NATIONAL CENTER FOR EDUCATION STATISTICS
Institute of Education Sciences

The Nation's Report Card

# Algebra I and Geometry Curricula

## Results From the 2005 High School Transcript Mathematics Curriculum Study

NCES 2013-451

U.S. DEPARTMENT OF EDUCATION

## WHAT IS THE HIGH SCHOOL TRANSCRIPT STUDY?

The High School Transcript Study (HSTS) collects and analyzes transcripts from a representative sample of America's public and private high school graduates. The study is designed to inform the public about the types of courses that graduates take during high school, how many credits they earn, and their grade point averages (GPAs). The HSTS also explores the relationship between coursetaking patterns and student achievement, as measured by the National Assessment of Educational Progress (NAEP). High school transcript studies have been conducted periodically for nearly two decades, permitting the reporting of trends in coursetaking and GPA, as well as providing information about recent high school graduates. In addition to collecting transcripts, the HSTS collects student information such as gender, graduation status, race/ethnicity, and information about the schools studied.

## WHAT IS THE NATION'S REPORT CARD™?

The Nation's Report Card™ informs the public about the academic achievement of elementary and secondary students in the United States. Report cards communicate the findings of the National Assessment of Educational Progress (NAEP), a continuing and nationally representative measure of achievement in various subjects over time.

Since 1969, NAEP assessments have been conducted periodically in reading, mathematics, science, writing, U.S. history, civics, geography, and other subjects. NAEP collects and reports information on student performance at the national, state, and local levels, making the assessment an integral part of our nation's evaluation of the condition and progress of education. Only academic achievement data and related background information are collected. The privacy of individual students and their families is protected.

NAEP is a congressionally authorized project of the National Center for Education Statistics (NCES) within the Institute of Education Sciences of the U.S. Department of Education. The Commissioner of Education Statistics is responsible for carrying out the NAEP project. The National Assessment Governing Board oversees and sets policy for NAEP.


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# Executive Summary

The 2005 National Assessment of Educational Progress (NAEP) High School Transcript Study (HSTS) found that high school graduates in 2005 earned more mathematics credits, took higher level mathematics courses, and obtained higher grades in mathematics courses than in 1990. The report also noted that these improvements in students' academic records were not reflected in twelfth-grade NAEP mathematics and science scores. Why are improvements in student coursetaking not reflected in academic performance, such as higher NAEP scores?

The Mathematics Curriculum Study (MCS) explored the relationship between coursetaking and achievement by examining the content and challenge of two mathematics courses taught in the nation's public high schools—algebra I and geometry. Conducted in conjunction with the 2005 NAEP HSTS, the study used textbooks as an indirect measure of what was taught in classrooms, but not how it was taught. In other words, the textbook information is not used to measure classroom instruction. Textbooks served as an indicator of the intended course curriculum (Schmidt, McKnight, and Raizen 1997). The chapter review questions in each textbook were used to identify the mathematics topics covered (or subject matter content) and the complexity of the exercises (or degree of cognitive challenge). Chapter review questions, and not the entire textbook, were coded because the questions have been found to be representative of the chapter content and complexity level in previous studies (Schmidt 2012). The study uses curriculum topics to describe the content of the mathematics courses and course levels to denote the content and complexity of the courses. The results are based on analyses of the curriculum topics and course levels developed from the textbook information, coursetaking data from the 2005 NAEP HSTS, and performance data from the twelfth-grade 2005 NAEP mathematics assessment. The study addresses three broad research questions:

1. What differences exist within the curricula of algebra I and geometry courses?
2. How accurately do school course titles and descriptions reflect the rigor of what is taught in algebra I and geometry courses compared to textbook content?
3. How do the curricula of algebra I and geometry courses relate to subsequent mathematics coursetaking patterns and NAEP performance?

In this report, curriculum topics, course levels, and grade 12 NAEP mathematics scale scores are used to describe the findings of the study. Curriculum topics are based on summaries of the textbook content that a school reported covering in an algebra I or geometry course. The six broad categories of curriculum topics used to describe the mathematics content found in both algebra I and geometry textbooks are: elementary and middle school mathematics, introductory algebra, advanced algebra, two-dimensional geometry, advanced geometry, and other high school mathematics. **Table A** lists the content found within these curriculum topics.

**TABLE A.** Defining curriculum topics

| | |
|---|---|
| Elementary and middle school mathematics | Basic arithmetic and pre-geometry |
| Introductory algebra | Pre-algebra, basic algebraic equations, and basic number theory |
| Advanced algebra | Advanced equations, basic functions, advanced functions, and advanced number theory |
| Two-dimensional geometry | Basic geometric concepts and properties of shapes |
| Advanced geometry | Three-dimensional geometry, coordinate geometry, and vector geometry |
| Other high school mathematics topics | Trigonometry, pre-calculus, statistics, validation and structuring, discrete mathematics, finite mathematics, and calculus |

NOTE: Curriculum topics in this report are defined as the mathematics topics found in textbooks used in algebra I or geometry courses in high schools.

Course levels are rankings of courses that high school graduates took based on the combination of content and challenge of each course, as determined by the textbooks used. Courses were assigned only one course level. These rankings were developed separately for algebra I and geometry courses. For both courses, the three levels are beginner, intermediate, and rigorous (**table B**).

**TABLE B.** Defining course levels

| | |
|---|---|
| Beginner | Covers more introductory material and less advanced material than an intermediate course. |
| Intermediate | Contains a balanced mix of both introductory and advanced material. |
| Rigorous | Covers more advanced material and less introductory material than an intermediate course. |

NOTE: Course levels are used to describe the rank of high school algebra I and geometry courses, based on the textbooks they used. The rankings are based on the curriculum topics covered and the level of challenge posed to the students.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, High School Transcript Study (HSTS), Mathematics Curriculum Study, 2005.

Results presented in this report are based on the 550 public schools and around 17,800 high school graduates selected for this study. This sample represents approximately two million public high school graduates from across the nation in 2005. Only high school graduates earning a regular or honors diploma are included in the analysis of this report, as is consistent with the reporting of the 2005 NAEP HSTS results. In addition, only graduates who took algebra I or geometry as high school courses were included in the study results. In 2005, 78 percent of all graduates took algebra I during high school and 20 percent of graduates took algebra I before entering high school. About 83 percent of all graduates took geometry during high school and 1.5 percent of graduates took geometry before entering high school.

The NAEP twelfth-grade mathematics results are reported as average scores on a scale of 0 to 300. The algebra and geometry scores are presented in the report to reflect performance on algebra I and geometry content, as opposed to overall mathematics performance. The MCS reports results using National Center for Education Statistics (NCES) statistical standards; findings from *t*-tests are reported based on a statistical significance level set at .05 without adjustments for multiple comparisons.

A few studies have analyzed textbook information and usage as a means to explain the apparent disconnect between coursetaking and achievement (Cogan, Schmidt, and Wiley 2001; Schiller et al. 2010; Tornroos 2005). The MCS adapted and built upon the methodology of these prior studies. See the Technical Notes of this report for a detailed description of the study methodology. While this study examined curriculum topics and course level of an algebra I or geometry course, it did not measure how well the curriculum was implemented in the classroom. In addition, only those graduates who took algebra I and geometry while in high school were included in the analyses. Therefore, students who took algebra I or geometry before entering high school were not included in the respective analyses because the textbook information was not collected. This limitation may be evident in the algebra I results, as those graduates who took the course in middle school were not included in the study results. Results from this study cannot be used to establish cause-and-effect relationships between mathematics textbooks and student mathematics coursetaking and performance.

## Core content made up about two-thirds of graduates' algebra I and geometry courses.

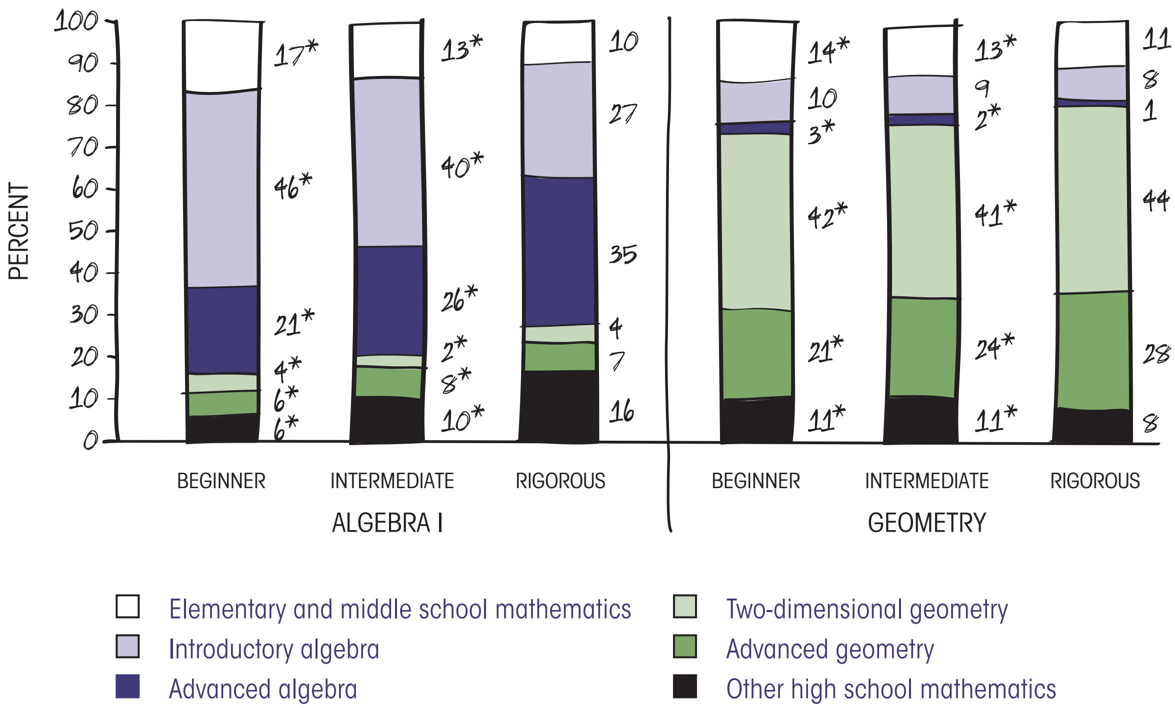
- In algebra I courses taken by high school graduates, about 65 percent of the material covered, on average, was devoted to algebra topics. About 35 percent of the material focused on elementary and middle school mathematics, geometry, and other high school mathematics topics typically taught in later mathematics courses.

- On average, about 66 percent of the material covered in geometry courses taken by high school graduates focused on geometry topics. About 34 percent covered elementary and middle school mathematics, algebra, and other high school mathematics topics.

## Graduates' courses varied widely in the mathematics topics covered.

- About 17 percent of the course content of graduates' beginner algebra I courses focused on elementary and middle school mathematics topics, compared to 10 percent for graduates who took rigorous algebra I courses (**figure A**).
- For graduates who took rigorous algebra I courses, about 16 percent of the course content was other high school mathematics topics that are generally taught in higher-level courses, compared to 6 percent for graduates in beginner algebra I courses.
- About 14 percent of the course content of graduates' beginner geometry courses covered elementary and middle school topics, compared to 11 percent for graduates who took rigorous geometry courses.

**FIGURE A.** Percentage of content in graduates' algebra I and geometry courses, by course level and curriculum topic group: 2005




* Significantly different ($p < .05$) from rigorous.

NOTE: Details may not sum to total because of rounding.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, High School Transcript Study (HSTS), Mathematics Curriculum Study, 2005.

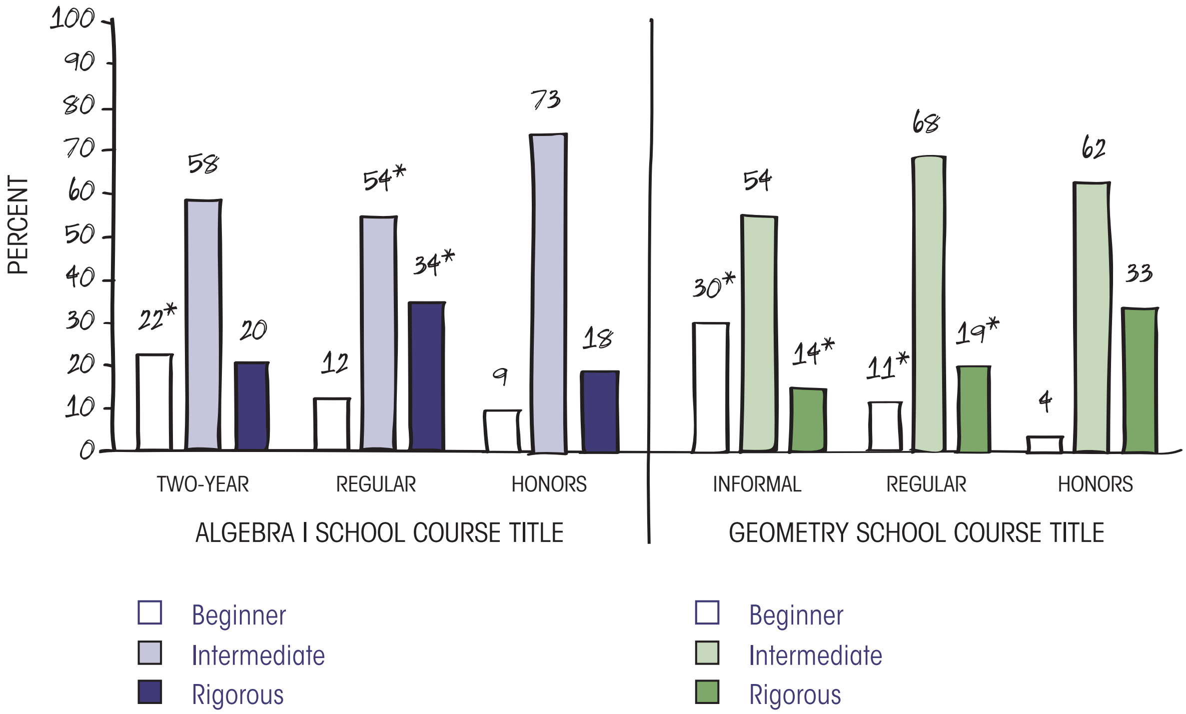
- For graduates who took rigorous geometry courses, 8 percent of their course content was other high school mathematics topics that are generally taught in higher level courses, compared to 11 percent for graduates who took beginner geometry courses.

## School course titles often overstated course content and challenge.

- Approximately 73 percent of graduates who took an algebra I class labeled "honors" by their school received a curriculum ranked as an intermediate algebra I course (**figure B**).

- A higher percentage of graduates who took an algebra I class labeled "regular" by their school (34 percent) received a curriculum ranked as a rigorous algebra I course, compared to graduates who took an algebra I class labeled "honors" by their school (18 percent).


**FIGURE B.** Percentage of graduates in algebra I and geometry course levels, by school course title and course level: 2005




* Significantly different (p < .05) from honors.

