp. 512-518; épreuve: *Revue de Math. Spéc.*, année 22, août, 1912, pp. 576-578; Composition in Analyse: *Nour. Annales de Math.*, tome 71, mai, 1912, pp. 231-240; Composition in mécanique: *Nour. Annales de Math.*, tome 71, décembre, 1912, pp. 549-567.

Agrégation 1912.—Composition in Math. Élém.: (1) *Journal de Math. Elem.*, année 37, juin, 1913, pp. 149-154; (2) *Nour. Annales de Math.*, tome 71, décembre, 1912, pp. 506-525; Composition in Math. Spéc.: *Revue de Math. Spéc.*, année 23, sept., 1913, pp. 289-294; Composition in Analyse: *Nour. Annales de Math.*, tome 72, juli, 1913, pp. 316-323.

Agrégation 1913.—Composition in Math. Élém.: (1) *Jour. de Math. Elem.*, année 38, juin, 1914, pp. 143-144; (2) *Nour. Annales de Math.*, tome 73, fév., 1914, pp. 81-89; Composition in Math. Spéc.: (1) *Revue de Math. Spéc.*, année 24, juli, 1914, pp. 553-558; (2) *Nour. Annales de Math.*, tome 73, juli, 1914, pp. 316-324; Composition in Analyse: *Nour. Annales de Math.*, tome 73, jan., 1914, pp. 26-34; épreuve pratique: *Revue de Math. Spéc.*, année 24, juli, 1914, pp. 559-561.

The programs for each year are usually published during the preceding July in the *Bulletin administratif du ministère de l'instruction publique*.

APPENDIX E.

GERMANY.

A. REIFEPRÜFUNGEN.

I. WÜRTTEMBERG.

Questions of a (1) Gymnasium; (2) Realgymnasium; (3) Oberrealschule.

(1) GYMNASIUM:

ALGEBRA AND TRIGONOMETRY.

1. Two places M and N are 119 km. apart; A goes from M to N and travels 20 km. on the first day, 18 on the second, etc.; B starts from N toward M two days after A 's departure and travels 10 km. on the first day, 13 km. on the second, etc. When and where do they meet each other?

2. Solve:

$$\begin{aligned}x - y &= 1 - z. \\x^2 - y^2 &= \frac{1}{4}(1 - z^2). \\x^2 - y^2 &= \frac{1}{4}(1 - z^2).\end{aligned}$$

3. A certain capital brought 4 per cent interest, and, although 420 marks were annually withdrawn, was doubled in 18 years. How large was the capital?

4. Solve the triangle given

$$a=450.34, \rho=92.45, \rho_a=367.52.$$

[ρ is the radius of the inscribed circle and ρ_a the radius of the escribed circle opposite the angle A .]

PLANE AND SOLID GEOMETRY.

1. Construct a triangle given h_a , t_a and the condition that the projection of t_a on a is equal to $b-c$. [h_a =altitude of triangle from angle A ; t_a =length of the median from the angle A .]
2. On a given line segment a , draw two similar rectangles such that the portion of one outside the other is a square.
3. On a given line segment AB a semicircle is described. Find a point X on the diameter AB produced and a point Y on the tangent at A such that XY is divided by the semicircle into three equal parts.
4. A given rectangle $ABCD$ rotates about the side AB as axis. It is required to divide the rectangle into three parts by lines drawn from D such that the solids generated by these parts shall have equal volumes.

(2) REALGYMNASIUM:

HIGHER ANALYSIS.

1. Find the value of $\frac{e^x - e^{-x}}{x \cos x}$, for $x=0$.

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- The total surface of a cylinder is 924 square meters. What must be its height and the length of the radius of its base in order that the cylinder should have a maximum volume?
- For the curve $y=x^2(3-x)$ determine the maxima and minima points and point of inflexion, also the equation of the inflexional tangent. Graph the curve. Calculate the area between the curve and the positive x -axis.
- Determine the area bounded by the curves:
 $y^2 = -\frac{1}{2}(x-8)$ and $(x+4)(y+3)=36$,
 and lying within the first curve.

ANALYTIC GEOMETRY.

