DEPARTMENT OF THE INTERIOR
BUREAU OF EDUCATION

BULLETIN, 1924, No. 18

INTRODUCTION OF ALGEBRA INTO AMERICAN SCHOOLS IN THE EIGHTEENTH CENTURY

By

LAO GENEVRA SIMONS

WASHINGTON
GOVERNMENT PRINTING OFFICE
1924

ADDITIONAL COPIES

OF THIS PUBLICATION MAY BE PROCURED FROM
THE SUPERINTENDENT OF DOCUMENTS
GOVERNMENT PRINTING OFFICE
WASHINGTON, D. C.
AT
15 CENTS PER COPY

CONTENTS

PREFACE

Certain periods in the making of history have been deficient in contemporary chronicles. This is notably true in the history of American education during the eighteenth century. Such history presents lessons to the educator of a later generation. As we follow the growth of the American people from the status of settlers in a new country to that of a distinctive nation with its own life to provide for by training and education, we are led to an understanding of the American character and civilization of our own day. This understanding is necessary for all those who are engaged in the attempt to prepare boys and girls to take their places in the present social structure.

The history of education is made up in part of accounts of various subjects which have developed into courses of study. Mathematics of some kind has always been included in such courses. In the American Colonies arithmetic was an important subject for practical reasons. It was needed for trade and commerce. With sailing vessels plying between Europe and America and the only means of communication with the "homeland," navigation and all the kinds of sailing that had to be put to daily use came to be a continuation of the course in arithmetic. Astronomical observations were an important feature in laying out a course at sea, and so astronomy is found in connection with arithmetic. Some elementary trigonometry, logarithms, and geometric constructions played a necessary part in the calculations incident to both navigation and astronomy. With this list the practical uses of mathematics in that day are exhausted.

It is the purpose of this study to show that algebra, another branch of mathematics, entered into the American education of the eighteenth century, and to show further that we must seek some other reason for its presence than a practical need for it.

The research connected with this work has been made from original sources found in many libraries, both public and private, in

the East. It would be a pleasure, if it were possible, to acknowledge in detail the cordial helpfulness that has been extended in every one of these libraries. In particular the writer is indebted to Miss Isadore G. Mudge, reference librarian of Columbia University Library, for her efforts in many directions.

To Prof. David Eugene Smith, of Columbia University, who suggested the problem, and whose interest and appreciation have been unfailing, the writer acknowledges inspiration in this study as well as in her whole professional life.

LAO GENEVRA SIMONS.

Hunter College of the City of New York.

INTRODUCTION OF ALGEBRA INTO AMERICAN SCHOOLS IN THE EIGHTEENTH CENTURY

Chapter I

FOREIGN INFLUENCES LEADING TO THE INTRODUCTION OF ALGEBRA INTO AMERICAN EDUCATION

English influence.—Education in the American Colonies for more than a century after its beginnings was an attempt at a reproduction of the education of the countries from which our forefathers came. The influence of foreign universities appears in every aspect of the school life before the American Revolution.

The first colleges were modeled entirely on the English universities, so far as the limited resources of the founders allowed. Courses of study, textbooks, and organization were for a long time almost exclusively English. Presidents went to England to raise money to carry on the work on the home field. College professors were either imported from England or Scotland, and in many instances returned there, as was the case with a long line of men at the College of William and Mary, or they traveled to England to obtain the education necessary to fill their positions.

Professors educated at Oxford or Cambridge or any other university must have been interested in the application of their foreign experiences on their return and would naturally have transplanted the traditions of the day, so far as it was possible, to the American institutions to which they came. At the same time they were engaged in correspondence with foreign leaders, and so all through the period under consideration reflections of the intellectual life abroad will be apparent.

English algebra.—From the middle of the seventeenth century works on algebra were being published, and prominent teachers were presenting the subject. In 1707 Sir Isaac Newton's work on algebra and the theory of equations, the *Arithmetica Universalis*, appeared; and it was followed by an English translation in 1720. With the unparalleled reputation of Newton and the genius of his discoveries this work must have taken a strong hold on the algebraically minded teacher. It seems inevitable that the first quarter of the eighteenth century should have seen some beginnings of algebra as well as fluxions in American schools and colleges.

The earliest formal piece of work in American algebra growing out of this English influence is found in a set of notebooks prepared under the direction of a native American who went to England to continue studies begun in his undergraduate days.

Chapter II

ALGEBRA AT HARVARD IN 1730

Arithmetic notebooks.—The custom of keeping notebooks for school subjects was well-nigh universal during the eighteenth century. One of these subjects was arithmetic, and arithmetic notebooks are commonly found in libraries and collections of Americana. They are sometimes so exquisitely prepared by hand that they appear to be the work of a craftsman in printing. One [1] of the earliest of these books is that of Robert Hale, and it bears the colophon: "The End of this Treatise of Arithmetick, begun by me Robert Hale, Feb. 23, 1718/9 & Ended Nov. 5, 1719." It contains treatises on arithmetic, logic, metaphysics, physics, geography, and geometry.[2] Only occasionally is a section on algebra numbered among the contents of such a book.

Algebra notebooks.—With the comparative rarity of algebraic material, the discovery [3] of a complete manuscript notebook on algebra as early as 1739 greatly enriched our knowledge of that subject in America. The manuscript referred to was found in the museum housed in the old jail in York Village, Maine. It is a manuscript on algebra by Samuel Langdon, at one time president of Harvard College; and the original of it was prepared, as later evidence will show, by Isaac Greenwood, for several years professor of mathematics and natural philosophy of the same institution.

The value of this manuscript was considerably enhanced by the further discovery, in the Manuscripts Americana at Harvard University, of another manuscript on algebra which so closely resembles the Langdon one as to leave no doubt that the two notebooks were taken from the same original. The Harvard manuscript was written

[1] American Antiquarian Society.

[2] Robert Hale was graduated from Harvard in 1721, and this probably represents the mathematical work that he took during his freshman or sophomore year.

[3] David Eugene Smith. "A Glimpse at Early Colonial Algebra." *School and Society*, Jan. 5, 1918. This article fully describes the manuscript, and the description herein contained will necessarily cover the same ground, although it is the result of a study made by the author.

by James Diman, who was for a time the librarian of Harvard College, and contains the date 1730.

The existence of two such manuscripts is a matter of great importance in the history of American mathematics. One manuscript might have been the work given to a private pupil by the professor at Harvard, but two similar manuscripts written at different times during the same professorship afford unmistakable proof that such work was being taught during the period of this particular man.

The Langdon manuscript consists of 75 numbered pages, with 2 unnumbered pages forming a front cover and 18 unnumbered pages at the end. The latter contain no notes, except one leaf. The pages are 14.7 cm. by 18.7 cm., and the book consists of 48 leaves, three-fourths of one leaf having been cut out by the author. On the front cover appears the inscription: "Samuel Langdon's Book, July 25, 1739," and on the reverse of this leaf are the words: "Algebra by Isaac Greenwood, M. A. Began July 25, 1739." A colophon reads: "Finished writing Algebra August 17, 1739. Algebrae Finis."

The Diman manuscript consists of 4 unnumbered pages followed by 125 numbered pages, 16 cm. by 19.3 cm. In the upper right-hand corner of the first page is the inscription: "James Diman's Book 1730/31." In large writing on this same page appears the title: "Algebra or Universal Mathematics reviewed 1738 with Notes and Additions." The third page has the following note: "Books perused in y^e^ review of my Algebra made in 1738. 1. Harris Lexicon Technicum. 2. Chambers Cyclopaedia. 3. Wolfius Elementa Matheseos Univers." The work ends on page 125 simply with "Finis." The apparent difference in the lengths of the two books is due to a difference in the size and closeness of the writing and not to a difference in the amount of subject matter, the Diman manuscript having only six pages not found in Langdon.

Isaac Greenwood.—Interest always attaches to the personalities of men connected with any work, and so we shall first give some account of the three men whose names appear on these two manuscripts. Isaac Greenwood was born in Boston in 1702, and was graduated at Harvard in 1721. Three years later he received the A. M. degree. In 1727 a professorship of mathematics was created at Harvard through the benefaction of Thomas Hollis, of London. As is still and probably ever will be the custom, the college authorities sought among their own graduates a brilliant student to fill the chair. They were unanimous in their choice of Greenwood. A visit to England about this time enabled him to qualify himself more perfectly for the expected appointment. He received it, and an entry in the "Minutes of the

College Officers" under date of 1728, February 13, reads, "Mr. Isaac Greenwood installed professor."[4]

In 1729 Greenwood published anonymously a work on arithmetic.[5] This was the second arithmetic to appear in the American Colonies and the first by a native American. The college notes above referred to contain another entry, "N. B. dismissed July 13, 1738, died at Charleston, S° Carolina, Octo. 22, 1745." His dismissal came as the result of his having been "guilty of many acts of gross intemperance, to the dishonor of God and the great hurt and reproach of the society." There seems to have been no question about Greenwood's abilities. Both the cause of education which he was serving so well and his own career in life were the losers by his weakness of will and unfortunate appetites.

Samuel Langdon.—Samuel Langdon[6] was born in Boston in 1723, entered Harvard in 1736, graduated in 1740, and received the degree of A. M. in three years. He studied divinity for a time at the college itself, and in 1745 was licensed to preach. The University of Aberdeen conferred upon him the honorary degree of S. T. D. In 1774 Langdon became president of Harvard College, and his name appears for the first time on the Harvard commencement program for 1776. It must have been a satisfaction to have dedicated to him as president the first sheet of commencement theses to appear after the Declaration of Independence, for he was heart and soul in sympathy with the principles of the American Revolution. These patriotic sympathies led in the end to his forced resignation as president in 1780. He spent the remainder of his life as minister at Hampton Falls, N. H., where he died and was buried in 1797.

James Diman.—James Diman,[7] was born in 1707 in East Hampton, Long Island. In 1730 he graduated from Harvard, and in 1733 received the degree of A. M. He was appointed librarian in 1735 and served until the spring of 1737. In February of that year he was called to the pastorate of the Second, or East, Church in Salem, where his ministry continued for over 50 years. We see, therefore, that Diman took part or all of his college course during Greenwood's term of service and later was associated with him in an official capacity.

The earlier date, 1730–31, on the Diman manuscript occurred during Greenwood's first years in his professorship, and hence there is

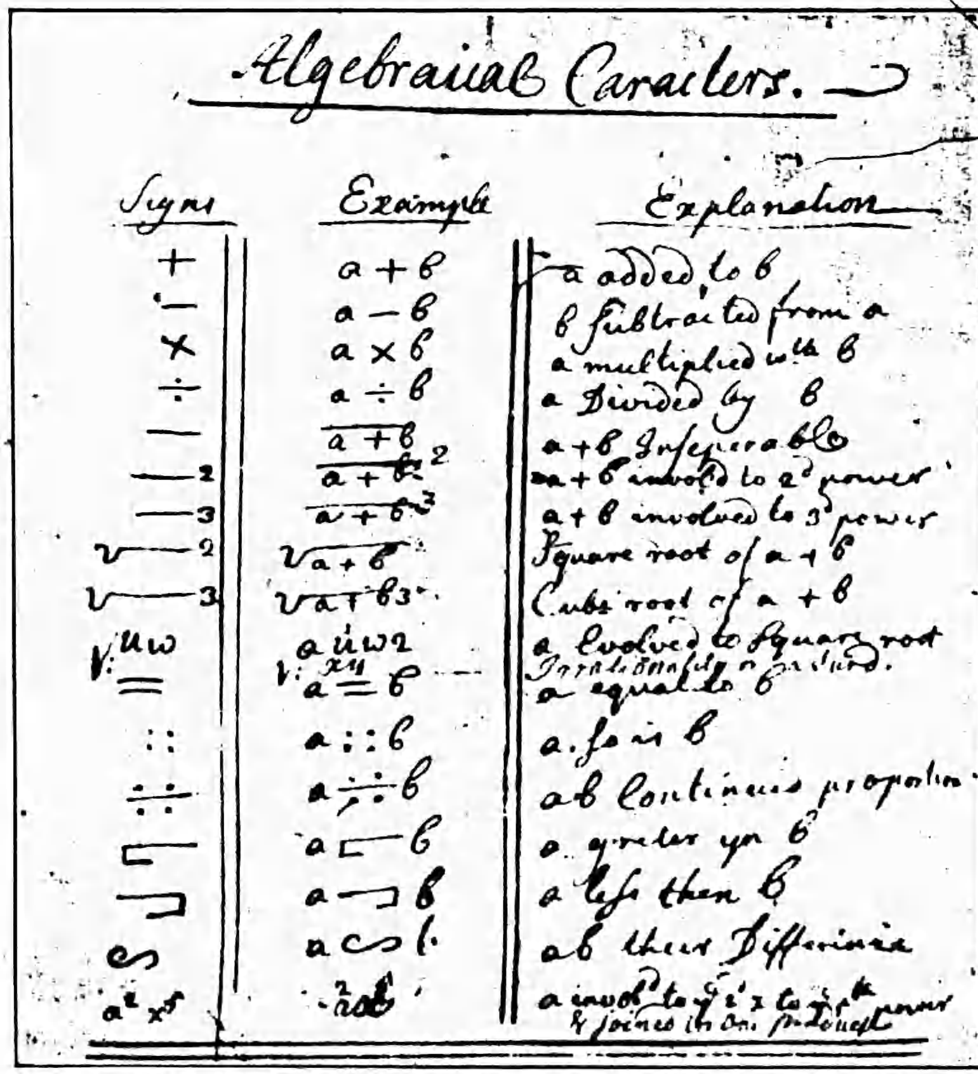
[4] Harvard University Library.

[5] See page 68 for conclusive evidence on the authorship of this arithmetic. The copy of this work in the New York Public Library contains, in several places, "Eliakim Willis his Book 1733." Willis graduated from Harvard in 1735 and no doubt used the book in his college course.

[6] See F. B. Sanborn. *Dr. Langdon (1723–1797)*, 1904, for a full description of Langdon's life and connection with Harvard College.

[7] Account taken from A. C. Potter and C. K. Bolton. *Librarians of Harvard College, 1667–1877*, Cambridge, 1897.

every reason to believe that Greenwood was using this algebra material at least as early as 1730. The other date, August, 1738, on this same manuscript was just after Greenwood's withdrawal from the college and during Diman's pastorate at Salem. A comparison of the two shows that the Diman manuscript contains just a small section that is not found in the Langdon one, and so Diman added little to the original in his review. Langdon's book was made about a



Algebraical Caracters.

Signs	Example	Explanation
+	a + b	a added to b
−	a − b	b Subtracted from a
×	a × b	a multiplied with b
÷	a ÷ b	a Divided by b
—	$\overline{a+b}$	a+b Inseperable
—2	$\overline{a+b}^2$	a+b involved to 2d power
—3	$\overline{a+b}^3$	a+b involved to 3d power
√—2	√a+b	Square root of a + b
√—3	√a+b3	Cube root of a + b
√:uw	a uw2	a evolved to Square root
=	a = b	a equal to b
::	a :: b	a. [illegible] b
∺	a ∺ b	ab Continual proportion
⊏	a ⊏ b	a greater yn b
⊐	a ⊐ b	a less then b
∽	a ∽ b	ab their Difference
a^2x^3	[illegible]	a invol'd to 2 & x to 3th power & joined in one product

Symbols given in the Diman (1730) manuscript from Harvard College

year later, in July, 1739. Both manuscripts seem to be careful copies of work done earlier at Harvard.

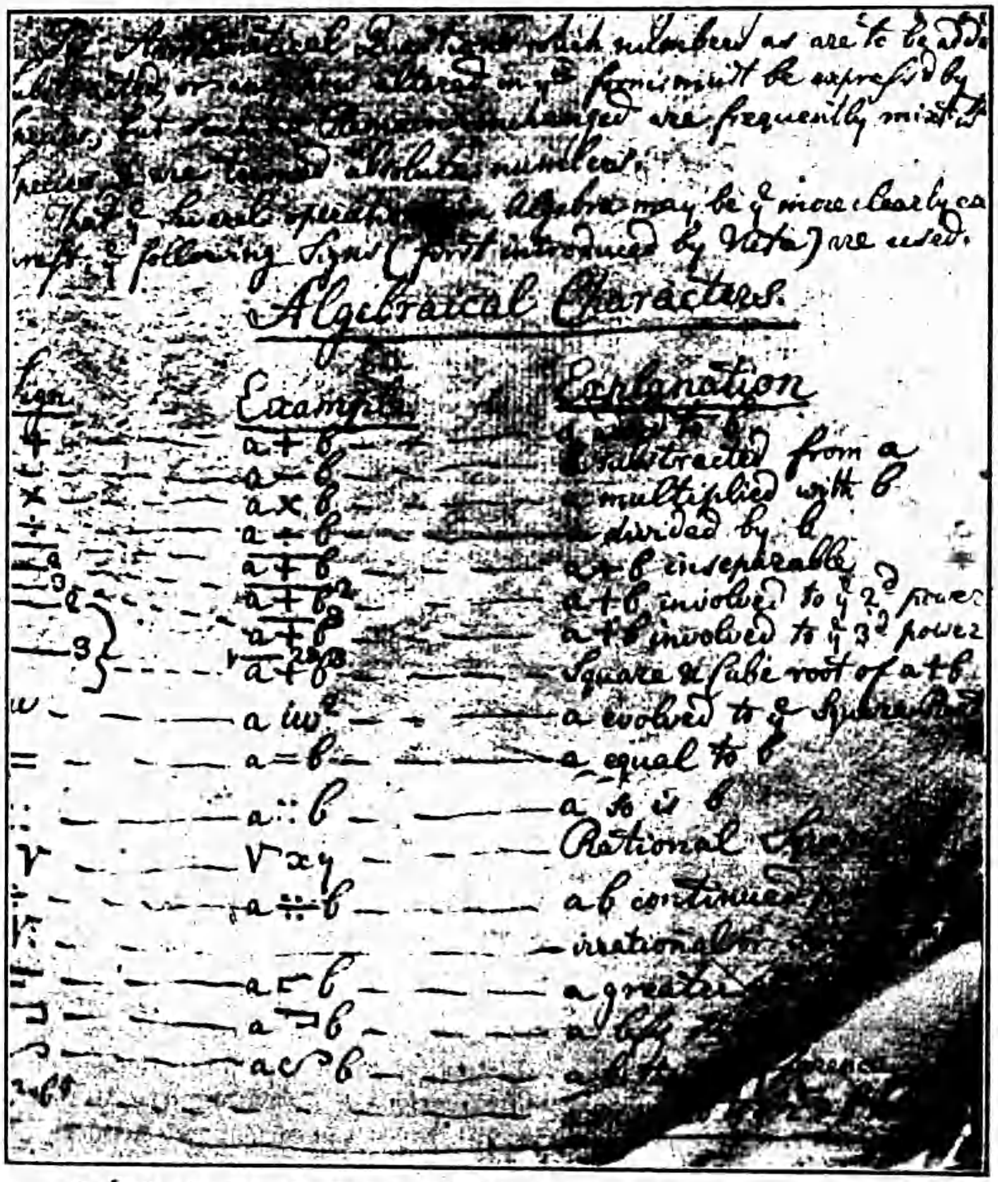
Introduction of the Harvard manuscripts.—We turn now to a description and comparison of the Diman and Langdon manuscripts.[8] Both algebras open with an "Introduction," which begins as follows:

1. This Science is called Algebra from two or three words en ye Arabian Language, wch may be interpreted either the Art of Restitution, & Comparison; or ye Art of Resolution & Equation. It is also known by various other Names.

[8] The quotations given are from the earlier manuscript, that by Diman; citations from the Langdon one would be practically the same.

The first y^t wrote upon this Subject, in Europe, termed it y^e Rule of Restitution & Opposition; Since, it has been called by some, the Analytick Art; by others, Specious Computation; Regula Rei et Census; y^e great Art; Modern Geometry; Universal Mathematicks &c. (Diman, p. 1; Langdon, p. 1.)

This introduction shows an interest on the part of the author of the original in the history of the subject. For this history he drew directly from John Wallis,[9] or from some work founded on Wallis.



Algebraical Characters.

Sign	Example	Explanation
+	a + b	a added to b
−	a − b	subtracted from a
×	a × b	a multiplied with b
÷	a ÷ b	a divided by b
=	a = b	a equal to b
√	√xy	Rational

Symbols given in the Langdon (1730) manuscript from Harvard College

The paragraph cited shows a strong resemblance to the opening paragraph in the article on algebra in Chambers's Cyclopaedia.[10]

Symbols.—A page of "Algebraical Characters" follows the introduction. Attention is called to the interesting features of this table of symbols; to the bar which is the only sign of aggregation

[9] John Wallis. *A Treatise of Algebra both Historical and Practical.* London, 1685.
[10] E. Chambers, *Cyclopaedia.* Second edition, London, 1738.

used; to the symbol for continued proportion; to those for inequality, unequal parallel lines met at one set of extremities by a vertical transversal;[11] to the capital S turned on its side to indicate the difference between a and b;[12] a^2x^5 is explained to mean "a invol^d to y^e 2^d x to y^e 5th power & joined in one product," although the more common form aa is found, and frequently even such forms as $aaaaa$, showing the difficulty in adopting the representation by the exponent. Later in the manuscripts, it is curious to find the powers of $x - y$ up to the sixth in this latter form, so that the sixth power reads $xxxxxx - 6xxxxxy - 15xxxxyy - 20xxxyyy + 15xxyyyy - 6xyyyyy + yyyyy$ (sic) and then to find the seventh, eighth, and ninth powers in the present day form. (Diman, p. 27; Langdon, p. 19.)

Topics.—The topics reproduced as they appear in the two manuscripts are set down to show the similarity between them. They are paired except when the topics are identical, a blank space indicating that the topic is omitted. The first member of each pair is taken from the Diman manuscript, the second from that by Langdon.

Notation; Algebraical Caracters, Algebraical Characters; Addition of Integers; Subtraction, Substraction;[13] Multiplication of Algebraic Integer, Multiplication of Algebraick Integers; Division; Algebraical Fractions, Algebraick Fractions; Addition & Subtraction of Fractions, Addition & Substraction of Fractions; Multiplication of Fractions; Division of Fractions; Involution of whole Quantities; Involution of Fractional Quantities, Fractional Involution; Evolution of whole Quantities; Fractional Evolution; Binomial Quantities; Involution [of binomial quantities]; Promiscuous Examples [the examples are found but not the heading]; [Heading not given, but two fractions are included in the set of examples], Involution of Binomial Fractional Quantities; Multinomial Quantities; Involution [of multinomial compound quantities]; [No heading but the statement: "Fractional Compound Quantities are also Involved in y^e same maner"], Fractional Compound Quantities; Evolution, Evolution of Multinomial Quantities; Surd Quantities; Notation [of surds]; Reduction of Surds; Multiplication of Surds; Division of Surds; Addition and Subtraction of Surds, Addition and Substraction of Surds; Compound Surds; Multiplication of Binomial Surds, ————; Division in Compound Surds, ————; Equation; Reduction of Equations; Reduction by Addition; Reduction by Subtraction, Reduction by Substraction; Reduction by Multiplication; Reduction by Division; Reduction by Involution; Reduction by Evolution; Reduction by Analogies to Equations & e Contra; The Method of Resolving Algebraical Questions; General Rules Concerning y^e Reduction of Equations; Simple Equations; The Solution of Adfected Quadratick Equations, ————; Mr Oughtreds method of solving adfected Quadraticks, Mr. Oughtreds method of solving adfected Quadratics; The Solution of Adfected Equations by taking away y^e Second Term, The Solution of adfected Equa-

---

[11] These symbols are used by William Oughtred in his *Clavis Mathematicæ*, p. 166, London, 1648.

[12] This symbol was used first by Oughtred, loc. cit., Ed. 2, 1652.

[13] Diman uses the spelling "subtraction," while Langdon uses "substraction." The latter is used throughout Greenwood's Arithmetic. For a discussion of the spelling of these words, see David Eugene Smith, *History of Mathematics*, 2 vols., Boston, 1923–24, hereafter referred to as Smith, *History*, II, 95.

tions by taking away the second term; The Solution of Adfected Quadratick Equations by y^e method of Compleating y^e Square, The Solution of Adfected Quadratic Equations by y^e Method of compleating ye square; Questions, Questions producing adfected Quadratick Equations; The Resolution of Cubic Equations, The Resolution of Cubick Equations; Cubic Equations by Substitution, Cubick Equations by Substitution; Cubic Equations by Tryalls & Depression, Cubick Equations by Tryals & depression; The Solution of Irregular Cubics, The Solution of irregular Cubicks; The Method of Converging Series, The method of converging Series and Approximation; ———. 1 of Simple Roots; ———. II of Adfected Equations; Mr Raphson's Theorems for Simple Powers [not so designated, but all the Raphson formulas are given]; Mr. Raphson's Theorems for adfected Equations; Dr Halley's Theorems for Solving Equations of all sorts, Dr. Halley's method for solving equations of all sorts; Concerning the Method of resolving Geometrical Problems algebraically, Concerning y^e method of solving Geometrical Questions Algebraically.

Treatment of topics.—Many interesting passages show the spirit and subject matter of these two notebooks, and at the same time multiply the evidence bearing on their common source. Some important ones will be touched upon.

The clearness of explanation throughout may be illustrated by the treatment of signs in addition and subtraction, which begins as follows:

The reason of y^e Operation in Algebraical Addition and Subtraction may be easily understood by considering y^e affirmative and negative Quantities like opposites as y^e Case is in Ballancing accompts. (Diman, p. 7; Langdon, p. 5).

Involution of Binomial Quantities. . . . Consequently if y^e Numeral figures of Coefficients could be found y^e whole might be performed without multiplication and this is done by y^e following Problem. To find y^e Coefficients in Binomial Powers. Rule. Multiply y^e Coefficient into y^e Index of y^e Power and Divide that Product by y^e Number of terms, counting from y^e left hand, and y^e Quotient will be y^e Coefficient or Numeral Figure of y^e next successive Quantity. (Diman, p. 27; Langdon, p. 18).

