Grades 6–8

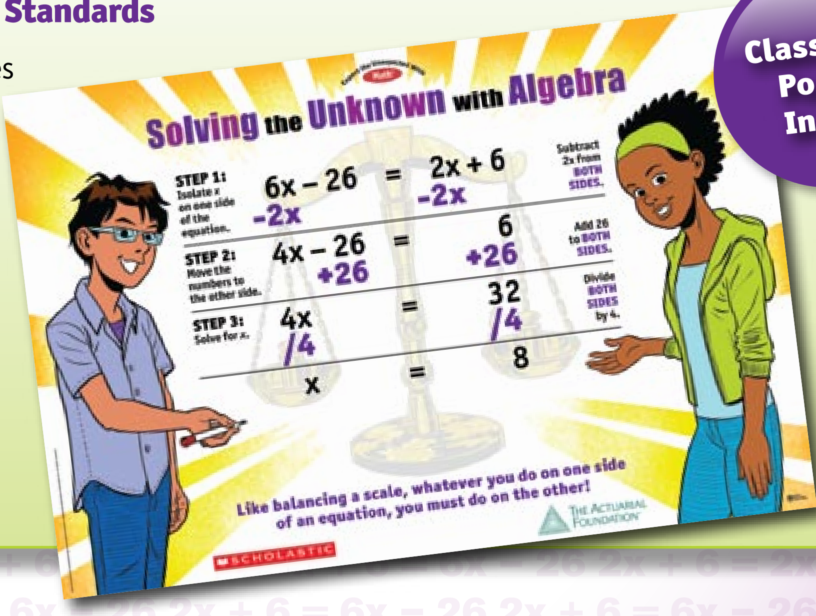
Expect the Unexpected With Math®

Solving the Unknown with Algebra

Poster/Teaching Guide for **Pre-Algebra** Students

Aligns with NCTM Standards

Follow the adventures of Rick and Athena as they solve real-world questions through powerful mathematical thinking!



Classroom Poster Inside

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Dear Teacher:

Welcome to **Solving the Unknown with Algebra**, a new math program aligned with NCTM standards and designed to help students practice pre-algebra skills including using formulas, solving for unknowns, and manipulating equations.

Developed by The Actuarial Foundation with Scholastic, this program provides skill-building activities that use mathematics for real purposes, while motivating students to achieve success in the classroom and in real-world situations outside of school. We hope you enjoy this new program!

Sincerely,
The Actuarial Foundation   Scholastic Inc.

Getting Started:

This program is designed to supplement your existing pre-algebra instruction in these areas:

1. solving for unknowns;
2. manipulating equations while keeping them in balance;
3. using formulas to solve real-world challenges; and
4. working with proportions, radicals, and exponents.

The materials are taught through this story line: Athena and Rick are two students frequently called upon to use their powerful mathematical thinking to solve seemingly unsolvable mysteries, problems, and questions. They post case files on a math blog, and the worksheets inside engage students to follow along with the duo and solve problems by applying mathematical skills and a logical, systematic approach.

Three lesson plans teach basic pre-algebra concepts; each lesson features a **worksheet**, and is also supplemented by a **bonus worksheet** and **take-home activity**. The materials are designed with flexibility to allow teachers to use them in whatever order best supplements the scope and sequence of his/her pre-algebra instruction and curriculum. The worksheets also include additional math content connections, highlighted in the materials section of each lesson.

The **take-home activities** further encourage students to apply their math skills to real-world situations. The **classroom poster** displays the important concept of keeping equations in balance while solving for unknowns and reminds students, particularly visual learners, of this fundamental pre-algebra concept.

Lesson 1

Using Mathematical Models/Proportions

OBJECTIVES:

- Students will understand that an equation works like a scale with the equal sign serving as the balancing point; that the equal sign means "is the same as" rather than "the answer to the problem is"; that, like balancing weight on a scale, what is done to one side of an equation must be done to the other
- Students will understand how scale keeps a map's distances in proportion to real-world areas depicted in the map; use a map's scale and coordinates to calculate distance; identify the variables in the formula $d = r \cdot t$; and manipulate the formula to solve for variables

Time: 30–40 minutes, depending on class review needs; additional time for worksheets

Materials:

- Balance scale with weights (drawings on the board will also suffice)
- Map (e.g., world, state, etc.) with longitude/latitude and/or coordinates, and a key including scale
- Worksheet 1 (Content Connections: Ordered Pairs)
- Bonus Worksheet 1 and Take-Home Activity 1 (Content Connections: Measurement)

DIRECTIONS:

1. Show students a balancing scale without weights on either arm. Ask why the scale is in balance (because each side has the same weight). Ask what would happen if 5 grams were added to both sides. Demonstrate, reiterating that the scale is still balanced because the same amount was added to each side.
2. Ask what would happen if we only operated on one side of the scale. What if 2 grams were taken off one side? Demonstrate that the scale is out of balance because the action taken on one side wasn't repeated on the other. Return the 2 grams to balance the scale.
3. Ask what would happen if we tripled both sides. Demonstrate that the scale is still balanced. Then, to demonstrate inverse operations, divide the weight on each side by 3, showing that inverse operations "cancel each other out."
4. Indicate that equations work like the balancing scale, with the equal sign acting as the fulcrum. (Many students have the common misconception that the equal sign means "the answer to the problem is" rather than "is the same as.") Before proceeding, make sure students understand this distinction and the goal of keeping things in balance.
5. Write a simple equation on the board, e.g., $x + 7 = 9$. Indicate that the goal is, as with the scale, to keep the equation in balance. Indicate that we can solve for x if we isolate it on one side of the equation through manipulations that keep the equation in balance. Depending on class support needed,

Lesson 1 continued Using Mathematical Models/Proportions

show how to solve for x using the scales metaphor, and keeping the equation in balance by taking the same action on both sides. Write –7 under the 7 on the left side of the equation and –7 under the 9 on the right side. Complete the subtraction, leaving $x = 2$. Repeat with other one-step equations, such as $x - 4 = 7$, $5x = 45$, and $x/6 = 7$. Show how to check the accuracy of the calculation by incorporating the value of x into the original equation. Both sides will be equal.

6. Follow up with an equation requiring more than one manipulation, e.g., $2x + 4 = 8$. Ask students to solve and explain, step by step, how they kept the equation in balance. For something more complex, try $2x + 4 = 6x - 8$. Again, ask students to solve and explain. Show the step-by-step manipulation, first adding 8 to both sides, then subtracting $2x$ from both sides, leaving $12 = 4x$. Show how dividing both sides by 4 keeps the equation in balance, leaving $x = 3$. Incorporate the value of x back into the original equation to show both sides being equal.
7. Mention that keeping an equation in balance applies to many real-life situations. Show students the map and ask them to explain how a map is a representation of the area it depicts. Answers should include that map distances are proportional to the real distances they represent. Demonstrate by measuring the distance between two map locations and using proportions to calculate the actual distance. For example, if 1 inch = 200 miles and you measured 2.5 inches, show how 1 in./200 mi. = 2.5 in./x mi. or 500 miles.
8. Using distances calculated, ask how long it would take to travel between the two points. Students should indicate that the time depends on how fast one is traveling. Make an appropriate assumption depending on the means of travel (e.g., a driving speed of 55 mph or flying speed of 600 mph). Divide distance by rate to determine trip length.
9. Write "Distance = Rate · Time" on the board and explain what each term means. Indicate that a formula is a representation of a relationship in the real world, just as a map is a representation of a real place. Show how it's possible to determine one of the terms if the other two are known, e.g., one could calculate rate if time and distance are known. Write the formula as $d = r \cdot t$ and show how to manipulate the formula, e.g., $r = d/t$ and $t = d/r$.
10. Point out coordinates and/or longitude/latitude markings on the map. Determine whether students know how to express a location's coordinates in x,y format (and longitude/latitude format if desired). Make sure they can determine horizontal and vertical distance using coordinates.
11. Distribute **Worksheet 1**. Read the introduction as a class and review the "Additional Clues" (key facts) before students complete the worksheet. Make sure students are able to determine each route's distance using the coordinates and the map's scale. Review answers as a class.
12. Students will build on these skills in **Bonus Worksheet 1** and **Take-Home Activity 1**. If students require additional support, review worksheets as a class.

