

# **Lesson Overviews**



### **Dear Teacher:**

Welcome to **Bars, Lines, & Pies**, a dynamic math program designed to build graphing skills in students, while also showing them the relevance of math in their lives. Developed by The Actuarial Foundation along with Scholastic, the graphing lessons and activities inside involve engaging, real-world examples about the environment and recycling.

The Actuarial Foundation seeks to improve and enhance student math education today and in the future. We hope you enjoy this great new program!

Sincerely,

The Actuarial Foundation Scholastic

## **Getting Started**

In the following lessons, students will create, apply, and analyze **pie charts**, **bar graphs**, and **line graphs**.

The **fold-out poster** provides a good **discussion starter** to the topic of graphs.

Review the **definition of a graph** with students: a diagram that visually shows the relationship between numbers or amounts. Ask students what they think it means to show numbers. Where have they seen graphs before? What are the purposes of graphs?

Photos, top to bottom, left to right: © Corbis Veer;

## **Lesson 1: Peering into Pie Charts**

**OBJECTIVES: Students will understand**—

- that a pie chart is used to represent a part-to-whole relationship;
- that the size of each segment represents the segment's proportion to the whole set of data:
- how to critically read pie charts and use information to perform calculations and make predictions.

Time Required: 45 minutes

Materials: Reproducible Activity 1, rulers, protractors, colored pencils, calculators DIRECTIONS:

- 1. On the board, draw a simple pie chart with eight sections. Ask students what the circular image looks like. Guide students to compare the circle to a pie, with each "slice" representing a portion of the whole pie. Tell students that pie charts (or circle graphs) are used to represent data as portions (or segments) of a whole. Explain that just as they would see a pizza pie cut up into pieces, a pie chart is divided into different pieces of data. Each portion represents a percentage of the pie; all portions add up to 100%. Explain that if a pie chart is divided evenly, each portion is the same. Show students how the pie chart on the board has eight even segments and that each segment represents 12.5% (100 ÷ 8 = 12.5).
- Draw another circle on the board. Ask students to list five different percentages that add up to 100%. Write the percentages on the board. Show the students a rough estimation of how to divide the circle to match the provided percentages.
- 3. Explain to students that by using the total number of degrees of a circle (360°), they can calculate the degree of the angle for each segment. Write this simple formula on the board: 1% = 3.6°. Demonstrate how, with simple multiplication, the percentage of a data set can be converted into a degree figure. For example: 25% = 3.6° x 25 = 90°. Ask students for five other percentages that add up to 100%. Draw a new pie chart with these percentages, using the formula to generate the correct angles. Provide additional examples if needed.
- 4. Explain that once a pie chart is divided into segments, each segment should be colored and labeled with the percentage it represents. Point out that circle graph segments are ordered by size from smallest to largest in a clockwise direction (usually starting at "12 o'clock") in order to help people more quickly compare the data.
- 5. Distribute Reproducible Activity 1. Read the introductory text and discuss the table. Instruct students to review the table and answer questions 1 and 2. Then direct them to the "Make a Pie" question and the location of the pie graph template. If necessary, provide guided practice by showing students how to compute the size of the first segment (percentage of old homework paper). Review method for determining segment sizes if needed. Point out the radius line that runs through the circle. Instruct students to use this line as a starting point for creating their segments.
- **6.** Ask students to give examples of the type of data illustrated with a pie chart and have a volunteer describe how the segment sizes in a pie chart are calculated using a protractor.
- Instruct students to answer the questions on the reproducible. When they are finished, review the answers as a class.

## Math Resources Available from The Actuarial Foundation

**Math Program Grants** 



Advancing Student Achievement is a grant program that brings together actuaries and educators in local classroom environments with the belief that interaction with realworld mentors will boost students' interest and achievement in math. Apply for an ASA Grant at: www.actuarialfoundation.org/grant/index.html

### **Best Practices Guide**



The *Best Practices Guide* features a compilation of research on the value of mentoring combined with 15 case histories of programs funded by The Actuarial Foundation, each of which

includes information on program design and results. Free copies are available at: www.actuarialfoundation.org/grant/bestpractices.html

### **Math Academy**



The *Math Academy* series includes hands-on activities for grades 3-6, which you can use to enhance your math instruction while staying true to the academic rigor required by the state standards framework.