Irrational Quantities are noted thus; $\sqrt{\ }:2$ w^{ch} is 2 w^{th} y^e Sign of Irrationality $\sqrt{\ }:$ before it. . . . There is also another way of marking surd Quantities where Roots are expressed without y^e Radical sign by their Index, this is founded upon y^e manner of expressing Powers, thus as x^2, x^3, x^4 signifies y^e Square, Cube & Biquadratick of x: so $x^{\frac{1}{2}}$, $x^{\frac{1}{3}}$, $x^{\frac{1}{4}}$ will accordingly signify y^e Square, Cube and Biquadratick Root of x. . . . and w^n at any time there is y^e Sign of Irrationality prefixt to mixt Quantities with y^e sign of Inseperation (sic) over y^m thus: $\sqrt{\ }:\,^{3}\overline{7+\surd:2}$ it is called a universal Root. (Diman, p. 33f; Langdon, p. 23f.)

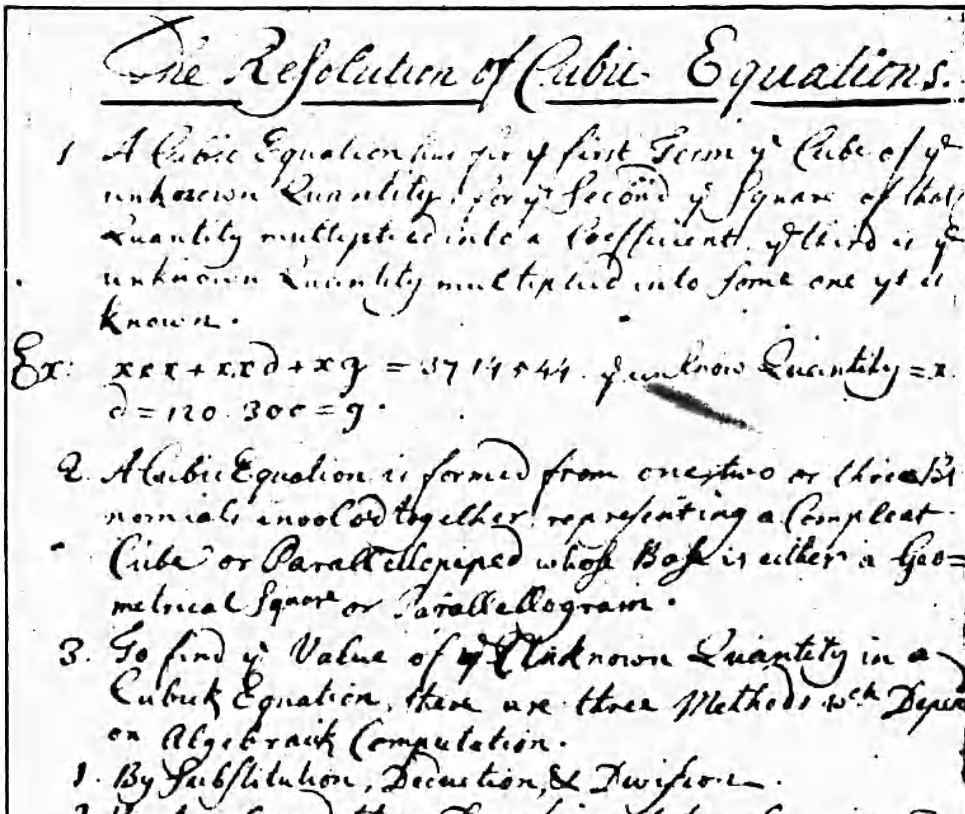
There are seven methods of "Reduction of Equations." The seventh, "Reduction of Analogies to Equations & é Contra," is illustrated by Ex. 1:

Reduce y^e Analogy $x:4:2x$, $4\times4=16$, $x\times2x=2xx$, $\frac{2xx}{2}=\frac{16}{2}=8$ per 16 : 6 Euclid, $xx=8$ (Diman, p. 59; Langdon, p. 31).

Under "The Method of Resolving Algebraical Questions" we find:

This part of Algebra is wholly arbitrary & everyone is left to himself to pursue his own particular Genius and way of thinking, which is so far from being a Defect y^t it is one of y^e Chief Excellencies of this Science, which may from hence not unjustly be called a sublime way of Reasoning. (Diman, p. 62; Langdon, p. 33).

Eight rules precede the set of questions under "Simple Equations," and these are accompanied by illustrative examples, employing letters throughout. The method employed in the questions can best be understood by examining one of them. This one bears the same number in the two manuscripts.


The Resolution of Cubic Equations.

1 A Cubic Equation has for ye first Term ye Cube of ye unknown Quantity, for ye Second ye Square of that Quantity multiplied into a Coefficient, ye third is ye unknown Quantity multiplied into some one yt is known.

Ex: $xxx + xxd + xg = 3714 \cdot 44$. ye unknown Quantity $= x$. $d = 120$. $300 = g$.

2. A Cubic Equation is formed from one, two or three Binomials involved together, representing a Compleat Cube or Parallelepiped whose Base is either a Geometrical Square or Parallellogram.

3. To find ye Value of ye Unknown Quantity in a Cubick Equation, there are three Methods wch Depend on Algebraick Computation.

1. By Substitution, Deduction, & Division.
2. By trial and then Depressing it to a Square.
3. By approximation or Converging Series, which also is a universal Method and Reaches all other Powers whatsoever —

Introduction to cubic equations in the Diman (1730) manuscript from Harvard College

Quest. 25. I am a Brazon Lyon, my two Eyes, my Mouth & y^e Sole of my Right foot, are so many several Pipes, which fill a Cistern, y^e Right Eye in 2 Days, y^e Left in 3, & y^e Sole of my foot in 4, but my Mouth can fill it in 6 hours, tell me in w^t. time all these to-gether, my mouth, my Eyes and my foot will fill y^e Cistern. (Diman, p. 77; Langdon, p. 41.)

The unknown x is taken for the number of hours sought, and $\frac{x}{a}, \frac{x}{b}, \frac{x}{c}, \frac{x}{d}$ for the part of the cistern filled by the respective pipes. From these fractional values an equation is formed and a general solution in terms of a, b, c, d effected. In the value of x so found, numerical substitutions are made. This is the usual procedure in the solution of the questions. Diman gives 26 such problems to be solved, while Langdon gives 30. Twenty-two problems are alike in the two works. The problems are mixed in their nature, Diman favoring somewhat the more mechanical kind and Langdon the more practical—practical for the day in which they arose, if not for theirs nor for ours. They include age, merchants trading for linen and pepper, numbers multiplied, divided, and operated upon in a variety of ways, the vintner, the man who found poor persons at his door ready to receive alms, clocks, the shepherd in time of war, cisterns, noblemen traveling for pleasure, the gentleman who hired a servant, a general setting his army in square array, two persons discoursing about their money, and partnership.

"Adfected Quadratick Equations" are considered under three forms, and "Each of these Forms may be Resolv'd 3 several ways." The first of the methods shown is:

> Mr. Oughtreds[14] Method of Solving Quadraticks . . . Rule. Multiply ye absolute Number by four & add thereunto ye Square of ye Differential Quantity, ye Square Root of ye minus ye Differential Quantity being divided by two, is ye Quantity Sought. (Diman, p. 78; Langdon, p. 43).

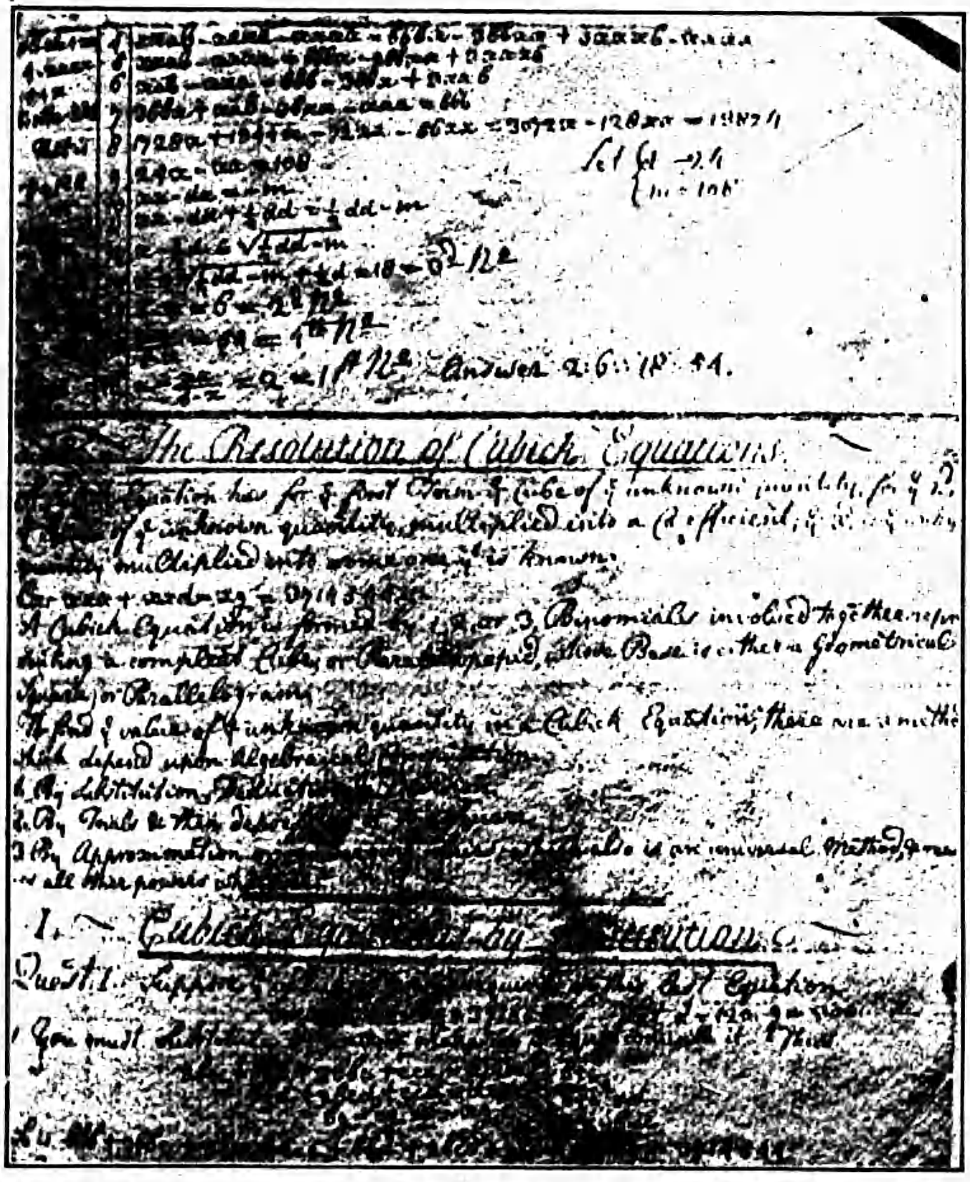
The second method is "The Solution of Adfected Equations by taking away ye Second Term." (Diman, p. 81; Langdon, p. 46.) In this method $y-\frac{1}{2}d$ is substituted in the equation $x^2+dx=m$; whence $y^2-yd+\frac{1}{4}d^2=x^2$ and $y^2=m+\frac{1}{4}d^2$ or $y=\sqrt{m+\frac{1}{4}d^2}$.

The labor involved in removing one term is admissible only from the standpoint of the interest inherent in a different manner of solution.

The third of the three methods given for the treatment of the quadratic equation is "The Solution of Adfected Quadratick Equations by ye method of compleating ye Square." This method is the familiar one known by the same title at the present time. A set of 25 problems follows these three methods, and the two manuscripts agree in the problems and in the order of them with a single exception. It is of especial interest to note that, in connection with the solution of question 7 of this set, the Langdon manuscript (p. 49) gives an imaginary number in the result. This is the only approach to an imaginary in either of the manuscripts, and indicates scholarship on the part of the author of the notes as well as ability on the part of his pupil.

[14] Oughtred's *Clavis Mathematicae* consists for the most part of the solution of quadratic equations. On his treatment of these equations, see F. Cajori, *William Oughtred*, p. 29, Chicago, 1916.

Subject matter of an advanced nature.—Up to this point the subject matter has been largely that of the latter-day secondary schools, although the spirit of the texts is in many respects more mature. The next topic is usually included to-day in a college course in algebra and shows that work of a high order was being done at


The Resolution of Cubick Equations

Introduction to cubic equations in the Langdon (1730) manuscript from Harvard College

Harvard in 1730. It begins with "The Resolution of Cubic Equations."

The first method employed in the solution of these equations is that of Substitution. (Diman, p. 94; Langdon, p. 54.) An outline of the problem solved in modern form reads as follows:

Given $x^3+120x^2-300x=3714544$. Let $x=b+c$.
Then $(b+c)^3+120\ (b+c)^2-300\ (b+c)=3714544$

Or $b^3+3b^2c+3bc^2+c^3+120b^2+240bc+120c^2-300b-300c=3714544$

$\dot{3}71\dot{4}54\dot{4}$ $b=100$

Substituting in all terms containing b alone,

$b^3+120b^2-300b=2170000$

$3714544-2170000=1544544$ 1st Dividend

Substituting for b in all terms which contain c with some other letter or number

$3b^2+3b+240b+120=54120$

Then $1544544 \div 54120=20=c$

Substituting values of b and c in all the terms containing c,

$3b^2c+3bc^2+c^3+240bc+120c^2-300c=1250000$

$1544544-1250000=294544$ 2nd Dividend

Then make b equal the former $b+c$ or 120

Substituting for b in the terms containing c,

$3b^2+3b+240b+120-300=72180$

$294544+72180=4$, a new c

Substituting for b & c again in the terms containing c,

$3b^2c+3bc^2+c^3+240bc+120c^2-300bc=294544$

$294544-294544=0$ Hence $x=124$.

The second method of solving cubic equations is by "Tryalls and Depression," and this is virtually an application of the remainder theorem. With the third method, "The Method of Converging Series," the more difficult handling of these equations appears. As defined in the book,

> The Method of Converging Series is an Approximation, or orderly approach nearer & nearer y^e Truth y^e more one works on ad Infinitum, by w^{ch} means any Equation w^t soever either Quadratick, Cubic, Biquadratick, Sursolid &c may be answer'd to any degree of exactness. (Diman, p. 102; Langdon, p. 60).

After some further general remarks on this method, there follows "M^r Raphson's[15] Theorems for Simple Powers." The theorem for "y^e Biquadratick" is applied to extract "y^e Biquadrate of 90." In addition, there are "M^r Raphson's Theorems for adfected Equations. (Diman, p. 108; Langdon, p. 62.) The biquadratic equation $aaaa-4aaa=13824$ is solved." The last section of this part of the work is devoted to "D^r Halley's[16] Theorems for Sôlving Equations of all sorts." (Diman, p. 110; Langdon, p. 64.) They are theorems for obtaining approximate roots of numerical higher equations.

Geometric problems.—The book closes with a section "Concerning the Method of resolving Geometrical Problems Algebraically," a section which shows how completely geometry was under the sway of algebra during the eighteenth century. In the opening paragraph, the author writes:

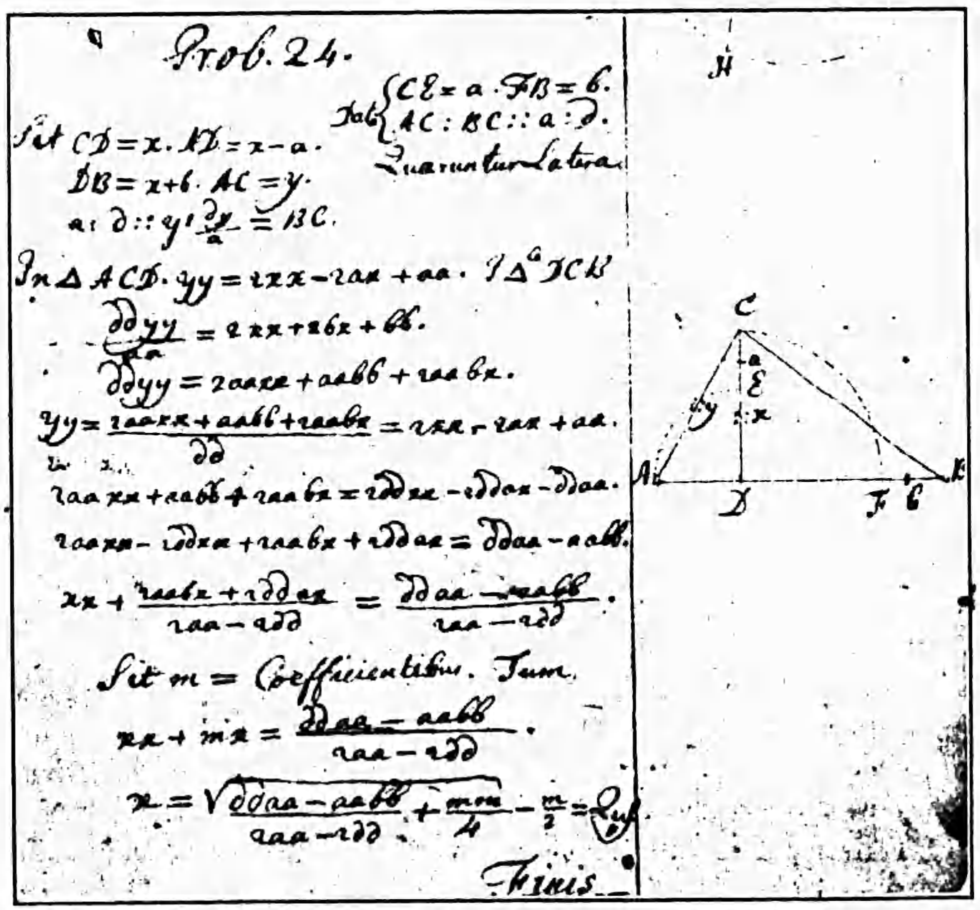
> But in Geometrical Problems is tho't sufficient to note only such Particulars as are necessary to lead y^e Geometritian to some known Theorem, whereby

[15] Joseph Raphson, who published in 1690 a work entitled "*Analysis aequationum universalis*," in which he modified Newton's method of finding the approximate roots of a numerical equation. For a discussion, see F. Cajori, *Oughtred*, p. 140.

[16] Edmund Halley, the great astronomer. He was also deeply interested in geometry and algebra.

y^e solution may be made. And to facilitate this I must advise y^e Student always in Geometrical affairs, to consider y^e unknown Quantity as really known. Then Comparing y^e several Quantities in y^e Problem, note how they are related either directly or by Consequence of any of Euclid &c Demonstrations. (Diman, p. 114; Langdon, p. 65).

Twenty-four problems of a geometric nature follow. Diman uses Latin throughout this set, while Langdon continues to use the English language. The problems are the same and are numbered alike in the two manuscripts. No statement of the problem in words precedes the solution, except in one instance. The diagram in each case is clearly marked, and the data given by reference to the dia-



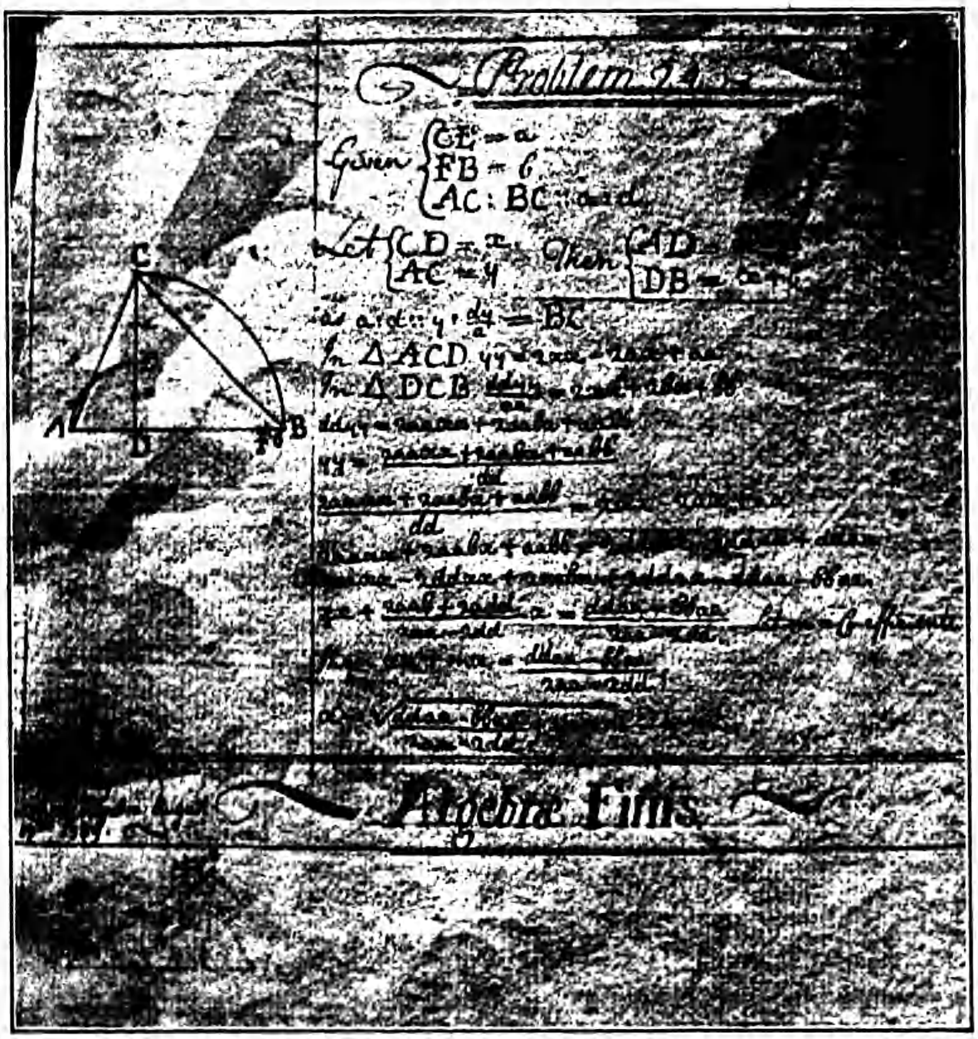
Geometric problems solved by means of algebra in the Diman (1730) manuscript from Harvard College

gram. The problem shown would read, "Given the triangle ABC, with the altitude CD upon the base AB. Let a circle with center D and radius DA cut CD in E, and a circle with center D and radius DC cut DB in F."[17] Given CE and FB equal to a and b respectively and $AC : BC :: a : d$. Required the value of CD in terms of a, b, and d. AC is represented by y and CD by x, and the problem is solved by applications of the Pythagorean theorem and the given data.

[17] Langdon uses a triangle in which CD equals DB, and hence FB is incorrect.

There are scattered throughout the exercises many facts from Euclid, both definitely referred to and implied. We can not doubt that the pupils who studied this algebra, writing out these manuscripts as textbooks, had already taken a good course in geometry.

Sources of material.—The sources from which Greenwood drew his material can not, of course, be known with any assurance. Certain striking resemblances to texts of the day which appear to be



Geometric problems solved by means of algebra in the Langdon manuscript from Harvard College

more than mere coincidences may be indicated and indeed some of these have been referred to in the description of the manuscripts. Hints are given by the writers. As already noted, Diman states that he used in his review Harris, Wolfius, and Chambers's Cyclopædia. Langdon at the end of the manuscript, on the inside of the last cover leaf, states that "The Diameter of a Circle being supposed, the Circumference is expressed by the following figures—vide Chamb. Dict.," and then gives the ratio of the circumference of a circle to the diameter correct to 32 decimal places but without using any symbol for the ratio.

The bodies of the manuscripts furnish clues to further sources of material, for in them appear the names of Raphson, Oughtred, and Dr. Halley. In all probability Greenwood had access to the original works of these algebraists and may have brought copies of them back from England. In addition to these names, we find "Such Demonstrations as may be found in approved Authors such as Euclid, Apollolius, Archimedes." (Diman, p. 62; Langdon, p. 33.)

A reference to Chambers has already been made, showing that this cyclopædia may have been drawn on for the history of algebra. The meaning of universal root is given in practically the same words as those used in Chambers, viz "$\sqrt{^3 : 7+\sqrt{:2}}$, which last is called an universal root." Under the topic "Reduction of Surds" two rather unusual terms *heterogeneal* and *homogeneal*, are employed, which seem to have been taken from this source.

It is easy to believe that Greenwood drew some inspiration, if not actual subject matter, for the section on "Geometrical Problems" from Raphson's translation [18] of Sir Isaac Newton's *Arithmetica Universalis*. A comparison of certain introductory matter to the geometrical exercises found in the two works follows:

In the first Place, therefore, the Calculus may be assisted by the Addition and Subtraction of Lines, so that from the Values of the Parts you may find the Values of the whole, or from the Value of the whole and one of the Parts, you may obtain the value of the other Part.[19]

I. The Calculus may be assisted by y^e Addition and Subtraction of Lines, so y^t from y^e values of y^e Parts y^e value of the whole, or from y^e value of y^e whole and one of y^e Parts, y^e Value of y^e other Part may be obtained. (Diman, p. 114 f.; Langdon, p. 66 f.)

The other two parts show the same similarities.

One of the successful and well-known teachers of mathematics of the seventeenth century was an Englishman by the name of John Kersey (1616–1677). He was the author of an algebra which was entitled: *The Elements of that Mathematical Art commonly called Algebra.* The first edition appeared in 1673, and it was followed by other editions. On its publication, this work became a standard authority.

An indication that Kersey's algebra was used by Greenwood in preparing his algebra notes was given by a reference in one of the two manuscripts. In stating the rule for the coefficients in the ex-

[18] *Universal Arithmetick* . . . Translated from the Latin by the late Mr. Raphson and revised and corrected by Mr. Cunn, London, 1720.