Lesson 2 Analyzing Change/ Growth and Decay Formula

OBJECTIVES:

Students will be able to identify what interest is as it pertains to saving and investing; calculate simple and compound interest; apply the compound interest formula (i.e., the growth and decay formula) to nonfinancial situations

Time: 20–30 minutes, depending on class review needs; additional time for worksheets

Materials:

- Ads from financial institutions indicating interest rates paid on savings and investment accounts
- Student calculators (for Worksheet 2; can also be done without calculators)
- Worksheet 2 (Content Connections: Percentages, Exponents)
- Bonus Worksheet 2 (Content Connections: Conversion of Percentages to Decimals, Exponents)
- Take-Home Activity 2 (Content Connections: Percentages, Exponents)

DIRECTIONS:

1. Show a savings or investment ad to the class. Ask students to explain what the interest rate (e.g., 1.75%) means. Establish that the *interest rate* represents the rate of payment by the financial institution to the depositor in return for depositing money with the institution.
2. Using the example of a $1,000 CD deposited for one year with a 2% interest rate, show the calculation $1,000 · .02 · 1 = $20. Work the calculation as necessary and generalize the formula by writing the following formula for simple interest: $I = p \cdot r \cdot t$, where I = interest, p = principal (amount deposited), r = rate (of interest), and t = time (in years). It might also be useful to show the class that the total value of the CD at maturity equals Principal + Interest (Principal · Rate · Time). If necessary, explain to the class that the interest rate is in effect the entire term of the CD. (If a student asks, it may be necessary to explain that interest also comes into play when a financial institution loans money. In these cases, the borrower pays interest to the financial institution.)
3. Ask students what they think would happen if the bank offered a two-year CD. Because this is a case of simple interest, indicate that the bank will pay the interest at the end of each year. This results in a $20 payment at the end of each year. Plug these numbers into the formula $I = p \cdot r \cdot t$ to show how a total of $40 in interest will be paid. To show how much the depositor has in total, show how to add the interest to the principal, i.e., $1,000 + $40 = $1,040.
4. Ask what would happen if the depositor kept the first year's interest in the account to "grow." Show how the depositor would earn an extra 40¢ by using the $20 as principal for one year (the second year of the CD's term) at 2%. Explain that this is an example of *compound interest*.
5. Indicate that there is a formula that can be used to calculate how money grows with compound interest: $y = a(1+r)^n$ where

Lesson 2 continued
Analyzing Change/ Growth and Decay Formula

y = ending value, a = starting amount (in this case principal or amount invested), r = interest rate, and n = the number of time periods. The amount of interest can be determined by subtracting the starting principal amount from the ending principal amount. Demonstrate with calculations from the two-year CD.

6. Distribute **Worksheet 2** and classroom calculators. Read the introduction with the class and review the key facts before students complete the worksheet. Review answers as a class.
7. Students will build on these skills in **Bonus Worksheet 2** and **Take-Home Activity 2**. If students require additional support, review worksheets as a class.

Lesson 3
Functions and Formulas/Square Roots

OBJECTIVES:
Students will understand what a square root is and that squares and square roots are inverse operations and can be used to manipulate equations as long as "whatever is done to one side of the equation is done to the other."

Time: 20–30 minutes, depending on class review needs; additional time for worksheets

Materials:
- Student calculators (for Worksheet 3; can also be done without calculators)
- Worksheet 3 (Content Connections: Square Roots)
- Bonus Worksheet 3 (Content Connections: Probability)
- Take-Home Activity 3 (Content Connections: Variables)

DIRECTIONS:
1. Ask the class to define what a square is. After students mention that a square has four equal sides and four right angles, draw a square on the board.
2. Label one of the sides "4 feet." Ask how the area of the square can be determined (by multiplying $4 \cdot 4$ or, more generally, squaring one of the sides) and write $A = s^2$ where A represents area and s represents the length of a side. If necessary, explain how the superscript "2" is used to note exponents.
3. Ask the class how to find the length of a side of a square if we know its area, for example when $A = 25$. Depending on the level of class support needed, indicate that the problem can be solved by finding out what number times itself equals 25.
4. Introduce the radical sign notation, e.g., that $5 = \sqrt{25}$.
5. Go back to the formula for area, $A = s^2$. Then write $25 = s^2$.
6. Ask how we can find out what s equals. Indicate that we can take the square root of both sides. Referring to the scale example, remind students that what we do to one side of an equation we must do to the other. Provide other examples as needed using perfect squares for the area.
7. Introduce an example where the area is not a perfect square, for example, $A = 29$. Using a calculator, show that $\sqrt{29}$ is approximately equal to 5.385.
8. To show how squares are inverse operations of square roots, ask what $\sqrt{9^2}$ is equal to. Break down the problem for students as needed, showing step-by-step how $9^2 = 81$ and $\sqrt{81} = 9$. To generalize, this means that $\sqrt{x^2} = x$. Provide other examples with perfect squares as needed.
9. Finally, provide an example of an equation involving taking the square root of both sides of an equation. Write $x^2 + 6 = 15$. Ask students to solve and explain. Show the solution on the board, first subtracting 6 from both sides, leaving $x^2 = 9$. Indicate that the equation is still in balance and that it will stay in balance if we take the square root of both sides, leaving $x = 3$. Remind students that whatever is done to one side of an equation must be done to the other.
10. Distribute **Worksheet 3** and calculators. Read the introduction and review the key facts. Depending upon student support needed, it might be necessary to compute the first vehicle's speed as a class and/or review the calculator's square root function.
11. Ask students to complete the worksheet. Review answers as a class.
12. Students will further develop skills at working with equations in **Bonus Worksheet 3** and **Take-Home Activity 3**. If students require additional support, review worksheets as a class.