- Through the vertices B_1 and B_2 of the minor axis of an ellipse and a focus F_1 , a parabola is drawn whose axis coincides with the major axis of the ellipse. (a) Find the equation of the parabola; (b) Construct the tangents to the parabola at B_1 or B_2 and find their equations.
- The tangent to the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

at a point P meets the Y -axis in P_2 ; and the normal at P meets it in P_1 . One focus F_1 is joined to P_1 , the other to P_2 . What is the geometric locus of the point of intersection of F_1P_1 and F_2P_2 ? (F_1 and F_2 on the X -axis.)

- Consider whether or not the line $y=4x-2$, $z=3x+2$. Cuts the circle
 $x^2+y^2+z^2=30$.
 $3x-5y+z+2=0$.
- Given a sphere with center at the origin and radius equal to 5. A cylindrical surface tangent to this sphere is described with generating line parallel to the line.
 $y=2x, z=3x$.

Find the equation of the cylindrical surface.

(3) OBERREALSCHULE.

DESCRIPTIVE GEOMETRY.

A sphere is surrounded by a plane concentric ring (Saturn); the plane of the ring is parallel to the horizontal plane. Construct the shadow of the sphere on the ring, the shadow of the ring on the sphere; the shadow of the sphere on itself, the shadow of the whole system on the horizontal plane. (The light comes from above, the left and is parallel to the vertical plane.) The various measurements are supposed given.

TRIGONOMETRY.

- In a place A , whose eastern longitude is $\lambda=9^\circ 59'$, it was observed on the 19th of June that the sun rose at 24 minutes, 8 seconds past 4 (middle European time) and culminated at 48 minutes, 14 seconds past 5 (star time). Hence determine the geographical latitude of A , the M. E. T. of sunset, and the declination of the sun if the time equation is $+0' 24''$; and the inclination of the ecliptic is $\epsilon=27^\circ 27' 10''$.
- If $\alpha+\beta+\gamma+\delta=360^\circ$, change the expression.

$$\cot \frac{\alpha}{2} + \cot \frac{\beta}{2} + \cot \frac{\gamma}{2} + \cot \frac{\delta}{2}$$

into a product in which δ is lacking.

PHYSICS.

(Mathematical questions only.)

1. A factory power canal 2 meters broad and 1 m. deep is closed by a heavy lock, of weight $G=110$ kg., and raised upward. How many horsepower must an electric motor possess in order to raise the gate entirely out of the water, in $t=3$ seconds, the coefficient of friction of the gate being $f=0.2$.
2. A plane convex lens stands centered opposite a telescope focused to infinity, and with its plane surface next the telescope. A luminous point lying on the other side of the lens, on its axis, is clearly seen in the telescope if it is at a distance $l=16.3$ from the vertex of the curved surface of the lens. What significance has l for the lens? How can the radius of curvature and the position of the principal points be calculated from this, if the thickness of the lens is $d=0.6$ cm. and the refraction quotient is 1.53?

II. HAMBURG.

(1) WILHELM GYMNASIUM. Spring, 1909.

1. An ellipse with semi-axes $a=7$, $b=5$ cm. is rotated about (i), the axis $2a$ (ii), the axis $2b$, and (iii), a tangent parallel to a . Compare the volumes of the three surfaces of revolution with one another.
2. Given the function $y=\log \frac{a+x^2}{2bx}$.
Find the first and second derivatives, and the value of x for which the function is a minimum.
3. A square pyramid is to be circumscribed about a sphere of radius r , so that its volume shall be a minimum. What are the lengths of the edge of the pyramid and of a side of the square base?
4. How high is the sun on the longest day in Hamburg if it is exactly in the west and at what time (local time) does this occur?

Special exercises.¹

1. $\int_0^a 4\sqrt{x^3} dx = ?$

2. Integrate $se^x \sin x dx$.

(2) REALGYMNASIUM. Spring, 1907.