Download a free copy at: www.actuarialfoundation.org/grant/ index.html

Free printable copies of this program are available at: www.scholastic.com/barslinespies

Also available: Download free copies of Shake, Rattle, & Roll at www.actuarialfoundation.org/grant/index.html

(for grades 7-8, and gifted 6)

# **Lesson Overviews**

## Lesson 2: Bar Graphs: A Statistical Skyline

### **OBJECTIVES: Students will understand**

- how to use bar graphs to represent, analyze, and generalize data patterns;
- that bar graphs show trends in data and how one variable is affected as the other rises or falls;
- how to propose and justify predictions based on bar graph analysis.

#### Time Required: 45 minutes

Materials: Reproducible Activity 2, graph paper, rulers, colored pencils, calculators

#### **DIRECTIONS:**

- Distribute copies of Reproducible Activity 2. Read the "Raise
  the Bar" sidebar as a class. Explain to students that they will be
  learning about bar graphs in this activity. Tell students a bar graph
  is used to display and compare information. Explain that the height
  of each bar is proportional to the amount of data the bar represents.
  The higher the bar the larger the number or amount of data.
- 2. Draw an X- (horizontal) and a Y-axis (vertical) on the board. Label each axis. On the X-axis write the different months of the year and on the Y-axis a sequence of numbers from 0 to 35 at intervals of 5. Use a show of hands to record the number of students born in each month of the year. Use this data to create an example of a bar graph. For example, 3 students were born in January, 7 in February, and so on.
- Explain that one axis of the graph is where the grouped data (months) is presented while the other is a frequency scale (number of students) showing the quantity of each group.
- 4. When making a bar graph the data to be presented is used to create an appropriate interval scale. This scale helps people visualize and understand the data. Point out the interval scale of the bar graph that you created. Ask students how the graph would change in appearance if the scale were made of smaller intervals or larger intervals. A scale made of smaller intervals is better at illustrating small differences in bar height.
- 5. Direct students to questions under "Work the Math." Instruct them to create their first bar graph (Question 1) in the space provided and the second bar graph (Question 2) on a separate sheet of graph paper using the data provided in the table. Remind students to include a title and labels on their graphs and to neatly color in each bar. Once students have finished both graphs, instruct them to answer the remaining questions on the reproducible.
- 6. When students are done, review the answers to the reproducible and invite them to share their bar graphs with the class.
- 7. Have students explain differences in the data sets of bar graphs and pie charts. Ask how segment size and bar length perform similar functions in the two types of graphs.

## **Lesson 3: Looking Through Line Graphs**

### **OBJECTIVES: Students will understand—**

- that line graphs show how two pieces of information are related and how data changes over time;
- the dependent variable of a line graph typically appears on the Y-axis, and the independent variable appears on the X-axis;
- that line graphs are used to analyze the nature of changes in quantities.

#### Time Required: 45 minutes

Materials: Reproducible Activity 3, rulers, colored pencils, calculators DIRECTIONS:

- Draw and label an X- and a Y-axis on the board. Tell students that
  they will be learning about *line graphs* in this activity. Explain that
  a *line graph* uses *points* and *lines* to examine changes over time.
  Line graphs are often used when examining relationships between
  two types of information.
- 2. Tell students that, like the bar graph, the line graph has an X- and a Y-axis. The dependent variable is plotted on the Y-axis and usually measures quantity (percentage, dollars, liters, etc.). The independent variable is plotted on the X-axis and usually measures time. Use the following data to complete your line graph on the board:
  - Y-axis: \$0.50, \$0.75, \$1.00, \$1.25
  - X-axis: year: 2005, 2006, 2007, 2008
- 3. Write the title "Cost of Milk at School" above the graph. Ask students where the first point in the graph should go if the cost of milk was 50 cents in 2005. Mark this point on the graph. Have students point out where other points should be marked. When done, connect the points with a line.
- 4. Distribute Reproducible Activity 3. Read the first paragraph as a class. When you are finished, point out the data group that students will be using to make their graph.
- 5. Direct students to the "Line It Up" question on the reproducible. Instruct them to use the data from the table to create a line graph in the space provided. Tell students they will need to create a *frequency scale* on the Y-axis to illustrate the frequency of each group of data. Remind students to include a title and labels on their graph. Once students have finished their graphs, instruct them to move on to the questions on the reproducible.
- **6.** Once complete, review the answers to the reproducible as a class and invite students to share their line graphs with the class.
- 7. Ask the class when a line graph would be chosen to illustrate a data set. Have a volunteer give an example of when percentages might be illustrated using a line graph rather than a pie chart.