NOTE: Details may not sum to total because of rounding and the use of integrated mathematics textbooks in nonintegrated mathematics courses. "Two-year" algebra I is a course that is completed in two years. "Informal" geometry is a course that does not emphasize proofs.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, High School Transcript Study (HSTS), Mathematics Curriculum Study, 2005.

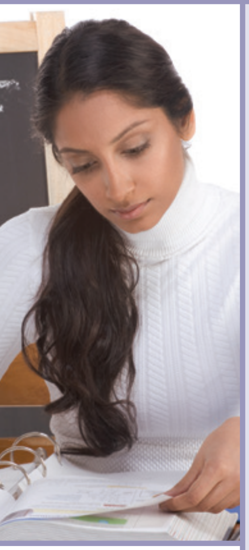
- Of the graduates who took a geometry course labeled "honors" by their school, approximately 33 percent received a curriculum ranked as rigorous geometry, whereas 62 percent received a curriculum ranked as intermediate geometry.

## Few racial/ethnic differences by course level were found among subgroups who took similarly titled courses.

- Of the graduates who completed "two-year" algebra I courses, about 37 percent of Hispanic graduates received a curriculum equivalent to a beginner algebra I course, compared to 19 percent each of White and Black graduates.

- Of the graduates who completed "honors" geometry courses, about 37 percent of White graduates received a curriculum equivalent to a rigorous geometry course, compared to 17 percent of Hispanic and 21 percent of Black graduates.

- No racial/ethnic differences by course level were found among graduates who took classes labeled as "honors" algebra I. There were no measurable differences at any course level among White, Black, and Hispanic graduates who took either "informal" or "regular" geometry.



## Fewer graduates who had beginner algebra I or geometry courses went on to complete advanced mathematics courses.

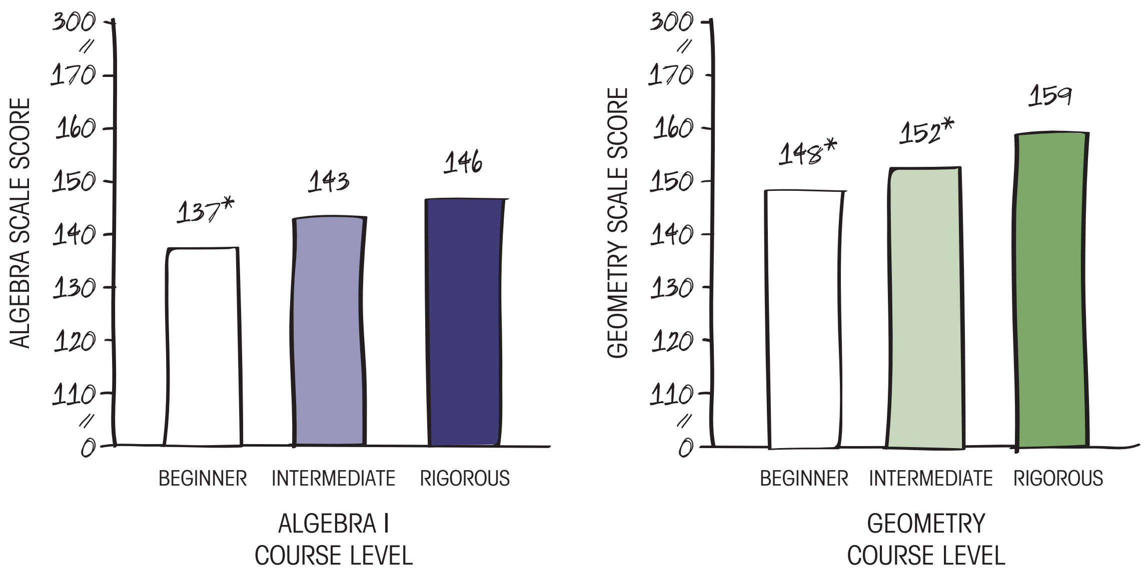
- About 60 percent of graduates who completed beginner algebra I courses went on to complete an algebra II course or higher as their highest level mathematics course, less than the 74 percent of graduates who had intermediate high school algebra I courses and 79 percent of graduates who had rigorous high school algebra I courses.

- Of the graduates who had a rigorous geometry course, about 50 percent took an advanced mathematics or calculus course as their highest mathematics course, comparatively higher than the 38 percent of graduates who had a beginner geometry course or the 42 percent who had an intermediate geometry course.

## Graduates in rigorous algebra I and geometry courses scored higher on NAEP.

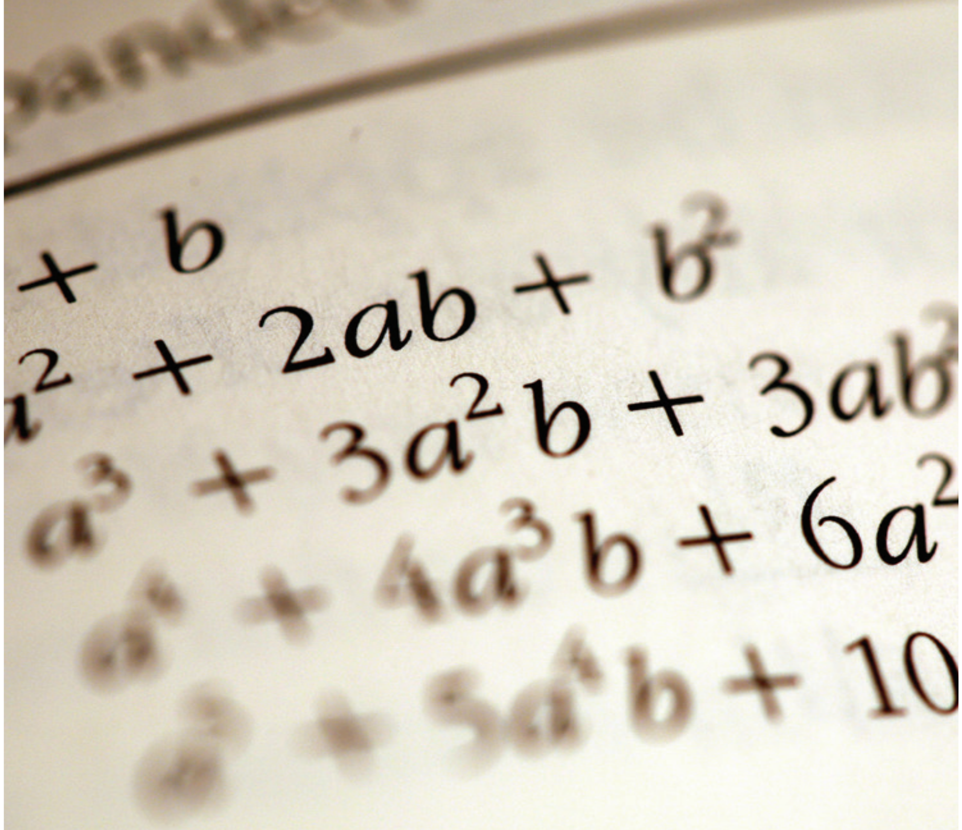
- Graduates who took rigorous algebra I courses obtained higher NAEP algebra scores (146) than graduates who took beginner algebra I courses (137) (**figure C**).
- Graduates who took rigorous geometry courses obtained higher NAEP geometry scores (159) than graduates who took beginner (148) or intermediate (152) courses.

**FIGURE C.** Average NAEP mathematics score of graduates in algebra I and geometry, by course level: 2005




* Significantly different ($p < .05$) from rigorous.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress, High School Transcript Study (HSTS), Mathematics Curriculum Study, 2005.



# Introduction and Overview

The relationship between student coursetaking and academic performance has long been established. There is evidence that students who take advanced courses perform better academically than those students who do not take advanced courses (Shettle et al. 2007; Grigg, Donahue, and Dion 2007). Therefore, many reform efforts have focused on increasing the number of course credits required for high school graduation, including mathematics credits (Medrich et al. 1992; Chaney, Burgdorf, and Atash 1997; Stevenson and Schiller 1999). Results from the 2005 National Assessment of Educational Progress (NAEP) High School Transcript Study (HSTS) report (Shettle et al. 2007) found that 2005 high school graduates earned more credits, took a range of higher level courses, and earned higher grade point averages in mathematics than graduates in 1990.

The average number of credits in mathematics earned by 2005 graduates (3.8) was significantly higher than the average number of credits earned by graduates in 1990 (3.2). Graduates in 2005 earned a higher grade point average in mathematics courses (2.63) than graduates in 1990 (2.34). In addition, a higher percentage of graduates in 2005 than in 1990 completed a rigorous curriculum level. The rigorous curriculum level is used to report HSTS results (Shettle et al. 2007) and requires a graduate to take more advanced mathematics courses such as pre-calculus and calculus, advanced science courses, and more foreign language courses. Curriculum levels are based on the number of credits earned and the types of courses taken by graduates. Curriculum levels differ from the course levels discussed in this report.

The report also highlighted a lack of congruence between the HSTS and the NAEP. Improvements in student coursetaking that were shown in the 2005 NAEP HSTS report were not reflected in NAEP score trends (Shettle et al. 2007, p. 34).

For example, there was no measurable difference in the percentage of White and Black graduates who completed at least a midlevel curriculum in 2005. One of the requirements for achieving a midlevel curriculum is the completion of at least three years of mathematics courses, which include both algebra and geometry. The six-point percentage gap in 1990 between White and Black graduates completing at least a midlevel curriculum closed in 2005. However, performance gaps on the NAEP mathematics assessment remained (Shettle et al. 2007).

There are several plausible explanations for the lack of relationship between changes in high school coursetaking and NAEP score trends. The following are a few factors that might mitigate this relationship: changes in the population of students tested; declines in twelfth-graders motivation to do well on NAEP, a low-stakes assessment; and differences in course content (Shettle et al. 2007). Given all of these possible explanations, more in-depth analyses of these data are needed to understand the trends in student performance. The current study examines mathematics course content to further understand this relationship.

## THE MATHEMATICS CURRICULUM STUDY

The Mathematics Curriculum Study (MCS) explores the relationship between student coursetaking and achievement by investigating the content and challenge of two core high school mathematics courses—algebra I and geometry. The study was conducted in conjunction with the 2005 NAEP HSTS.

### Sample

The MCS brings together information from three sources—students, schools, and textbooks—to provide a more in-depth look at high school graduates' mathematics courses. During the 2005 NAEP HSTS data collection, 550 public schools provided textbook data for the study. The student sample included 17,800 graduates, which is representative of about 2 million graduates from across the nation. The analyses are limited to only those high school graduates who earned a regular or honors diploma, and completed an algebra I or geometry course during high school. In 2005, 78 percent of all graduates took algebra I during high school and 20 percent of graduates took algebra I before entering high school (see **table 1** on page 15). About 83 percent of all graduates took a geometry course during high school and 1.5 percent of all graduates took a geometry course before entering high school.

### Methodology

Information from about 120 algebra I, geometry, and integrated mathematics textbooks was collected and coded for the study. Only those graduates who took an algebra I and/or geometry course that was linked to a textbook were included in the study analyses. Incorporating the textbook data with the transcript data, student and school demographic characteristics, and the NAEP mathematics assessment data allows for a comprehensive analysis of mathematics course-taking and achievement.

The study's analyses are limited to textbook data linked to algebra I and geometry courses taken by public high school graduates. Public schools that did not offer algebra I and/or geometry courses, or comparable courses such as integrated mathematics, were not included in the study since none of the students in these schools could be connected to a textbook. Graduates who took algebra I during high school were included in the algebra analysis, while graduates who took geometry during high school were included in the geometry analysis.

The inclusion criteria for courses completed are independent of each other. For example, a graduate who took algebra I in eighth grade and geometry in ninth grade would be included in the geometry analyses, but not the algebra I analyses.

## Textbook Coding

In this study, textbooks serve as indicators of the intended course curriculum as defined by Schmidt et al. (1997). The content of the textbook was used as an indirect measure of what was taught in classrooms (Tornroos 2005) because classroom instruction could not be measured in this study. That is, textbooks indicate the mathematics topics and types of skills a student will be exposed to in a course. Because textbooks are the main source of instructional material, they are used to measure what is taught in a course. About 120 textbooks were collected and analyzed for this study. The chapter review questions in each collected textbook, and not the entire textbook, were coded to determine two curriculum measures—the mathematics topic content and the level of cognitive challenge. The chapter review questions have been found to be representative of the chapter content and challenge level based on previous studies by Schmidt (2012). Both content and challenge were used in classifying graduates' classes into course levels. Content and challenge are not always directly related; that is, not all questions focused on low-level content have low degrees of challenge, and not all high-level content questions have high degrees of challenge. Coding textbooks at the chapter level allowed the study to distinguish between courses that covered the entire textbook and courses that only covered selected chapters from the textbook. Trained coders used a comprehensive framework of over 200 mathematics topics describing elementary and secondary education mathematics curriculum to identify the content covered in each textbook. (See **chart A1** in the Technical Notes for more details.) Information for chapters used in each course was aggregated by summing the mathematics topics covered and then connected to the graduates who took the courses.

The level of challenge for each textbook was determined by coding the chapter review questions, using about 25 major student performance expectations. Performance expectations are the activities or skills a student was expected to use to correctly answer a review question. The performance expectations for each chapter review question were ranked, and these ranks were averaged to create a level of cognitive challenge for the chapter. The overall cognitive challenge level for a course was aggregated by averaging the cognitive challenge level for the chapters covered within each textbook used in the course. These measures were then connected to the graduates who took the courses.

These two textbook curriculum measures were used to create two new measures that are used to describe the results of the study—curriculum topics and course levels. The curriculum data were analyzed along with coursetaking data from the 2005 NAEP HSTS and achievement data from the NAEP 2005 twelfth-grade mathematics assessment. For more detail on the textbook coding, refer to the 'Textbook Analyses' section of the Technical Notes.

## Purpose

The MCS used measures of curricular content and challenge to address the following research questions:

1. What differences exist within the curricula of algebra I and geometry courses?
2. How accurately do school course titles and descriptions reflect the rigor of what is taught in algebra I and geometry courses compared to textbook content?
3. How do the curricula of algebra I and geometry courses relate to subsequent mathematics coursetaking patterns and NAEP performance?

Only a few studies have taken the approach of looking at textbook information and usage as a means to explain the lack of congruence between coursetaking and achievement (Cogan, Schmidt, and Wiley 2001; Schiller et al. 2010; Tornroos 2005). These three studies were limited by the number of textbooks examined, the number of schools participating, or the measures of achievement. Therefore, the present study builds on the methodology of prior studies by using a large national sample and the NAEP mathematics assessment data to measure achievement.

# REPORTING THE RESULTS

In this report, curriculum topics, course levels, and NAEP mathematics scale scores are used to describe the findings of the study. The six categories of curriculum topics and three course levels referred to throughout this report were developed specifically for this study.