1. *Projective geometry:* The vertices A and A_1 of the major axis of an ellipse are joined to a vertex B of the minor axis; on these lines BA and BA_1 , perpendiculars are let fall from the points D and D_1 , in which a movable tangent to the ellipse cuts the tangents at the vertices A and A_1 . Show that the locus of the point of intersection Δ of these perpendiculars is a hyperbola, of which the asymptotes are the perpendiculars through A and A_1 to BA and BA_1 .
2. *Analytic geometry:* Given a hyperbola with center O and a line s perpendicular to the transverse axis and cutting it at a distance c from O ; the polar of a point P_1 of s cuts the diameter OP_1 in Δ . What is the locus of Δ as P_1 moves in s ?
3. *Cubic equation:* In order to compare the specific heat of copper and lead one wants two quantities of equal superficial area and weight. If a quantity of lead in the form of a cylinder with ends capped by two hemispheres is available, then a similarly formed copper cylinder with hemispheres capping the ends must be

¹ In practice, as a matter of fact, solutions of special exercises, which are very little harder than others, compensate for failure in one of the required questions. Two faultless solutions of questions are usually necessary and sufficient for the predicate "satisfactory." For "very good," four solutions without error are expected.

formed such that its weight $p=162\pi g$ and surface $O=27\pi \text{ cm.}^2$ are equal to the piece of lead. Determine the height of the cylinder and the radius of the end hemispheres if the specific heat of copper is 9 cm.^{-3} .

4. *Differential calculus:* The curve

$$y^2 = \frac{4}{3}x(x^2 - \frac{1}{2}x^2)$$

has a point of inflexion, a maximum, and a minimum point. Determine these points and draw the curve.

Special exercises.

- (i) Solve the first question by analytic geometry.
 (ii) Solve the second question by pure geometry.
 (iii) The intrinsic equation of Van der Waals is

$$p\theta = \left(p + \frac{a}{v^2}\right)(v - b)$$

where p is constant and the absolute temperature θ is a function of the volume v . For what value of v has the corresponding curve an inflexional tangent? What value must p have in order that the inflexional tangent be parallel to the axis of volume?

- (3) *OBERREALSCHULE* (one of four), spring, 1907.

- Through a payment of 6,000 marks a man 30 years of age purchases life insurance. How much is this insurance on the basis of 4 per cent interest and Schubert's table of vital statistics? What age must the man reach in order that an equal sum would be paid to his heirs from a savings bank in which the 6,000 marks at 3½ per cent had been deposited till the termination of the contract?
- What regular spherical polygons can be made up of equilateral triangles and how large are the radii of the inner and outer tangent circles of these polygons?
- Find the first polar of the curve

$$2x^2 + 2x^2 - 8x - y + 8 = 0$$

with respect to the origin. Draw the curve and discuss it. Find its intersections with the x -axis and the tangents to the curve in these points.

4. What is the value of the difference

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{2 \sin x - \sin 2x}{\cos x} dx - \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\cos x - \tan \pi/4}{\sin x} dx$$

Special exercises.

- (a) Find the locus of the vertex of a triangle with given base $2c$ and sides of given product p^2 ?

- (1) What is the form of this locus according as p is greater than, equal to, or less than c ?

Example for sketching: $c=3$

$$p_1 = 2\sqrt{2}, p_2 = \sqrt{10}, p_3 = 3$$

- (2) In what case is there a double point, and what are the tangents at this double point?
 (3) Find the first polars of the point at infinity on the base of the triangle, of the point at infinity on the perpendicular bisector of the base, and also of the middle of the base with respect to the locus in question. Indicate the significance of these lines for the locus, and draw a sketch for the numerical values $c=3$ and $p=2\sqrt{2}$.

(b) Evaluate the integral

$$\int_{b:c}^{(b+\sqrt{b^2+ac}):c} \frac{dx}{\sqrt{a+2bx-cx^2}}$$

B. LEHRAMTSPRÜFUNGEN.

WURTEMBERG.

I. Questions in the Mathematics-Physics Division.

HIGHER ALGEBRA.

1. Given the equation

$$x^4 + a_1x^3 + a_2x^2 + a_3x + a_4 = 0$$

whose roots are all different from one another. Find the equation of the fourth degree in y , which is satisfied by the unsymmetric functions of roots $y = x_1 + x_2 + x_3$, etc. Prove that the discriminants of the two equations are identical.

