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Process Inquiry: Analysis of Oral Problem-Solving Skills in Mathematics of Engineering Students*

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This paper presents another effort in determining the difficulty of engineering students in terms of solving word problems. Students were presented with word problems in algebra. Then, they were asked to solve the word problems orally; that is, before they presented their written solutions, they were required to explain how they understood the problem, and to give the processes they wanted to use in order to obtain the answer. Responses of students for each word problems would be noted. Discussions were recorded so that all responses were accounted for. Using NEA (Newman's error analysis), student's problems on reading, comprehension, transformation, and process skills can be determined by the teacher before the encoding of the solution is done. Also, the teacher directly addresses whatever misconceptions are made by the student in the process as well as of other students who are thinking the same way. More than 70% of the errors found were comprehension and transformation errors. Thus, students were given remedial classes to minimize their comprehension and transformation errors.

Keywords: engineering mathematics, oral problem-solving skills, difficulties

Introduction

From 1995 to 2011, the IEA (International Association for the Evaluation of Education Achievement) that has been conducting the TIMSS (Trends in Mathematics and Science Study) reported that Filipino students had poor performance in mathematics.

In fact, the 1999 and 2007 TIMSS reports provide very revealing results. The 1999 TIMSS reports indicated that Filipino students' performance in mathematics was 29% lower than the international mean (UP—NISMED (University of the Philippines—National Institute of Science and Mathematics Education), 2000). Almost 10 years later, in 2007 TIMMS advanced report, almost the same observation was noted when the Philippines ranked last among the 10 participating countries (2008 TIMSS advanced international report).

This condition of mathematics education in the country was clearly highlighted in the 2009 National Education Testing and Research Center of the Philippines report. Among other subjects, it pointed out that Filipino students have not reached mastery in mathematics.

For a country like the Philippines where mathematics has been recognized as a major factor in national

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development, these findings have posed several issues concerning the teaching and learning of mathematics in the country. Thus, national policy-makers and educators have expressed concerns how students learn mathematics.

One of the many areas explored to address this concern is to find out what misconceptions students have and where they take place in the process of solving a mathematical word problem. Through an oral examination with the use of NEA (Newman's error analysis), teachers would be able to identify what errors students commit, where they occur and what strategies teachers need in teaching mathematical word problems (Newman, 1977; 1983).

Literature Review

Lester and Kehle (2003, p. 510) defined problem-solving as an activity that involves the students' engagement in a variety of cognitive actions including accessing and using previous knowledge and experience.

Hence, it is understood that students who are looking for a solution to a given problem must think consistently with the contexts and content, among others. Non-routine problems are believed to have given opportunities for students to develop higher-order thinking in the process of understanding, exploration, and application of mathematical concepts (Polya, 1973).

Since it is a process, it uses different forms of knowledge that lead to the goal of solving the problem. According to Mayer (1982; 1987), such knowledge consists of the following: (1) linguistic and factual knowledge; (2) schema knowledge; (3) algorithmic knowledge; and (4) strategic knowledge.

This knowledge is necessary when a person attempts to answer a standard, written mathematics problem. Newman (1977; 1983) argued that any person wanting to arrive at the correct answer must go through a number of successive hurdles, such as reading, comprehension, transformation, process skills, and encoding.

A number of studies have indicated that difficulties in solving word problems lie not only in one stage but in two or more stages. For example, the study of Kaur (1995) found that: (1) the lack of comprehension of the problem; (2) the inability to translate the problem into mathematical form; and (3) the lack of strategy in solving the problem, were problem-solving difficulties that Singaporean students experienced.

Similarly, Marinas and Clements (1990), Ellerton and Clements (1996), and Singhatat (1991) have noted a large proportion of errors first occurred at the comprehension and transformation stages. In fact, they reported that approximately 70% of errors made by Grade-7 students on typical mathematical questions were at the comprehension or transformation levels.

Meanwhile, the same researchers found out that reading (decoding) errors accounted for less than 5% of initial errors and the same was true for process skills errors which were mostly associated with standard numerical operations (Ellerton & Clarkson, 1996).

However, there are those who have attributed problem-solving difficulties to the problem-solvers' cognitive and affective abilities. Schoenfeld (1985) suggested four aspects that affect one's performance, namely, mathematical knowledge, knowledge of heuristics, affective factors that affect the way the problem-solver views problem-solving, and managerial skills connected with selecting and carrying out appropriate strategies.