### **Real-World Math Extensions**

- 1. Ask students to think of graphs that they have seen in the real world. For what purposes were they used? Have students hunt for examples in books, in magazines, on the Internet, in newspapers, and in business documents.
- 2. Review with students the definition of *actuary* on the poster. How can statistics help someone plan for the future? How might an actuary use graphs and math in the following real-world situations?
  - Help a school principal plan a recycling program. How could math and graphs show what the school has used
    in the past, and how much could be saved in the future by recycling? (Use past data to figure out future data
    [extrapolate], and compare results in a graph.)
  - Help the manager of a city plan for a second landfill. How much space would be needed for the new landfill? (Use past data from the first landfill, as well as data that reflects current use and extrapolate for future data. Display the findings in a graph.)
  - Help the manager of a company figure out how much money could be saved by recycling over a period of 10 years. (A line graph would reflect the increase of money saved over a period of time.)

# Make a Paper Pie

Name: \_\_\_\_\_ Da



Work the Math:

In the table below, calculate the amount of material saved if 1,000 tons of paper were recycled. Add your answers to the last column.

# **Materials Saved by Recycling Paper**

Millions of trees are used each year to make paper. Study the table below and then answer the questions to see how math

can show you the way that recycling adds up.

3 9 9 9 9					
Material Saved	Amount Saved per Ton of Recycled Paper	Amount Saved per 1,000 Tons of Recycled Paper			
Water	7,000 gallons				
Oil	380 gallons				
Air Pollution	60 pounds particulate matter (e.g., dust, pollen, acid droplets)				
Landfill Space	3.3 cubic yards				
Energy	4,100 kilowatt-hours				

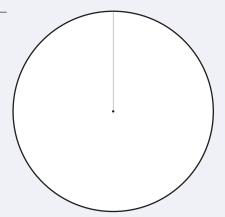
# Slice the Pie!

What you need to know about pie charts:

- Pie charts are used to represent data as portions of a whole.
- All segments of a pie chart added together equal 100%.
- Segments are organized by size from smallest to largest in a clockwise direction.
- Use the following formula to convert a percentage (X) into a degree (Y):

 $3.6^{\circ} \times X\% = Y^{\circ}$ 

- **a.** Water costs \$11.90/7,000 gallons. What are the total water cost savings if 1,000 tons of paper are recycled?
  - **b.** Energy costs \$0.15/kilowatt-hour. What are the total energy savings if 1,000 tons of paper are recycled? \_\_\_\_\_
- Make a Pie Imagine a school that recycles its paper every year. 40 tons of the paper are old homework. 15 tons are lunch menus. 35 tons are posters and artwork. 10 tons are permission slips. On a separate sheet of paper, calculate the percentages of the different paper categories. Then, using the blank chart to the right, create a pie chart to show the percentages.



- Look above at the table from question 1 and the pie chart from question 3 to answer the following:
  - **a.** How many gallons of water are saved by recycling permission slips? \_\_\_\_\_
  - **b.** How much landfill space is saved by recycling old homework?

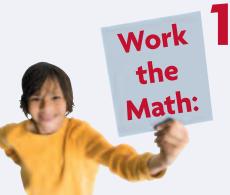


# **ACTIVITY 2: Bar Graphs—A Statistical Skyline**

# Raising the Recycling Bar

Name:	Date:

Americans are recycling more than ever before, but some items are still thrown away—especially plastic bags. Plastic bags are a serious problem because they take a long time to decompose. Complete the questions below to see how recycling can subtract plastic bags from the environmental equation.