For more free math resources, visit The Actuarial Foundation Web site at: www.actuarialfoundation.org/programs/for_teachers.shtml

- **The Math Academy Series: Using Math in the Real World**
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  Probability: *Shake, Rattle, & Roll*

  Graphing: *Bars, Lines, & Pies*

  Converting decimals, fractions, and percents: *Conversions Rock*

  Perimeter, area, surface area, and volume: *Setting the Stage with Geometry*

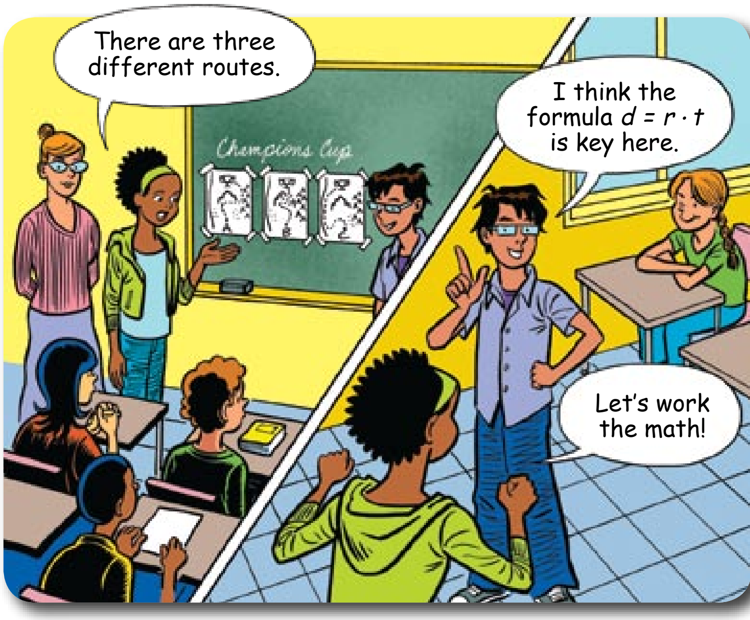
Printable copies also available at:
www.scholastic.com/unexpectedmath

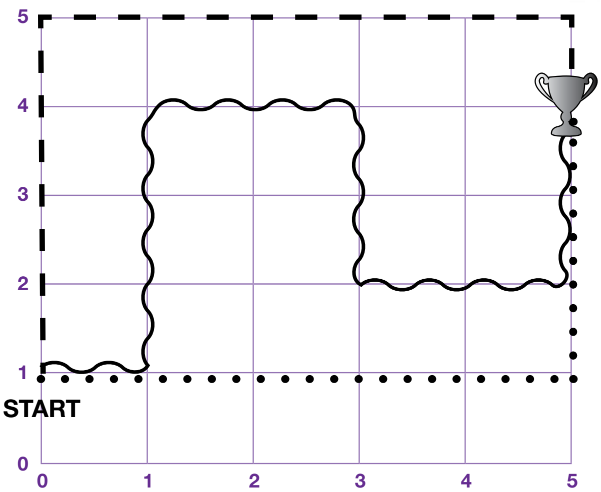
The Case of the Doubtful Distance

Solving the Unknown with Algebra—a new blog by 8th-graders Rick and Athena to solve questions and mysteries by using math—has received its first e-mail request!