2. According to the general interpolation formula find an integral algebraic function y_x (fourth order in x), which for
 $x=0, 2, 3, 6, 9$
 $y=0, 0, 3, 105, 630$

(Indicate also a second method, without details; evaluate the determinants which here enter.) Further show that the value y_x can be changed into an "Überzweizahl," if for numbers of the form $\binom{n}{r}$ the name "Über- r -Zahl" is employed. Write down the values of the numbers y_{-1} to y_0 . Further calculation of y_x checked by these values; calculation of the sum y_{-1} to y_x . Checks.

HIGHER ANALYSIS.

1. Evaluate and give the geometric meaning of the double integral

$$\int_0^{2a} \sqrt{\frac{2a-x}{4a-x}} dx \int_0^{\sqrt{x(4a-x)}} [\sqrt{x(4a-x)} - y] dy;$$

give a plan and a variation sketch.

$$2. y = x \left[\frac{dy}{dx} - \left(1 - \left(\frac{dy}{dx} \right)^2 \right) \right]^{\frac{1}{2}}.$$

$$3. 12xy^2 \frac{\partial^2 z}{\partial x^2} + 3y(x^2 - 2y^2) \frac{\partial^2 z}{\partial y^2} = 2.$$

4. Derivation of the properties of the logarithm and of the exponential function in the complex field. Give three examples.

ANALYTIC GEOMETRY.

1. Discuss the curve

$$x^3 - x^2y - 2x^2y^2 + 2x^2y^3 + xy^4 - y^5 - 2x^4 + 4x^3y - 4xy^3 + 2y^4 - 4x^2y + 3xy^2 - y^3 = 0.$$

2. Find the equation of the tangent-surface of the space curve $x=\lambda, y=\lambda^2, z=\lambda^3$ and determine the orthogonal trajectories of the tangents on the surface. Without calculation, what may be stated with regard to the lines of curvature, the asymptotic lines, and the geodesic lines of the surface? (If time is lacking it is only necessary to write the differential equation of the trajectories.)

SYNTHETIC GEOMETRY.

1. Determine the center of a hyperbola given four points and the direction of an asymptote.

2. To four planes through a point O , no three of which cut in a straight line, the normals in O are given. How may the normal at O to any plane through O be con-

structed through simply joining and intersection, and in particular without the further use of a right angle?

DESCRIPTIVE GEOMETRY.

In a vertical plane of projection a semi-ellipse is given, of which the shadow is a semicircle. It is subjected to a rotation in space about a vertical axis. Construct the normal curve and the meridian of the surface of rotation so constructed, i. e., the section of any plane (1) perpendicular to the axis of rotation or (2) through the axis. (Suppose the proportions given.) Give a free drawing in india ink of the two curves sought, also a short description of the corresponding construction.

ANALYTIC MECHANICS.

1. On three mutually perpendicular weightless sticks, rigidly fastened together at a point O , "mass-points" P , Q , R are placed at distances a , b , c from O .

The system is turned about an axis through O which at a given instant, when the angular velocity is ω , makes angles α , β , γ with the three sticks.

How great is the *vis viva* of the turning motion of the system at this instant? Through what single force at O and what pair of forces (what axis-moment) may the three centrifugal forces be replaced?

Discussion of the results for the case $\alpha = \beta = \gamma$.

2. The velocities of three noncollinear points of a rigid body moving with freedom of the first order, are given for a certain instant as vectors, i. e., in size and direction. Show how to find, either by drawing or by calculation, for the same instant: (a) the velocity of a fourth point of the body, also considered as a vector, and (b) the central axis or rotation axis, i. e., the locus of the points of the body which, for the instant, of all of its points has the least velocity.

TRIGONOMETRY AND MATHEMATICAL GEOGRAPHY.

1. On two points A and B of equal height and 25 meters from each other, rests a smooth thin band of steel which sags 50 centimeters in the middle. How many millimeters (to the nearest tenth of a millimeter) is the band longer than 25 meters?

The calculation is to be carried through, and the exactitude of the result proved, without employing any tables. (The calculation is to be carried through in numbers.)