On the one hand, Lester (1994) expressed that difficulties experienced during problem-solving could also be caused by the problem-solvers' characteristics, such as traits, dispositions, and experiential background.

In fact, Lee (2001) enumerated these difficulties, such as: (1) the lack of experience in defining problems, a tendency to rush toward a solution before the problem has been clearly defined; (2) the tendency to think convergently; and (3) the lack of specific domain-specific knowledge.

Kaur and Yap (1996; 1997; 1998) likewise believed that students did not lack any effort in attempting the problems, but they appeared to lack motivation and confidence in attempting unfamiliar problems.

For McGinn and Boote (2003), there were four primary factors that affected perceptions of problem difficulty: categorization, goal interpretation, resource relevance, and complexity. They suggested that the level of difficulty of the problem depended on problem-solvers' perceptions of whether they had suitably categorized the situation, interpreted the intended goal, identified the relevant resources, and executed adequate operations to lead toward a solution.

As one of the many implications of these studies, questions have been raised on whether too much emphasis is placed in schools on basic arithmetic skills, and not enough on the peculiarities of the language of mathematics and on the characteristics of the learners (Clements & Ellerton, 1992).

Paying attention to this concern, one established method employed in identifying students' errors and the stage they occur is the NEA. Prackitipong and Nakamura (2006) have explored the procedure proposed by Newman (1983) and noted that it can be conducted through oral interviews or examinations.

Oral retelling has long been recognized as a strategy for improving students' comprehension. However, a few used it to address mathematical misconceptions (Gambrell, Kapinus, & Koskinen, 1991).

Through oral retelling, students would be able to share their experiences and thoughts in solving word problems in mathematics without being forced to answer them under time pressure. Hence, documenting these oral testimonies during word problem exercises greatly yields significant data on what their errors are and where they occur.

It must be emphasized that linguistic issues in children's understanding of problems is important (LeBlanc & Weber-Russell, 1996). Some even say there is a blend of linguistic skills and practical knowledge involved in solving word problems (Fuson, Caroll, & Landis, 1996).

Countryman (1992) offered an explicit explanation: Words are instruments that facilitate thought. But word problems can be difficult for children to comprehend, because the language of word problems is different from the language they use in describing their own problems and experiences.

For Askew (2003), language provides a vehicle for rich classroom discussions and assist teachers and students to appreciate the power of mathematics in making sense of their world. Thus, oral testimonies are said to get into what was, is, and will be the students' appreciation, understanding, and application of mathematics in their lives.

Methodology

Participants

One hundred fourteen students (70 men and 44 women; 68 fourth year students and 46 fifth year students) with B.S. (bachelor of science) in ECE (electronics engineering) from the College of Engineering and Architecture of WVCST (Western Visayas College of Science and Technology), Iloilo City, Philippines participated in the study. They were enrolled in ECE 130 and ECE 330, both subjects have a descriptive title "Review of Mathematics and Allied Subjects", under the same professor. All speak the Hiligaynon language but had taken and studied Filipino and English languages. Some of them can also speak other dialects, such as "Kinaray-a" and "Sina", both are native dialects of Panay Island where Iloilo City is located.

Materials

This study used non-routine word problems in Algebra. They were categorized as “easy”, “moderate”, and “difficult” questions based on the number of steps a participant has to perform in answering a particular question. A total of 10 word problems consisting of four, three, and three “easy”, “moderate”, and “difficult” questions, respectively were given to the participants through a PowerPoint presentation. Algebra topics included were: Prime Factorization, Digit-Number Problem, Remainder Theorem, Theory of Equations, Rate-Time-Distance Problem, Inequalities and Order Axiom, Mixtures, Fractions, Ratio and Proportion, Sum, Square, and Cube of a Binomial, Combinatorics, Arrangements, Factoring, Logarithm, Laws of Exponents, and other basic concepts. Table 1 presents the topic included in each word problem raised and its corresponding category.