"We've been asked to be advisors for the annual 7th-grade field trip to the state park," said Athena. "One of the homerooms is not sure of the best route to take to win the Champions Cup race!"

For the Champions Cup race, each homeroom receives a map showing alternate routes to the cup. "I think we should consider the formula ***Distance* = *Rate* · *Time* (*d* = *r* · *t*)**," said Rick. "Check out this map":





KEY:

• • • • = Route A

— — — = Route B

∼∼∼ = Route C

0 — 1 = 1,000 feet

ADDITIONAL CLUES:

Rates of speed for each route based on previous years:

Route A (rough terrain): 2,000 feet per hour

Route B (swampy): 3,000 feet per hour

Route C (flat and dry): 6,000 feet per hour

WORK THE MATH

Show your work—use separate paper as needed.

1 **How long is each route?**

Route A ____________

Route B ____________

Route C ____________

2 **Using the formula $d = r \cdot t$, how long should it take to complete each route?**

Route A ____________

Route B ____________

Route C ____________

A Case of Interest

"Looks like we have another case!" shouted Rick as he scanned his e-mail. Athena and Rick have received an anxious query from the president of the 6th-grade class. The middle school has a proud tradition which involves each 6th-grade class raising money that they will use at the end of their 8th-grade year for a community service project. This year, the 6th-graders held a bake sale which raised $500. The class treasurer is concerned because the class wants to build flower beds at the town's senior center at a cost of $545. Athena researches savings opportunities for the class and finds that:

- First National Bank is offering a two-year CD with **4.9% simple interest**.
- Second National Bank is offering a two-year CD with **4.8% compound interest (compounded yearly)**.

Do the students need to hold another fund-raiser? Rick thinks they might, but Athena has another idea and opens her laptop to get to work.

WORK THE MATH

Show your work—use separate paper as needed.

Hint: **Remember these formulas:**
$I = p \cdot r \cdot t$ (simple interest)
$y = a(1 + r)^n$ (compound interest)

1 **How much will the class have in two years if it buys a First National Bank CD?**

2 **How much will the class have in two years if it buys a Second National Bank CD?**

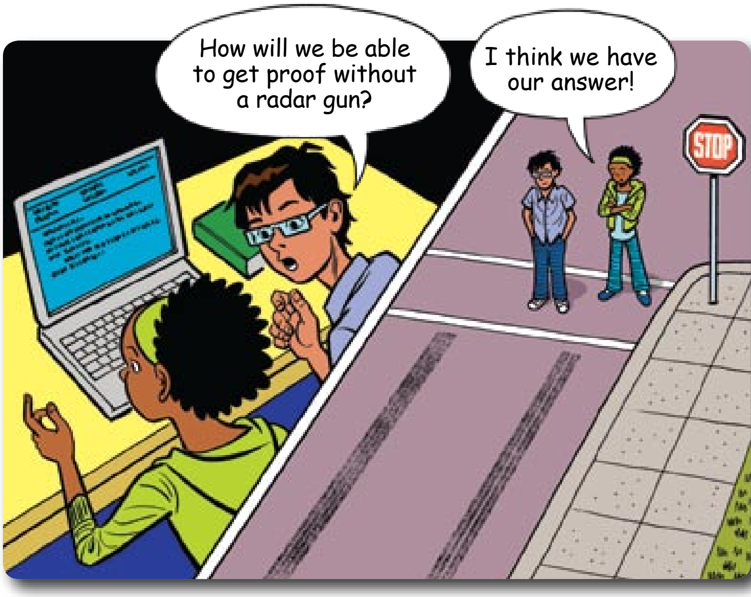
3 **Which CD is the better deal? Explain your thinking. Did your calculations surprise you?**

NOW TRY THIS:

Assume next year's 6th-grade class needs $600 for its service project. If it can buy a two-year CD with a compound interest rate of 5%, how much does it need to raise at its fund-raiser?

The Case of the Screeching Tires

"SCREECH—help!" was the subject line of an e-mail Rick and Athena received from their principal, who suspects that drivers are exceeding the 15 mph speed limit along the road leading to the school. The principal has urged the city to install speed bumps, but wants to provide proof that cars are speeding. Rick thinks a speed radar gun is the only way, but Athena suggests they first inspect the scene. The pair discover different sets of tire tracks where cars had slammed on their brakes before a stop sign. "I think our answer is right here!" Athena declares.