Type of

Waste

**Paper** 

Yard

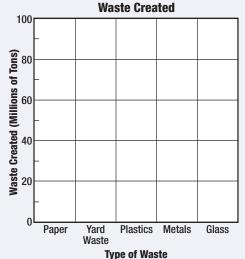
Waste Plastics

Metals

Glass

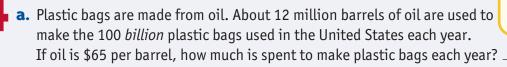
**Graph It** The table on the left below shows different types and amounts of materials commonly found in the average landfill, as well as the amount of each material recovered by recycling. In the blank graph on the right below, create a bar graph showing the amount of each type of waste created.

#### Millions of Millions of Millions of Tons of **Tons Recovered Tons Sent Waste Created** to Landfill by Recycling 84 42 32 20 12 30 2 28 19 7 12 13 10



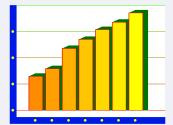
On separate graph paper, create a second graph showing the *percentage* of each material that is being recycled. (*Round your answer to a whole percentage*.) For example, if 22 million tons of waste was created and 8 million was recycled, 36% would have been recovered (8 ÷ 22 = .36).

If <sup>3</sup>/4 of all plastic waste created was recycled, how many tons of plastic would *end up* in the landfill? \_\_\_\_\_\_



**b.** Calculate the quantity of oil saved if 25% less bags were produced. \_\_\_\_\_

### Reproducible 2



# Raise the Bar!

What you need to know about bar graphs:

- Bar graphs are used to display and compare data.
- Bar graphs have a horizontal X-axis and a vertical Y-axis. The X-axis represents the group of data being graphed. The Y-axis represents the value or number of each group.
- The height of each bar represents a certain amount of data of each group. The higher the bar, the bigger the value or number of each group.



## **Fun Fact!**

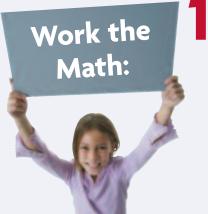
In 1990, 16% of waste in the U.S. was recycled. In 2005, that number doubled to 32%. An actuary could calculate that, if the increase remains the same, by 2020, we could be recycling 64% of waste in the U.S.

# **ACTIVITY 3: Looking Through Line Graphs**

# Is That Trash You're Wearing?

Name: \_\_\_\_\_ Date: \_\_

Plastic bottles are made from a recyclable plastic called PET. PET is a versatile material used in everything from soda bottles to sails. Best of all, it is easy to recycle. PET bottles can be recycled and even made into clothing! Study the facts in the table at left below and then answer the questions below.



**Billions of** 

**Bottles** 

Sold

29

49

55

Year

1999

2001

2003

2005

% of

**Bottles** 

Recycled

24%

24%

20%

22%

**Billions of** 

**Bottles** 

Recycled

Line It Up In the table below on the left, calculate the *number* of bottles recycled between 1999 and 2005. (*Round your answers to the nearest whole number*.) Write your answers in the last column of the table. Then use these numbers to create a line graph in the blank graph on the right below.



## Reproducible 3



# **Draw the Line!**

What you need to know about line graphs:

- Line graphs are used to show how data changes over a period of time.
- Like bar graphs, line graphs have an X-axis and a Y-axis. The X-axis usually represents time. The Y-axis represents quantity.
- Line graphs are made up of points on the graph that are connected by a single line.



In 2006, 13 billion of these containers were recycled but 47 billion were not. What percentage was recycled in 2006? (Round your answer to a whole percentage.)

Many companies are now recycling PET. One company makes many items from recycled PET, including fleece jackets! It takes 5 two-liter PET bottles to create the fill for a man's ski jacket and 36 bottles for a sleeping bag.