[19] Ibid., p. 89 f.

pansion of a binomial, Langdon (p. 19) says : "This is done by y^{e} following rule, given by S^{r}. Isaac Newton, see p. 139." Kersey's algebra is without doubt the authority referred to. Page 139 of that work consists of "A Table of Powers produced from the Binomial Root $a+e$." Further, the order of the chapters and the subheadings under them are in practically every detail those given by Kersey, and the manuscripts agree with this algebra in every one of the symbols used. All of the 28 problems under "Simple Equations" in Kersey are found either in Dinan or Langdon.

Greenwood was eclectic in his teaching, gleaning here and there the material best suited for the instruction of his students. Evidence of his wide acquaintance with writers of books on mathematics and of his wisdom in the choice of subject matter are shown by this comparison of his work with some of the authors of the day.

Algebra in the Harvard course.—The two manuscripts under discussion bear a close resemblance to each other, but they differ enough to indicate that the individualities of the students themselves played a part in the final productions. They are both unquestionably based on a set of notes prepared by Professor Greenwood to use in a course at Harvard. Had he lived out his professional life, it is altogether probable that we should have had from his pen the first printed work on algebra written by a native American.

Chapter III

THE NOTEBOOK OF A PRINCETON STUDENT[1]

Philip Vickers Fithian.—The journal and letters of Philip Vickers Fithian[2] have been drawn upon extensively for pictures of life in Virginia during the time that he spent there as tutor in the home of Colonel Carter at Nomini Hall. It is not generally known, however, that among the papers[3] relating to Fithian there is to be found a collection of problems which gives an insight into the work in algebra that was being taught in the College of New Jersey (now Princeton University) while he was a student there.

Philip Fithian was a student at the College of New Jersey from 1770–1772, studied theology 1772–73, and taught in Virginia for the year 1773–74. In July, 1776, he enlisted as a chaplain in the American Revolution and served under Washington during the battles of Long Island and Harlem Heights. He died near Fort Washington, October 8, 1776, in his twenty-eighth year.

Fithian kept a careful diary up to the time of his entrance to college and again after he graduated, especially during his stay in Virginia. It is unfortunate that there is no trace of a journal kept during the years that he spent in college. Some incidental references throw light on the studies that he was pursuing. That he was familiar with several branches of mathematics is shown by the following extract from his Miscellany:

> A Declamation on the Difficulty of Composition; the second in the Junior Winter pronounced at Nassau Hall. January 10th Anno 1771 . . . And when all is done, I think it full as hard to become a compleat Master of the first of those divisions, that is to be able to make a proper and concise Introduction to a Subject as to become a compleat Proficient in Algebra and Fluctions; . . . as old Euclid tells us in his 16: prop: Lib. 3 is sufficiently evident.

[1] The earliest reference to algebra at Princeton is found in a set of letters written in 1750, 1751, 1752, 1753, by Joseph Shippen, a student at the college, to his father and other friends. In a letter of the 8th of June (1750) he says: "I shall learn Horace in a little while, . . . but my time is filled up in studying Virgil, Greek Testament, and Rhetoric, so that I have no time hardly to look over any French, or Algebra, or any English book for my improvement." John Maclean. *History of the College of New Jersey* . . . I, p. 141, Philadelphia, 1877.

[2] *Philip Vickers Fithian. Journal and Letters 1767–1774.* Edited by John Rogers Williams, Princeton, 1900.

[3] P. V. Fithian, letters and miscellaneous papers, 1766–1776, MS. Princeton University library.

It is apparent from this speech that algebra, Euclid, and fluxions were a part of the course in mathematics at the College of New Jersey during the period from 1770 to 1772.

Fithian's papers do not show any subject matter of these branches of mathematics except a set of problems in algebra, but such a set implies an excellent foundation along other mathematical lines. There are 99 problems altogether. The first 48 are headed "Problems of Saunderson," and problems 49 through 99 are indicated as coming from Hill.

Saunderson problems.—The Saunderson problems are taken from a work entitled: *Select Parts of Professor Saunderson's Elements of Algebra.*[4] This work is an abstract of a long treatise on algebra by Nicholas Saunderson, the blind mathematician. The problems in the Fithian notebook are found on pages 121–149 of the Saunderson work, under the heading, "The solution of some problems producing simple equations." They resemble the usual problems of the day, problems relating to numbers, age, a fish, distribution of money, debtors, hiring a servant who was to forfeit money the days that he did not work, couriers, shepherds in time of war, and so forth.

Hill problems.—The second group of problems is drawn from a work by John Hill which ran through many editions, although it is a curious collection of mathematical topics under the title: "*Arithmetick*, both in the theory and practice . . . with the addition of several algebraical questions. The like not extant."[5] De Morgan[6] says concerning the 1745 edition of this arithmetic:

> This is the seventh edition of a work of much celebrity. It seems to have owed its fame partly to a recommendation by Humphrey Ditton, prefixed to the first edition (about 1712), praising it in the strongest terms. Perhaps at this time the only things which would catch the eye are the tables of logarithms at the end, and the powers of 2 up to the 144th, very useful for laying up grains of corn on the squares of a chess-board, ruining people by horse-shoe bargains, and other approved problems.

Hill gives, on pages 365–382, ninety-nine "Problems or Questions in Algebra," with neither explanations nor solutions. Fithian's set begins at the forty-ninth and continues to the end, problem by problem agreeing with the like-numbered one in Hill. Most of these problems lead to quadratic equations. Other general features of them include radicals, proportion, fractions, and equations of the quadratic form. Hence it is obvious that we have in Fithian's notebook a good set of exercises under the two topics, "Simple and Quadratic Equations."

---

[4] First edition, London, 1756. The second edition, with which this comparison is made, appeared in 1761.

[5] Comparison made on basis of the eleventh edition, London, 1772.

[6] Augustus DeMorgan, *Arithmetical Books*, p. 70, London, 1847.

William Churchill Houston.—Fithian studied mathematics under William Churchill Houston.[7] Houston received his early education in North Carolina at Alexander's Academy. He was appointed a teacher in the Princeton Grammar School just as soon as he had matriculated and retained his position during most of his college course.

In 1768 Houston was appointed senior tutor, and that same year he graduated with the highest honors. In 1771 he received the A. M. degree. Now, in 1768, John Witherspoon became president of the College of New Jersey. Under his leadership the curriculum was enlarged by the introduction of new courses in Hebrew, French, and mathematics. It was at this time that the professorship of mathematics and natural philosophy, "as most immediately requisite," was established, and Mr. Houston was the first man to occupy the chair. His work went on from 1771 until the outbreak of the American Revolution when, owing to the abandonment in the autumn of 1776 of all work at the college, it was discontinued. He was a member of the Continental Congress, clerk of the supreme court of New Jersey, and delegate to the Constitutional Convention of 1787 and to Annapolis. By the winter of 1778 Houston was back again in his old position, which he held until 1783.

The scholarship and character of this mathematics professor must have made a deep impression on his students, with whom he came into daily and close contact. And this is the man under whom Fithian did the problems which have been preserved with his journal and letters.

---

[7] Account taken from Thomas Allen Glenn, *William Churchill Houston, 1746-1788*, Norristown, Pa., 1903.

Chapter IV

A MATHEMATICAL NOTEBOOK FROM THE UNIVERSITY OF PENNSYLVANIA

"*Mathematica Compendia.*"—Mathematics held an important place in the curriculum of the University of Pennsylvania during the eighteenth century, as is shown elsewhere, and the existence of a manuscript notebook which belonged to a student at the college emphasizes the strength of this position. This notebook consists of two volumes and bears on its first page the words: "Mathematica Compendia: or A Short System of Mathematical Literature as it is taught by Robert Patterson, A. M., Professor of Mathematics in The University of Philadelphia, Aug^{t} 26, 1788. Sam^{l}. Miller."[1]

The first volume of this work contains 170 pages, 33 cm. by 19.2 cm. It begins with "Algebra" under the date "Aug^{t}. 26, 1788." This is followed by "Practical Geometry," "Trigonometry, begun September, 1788," five-place "Logarithms," "Oblique Spheric Trigonometry," "Mensuration of plane figures" including "Area of ellipsis," "Mensuration of solids," and "Stereographic Projection of the sphere." The second volume contains 175 pages of the same dimensions as the first. The subjects treated are "Conic Sections," "Surveying," "Navigation," and "Spheric Geometry." Fluxions are used to find the length of a curve, but there is no formal presentation of the subject.

Out of 170 pages of the first volume of this compendium, 71 treat of algebra. The topics in the words of the text are as follows:

> Definitions, Praxis on the Signs, Addition, Subtraction, Multiplication, Division, Involution, Evolution, Formation & Resolution of Equations, Method of resolving Questions that contain two Equations and two unknown quantities, Quadratic Equations, Method of resolving questions that contain three Equations and three unknown quantities, Resolution of Adfected Equations by the universal method of Converging Series, The manner of resolving Equations where the unknown quantity is to several powers in both Equations, The method of resolving questions which contain four Equations & four unknown quantities, and Promiscuous Questions.

Throughout the text, rules are given, and these are followed by examples. Under equations, "Simple, Quadratic & Adfected" are defined in detail. Fractional and radical equations are found in

[1] University of Pennsylvania library. Another notebook in this library contains no name or date but gives evidence of being at least as early as this one.

the questions which follow these definitions, but no explanation is offered of the special treatment which must be accorded them. Eighteen questions exemplify simple equations. The treatment of the problems is made unnecessarily hard by the insistence on the use of letters in the first statement of the equation and its later working out, with the substitution of the numbers given in the problem at the end. There are many purely literal equations among the sets of problems. No attempt is made to place the problems of a geometric nature in a special section.

The most advanced work is that under the topic "The Resolution of Adfected Equations by the universal method of Converging Series." The solution of the ninety-fourth question illustrates this method and shows also the kind of work in algebra that was being undertaken in the University of Pennsylvania in 1788. Professor Patterson, who compiled the material for this notebook, must have believed in problems more than in mechanical work. This section on algebra, covering 71 pages, contains 110 problems. The solutions of many of them are very long, unnecessarily so in some instances. But it is significant that so much space should have been devoted to the application of the algebra to problems rather than to tedious work in operations.

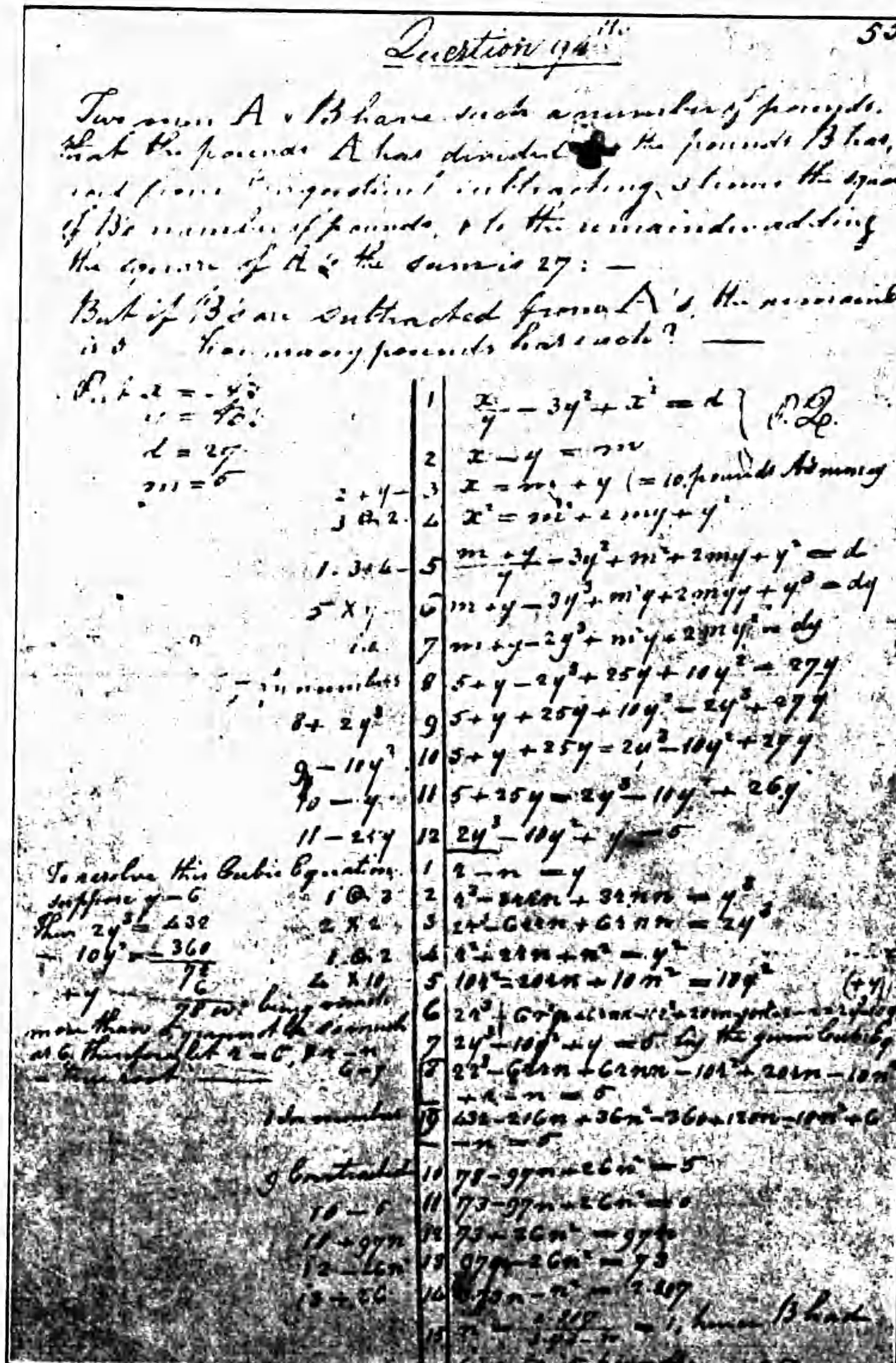
Robert Patterson.—This account would be incomplete without some record of the man who worked out such a good course in mathematics for his students. Robert Patterson [2] was a Scotchman, although he was born in Ireland in 1743. Robert early showed promise of his mathematical ability. When he had completed his first formal schooling, he gave evidence of his adventurous and ambitious spirit by setting sail for the New World. He came to the Colonies with the firm intention of becoming a schoolmaster and soon succeeded in impressing his qualifications on the proper authorities in Buckingham, near Philadelphia. He later removed to Philadelphia and offered instruction to navigators in methods of calculating longitude from lunar observations, and related matters.

During the American Revolution, Patterson devoted himself to the cause of liberty, and acted for some time as military instructor. He entered the University of Pennsylvania in 1779, receiving an appointment first as professor and later as vice provost, and remained there for 35 years. The college honored him with the degree of LL. D. in 1816. After holding various offices in the American Philosophical Society, Patterson became its president in 1819, which office he filled until his death in 1824.

Professor Patterson was a great teacher, but he seems to have written little for publication. In 1818 there appeared a work en-

[2] Account taken from *Transactions of the American Philosophical Society*, Vol. II, New Series, Philadelphia, 1825. Obituary notice of Robert Patterson, LL. D.

titled: "*A Treatise of Practical Arithmetic*, intended for the use of Schools; in two parts." The "Explanatory Notes" of Part I have some interest in connection with algebra. Patterson edited American



Solution of a cubic equation from the notebook of a University of Pennsylvania student in 1788. Doctor Pell introduced the "method of registering the steps" shown here

editions of several English works on arithmetic, natural and experimental philosophy and astronomy, and in some cases added an appendix to the edited work.[3] Besides the Mathematica Compendia, he prepared a work on navigation for the use of his students which is also extant in notebook form. The title-page of this book carries the superscription: "Note-book of Samuel Hayes. Philadelphia, 1789. Navigation by Robert Patterson, A. M."[4]

If Robert Patterson had lived in a day when paper and printing were relatively inexpensive, he would have published both the "Mathematica Compendia" and the "Navigation." The character of these works called for their publication. In his long years of service as professor in the University of Pennsylvania, he might have accomplished greater things by the use of textbooks than by the more tedious dictation of lectures.

[3] Professor Cajori states that Patterson wrote a small astronomy entitled "*The Newtonian System*," which was published in 1808. F. Cajori. *The Teaching and History of Mathematics in the United States*, p. 66, Washington, 1890.

[4] Manuscript Department, Historical Society of Pennsylvania.

Chapter V

MANUSCRIPT MATERIAL FROM MISCELLANEOUS SOURCES

Manuscripts by Robert Brooke.—Another set of notebooks which were probably prepared, at least in part, under the direction of Professor Patterson is that by Robert Brooke. He is given as a non-graduate of the University of Pennsylvania, presumably of the class of 1793.[1]

The first[2] of these books bears the superscription, "Robert Brooke, His Book—March 10, 1783. Arithmetick." It contains 180 pages 20.5 cm. by 33 cm., covering the subject of arithmetic in a complete fashion. A second book, dated 1792, is on "Practical Geometry." While Brooke was given to dating his work rather freely, he failed to set down any indication of the time at which he began to prepare the material on algebra. This part of the notebook consists of 41 pages, 19 cm. by 33 cm., and has a complete set of algebra topics. They bear a resemblance to the Mathematica Compendia by Robert Patterson, and the work was probably done under him, although this is a briefer course. The topics are as follows:

Definitions, Praxis on the Signs, Addition, Subtraction, Multiplication, Division, Involution, Binomials, Multinomials, Evolution, Fractions, Simple Equations, Equations with Two Unknowns, Quadratic Equations, Adfected Equations.

The survival of earlier symbols and forms may be noted in this work. The signs for inequality are the Oughtred symbols found in the Harvard manuscripts, but in the opposite order, and hence they resemble the present-day symbols.

Still another undated book contains no name, but is unmistakably in the handwriting of Robert Brooke. It consists of 36 pages and deals with the "Application of Algebra to the Invention of Theorems." These applications are made to—

Arithmetic Progression, Geometric Progression, Annuities, Compound Interest, Extraction of Square Root, Mensuration of Superficies, Mensuration of Solids, and Spheric Trigonometry.

---

[1] *University of Pennsylvania. Biographical Catalogue of the Matriculates of the College 1749–1893*, XIII, Philadelphia, 1894.

[2] This notebook and all the others by Robert Brooke herein described are in one package in the manuscript department, Historical Society of Pennsylvania, hereafter referred to as Hist. Soc. Pa.

Other notebooks by the same man are devoted to: "Applications of Plain Trigonometry to Astronomy," "Spheric Geometry," "Applications of Astronomy," "Dialing," and "Fluxions."

There is sufficient resemblance between the notebook on algebra by Robert Brooke and one by Andrew Porter, jr., to warrant the inclusion of the Porter book at this point. The latter was "Andrew Porter jun.r.'s Book 1792."[3] Thirty-nine pages, 19 cm. by 31 cm., show a fair knowledge of elementary algebra and constitute an abbreviated edition of the algebra work by Brooke. The order of topics agrees with this same book as far as "Quadratic Equations." Only the heading for that topic appears, and the work stops there. Symbols and forms and, in some cases, exercises are the same.

"*Practical Mathematics*."—Brief mention will now be made of such other notebooks containing algebraic material as have been found during a careful search in many libraries of the East. The number of these books extant does not compare with the number containing arithmetic, and yet a bit of algebraic material crops up in the most unlooked for places. A set of five volumes beautifully printed by hand, by one Thomas Sullivan, in 1796, is entitled "Practical Mathematics."[4] Virtually every topic of interest in applied mathematics at that time is included in these volumes, as well as some purely theoretical work. A collection of algebraic problems in such a series could not prove to be other than an unexpected treasure. Algebra is used here for its applications under the heading: "The Application of Algebra to the Solution of Problems." These 24 problems are drawn from geometry, but are not of a really useful nature.

In a good piece of work on trigonometry by one Joseph King,[5] which contains the date 1740, although in another hand than that of King, familiarity with the use of letters in algebraic expressions is shown. The proportion $x:y::y:z$ appears, followed by "Then $xz=y^2$ and $\sqrt{xz}=y$." Ten principles of trigonometry are given, and the tenth one reads: "By Algebra or Analytical Investigation."

Somewhat more extended is the algebra in "William Winthrop's Book, began June 6, 1769,"[6] and carried over into the year 1770. This book has 20 problems and a few statements, such as "The Sum and Difference of any two Numbers multiply'd to-gether produces the Difference of their squares." Later in the book there are several pages on algebra in Latin.

Some of these notebooks give us side lights on the attitudes of the college boys who were compelled to prepare them. One[7] such book

[3] Hist. Soc. Pa.
[4] Manuscript Department, Library of Congress.
[5] Hist. Soc. Pa.
[6] Harvard University Library.
[7] American Antiquarian Society.

which starts: "Bought Jan. 3, 1786, Doctor Webber Philomath.[8] Samuel Haven's Book, Cambridge University," ends with a telling comment by Haven on the well-known quotation from Pope, as follows:

> A little learning is a dangerous thing
> Drink deep, or taste not the pyerian Spring
> For shallow draughts intoxicate the brain
> But drinking largely sobers us again.

N. B. I had rather be intoxicated than drink deeply of mathematical learning. The AUTHOR.

The author had not drunk deeply of algebra, because only 17 out of 167 pages refer to that subject; and only definitions, axioms, and the four fundamental operations are covered.

A later student at Harvard in 1794 left even a briefer record[9] of his work in algebra. This student was Joseph McKean, who was afterwards professor of rhetoric and oratory at this same college.

Nathaniel Bowditch.—A notebook[10] by Nathaniel Bowditch[11] should be mentioned on account of its intrinsic interest rather than because of its bearing on the mathematical education of his day.

When Bowditch was about 14 years of age he heard a vague account of a method of working out problems by *letters* instead of *figures;* he succeeded in borrowing the book, and was so much interested and excited by this, his first glance at algebra, that he could not get the least sleep during the whole of the next night.[12] Bowditch transcribed an immense number of mathematical papers from the printed *Transactions of the Royal Society of London* and many other scientific works in order that he might have this material in his personal possession, but his book on algebra gives no evidence of being so transcribed.

This notebook has the title: "NATHel BOWDITCH his Book Aug.t 23d 1788. He began to learn algebra on the 1st of August, 1787." The contents are the usual ones from "Addition of Algebra" through "Solution of high adfected Equations." The peculiar character of this notebook of 114 pages is that through 80 pages of it the catechism method is used. And this seems to indicate that Bowditch worked out a textbook of his own, using the subject matter of other works. Indeed, he refers to "Mr. Ward" in

[8] Samuel Webber. Tutor of mathematics at Harvard from 1786–1789. Later professor of mathematics and president of the university.

[9] Boston Public Library.

[10] Boston Public Library.

[11] Nathaniel Bowditch (1773–1838). Best known as the translator of Laplace's *Mécanique Céleste* and by another work that was largely his own, *The New American Practical Navigator, 1802.*

[12] Memoir of the translator of the *Mécanique Céleste* by his son, Nathaniel Ingersoll Bowditch, in Vol. IV, Boston, 1839.

several places, to "Cocker" and to "James de Billy's algebra." Elsewhere in his mathematical material he refers also to "Dilworth," and Dilworth did employ this method.

This is such a late survival of an earlier method that one illustration of it is cited.

Dialogue 1.
Between Philomathes & Tyrannaculus concerning addition, Substraction, Multiplication & Division of algebra.
P. as you understand Vulgar fractions & know the Signs & characters (sic) you say the very first thing that I shew you will be addition.
T. how is addition in algebra performed.
P. the same as common addition provided the signs be both affirmative or both negative as you will soon find by the four following cases.
T. I understand all but the fifth for I cannot at present conceive that +42 added to −42 can be equal to nothing. I should think rather that substracting them they would be equal to nothing.
P. that is your mistake for their difference is 84 . . . . because the Negative sign makes void the affirmative.
T. I ask pardon but I do not rightly apprehend it.
P. I think you are a little Dull now. Do you not remember that I told you that this Sign (−) signifies a want or Deficiency so many times less than nothing as the figures after it express.
T. Yes I do.
P. Observe then suppose you stood Indebted to a person £42 and had no effects of any sort to pay the debt there it is plain you would be 42 times worse than nothing that is have 42 times less than a real property of your own. Now if a friend should give you £42 to pay of the debt and you do so still it is plain you would have Nothing in hand to begin the world again with consequently then +42 added to −42 = * or Nothing.
T. I am very thankful Philomathes for so plain a demonstration.

A generous portion of the notebook is devoted to problems of all possible varieties. The one quoted is in rhyme, and so the book illustrates the old rhyming arithmetics as well as the dialogue method.

Problem 114th. four Virtuous Damsels would be joined for life
Come Batchelors Come now and chuse a wife
Phoebe is young. Stella has charm—but hold
Astrea has Virtue—old Aurilea Gold
Proportion Geometril their fortunes claim
Phoebe has least Aurilea, a Rich Dame
Their fortunes sum does in the margin Stand
Take Beauty Virtue youth or house and land
Divide each fortune by just twenty-four
And youl their ages Easily Explore.

End of the notebook custom.—This custom of keeping manuscript mathematical books does not seem to have died out until well into the nineteenth century. "Conic Sections. For the use of students in Union College by William Allen, A. M. Prof. Math. et. Nat. Phil. in U. College Schenectady state of New York. Transcribed by and for John S. Mabon, Jan. 1st, 1805," contains 60 pages on advanced algebra.[13] "Ebenezer Gay's Property. Algebra 1807–08" is a student notebook which extends through cubic equations.[14] A notebook which records only the date when it was finished reads on its title page: "End of Algebra 4 February 1813 Elisha Fuller Merrimack AD 1813."[15] Fuller's work is excellent and is carried through the solution of cubic and higher degree equations.

By the year 1814 several reprints of English algebras had appeared, and in that year an algebra by an American author was published.[16] Textbooks were in the hands of students, and the universal custom of student textbook making had practically died out. Notebooks were kept, as a student might keep a book to-day, for work supplementary in its nature, or to meet the whims of a particular professor.

---

[13] Manuscript collection of Mr. George A. Plimpton.
[14] *Ibid.*
[15] American Antiquarian Society.
[16] Jeremiah Day, *The Elements of Algebra*, New Haven, 1814.