WORK THE MATH

Athena's first step is to assemble the key facts. She connects with police investigators and discovers that the formula used to analyze tire tracks is $s = \sqrt{24 \cdot d}$ where **s = speed** in miles per hour and **d = length of the tire tracks in feet**. She then takes measurements of different tire tracks.

Show your work—use separate paper as needed.

1 For each set of measurements below, calculate each vehicle's speed:

TIRE TRACKS	VEHICLE SPEED
#1: **9.375 feet**	
#2: **6 feet**	
#3: **37.5 feet**	
#4: **24 feet**	
#5: **150 feet**	

2 What should Rick and Athena report to the principal about speeding cars?

NOW TRY THIS:

A principal in another school submits different tire track data to analyze possible speeding on a road near her school:

Tire Tracks #1: 16 2/3 feet

Tire Tracks #2: 66 2/3 feet

Tire Tracks #3: 104 1/6 feet

The speed limit is 20 mph. For each measurement above, calculate each vehicle's speed to determine whether or not there is a speeding issue.

Worksheet 1: The Case of the Doubtful Distance

1. Route A = 8,000 ft. (5 units + 3 units = 8 units · 1,000 feet)
   Route B = 10,000 ft. (4 units + 5 units + 1 unit = 10 units · 1,000 feet)
   Route C = 12,000 ft. (1 unit + 3 units + 2 units + 2 units + 2 units + 2 units = 12 units · 1,000 feet)
2. Route A = 8,000 ft. / 2,000 ft. per hr. = 4 hours
   Route B = 10,000 ft. / 3,000 ft. per hr. = 3.33 hours = 3 hours, 20 minutes
   Route C = 12,000 ft. / 6,000 ft. per hr. = 2 hours

Bonus Worksheet 1: The Case of the Perilous Planting

1. To calculate the height of a mature tree, set up a proportion showing the relationship between a tree and its shadow. A one-foot shadow is cast by a three-foot tree, so a 30-foot shadow would be cast by a 90-foot tree. As a proportion this is shown as 30 ft./x feet = 1 foot /3 feet. Solving for x leaves x = 90 feet. Therefore, 60 feet from the school building is not a safe distance.
2. 180 feet. Set up a proportion 6 ft./3 ft. = x ft./90 ft. Solving for x leaves $x = 180$

Now Try This: The length of a shadow of a 45-foot-tall building can be determined by setting up a proportion. In this case, $1/3 = x/45$. Solving for x leaves $x = 15$ feet.

Take-Home Activity 1: The Case of Sweet Proportions

For 32 servings:

- Butter: 1 2/3 cups
- Sugar: 1 cup
- Flour: 3 1/3 cups
- Whipping Cream: 6 2/3 tablespoons
- Almonds: 4 2/3 cups
- Coconut: 4 cups
- Chocolate: 10 2/3 squares

Now Try This: For 8 servings:

- Butter: 5/12 cup
- Sugar: 1/4 cup
- Flour: 5/6 cup
- Whipping Cream: 1 2/3 tablespoons
- Almonds: 1 1/6 cups
- Coconut: 1 cup
- Chocolate: 2 2/3 squares

Worksheet 2: A Case of Interest

1. After two years students will have the \$500 they deposited plus interest calculated using the formula $I = p \cdot r \cdot t$. Interest = \$500 · .049 · 2 = \$49, so the students will have \$500 + \$49 = \$549

2–3. Using the growth formula $y = a(1+r)^n$, after two years the students will have $\$500(1+.048)^2$ or \$549.15. So this is the better deal.

Now Try This: Students will need to solve the following equation to figure out how much they need to raise: $600 = a(1.05)^2$, $600 = a(1.1025)$, $600/1.1025 = a$, $a = \$544.22$

Bonus Worksheet 2: The Case of the Smelly Sandwich

1. Growth rate = 100% per week, written as 1.0 as a decimal
2. 10 bacteria per cubic centimeter
3. 1,000 bacteria per cubic centimeter
4. Weeks
5. $10(1 + 1.0)^5 = 10 \cdot 32 = 320$ bacteria per cubic centimeter, so the sandwich was in the locker at least five weeks which is more than a month.
6. $10(1 + 1.0)^6 = 640$ bacteria per cubic centimeter and $10(1 + 1.0)^7 = 1{,}280$ bacteria per cubic centimeter, so the sandwich has been in the locker between six and seven weeks.