2. On a simple rod standard a vertical reference-plane F is determined by a strong white endless thread passing through two rings O_1 and O_2 , and stretched with the help of a hanging stone O_3 . The azimuth of the two directions of the thread differs by exactly 180° . The azimuth is not exactly known, but it corresponds about (according to the compass within 5°) to the prime vertical.

At the point of observation (in middle Germany), determining the northern latitude ϕ on an evening toward the end of November, 1895, the following times of transit through the plane of the thread are observed in sidereal time:

γ Lyra (about westward) $S_1 = 22^h 29^m 59^s$,

γ Andromeda (about eastward) $S_2 = 22^h 35^m 45^s$.

The apparent AR [right ascension] and δ [declination] of the two stars on the evening of observation were:

γ Lyra $\alpha_1 = 18^h 55^m 2^s$, $\delta_1 = +32^\circ 32' 3''$,

γ Andromeda $\alpha_2 = 1^h 57^m 32^s$, $\delta_2 = +41^\circ 50' 1''$.

What is the northern latitude of the place of observation? What was the azimuth of the plane of the thread? What kind of stars are to be chosen for such determination of the northern latitude by transits through one and the same vertical first eastward and then westward? Calculation of ϕ in numbers is desired.

(The candidate is allowed a logarithm table.)

THEORETICAL PHYSICS.

1. Discussion of the physical foundations of the Newtonian definition of the quantities "force" and "mass."
2. Derivation of the formula of an adiabatic curve.
3. Foundations of a theory of plane diffraction grating for parallel light rays.
4. How and with what exceptions is Helmholtz's law of induction derived on the basis of the law concerning the conservation of energy?

II. *Mathematical Questions of the Chemistry-biology Division.*

ALGEBRA AND LOWER ANALYSIS.

1. A merchant buys wares for a certain sum, has in addition 5 per cent expenses, and sells them again for 504 marks, and thereby gains a twentieth part of the purchasing price. What did this amount to?
2. A left a fortune of 100,000 marks. From this his children must receive 6,810 marks annually for 10 years from the first payment within a year after his death. The capital after 10 years is to be devoted to school purposes. How large will this amount be?
3. Express the fraction $\frac{1}{13 \cdot 23}$ as the sum of two positive fractions with denominators 13 and 23. What are possible solutions?
4. In the equation $x^4 - 11x^3 + px^2 + qx - 60 = 0$ the root $2+i$ is given. Find the other roots and the coefficients p and q . (Check by means of Horner's method).

DIFFERENTIAL AND INTEGRAL CALCULUS.

1. $\lim_{x \rightarrow 0} \frac{e^x - e^{\sin x}}{x - \sin x} = ?$
2. Express $y = \log(x + \sqrt{1+x^2})$ as a power series in x and discuss the convergence of the series.
3. Given the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} - 1 = 0$.

Through the origin and with different points of the hyperbola as centers, circles are described. Find the equation of the envelope of these circles.

ELEMENTARY GEOMETRY.

1. Describe a circle which touches a given line in a given point P and cuts a given circle K at the ends of a diameter. (Analysis, construction, proof.)
2. Given a point and two lines L and L' , which cut one another in A . On L find a point X such that the perpendicular XY from X on L' is a mean proportion between AY and PX . (Construction and proof.)
3. Given a right circular cone of which an axial section is an equilateral triangle. Produce the surface beyond the circular base, such that the whole surface of the added conical shell is to the surface of the whole cone as 5 is to 6 (including construction of the calculated result).
4. B and C are the middle points of two spheres of radii r_1 and r_2 (10 and 14 cm.). To an observer at A the spheres appear under angles of sight S_1 and S_2 ($3^\circ 37' 20''$ and $4^\circ 16' 30''$). Angle BAC is $71^\circ 4' 40''$. How great is the distance between B and C , and how large are the angles ABC and ACB ?

ANALYTIC GEOMETRY.

1. The curve with equation

$$x^3 - 2x^2y + 2y^3 - y^2x - 4y^2 + 2xy - x + 2y = 0$$

breaks up into three lines, of which one is $x - 2y$. What are the equations of the other two? What are the coordinates of the vertices of the corresponding triangle; find its area.