Chapter VI

COMMENCEMENT THESES

A commencement custom.—Commencement theses constitute original source material of quite another nature from that of student notebooks. A thesis has come to mean generally a completed essay or dissertation presented by a candidate for a degree. Another meaning, in less common use, attaches to it, however. This meaning makes a thesis to be a statement of a proposition which is to be established by argument, implying that objections may be raised and answered. It is with this latter significance that the commencement theses[1] of Harvard, Yale, Rhode Island College, and the College of New Jersey during the eighteenth century will be considered.

Several accounts of these early commencements have come down to us. The earliest one relates to the custom at Harvard in the seventeenth century, and is found in two references from the *Magnalia*, by Cotton Mather. It is as follows:

> When the *Commencement* arrived, . . . they that were to proceed Bachelors, held their *Act* publickly in *Cambridge*; whither the *Magistrates* and *Ministers*, and other *Gentlemen* then came, to put Respect upon their Exercises; And these Exercises were besides an *Oration* usually made by the *President*, *Orations* both *Salutatory* and *Valedictory*, made by some or other of the Commencers, . . . But the main Exercises were *Disputations* upon *Questions*, wherein the *Respondents* first made their Theses.

> At the *Commencement*, it has been the Annual custom for the Batchelors to publish a Sheet of *Theses*, *pro virili Defendendae*, upon all or most of the *Liberal Arts*; among which they do, with a particular character, distinguish those that are to be the Subjects of the *Publick Disputations* then before them; and those *Theses* they *dedicate* as handsomely as they can, to the Persons of Quality, but especially the Governour of the Province, whose Patronage the Colledge would be recommended unto.[2]

---

[1] The best collection of commencement theses of the various universities are to be found in their libraries as follows: Harvard, 1642–1810; Yale, 1718–1797; Brown, 1769–1811; Princeton, 1752. Only one commencement broadside could be located at Princeton. The library possesses, however, a manuscript volume, compiled by John Rogers Williams, and containing all the newspaper notices of commencements from 1748 to 1865. The account for 1748 records that the candidates entered upon public disputations in Latin of theses which had been distributed in printed form. Similar accounts for many years show that the same custom prevailed at this college as at the other colleges of the day.

[2] *Magnalia* (1702) Book IV, 128, 131. Quoted in Henry W. Edes "Harvard Theses of 1663," *Transactions of the Colonial Society of Massachusetts*. Vol. VI, 1897, 1898, Boston, 1902, p. 323.

These commencement theses were printed in the form of broadsides.[3] The programs start with a dedication, the last part of which is a variation on the statement:

Theses hasce jam in Lucem editas, quas, Deo favente, Pro Viribus defendere, conabuntur Juvenes in Artibus Initiati, tam submisse, quam humillime. (These theses now brought forth to light, which with the favor of God, young men who have been admitted to the art will try to defend as humbly and modestly as possible).


Theses RHETORICÆ.

1 RHETORICA est Ars [illegible] Vocabula, Sententias, & Vocem ad persuadendum.
2 Tropi & Figuræ sunt ornamenta orationis [illegible].
3 [illegible] rerum Affectio est cor tropi.
4 Metaphora est Troporum maxime persuasoria.
5 Figuræ dictionis potius suavitati, quam Affectui inserviunt.
6 [illegible] Temperamentum est maximum in orationis Ornamentum.
7 [illegible] Compositio orationem plurimum condecorat.

Theses MATHEMATICÆ.

1 MATHEMATICA est Quantitatem computandi regula.
2 Quantitas discreta est Arithmeticæ Objectum, Continua autem Geometriæ.
3 [illegible] est pars Numeri
4 Cyphræ ad sinistram numeri Integri nihil valent; Decimalis autem Fractionis æstimationem minuunt.
5 Fractionis decimalis æstimationem Multiplicatio minuit, Divisio auget.
6 Algebra est Doctrina qua per Comparationem quantitatum cognitarum, cum incognitis, quæstiones occultissimæ Arithmeticæ ac Geometriæ, facillime resolvuntur.
7 Partes Algebræ integrantes sunt Numeratio & Æquatio.
8 Neque datur Numerus algebraicus maximus neque minimus.
9 Subductio Negativarum ex Affirmativis, quantitatem auget.
10 Multiplicatio duarum Quantitatum Negativarum, producit Affirmativam.
11 Quum quantitates et in Dividendo, & in Divisore sunt eædem, Quotum est Unitas.
12 Fractio algebraica tanto multiplicatur, quanto e denominatore ejicitur.
13 Quod Involutione involvitur, Evolutione solvitur
14 In Quantitatibus proportionalibus Extremorum productum producto mediorum est æquale.
15 Æquatio perficitur transferendo omnes Quantitates notas ad unum æquationis Latus
16 Primarii Logarithmi reiterata radicis quadratæ extractione formantur.
17 Secundarii Logarithmi additione & Subductione primariorum producuntur.
18 Omnis Triangulus rectilineus duos rectos continet Angulos.
19 Data Basi et Perpendiculo tantum, Angulus ad basin usu Lineæ sinuum inveniri non potest.
20 Sinus Angulorum Lineis oppositis sunt proportionales.
21 Casus Trigonometrici Logarithmorum usu accuratissime resolvuntur.
22 Circulus eodem modo cum Triangulo rectangulo, mensurari potest.
23 Area Sectoris sine notitia area Circuli totius, inveniri potest.
24 Superficies est Latitudo et Longitudo sine Profunditate.
25 Superficies anius cujusque Globi, Circulo ejusdem maximo est Quadruplex.
26 Declinatio Stellæ est distantia ejus ab Æquatore, Latitudo ab Ecliptica.
27 Ascensio stellæ recta est Distantia ejus meridionalis ab initio Arietis numerata per Gradus super Æquatorem, Longitudo vero super Ecliptica.
28 Spatium Temporis a Meridie ad Meridiem non est semper æquale.
29 Situs anguli basis Gnomonis Solarii horizontalis debet esse Secundum Poli Elevationem.
30 Solarium polare elevari debet secundum loci Latitudinis Complementum.

Theses PHYSICÆ.

1 PHYSICA est Ars Naturæ vestigia, investigandi.
2 Materia est divisibilis, impenetrabilis, ac [illegible] passiva.
3 [illegible] non est [illegible] Principium.
4 [illegible] spatium ubi non datur Locus.
5 Duratio unius est mensura durationis alterius.
6 Motus est Impetui motivo impresso proportionalis.
7 Motus non fit Gurgite accelerante.
8 Calor, Frigus, Humiditas, Siccitas non sunt Qualitates primæ.
9 Qualitates secundæ oriuntur ex Magnitudine, Situ, Motu, Figura, & Numero particularum.
10 Calor per transversam particularum minimarum Agitationem extrinsecam, producitur.
11 Generatio est productio rei ex Principiis præexistentibus per occultam naturæ Operationem.
12 Ignis fatuus non est Meteorum Ignitum.
13 Terræ motus fit ab Igne subterraneo
14 Elevatio Vaporum fit Rarefactione.
15 Datur Transmutatio Metallorum.
16 Non datur Generatio spontanea.
17 Omnes Corporis animati partes in Semine continentur.
18 Sensatio fit per concussionem, Nervorum.
19 Diversitas sensuum a diversitate Nervorum pendet.
20 Paucitas Lucis extensionem Pupillæ requirit, & contra.
21 Respiratio necessaria est ad Circulationem Sanguinis continuandam
22 Dantur Insectorum Metamorphoses
23 Anima rationalis potest agere inorganice.
24 Voluntas Coactioni non Subjicitur.
25 Omnia Corpora cælestia non fuerunt in Massa prima contenta.
26 Ut sol est Centrum hujus Systematis, sic stellæ fixæ aliorum.
27 Probabile est stellas novas esse aliorum systematum Planetas.
28 Satellites circumjoviales & circumsaturnini cum cæteris circumstantiis, illos Planetas habitari probabile reddunt.
29 Vis Centripeta rectilineum Planetarum motum in curvilineum convertit.
30 Omnes Planetarum primariorum Orbes sunt Elliptici.
31 Motus stellarum Platonicus, per Recessionem Terræ orbis Ellipsis in Eclipticam, Solvitur.
32 Cometæ sunt massæ indigestæ, Orbe parabolico circa Solem revolventes.
33 Prædictiones omnes Astrologicæ de futuris contingentibus, sunt fallaces & vanæ.
34 Mundus non est Infinitus, sed Indefinite extensus.

His Antecedit Oratio Salutatoria.

Habita in Comitiis NOVI-PORTI CONNECTICUTENSIS, Die Decimo Septembris, MDCCXVIII.

Theses from the commencement program of Yale College in 1718. Eight of the theses relate to algebra

Then follows a list of the candidates for the bachelor's degree. The theses are next set down and are classified under "Technologicae," "Logicae," "Grammaticae," "Rhetoricae," "Mathematicae," and "Physicae," usually a hundred or more on one sheet. As time goes on other classifications are included. Beginning with 1751, "Theses Metaphysicae," "Ethicae," and "Theologicae" are found. Not until 1778 do "Theses Politicae" appear, and still later "Theses Geographicae," Historicae" and "Astronomicae" enter.

[3] For an excellent general account of these publications, see William Coolidge Lane, "Early Harvard Broadsides," in *Proceedings of the American Antiquarian Society*, Worcester, Oct., 1914, pp. 264–304.

These theses more than any extant material show the content and character of undergraduate studies during the early years in which they were printed. It is admissible in this work, however, to enlarge only upon the course in mathematics implied in them, at the same time laying emphasis upon the early appearance of algebraic truths and their subsequent inclusion in most of the commencement programs.

Yale mathematical theses in 1718.—The two broadsides which have been chosen for discussion are the earliest ones from Yale and Harvard which contain algebra theses. The broadside for Yale of 1718 is also the earliest one from Yale extant.[4] The English translations of the Latin theses on mathematics are introduced at this point.

MATHEMATICAL THESES, YALE, 1718

1. Mathematics is a set of rules for computing quantity.
2. Discrete quantity is the object of arithmetic, continuous, however, of geometry.
3. Unity is a part of number.
4. Ciphers to the left of a whole number have no value, but they decrease the value of a decimal fraction.
5. Multiplication by a decimal fraction decreases the value of any number; division increases it.
6. Algebra is the doctrine in which by comparing known quantities with unknown, difficult questions of arithmetic and geometry are easily resolved.
7. The fundamental parts of algebra are numeration and equation.
8. Algebraic number gives neither greatest nor least.
9. Subtracting a negative from an affirmative increases quantity.
10. The product of two negative quantities produces an affirmative.
11. When quantities in both the dividend and divisor are the same, the quotient is unity.
12. An algebraic fraction is multiplied by taking out a factor of the denominator.
13. What is involved by involution is resolved by evolution.
14. In a proportion, the product of the extremes is equal to the product of the means.
15. An equation is solved by transferring all known quantities to one side of the equation.
16. Primary logarithms are formed by the repeated extraction of the square root.
17. Secondary logarithms are produced by the addition and subtraction of primary logarithms.
18. All rectilinear triangles contain two right angles.
19. Given the base and altitude, the angle at the base can not be found by the use of a line of sines.
20. Sines of angles are proportional to the opposite sides.
21. Trigonometric problems can be solved most accurately by the use of logarithms.

---

[4] Franklin B. Dexter. *Biographical Sketches of the Graduates of Yale College with Annals of the College History, October, 1701–May, 1745*, p. 179, New York, 1885.

22. A circle can be measured in the same manner as a right-angled triangle.
23. The area of a sector can be found without knowing the area of the whole circle.
24. Surface is width and length without depth.
25. The surface of a sphere is four times the area of its largest circle.
26. The declination of a star is its distance from the equator, the latitude its distance from the ecliptic.
27. The right ascension of a star is its meridian distance from the beginning of the Ram [vernal equinox] numbered by degrees on the equator, the longitude on the ecliptic.
28. The amount of time from midday to midday is not always the same.
29. The angle at the base of a horizontal sundial must agree with the elevation of the pole.
30. The solar pole must be elevated in agreement with the complement of the latitude of the locality.

Let us now examine in some detail these mathematical theses. They include two general [1, 2], four arithmetic [3, 4, 5, 11], eight algebraic [6, 7, 8, 9, 10, 12, 13, 15], seven geometric [14, 18, 19, 22, 23, 24, 25], two logarithmic [16, 17], two trigonometric [20, 21], and five astronomical [26, 27, 28, 29, 30] statements. Here, then, we have in the year 1718 algebra and trigonometry in the college course at Yale along with arithmetic, plane and solid geometry, and astronomy, the last three of which had been looked upon for centuries as essential parts of a liberal education.

The theses show a wide divergence in degree of difficulty. The fourth thesis, with its statement concerning the placing of ciphers to the left of significant figures in whole numbers and decimals, is probably known to every first-year grammar-school child to-day, while the twenty-fifth, which gives the area of the surface of a sphere, is known only at the end of a high-school course or during the first year in college.

The statements in geometry show that demonstrations were included in that branch of mathematics, and not merely geometric constructions with no accompanying proofs such as form a part of many of the student notebooks of the eighteenth century. The defense of the thesis that "all rectilinear triangles contain two right angles" involves a geometric demonstration, as do most of the other geometric statements in this set. Logarithms might have been taught mechanically, but the law of sines must have been developed in connection with a course in trigonometry, even if it was a limited one. Declination, right ascension, latitude, longitude, and gnomon are all terms that imply familiarity with mathematical and not merely descriptive astronomy.

Algebra theses in 1718.—The algebra theses on this 1718 program possess peculiar interest, and emphasis must be laid on their presence among commencement theses as early as that year. This is, as far as research so far conducted shows, the first direct evidence of the teaching of algebra in the American Colonies. By 1718 a class of boys had studied this subject one or two or three years, and these students were now ready to argue for the truth of certain algebraic relations.

On this particular program the number of algebraic truths is greater than that of any other mathematical subject. Algebra appears in the sixth statement in all its power when it takes the form of a magic wand waved over abstruse questions of arithmetic and geometry. A modern dictum, that the equation is the central topic of algebra, occurs in the seventh thesis. In number eight is given one of the distinguishing characteristics of the subject, the extension of the number system to include positive and negative quantities without limit. The other statements lead to the conclusion that only the initial steps had been taken at this early date. But any beginning at all was a hopeful sign that the tutor of mathematics at Yale had the courage to break away from the accepted curriculum and to introduce the subject which had developed so rapidly in the preceding century.

Samuel Johnson and Algebra.—At this time Samuel Johnson (1696–1772), who was later called to be the first president of King's College (later Columbia College and at present Columbia University), was the sole tutor at Yale College.[5] He had been sole tutor for two years since his appointment in the fall of 1716, and his name appears among the graduates of 1714 as "Samuel Johnson, Mr. Tutor," on the Yale program of 1718. Certain notebooks and printed books[6] contribute to our knowledge of the lines of work that he was following as tutor.

One notebook in Johnson's handwriting carries the words: "F Libris Sam[el] Johnsoni Anno Dom. 1717." It contains the following scheme of mathematical subjects, which he doubtless gave to his pupils:

Arithmatic is the Art of Numbering both of Integers & Fractions. Appendages hereunto are 1. Decimal Fractions. 2. Logarithms. 3. The Extraction of Roots. 4. Algebra.

Geometry is the Art of measuring hereto belong the Treatises 1. of Trigonometry Plain & Spherical 2. Geodesia of Surfaces. 3. Stereometry of Solids.

In another place in this same notebook, there occurs this list:

"Mathematicks, Arithmatic, Geometry, Algebra, Trigonometry, of Numbering & Measuring."

Several very detailed outlines of the parts of mathematics are written on the first blank pages of a copy of Sturm's *Mathesis Juvenilis*[7] which has on its last page, "Libris Samuelis Johnsoni. Col.

[5] F. B. Dexter, *Biographical Sketches*, p. 123.

[6] Samuel Johnson Collection, Columbia University library.

[7] *Mathesis Juvenilis:* Made English from the Latin of Jo. Christopher Sturmius. By George Vaux, M. D., London, 1709.

legii Yalensis Nov. Port. Anno Dom. 1718." This work must have yielded material for Johnson's classroom use. It is quite certain that his pupils had no textbooks, but it is to be hoped that no one of them wrote concerning him what he wrote while he was a student at Yale. This frank confession is found in a notebook dated 1714:

> Ho when I was at Colledg I was taught nothing but to be a conceited Coxcomb like those that taught me. Indeed we had no Books & our Ignorance made us think we needed none So that we dug everything almost out of our own Brains as a certain Gent of those times, used to say was his way.

In September, 1718, Daniel Brown, a classmate and close friend of Johnson, was chosen junior tutor at Yale, and together they carried on the work of instruction until the following year. On the first blank page of a copy of Euclid's Elements[8] is found the following interesting indication of the ownership of the book:

> Daniel Brown's Book Decr 25 1717
> Samuel Johnson's Book 1718 bought of Mr. Brown

Most of the propositions in Book II of the Elements have marginal notes in Brown's handwriting, giving a complete algebraic treatment of them. At the time that Brown owned this book, he was rector of the Hopkins Grammar School, of New Haven.

Nothing can be said definitely about the bearing of this algebraic material on the course of study in either the grammar school or Yale. Both Johnson and Brown were excellent scholars and students. They were also successful teachers, and enthusiastic study led, in Johnson's case, to a reaction in his classroom which took at least one form, that of the first presentation of algebra at Yale College.

Mathematical theses at Harvard.—The earliest of the extant theses of Harvard College which include any mathematics were printed in 1653 and cover "Arithmeticae" and "Geometricae." On the set of theses for 1708, we find ones which read: "Arithmetica et Geometria tantum sunt artes pure Mathematicae" (Arithmetic and geometry are pure mathematical arts), and "Astronomia est Scientia Mathematica mixta" (Astronomy is a combination of science and mathematics). Conic sections enter in 1711 with the statement: "Nullus Excentricitatis Gradus Circulum producit, Indefinito grado Parabola producitur" (No degree of eccentricity produces a circle, but a parabola is produced by an indefinite degree of eccentricity). In 1719 two statements show that fluxions had gotten a foothold; one is to the effect that "Fluxio est Augmentationis vel Diminutionis

[8] *Euclide's Elements* . . . By Isaac Barrow, D. D. London, 1705. Johnson's library contains also "The Elements of Euclid . . . Written in French by . . . de Chales. Now made English . . . Oxford, 1704." It has on the front page "Thomas Prince 1707 Books omitted viz. 7, 8, 9, 10, 13 & Supra. Books contained viz: 1, 2, 3, 4, 5, 6, 11, 12." Then follows a list of omitted propositions in the same handwriting. Prince graduated from Harvard in 1707. Was this the textbook in geometry at Harvard in the same year?

Theses MATHEMATICÆ.

1 MATHEMATICA, Quantitates Materiæ, Spatii, et Motionis investigat, et definit.
2 Partes Mathematicæ sunt duæ, Pura et mixta.
§ I. 3 Mathematica Pura, Quantitatem, a Materia, Spatio, et Motione abstractam considerat.
4 *Quantitas* duobis Modis consideranda est, viz. quoad *Numerum*, quoad *Magnitudinem*, alterum *Arithmetica*, alterum *Geometrica*, et alterutrum *Algebra* objectum est.
I. 5 Arithmetica *Numeros*, Quantitatum cujusvis Generis, in alia ejusdem Quantitate, contentos, exponit.
6 In Numeris *Arithmetice* proportionalibus, summa numerorum extremorum æquatur summæ duorum mediorum, æqualiter ab Extremis distantium.
7 Numeris *Geometrice* proportionalibus, Rectangulum Extremorum æquatur quadrato Terminorum mediorum.
8 *Logarithmi* sunt Numerorum artificialium series, in Proportione Arithmetica, sicuti numeri respondentes in Geometrica progredientes.
9 Additio et Subtractio in Arithmetica, Multiplicationi et Divisioni in Geometrica, Proportione respondent.
II. 10 Geometria Totum, vel aliquam Partem *Magnitudinis* datæ indicat, et Determinat.
11 Parallelogramma super æqualibus Basibus, et in iisdem Parallelis constituta inter se æqualia sunt.
III. 12 Algebra est Ars ratiocinandi, *Quantitatibus* ignotis, ut earum Habitudinem Quantitatibus notis definiat.
13 *Arithmetica* a Quantitatibus datis, ad quæsitas; *Algebra* autem a quæsitis, ad datas progreditur.
14 Multiplicatio Quantitatum concretarum, est quando aliqua Quantitas involvit talem Rationem Multiplicando, qualem Multiplicator, unitati.
15 In Divisione, Quotiens eandem Rationem Dividendo, quam Divisor unitati involvit.
16 Momenta alicujus Quantitatis generatæ æqualia sunt Momentis omnium generantium, cum Indicibus suarum Potestatum, et Coefficientibus continuo multiplicatis.
§ II. 17 Mathematica Mixta est tantum Puræ, ad Materiam, vel Spatium, vel Motionem, Applicatio.
18 *Projectio* sphæræ in plano, est omnium Circulorum sphæræ super alicujus Circuli planum Delineatio Geometrica.
19 *Astronomia* est Scientia Corporum Cœlestium, quoad eorum Magnitudinem, Distantias, Eclipses, Positiones, et Motiones.
20 Eclipses Lunares efficiuntur per Atmospheram Corporis obscurantis.
21 Longitudo Locorum, Regulis Astronomicis, facile determinari potest.
22 Partes *Musicæ* tantum sunt tres revera distinctæ.

Theses PHYSICÆ.

1 PHYSICA est Rerum naturalium, earumque Phænomenon Tractatus.
2 Res naturales sunt, quæcunque Universum constituunt.
3 Phænomena sunt situs, motusque, Corporum naturalium, quatenus a Mente intelligenti non dependent.
☞ *OmneCorpus ad motum vel Quietem est indifferens.*
☞ *Sol et Luna sunt Luminaria, gubernantia Corpora humana.*
6 Ignis est materia vivide agitata.
7 Motio Corporis est ejusdem continua et successiva Loci mutatio.
8 Momentum Motionis, Causæ motionem producenti, est proportionatum.
9 Actio et Reactio semper sunt æquales.
10 Æquales materiæQuantitates, eademVelocitate actæ, æqualia habebunt momenta.
11 Vires æquales et contrariæ in idem Corpus agentes, mutuum effectum tollunt.
12 Corporum magnitudine inæqualium sed æqualium Velocitatum momenta, sunt Materiæ in iis contentæ proportionalia.
13 Gravitas materiæ non est Essentialis.
14 Secundum Gradus differentes Attractionis, sunt Gradus Elasticitatis et Cohæsionis.
15 Ex gravitanti natura omnium Corporis particularium sequitur Cohæsio.
16 Colores oriuntur e Corporum aptitudine, Radios Lucis reflectendi vel transmittendi.
17 Generatio non fit in Instanti.
18 Omne Animal præexistit in Animalculo.
19 Corporum Porrorum Parvitas, Radios Solis excludere non possunt.
20 Plantæ non præcipue constituuntur ex Aqua.
21 Materia non est divisibilis in Infinitum.
22 Terræ motus fit ex motione et mixtura Differentium particularum, intra Terræ Viscera.
23 Corpora fluida, Vasis continentis Latera, premunt Directione eis perpendiculari.
24 Essentia Lucis in Corporeitate et vivida motione consistit.
25 Omne Corpus in Terra superficiem, continuam patitur mutationem.
26 Mundus per ultimam Conflagrationem, tantum est purificandus.
27 Modus quo Anima et Corpus uniuntur, nobis prorsus est ignotus.

O SALUTATORIA.

MDCCXXI.

Theses from the commencement program of Harvard College in 1721. Six of the theses relate to algebra quantitatum fluentium Velocitas" (A fluxion is the velocity of an increasing or diminishing flowing quantity), and the other that "Fluxio ex quantitate fluente Invenitur" (A fluxion is found from a flowing quantity). The theses of this same year include one on the fourth subject of the quadrivium, denoting that music was still being taught as a branch of mathematics. The proposition to be argued is: "Dias et Trias harmonica sunt fundamenta contrapuncti musici" (Harmonic diads and triads are the foundation of musical counterpoint.)

The first set of extant Harvard theses to contain algebra is that of 1721. The theses in this broadside are classified and arranged in the same order as those of Yale in 1718. Most of the mathematical theses are general in their nature, with few detailed facts from the various subjects. These cover about the same range as those on the Yale paper already discussed. They are presented in two divisions, "Mathematica Pura," and "Mathematica Mixta." Under the former there are two general statements and three subdivisions, viz, "Arithmetica," "Geometria," and "Algebra"; under the latter are found "Astronomia" and "Musica."

Algebra theses at Harvard in 1721.—The English translations of the statements relating to algebra are as follows:

4. Any quantity has to be considered from two viewpoints, viz, either as a number or as a magnitude; the one is the object of arithmetic, the other of geometry, and both of algebra.
12. Algebra is the art of reasoning with unknown quantities in order to define their relations to known quantities.
13. Arithmetic proceeds from given to required quantities; algebra, however, from quantities sought to those given.
14. Multiplication of concrete ["concretarum" means here "that which is grown together"] quantities is when some quantity involves the multiplicand in such a relation as the multiplier [involves] unity.
15. In division the dividend involves the quotient in the same relation as the divisor [involves] unity.
16. The moments of any generated quantity are equal to the moments of all the generators with the exponents of their powers and their coefficients in continued multiplication.

These theses give no indication of the amount of subject matter that was being undertaken in algebra. The inclusion of number 16 shows that algebra and fluxions were closely associated. A greater degree of maturity in the student or in his familiarity with the subject was needed to handle these truths than would be needed to prove, for instance, that the product of two negative quantities is an affirmative quantity. A power of generalization on the basis of algebraic facts was required to defend such theses as are here included.

Attention has already been called to one graduate of this class of 1721, "Isaacus Greenwood," as his name appears on the program.