## Curriculum Topics

Curriculum topics refer to broad categories of mathematics content topics that are covered in algebra I and geometry courses. Mathematics content topics were grouped by using a hierarchical structure of the curriculum framework and the grade level in which topics are introduced. (See the Technical Notes for more information on how the topics are aggregated.) Six main categories of curriculum topics were developed based on the content identified by the coding of textbook chapter review questions, as described in the previous section. Each is used to describe the mathematics content found in both algebra and geometry textbooks. These categories are as follows:

**Elementary and middle school mathematics** includes mathematics topics that are traditionally taught before a student takes an algebra I course. These topics include elements of basic arithmetic (e.g., addition, subtraction, fractions, and rounding) and pre-geometry (e.g., patterns, perimeter, area, and proportion).

**Introductory algebra** includes mathematics topics needed to understand the basics of algebra and provide the foundation for learning advanced algebra. These topics include pre-algebra, basic algebraic equations (e.g., algebraic expression, simple linear equations, and simple inequalities), and the basic elements of number theory (e.g., integers, absolute value, and rational numbers).

**Advanced algebra** includes mathematics topics that cover the more complex elements of algebra. These topics include advanced equations (e.g., quadratic equations, polynomial equations, and matrix solutions), basic functions (e.g., representation of relationships and functions, and graphing functions), advanced functions (e.g., functions of several variables and quadratic functions), and advanced number theory (e.g., real numbers, exponents, roots, radicals, and matrices).

**Two-dimensional geometry** includes mathematics topics that focus on basic linear and planar geometric concepts. Examples of topics in this category include basic geometric concepts (e.g., points, angles, parallelism, and perpendicularity) and the properties of shapes.

**Advanced geometry** includes mathematics topics that cover advanced geometric concepts such as three-dimensional geometry (e.g., three-dimensional shapes, conic sections), coordinate geometry (e.g., equations of lines, planes, and surfaces in space), and vector geometry (e.g., vectors, transformation, congruence, and similarity).

**Other high school mathematics** includes mathematics topics that are traditionally taught in courses taken after geometry and algebra II. Examples of topics in this category include trigonometry, pre-calculus, statistics (e.g., data representation and analysis, uncertainty and probability), validation and structuring (e.g., logic, set theory, and axioms), discrete mathematics (e.g., tree diagrams and binary arithmetic), finite mathematics, and calculus.

## Course Levels

Course levels are rankings of students' algebra I and geometry coursework. They are based on both the curriculum topics covered and the level of challenge of high school graduates' courses, as determined by the content of their textbooks. Performance expectations were used to determine the degree of challenge.

Algebra I and geometry courses were grouped into three course levels—beginner, intermediate, and rigorous. Courses were assigned only one course level. While the rigorous course level is the highest level, it is not intended to denote an advanced course. The term "rigorous" is used to differentiate the course level from courses schools label "advanced." These levels are as follows:

**Beginner level courses** cover more introductory material and less advanced material than intermediate level courses.

**Intermediate level courses** contain a balance of both introductory and advanced material.

**Rigorous level courses** cover more advanced material and less introductory material than intermediate level courses.

Graduates who took integrated mathematics courses were not assigned a course level, but to a separate integrated mathematics category.

## NAEP Scale Scores

The HSTS is conducted in conjunction with the NAEP. Therefore, the coursetaking patterns of the graduates can be examined relative to their educational achievement as measured by NAEP. Instead of looking at the overall mathematics scores, however, this study uses the content area scale scores—also called "subscale scores"—that focus on algebra and geometry as the achievement measure. These subscale scores correlate highly with the overall mathematics scores and are closely associated with the content taught in algebra I and geometry courses (http://nces.ed.gov/nationsreportcard/tdw/analysis/2004_2005/scaling_determination_correlations_math2005-conditional.asp). The 2005 NAEP twelfth-grade mathematics results—both overall and subscale scores—are reported as average scores on a scale of 0-300.

## INTERPRETING THE RESULTS

The MCS presents subgroup comparisons. NCES uses widely accepted statistical standards when reporting results; findings from $t$-tests are reported based on statistical significance level set at .05 without adjustments for multiple comparisons (see the Technical Notes for more information). The symbol (*) is used in tables and figures to indicate that the percentage or performance of one group is significantly different from another group. Only those differences that are found to be statistically significant are discussed as higher or lower.

When scores are significantly different, then student performance is different. However, the MCS was not designed to identify the causes of these differences. More information about interpreting statistical significance can be found in the Technical Notes.

Although comparisons are made in students' performance based on demographic characteristics, the results cannot be used to establish cause-and-effect relationships between student characteristics and achievement. Many factors may influence student achievement, including educational policies and practices, available resources, and demographic characteristics of the student body.

# High school graduates took algebra I courses before and during high school

It is important to keep in mind that the study analyses are limited to high school graduates who took algebra I or geometry courses while they were in high school. That is, only graduates who took algebra I during high school were included in the analysis sample for algebra I, just as only graduates who took geometry during high school were included in the analysis for geometry. Textbook information for courses taken before entering high school was not collected as a part of this study. Therefore, the lack of information on the content and challenge of algebra I courses taken before high school may impact the algebra I findings of this report. There are differences in the academic profiles of high school graduates who took algebra I before and during high school. **Table 1** below compares the academic and demographic characteristics of these two student groups. In 2005, one in five high school graduates had completed an algebra I course before entering high school. These graduates earned more total course credits, higher GPAs, and a higher overall score on the NAEP mathematics assessment than students who took algebra I in high school.

**TABLE 1.** Profiles of graduates who took algebra I before and during high school: 2005

| | Before high school | During high school |
|---|---|---|
| Percent of all graduates | 20 | 78 |
| Student race/ethnicity | | |
| Percent of White graduates | 23 | 74 |
| Percent of Black graduates | 8 | 89 |
| Percent of Hispanic graduates | 10 | 87 |
| Percent of Asian/Pacific Islander graduates | 30 | 69 |
| Coursetaking and performance | | |
| Average total course credits earned | 28.0* | 26.6 |
| Average credits earned in mathematics courses | 4.2* | 3.7 |
| Average overall GPA | 3.43* | 2.88 |
| Average GPA in mathematics courses | 3.10* | 2.52 |
| Average overall NAEP mathematics score | 182* | 145 |

* Significantly different ($p < .05$) from graduates who took algebra I during high school.

NOTE: Data for graduates who did not take an algebra I course are not shown. Black includes African American, Hispanic includes Latino, and Asian/Pacific Islander includes Native Hawaiian. Race categories exclude Hispanic origin.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress, High School Transcript Study (HSTS), Mathematics Curriculum Study, 2005.

# Understanding textbook coding

The four examples that follow illustrate how chapter review questions were coded to create the curriculum measures—curriculum topics and content levels—used to describe findings in this report. Chapter review questions from algebra I and geometry textbooks were coded for mathematics content and performance expectations, the latter measure being used to develop a degree of challenge. Both content and challenge were used in classifying graduates' classes into course levels. Content and challenge are not always directly related; that is, not all questions focused on low-level content have low degrees of challenge, and not all high-level content questions have high degrees of challenge. For detailed information on the curriculum topics and performance expectation coding, see the "Textbook Analyses" subsection on page 44 of the Technical Notes.

## ALGEBRA I: ILLUSTRATIVE QUESTIONS

### Question 1

### Introductory algebra content with high degree of challenge

Curriculum topic: Basic algebraic equations.

Performance expectation: Communicating mathematical ideas and problem solving.

Question 1: Identify and correct any errors in the solution shown below.

Solve: 2x–(5x–2) = 4

Solution: 2x–(5x–2) = 4
-3x–2 = 4
-3x = 6
x = -2

Answer: An error occurs in the first step of the solution because the negative sign before the parentheses is not evenly distributed. After removing the parentheses, the first step should read -3x + 2 = 4. The next step should be -3x = 2. The answer is x = -2/3.

### Question 2

### Advanced algebra content with low degree of challenge

Curriculum topic: Advanced algebraic equations.

Performance expectation: Using algebraic procedures to manipulate formulas.

Question 2: At what rate would you have to invest to double your money in 20 years?

Compound interest formula:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Answer: 3.47 percent.

## GEOMETRY: ILLUSTRATIVE QUESTIONS

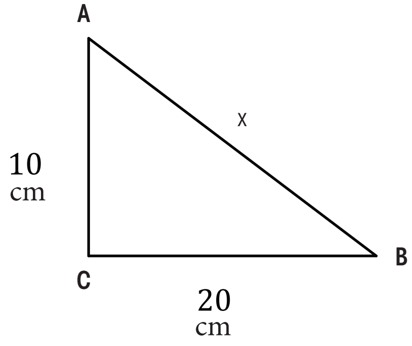
### Question 3

### Two-dimensional geometry content with low degree of challenge

**Curriculum topic:** Pythagorean Theorem.

**Performance expectation:** Recalling the Pythagorean Theorem and computation.

**Question 3:** In right triangle ABC, with the right angle at C, find x to the nearest tenth decimal place.




**Answer:** 22.4 cm

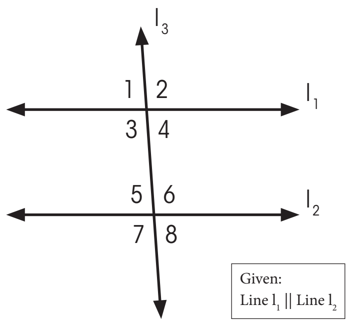
### Question 4

### Two-dimensional geometry content with high degree of challenge

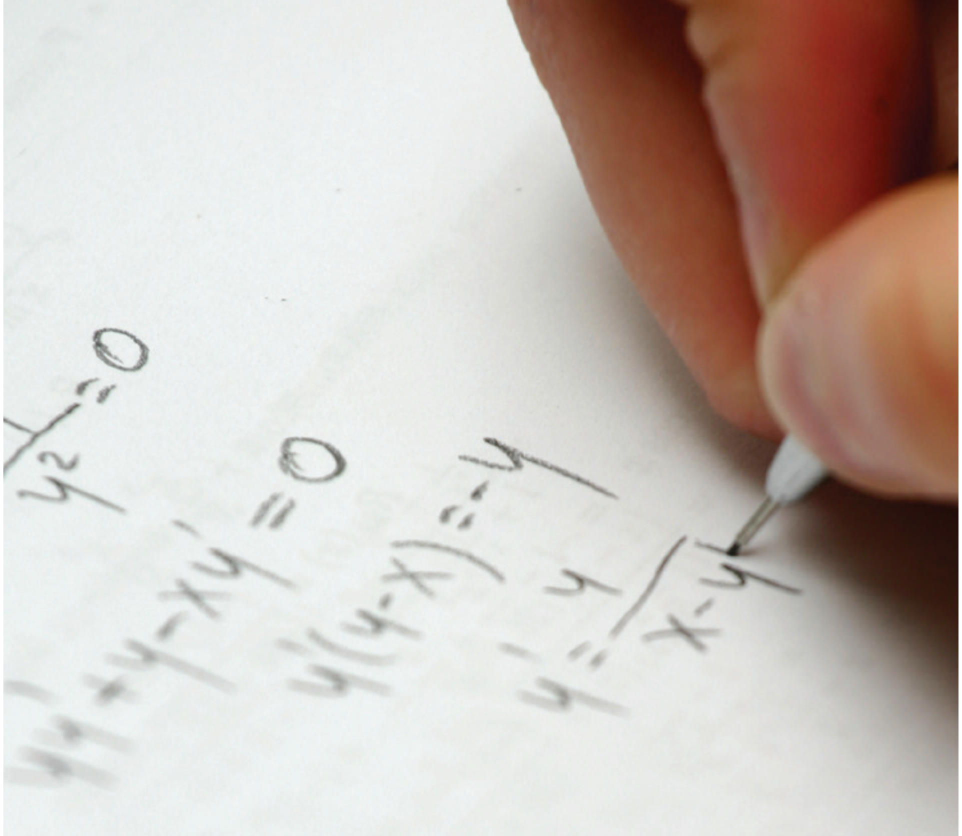
**Curriculum topic:** Angles and parallelism.

**Performance expectation:** Investigating and problem solving.

**Question 4:** Using the diagram below, if $m\angle 1 = 2x+30$ and $m\angle 6 = 3x+10$, where m denotes the measurement of an angle, find the measure of each angle.




**Answer:** $m\angle 1 = m\angle 4 = m\angle 5 = m\angle 8 = 86^{o}$
$m\angle 2 = m\angle 3 = m\angle 6 = m\angle 7 = 94^{o}$



# Mathematics Course Profiles

A profile of high school algebra I and geometry courses using six curriculum topics and three course levels is presented in this section of the report. Two-thirds of the content of algebra I and geometry courses focused on curriculum topics principal to the course, algebra I and geometry, respectively. The remaining one-third covered different mathematics topics. Across the nation, there was wide variation in the mathematics topics covered in graduates' algebra I courses, in particular in the percentage of content that is devoted to elementary and middle school mathematics. When disaggregated by race/ethnicity and course level, few measurable differences were found. Higher percentages of Hispanic and Asian/Pacific Islander graduates took courses ranked as beginner algebra I courses compared to White graduates.

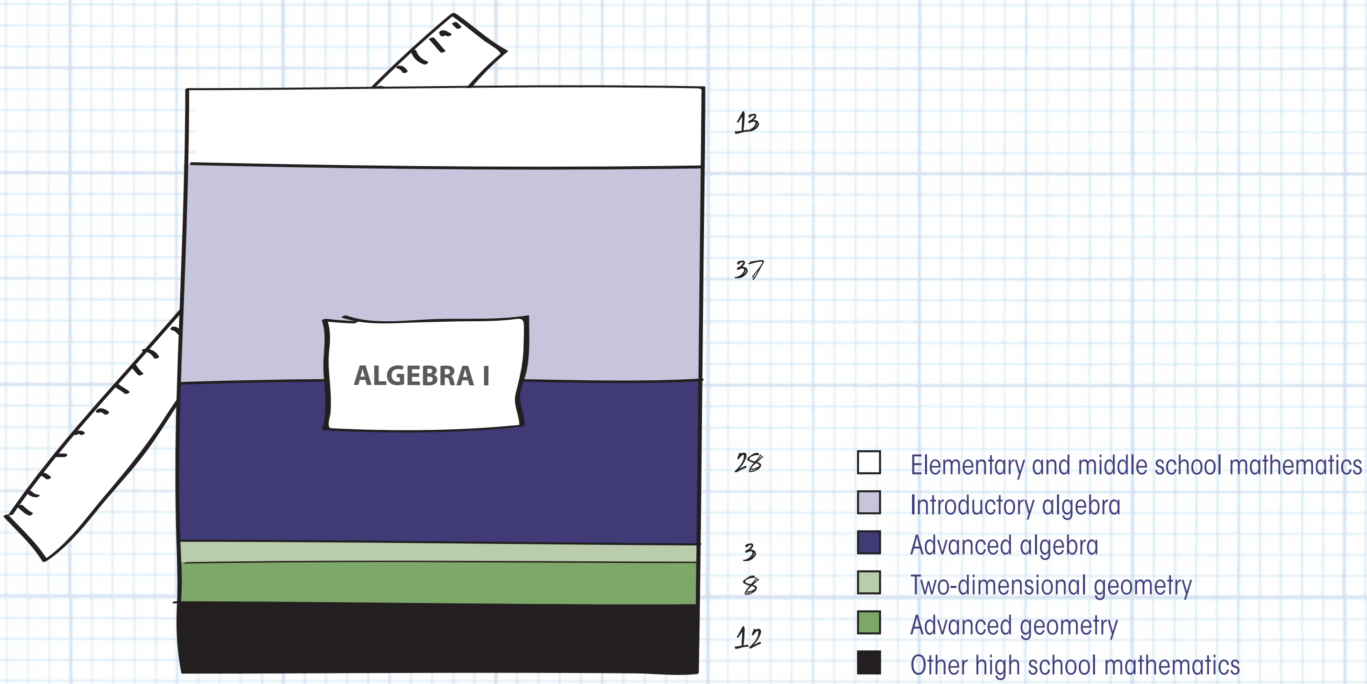
## ALGEBRA I

What is algebra I? In general, algebra I courses focus on using symbols to express numbers and mathematical operations in equations, and manipulating mathematical expressions to solve for inequalities. Courses also concentrate on using functions to describe situations where one quantity determines another, such as rates of growth and decline. The mathematical operations that students are expected to perform become increasingly complex over the duration of a course. The following results describe high school algebra I courses, based on the textbooks used in the courses, using curriculum topics and course levels. It is important to keep in mind that textbook information was used as an indirect measure of the topics to be taught in a course, but does not reflect classroom instruction.