2. In the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} - 1 = 0$ a diameter with slope μ is given. What is understood by a diameter conjugate to this diameter and how is the slope μ' of this diameter derived? How large are the semi-diameters corresponding to these two directions?

3. Discuss and sketch the curve with equation

$$x^3 + y^2x - 2x^2 + y^2 = 0.$$

4. Given a circle of radius r and a line which is at a distance a from the middle point of the circle. A second circle touches the line in its intersection with the line drawn through the middle point of the first circle perpendicular to the line; the common tangents of the circles touch the second circle in points, the locus of which is required. (The given line is to be taken as x -axis, and the foot of the perpendicular as origin of coordinates.)

DESCRIPTIVE GEOMETRY.

An ellipse is projected horizontally as a circle of 10 cm. diameter, vertically as a line (30° toward the right with the base line). The horizontal trace of a given plane makes an angle of 60° , the vertical trace of 45° ; both toward the right with the base line. Construct the shadows of the ellipse on these planes. (Both projections of rays of light coming from the left make angles of 45° with the base line.)

The two projections of the shadows are to be constructed independently of one another; at any given point construct the tangent and find the nature of the curve in order that its conjugate diameters can be determined.

EXPERIMENTAL PHYSICS.

1. A locomotive weighing 20,000 kilograms moves on a track $1\frac{1}{2}$ meters wide and its center of gravity is $1\frac{1}{2}$ meters above the rails; what is the greatest velocity that the locomotive may attain in order that, on a curve of 80 meters radius, it shall not leave the rails? What is the maximum velocity, if the outer rail be so raised that the plane of the rails is inclined to the horizontal plane with an angle of 5° ?

2. For determining the temperature of a smelting furnace, a platinum sphere of 100 grams is put in it and then thrown into a mixing calorimeter which contains 800 grams of water at 10° C. What is the temperature of the furnace if the brass calorimeter tub weighed 250 grams and the final temperature reached 14.8° ? (Specific heat of brass, 0.0926; of platinum, 0.0326.)

3. Two biconvex lenses with focal lengths $f_1 = 4$ cm. and $f_2 = 6$ cm., are arranged from left to right, such that the distance of their optical middle point amounts to $\delta = 1$ cm.; the thickness of the lenses may be neglected. To the left of the first lens is a luminous substance AB 1 cm. high. Construct the picture of the object which is thrown through the pair of lenses and also determine the distance b_1 of the picture from the second lens. How great is the common focal distance f of the system of lenses counted from the second lens; and what advantage is there in such a combination of lenses over a simple lens with the same focal distance?

APPENDIX F.

JAPAN.

The following mathematical papers were set in Tokyo for the twenty-fifth examination for teachers' licenses, in 1911.

PRELIMINARY EXAMINATION QUESTIONS.

ARITHMETIC (3 hours).

1. Find three fractions, A , B , and C equal to $\frac{3}{5}$, $\frac{4}{7}$, and $\frac{5}{11}$, respectively, such that A 's denominator is equal to B 's numerator, and B 's denominator to C 's numerator. Find the simplest forms of such three fractions.

2. A certain company, dividing its capital in the ratio of 3 : 5 : 7, carried on its business in three divisions. At a semiannual settlement it was found that the first division had made 2,600 yen, and the second had earned 8 per cent a year on its capital, but that the third had suffered a loss of 5 per cent a year of its capital. However, the net result was found to be a gain of 6 per cent a year on the total capital. What was the amount of the capital?

3. A steamer, bound for a certain port, had its engine damaged when one-fifth of its voyage had been completed. As it had to reduce its speed by 10 knots for the rest of its course, the average speed was found to be less than the first by 4 knots. What was the initial speed?

4. By evaporating 600 grams of water containing 3 per cent of salt, one containing 5 per cent of salt was to be obtained. It was found, however, that 70 per cent of the water had already evaporated. How much water containing 3 per cent of salt must be added in order to obtain the solution of required strength?