Whatever his later study, Greenwood must have gotten a start on algebra in his college course that awakened a love for the subject. This interest he passed on to his students when he became the professor of mathematics at Harvard College.

The recurrence of certain theses on the commencement program of the same college and of the different colleges has an element of interest. A favorite one which occurs, with slight variations in wording, at Yale in 1720, Harvard in 1731, Brown in 1773, and Harvard again in 1780, is the following: "Duo numeri biquadrati summatim sumpti numerum quadratum constituere non possunt." (Two biquadratic numbers taken at random can not constitute a square, Yale, 1720, 4).[9]

The custom of presenting theses at commencement time continued in force at all the principal colleges until well into the nineteenth century. A careful examination of the theses of Harvard, Yale, Rhode Island College, and the College of New Jersey shows that algebra appears among them at frequent intervals until the custom passed away. Its omission is apparently due to a greater emphasis along another line in that year.

At Yale, from 1739 for several years, conic sections and higher plane curves were stressed with such theses as: "Cycloidis area triplex est ejus circuli generantis" (the area of a cycloid is three times that of the generating circle, Yale, 1739, 6), and "Parallelogramma omnia, circa datae ellipseos vel hyperbolae diametros quaevis conjugacas descripta, sunt inter se aequalia." (All parallelograms described on conjugate diameters of ellipses or hyperbolas are equal, Yale, 1740, 17.)

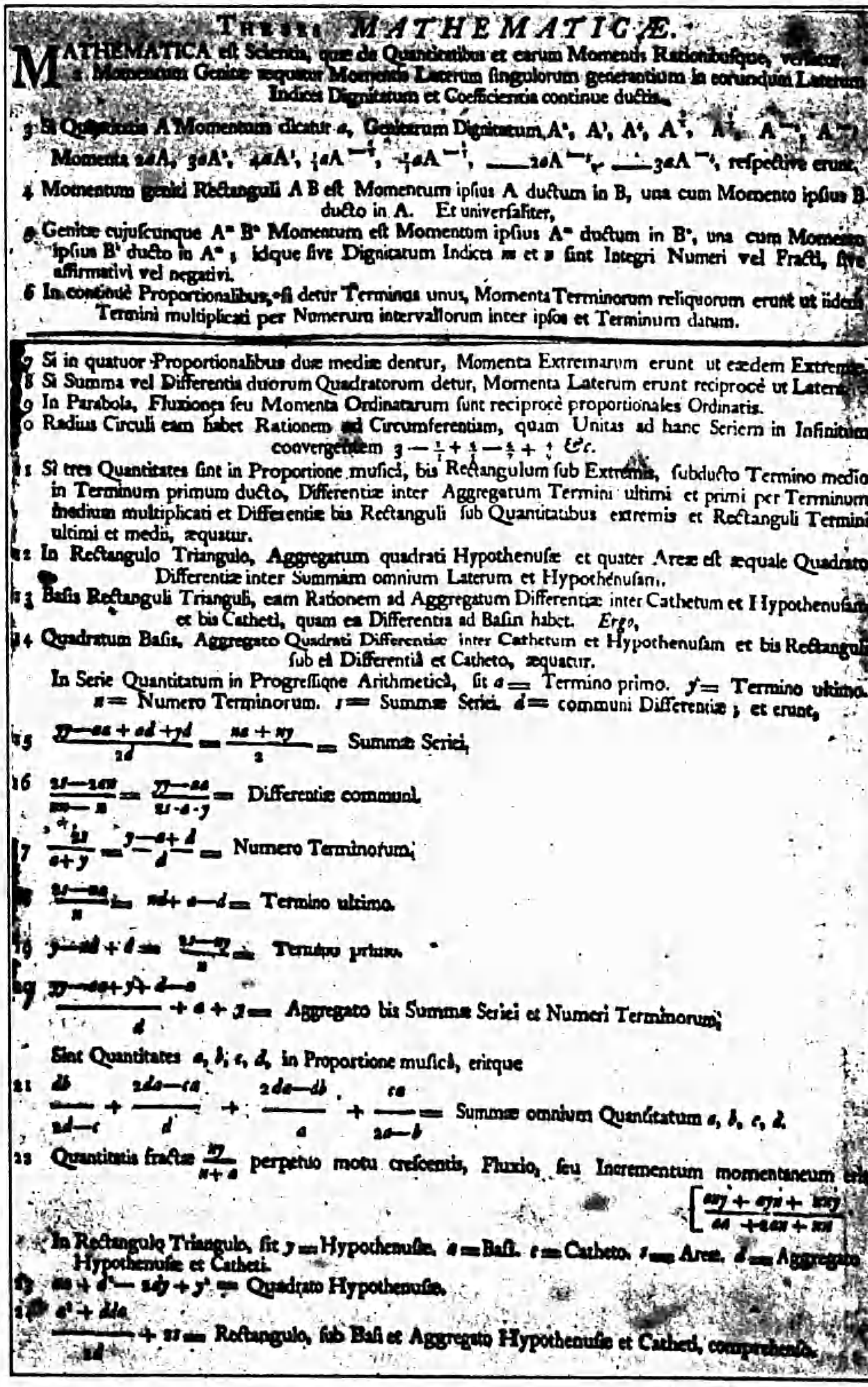
Beginning at an earlier date, 1735, Harvard parallels this Yale record. From that year through 1750, a prominent place is given to statements concerning conic sections, cycloids, semicycloids, cissoids, spirals, and other higher plane curves. One instance of this is the thesis: "Conchoides, Cissoides et Curva Logarithmica habent unum Asymptoton." (Conchoids, cissoids, and logarithmic curves have one asymptote, Harvard, 1746, 24.)

Fluxions.—Then 1751 shows a radical change and a very interesting one. Fluxions occupy a leading place among the mathematical commencement theses to the exclusion of practically all the mathematical topics that had hitherto appeared, and this subject did not recede from its prominent position at Harvard during the eighteenth century. The fact that all problems in fluxions require a familiarity with the handling of algebraic expressions and operations should not be lost sight of at any time.

---

[9] This is one of Fermat's theorems. See Leonard Eugene Dickson, *History of the Theory of Numbers*, Vol. II, p. 616, Washington, 1920.

At Yale, fluxions appeared in 1758, and during 25 years thereafter only a few sets of these lack problems in this subject. Some stu-


THESES MATHEMATICÆ.

MATHEMATICA est Scientia, quæ de Quantitatibus et earum Momentis Rationibusque, versatur.
2 Momentum Genitæ æquatur Momentis Laterum singulorum generantium in eorundem Laterum Indices Dignitatum et Coefficientia continue ductis.
3 Si Quantitatis A Momentum dicatur a, Genitarum Dignitatum A^2, A^3, A^4, $A^{\frac{1}{2}}$, $A^{\frac{1}{3}}$, A^{-1}, A^{-2} Momenta $2aA$, $3aA^2$, $4aA^3$, $\frac{1}{2}aA^{-\frac{1}{2}}$, $\frac{1}{3}aA^{-\frac{2}{3}}$, $-aA^{-2}$, $-2aA^{-3}$, respective erunt.
4 Momentum geniti Rectanguli A B est Momentum ipsius A ductum in B, una cum Momento ipsius B ducto in A. Et universaliter,
5 Genitæ cujuscunque $A^m B^n$ Momentum est Momentum ipsius A^m ductum in B^n, una cum Momento ipsius B^n ducto in A^m; idque sive Dignitatum Indices m et n sint Integri Numeri vel Fracti, sive affirmativi vel negativi.
6 In continuè Proportionalibus, si detur Terminus unus, Momenta Terminorum reliquorum erunt ut iidem Termini multiplicati per Numerum intervallorum inter ipsos et Terminum datum.
7 Si in quatuor Proportionalibus duæ mediæ dentur, Momenta Extremarum erunt ut eædem Extremæ.
8 Si Summa vel Differentia duorum Quadratorum detur, Momenta Laterum erunt reciprocè ut Latera.
9 In Parabola, Fluxiones seu Momenta Ordinatarum sunt reciprocè proportionales Ordinatis.
10 Radius Circuli eam habet Rationem ad Circumferentiam, quam Unitas ad hanc Seriem in Infinitum convergentem $4 - \frac{4}{3} + \frac{4}{5} - \frac{4}{7} + \frac{4}{9}$ &c.
11 Si tres Quantitates sint in Proportione musicâ, bis Rectangulum sub Extremis, subducto Termino medio in Terminum primum ducto, Differentiæ inter Aggregatum Termini ultimi et primi per Terminum medium multiplicati et Differentiæ bis Rectanguli sub Quantitatibus extremis et Rectanguli Termini ultimi et medii, æquatur.
12 In Rectangulo Triangulo, Aggregatum quadrati Hypothenusæ et quater Areæ est æquale Quadrato Differentiæ inter Summam omnium Laterum et Hypothenusam.
13 Basis Rectanguli Trianguli, eam Rationem ad Aggregatum Differentiæ inter Cathetum et Hypothenusam et bis Catheti, quam ea Differentia ad Basin habet. *Ergo*,
14 Quadratum Basis, Aggregato Quadrati Differentiæ inter Cathetum et Hypothenusam et bis Rectanguli sub eâ Differentiâ et Catheto, æquatur.
In Serie Quantitatum in Progressione Arithmeticâ, sit $a =$ Termino primo. $y =$ Termino ultimo. $n =$ Numero Terminorum. $s =$ Summæ Seriei. $d =$ communi Differentiæ; et erunt,
15 $\frac{yy-aa+ad+yd}{2d} = \frac{na+ny}{2} =$ Summæ Seriei.
16 $\frac{2s-2an}{nn-n} = \frac{yy-aa}{2s-a-y} =$ Differentiæ communi.
17 $\frac{2s}{a+y} = \frac{y-a+d}{d} =$ Numero Terminorum.
18 $\frac{2s-na}{n} = nd+a-d =$ Termino ultimo.
19 $y-nd+d = \frac{2s-ny}{n} =$ Termino primo.
20 $\frac{yy-aa+y+d-a}{d} + a + y =$ Aggregato bis Summæ Seriei et Numeri Terminorum.
Sint Quantitates a, b, c, d, in Proportione musicâ, eritque
21 $\frac{db}{2d-c} + \frac{2da-ca}{d} + \frac{2da-db}{a} + \frac{ca}{2a-b} =$ Summæ omnium Quantitatum a, b, c, d.
22 Quantitatis fractæ $\frac{xy}{x+a}$ perpetuo motu crescentis, Fluxio, seu Incrementum momentaneum erit $\left[\frac{a\dot{x}y + a\dot{y}x + x\dot{y}x}{aa+2ax+xx}\right.$
In Rectangulo Triangulo, sit $y =$ Hypothenusæ. $a =$ Basi. $c =$ Catheto. $s =$ Areæ. $d =$ Aggregato Hypothenusæ et Catheti.
23 $aa + d^2 - 2dy + y^2 =$ Quadrato Hypothenusæ.
24 $\frac{a^3+dda}{2d} + 2s =$ Rectangulo, sub Basi et Aggregato Hypothenusæ et Catheti, comprehenso.

Mathematical theses from the commencement program of Harvard College in 1751, the first year in which fluxions were prominent

dent defended the following truth: "Quatenus Algebra arithmeticae, eatenus doctrina Fluxionum algebrae antecellit" (As far as algebra is superior to arithmetic, so far is the doctrine of fluxions superior to algebra, Yale, 1782, 16), a thesis which required a knowledge of all three branches of mathematics.

This discussion may well conclude with a pronouncement which the changes in psychological fashions have in no way disturbed. It is as follows: "Mathesis studium disciplinam mentis optimam praebet." (The study of mathematics provides the best mental discipline, Harvard, 1808, 30.)

Chapter VII

MATHEMATICAL THESES OF HARVARD COLLEGE

In quite a different category from the commencement theses discussed in an earlier chapter are the problems presented to the department of mathematics at Harvard College by the members of the junior and senior classes. There are 406 sets of these problems extant,[1] the earliest one dated 1782 and the latest 1839.[2] Many of them show the signatures of men well known in later years in various walks of life. Some provide information of conditions at Harvard which no longer exist. All give to an unusual degree a picture of the work in mathematics done by the students whose names they bear.

Each set of problems is worked on one side of a sheet of heavy linen paper with body enough to make it resemble a light-weight cardboard. The sheets are of slightly varying sizes, 18 by 14 inches, 19.5 by 15 inches, 23.5 by 18 inches, being the measurements of a few chosen at random. The work is done by hand in ink, and at times the workmanship is exquisite. Many of the trigonometry problems are accompanied by beautiful illustrations in color.

There are 79 sets of problems from 1782 to 1800. The majority of these sets are devoted to calculations of eclipses or to plans of buildings. A graduate of 1796 took for his mathematical thesis: "A North East View of the House of Samuel Webber, A. A. S.,[3] and of the Court House in Cambridge, by an actual Survey." Of the 79 sets, however, 13 are designated "Algebraic Problems" or "Algebraical Solutions of Problems" or "Application of Algebra to the Geometrical Solution of Problems," while 3 are simply "Mathematical Problems." Algebra problems continued to occupy a prominent place among those in pure mathematics for many years.

Algebra theses.—The heading "Algebraic Problems" appears for the first time in 1786, and in this set of exercises are found four problems, a simple one in geometry, two in surveying, and one in trigonometry. In the following year four students presented mathematical theses, three of which are entitled "Mathematical Problems

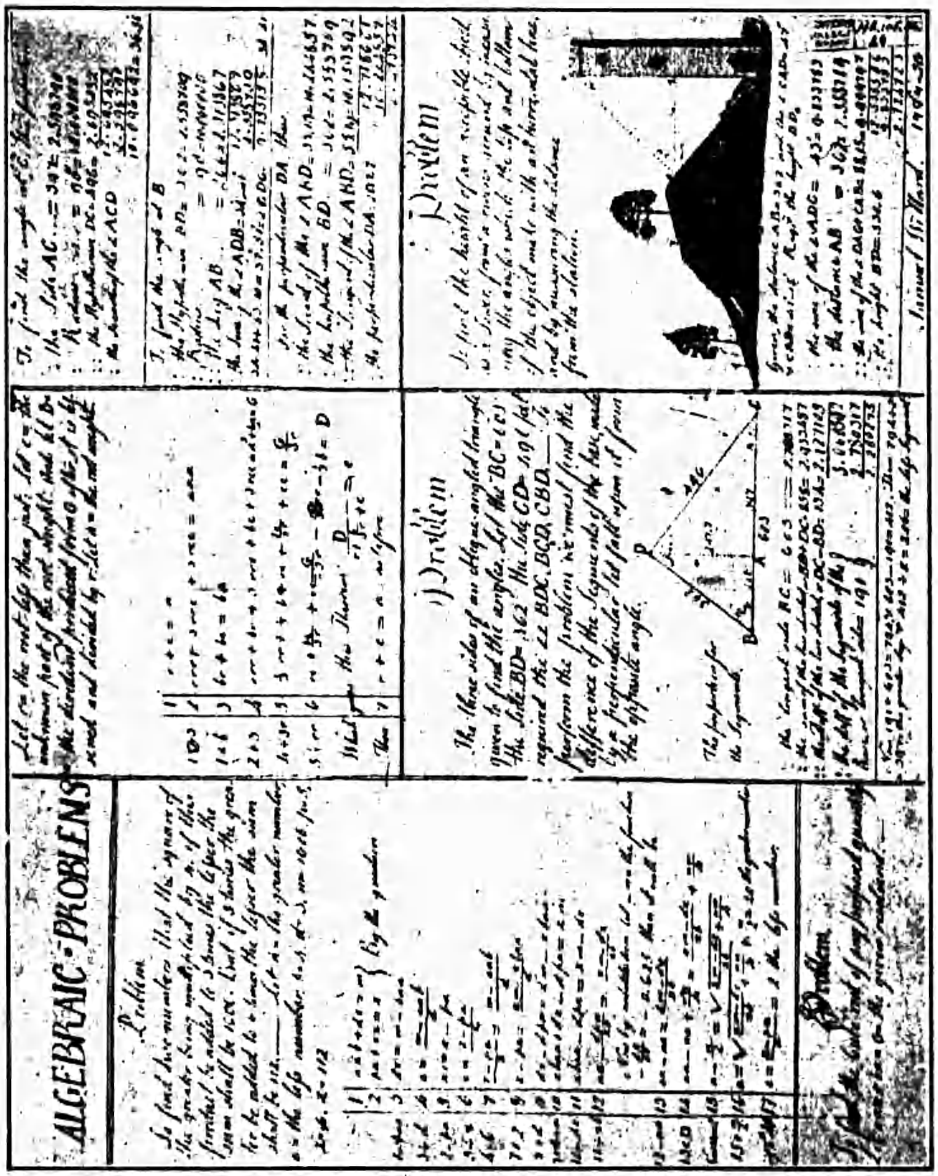
---

[1] Harvard University Library.

[2] A complete list of the headings found on these sets of problems is given in Henry C. Badger, "Mathematical Theses of the Junior and Senior Classes, 1782–1830," in *Biographical Contributions*, No. 32, Library of Harvard University, Cambridge, 1888.

[3] Samuel Webber was the professor of mathematics at the time.

and their Solutions," and the fourth is again "Algebraic Problems." This last set of 1787 will be discussed as an illustration of this type of work. Samuel Willard, whose signature is affixed to this mathematical thesis, graduated in that year, and so this group of four exercises is probably representative of his course in mathematics at Harvard. It is in many respects the work that would be



Theses presented to the mathematics department at Harvard College in 1787

expected from a good secondary school pupil of the present day. The treatment of the algebra problem is that which, as pointed out in connection with the notebooks from Harvard and the University of Pennsylvania, was accorded problems so universally in the eighteenth century. Here a and e are used for the unknown quantities, a usage occurring in Ward's *Young Mathematician's Guide.*[4] The next problem, "To find the Cube root of any proposed quan-

[4] John Ward. *The Young Mathematician's Guide*, London, 1709; and later editions.

PROBLEM II.

It is required to find four numbers in $\because$ whose sum may be $= b = 30$; and the sum of their squares $= c = 340$.

1. Let the two means be $= a$ and e.
2. Then the two extremes will be $= \frac{aa}{e}$ and $\frac{ee}{a}$.

	3	$\frac{aa}{e}+a+e+\frac{ee}{a}=b.$
	4	$\frac{a^4}{ee}+aa+ee+\frac{e^4}{aa}=c.$
$3\times ae$	5	$a^3+aae+aee+e^3=bae.$
$4\times aaee$	6	$a^6+a^4ee+aae^4+e^6=caaee.$
$5\div\overline{a+e}$	7	$aa+ee=\frac{bae}{a+e}.$
$6\div\overline{aa+ee}$	8	$a^4+e^4=\frac{caaee}{aa+ee}.$
7 sq.	9	$a^4+2aaee+e^4=bb\times\overline{\frac{ae}{a+e}}\Big\rvert^2$
$9-2aaee$	10	$a^4+e^4=bb\times\overline{\frac{ae}{a+e}}\Big\rvert^2-2\times\overline{ae}\rvert^2$
7, 8	11	$a^4+e^4=caaee\div\frac{bae}{a+e}=\frac{cae\times\overline{a+e}}{b}.$
$7+2ae$	12	$aa+2ae+ee=\overline{a+e}\rvert^2=\frac{bae}{a+e}+2ae.$
Subst.	13	$x=ae$, and $z=a+e.$
10, 11, 12, 13	14	$bb\times\frac{xx}{zz}-2xx=\frac{cxz}{b}$
12, 13	15	$zz=\frac{bx}{z}+2x.$
$14\times\frac{bzz}{x}$	16	$b^3x-2bzzx=cz^3.$
$15\times cz$	17	$cz^3=cbx+2czx.$
$16=17$	18	$b^3x-2bzzx=cbx+2czx.$
$18\div x$	19	$b^3-2bzz=cb+2cz.$
19 Transp.	20	$2bzz+2cz=b^3-cb.$
$20\div 2b$	21	$zz+\frac{c}{b}z=\frac{1}{2}bb-\frac{1}{2}c.$
$21+\frac{cc}{4bb}$	22	$zz+\frac{c}{b}z+\frac{cc}{4bb}=\frac{bb}{2}-\frac{c}{2}+\frac{cc}{4bb}$
22 rt.	23	$z+\frac{c}{2b}=\sqrt{\frac{bb}{2}-\frac{c}{2}+\frac{cc}{4bb}}.$
$23-\frac{c}{2b}$	24	$z=\sqrt{\frac{bb}{2}-\frac{c}{2}+\frac{cc}{4bb}}-\frac{c}{2b}==12.$
17.	25	$x=\frac{z^3}{2z+b}=\frac{1728}{54}=32.$
$13\div e$	26	$\frac{x}{e}=a.$
13, 26	27	$z=\frac{x}{e}+e.$
$27\times e$	28	$ze=x+ee.$
28 Transp.	29	$ee-ze=-x.$
29 Reduced.	30	$e=\frac{1}{2}z\pm\sqrt{\frac{1}{4}zz-x}=6+2=8.$
$13-e$	31	$z-e=\frac{1}{2}z\mp\sqrt{\frac{1}{4}zz-x}=a=4.$
2	32	$\frac{aa}{e}=2$, and $\frac{ee}{a}=16.$

"Algebraical solutions of problems" from the thesis presented by Luther Richardson to the mathematics department of Harvard College in 1799 tity," might have been copied from that same work.[5] The third problem employs six-place logarithms, but shows no use of interpolation. It combines a knowledge of geometric facts with certain trigonometric relations. The fourth problem is the real applied work and shows the use of the law of sines. In the original the diagram is colored with excellent taste.

Evidence drawn from the commencement theses goes to show that this work by Samuel Willard does not cover all of the mathematical instruction at Harvard in 1787, for the theses of that year include a number of statements concerning fluxions. It may represent the required work, while the more difficult courses were open only to exceptional students.

Other sets of algebraic problems show such differences as would arise from the need of the professor of mathematics to vary his subject matter and from the inclinations of the individual pupils. Francis Cabot Lowell, on October 30, 1792, presented two problems which show a love for long mechanical operations. In one the value of the unknown is given in a number containing 38 digits. In the other, the equation $45x^4 - 1800x^3 + 21564x^2 - 324000 = 324000$ is brought to a successful solution by a method of approximation, but there is a great deal of tedious work before that end is attained. The persistence of the Oughtred symbols for inequality is shown by their use in a paper of 1793. Work of an involved nature and even of an advanced character is that on the paper of Luther Richardson in 1799.

Such direct evidence as the foregoing mathematical theses is worth more in an account of the mathematics in American colleges during the eighteenth century than all the statements about that subject in catalogue or college president's report. It warrants the conclusion that algebra had obtained an established place in the curriculum, which place it has filled from that time until the present day.

[5] *Ibid*, p. 238 (1719).

Chapter VIII

COLLEGE RECORDS AND WRITINGS OF PROFESSORS AND PRESIDENTS

Algebra in manuscripts and printed works.—References to algebra in the manuscript and printed works of college presidents and professors, as well as in regulations and laws governing courses of study, constitute another link in the chain of evidence showing the recognition granted to the subject during the eighteenth century.

Harvard requirements.—The earliest laws for Harvard College were prepared in 1642 by President Dunster and included the study of mathematics but with no mention of specific branches.[1] These laws governed the Harvard curriculum with no material changes during the seventeenth century. In the early part of the eighteenth century a professorship in mathematics was established by Thomas Hollis. The first definite requirement of algebra at Harvard is included in the principles on which the chair was founded. These principles are set down thus:

> Rules and Orders relating to a Professor of the Mathematics, of Natural & Experimental Philosophy in Harvard College in Cambridge in New England, appointed by Mr. Thomas Hollis of London Merchant.
>
> 1. That the Professor be a Master of Arts and well acquainted with the several parts of the Mathematics & Experimental Philosophy.
>
> 2. that his Province be to Instruct the Students in a System of Natural Philosophy & a course of Experimental, in which to be comprehended, Pneumaticks, Hydrostaticks, Mechanicks, Staticks, Opticks, &c in the Elements of Geometry to-gether with the doctrine of Proportions the Principles of Algrebra (sic) Conic Sections, plain & Sperical (sic) Trigonometry with the general principles of Mensuration, Plain & Solids, in the Principles of Astronomy & Geometry, viz, the Doctrine of the Spheres the use of the Globes, the motions of the Heavenly Bodies according to the different Hypotheses of Ptolomy (sic) Tycho Brahe & Copernicus with the general Principles of Dialling the Division of the World into its various Kingdoms with the use of the Maps, &c.[2]

The second rule is the only one which bears directly on the subject in hand. It shows that the principles of algebra were accepted as a necessary part of the instruction in mathematics as early as the date on this paper, 1726. Isaac Greenwood, the first man who was

[1] Louis Franklin Snow. *The College Curriculum in the United States*, p. 25. [no place], 1907.

[2] Harvard College Papers, Vol. I, 1650–1763, Jan. 18, 1726, Harvard University Library. There are 13 rules.

called upon to live up to this fine set of rules, may have had something to do with formulating them. He was one of five men to respond to a call from Mr. Hollis to furnish plans for his projected chair of mathematics.[3] The two Harvard notebooks on algebra already set forth attest the success of Greenwood in fulfilling the requirement to teach algebra.

These rules guided the professor of mathematics at Harvard for many years. Not until 1787 are there any more specific directions adopted for his control. A committee appointed to revise the course of instruction voted on August 16, 1787,[4] with respect to the sophomores—

> that at eleven o'clock on Friday they attend the Professor of Mathematics to be instructed in Algebra, and to be carried forward to other branches of the Mathematics if the time allow.

And again on October 16, 1788,[5] that future Hollis professors of mathematics were to carry the classes forward by private lectures,

> in Algebra as far as through affected quadratic equations and infinite series.

Algebra, then, receives specific mention in all the laws and regulations formulated at Harvard in the eighteenth century.