### About two-thirds of an algebra I course consisted of algebra topics.

**Figure 1** depicts the average mathematics content of high school algebra I courses by curriculum topics (see pages 12 and 50 for details). On average, 65 percent of a graduate's high school algebra I

**FIGURE 1.** Percentage of content of graduates' algebra I courses, by curriculum topic group: 2005




NOTE: Details may not sum to total because of rounding.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, High School Transcript Study (HSTS), Mathematics Curriculum Study, 2005.

course focused on algebra topics, including 37 percent on introductory algebra topics (e.g., pre-algebra and basic equations) and 28 percent on advanced algebra topics (e.g., advanced equations, basic and advanced functions, and advanced number theory). The remaining one-third of the content covered in a graduate's course was elementary and middle school mathematics (13 percent), two-dimensional geometry (3 percent), advanced geometry (8 percent), and other high school mathematics topics (12 percent) that are generally the focus of courses taken later in high school, like trigonometry, pre-calculus, and statistics.

## Graduates who took rigorous courses had less review material than graduates who took beginner or intermediate courses.

**Table 2** shows the percentage of content in algebra I courses taken by high school graduates broken down by course level. Graduates' algebra I courses varied widely in the mathematics topics covered. While all levels of algebra I courses contained some review material on elementary or middle school mathematics, on average, the percentage of this content was lower in rigorous level courses. For example, high school graduates in beginner level algebra I courses had, on average, 17 percent of their content focused on elementary and middle

**TABLE 2.** Percentage of content of graduates' algebra I courses, by course level and curriculum topic group: 2005

| | Algebra I course levels | | | |
|---|---|---|---|---|
| Mathematics curriculum topic group | All levels | Beginner | Intermediate | Rigorous |
| Elementary and middle school mathematics | 13 | 17* | 13* | 10 |
| Introductory algebra | 37 | 46* | 40* | 27 |
| Pre-algebra | 9 | 18* | 9* | 7 |
| Basic equations | 27 | 28* | 31* | 21 |
| Advanced algebra | 28 | 21* | 26* | 35 |
| Advanced equations | 15 | 12* | 15 | 16 |
| Basic functions | 4 | 2* | 2* | 6 |
| Advanced functions | 2 | #* | 3* | 2 |
| Advanced number theory | 8 | 6* | 6* | 11 |
| Two-dimensional geometry | 3 | 4* | 2* | 4 |
| Advanced geometry | 8 | 6* | 8* | 7 |
| Other high school mathematics | 12 | 6* | 10* | 16 |

# Rounds to zero.

* Significantly different ($p < .05$) from rigorous.

NOTE: Details may not sum to total because of rounding. The categories that are indented are subcategories within the six broad curriculum topics: elementary and middle school mathematics, introductory algebra, advanced algebra, two-dimensional geometry, advanced geometry and other high school mathematics.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, High School Transcript Study (HSTS), Mathematics Curriculum Study, 2005.

school mathematics, compared to 10 percent of the content for graduates in rigorous courses. The percentage of introductory algebra content followed the same pattern. About 46 percent of the content graduates covered in beginner courses was introductory algebra, compared to 27 percent of content for graduates in rigorous courses.

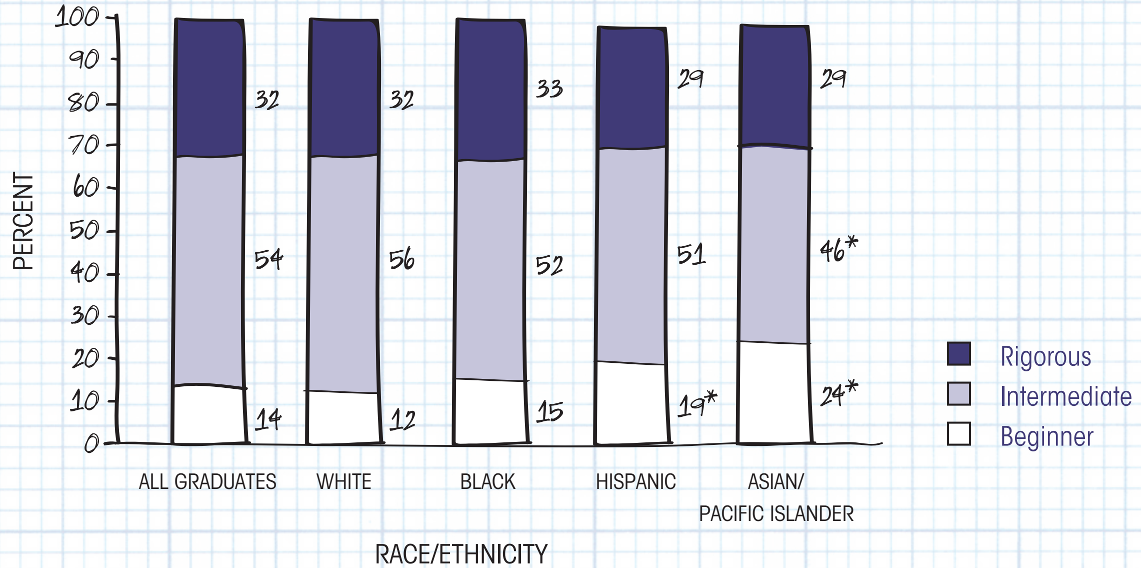
Conversely, the percentage of other high school mathematics topics introduced to high school graduates in algebra I courses was higher in intermediate and rigorous courses. Graduates in intermediate and rigorous courses received, on average, a larger percentage of content in other high school mathematics topics (10 and 16 percent, respectively) than graduates in beginner courses (6 percent). Graduates in rigorous courses had a larger percentage of advanced algebra topics (35 percent) than graduates in beginner courses (21 percent).

## Most graduates, regardless of race/ethnicity, took an intermediate level algebra I course.

The percentage of high school graduates who took algebra I courses, by course level, is shown in **figure 2** for all graduates and by race/ethnicity. More than one-half (54 percent) of all graduates took an intermediate algebra I course. Approximately 14 percent had a beginner course and 32 percent took a rigorous course.

When comparing across racial/ethnic subgroups, there were no measurable differences among White, Black, and Hispanic graduates who took intermediate and rigorous algebra I courses. However, some differences were seen in the percentage of graduates who took beginner courses. A larger percentage of Hispanic (19 percent) and Asian/Pacific Islander (24 percent) graduates took a beginner algebra I course, compared to White graduates (12 percent). It is important to keep in mind the differences in percentages of students who took algebra I before entering high school across race/ethnicity. For example, 30 percent of Asian/Pacific Islander graduates took algebra I before entering high school (see **table 1**).

**FIGURE 2.** Percentage of graduates who took algebra I courses, by student race/ethnicity and course level: 2005




* Significantly different ($p < .05$) from White graduates.

NOTE: Details may not sum to total because of rounding. Black includes African American, Hispanic includes Latino, and Asian/Pacific Islander includes Native Hawaiian. Race categories exclude Hispanic origin.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, High School Transcript Study (HSTS), Mathematics Curriculum Study, 2005.

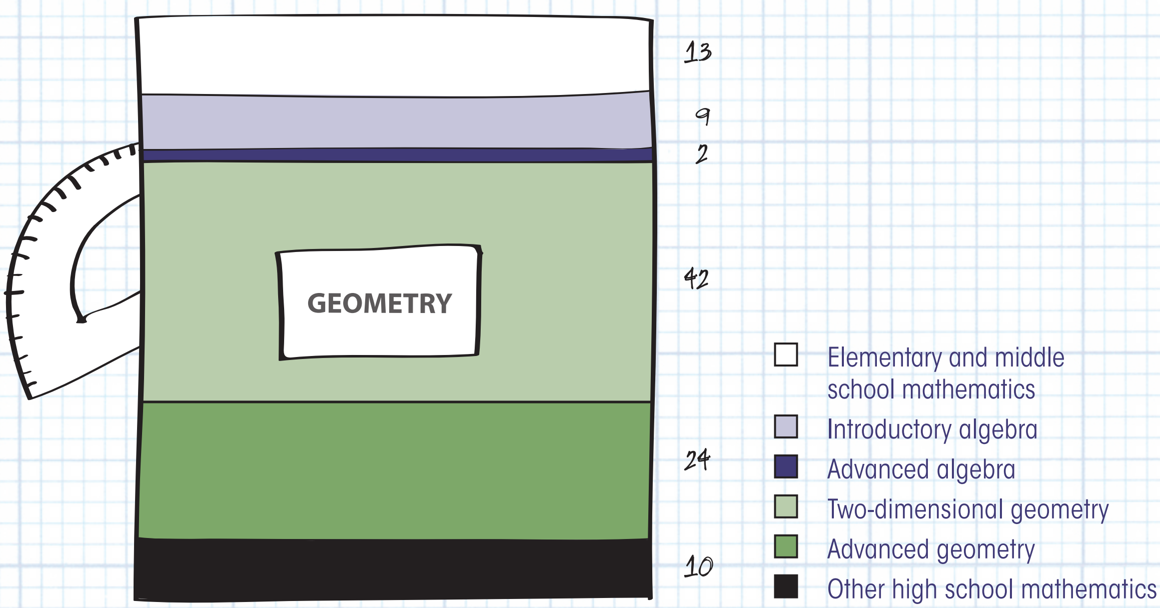
## GEOMETRY

What is geometry? High school geometry courses are devoted to the formal analysis of two-dimensional shapes, the understanding of which can be applied in real-world contexts. Students are expected to use precise definitions and equations for analysis, which is more complex than the expectations of elementary and middle school classes. The results that follow describe high school geometry courses, based on the textbooks used in the course, using curriculum topics and course levels. It is important to keep in mind that textbook information was used as an indirect measure of the topics to be taught in a course, but does not reflect classroom instruction.

### Geometry topics made up two-thirds of the content of geometry courses.

The mathematics content of high school geometry courses is shown in **figure 3**. On average, 66 percent of a graduate's geometry course focused on the core geometry topics of two-dimensional geometry (42 percent) and advanced geometry (24 percent), such as three-dimensional and coordinate geometry. The remaining one-third (34 percent) covered elementary and middle school mathematics review, algebra, and other high school mathematics topics.

**FIGURE 3.** Percentage of content of graduates' geometry courses, by curriculum topic group: 2005




NOTE: Details may not sum to total because of rounding.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, High School Transcript Study (HSTS), Mathematics Curriculum Study, 2005.

**TABLE 3.** Percentage of content of graduates' geometry courses, by course level and curriculum topic group: 2005

| Mathematics curriculum topic group | Geometry course level | | | |
|---|---|---|---|---|
| | All levels | Beginner | Intermediate | Rigorous |
| Elementary and middle school mathematics | 13 | 14* | 13* | 11 |
| Pre-geometry | 10 | 11* | 10 | 10 |
| Introductory algebra | 9 | 10 | 9 | 8 |
| Advanced algebra | 2 | 3* | 2* | 1 |
| Two-dimensional geometry | 42 | 42* | 41* | 44 |
| Advanced geometry | 24 | 21* | 24* | 28 |
| Three-dimensional geometry | 6 | 7* | 6 | 5 |
| Coordinate geometry | 4 | 3* | 4* | 7 |
| Vector geometry | 14 | 10* | 14* | 16 |
| Other high school mathematics | 10 | 11* | 11* | 8 |
| Validation and structuring | 6 | 7* | 7* | 5 |

* Significantly different ($p < .05$) from rigorous.

NOTE: Details may not sum to total because of rounding or omitted categories. The categories that are indented are subcategories within the six broad curriculum topics: elementary and middle school mathematics, introductory algebra, advanced algebra, two-dimensional geometry, advanced geometry,
and other high school mathematics. Pre-geometry covers basic patterns, perimeter, area, volume, and proportionality.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, High School Transcript Study (HSTS), Mathematics Curriculum Study, 2005.

## Graduates' beginner geometry courses contained more review content; rigorous courses had more geometric content.

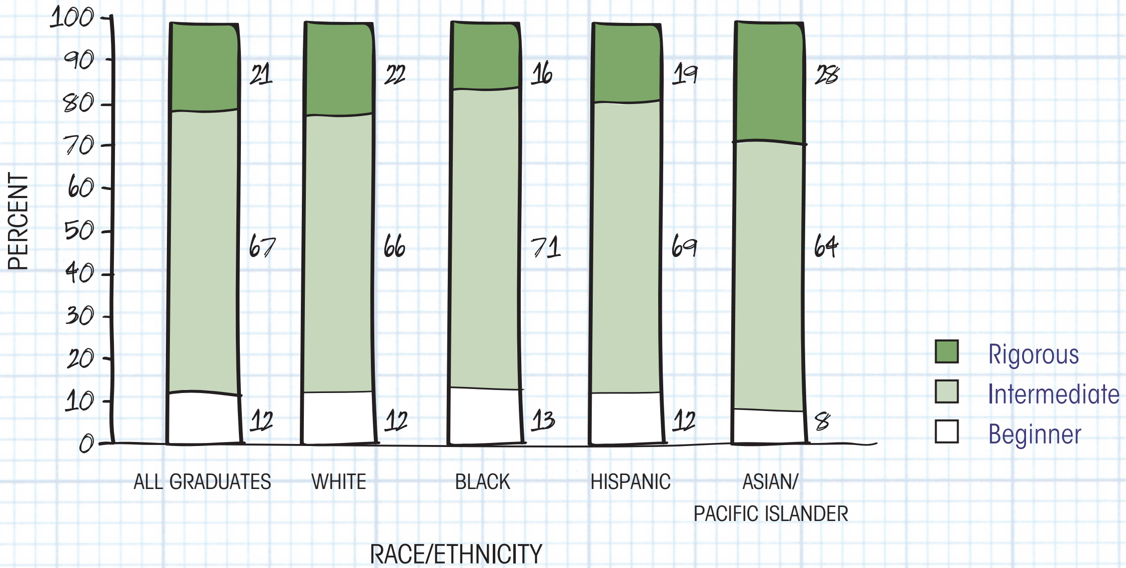
**Table 3** shows the percentage of mathematics content of high school graduates' geometry courses broken down by course level. On average, graduates in beginner level geometry courses received a higher percentage of content in elementary and middle school mathematics topics than graduates in any other course level. In addition, graduates in beginner and intermediate level courses covered a higher average percentage of content in other high school mathematics topics (11 percent for both), compared to graduates in rigorous level courses (8 percent).