5. Of a cylindrical vessel holding one *shō*, the height and diameter of which are equal, find the height to the hundredth place.

ALGEBRA (3 hours).

1. When $a+b+c=0$, prove

$$\left(\frac{b-c}{a} + \frac{c-a}{b} + \frac{a-b}{c}\right) \left(\frac{a}{b-c} + \frac{b}{c-a} + \frac{c}{a-b}\right) = 9.$$

2. Solve and discuss the following simultaneous equations:

$$ax - by - z + 1 = 0,$$

$$x + y - az - b = 0,$$

$$x + y - z + 1 = 0.$$

3. Cut a triangle and a rectangle, having equal bases on a straight line, by another straight line [parallel to it] so that the sum of the areas cut out between the parallel lines shall be equal to the area of the triangle. Find the distance between the parallel lines.

4. In how many different ways can 10 balls be arranged in a straight line, provided that 2 special balls must in all cases be placed so as to occupy alternate positions?

5. Let a_1, a_2, a_3, \dots be an arithmetic progression, and b_1, b_2, b_3, \dots be a geometric having all its terms positive. Prove that a_n is not greater than b_n , if $a_1 = b_1$ and $a_2 = b_2$.

¹These words do not occur in the original.

²It is not clear what is meant by "alternate positions."

GEOMETRY (3 hours).

1. Let two circles touch internally at A . From any point P in the circumference of the external circle draw a tangent PM to the internal one, and prove that $PA:PM$ is constant.
2. Given a vertical angle, the radius of the inscribed circle, and the area, construct the triangle.
3. The vertex A of the rectangle $ABCD$ is a fixed point, and B and D are on the circumference of a fixed circle. Find the locus of the point C .
4. Find the limit of the position of a point, the ratio of whose distances from two fixed points is less than a given ratio.
5. Of a quadrilateral whose four vertices are not all in one plane, three are fixed and one moves along a straight line. Find the locus of the intersection of the lines joining the middle points of its opposite sides.

FINAL EXAMINATION QUESTIONS.

ARITHMETIC, ALGEBRA, AND GEOMETRY (written).

Part I (3 hours).

1. The sum of a certain irreducible fraction and its reciprocal is equal to $\frac{138794}{400115}$. Find the irreducible fraction.

2. Eliminate x , y , and z from

$$\frac{x}{a} + \frac{a}{x} = \frac{y}{b} + \frac{b}{y} = \frac{z}{c} + \frac{c}{z},$$

$$xyz = abc,$$

$$x^2 + y^2 + 2(ab + bc + ca) = 0.$$

3. If a , b , p , and q be real, prove that the following equation has real roots,

$$\frac{p^2}{a+x} + \frac{q^2}{b+x} - 1 = 0.$$

4. Solve the following inequality, $x - b > \sqrt{a(a-2x)}$, where a and b are positive, and $\sqrt{\quad}$ represents the positive square root.
5. Prove that the following three equalities are consistent with one another,

$$x = 10^{1-\log z}, \quad y = 10^{1-\log x}, \quad z = 10^{1-\log y},$$

\log representing the common logarithm.

Part II (3 hours).

1. If rectangles $ABDE$ and $ACFG$ be externally constructed on the two sides AB and AC of the right angle A of a right-angled triangle ABC , prove that the straight lines BF and CD intersect with each other on the perpendicular from A to the hypotenuse BC .
2. Draw a circle with its center on a straight line passing through the center of a given circle, intersecting this circle at right angles and passing through a given point.
3. Of a triangle ABC , the vertex A is a fixed point on an edge of a trihedral angle and the other two vertices B and C move respectively along two other edges. Find the locus of the center of gravity of the triangle.

ARITHMETIC (oral).

A boat is rowed over a certain distance, when there is no tide, in 24 hours. With the tide, however, the same distance can be rowed over in 15 hours. Against the tide the boat can be made to go over 32 knots¹ in 2 hours. Find the speed, accordingly, when it is rowed with the tide.

ALGEBRA (oral).

Solve the following simultaneous equations,

$$(1+2k)x - (1+k)y = 1-k,$$

$$3(1+k)x - (3+k)y = 3+k.$$

GEOMETRY (oral).