Table 1

Topics Included in Every Question

Question number	Topic	Category
Q1	Prime factorization	Easy
Q2	Digit-number problem	Easy
Q3	Remainder theorem	Easy
Q4	Theory of equations	Easy
Q5	Rate-time-distance problem	Moderate
Q6	Inequalities and order axiom	Moderate
Q7	Mixtures, fraction, ratio, and proportion	Moderate
Q8	Sum, square, and cube of a binomial	Difficult
Q9	Combinatorics and arrangements	Difficult
Q10	Factoring, logarithm, and laws of exponents	Difficult

Instructions for the Randomly Selected Students

The instructions for the randomly selected students are as follows:

(1) Kindly read each question clearly;

(2) Briefly explain (orally) what the problem is all about;

(3) Also, explain how you would obtain the answer(s) by mentioning the processes you want to use;

(4) You can likewise mention specific formulas, methods, concepts or principles as well as assumptions you need to establish in order to support your processes/procedures;

(5) After you are done with the questions, you are required to pass the encoded solutions.

Procedures

Just before the period, participants were asked not to get any notes, books, pens, and papers nor write in the air while the experiment was going on. Also, directions on what to do during the period were cleared and discussed with the participants. Since the participants came from two year levels, the projected questions were shown in two periods. The first period was shown on Monday for the fifth year students during their class in ECE 130 and the second period was shown the following day for the fourth year students during their class in ECE 330. Students in the first period were told not to share whatever they have seen in their period in order to have secrecy in the questions.

Also, before the period starts, students were in groups. Students’ previous performances in quizzes were the basis for grouping. Random numbers were generated using the MS (Microsoft) Excel based on the number of students in that period. Some students in the lower group were then chosen using these random numbers.

They were separated from the class and accommodated in another room to answer the same questions individually, one at a time.

Using an LCD (liquid crystal display) projector, each question in all categories was flashed one after the other. With each question, students were asked to solve the word problem orally. Any student may volunteer to read the question and then answer it orally. He/she will tell the class if he/she understands the problem or not. If he/she understands the problem, he/she will retell what the problem is all about, what is asked, as well as any related concepts and principles that may be used in solving the problem. He/she may propose a solution to the problem, saying the translation of the word problem into a mathematical expression or equation. Then, some manipulations may follow. If he/she does not understand the problem, he/she will tell what things, words, phrases, or statements does he/she find confusing. Other students may agree or disagree with his/her answers. The professor will then ask another student to give his/her thoughts and opinions regarding the word problem. After several attempts of the students, the professor will stop the discussion. The next problem will then be flashed and the same process will happen.

In the whole of the two periods, discussions made by the professor as well as responses of students were recorded both using manual listing and a video camera. An alternative camera (Webcam) was used, so that other behaviors inside the classroom will be accounted for.

After the presentation, each student was given a copy of all the questions asked in the presentation. They were also asked to make a written solution in separate sheets of paper and have to submit their papers with solutions before the end of the period.

Data gathered, such as the video of the discussions and responses as well as the manual listing were analyzed using NEA. Errors were classified into five: reading, comprehension, transformation, process skills, and encoding. A tally sheet was made by the researcher to make the classification and categorization easier to interpret. If a student made an error in comprehension in a problem, the researcher will then mark the comprehension part of that question. That means that the students’ first error starts in that part. Errors were then tabulated to show a more meaningful thought.

NEA

The Newman’s procedure was employed to classify and categorize students’ errors (Prackitipong & Nakamura, 2006). This procedure has many aspects which are described as following:

(1) Reading: Does the student know how to read the question?

(2) Comprehension: Does the student understand the question?

(3) Transformation: Can the student select appropriate mathematical representations, operations, and procedures?

(4) Process skills: Can the student perform mathematical calculations accurately?

(5) Encoding: Can the student represent the answers appropriately?

In this method, there are two kinds of difficulties that hinder students from reaching the correct answers in the process of problem-solving:

(1) Problems of fluency in linguistic and conceptual understanding corresponding to the level of simple reading and understanding meaning of problems;

(2) Problems in mathematical processing consisting of transformation, process skills, and encoding answers.

Results

A total of 132 errors were found in the students' oral responses to word problems after using the NEA. Table 2 summarizes the type of errors committed, its frequency, and percentage distribution. It shows that the most number of errors were on transformation, which accounted for almost 50% (47.69%) while the least number of errors were on reading with almost 4% (3.85%). One significant observation to note is that reading and encoding errors are still possible despite the fact that the participants were already in their fourth and fifth years of study.