All graduates, regardless of the course level of their geometry class, had courses with a higher percentage of two-dimensional geometry topics than any other curriculum topic. However, graduates who had rigorous geometry courses received larger percentages of two-dimensional geometry and advanced geometry content than graduates in other course levels.

## Most graduates took an intermediate geometry course.

The percentage of high school graduates who took a geometry course, broken down by course level and student race/ethnicity, is shown in **figure 4**. Approximately 12 percent of graduates took a beginner geometry course. Sixty-seven percent of graduates took an intermediate course, and 21 percent of graduates took a rigorous course. When compared to White graduates, there were no differences in the percentages of Black, Hispanic, or Asian/Pacific Islander graduates who took beginner, intermediate, or rigorous courses.

**FIGURE 4.** Percentage of graduates who took geometry courses, by student race/ethnicity and course level: 2005




* Significantly different ($p < .05$) from White graduates.

NOTE: Details may not sum to total because of rounding. Black includes African American, Hispanic includes Latino, and Asian/Pacific Islander includes Native Hawaiian. Race categories exclude Hispanic origin.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, High School Transcript Study (HSTS), Mathematics Curriculum Study, 2005.

## INTEGRATED MATHEMATICS: AN ALTERNATIVE APPROACH TO TEACHING MATHEMATICS

Some states and school districts offer a different approach to teaching high school mathematics topics by integrating them into a single curriculum. Integrated mathematics (or unified mathematics) courses cover several mathematics topics or strands in one course, such as algebra, geometry, trigonometry, statistics, and analysis. Instead of separating these topics into individual courses, integrated mathematics programs interweave the topics taught. First-year integrated mathematics courses are generally taken at the same time most students take algebra I courses, while second-year integrated mathematics courses are taken when most students take geometry courses. There are textbooks designed specifically for integrated mathematics courses, although not all schools with integrated mathematics programs use them. Conversely, some schools adopt integrated mathematics textbooks for algebra I and geometry courses.

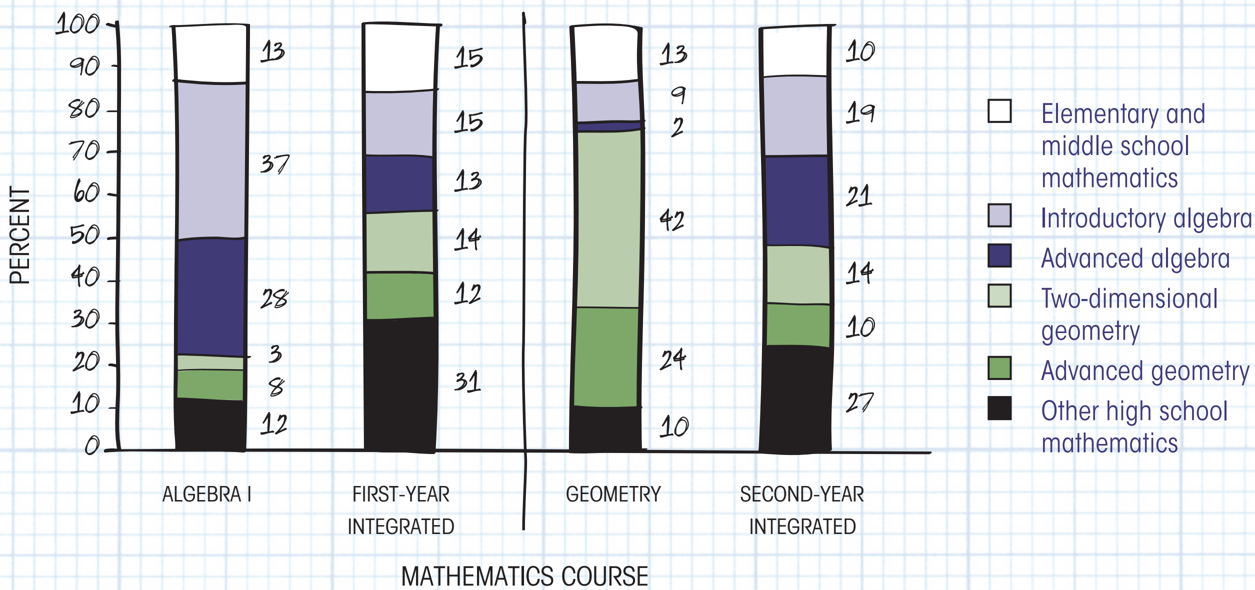
Only 6 percent of high school graduates completed a first-year integrated mathematics course, and 5 percent of graduates completed a second-year course. Due to the small number of graduates who took integrated mathematics courses, it was not possible to differentiate the course levels of integrated mathematics courses and meet reporting standards. Integrated mathematics courses were not ranked using course levels.

### Advanced topics made up a quarter or more of integrated mathematics course content.

**Figure 5** shows the mathematics profile of first- and second-year integrated mathematics courses taken by high school graduates. Graduates in first- and second-year integrated mathematics courses were exposed to more other high school mathematics topics, such as trigonometry, statistics, and calculus, than graduates in algebra I and geometry courses. These topics are typically the focus of courses taken after geometry.

Compared to traditional algebra I and geometry courses, integrated mathematics courses are not as focused on the core content of algebra or geometry. Whereas graduates in the average algebra I course had two-thirds of the course focused on algebra topics, graduates in first-year integrated mathematics courses had less than a third of the course devoted to this content (an average of 15 percent on introductory algebra and 13 percent on advanced algebra). Similarly, about a quarter of the content received by graduates in second-year integrated courses focused on

**FIGURE 5.** Percentage of content of graduates' algebra I, geometry, and integrated mathematics courses, by curriculum topic group: 2005




NOTE: Details may not sum to total because of rounding.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, High School Transcript Study (HSTS), Mathematics Curriculum Study, 2005.

either two-dimensional geometry (14 percent) or advanced geometry (10 percent), compared to an average of 66 percent for graduates in traditional geometry courses.

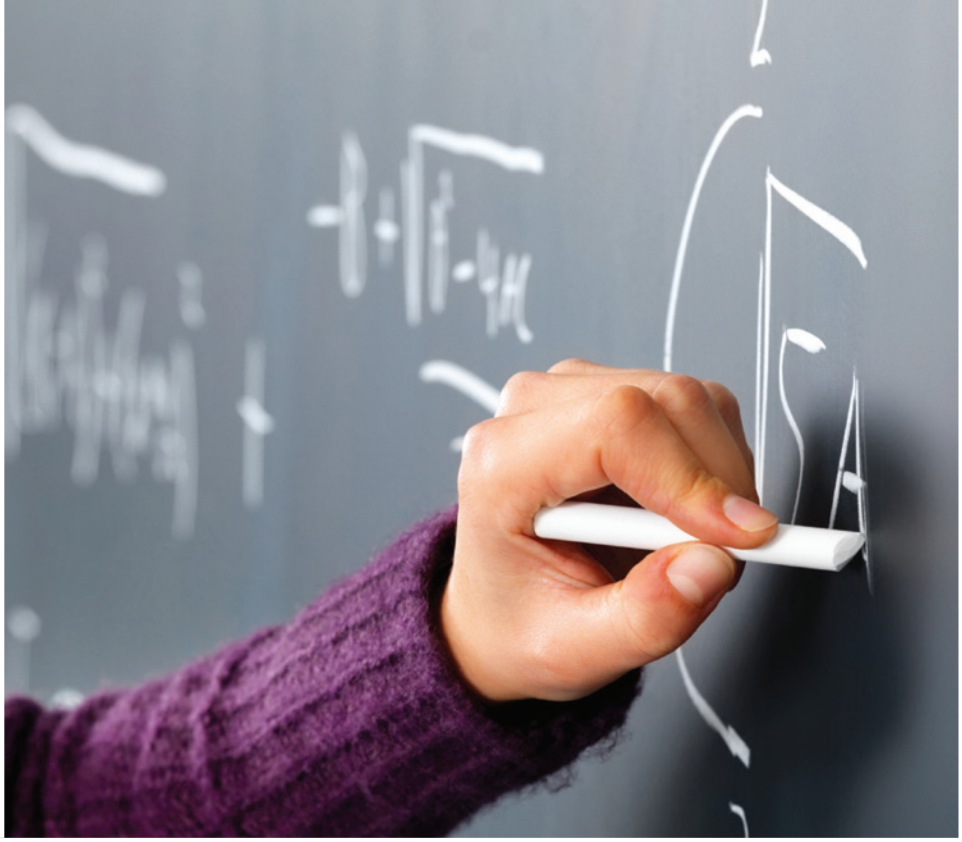
The average algebra and geometry content of high school graduates' first-year and second-year integrated mathematics courses is shown in **table 4**. Similar to traditional algebra I courses, the largest percentage of algebra content within graduates' first-year integrated course was basic equations (11 percent). The largest percentage of the geometry content for graduates in either a second-year integrated course or a traditional geometry course was two-dimensional geometry (14 percent and 42 percent, respectively).

**TABLE 4.** Percentage mathematics content of graduates' integrated mathematics courses, by curriculum topic group: 2005

| Mathematics curriculum topic group | Integrated mathematics course | |
|---|---|---|
| | First-year course | Second-year course |
| Elementary and middle school mathematics | 15 | 10 |
| Pre-geometry | 6 | 5 |
| Introductory algebra | 15 | 19 |
| Pre-algebra | 4 | 2 |
| Basic equations | 11 | 17 |
| Advanced algebra | 13 | 21 |
| Advanced equations | 4 | 7 |
| Basic functions | 4 | 3 |
| Advanced functions | 2 | 2 |
| Advanced number theory | 2 | 9 |
| Two-dimensional geometry | 14 | 14 |
| Advanced geometry | 12 | 10 |
| Three-dimensional geometry | 2 | 3 |
| Coordinate geometry | 5 | 4 |
| Vector geometry | 5 | 4 |
| Other high school mathematics | 31 | 27 |
| Validation and structuring | 1 | 4 |

NOTE: Details may not sum to total because of rounding or omitted categories. The categories that are indented are subcategories within the six broad curriculum topics: elementary and middle school mathematics, introductory algebra, advanced algebra, two-dimensional geometry, advanced geometry, and other high school mathematics. Pre-geometry covers basic patterns, perimeter, area, volume, and proportionality.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, High School Transcript Study (HSTS), Mathematics Curriculum Study, 2005.




# Comparison of School Courses

School course titles and descriptions are compared to course levels in this section of the report. The majority of algebra I and geometry classes were ranked as intermediate, regardless of the label given by the school. Seventy-three percent of graduates in classes the school labeled "honors" algebra I and 62 percent of graduates in classes the school labeled "honors" geometry were in courses ranked as intermediate. A larger percentage (37 percent) of White graduates in geometry classes labeled "honors" were enrolled in rigorous courses, compared to the percentage of Black and Hispanic graduates (21 and 17 percent, respectively) in similarly titled courses.

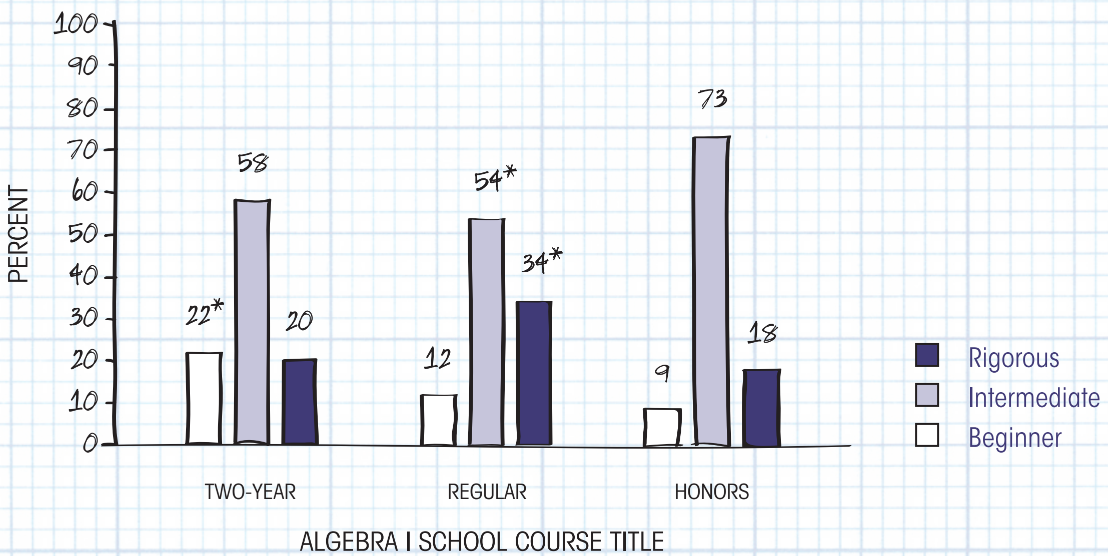
## ALGEBRA I

### Most graduates' algebra I courses ranked at the intermediate level, regardless of the course title.

The percentages of graduates who took different types of algebra I courses, by school course descriptions and course levels, are shown in **figure 6**. Only 18 percent of high school graduates who took classes that schools labeled as "honors" algebra I were in courses ranked as rigorous, based on textbooks used by the schools. Most graduates who took classes schools labeled as "honors" algebra I (73 percent) were in intermediate courses. The percentage of graduates who took "honors" classes that were ranked as intermediate (73 percent) is larger than the percentage of graduates who took "regular" algebra I classes that were ranked intermediate (54 percent). About 9 percent of graduates in classes that schools labeled "honors" algebra I were in courses ranked as beginner.

Conversely, a larger percentage of graduates who took "two-year" algebra I classes (22 percent) were in courses ranked as beginner, compared to graduates who were in "honors" algebra I classes (9 percent). ("Two-year" algebra I is a course that is completed in two school years.)