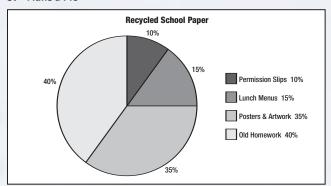
- a. How many bottles would be needed to make: 25,000 jackets? \_\_\_\_\_\_\_
  25,000 sleeping bags? \_\_\_\_\_\_
- **b.** If the company saves \$4 per jacket and \$12 per sleeping bag by using recycled bottles, how much money would they save if they made: 25,000 jackets? \_\_\_\_\_\_\_\_ 25,000 sleeping bags? \_\_\_\_\_\_

Reproducibles Answer Key

### **ACTIVITY 1: Peering into Pie Charts**

- **1.** Water: 7,000,000; Oil: 380,000; Air Pollution: 60,000; Landfill Space: 3,300; Energy: 4,100,000.
- 2. **a.**  $$11.90 \times 1,000 = $11,900$ ; **b.** 4,100 kw-hrs x 1,000 tons = 4,100,000 x \$0.15 = \$615,000

#### 3. Make a Pie



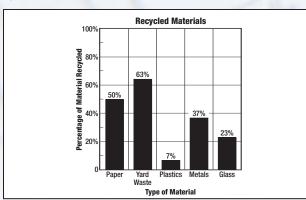
- **4. a.** 7,000 gallons x 10 tons = 70,000 gallons
  - **b.** 3.3 cubic yards x 40 tons = 132 cubic yards

### **ACTIVITY 2: Bar Graphs—A Statistical Skyline**

### 1. Graph It



#### 2.

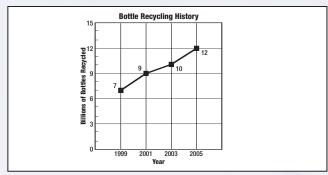


- 3. 3/4 x 30 million tons = 22.5 million tons recycled; 30 million tons 22.5 million tons = 7.5 million tons left in landfill
- **4. a.** 12 million barrels x \$65 = \$780 million
  - **b.** 12 million barrels x 0.25 = 3 million barrels

### **ACTIVITY 3: Looking Through Line Graphs**

### 1. Line It Up

Billions of bottles recycled: 1999: 7 (6.96); 2001: 9 (9.12); 2003: 10 (9.8); 2005: 12 (12.1)

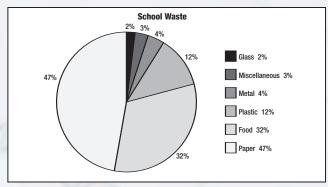


- 2. 13 billion + 47 billion = 60 billion, 13 ÷ 60 = 21.7% (22% rounded up)
- **3. a.** 25,000 jackets x 5 bottles = 125,000 bottles; 25,000 sleeping bags x 36 bottles = 900,000 bottles
  - **b.** 25,000 jackets x \$4 = \$100,000; 25,000 sleeping bags x \$12 = \$300,000

### **BONUS ACTIVITY 1: Recycling by the Numbers**

### 1. Make a Pie

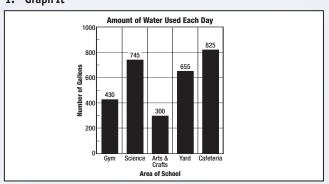
Paper = 169.2°; Food = 115.2°; Plastic = 43.2°; Metal = 14.4°; Glass = 7.2°; Miscellaneous = 10.8°



- 2. paper and food
- 3. a. 600 cases  $\div$  40 cases/ton = 15 tons; b. 15 tons x  $\frac{1}{3}$  = 5 tons
- **4.**  $350 \text{ tons } \times 0.32 = 112 \text{ tons}$ ;  $350 \text{ tons } \times 0.04 = 14 \text{ tons}$
- 5. 3,500 cases ÷ 40 cases per ton = 87.5 tons x 17 trees per ton = 1,487.5 trees. 1,487.5 trees x 9 months = 13,387.5 trees saved