Hugh Jones at William and Mary.—The College of William and Mary,[6] the second oldest college in the United States, was founded in 1693 at Williamsburg, Va., and the charter provided at the outset for a president and six professors. The published records until recently named the Rev. Hugh Jones as the first professor of mathematics. An earlier name in the faculty of the college is that of Mr. Le Fevre, as shown by the following extracts from the letters of Governor Spotswood, of Virginia:[7]

VIRGINIA, *July 28, 1711.*

To Mr. BLATHWAYT:

SIR: I have not had the honor of any from you since my last, but having seen a Letter that you writt to Collo. Diggs in behalf of Mr. Le Fevre, I very gladly embraced the Opportunity of doing hon'r to your Recommendation by getting the Governor of the College to receive him as a Mathematick Professor. . .

VIRGINIA, *May 8th, 1712.*

To the B'P OF LONDON:

. . . I gave your Lord'p an account of Mr. Le Fevre's admission into the College upon your Lord'p's recommendation, and am now to acquaint you that after a Tryal of three-quarters of a year he appeared so negligent in all the

[3] Josiah Quincy, *The History of Harvard University*, Vol. I, p. 399, Boston, 1860.

[4] L. F. Snow, *College Curriculum*, p. 84, Quoted from Col. Book 8, p. 243.

[5] Ibid., p. 270 ff.

[6] The researches of Dr. Lyon Gardiner Tyler, for 31 years the president of William and Mary, have been most helpful in the study of that college. They are published in *The William and Mary Quarterly*.

[7] *The Official Letters of Alexander Spotwood, Lieutenant Governor of the Colony of Virginia, 1710–1722.* Now First Printed from the Manuscript in the Collections of the Virginia Historical Society with an introduction and notes by R. A. Brock . . . I, pp. 103, 156, Richmond, 1882.

post of duty and guilty of some other very great irregularities, that the Governors of the College could no longer bear with him, and were obliged to remove him from his Office. . . .

However unworthy the gentleman was, Mr. Le Fevre was the first professor of mathematics in an American college.

Another letter[8] from Governor Spotswood settles definitely the fact that the Rev. Hugh Jones was established at William and Mary in 1717, some 10 years before Hollis had created a professorship of mathematics at Harvard. The part of the letter referring to Jones is as follows:

JUNE 13, 1717.

To the BISHOP OF LONDON:

. . . and I doubt not y'r Lord'p is already informed that Mr. Jones is admitted into the College according to y'r Lo'p's Recommendation : . . .

Jones was an Englishman of university training and kept very closely in touch with his native land.[9] His connection with William and Mary as professor of mathematics was a brief one, for he left the Colonies for England in 1722. On his return to America he devoted himself exclusively to the ministry. In 1724 Jones published in London a most interesting history in whose making he had participated, entitled *The Present State of Virginia*. This is the work which holds interest for the student of the history of mathematics as well as the student of colonial history. Referring to the Virginians, Jones says in this work:

They are more inclinable to read Men by Business and Conversation, than to dive into Books, and are for the most part only desirous of learning what is absolutely necessary, in the shortest and best Method.

Having this knowledge of their Capacities and Inclinations from sufficient Experience I have composed on Purpose some short Treatises adapted with my best Judgment to a Course of Education for the Gentlemen of the Plantations; consisting in a short *English Grammar;* an *Accidence to Christianity;* an *Accidence to the Mathematicks*, especially to *Arithmetick* in all its Parts and Applications, *Algebra, Geometry, Surveying of Land*, and *Navigation*.

These are the most useful Branches of Learning for *them*, and such as they willingly and readily master, if taught in a plain and short Method, truly applicable to their *Genius;* which I have endeavoured to do, for the use of *them*, and *all others* of their Temper and Parts.[10]

It will be noted that algebra is to be found among these treatises which Jones states that he composed. Since he holds "from sufficient Experience" that the people of Virginia are capable of mastering this and other branches of mathematics, "if taught in a plain and short Method," he must have reached that opinion by actual experience in teaching the subjects enumerated to them. And

[8] Spotswood's Letters, loc. cit., II, p. 253.

[9] Many letters of his are listed in British Museum catalogues of manuscripts.

[10] Hugh Jones, *The Present State of Virginia*, p. 44, London, 1724.

so it is fairly obvious that algebra was included in the curriculum of the College of William and Mary before the year 1722.

The treatise on grammar[11] is extant, and a copy of it is in the British Museum. The "Accidence to the Mathematicks" does not appear to have been preserved in like manner, but the British Museum does possess a manuscript on mathematics by Hugh Jones,[12] entitled: "Reasons and uses of the Georgian Calendar and of Octave Computation or Natural Arithmetic." The date of it is evidently 1752, as "this year 1752" occurs in the text. The first part of this work is concerned with a calendar composed of 13 seasons of lunations of 28 days, one day for the Nativity and a bissextile every fourth year for public prayer. The second parts starts with a dissertation on the disadvantages of numbering by 10 and proposes substituting 8 as a radix. It gives a complete numeration table and lays down rules for the reduction of decades to octaves and other matters. It also points out methods for dividing weights and measures on this basis, so making a universal standard.[13]

Thomas Clap at Yale.—We turn next to the third college established in the Colonies. The keen interest of the president of a college in any subject when that college was little more than a collegiate school was certain to have a strong influence on the place which that subject occupied in the curriculum. Thomas Clap brought such an interest in mathematics to Yale when he became rector or president in 1739. Four years later, he published a catalogue of the college library[14] which he had prepared himself. In the preface to this catalogue, he recommends a plan of studies for the college course, as follows:

In the First year to study principally the Tongues, Arithmetic, and Algebra; the Second, Logic, Rhetoric, and Geometry; the Third, Mathematics and Natural Philosophy; and the Fourth, Ethics and Divinity.

Mathematics is found in three years of this plan, with algebra in the first year.

That Clap carried out a still more ambitious plan of mathematical instruction than he had advocated is shown in the history of Yale which he wrote in 1766. Referring to the undergraduate students, he says:

They are divided into four classes; according to the respective years in which they are admitted. At their admission they are able well to construe and parse Tully's Orations, Virgil and the Greek Testament; and understand the Rules of common Arithmetick. In the first year, they learn Hebrew, and principally

[11] Jones (Hugh), A. M., Minister of James Town, Virginia. *An Accidence to the English Tongue*, etc., London, 1724. Given in British Museum catalogue.

[12] British Museum Additional Manuscript 21, 893.

[13] Account, of which the above is a résumé, furnished by B. F. Stevens & Brown, London, through the courtesy of Columbia University library.

[14] [Thomas Clap] *A Catalogue of the Library of Yale-College in New-Haven*, N. London, 1743.

pursue the Study of the Languages, and make a Beginning in Logick, and some Parts of the Mathematicks. In the second year, they study the Languages; but principally recite Logick, Rhetorick, Oratory, Geography and natural Philosophy. And some of them make good Proficiency in Trigonometry and Algebra. In the third year, they still pursue the Study of Natural Philosophy, and most Branches of the Mathematicks; many of them well understand Surveying, Navigation and the Calculation of the Eclipses; and some of them are considerable Proficients in Conic Sections and Fluxions. In the fourth year they principally study and recite Metaphysicks, Ethicks and Divinity.[15]

The printed statement of a projected plan or of a completed one may always be taken with some reservations, but the commencement theses during Clap's presidency of Yale give added confidence in the acceptance of this one at its face value.

In 1753, during Clap's administration, the "Linonian Society" was founded. At one time it was the custom to have a curious question brought in at each meeting. From the records [16] of this society is taken one specimen which relates to algebra:

Dec. 5th A D 1770. 2. How do you solve Questions, when the unknown quantity has several Powers in one Equation, and only the first Power in the other Equation.

The recording of the question and its answer means that algebra engaged the interest of eighteenth century Yale boys.

University of Pennsylvania.—The first page of the records of the institution now known as the University of Pennsylvania shows that algebra is included among the subjects intended to be taught. The seventh page contains this entry, showing the fulfillment of the intention:

Mr. Theophilus Grew having offered himself as a Master in the Academy to teach Writing, Arithmetick, Merchant's Accounts, Algebra, Astronomy, Navigation, and all other Branches of the Mathematicks, it is ordered that he be received . . . his Service to commence on the seventh day of January next. July 27, 1750.[17]

In 1753 Provost William Smith published a work entitled "A General Idea of the College of Mirania, with an account of the College and Academy of Philadelphia" [University of Pennsylvania]. His belief in mathematics in general is indicated in the preface to this work, where he sets forth the plan of studies covering five classes. He states further that he is now endeavoring to realize these plans in the seminary over which he has the honor to preside, and so we know that he was actually working out his own fruitful ideas. The portions relating to mathematics are found in the first two classes and read:

---

[15] Thomas Clap, *The Annals or History of Yale-College in New Haven.* (New Haven, 1766.) Appendix, p. 81.

[16] Yale University library.

[17] Minutes of the Trustees of the College, Academy, Charitable Schools of Philadelphia, Vol. I, 1749–1768.

The First Class of the College . . . In the afternoon they learn arithmetic, vulgar and decimal, merchant's accounts, some parts of algebra, and some of the first books of Euclid.

The Second Class. The next year is spent in this class; the master of which is styled Professor of Mathematics. He carries the youth forward in algebra, teaches the remainder of the first six books of Euclid, together with the eleventh and twelfth, and also the elements of geometry, astronomy, chronology, navigation, and other most useful branches of the mathematics.[18]

The plan here suggested was formulated at the request of the trustees. It was adopted in 1756 and continued in use while Smith was provost and, as far as records show, until the early part of the next century. It influenced the later curricula of all the American colleges. The prominence given to algebra in the mathematical scheme is indicated by the inclusion of that subject in two years of the college course.

Columbia University.—Samuel Johnson brought from his experience at Yale sufficient interest in mathematics to cause him to give to that subject a place in the plans which he, to a large extent, formulated for King's College [Columbia University]. The first professorship established was that of mathematics.[19] The laws and orders which were adopted in June, 1755, placed "Mathematics and the Mathematical and Experimental Philosophy in all the several branches of it" in the second and third years of the course. In 1785 the plan of education included "Algebra as far as quadratic equations" for the freshman class and the "higher branches of Algebra" for the sophomore class. In 1789 the professor of mathematics and natural philosophy was authorized to give, as the algebra course, "Algebra as far as Cubic Equations" to the freshman class and "the higher parts of Algebra" with "the application of Algebra to Geometry" to the junior class. This was an advance in the subject matter in algebra which continued in force until the end of the century.[20]

Evidence goes to show then that from the very early years of the eighteenth century until its close, algebra is mentioned by professor and president alike and is included in plans proposed and plans adopted for the curricula of the colleges of this period.

---

[18] *The Works of William Smith, D. D.*, Late Provost of the College and Academy of Philadelphia, Vol. I, Part II, p. 183, Philadelphia, 1803.

[19] *A History of Columbia University* (New York, 1904), p. 22 . . . "the Governors, on the 8th of November (1757) appointed Mr. Daniel Treadwell, 'a young gentleman of a very excellent character educated at Harvard College, and recommended by Professor Winthrop as eminently fitted for that station.'"

[20] L. F. Snow, *College Curriculum*, p. 93 ff.

Chapter IX

EVIDENCES OF THE USE OF FOREIGN TEXTBOOKS

Scarcity of printed books.—In the twentieth century, when inexpensive books are procurable in literature and science alike, it is difficult to realize the almost total lack of printed classroom subject matter during practically the whole extent of the eighteenth century. Authorship of texts had not yet become a trade, and the cost of printing and paper made the publication of works for pupils in schools well-nigh an impossibility. Books were in the hands of professors and tutors, and their contents, as already pointed out, were passed on in the form of lectures taken down almost verbatim by the members of a class. But some books were imported from abroad in sufficient numbers to permit of putting them into the hands of the students themselves.

Such foreign textbooks as were used in American schools of the colonial period and for some years thereafter by the students were books by English authors, published in England. In the first quarter of the nineteenth century French and German authors began to exert an influence, and reprints or translations of works from England, France, and Germany were published in America for use in American colleges.

The Young Mathematician's Guide.—We shall speak first of the English texts. Among the most popular but not the most worthy was "The Young Mathematician's Guide. Being a Plain and Easie Introduction to the Mathematicks" by John Ward. This seems to have had a use out of all proportion to its merits, and direct evidence of the position held by Ward's book is extensive.

An intimate record of the use of this work is found in the commonplace book of Eleazer May,[1] of the class of 1752, Yale. A long list of this student's readings is given on the seventh page of his book. This list is headed:

> An Memorandum of the Books red in my freshmanship viz: A. D. 1749.
>
> . . .
>
> In my Sophimore Ship viz. A. D. 1750
>
> . . .
>
> recited. Wards Mathematick.

[1] "Eleazer May's Notebooks." Interesting manuscripts of the middle and last quarter of the eighteenth century now in the university library. *Yale Alumni Weekly*, January 10, 1923, p. 501 f. Some additional material, as the result of a personal study is noted.

In another place young May copied a quotation from Ward that had struck his fancy. It was:

> Of time it is not an Easy thing to give a true Divinition of time for according to the philosophick Poet
>
> Time of itself is nothing but from thought
> Recieves its rise by labouring fancy wrought
> from things considered whilst wee think on some
> as present some as past or yet to come
> no thought can think on time that stil confest
> but thinks on things in motion or at rest:
>
> Ward's Mathema:[2]

A graduate of Harvard, class of 1746, Dr. E. A. Holyoke, says in a letter to Prof. Benjamin Peirce that Ward's Mathematics and Euclid's Geometry were used during his college course. Without referring to any other authority, Peirce states that:

> In the early part of this presidency (that of President Edward Holyoke, which began in 1737) and probably for many years before, the textbooks were the following . . . Ward's Mathematics . . . Euclid's Geometry.[3]

As late as 1794 a Harvard student notebook[4] contains the name of Ward in connection with books to be consulted on mathematical subjects. Catalogues of libraries of gentlemen of this period show that Ward's book was one of those on mathematics to be generally acquired.[5] Samuel Johnson prepared, "A Catalogue of my Library with the vallue of each Book Aug. 15, 1726."[6] This catalogue included "Mr. Ward's Young Mathematician," which he may have been using while he was a tutor at Yale. In 1743 there was published a book with the title: "An Introduction to the Study of Philosophy, Exhibiting a General View of all the Arts and Sciences. By a Gentleman Educated at Yale-College." This gentleman was Samuel Johnson, and he gives the advice (p. 28): "On Mathematics, read Ward's Young Mathematician's Guide, . . ."

A 1764 "Catalogue of Books belonging to the Public English School of Friends at Philadelphia" has one copy of Ward's Mathematician's Guide. In 1778 a "Catalogue of Books granted the College [Harvard] from the sequestered Libraries"[7] contains Ward's Mathematics. In the libraries of universities and in libraries connected with organizations of various sorts are to be found to-day from one to four copies of this popular old work.

The curriculum of the colleges at certain times was stated in terms of the texts used rather than in terms of subjects. In 1756, in con-

---

[2] John Ward, *Young Mathematician's Guide*, p. 37.
[3] Benjamin Peirce, *A History of Harvard University*, p. 237, Cambridge, 1833.
[4] Joseph McKean, Boston Public Library.
[5] "Libraries of Colonial Virginia in *William and Mary Quarterly*. Vol. III, p. 133, Vol. IX, p. 167, Vol. X, p. 232.
[6] Notebook in Samuel Johnson Collection, Columbia University Library.
[7] Harvard College Papers, Vol. II, 1764–1785. Harvard University Library.

nection with the plan of studies prepared for the trustees of the College of Philadelphia, Provost Smith recommended certain books to be read "for improving youth in various branches." He suggested "Maclaurin's Algebra" (London, 1748) and "Ward's Mathematics" for the mathematics in the first year.[8]

At Yale in 1778, President Stiles on assuming the executive office gave as the mathematical part of the program of studies then in force:

> Freshman Class—Ward's Arithmetic
> Sophomore Class—Hammond's Algebra
> Ward's Mathematics
> Junior Class—Ward's Trigonometry[9]

Since the *Young Mathematician's Guide* was considered worthy of a place in American schools for so long a time, it may be worth a brief presentation.[10] This work on "Mathematicks" is in five parts, "I Arithmetick, II Algebra, III The Elements of Geometry, IV Conick Sections, V The Arithmetick of Infinites," and to these sections is added an "Appendix of Practical Gauging."

The book was put out with a recommendation that might well have started it upon a successful career. This recommendation reads:

> Upon Careful Perusal of this Book, we think it a good Introduction to the *Abstracted* Parts of *Mathematicks*, and as such we recommend it to the Studious and Industrious Reader.
>
> J. Raphson, A. M. & R. S. S.
> H. Ditton,[11] Master of the New Mathematical School in Christ's Hospital.

Samuel Cunn, who revised and corrected Raphson's translation of the *Arithmetica Universalis* by Sir Isaac Newton, expressed his appreciation in a poem, "To the Ingenious Mr. John Ward Upon His Most Useful Piece, the Young Mathematician's Guide," which is printed at the back of the book.

Part II is entitled:

> Algebra, or Arithmetick in Species; wherein the Method of Raising and Resolving Equations is rendered easie; and Illustrated with a Variety of Examples, and Numerical Questions. Also the whole Business of Interest and Annuities, &c. perform'd by the Pen, and a small Table, with several new Improvements.

The algebra covers pages 143 to 277, or 134 pages, and so ranks as to space occupied about on a footing with the arithmetic of the book. The table of contents gives the topics treated as follows:

> The Method of Noting down Quantities, and Tracing of the Steps used in bringing them to an Equation; The Six Principal Rules of Algebraick Arith-

---

[8] Horace W. Smith, *Life and Correspondence of the Rev. William Smith*, p. 124 f., Philadelphia, 1879.

[9] L. F. Snow, *College Curriculum*, p. 79. Quoted from Stile's Diary, Nov. 9, 1779.

[10] Account taken from the "Third Edition, Corrected," 1719.

[11] Humphrey Ditton was highly regarded by Sir Isaac Newton, and exercised an influence on the Newtonian Philosophy. See Smith, *History*, I, p. 456, Boston, 1923.

metick in whole Quantities; Of Algebraick Fractions, or Broken Quantities; Of Surds, or Irrational Quantities; Concerning the Nature of Æquations, and how to prepare them for a Solution, &c.; Of Proportional Quantities, both Arithmetical and Geometrical Continued; Also of Musical Proportion; Of Proportional Quantities Disjunct, both Simple, Duplicate, and Triplicate; And how to turn Æquations into Analogies, &c.; Of Substitution; And Resolving Quadratick Æquations; Of Analysis, or the Method of Resolving Problems, Exemplified by Forty Numerical Questions; The Solution of all Kind of Adfected Æquations in Numbers; Of Simple Interest, and Annuities in all their various Cases; Of Compound Interest, and Annuities, both for Years and Lives; And of Purchasing Free-hold Estates.

There is nothing of an original nature to which attention need be called in the subject matter of this algebra. Rules and solutions of problems follow one another in a presentation that strikes the reader of to-day as tedious. No exercises are added in any topic for the sake of the student. The application of algebra to the solution of geometric problems is found in the section on geometry.

Textbooks entitled Algebra.—The first textbook designated algebra that was used in any American college seems to have been *The Elements of Algebra*, by Nathaniel Hammond.[12] In the list of studies given by President Stiles, of Yale, in 1778,[13] it will be noted that Hammond's algebra occurs in the sophomore year.[14]

Harvard in 1778 received from the sequestered libraries, in addition to Ward's Mathematics already referred to, Hammond's algebra. These books were put at the disposal of the professor of mathematics, but no record remains to show that Hammond was placed in the pupils' hands.[7]

In the laws of Rhode Island College [Brown University] for 1783, Hammond's algebra was required in the third year, and in the laws for 1793 the same requirement appears in the second year. Memoranda collected by a descendant from the papers of Solomon Drowne, of the class of 1773, give the only knowledge of the curriculum before 1783.[15] These papers show that Drowne began Hill's arithmetic in October, 1771, and Hammond's algebra in December of the same year, with Euclid's Elements and trigonometry in February, 1772. From this information it appears that the first printed laws of Brown set down requirements in algebra which had already been in operation for over 10 years.

---

[12] Nathaniel Hammond: *The Elements of Algebra in a New and Easy Method, with their Use and Application in the Solution of a great Variety of Arithmetical and Geometrical Questions; By general and universal Rules.* To which is prefixed an Introduction, containing a Succinct History of this Science. By Mr. Nathaniel Hammond, of the Bank. London, 1742.

[13] See p. 53.

[14] In "A partial list of textbooks used in Yale College in the eighteenth century" (Yale University Library) occurs "Nathaniel Hammond. Elements of Algebra (used by Sophomores, 1774–1781)."

[15] Walter C. Bronson, *History of Brown University, 1764–1914*, p. 102, Providence, 1914.

We open a real algebra when we turn to an examination of Hammond, even though the scope of it is limited as measured by present-day standards. In the preface the author states the prerequisite for the student of his book and his confidence in its pedagogical principles. A good sketch of the history of algebra follows this preface. After the first 75 pages, which are given over to the presentation of the four fundamental operations, involution, evolution, and surd quantities, there is not a page (except for a very occasional digression to introduce a needed process) in the whole extent of 328 pages which does not contain a problem, or some steps in the working out of a problem. In this way Hammond takes up simple equations, equations with two or three unknowns, quadratic and adfected equations, and gives explanations for their solution in the greatest detail. Some 109 problems are solved, marking the progress from the solution of the simplest equation to the solution of cubic and biquadratic equations by the method of converging series. There is the almost irreducible minimum of rules and the maximum of illustrative material which must have made an appeal to pupil and teacher alike. It is curious that a man who could write a work as good as this one should have been so prejudiced as to make no mention of exponents until page 304, thereby perpetuating in the many schools in which it was used the long form of writing powers of a base.

If this whole book was covered in the colleges which required it, the pupils had as good a course in the solution of equations as they would get to-day in a secondary school, plus some of the course in college algebra.

Another good algebra that was, without doubt, in the hands of teachers as soon as it appeared is entitled *A Treatise of Algebra.*[16] Its author was Thomas Simpson,[17] a man of undoubted genius. The first mention of this work as a textbook occurs in a pamphlet published in 1794 and containing the charter and the laws of the College of New Jersey.[18]

In addition to the usual list of topics, this work covers the following: Resolution of equations of several dimensions, Sir Isaac Newton's method of divisors, Cardan's cubic and higher equations, Descartes's biquadratic equations, converging series, indeterminate problems, investigation of sums of powers, figurate numbers, interest and annuities, plane trigonometry, and application of algebra to the solution of geometrical problems.

---

[16] Thomas Simpson, *A Treatise of Algebra: wherein the Principles are demonstrated, and applied in many useful and interesting inquiries, and in the resolution of a great variety of problems of different kinds* . . . London, 1745.
[17] Smith, *History*, I, 457.
[18] John Maclean, *History of the College of New Jersey*, p. 307.

In the editions of this work after the first one the treatment is a peculiar one. Rules with illustrations are given, and in a footnote arrangement "demonstrations." These demonstrations are complete explanations of the rules.

An American reprint of Simpson's algebra from the eighth London edition came from a Philadelphia press in 1809 and a second one in 1821. The change from the use of imported books to the use of books printed at home was only a question of time. It involved the establishment of printing presses and the demand on the part of a sufficient number of influential persons that these presses be put to work on textbooks.

Chapter X

THE FIRST BOOKS CONTAINING ALGEBRA PUBLISHED IN THE NEW WORLD

Mexican algebra.—In very widely separated sections appeared the first works printed on the American Continent and containing algebra. As early as 1556 there was published in Mexico a book entitled *The Sumario Compendioso.*[1] The most interesting feature of this work consists of six pages devoted to algebra.

Under the title "Arte Mayor," the author gives a number of examples generally involving quadratic equations, of which the following are types:

1—Find a square from which if 15¾ is subtracted the result is its own root.

Rule: Let the number be cosa (x). The square of half a cosa is equal to ¼ of a zenso (x^2). Adding 15 and ¾ to ¼ makes 16, of which the root is 4, and this plus ½ is the root of the required number.

Proof: Square the square root of 16, plus half a cosa, which is four and a half, giving 20 and ¼, which is the square number required. From 20¼ subtract 15 and ¾ and you have 4 and ½, which is the root of the number itself.

2—A man takes passage in a ship and asks the master what he has to pay. The master says that it will not be any more than for the others. The passenger on again asking how much it would be, the master replies: "It will be the number of pesos which, multiplied by itself and added to the number, will give 1260." Required to know how much the master asked.

Rule: Let the cost be a cosa of pesos. Then half of a cosa squared makes ¼ of a zenso, and this added to 1260 makes 1260 and a quarter, the root of which less ½ of a cosa is the number required. Reduce 1260 and ¼ to fourths; this is equal to 5041 divided by 4; the root of which is 71 halves; subtract from it half of a cosa and there remains 70 halves, which is equal to 35 pesos, and this is what was asked for the passage.

Proof: Multiply 35 by itself and you have 1225; adding to it 35, you have 1260, the required number.

Far removed in time and language as well as place was the next book to be printed on the Western Hemisphere, which bears in part of its contents on the subject of algebra. This was a Dutch textbook of 1730, published in New York City, and it was preceded in the American Colonies by only two published works on mathematics, both of which were arithmetics.

[1] For a complete account and facsimile, see David Eugene Smith, *The Sumario Compendioso of Brother Juan Diez*, Boston, 1921.

A Dutch algebra.—It is natural that arithmetic should have been the first mathematical subject to appear in print in the American Colonies. It is, on the contrary, surprising that algebra should occupy over one-third of the space in the third book on arithmetic published in this country when nearly 60 years were to elapse before the appearance of another book containing any algebra. Some 4,500 titles[2] of publications in Pennsylvania before 1785 show many almanacs, but few works on mathematics, and none containing algebra. A complete bibliography[3] of all American books up to 1792 reveals, among those on mathematics, only three which include algebra in their contents. One of the three is this Dutch textbook, and while the extent of its influence was probably very limited, it has interest as the earliest and, for a long period, the only work on algebra printed here.