**FIGURE 6.** Percentage of graduates who took algebra I classes, by school course title and course level: 2005




* Significantly different (p < .05) from honors.

NOTE: Details may not sum to total because of rounding and the use of integrated mathematics textbooks in nonintegrated algebra I courses. "Two-year" algebra I is a course that is completed in two school years. "Honors" algebra I is a course that covers more advanced algebra topics and/or more in-depth analysis of algebra topics, including courses labeled honors, gifted and talented, and college preparatory.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, High School Transcript Study (HSTS), Mathematics Curriculum Study, 2005.

**TABLE 5.** Percentage of graduates who took algebra I classes, by course level, school course description, and student race/ethnicity: 2005

| School course description and student race/ethnicity | Algebra I course level | | |
|---|---|---|---|
| | Beginner | Intermediate | Rigorous |
| Two-year algebra I | 22 | 58 | 20 |
| White | 19 | 60 | 21 |
| Black | 19 | 55 | 25 |
| Hispanic | 37* | 53 | 11 |
| Asian/Pacific Islander | ‡ | ‡ | ‡ |
| Regular algebra I | 12 | 54 | 34 |
| White | 10 | 55 | 34 |
| Black | 14 | 52 | 34 |
| Hispanic | 17 | 52 | 32 |
| Asian/Pacific Islander | 22* | 46 | 32 |
| Honors algebra I | 9 | 73 | 18 |
| White | 7 | 76 | 18 |
| Black | 15 | 67 | 18 |
| Hispanic | 10 | 56 | 35 |
| Asian/Pacific Islander | ‡ | ‡ | ‡ |

‡ Reporting standards not met.

* Significantly different ($p < .05$) from White graduates.

NOTE: Details may not sum to total because of rounding and the use of integrated mathematics textbooks in nonintegrated algebra I courses. "Two-year" algebra I is a course that is completed in two school years. "Honors" algebra I is a course that covers more advanced algebra topics and/or more in-depth analysis of algebra topics, including courses labeled honors, gifted and talented, and college preparatory. Black includes African American, Hispanic includes Latino, and Asian/Pacific Islander includes Native Hawaiian. Race categories exclude Hispanic origin.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, High School Transcript Study (HSTS), Mathematics Curriculum Study, 2005.

## Few racial/ethnic differences by course level found among graduates in similarly titled algebra I courses.

**Table 5** shows the course level breakdown of algebra I classes by school course descriptions and student race/ethnicity. Overall, few significant differences were found when comparing the percentages of White graduates who took algebra I courses by school course description and course level to Black, Hispanic, and Asian/Pacific Islander graduates who took similar courses, as noted in **table 5**. Approximately 37 percent of the Hispanic graduates in "two-year" algebra I classes were in beginner courses, compared to 19 percent each of White and Black graduates in similarly titled classes. When looking at graduates who were in courses titled "regular" algebra I classes, a larger percentage of Asian/Pacific Islander graduates, 22 percent, were in beginner courses compared to White graduates (10 percent). No racial/ethnic differences by course level were found among graduates who took classes labeled as "honors" algebra I.

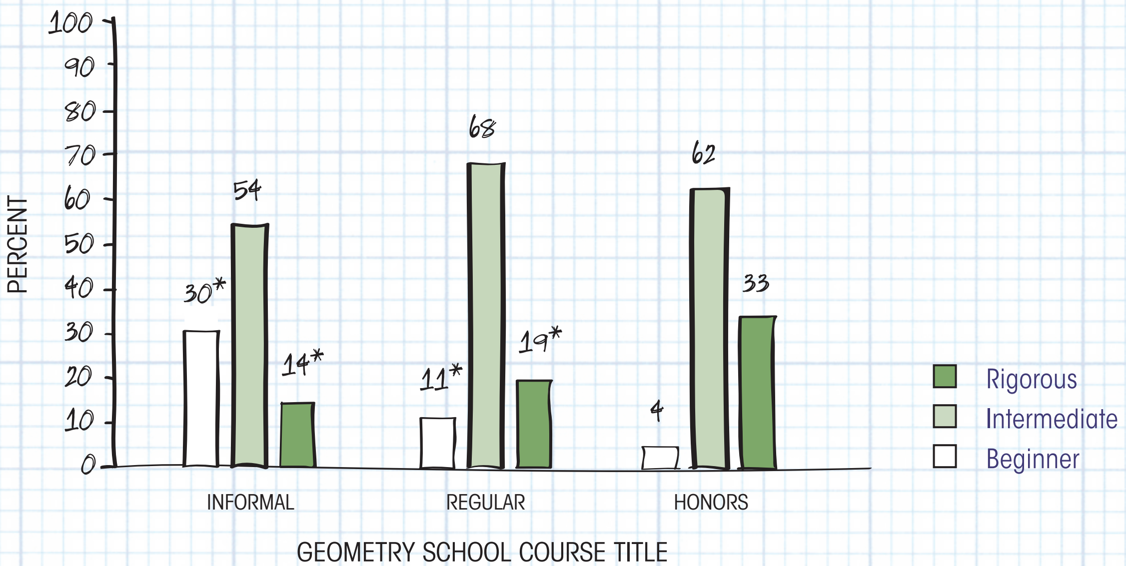
# GEOMETRY

## Most graduates received intermediate level geometry courses, regardless of course title.

The percentages of graduates who took different types of geometry courses, by school course descriptions and course levels, are shown in **figure** 7. One-half or more of the high school graduates enrolled in “informal,” “regular,” and “honors” geometry classes had an intermediate course. (“Informal” geometry is a course that does not emphasize proofs.) Fifty-four percent of graduates in classes that schools described as “informal” geometry, 68 percent of graduates in “regular” geometry, and 62 percent of graduates in “honors” geometry were in courses ranked as intermediate.

For those graduates in “honors” geometry classes, only 33 percent were in rigorous geometry courses. A larger percentage of graduates who took “honors” geometry classes had rigorous coursework, compared to graduates who took “informal” and “regular” geometry classes (14 and 19 percent, respectively). A larger percentage of graduates who took classes labeled “informal” geometry received coursework ranked as beginner level (30 percent), compared to graduates who took classes labeled “regular” or “honors” geometry (11 and 4 percent, respectively).

**FIGURE 7.** Percentage of graduates who took geometry classes, by school course title and course level: 2005




* Significantly different (p < .05) from honors.

NOTE: Details may not sum to total because of rounding and the use of integrated mathematics textbooks in nonintegrated geometry courses. “Informal” geometry is a course that does not emphasize proofs. “Honors” geometry is a course that covers more advanced geometry topics and/or more in-depth analysis of geometry topics, including courses labeled honors, gifted and talented, and college preparatory.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, High School Transcript Study (HSTS), Mathematics Curriculum Study, 2005.

## Racial/ethnic differences by course level were evident only among graduates in courses titled "honors" geometry.

**Table 6** shows the breakdown of geometry classes by course level, school course descriptions, and student race/ethnicity. For graduates in classes labeled "honors," 37 percent of White graduates had rigorous courses, compared to 21 percent of Black graduates and 17 percent of Hispanic graduates in similarly titled courses. One-half or more of graduates in each racial/ethnic subgroup who took "honors" geometry classes were in courses that were ranked as intermediate (57 percent of White, 73 percent of Black, 81 percent of Hispanic, and 63 percent of Asian/Pacific Islander graduates). A larger percentage of Black (73 percent) and Hispanic (81 percent) graduates who took "honors" geometry classes were in intermediate courses, compared to White graduates (57 percent). There were no measurable differences at any course level among White, Black, and Hispanic graduates who took either "informal" or "regular" geometry.

**TABLE 6.** Percentage of graduates who took geometry classes, by course level, school course description, and student race/ethnicity: 2005

| School course description and student race/ethnicity | Geometry course level | | |
|---|---|---|---|
| | Beginner | Intermediate | Rigorous |
| Informal geometry | 30 | 54 | 14 |
| White | 29 | 53 | 17 |
| Black | 30 | 62 | 4 |
| Hispanic | 37 | 49 | 10 |
| Asian/Pacific Islander | ‡ | ‡ | ‡ |
| Regular geometry | 11 | 68 | 19 |
| White | 11 | 68 | 20 |
| Black | 12 | 72 | 16 |
| Hispanic | 12 | 69 | 19 |
| Asian/Pacific Islander | 9 | 64 | 27 |
| Honors geometry | 4 | 62 | 33 |
| White | 4 | 57 | 37 |
| Black | 6 | 73* | 21* |
| Hispanic | 2 | 81* | 17* |
| Asian/Pacific Islander | 2 | 63 | 35 |

‡ Reporting standards not met.

* Significantly different ($p < .05$) from White graduates.

NOTE: Details may not sum to total because of rounding and the use of integrated mathematics textbooks in nonintegrated geometry courses. "Informal" geometry is a course that does not emphasize proofs. "Honors" geometry is a course that covers more advanced geometry topics and/or more in-depth analysis of geometry topics, including courses labeled honors, gifted and talented, and college preparatory. Black includes African American, Hispanic includes Latino, and Asian/Pacific Islander includes Native Hawaiian. Race categories exclude Hispanic origin.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, High School Transcript Study (HSTS), Mathematics Curriculum Study, 2005.

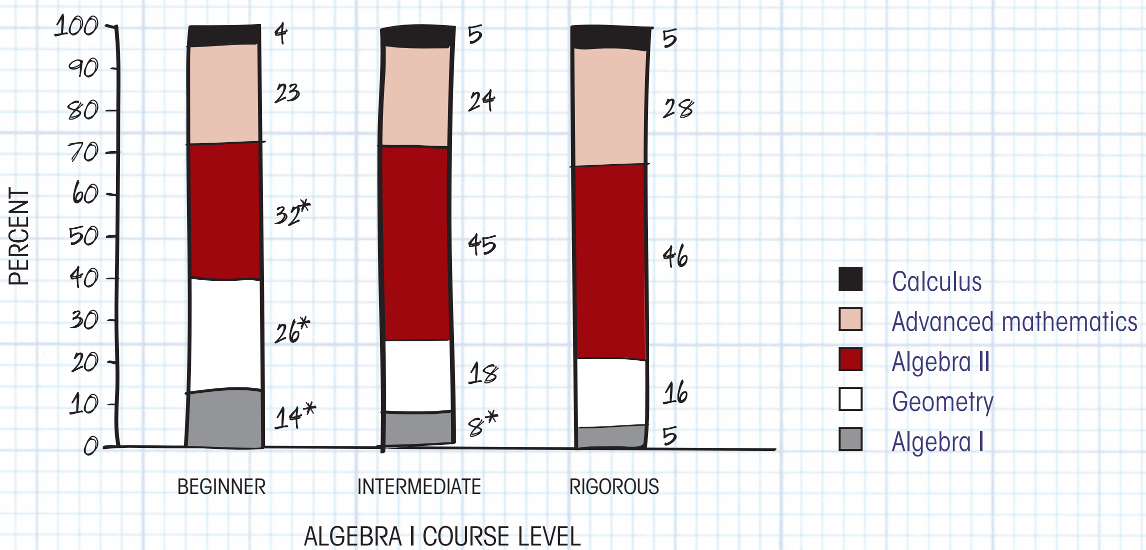

# Coursework and Performance

**Graduates who** took beginner, intermediate, or rigorous algebra I or geometry courses and their subsequent mathematics coursetaking and performance on the NAEP mathematics assessment are shown in this section of the report. The pattern for subsequent mathematics coursetaking is associated with both algebra I and geometry course level. Graduates who took rigorous algebra I courses had a higher average NAEP algebra score (146) than graduates who took beginner algebra I courses (137). Graduates who took rigorous geometry courses more often took a calculus course and achieved higher NAEP mathematics scores than graduates who took beginner or intermediate geometry courses.

## Graduates who took beginner algebra I courses more likely had algebra I or geometry as their highest mathematics course.

**Figure 8** shows the highest level mathematics course taken by graduates given the course level of the algebra I class they took while in high school. A higher percentage of graduates who took a beginner algebra I course went on to have that course or a geometry course as their highest level mathematics course than graduates who took an intermediate or rigorous algebra I course. Of the high school graduates who had a beginner algebra I course, 14 percent had that class as their highest level mathematics course, which was higher than graduates who had an intermediate or a rigorous course. Similarly, a larger percentage of graduates (26 percent) who took a beginner algebra I course had geometry as their highest level mathematics course compared to graduates who had an intermediate (18 percent) or a rigorous course (16 percent). Fewer graduates who took beginner algebra I courses had algebra II as their highest level mathematics course taken than graduates who had intermediate or rigorous courses. About 32 percent of graduates who took a beginner algebra I course had algebra II as their highest mathematics course taken, compared with 45 percent of

**FIGURE 8.** Percentage distribution of highest level mathematics course taken by graduates, by algebra I course level: 2005

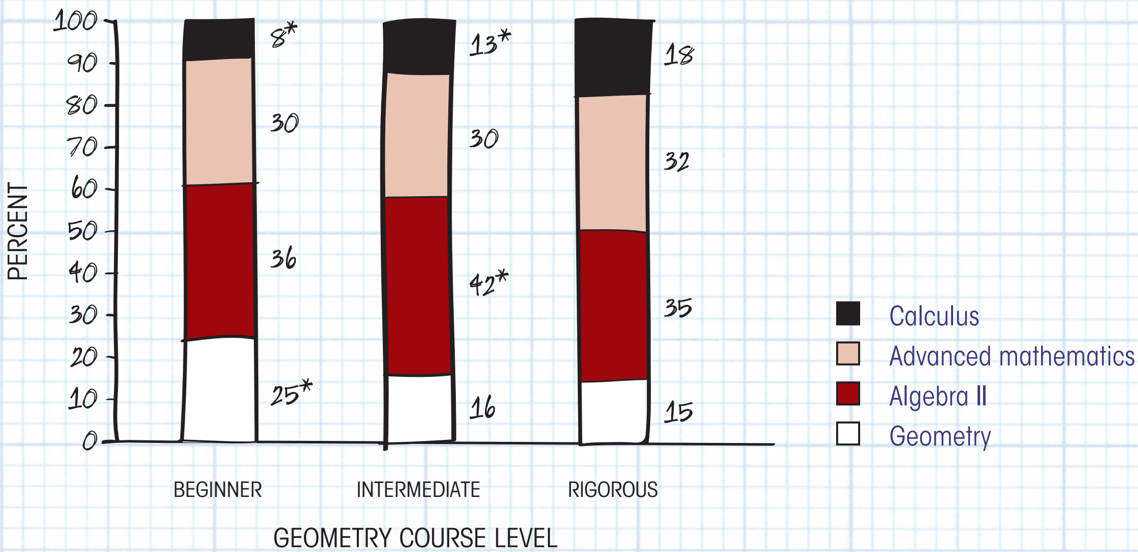



* Significantly different ($p < .05$) from rigorous.

NOTE: Details may not sum to total because of rounding.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, High School Transcript Study (HSTS), Mathematics Curriculum Study, 2005.