Table 2

Composition of Errors

Error	Number of errors	Percentage (%)
Reading	5	3.85
Comprehension	32	24.62
Transformation	64	47.69
Process skills	24	18.46
Encoding	7	5.38
Total	132	100.00

Table 3 presents the percentage distribution of the type of error students committed in 10 word problems. It reveals that at least 45% of the errors students made in six out of the 10 given questions were on transformation: Q1—69.2%, Q2—46.2%, Q3—53.8%, Q4—76.9%, Q6—61.5%, and Q8—53.8%. Furthermore, it can be observed that the next prevalent type of error made was on comprehension. It accounted for at least 8% to 62% of errors committed in all word problems.

Table 3

Errors in Every Question Categorized Using NEA (%)

Error	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Reading	-	-	-	-	-	15.4	-	-	15.4	7.7
Comprehension	23.1	30.8	7.7	7.7	30.8	7.7	61.5	15.4	46.1	15.4
Transformation	69.2	46.2	53.8	76.9	38.5	61.5	30.8	53.8	23.1	23.1
Process skills	-	23.1	23.1	7.7	30.8	15.4	7.7	7.7	15.4	53.8
Encoding	7.7	-	15.4	7.7	-	-	-	23.1	-	-

However, one interesting result to highlight is that almost all types of errors could be made in all word problems given. That is, a student is likely to commit an error not due to the topic included but to the problem-solving process.

On the other hand, Table 4 presents the words, phrases, and statements that students found confusing in each word problem. It is important to mention that a number of students did not understand well these words or phrases in spite of having taken, studied, and passed English courses.

When asked about the topics or concepts and principles related to the questions as well as the methods, they were going to use to arrive at the solution, the participants responded the following: logarithms, long-division of polynomials, substitution, permutation, inequalities, trial-and-error, quadratic equations, linear functions, factoring, decimals, synthetic division, distributive property using laws of exponents, remainder theorem, prime factorization, positive integer, roots of polynomials, completing the squares, three equations—three unknowns, elimination method, and even specific formulas.

Table 4

Confusing Items in the Questions Presented

Question	Words, phrases, and statements
Q2	Five-digit positive integer
Q7	Mixing of colors to produce another color Parts of blue and green, parts of blue and violet
Q9	Over peg A is worth one point Peg If all three rings land on pegs If the rings will be released one at a time or at the same time

Equally important as the quantitative results are the observations recorded during the study period. The researcher noted that students who did not have comprehension skills read the particular question more than thrice before attempting to answer it. After that, they were observed to have repeated some words from the word problem until they gave up computing or giving the correct answer. Some would even laugh when asked about their answer.

Moreover, it was evident that some students preferred to answer the word problems by writing their solutions than explaining the process they applied to arrive at the correct answer. Perhaps, this was due to the fact that they were shy to answer when they were taped-recorded. In fact, students' responses on papers show lesser errors committed compared to the ones they made in a face-to-face interview.

Discussions

This study identified students' errors and the stage they occur in solving word problems using the NEA. This was carried out through the oral retelling method, which proved to be an indispensable tool in determining students' difficulties and experiences in word-problem-solving.

Among others, one significant research finding is the high proportion of errors made on the transformation stage that accounted for almost 50% of the total number of errors committed. In fact, at least 45% of students' errors in six out of the 10 given questions were on the transformation stage. Furthermore, it was noted that the second most prevalent type of error made was on comprehension stage that made up at least 8% to 62% of errors committed in all word problems.

This is in agreement with the findings of Marinas and Clements (1990), Singhatat (1991), and Ellerton and Clements (1996) who have noted a large proportion of errors first occurred at the comprehension and transformation stages. They likewise reported that approximately 70% of errors made by Grade-7 students on typical mathematical questions were at the comprehension or transformation levels.

Considering all variables, the high percentage of errors in comprehension and transformation levels suggests that students have considerable difficulty in understanding and developing appropriate mathematical representation of word problems.

Kaur (1995) echoed the same concern when he found out that Singaporean students experienced at least three problem-solving difficulties: (1) lack of comprehension of the problem; (2) inability to translate the problem into mathematical form; and (3) lack of strategy in solving the problem.

Such difficulties are very evident when students' errors were classified into categories by algebra topics. Students have not related well to basic topics taught at elementary level like prime factorization, inequalities, mixtures, fractions, ratio and proportion, and operations on binomials as shown in the 69% and 23% of errors made at the transformation and comprehension levels, respectively in Q1, Q6, Q7, and Q8.