The title of the book is: *Arithmetica of Cyffer-Konst, . . . Als Mede Een kort ontwerp van de Algebra.*[4] The names which appear on the title-page are the names of three men who, as we shall see, were kindred spirits in their independence of authority.

The name of John Peter Zenger is inseparable from the history of the freedom of the press. He was the second printer of New York, and his newspaper was the instrument by means of which active protest was made against the tyranny of the royal governors, which eventuated in the American Revolution. The trial of Zenger is significant in all history, and the outcome of it was that liberty of the press which gave people in this country the right to freely criticize the conduct of public officials. Zenger's press was established in 1726, and his newspaper had its beginning in 1733. The Venema book appeared between these two dates.

Jacob Goelet appears in a minor way as breaking away from the authority of the church. He is referred[5] to as expressing himself in opposition to a stand taken by the ecclesiastical body in several instances. One of these was connected with the licensing of private-school teachers.[6] At any rate, he succeeded in having printed this textbook in Dutch by a man who, as will appear later, was under the ban of the local church leaders.

---

[2] Charles R. Hildeburn. *A Century of Printing. The Issues of the Press in Pennsylvania, 1685–1784*, Philadelphia, 1885.

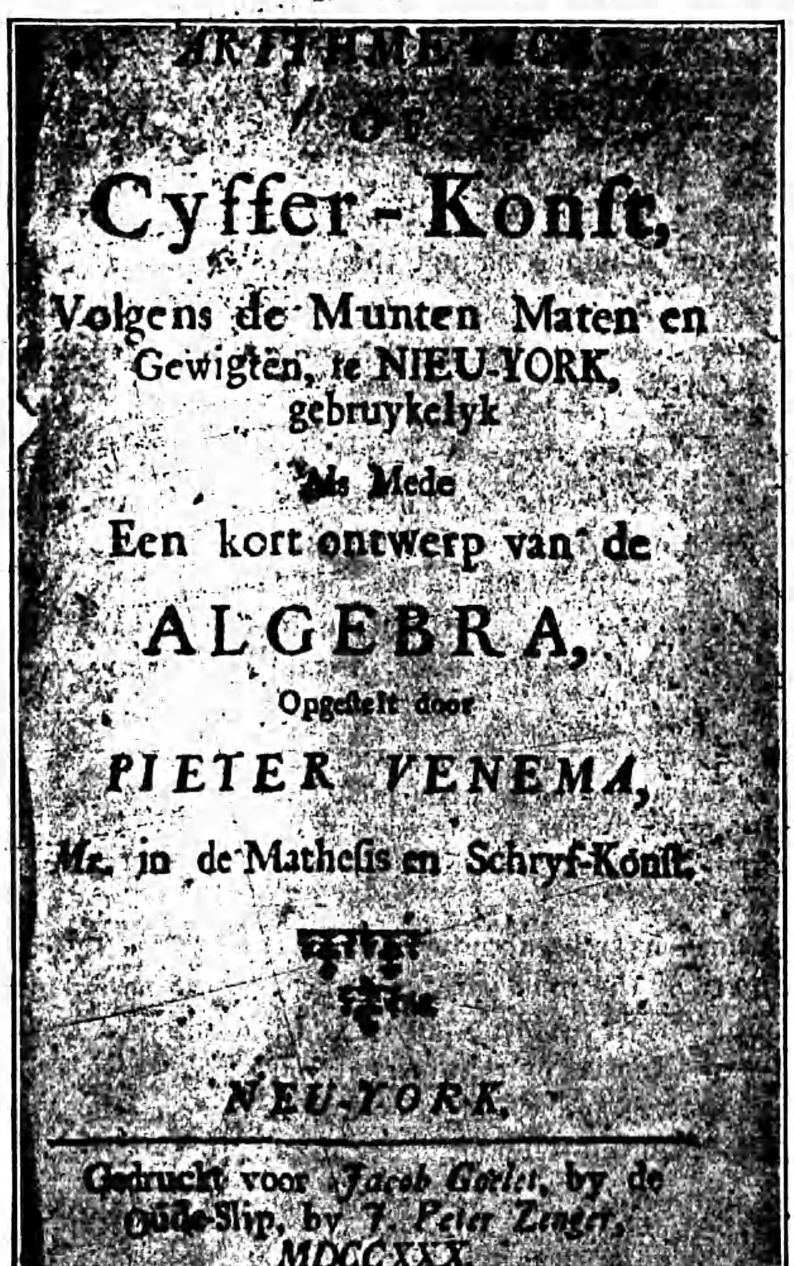
[3] Charles Evans. *American Bibliography, 1639–1820*, Chicago, 1903–1914.

[4] Only two copies have been located. They are both in the library of the New York Historical Society. The copy in the New York State Library, listed in Evans, loc. cit., was destroyed in the fire of March 29, 1911.

[5] *Ecclesiastical Records of the State of New York*, Vol. IV, p. 2333, Albany, 1901–1916.

[6] H. W. Dunshee. *History of the School of the Collegiate Reformed Dutch Church from 1633–1883*, p. 38, New York, 1883.

Pieter Venema was repeating history by making trouble in the church. In a letter[7] from the Rev. Gaulterus DuBois to the Rev. Classis, of Amsterdam, May 14, 1741, we find this complaint:



Cyffer-Konst,

Volgens de Munten Maten en Gewigten, te NIEU-YORK, gebruykelyk

Als Mede

Een kort ontwerp van de

ALGEBRA,

Opgestelt door

PIETER VENEMA,

Mr. in de Mathesis en Schryf-Konst.

NIEU-YORK,

Gedrukt voor Jacob Goelet, by de Oude-Slip, by J. Peter Zenger, MDCCXXX.

"Arithmetic or the art of ciphering, according to the coins, measures and weights of New York, together with a short treatise on algebra drawn up by Pieter Venema, master in mathematics and the art of writing. New York, printed for Jacob Goelet, near the Old Slip, by J. Peter Zenger, 1730." The title page of the first book containing algebra that was printed in the American Colonies.

Inasmuch as the Rev. Consistory of New York several years ago exhorted their ministers to be on their guard, and oppose the artful misleadings of one

[7] *Ecclesiastical Records*, loc. cit., p. 2756.

Pieter Venema, a crafty free-thinker of Groeningen, who had previously been a Reader and School-master just outside that city. I, therefore, determinedly set myself against him. Under God's blessing my efforts accomplished much good, although some still adhere to him. Among these is one Jacob Goelet, who with his conventicles, endeavors to do all possible harm to our church.

We therefore see that Venema came from a city in Holland which offered university privileges, and in which he had been a school-teacher.

More definite evidence that Venema was a schoolmaster is given in the dedication to a work of his published in Holland in 1714.[8] In this dedication, which is signed Pieter Venema, appears the statement:

Is de Gunst van U Ed. Mog. geweest dat ik eenige Jaren herwaarts in U Ed. Mog. niet min vermaarde Stad Groningen, mijn bedieninge als School-Meester hebbe waargenomen. (Through your favor I had the chance of serving as a schoolmaster at Groningen for several years.)

Farther on he states that he had the honor of enjoying for several years the teachings of "Heer J. Bernoulli."[9]

The high regard in which he was held by the mathematicians of his own country and time is shown by an inscription to him in the work[10] already referred to. This inscription contains such phrases as:

. . . the talent which God has granted you.
. . . we need no teacher, the book is a guide in itself. We thank you, Venema, we thank you, brave teacher, give us more of your knowledge! You have won so much distinction at Groningen that it is impossible that you should be forgotten.

Venema must have been known during the eighteenth century and the early part of the nineteenth century, because he is cited repeatedly in collections of problems solved and published by Dutch mathematicians and societies of that period.[11]

Venema's reasons for writing the book under consideration appear in the "Konst Lievende Leser." He says:

Because I realized that there was here no ciphering book in the Low Dutch concerning trade or merchandise, and for the sake of the teaching of inquiring youth and of all lovers of the teaching of arithmetic, I have undertaken to make a clear and succinct ciphering book upon that excellent science which flourishes in this city and country. To this are added the elements of algebra.

[8] Pieter Venema, "*Een kort en klare Onderwysinge in de Beginselen van de Algebra ofte Stel-konst.*" Te Groningen. 1714. Other editions Amsterdam 1730, 1756, 1768, 1783, 1794, 1803.

[9] This was Jean (I), who was professor at Gröningen, 1695–1705.

[10] P. Venema, 1714, loc. cit. No bibliography consulted revealed the existence of this edition, but a copy was found in the collection of Mr. George A. Plimpton, to whom the writer is indebted for the privilege of examining the book. Mr. Plimpton also has a copy of the 1756 edition.

[11] *Wiskunstige verlustiging*, etc., Vol. I, pp. 29, 63, 70, 79, 81; Ontbindingen, etc., p. 213 ff. Vol. II, p. 8 (Amsterdam 1793, 1795), and other references.

whereby that which is not understood in arithmetic can be demonstrated by the clear words of algebra, so that algebra is the key to the obscure propositions of arithmetic. Should any one desire to go farther than the subjects included in this book, he can make use of my simple Algebra or Stel-konst published in the year 1714, in my native city, Groeningen.[12]

The book consists of 120 pages, of which 75 are devoted to arithmetic and 45 to algebra.

The section devoted to arithmetic starts with addition tables, leading to the multiples of numbers up to 9 by 9. This is followed by addition, subtraction, multiplication, division, tables of weights and measures, the operations with money, rule of three, reduction of fractions to lowest terms, to common denominators, operations with fractions, inverse rule of three, rule of five, compound rule, conjunct rule, rule of partnership, partnership with time, and alligation.

The second section bears the heading, "Algebra ofte Stel-konst," that is, "Algebra or the art of place." The reason for the use of the Stel-konst is stated thus:

This science is called by the word Stel-konst because that means, for the unknown, place x, y, z, the last three or more letters of the alphabet, and for the known, the letters a, b, c, d, and so forth.

The contents of the algebra text are as follows: Signs of operation, general notions, axioms, addition, subtraction, multiplication, including product of $a+b$ by $a+b$, $a-b$ by $a-b$, and $a+b$ by $a-b$, division, reduction of fractions to lowest terms and to a common denominator, addition, subtraction, multiplication and division of fractions, solution of simple equations and simultaneous equations in two unknowns, and problems.

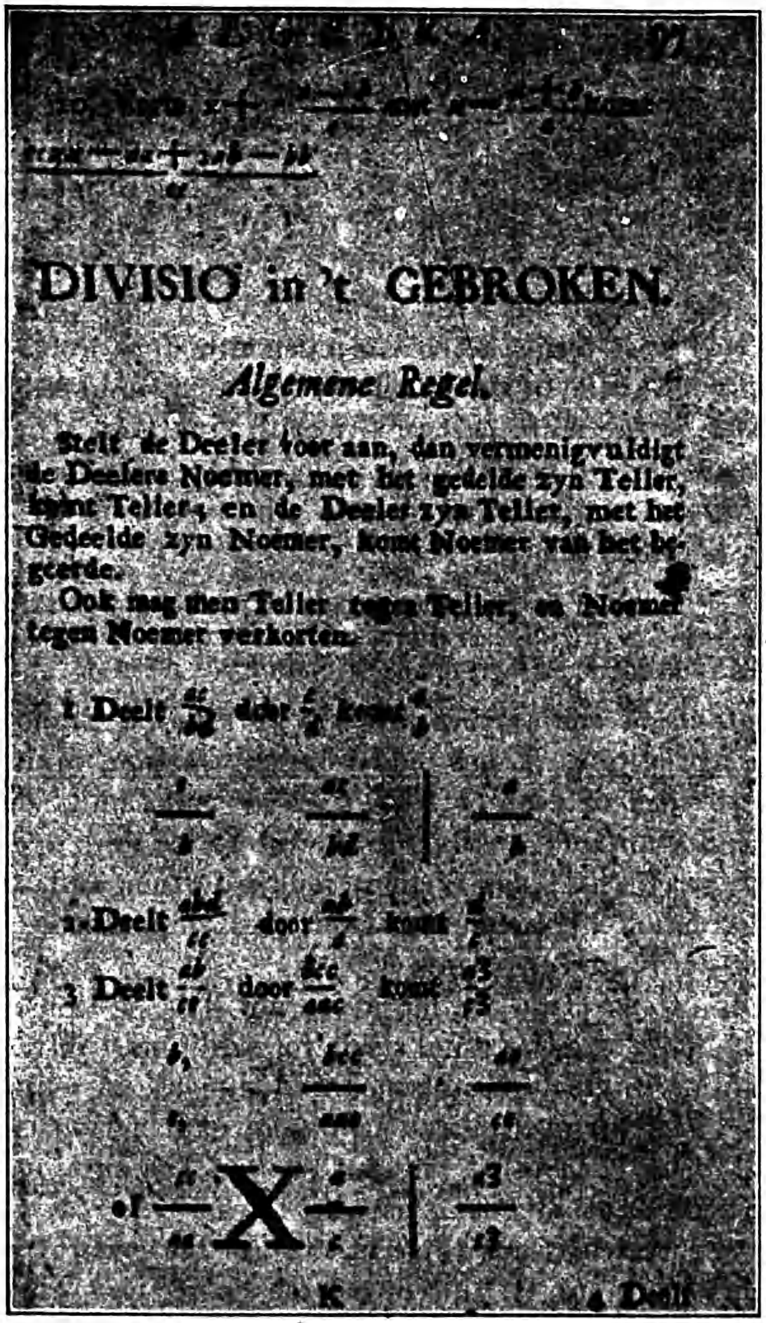
The signs of operation are not given until the section on algebra is reached. They are the usual signs for addition, subtraction, and equality and a • which "betekent tot" (denotes an empty space). No sign is given for multiplication, but an explanation is made for letters following each other without signs. The sign for equality is printed with unusually long parallel lines.

The procedure with each topic is to state the general rule, work out an illustration of it, and prove the correctness of the result by numerical substitutions. Sets of examples accompany all rules. Among the features of the book are some which would not now be found.

One of these is the form for the division of a fraction. The divisor, $\frac{cc}{aa}$, precedes the dividend $\frac{a}{c}$, and the two are separated by an exaggerated $\times$. Another feature is the repetition of a letter for the second power, and a large figure to the right of the base

[12] An examination of the 1714 algebra shows that the work of 1730 is an abbreviated treatment of the earlier work. Processes, examples, and problems of the latter are identical with parts of the former.

is the representation of higher powers; thus a^3 is written $a\ 3$.[13] The lowest common multiple of several expressions is found by a



DIVISIO in 't GEBROKEN.

Algemene Regel.

Stelt de Deeler voor aan, dan vermenigvuldigt de Deelers Noemer, met het gedeelde zyn Teller, komt Teller; en de Deeler zyn Teller, met het Gedeelde zyn Noemer, komt Noemer van het begeerde.

Ook mag men Teller tegen Teller, en Noemer tegen Noemer verkorten.

Division of fractions from Venema. The word for fraction is "broken"; for numerator, "numberer"; and for denominator, "namer."

method which is associated to-day with finding the lowest common multiple of small numbers.

[13] This form of the exponent may have been due to the convenience of the printer, since Venema uses the present-day form in his earlier algebra. [P. Venema, 1714, loc. cit., p. 74], except for the second power. However, such a form of the exponent may have been known to him. It is found in Pierre Hérigone, *Cursus Mathematicus, nova brevi et clara*, etc. Vol. II, section on algebra, p. 4, and consistently throughout the entire work, Paris, 1644.

The book ends with 24 problems, and the familiar age problem is among them. It takes the form of a curious son who asked his father his age:

The father answered, your age with the second part, the third part, and the fourth part of itself increased by 24½ years [is equal to mine]. I am as much over 40 years as you are under 40. How old was the son? Ans. 18 years.

Two unknown quantities are used in the solution. Other problems lead to indeterminate equations. One reads:

Three women bought apples, the first 100, the second 110, and the third 120. They sold, each a different number, the first day, at the same price, and the remainder the second day, also at a uniform price. In counting their money, they found that they had equal amounts. How many apples were sold on each day?

The unknowns x, y, z, respectively, are assumed for the number of apples sold the first day, v and w, respectively, for the price on the two days. By the conditions of the problem $y=x+\frac{10\,w}{w-v}$ and $z=x+\frac{20\,w}{w-v}$ are obtained. With the usual ingenuity in such problems, w, v, and x are taken so as to give one set of values for x, y, and z.

As simple as all this work seems, there is good stuff in it. The question arises as to the schools in which it was used, for there must have been a definite reason for printing it with the arithmetic. It is to be noted in the preface by the author that he looked upon algebra as necessary to the clearing up of doubtful points in arithmetic. But he, as a practical schoolmaster, must have known that the book was needed for instruction. Did Venema himself have a private school, and did he succeed, so early in the history of printing, in this country, in putting into print the material that he needed? It seems to be a safe conjecture to put him at the head of such a school, and the algebra of at least one secondary school of that period was not unworthy.

Chapter XI

EIGHTEENTH CENTURY BOOKS ON ALGEBRA BY AMERICAN AUTHORS

Algebra in the work of Nicolas Pike.—The first book written by an American and containing a section on algebra was the second book covering that subject to be published in what is now the United States. It appeared in 1788 at Newburyport, Mass., a town which seems off the beaten track of education, as is true of some other towns in which mathematics books were published. This book was entitled: *A New and Complete System of Arithmetic*,[1] and its author was Nicolas Pike, A. M., a graduate of Harvard College in 1766.

This work was probably the outgrowth of the author's practical experience. In 1773 the selectmen of Newburyport[2] chose Nicolas Pike (1743–1819) to take charge of the public school in that town. Later Pike opened an evening school and also a private school for young ladies. In 1786 it is known that he was a teacher of the grammar school, and in that same year he advertised the publication of his book.

The adoption of Pike's arithmetic as a collegiate textbook in Harvard, Yale, and Dartmouth seems to have been immediate. It had been brought to the attention of men prominent in public life before its publication, because Pike had submitted the manuscript to them. Letters of commendation from the professors of mathematics and philosophy at Harvard and Dartmouth, from the presidents of Harvard, Yale, and Dartmouth, and from Benjamin West were printed in the book.

The value that Pike's arithmetic holds for this present work is due to the inclusion in it of a section on algebra. In the preface to the first edition Pike credits this material to the proper source by saying:

---

[1] *A New and Complete System of Arithmetic, composed for the use of Citizens of the United States*: By Nicolas Pike, A. M., . . . Newbury-Port. 1788. A second enlarged edition revised and corrected by Ebenezer Adams, A. M., preceptor of Leicester Academy, came out at Worcester in 1797; still a third edition, this time revised, corrected, and improved by Nathaniel Lord, A. M., was published in Boston in 1808.

[2] Account taken from John J. Currier. *History of Newburyport, Mass., 1764–1905*, (1906).

The short introduction to algebra, which is subjoined, was abstracted principally from Bonnycastle, and that of Conic Sections, from Emerson's works.

The section on algebra is designated "An Introduction to Algebra. Designed for the use of academies," and covers only 39 out of 512 pages in the whole work. This material would be negligible were not its presence significant of some demand which led its author to include it. The usual start with definitions is made. The six operations follow, with all examples under them completely worked out. "Sir Isaac Newton's Rule for raising a binomial or residual quantity to any power whatever" is stated. Infinite series, arithmetical and geometrical proportion, simple and quadratic equations, all receive brief treatment. Only 18 problems are given, 12 under simple and 6 under quadratic equations. The section concludes with a "Recapitulation of the principles of Arithmetic & Algebra" under 9 so-called axioms.

Pike's arithmetic was the first work written by an American to have any extended use in the United States. It must be regarded also as the first printed work on algebra, written by an American, that was placed in the hands of students in colleges and academies. Another quarter of a century was to elapse after the first appearance of Pike's book before a book on algebra[3] alone and bearing that title was to be compiled by an American professor and published for the use of students in his classes and elsewhere.

The American Youth.—Another work containing algebra and published in the eighteenth century deserved more popularity than extant evidence shows it to have attained. Its authors followed the custom, quite common in these early years, of using a general title. The book appeared as *The American Youth.*[4] The authors, Consider and John Sterry, were apparently outside of university circles and engaged entirely in work with private pupils. They must have felt justified in going to the expense of publication, but it took courage on their part to put a book like this on the market in 1790.

Volume I is divided into books, much as geometry volumes are divided. Book II of this volume extends from page 241 to page 387, the end of the volume. All of the subject matter in an elementary algebra of the present day is covered, with the omission of involved exercises in factoring and fractions. The more advanced topics are quoted:

Infinite Series, Binomial Theorem, Proportion or Analogy Algebraically considered, Arithmetical, Geometrical, Harmonical Proportion, Genesis or Formation of Equations in General, Concerning the Transformation of Equations

[3] Jeremiah Day, *The Elements of Algebra*, loc. cit.

[4] *The American Youth: being a new and complete course of introductory mathematics: designed for the use of private students.* By Consider and John Sterry, v. 1 . . . Providence. Printed by B. Wheeler, for the authors, 1790.

and Exterminating their Intermediate Terms, Resolution of Equations by Divisors, Finding the Roots of Numerical Equations in General, by the Method of Approximations, Concerning unlimited Problems and Diophantine Problems.

It is an ambitious course in algebra set forth in this text at a time when students in some colleges were still dependent on taking mathematical notes from lectures and setting them down in notebooks. Perhaps its influence was more widespread than historical testimony shows. At any rate, copies of the book are to be found rather generally in the libraries of New England.

The findings of these two chapters lead to the conclusion that only three books containing algebra appeared in print in the American Colonies and the young American Republic during the eighteenth century. In each one of these books, it is treated in a section along with sections on other mathematical subjects.

Chapter XII

ALGEBRA AND ADVERTISEMENTS[1]

Algebra in the public press.—Perhaps the most unlikely source of information bearing on the teaching of algebra in the American Colonies during the eighteenth century would seem to be the files of early newspapers. And yet a number of advertisements relating to different phases of the subject are to be found among those dealing with the dates of the sailing of vessels and of the arrival of the post from Philadelphia, Boston, or New York, with runaway servants or slaves for sale, with the importation of good Cheshire cheese, or with lotteries for wharf, church, and college.

Private tutors and schoolmasters.—One form of advertisement which indicates educational activity is that in which a private tutor or the master of a school offers subjects to be taught. The earliest advertisement of this nature, and relating to mathematics, located is the following:

> *Boston News-Letter*, Mch. 21, 1709. Opposite to the Mitre Tavern in Fifth-street next to Scarlet's Wharff, Boston, are Taught, Writing, Arithmetick in all its parts; And also Geometry, Trigonometry, Plain and Sphaerical Surveying, Dialling, Gauging, Navigation, Astronomy; The Projection of the Sphaere, and the use of Mathematical Instruments: By Owen Harris. Who Teaches at as easie Rates, and as speedy as may be.

Isaac Greenwood, before he became the first professor of mathematics at Harvard College, used the newspaper as a means of obtaining pupils, as shown in these extracts:

> *Boston-News Letter*. Jan. 12, 1727. An Experimental Course of Mechanical Philosophy, wherein the Principles of that Noble Science, with the discoveries

[1] This study has been made from representative newspapers of Boston, New York, Philadelphia, and Virginia. The papers have been examined systematically from their beginnings to the dates indicated, such dates being in several instances the time at which publication of the paper ceased. The files examined consist of the originals or facsimiles in the New York Public Library, New York Historical Society, New York Society Historical Society of Pennsylvania, Library Company of Pennsylvania, and Virginia State Library. The following list of newspapers indicates the extent of the investigation:

Boston News-Letter, Apr. 24, 1704–Dec. 29, 1757.
New-York Gazette, Feb. 28, 1726–Oct. 15, 1744.
New-York Weekly Post-Boy, Jan. 19, 1747–Dec. 18, 1752.
New-York Weekly Journal, Oct. 5, 1733–Mar. 18, 1751.
New-York Evening Post, Dec. 17, 1744–Dec. 30, 1751.
(Philadelphia) American Weekly Mercury, Dec. 22, 1719–Jan. 1, 1746.
(Philadelphia) Pennsylvania Gazette, Oct. 1, 1728–Dec. 31, 1754.
Virginia Gazette, Jan. 1, 1767–Dec. 31, 1776.

of the incomparable Sir Isaac Newton therein are demonstrated by above Three Hundred Curious and Useful Experiments, accompanied with Experimental Lectures thereon in as easy Language as possible; . . . To be Performed by Isaac Greenwood, A. M. . . . (Repeated Feb. 1, 8, 15, 22.)

Boston News-Letter, July 6, 1727. The Experimental Course of Mechanical Philosophy which was intended to have been recapitulated this Summer . . . is deferred till the Fall . . . In the mean Time, if any Gentlemen are desirous of being acquainted with the Principles of Algebra; Sir Isaac Newton's incomparable Method of Fluxions, or the Differential Calculus, to-gether with any of the Universal Methods of Investigation used by the Moderns; the Elements of Euclid and Appollonius (sic); or, any Part of Speculation [changed to Speculative in next issue] or Practical Mathematicks, commonly taught in the Colleges or Schools in Europe; Attendance will be given by the Author of said Course at Mrs. Belknap's at the Upper End of Queen Street, Boston. Where, also, to such as are instructed in the Mathematical Sciences, the Principles of Sir Isaac Newton, and the Modern Discoveries in Astronomy and Philosophy will be explained and demonstrated in a concise and easy manner.

Ibid., July 13, 1727. To be taught by Mr. Greenwood . . . The Principles of Algebra . . . (Advertisement practically a repetition of the above. Repeated July 20.)

These are the first advertisements located on algebra which, as we have seen, Greenwood also included in the course at Harvard. About 15 months after his installation as professor, he advertised the publication of a work on arithmetic, as follows:

[Boston] *Weekly News-Letter*, May 29, 1729. Just Published Arithmetick Vulgar & Decimal; with the Application thereof to Variety of Cases in Trade & Commerce. By Isaac Greenwood, A. M. Hollisian Professor of the Mathematicks, and Philosophy. To be Sold by Thomas Hancock at the Bible & Three Crowns near the Town Dock, Boston. (Repeated June 5, 12.)[2]

After Greenwood's dismissal from Harvard, he again turned to private teaching and used his former means of informing the public:

Boston Weekly News-Letter, Nov. 9, 1738. Such as are desirous of learning any Part of Practical or Theoretical Mathematicks may be taught by Isaac Greenwood, A. M. . . . (Repeated Nov. 16, 24).