**FIGURE 9.** Percentage distribution of highest level mathematics course taken by graduates, by geometry course level: 2005




* Significantly different ($p < .05$) from rigorous.

NOTE: Details may not sum to total because of rounding.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, High School Transcript Study (HSTS), Mathematics Curriculum Study, 2005.

graduates who took intermediate algebra I courses and 46 percent of graduates who took rigorous algebra I courses. There were no significant differences in the percentages of graduates who took an intermediate or rigorous algebra I course who went on to take a geometry course or higher as their highest level mathematics course.

### A higher percentage of graduates who took rigorous geometry courses took advanced mathematics courses compared to graduates who took beginner and intermediate courses.

**Figure 9** shows the highest level mathematics course taken by high school graduates given the course level of the geometry class they took while in high school. A larger percentage of graduates who received a rigorous geometry course took a calculus course in high school (18 percent) than those graduates who received beginner or intermediate courses (8 and 13 percent, respectively). However, a larger percentage of graduates who received a beginner geometry course had that course as their highest level mathematics course (25 percent) than those graduates who took an intermediate or rigorous geometry course (16 percent and 15 percent, respectively).

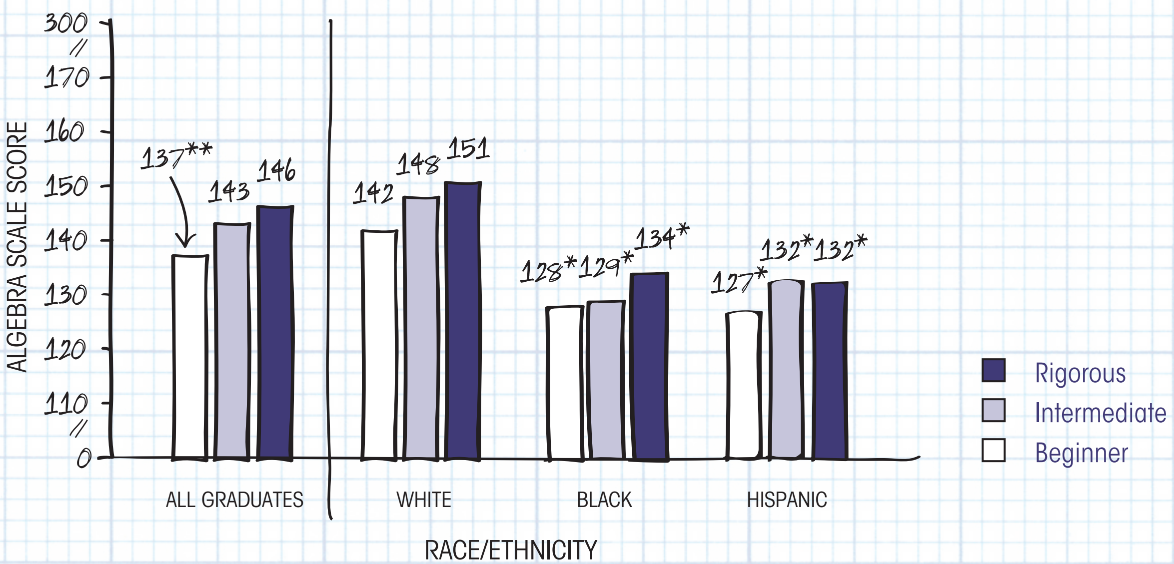
## Graduates in rigorous algebra I courses performed better on NAEP than graduates in beginner algebra I courses.

The average NAEP algebra scores for 2005 high school graduates, by the course level of the algebra I classes taken and student race/ethnicity, are shown in **figure 10**. Graduates who took rigorous algebra I courses earned an average NAEP algebra score of 146. This average score was nine points higher than the average score of 137 earned by graduates who took beginner algebra I courses.

## White graduates obtained higher scores across all algebra I course levels than Black or Hispanic graduates.

Score differences were also evident when the data were examined by student race/ethnicity. White graduates who took rigorous algebra I courses earned an average NAEP algebra score of 151, which was 17 points higher than the average scores obtained by Black graduates and 19 points higher than the average score obtained by Hispanic graduates who took rigorous algebra I courses. Significant achievement gaps were also

**FIGURE 10.** Average NAEP algebra score of graduates, by student race/ethnicity and algebra I course level: 2005




* Significantly different (p < .05) from corresponding White graduates.

** Significantly different (p < .05) from rigorous.

NOTE: Average NAEP algebra scale scores are shown. Asian/Pacific Islander graduates are included in the calculation of average NAEP algebra scores for "All Graduates" but are not reported separately because sample size does not meet reporting standards across course levels. Black includes African American, Hispanic includes Latino, and Asian/Pacific Islander includes Native Hawaiian. Race categories exclude Hispanic origin.

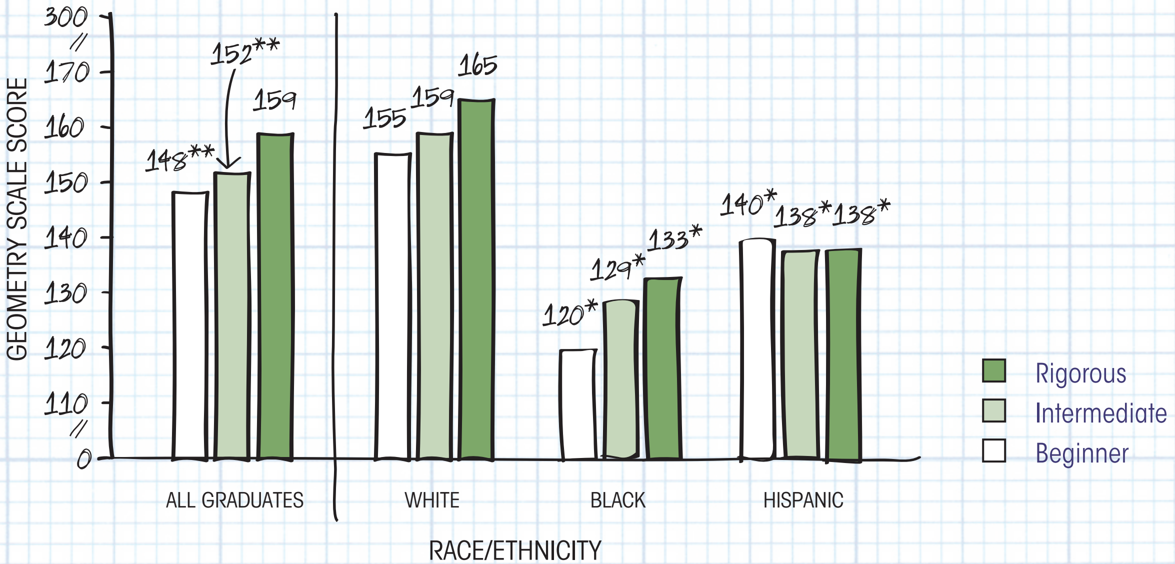
SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress, High School Transcript Study (HSTS), Mathematics Curriculum Study, 2005.

seen among White and Black graduates and White and Hispanic graduates who took beginner and intermediate algebra I courses.

## Graduates in rigorous geometry courses also performed better on NAEP.

**Figure 11** shows the average NAEP geometry score for 2005 high school graduates by the course level of the geometry classes they took and by student race/ethnicity. There was an eleven-point gap in the average geometry scores between graduates who took rigorous (159) and beginner (148) geometry courses. As was the case with algebra I, White graduates earned higher NAEP geometry scores than Black or Hispanic graduates, regardless of geometry course level.

**FIGURE 11.** Average NAEP geometry score of graduates, by student race/ethnicity and geometry course level: 2005




* Significantly different (p < .05) from corresponding White graduates.

** Significantly different (p < .05) from rigorous.

NOTE: Average NAEP geometry scale scores are shown. Asian/Pacific Islander graduates are included in the calculation of average NAEP geometry scores for "All Graduates" but are not reported separately because sample size does not meet reporting standards across course levels. Black includes African American, Hispanic includes Latino, and Asian/Pacific Islander includes Native Hawaiian. Race categories exclude Hispanic origin.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress, High School Transcript Study (HSTS), Mathematics Curriculum Study, 2005.

# Technical Notes

## Overview

The 2005 High School Transcript Study (HSTS) Mathematics Curriculum Study (MCS) brings together information from three sources—students, schools, and textbooks—to provide a more in-depth look at high school graduates' algebra I and geometry courses. The study used textbook data collected as part of the 2005 HSTS, which is associated with the National Assessment of Educational Progress (NAEP). Information from algebra I and geometry textbooks used by schools across the nation served as an indirect measure of the curriculum taught in these courses. Two mathematics curriculum measures—curriculum topics and course levels—were created from the textbook data and formed the basis of the study's results. These measures, along with the HSTS transcript data, student and school demographic data, and the NAEP 2005 twelfth-grade mathematics assessment data, were the data sources for the findings listed in this report.

A series of analyses were conducted for the textbooks collected, corresponding courses, and students who took the courses to create the curriculum topic and course level reporting measures. The textbook analyses, the first step, involved coding chapter review questions and creating chapter summary measures. Review questions for each chapter of a textbook, not the entire textbook, were coded for their curriculum topics (i.e., the mathematical topics covered) and performance expectations (i.e., the tasks required to answer the question). The chapter review questions have been found to be representative of the chapter content and challenge level (Schmidt 2012). Chapter summary measures were created to reflect the combined content of the chapter review questions for each textbook chapter.

The course analyses, the second step, involved generating course summary measures and creating categories of course content. The chapter summary measures of the textbook chapters used in each course were aggregated to create the course summary measures. A factor analysis of the course summaries was conducted to identify the patterns in the mathematics content coverage across courses, while a discriminant analysis was conducted to classify the courses into four content categories.

Lastly, the student analyses, the third step, involved generating student summary measures, classifying students' coursework into course levels, and defining broad categories of curriculum topics. The course summary measures were matched to students by the courses the students took, as listed on their transcripts, which allowed for the student summary measures to be created. Similar to the course analyses, a factor analysis of the student summaries was conducted to identify the patterns in the mathematics content coverage across students, while a discriminant analysis was conducted to classify the students' coursework into one of three course levels (beginner, intermediate, rigorous) and an integrated category. The student summary measures were also used to create six curriculum topic categories, which were also used as a reporting measure. The study methodology is summarized in this section of the report. The forthcoming Technical Report will provide full details of the study methodology.

## Target Population and Analytic Sample

The HSTS has been conducted periodically since 1990. The MCS is a component of the HSTS administered in 2005. The target population for the 2005 study included students in public schools in the United States who were enrolled in the twelfth grade during the 2004-05 school year, who graduated in 2005 with a regular or honors diploma, and who took an algebra I course and/or geometry course (or an equivalent course, such as integrated mathematics) in high school. Private schools were not included in the analysis. Graduates with special education diplomas, certificates of attendance, or certificates of completion were excluded from the analysis.

## Sampling

The 2005 NAEP HSTS sampling procedures were designed to achieve a nationally representative sample, including both public and private school graduates in the Class of 2005. Consistent with the 2005 NAEP national assessments, in the 2005 HSTS, students were selected for participation based on a stratified two-stage sampling plan. In the first stage of sampling, schools were selected, and students within schools were selected in the second stage. The weighting procedures for this study take into account the stratified sampling methods. The MCS uses graduates from the public school samples of the 2005 NAEP HSTS. The samples included the full student sample, which included approximately 37,900 public school students with transcript data, and a smaller sample, which only included the approximately 27,200 public school students who participated in the NAEP mathematics and science assessments. A full description of the sampling plan is beyond the scope of this Appendix. More sampling information about the 2005 NAEP HSTS can be found in The 2005 High School Transcript Study User's Guide and Technical Report (Shettle et al. 2008).

All public schools that participated in the 2005 NAEP HSTS were asked to fill out forms that identified the textbooks used for each mathematics course they offered. Schools that did not offer algebra I and/or geometry courses (or comparable courses such as integrated mathematics I and/or integrated mathematics II) or did not complete the textbook forms were not eligible for the MCS. High school graduates who did not take algebra I in high school were not included in the algebra I analysis. High school graduates who did not take geometry in high school were not included in the geometry analysis. About

17,800 public high school graduates from the 2005 NAEP HSTS were included in the MCS analyses. Around 12,500 graduates were part of the algebra I sample, while around 15,900 graduates were part of the geometry sample. The analyses for this report was limited to textbook data matched to algebra I and geometry courses. Approximately 5,700 students linked to the NAEP twelfth-grade mathematics assessment had textbook data matched to algebra I and/or geometry courses, of which 4,900 students were included in the MCS analyses.

As part of the 2009 NAEP HSTS, public schools were asked again to fill out forms that identified the textbooks used for each mathematics course offered. However, no further work has been conducted beyond the collection of these textbook lists. Therefore, no information from the 2009 data collection was included in this report.

## School and Student Response Rates

Among the 640 public schools that participated in the 2005 NAEP HSTS, 550 schools had textbook data matched to their algebra I and/or geometry courses. A weighted school response rate was calculated by comparing the weighted percentages of schools that had textbook data with all 2005 NAEP HSTS participating public schools. Among all 2005 NAEP HSTS public schools, the weighted school response rate was about 85 percent.

As this report includes an analysis of NAEP twelfth-grade mathematics assessment scores, and since not all public schools in the HSTS participated in that assessment, a separate weighted school response rate was calculated based on the 2005 NAEP HSTS public schools linked to the NAEP 2005 twelfth-grade mathematics assessment. Of the 590 public schools that participated in both the 2005 NAEP HSTS and 2005 NAEP twelfth-grade mathematics assessment, 520 schools had textbook data matched to their algebra I and/or geometry courses. Among 2005 NAEP HSTS public schools that participated in the NAEP twelfth-grade mathematics assessment, the weighted school response rate was about 88 percent.

In addition to the school response rates, weighted student response rates were calculated by comparing the weighted percentages of students that were analyzed with textbook data with all eligible students. For the full student sample, eligible students were defined as public high school graduates who attended the schools that had textbook data that could be matched to algebra I and/or geometry courses. From the 550 public high schools with textbook data matched to algebra I and/or geometry courses, there were 21,100 eligible students, of which 17,800 students were included in the MCS analyses. Among these eligible students, the weighted student response rate was around 85 percent.

Similar to the school response rates, a separate weighted student response rate was calculated for students in schools linked to the NAEP 2005 twelfth-grade mathematics assessment. For the smaller NAEP-linked sample, eligible students were defined as public high school graduates who attended the schools that had textbook data that could be matched to algebra I and/or geometry courses and participated in the NAEP twelfth-grade mathematics assessment. From the 520 public high schools in the NAEP-linked sample that had textbook data matched to algebra I and/or geometry courses, there were 5,700 eligible students, of which 4,900 students were included in the MCS analyses. Among these eligible students, the weighted student response rate was around 87 percent.