Thus, basic to one’s performance in solving a word problem are mathematical knowledge (Schoenfeld, 1985), experiential background (Lester, 1994), and categorization (McGinn & Boote, 2003).

Another important finding to highlight is reading and encoding errors are still possible despite the fact that the participants were already in their fourth and fifth years of study. Although they made up only at most 10%, committing errors in reading and encoding stages is very crucial since they are at the beginning and end of the problem-solving process as Newman (1977; 1983) emphasized. Thus, these types of errors could have a trickle-down effect in obtaining the correct answer.

A closer look at the given questions revealed that reading errors were highly recorded in Q6, Q9, and Q10. That is, many students could not answer them simply, because they did not know how to read the words or mathematical symbols included in such questions. In fact, Table 4 enumerates words, phrases, and statements that students found confusing.

As to errors committed in encoding stage, they substantially made up at least 8% to at most 23% of the total errors made in Q1, Q3, Q4, and Q8. Topics included in these questions are prime factorization, remainder theorem, theory of equations, and binomial operations.

Clearly, language comprehension plays a very crucial role in the problem-solving process. As what LeBlanc and Weber-Russell (1996) pointed out, linguistic issues in children’s understanding of problems are important. Fuson, Caroll, and Landis (1996) even asserted that solving word problems involves blend of linguistic skills and practical knowledge.

It was also found out that most participants were able to identify the topics, concepts, and principles related to the questions raised as well as the solutions that they had to apply. However, they relatively failed to convert this knowledge when highly needed as shown in the high proportion of errors made on transformation and comprehension stages.

This failure may be due to the fact that problem-solvers’ cognitive and affective abilities affect their performance. Lester (1994) considered the problem-solvers’ characteristics like traits, dispositions, and experiential background as determinants of difficulties experience during problem-solving.

Lastly, students’ behavior during the study period deserves a closer examination. They were noted to have shown comprehension difficulties, impatient attitude towards problem-solving, and preference on written exams over oral ones. Literature cited shows that the lack of experience in defining problems, especially the tendency to rush toward a solution before the problem has been clearly defined, is among students’ difficulties in problem-solving (Lee, 2001).

On the other hand, attitude towards problem-solving is a factor to deal with as shown in the study of Kaur and Yap (1996; 1997; 1998) who underscored that students did not lack any effort in attempting the problems but they appeared to lack motivation and confidence in attempting unfamiliar problems.

To put it explicitly, McGinn and Boote (2003) suggested that the level of difficulty of the problem depended on problem-solvers’ perceptions of whether they had suitably categorized the situation, interpreted the intended goal, identified the relevant resources, and executed adequate operations to lead toward a solution.

Preference on written exams over oral ones might be due to the fact that oral retelling is new to the students. Assessment of mathematics performance and other related measures in the Philippines are usually done in written form. Thus, students subjected to oral exams are relatively shy and fail to answer every question posed spontaneously.

The use of NEA, through the conduct of oral retelling in addressing mathematical misconceptions, has gained worldwide significance. The present study has explored the possibilities on using oral retelling as a method. Evidently, students' errors were identified and the stage where they occurred was made known.

Recognized as a strategy for improving students' comprehension, oral retelling offers good ways of knowing the peculiarities of the language of mathematics and on the characteristics of the learners (Clements & Ellerton, 1992). Indeed, it is a method that utilizes language at its core, which provides a vehicle for rich classroom discussions and assists teachers and students to appreciate the power of mathematics in making sense of their world (Askew, 2003).

Conclusion

Mathematics problem-solving errors of engineering students reveal that misconceptions occur at different levels. As Newman (1983) suggested, it must be categorized into five levels, namely, reading, comprehension, transformation, process skills, and encoding. Results showed that most errors committed were at the comprehension and transformation levels. Perhaps, the English language adds to the difficulty of the word problems, even though the Philippines are known to be an English-speaking country. Countryman (1992) explained that words are instruments that facilitate thoughts. However, word problems can be difficult for children, because the language of word problems is different from the language they use in describing their own problems and experiences (Kliman & Richards, 1992).

With many students to be handled, professors must devise a mechanism not only to help students learn but to know exactly what their misconceptions are.

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