### **BONUS ACTIVITY 2: Water Conservation**

### 1. Graph It



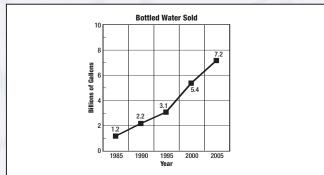
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Alignment with National Standards						
Math (National Council of Teachers of Mathematics)		LESSON 2	LESSON 3			
Represent, analyze, and generalize a variety of patterns with tables, graphs, and words (3-5,6-8)	~	<b>V</b>	V			
Use representations such as graphs, tables and equations to draw conclusions (3-5)	~	<b>V</b>	V			
Use graphs to analyze the nature of changes in quantities in linear relationships (6-8)			<b>V</b>			
Represent data using tables and graphs such as line plots, bar graphs, and line graphs (3-5)		<b>V</b>	<b>V</b>			
Propose and justify conclusions and predictions that are based on data (3-5)		<b>V</b>	<b>V</b>			
Use observations about differences between two or more samples to make conjectures about the populations from which the samples were taken (6-8)		<b>V</b>	<b>V</b>			
Recognize and apply mathematics in contexts outside of mathematics (3-5,6-8)		~	~			
Science (National Science Education Standards)		LESSON 2	LESSON 3			
Unifying Concepts and Processes: Evidence, models, explanations Change, constancy, and measurement		V	~			
Change, constancy, and measurement						
Science and Technology Understanding about science and technology	<b>✓</b>	~	~			
Science and Technology	<b>V</b>	V	V			
Science and Technology Understanding about science and technology Science and technology in society	V	•	<i>V</i>			
Science and Technology Understanding about science and technology  Science and technology in society Risks and benefits  Science in Personal and Social Perspectives	<i>V</i>	~	<i>V V V</i>			

# **Reproducibles Answer Key (continued)**

- 2. a. 745 gallons 655 gallons = 90 gallons
  - b. 300 gallons + 430 gallons = 730 gallons; Science Class and Cafeteria
- 3. a. 300 gallons x .25 gallons less = 75 gallons saved
  - **b.** 655 gallons x  $\frac{1}{2}$  yard not watered = 327.5 gallons saved
  - **c.** 745 gallons  $x^2/3$  students not using water = 496.7 gallons saved

### **BONUS ACTIVITY 3: Tap the Math Facts**

### 1. Line It Up



- 2. greater increase occurred between 1995 and 2000;  $5.4 3.1 = 2.3 \div 3.1 = 0.74 \times 100 = 74\%$  increase
- **3.** 85 gallons x \$1 = \$85; 85 gallons x 0.002 (fountain) = \$0.17; \$85 \$0.17 = \$84.83 saved
- **4. a.** 3,200 gallon-sized bottles x 0.018 gallons of oil per bottle = 57.6 gallons of oil; 57.6 gallons of oil x \$1.50 = \$86.40
  - **b.** 3,200 bottles x 2 gallons per bottle = 6,400 gallons of water

5. 3,200 x 0.25 = 800 additional bottles x 0.018 gallons of oil = 14.4 additional gallons of oil; 14.4 additional gallons + 57.6 gallons = 72 total gallons x \$1.76 = \$126.72

### **FAMILY ACTIVITY 1: Subtracting Waste**

- 1. Total: 83.9; Boxes/Packaging 46.5%, Newspaper 14.4%, Books/Magazines 4.3%, Printer/Copier 16.6%, Other 18.2%
- 2. somewhat likely
- 3. a. 3.6 million tons  $x \frac{1}{3} = 1.2$  million tons recycled; 39 million tons  $x \frac{3}{5} = 23.4$  million tons recycled
  - **b.** 1.2 + 23.4 = 24.6 million tons recycled total
  - **c.** 24.6 million tons x 17 trees per ton of paper = 418.2 million trees
- 4. Answers will vary.

### **FAMILY ACTIVITY 2: Water**

Answers will vary depending on individual water usage.

### **FAMILY ACTIVITY 3: Energy Equations**

- 1. a. winter: 3,200; summer: 2,400
  - **b.** winter
  - c. Answers will vary, depending on individual usage.
- 2. a. 1,210 kilowatt-hours x \$0.15 = \$181.50
  - **b.** \$181.50 x 62% savings = \$112.53 saved
- **3. a.** Answers will vary.
  - **b.** Answers will vary.
  - **c.** 450 kilowatt-hours x \$0.15 = \$67.50 saved per item