Ibid., Mch. 30, 1739. Such as are desirous of learning any Parts of the Mathematics whether Theoretical as the demonstrating Euclid, Appolonius (sic), &c., or Practical, as Arithmetic, Geometry, Trigonometry, Navigation, Surveying, Gauging, Algebra, Fluxions, &c. Likewise any of the Branches of Natural Philosophy, as Mechanics, Opticks, Astronomy, &c. may be taught by Isaac Greenwood, A. M. . . . (Repeated Apr. 5, 12, 19).

For 13 years from February 23, 1723, Nathan Prince was tutor of mathematics at Harvard. He also was dismissed from the college but carried on his teaching activities privately as shown in:

Boston Weekly News-Letter, Mch. 3, 1743. These may inform the Publick, that Nathan Prince Fellow of Harvard College proposes, on suitable Encourage-

[2] The name of Isaac Greenwood was not printed on the title page of this book. It has been placed there by hand in most copies to be found in libraries at the present time. This advertisement is conclusive evidence of the authorship of the book.

ment, to open a School in this Town for the instructing young Gentlemen in the most useful Parts of the Mathematicks . . . Particularly in the Elements of Geometry and Algebra; in Trigonometry and Navigation; in Geography and Astronomy . . . in the Arts of Surveying, Gauging and Dialing; and in the General Rules of Fortification and Gunnery . . . (Repeated Mch. 10, 17.)

These advertisements from former teachers of mathematics at Harvard increase the weight of evidence in favor of the presence of algebra in the course of study at that college from the early part of the eighteenth century; and also show attempts at including it in private school work.

New York City papers carry advertisements on algebra almost as early as the Boston paper cited. The first one runs:

New-York Gazette, Sept. 7, 1730. On the 15th of September next, at the Custom-House in this City . . . James Lyne designs to Teach in the Evenings (during the Winter) Arithmetick, in all its parts, Geometry, Trigonometry, Navigation, Surveying, Guaging, Algebra, and sundry other parts of Mathematical Knowledge . . .

In 1732, Alexander Malcolm was made the master of a public school to teach Latin, Greek, and mathematics in the City of New York, by an act of the general assembly of the Province, a school which flourished for only about seven years.[3] In 1734 a whole newspaper column was given to an announcement of Mr. Malcolm's school, of which the following is one section:

New-York Gazette, Jan. 7, 1734 . . . At the said School are Taught all the Branches of the Mathematicks, Geometry, Algebra, Geography, and Merchant's Bookkeeping after the most perfect manner. (Repeated Jan. 14, 21, 28.)

Malcolm wrote and published a book on arithmetic in the preface to which he says:

I have not supposed the Student of Arithmetick already acquainted with Algebra; but have gradually explain'd the Principles and Rules of it, as far as my purpose requir'd. As Algebra is nothing else but an universal method of representing Numbers, and reasoning about them, so it very naturally belongs to Arithmetick.[4]

Algebra was obviously a regular subject in the course of study in his school.

Other extracts from New York City papers are as follows:

New-York Weekly Post-Boy, Aug. 18, 1746. Arithmetic vulgar, decimal and algebraic . . . carefully and exactly taught by Joseph Blancherd . . .

New-York Gazette. Revived in the *Weekly Post-Boy*, June 17, 1751 . . . Robert Leeth, School-Master, from London, . . . in Wall-Street . . . teaches Latin & Greek As at the Academies in England as well as Reading, Writing & Arithmetick, Vulgar and Decimal, . . . and Algebra, also Logarithmical and Instrumental Arithmetick, Geometry and Trigonometry . . . (Repeated five times at intervals.)

[3] Daniel J. Pratt. *Annals of Public Education in the State of New York. From 1626 to 1746.* p. 124 ff. Albany, 1872.

[4] Alexander Malcolm. *A New System of Arithmetick*, p. XIII, London, 1730.

New-York Evening Post. Aug. 25, 1746. Arithmetic, Vulgar, Decimal and Algebrace (changed to Algebra in issue of June 15, 1747) . . . carefully and exactly taught by Joseph Blancherd . . . (This advertisement ran weekly through Nov. 9, 1747.)

Ibid., May 8, 1749. Writing, Arithmetick, Vulgar, Decimal, Duo-decimal, the Rules of Practice . . . the Elements of Euclid and Algebra, with their applications to Practical Geometry, Gauging, Surveying, Conick Sections . . . are Taught carefully and expeditiously . . . By John Wilson. (Ran through Sept. 18, 1749.)

Ibid., Oct. 7, 1751. A Young Man lately arrived from England proposes to teach Writing, Arithmetick, Merchants Accompts and the most useful Branches of the Mathematicks, viz., Algebra, Geometry, Trigonometry, Navigation, Astronomy, Surveying of Land, the Use of Mathematical Instruments, &c. in a publick manner . . . (Ran through Dec. 30, 1751.)

Philadelphia was another educational center during the eighteenth century. The *Pennsylvania Gazette* in a prospectus of October 1, 1728, covering a page, announced its policy in words which include the following:

. . . containing among many thousand other Things, such as the following: Algebra, or the Doctrine of Equations, Simple, Quadratick, Cubic, &c., Analyticks, or the Resolution of Problems by Species, or Symbolical Expressions, Geometry, or the Doctrine of extended or continuous Quantity.

The first advertisement on algebra located in this paper is one by Theophilus Grew, who kept himself before the public in this way for many years. It reads:

Pennsylvania Gazette. Aug. 15, 1734. In Second Street over against the Sign of the Bible is taught the Arts Mathematical, viz. Arithmetick in all its Parts, Geometry, (etc.) according to the most approv'd methods by Theophilus Grew. He also teaches Algebra, or the Analytical Art, with the Laws and Properties of Motion, a thing absolutely necessary to a right understanding of the Modern Philosophy, . . .

Variations on the above are found in issues of December 26, 1734; May 9–June 19, 1735; October 26–November 9, 1738; August 9, 1739. In some of them Grew appears also as an importer of silks and other goods. On May 14, 1741, his name is associated with that of James Houston, at the Free School of Kent County, in Chester Town, on Chester River. On September 2, 1742, he opened a school in Philadelphia and continued to advertise on October 14–November 25, 1742; March 17, 1743; September 17, 1744.

Grew had made himself so well known that on July 27, 1750, he began a term of service at the Academy of Philadelphia [University of Pennsylvania]. This connection was not long maintained, and the last series of advertisements, from September 21–October 5, 1752, shows him as a partner with Horace Jones in an evening-school venture. Algebra is included here, as in most of the other places in which Grew's name is found. All through these years this man was running advertisements in *The American Weekly Mercury*, another Philadelphia newspaper, at the same time that he was running them in the *Pennsylvania Gazette*. He showed himself to be an indefatigable advertiser, and he no doubt maintained some sort of a school from 1734 to the end of his life.

Another series of advertisements was run by Alexander Buller, the first one of which follows:

Pennsylvania Gazette, Nov. 5, 1741. Writing, Arithmetic, Merchant's Accounts, Navigation, Algebra, and other parts of the mathematics are taught by Alexander Buller, at the Public School in Strawberry Alley . . . (Repeated Nov. 12, 19, 26.)

Buller had received permission in October, 1738, to teach mathematics, among other subjects, in the "Publick School" [William Penn Charter School].[5] He had evidently been a pupil of Thomas Simpson. A letter dated "Philadelphia, Oct. 27, 1741," reads:

Friend Simpson . . . Abt. 3 years and half ago I got an insight into some difficult parts of ye Mathematics from thee . . . thy old friend Alex. Buller.[6]

This teacher in Philadelphia carried into his profession the inspiration received from his study under "that strange mathematical genius," Thomas Simpson.

Other advertisements of the teaching of algebra to be cited are:

Pennsylvania Gazette, Aug. 13, 1747 . . . are taught these Mathematick Sciences, viz., Arithmetick, algebra, geometry, plain and the sperical (sic) trigonometry, conick sections, arithmetick of infinites . . . by John Clare. (Repeated Aug. 20 and 27.)

Ibid. Nov. 2, 1752 . . . are still taught, these Mathematical Sciences, viz., Arithmetick in all its parts, Algebra, Geometry . . . by John Clare. (Repeated Nov. 9 and 16.)

Ibid. Dec. 8, 1748 . . . arithmetick, vulgar and decimal . . . algebra, all carefully taught . . . by Thomas Craven. (Ad. ran through Apr. 13, 1749.)

Virginia Gazette, May 2, 1771. A Clergyman of the Church of England, a sober young Man . . . proposes to teach . . . Algebra, Geometry, Surveying, Mechanics . . . the Reverend W. S. . . . Potowmack, Virginia . . .

In addition to these advertisements in which algebra is definitely named, some score different instances of the phrase "other parts of the Mathematicks" have been found. With the long list of mathematical subjects usually preceding this phrase algebra was undoubtedly covered by it.[7]

Teachers of mathematics wanted.—The advertisements located in which the services of a teacher of mathematics are called for are only three in number. They are found in the *Pennsylvania Gazette*, October 9, 1740, and in the *Virginia Gazette*, October 15, 1767, and

[5] Minutes of the Publick School, 1712–1770, Vol. I, p. 26.

[6] From Simpsoniana in the possession of Prof. David Eugene Smith.

[7] A number of advertisements of algebra have been located in newspapers between 1783 and 1800. On account, however, of the desultory nature of the search made during these years, they will not be cited.

June 14, 1776. Algebra is not specified, but probably was included in the requirement.

Solutions of algebra problems.—Only rarely were the newspapers used as the medium for the solution of problems. The *New York Weekly Journal*, Zenger's paper, in the issue of July 26, 1742, printed two questions sent in by a correspondent because, as he says, "There is little News at present to Entertain your Readers with." The solutions of these questions were printed in the next number, August 2, 1742. The success of this venture emboldened some reader of the paper to send in on August 23, 1742, the following communication:

Mr. Zenger: As you are under some Obligation of pleasing every Body, in your Station and according to your Capacity, be pleased to humour me for once in the Way. I am, yours, &c. 1+1=2. Two officers have each of them a Company of Men, the one has 40 less than the other; they divide amongst their Men each Officer 1200 Crowns. *Quere.* How many Men each Company consisted of if the Officer who had least gave 6 Crowns more to each Man than he who had most?

For the Benefit of those who don't understand *English* perfectly, but perhaps understand *Dutch*. *Twee Capiteinen hebben ieder een Compagnie Soldaten, de eene 40 minder als de andere: sy deelen ieder aan syn Volk 1200 Croonen. Vraag. Hoe veel Soldaten ieder in syn Compagnie heeft, so de gene die het minste Volk heeft 6 Croonen meer aan ieder Man [illegible]?*

Not until three months later did the solution of this problem appear. It is quoted in part:

New-York Weekly Journal, Nov. 22, 1742. *Antwoord op Peter Zenger's Vraag van August 23.* Stelt de eerste Compagnie Soldaten = x. So is de tweede Compagnie = x+40 Stelt het Geld voor ieder Man van tweede Compagnie = y.

Dan het Geld voor ieder Man van de eerste Compagnie = y+5

Elk met syn Geld gemultipliceert, komt xy+5x=1200 en xy+40y=1200

(Solution of these simultaneous equations follows.)

Opgelost door PETRUS VENEMA.

This is the Venema who was the author of the Dutch textbook already discussed. He was still in New York City or receiving this paper in some other place, in 1742.

Sale of algebra books.—Another phase of newspaper activity was the printing of lists of imported books placed on sale. The following algebras are found in such lists: Hammond's *Elements of Algebra* (*New-York Mercury*, Nov. 24, 1754);[8] Kersey's *Elements of Algebra* (*Pennsylvania Gazette*, Apr. 12, 1729); Maclaurin's *Treatise on Algebra* (*Virginia Gazette*, Jan 3, 1771); Newton's *Algebra* (*Pennsylvania Gazette*, May 30, 1751); Saunderson's *Algebra* (*Pennsylvania Ledger*, Oct. 22, 1777); Simpson's *Algebra* (*Virginia Gazette*, Sept. 17, 1772); Sturmius's *Elements of the Mathe-*

[8] The data given cover the issue of the newspaper located in which the book was advertised for the first time.

maticks (*Pennsylvania Gazette*, Apr. 12, 1729); Ward's *Young Mathematician's Guide* (*Pennsylvania Gazette*, May 25, 1738); Wolfius's *Algebra* (*Pennsylvania Gazette*, Aug. 4, 1748).

We see, then, that the public press of the eighteenth century bears witness to activity in the teaching of algebra, and in the sale of algebra textbooks, activity which must be interpreted in the light of a demand for this branch of mathematics.

Chapter XIII

SUMMARY

The direct evidence in defense of the thesis that algebra was an important part of the American education of the eighteenth century may be summarized as follows:

Manuscript notebooks on algebra:
- Harvard College, 1730–31, 1739, 1770.
- College of New Jersey [Princeton University], 1770.
- University of Pennsylvania, 1788.
- Miscellaneous.

Commencement theses containing algebraic truths:
- Yale College, 1718–1797.
- Harvard College, 1721–1810.
- College of New Jersey, 1752.
- Rhode Island College [Brown University], 1769–1811.

Mathematics theses:
- Harvard College, 1786–1839.

Statements from college records and writings of college presidents and professors:
- Hugh Jones, professor of mathematics at the College of William and Mary, 1724.
- Rules governing Hollis professorship at Harvard College, 1726, 1787, 1788.
- Thomas Clap, president of Yale College, 1743, 1766.
- Ezra Stiles, president of Yale College, 1778.
- Minutes of the Trustees of the College, Academy, Charitable Schools [University of Pennsylvania], 1749, 1750.
- William Smith, Provost of the same college, 1753, 1756.

Mathematical requirements in terms of textbooks.
- John Ward's *The Young Mathematician's Guide*, 1709.
- Nathaniel Hammond's *The Elements of Algebra*, 1742.
- Thomas Simpson's *A Treatise of Algebra*, 1745.

Sections on algebra in textbooks from American printing presses:
- Pieter Venema's *Arithmetica of Cyffer-Konst . . . Als Mede Een kort ontwerp van de Algebra*, 1730.
- Nicolas Pike's *A Complete System of Arithmetic with An Introduction to Algebra*, 1788.
- Consider and John Sterry's ***The American Youth***, 1790.

Advertisements in the public press from teachers in established schools and private tutors:
- **Boston. First date, July 6, 1727.**
- **New York City. First date, September 7, 1730.**
- **Philadelphia. First date, August 15, 1734.**

Algebra for its own sake.—**Nowhere are there found indications that a practical need for algebra actuated the teaching of it during this period. The inclusion of this subject in the curriculum of a** college of the eighteenth century, or the teaching of it as a special subject by some enthusiastic teacher, must be accounted for on the ground that it was done for the sake of the subject itself or for the theoretical aspects of fluxions. The fascination of this kind of analysis attracted teacher and pupil alike, and the simple joy of the intellectual life that it afforded was reward enough for its study, a reason that lies at the very heart of progress along any line of mental activity.

A Chronological List of American Algebra Textbooks to 1820,[1] the Year in which Algebra was First Required for Admission to an American College

1730 Venema, Pieter.

Arithmetica of Cyffer-Konst, Volgens de Munten Maten en Gewigten, te Nieu-York, gebruykelyk Als Mede Een kort ontwerp van de Algebra Opgestelt door Pieter Venema, Mr. in de Mathesis en Schryf-Konst. Neu-York. Gedruckt door Jacob Goelet, by de Oude-Slip, by J. Peter Zenger. MDCCXXX.

1788 Pike, Nicolas.

A New and Complete System of Arithmetic Composed for the use of the Citizens of the United States: By Nicolas Pike, A. M. Newbury-Port. MDCCLXXXVIII.

Second edition. Enlarged, revised, and corrected. By Ebenezer Adams, A. M., Preceptor of Leicester Academy. Worcester, Mass., 1797.

Third edition. Revised, corrected, and improved. By Nathaniel Lord, A. M., Boston, 1808.

1790 Sterry, Consider and John.

The American Youth: being a new and complete course of introductory mathematics; designed for the use of private students. By Consider and John Sterry, v. 1 . . . Providence. Printed by B. Wheeler, for the authors, 1790.

1798 Gough, John

Practical Arithmetick. By John Gough. Carefully revised by Thomas Telfair, Philomath. With an Appendix of Algebra. By the late W. Atkinson, of Belfast. Dublin: Printed. Wilmington: Reprinted and sold by Peter Brynberg. M.DCC.XCVIII. Second edition. 1800.

1801 Webber, Samuel

Mathematics compiled from the Best Authors and intended to be the Text-book of the Course of Private Lectures on these Sciences in the the University at Cambridge. Under the direction of Samuel Webber, A. M.–A. A. S. Hollis Professor of Mathematics and Natural Philosophy. In 2 vols. Boston. Printed by Thomas & Andrews. 1801.

Second edition, Cambridge. W. Hilliard, 1808.

1806 Bonnycastle, John

An Introduction to Algebra; with notes and observations designed for the use of schools and places of public education. First American edition. Philadelphia: Published by Joseph Crukshank, 1806.

Second American edition. Philadelphia: Kimber and Conrad, 1811. [Title as above] . . . to which is added an appendix on the application of algebra to geometry. First New York, from the tenth London edition. New York: E. Duyckinck, D. D. Smith & G. Long, 1818.

[1] Complete bibliography to 1850 in preparation.

1806 Chevigne, L. I. M.

Mathematical Manual for the use of St Mary's College of Baltimore containing four parts;—viz:—I Rational Arithmetic II Elements of Algebra III Practical Arithmetic IV Practical Algebra [L. I. M. Chevigne] Baltimore. Printed for St. Mary's College. By John West Butler. 1806.

1806 Vyse, Charles

The Tutor's Guide. By Charles Vyse. Philadelphia: Joseph Crukshank. 1806.

1807 Chevigne, L. I. M.

Mathematical Manual for the use of Colleges and Academies. Volume First. [Rest of the title same as 1806 edition.] Printed by John West Butler. 1807.

1809 Simpson, Thomas

A Treatise of Algebra: wherein the Principles are demonstrated and applied in many useful and interesting inquiries, and in the resolution of a great variety of problems of different kinds. To which is added, the geometrical construction of a great number of linear and plane problems, with the method of resolving the same numerically. By Thomas Simpson, F. R. S. First American, from the eighth London edition. Philadelphia: Printed for Mathew Carey by T. & G. Palmer. 1809.

1812 Hutton, Charles

A Course of Mathematics in two volumes for the use of academies, as well as private tuition. By Charles Hutton, LL. D. F. R. S. Late Professor of Mathematics in the Royal Military Academy. From the Fifth and Sixth London Editions. Revised and Corrected by Robert Adrain, A. M. Fellow of the American Philosophical Society and professor of Mathematics in Queen's College, New Jersey. New York, Samuel Campbell, . . . 1812.

Second edition, New York: Published by Samuel Campbell, . . . 1816.

Third Edition, New York: Published by Samuel Campbell, . . . 1818.

1814 Day, Jeremiah

An Introduction to Algebra, being the first part of a Course of Mathematics adapted to the method of instruction in the American colleges. By Jeremiah Day. New Haven: Howe & Spalding. 1814.

Second edition. New Haven: Howe & Spalding, 1820.

1818 Euler, Leonard

An Introduction to the Elements of Algebra designed for the use of those who are acquainted only with the First Principles of Arithmetic. Selected [by John Farrar] from the Algebra of Euler. Cambridge, N. Eng.: Hilliard & Metcalf. 1818.

1818 Lacroix, S[ilvestre] F[rancois]

Elements of Algebra, by S. F. Lacroix. Translated from the French for the use of the Students of the University at Cambridge, New England. Cambridge, N. E.: Printed By Hilliard & Metcalf, 1818.

1819 Day, Jeremiah

An Introduction to Algebra, being the First Part of a Course of Mathematics adapted to the method of instruction in the higher schools and academies in the United States. By Jeremiah Day, LL. D., President of Yale College, New Haven: Published by Howe & Spalding. 1819.

1820 Lacroix (& Bézout)

An elementry treatise on Plane and Spherical Trigonometry and on the application of Algebra to Geometry from the mathematics of Lacroix and Bézout. Translated from the French for the use of the students at the University at Cambridge, New England. Cambridge, N. E.: Printed by Hilliard and Metcalf, 1820.

BIBLIOGRAPHY

This study has been made largely from original sources and printed works in the libraries of the following institutions:

American Association of Arts and Sciences.
American Antiquarian Society.
Boston Athenaeum.
Boston Public Library.
Bowdoin College.
Brown University.
Bureau of Education, Washington, D. C.
Columbia University.
Essex Institute.
John Carter Brown Library.
Harvard University.
Historical Society of Pennsylvania.
Library Company of Pennsylvania.
Library of Congress.
Maine Historical Society.
Massachusetts Historical Society.
Newburyport Public Library.
New York Historical Society.
New York Public Library.
New York Society.
Peabody Institute.
Plimpton, George A., Private Library.
Princeton University.
Rhode Island Historical Society.
Salem Athenaeum.
Smith, David Eugene, Private Library.
University of Pennsylvania.
University of Virginia.
Virginia State Library.
Watkinson Library.
William and Mary College.
Yale University.
York Village, Maine, Museum.

Further sources of material are listed below:

American Philosophical Society. Transactions. Vol. II. New Series, Philadelphia, 1825.

Badger, H. C., *Mathematical Theses of Junior and Senior Classes, 1782–1839*, Cambridge, Mass., 1888.

Bowditch, N., *Mécanique Céleste*, By the Marquis de La Place, Translated by Nathaniel Bowditch, LL. D., Vol. IV. Memoir of the translator, by his son, Nathaniel Ingersoll Bowditch, Boston, 1839.

Bronson, W. C., *The History of Brown University, 1764–1914*, Providence, 1914.

Cajori, F., *The Teaching and History of Mathematics in the United States*, Washington, 1890.

Cajori, F., *William Oughtred*, Chicago, 1916.

Chambers, E., *Cyclopædia*. Second edition, London, 1738.

Clap, Thomas, *The Annals or History of Yale-College in New-Haven*, New Haven, 1766.

Clap, Thomas, *A Catalogue of the Library of Yale-College in New-Haven*, N. London, 1743.

Columbia University, *A History of Columbia University, 1754–1904*, New York, 1904.

Currier, J. J., *History of Newburyport, Mass., 1764–1905*, Newburyport, 1906.

De Morgan, Augustus, *Arithmetical Books*, London, 1847.

Dexter, F. B., *Biographical Sketches of the Graduates of Yale College with Annals of the College History, October, 1701–May, 1745*, New York, 1885.

Dickson, L. E., *History of the Theory of Numbers*, Vol. II, Washington, 1920.
Dunshee, H. W., *History of the School of the Collegiate Reformed Dutch Church from 1633-1883*. New York, 1883.
Ecclesiastical Records of the State of New York. Albany, 1901–1916.
Edes, H. H., "Harvard Theses of 1663." *Transactions of The Colonial Society of Massachusetts*, Vol. V, 1897, 1898, Boston, 1902.
Evans, Charles. *American Bibliography, 1639–1820*. Chicago, 1903–1914.
Glenn, T. A., *William Churchill Houston, 1746–1788*. Norristown, Pa., 1903.
Hérigone, Pierre. *Cursus Mathematicus, nova brevi et clara*. Vol. II, Paris, 1644.
Hildeburn, C. R., *A Century of Printing. The Issues of the Press in Pennsylvania, 1685-1784*. Philadelphia, 1885.
Jones, Hugh. *The Present State of Virginia*. London, 1724.
Lane, W. C., "Early Harvard Broadsides." Proceedings of *American Antiquarian Society*. Worcester, 1914.
Maclean, John, *History of the College of New Jersey, From its origin in 1746 to the Commencement of 1854*. Philadelphia, 1877.
Oughtred, William, *Clavis Mathematicæ*. London, 1631.
Peirce, Benjamin, *A History of Harvard University*. Cambridge, 1833.
Potter, A. C., and Bolton, C. K., *Librarians of Harvard College, 1667–1877*. Cambridge, 1897.
Pratt, D. J., *Annals of Public Education in the State of New York, From 1626 to 1746*. Albany, 1872.
Quincy, Josiah, *The History of Harvard University*. Boston, 1860.
Raphson, Joseph, *Universal Arithmetick: or, a Treatise of Arithmetical Composition and Resolution*. London, 1720.
Sanborn, F. B., *Doctor Langdon (1723–1797)*, 1904.
Smith, D. E., "A Glimpse at Early Colonial Algebra" in *School and Society*. January 5, 1918.
——— *History of Mathematics*, 2 vols., Boston, 1923–24.
——— *The Sumario Compendioso of Brother Juan Diez*. Boston, 1921.
Smith, H. W., *Life and Correspondence of the Rev. William Smith, D. D.* Philadelphia, 1879.
Smith, *The Works of William Smith, D. D.*, Late Provost of the College and Academy of Philadelphia, 1803.
Snow, L. F., *The College Curriculum in the United States* [No place], 1907.
Spotswood, *The Official Letters of Alexander Spotswood, Lieutenant Governor of the Colony of Virginia, 1710–1722*, Richmond, 1882.
University of Pennsylvania, *Biographical Catalogue of the Matriculates of the College, 1749–1893*, Philadelphia, 1894.
Wallis, John, *A Treatise of Algebra both Historical and Practical*, London, 1685.
William and Mary Quarterly, Vols. I–XXVII.
Williams, J. R., *Philip Vickers Fithian, Journal and Letters, 1767–1774*. Edited by John Rogers Williams, Princeton, The University Library, 1900.

INDEX

WASHINGTON : GOVERNMENT PRINTING OFFICE : 1924