Draw a straight line meeting two straight lines not in the same plane and normal to a given plane.

TRIGONOMETRY (written).

Theory (3 hours).

1. Solve and discuss the following equation,

$$\sin 3x = m \sin x.$$

2. Eliminate θ and φ from

$$c \sin \theta = a \sin (\theta + \varphi),$$

$$a \sin \phi = b \sin \theta,$$

$$\cos \theta - \cos \varphi = 2m.$$

3. Find the maximum value of

$$\frac{\operatorname{cosec}^2 \theta - \tan^2 \theta}{\cot^2 \theta + \tan^2 \theta - 1}.$$

4. If the length of three bisectors of the three angles A, B, C of a triangle ABC be respectively equal to p, q, r , prove that

$$\frac{\cos \frac{A}{2}}{p} + \frac{\cos \frac{B}{2}}{q} + \frac{\cos \frac{C}{2}}{r} = \frac{1}{a} + \frac{1}{b} + \frac{1}{c}.$$

5. Having given one angle, the perimeter, and the radius of the circumscribed circle of a triangle; solve the triangle.

Application (3 hours).

When the three sides of a triangle are known to be respectively,

$$a = 750.74 \text{ m.}, \quad b = 596.42 \text{ m.}, \quad c = 204.68 \text{ m.}$$

compute the three angles and the area.

ANALYTIC GEOMETRY (3 hours).

1. Given a point $(1, 1)$ and a straight line $3x + 4y - 6 = 0$, the axes being rectangular. Form the equation of the curve of the second degree, having the point and the straight line for its corresponding focus and directrix and 5 for its eccentricity, and reduce it to the standard form.

¹Such is the original.

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2. Let N be the point of intersection of the normal at any point M on an ellipse and its major axis. Prove that the orthogonal projection of MN on the line passing through M and one of the foci is constant.
3. Prove that the four vertices of a parallelogram circumscribing an ellipse and its two foci are on the same equilateral hyperbola.
4. Given an ellipse and a circle concentric with each other, the radius of the circle being equal to the sum of half the major axis and half the minor axis of the ellipse. Prove that the locus of the point of intersection of the two normals to the ellipse at the points at which two tangents are drawn to the ellipse from any point on the circle is a circle.
5. Let N be the point of contact at which a tangent is drawn from the center M of a fixed circle to the circumscribed circle of a triangle self-conjugate with respect to the fixed circle. Prove that MN is constant.

DIFFERENTIAL AND INTEGRAL CALCULUS (4 hours).

1. If $f'(x)$, $\varphi(x) - f(x)$, $\varphi'(x) \neq 0$ within the interval $a \leq x \leq b$, and $f(a) = 0$, $f(b) = 0$, then prove that $\varphi(x)$ will become zero within the given interval at least once. Here $f'(x)$ and $\varphi'(x)$ are continuous within the given limits.
2. Let Y be the point at which the line passing through any point X on the diagonal AC of a parallelogram $ABCD$ and the vertex B intersects the side AD or its extension. Find the minimum of the sum of the areas of the two triangles AXY and BXC .

3. Take z as the function of two independent variables x and y : substitute

$$x = r \sin \theta \cos \varphi, \quad y = r \sin \theta \sin \varphi, \quad z = r \cos \theta$$

in $\sqrt{1 + \left(\frac{dz}{dx}\right)^2 + \left(\frac{dz}{dy}\right)^2}$; taking θ and φ as independent variables eliminate x, y, z .

4. $B(l, m)$ represents $\int_0^1 x^{l-1}(1-x)^{m-1} dx$, l and m being positive.

Prove that

$$B(l, m) = \frac{l+m}{l} B(l+1, m).$$

5. Find the whole length of the space curve represented by the equations

$$ax = z(b+z), \quad a^2(x^2 + y^2) = b^2 z^2,$$

a and b being positive.

6. Take $x = \varphi(u, v)$ and $y = \theta(u, v)$, and change the variables of integration in

$$\iint dx dy \text{ from } x, y \text{ to } u, v.$$

¹ In the report this equation is given as $y = r \sin \phi$.

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