Take-Home Activity 2: The Case of the Decaying Car

1. Current value $= 20{,}000(1 - .20)^4 = 20{,}000 \cdot .8^4 = 20{,}000 \cdot .4096 = \$8{,}192.00$
2. Using the decay formula, the value after six years $= \$20{,}000(1 - .20)^6 = \$20{,}000 \cdot .8^6 = \$20{,}000 \cdot .262144 = \$5{,}242.88$. Since the car's value was \$8,192.00 after four years, the amount of the decline in value in the additional two years would be \$8,192.00 – \$5,242.88 = \$2,949.12

Now Try This:

1. $30{,}000(1 - .20)^3 = 30{,}000 \cdot .8^3 = 30{,}000 \cdot .512 = \$15{,}360.00$
2. $25{,}000(1 - .20)^1 = 25{,}000 \cdot .8 = \$20{,}000.00$
3. $40{,}000(1 - .20)^5 = 40{,}000 \cdot .8^5 = 40{,}000 \cdot .32768 = \$13{,}107.20$

Worksheet 3: The Case of the Screeching Tires

1. Tire Tracks 1 $= \sqrt{24 \cdot 9.375} = \sqrt{225} = 15$ mph
   Tire Tracks 2 $= \sqrt{24 \cdot 6} = \sqrt{144} = 12$ mph
   Tire Tracks 3 $= \sqrt{24 \cdot 37.5} = \sqrt{900} = 30$ mph
   Tire Tracks 4 $= \sqrt{24 \cdot 24} = \sqrt{576} = 24$ mph
   Tire Tracks 5 $= \sqrt{24 \cdot 150} = \sqrt{3{,}600} = 60$ mph
2. Three of the five cars analyzed were speeding, some more than twice the speed limit. The principal should request that the town install speed bumps.

Now Try This:

Tire Tracks 1 $= \sqrt{24 \cdot 16\ 2/3} = \sqrt{400} = 20$ mph

Tire Tracks 2 $= \sqrt{24 \cdot 66\ 2/3} = \sqrt{1{,}600} = 40$ mph

Tire Tracks 3 $= \sqrt{24 \cdot 104\ 1/6} = \sqrt{2{,}500} = 50$ mph

Two of the three cars exceeded the 20 mph speed limit.

Bonus Worksheet 3: The Case of the Tardy Transportation

1. *Elm Street route:* 5-minute run time + (2 minutes at light 1 · .5 probability at light 1) + (2 minutes at light 2 · .5 probability at light 2) + (2 minutes at light 3 · .5 probability at light 3) OR 5-minute run time + 2 minutes · .5 probability · 3 lights = 8 minutes

   Washington Road route: 5-minute run time + 1 minute · .1 probability · 4 lights = 5.4 minutes = 5 minutes, 24 seconds

   The Washington Road route is usually faster.
2. The probability of a red light plus the probability of a green light equals 100%. Expressed as a decimal, 100% = 1. So, the probability of a green light on Washington Road is 1 – the probability of a red light or 1 – .1 = .9
   The probability of two green lights in a row on Washington Road = .9 · .9 = .81 or 81.0%
3. The probability of four green lights in a row on Washington Road = .9 · .9 · .9 · .9 = .656 or 65.6%

Take-Home Activity 3: The Case of the Kid Bargain Hunter

1. Costs for each rental company:
   Let's Go: \$220 · 2 + .10 · 1,150 = 440 + 115 = \$555.00
   Smooth Ride: \$100 · 2 + .40 · 1,150 = 200 + 460 = \$660.00
   Cheap Wheels: .60 · 1,150 = \$690.00
   Uncle Teddy's: \$300 · 2 = \$600.00

   The family should use Let's Go because it's the cheapest.
2. $t = \$220w + .10n$, where t = total cost, w = number of weeks rented, n = number of miles driven

Now Try This:

Let's Go formula is $t = \$220w + .10n$
Develop a formula for Smooth Ride:
$t = \$100w + .40n$
Since the trip is for two weeks, substitute 2 for w in both formulas and set them equal to each other since they both equal t.
$220 \cdot 2 + .10n = 100 \cdot 2 + .40n$
$440 + .10n = 200 + .40n$
Subtract 200 and $.10n$ from both sides:
$240 = .30n$
Divide both sides by .3 leaving $n = 800$