## Textbook Collection

About 120 textbooks—50 algebra I or first-year integrated mathematics textbooks and 70 geometry or second-year integrated mathematics textbooks—were selected for coding from the textbook lists provided by schools as being used in algebra I and geometry courses across the nation. The textbook lists included an indicator of whether the textbook was the main or supplemental textbook used for the mathematics course. The lists also included what chapters from the textbook were taught in the course.

Around half of the textbooks selected for coding had been coded in prior mathematics textbook studies conducted by the Center for Research in Mathematics and Science Education, formerly the U.S. Trends in International Mathematics and Science Study (TIMSS) National Research Center at Michigan State University (MSU). These previously coded textbooks were included in the sample because (1) schools were still using the textbooks; and (2) when new editions of textbooks were compared to previously coded editions, no changes were found in the chapter review questions. In both instances, the curriculum topics and performance expectations coded in prior studies were unchanged and, therefore, adopted for the current study.

To maximize student coverage, textbooks were selected for coding as part of this study based on the number of 2005 NAEP HSTS participants in courses using those textbooks. Textbooks used by only one or two schools were not sampled for coding to prevent possible disclosure of schools and students that participated in the NAEP HSTS and the MCS. Because information about the textbooks used by schools is publicly available, at least three schools had to have used an algebra I, geometry, or integrated mathematics textbook for it to be included in the current study.

## Steps of the Analysis Process

The discussion of the series of analysis steps to conduct the MCS is broken into three sections: textbook analyses, course analyses, and student analyses. Figure A1 on the following pages displays an overview of the steps of the study analyses as a flow chart.

# Steps of the Analysis Process

The Mathematics Curriculum Study used information from algebra I and geometry textbooks used in schools across the nation to characterize the mathematics content of these two courses. A series of analyses were conducted for the textbooks collected, the corresponding courses, and the students who took the courses to create the curriculum topic and course level measures that were used for reporting. The following flow chart illustrates the overall analysis steps and the text briefly describes what occurred in each step. A more detailed description of the analysis steps follows in the remaining pages of the Technical Notes.

## STEP 1: Textbook Analysis

*Detailed on pages 44-45*

### Code Chapter Review Questions

The Trends in International Mathematics and Science Study (TIMSS) Framework, consisting of over 200 mathematics topics and 20 performance expectation codes, was used to code each chapter review question in the textbooks.

### Create Chapter Summary Measures

The assigned curriculum topic and performance expectation codes were used to create instructional measures, total content page counts and weighted topic content page counts, for each textbook chapter. The over 200 mathematics topics were aggregated to 32 groupings and the 20 performance expectation codes were ranked on a 4 point scale.

## STEP 2: Course Analysis

*Detailed on pages 45-48*

### Create Course Summary Measures

After the chapter review questions were coded and the information was summarized for textbook chapters, measures of total content page counts, weighted topic content page counts, and overall cognitive challenge were created for algebra I and geometry courses.

### Course Factor Analysis

Factor analysis was used to identify distinct patterns of mathematics topics across courses. The 32 general topic groupings were aggregated to 17 and used as input for the factor analyses. A separate factor analysis was conducted for the algebra I and geometry courses. The resulting factors, five for algebra I and six for geometry, were used in the course discriminant analysis.

### Course Discriminant Analysis

Discriminant function analysis was conducted to classify the courses into four distinct categories that identified the differences in curriculum content across the nation—low, medium, high, and integrated. The course summary measures of total content page counts, overall cognitive challenge ratings, and factors from course factor analysis were used as input. The resulting categories were used in the student discriminant analysis.

### Link to Student Transcripts

To assist in creating the student summary measures, the course summary measures were matched to students through the algebra I and geometry courses on the students' transcripts.

## STEP 3: Student Analysis

*Detailed on pages 48-49*

### Create Student Summary Measures

After the course summaries were matched to the students who took the algebra I and geometry courses, measures of total content page counts, weighted content page counts, and overall cognitive challenge were created for students.

### Student Factor Analysis

Factor analysis was used to identify the patterns of mathematics topics across students who took algebra I and geometry courses. The 32 general topic groupings were aggregated to 17 and used as input for the factor analyses. A separate factor analysis was conducted for the algebra I and geometry courses. The resulting factors, five for algebra I and six for geometry, were used in the student discriminant analysis.

### Student Discriminant Analysis

Discriminant function analysis was conducted to classify student coursework into three distinct course levels that identified the differences in curriculum content and complexity across the nation—beginner, intermediate, and rigorous. An integrated category was also defined. The student summary measures of total content page counts, overall cognitive challenge ratings, and factors from student factor analysis were used as input.

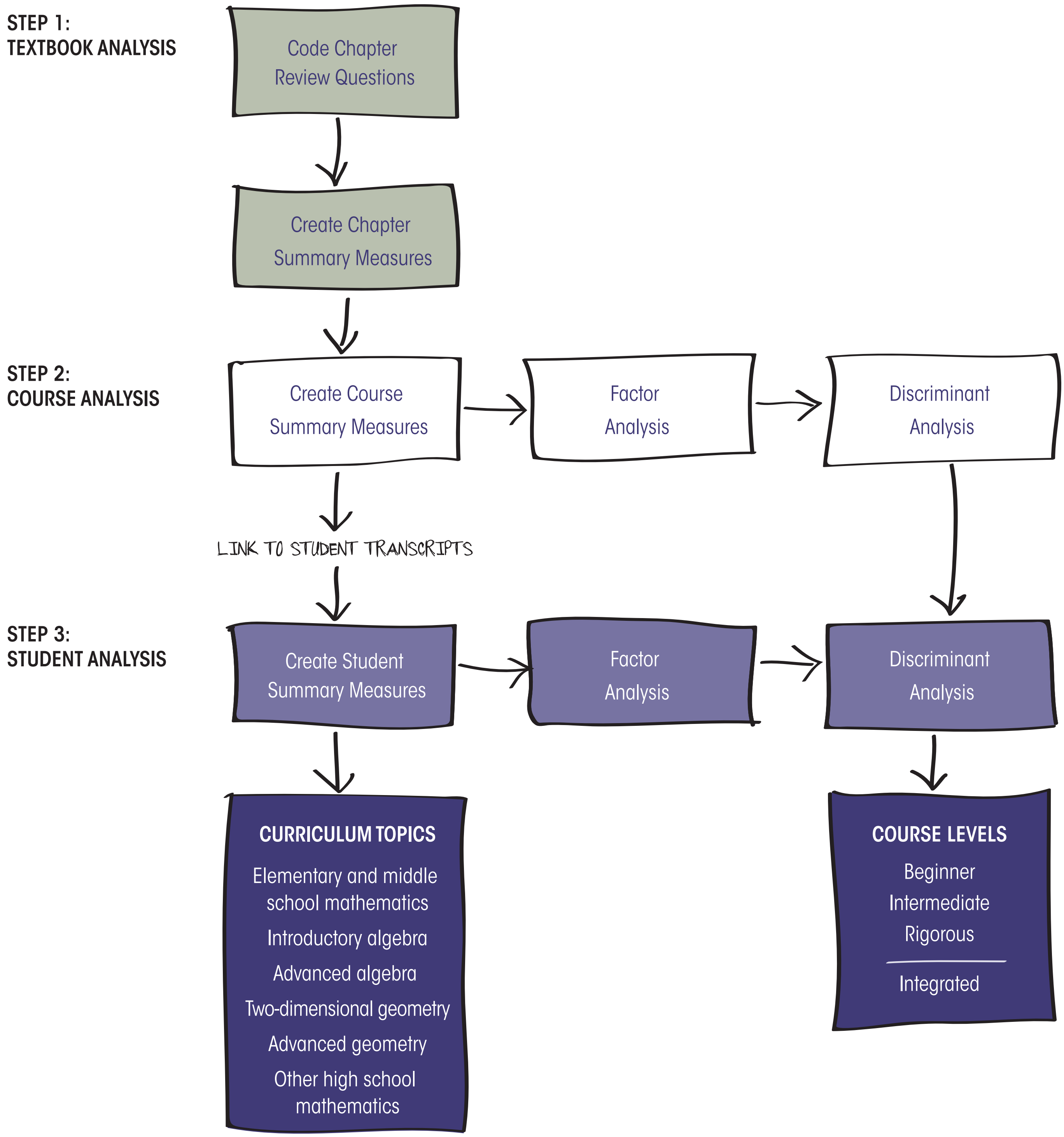
### Define Course Levels

Based on the results of the student discriminant analysis, students' algebra I and geometry coursework was classified as one of three course levels—beginner, intermediate, and rigorous. An integrated category was also used for reporting.

### Define Curriculum Topics

The 17 mathematics topic groupings used in the course and student factor analyses were collapsed into six broad categories for reporting students' algebra I and geometry coursework—elementary and middle school mathematics, introductory algebra, advanced algebra, two-dimensional geometry, advanced geometry, and other high school mathematics.

**FIGURE A1.** Overall steps of the analysis process for the Mathematics Curriculum Study: 2005




SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, High School Transcript Study (HSTS), Mathematics Curriculum Study, 2005.

## Textbook Analyses

The first step of the analyses for this study began with the textbooks. Two processes occurred during this step: coding of the chapter review questions, and creating chapter summary measures.

### Code Chapter Review Questions

The textbook coding process for this study is based on frameworks developed for the Third International Mathematics and Science Study (TIMSS) (Robitaille et al. 1993; Schmidt, McKnight, and Raizen 1997) and was used in two other major national studies—the Longitudinal Study of American Youth (LSAY) and the Adolescent Health and Academic Achievement (AHAA) project (Muller et al. 2007; Schiller et al. 2008). The TIMSS framework provided the procedures for coding each review question found at the end of a textbook chapter for mathematics topics covered (curriculum topics) and the tasks required for students to answer the question (performance expectations). These data were aggregated to create summary measures for each textbook chapter, and the chapter summary measures were aggregated to create course summary measures.

Chapter review questions have been found to be representative of the chapter material based on previous work by Schmidt (2012); therefore, only chapter review questions were coded, not the entire textbook. By coding each review question at the end of each chapter, textbook contents were classified by the mathematics topics represented in the review question and the activities explicitly required to answer the question. Coding the activities as well as the topics provides a measure of textbook content that takes into account both contributions to student learning—the amount of instructional material and level of cognitive engagement (Gamoran et al. 1997). The topics and activities that were coded followed the TIMSS curriculum framework (Schmidt et al. 2001; Peak 1996). The procedures for coding the chapter review questions were the same procedures that were established for TIMSS. The procedures of recording topics and activities in the chapter review questions were intentionally designed for TIMSS to increase the reliability of coding across grade levels and nations. The procedures were described as low-inference (Schiller at al. 2010) because coders only record topics and tasks explicitly asked about in each chapter review question.

The coders used in the MCS were trained and supervised by the Center for Research in Mathematics and Science Education at MSU. The coders were required to have obtained graduate-level education in mathematics or mathematics education. The MCS used the same standardized coder training methods and quality control procedures used in the TIMSS (Schmidt, McKnight, and Raizen 1997) and AHAA (Schiller et al. 2008) studies.

Each potential coder for the MCS participated in a training session. During the training session, the coding supervisor introduced the coders to the TIMSS framework, gave instructions on how to code curriculum topics and performance expectations, and instructed the coders on how to complete the textbook coding forms. For hands-on coding practice, each potential coder was given an algebra I or geometry textbook to code during the training session. At the end of the training session, the coding supervisor reviewed each coder's work and provided feedback. Coders who demonstrated at least 80 percent accuracy in their textbook coding during the training session were eligible to serve as coders during the MCS coding session. The standard of 80 percent accuracy was the same standard that was set for the TIMSS study (Schmidt, McKnight, and Raizen 1997). Coders who did not meet this standard were given additional training and feedback. If these coders still could not meet the 80 percent accuracy threshold, they were not eligible to serve as coders during the MCS coding session. All coding information for chapter review questions that were coded by potential coders who were not retained for the MCS coding session was discarded and the questions were reassigned for recoding. During the MCS coding session, the coding supervisor also monitored the coders' work. Frequent random checks of their coding were conducted throughout the textbook coding process to ensure that the standard of 80 percent accuracy was maintained. Because the MCS uses previously coded textbooks from the LSAY and AHAA textbook studies, and coding accuracies for the textbooks MCS used cannot be disaggregated from the overall study accuracies, an overall coding accuracy rate for the MCS could not be determined. Inter-rater reliability was not measured for this study.

The TIMSS curriculum framework of mathematics topics used for the textbook coding is a comprehensive listing of topics taught in mathematics in elementary and secondary schools around the world (Robitaille et al. 1993). The framework was designed by a committee of international education researchers. The framework contains over 200 topic codes and 20 performance expectation codes (Robitaille et al. 1993). In the textbook coding process, up to five main topics and three supplementary topics could be recorded for each chapter review question. Up to three performance expectation codes could also be recorded. Each topic and performance expectation code was assigned a Boolean value to indicate whether the topic or code was present or absent for the question. Chart A1 lists the topic and performance expectation codes used in the study.

Since the AHAA study (Schiller et al. 2008), the TIMSS mathematics curriculum framework was updated to reflect the latest trends in mathematics curriculum. The framework was designed so that emerging trends in mathematics curriculum could be tracked (Robitaille et al. 1993). Changes made to the framework included adding new mathematics topics, expanding established topics, and re-ordering the algebra topics. Chart A1 reflects all of the changes made to the TIMSS mathematics curriculum framework since the AHAA study was conducted in 2004.

The following examples show how the illustrative textbook questions of the "Understanding textbook coding" section (found on pages 16 and 17 of this report) would have been coded using the TIMSS curriculum framework, including the index numbers of the topics and performance expectations from chart A1:

- Question 1 (page 16): The topic code for this example is "Linear equations and their formal (closed) solutions" (1.6.2.5). Performance expectation codes for this example are (a) "Performing routine procedures" (2.2.2); (b) "Critiquing" (2.5.4); and (c) "Verifying" (2.3.5).

- Question 2 (page 16): The topic codes for this example are (a) "Logarithmic and exponential equations and their solutions" (1.6.2.9); (b) "Growth and decay" (1.8.2.1); and (c) "Substituting into or rearranging formulas" (1.6.2.15). The performance expectation code for this example is "Performing routine procedures" (2.2.2).

- Question 3 (page 17): The topic codes for this example are (a) "Pythagorean Theorem and its applications" (1.3.3.2); and (b) "Rounding and significant figures" (1.1.5.2). Performance expectation codes for this example are (a) "Recalling mathematical objects and properties" (2.1.3); (b) "Performing routine procedures" (2.2.2); and (c) "Using more complex procedures" (2.2.3).

- Question 4 (page 17): The topic codes for this example are (a) "Angles" (1.3.2.2) and (b) "Parallelism and perpendicularity" (1.3.2.3). Performance expectation codes for this example are (a) "Relating representations" (2.5.2); (b) "Formulating and clarifying problems and situations" (2.3.1); and (c) "Solving" (2.3.3).

### Create Chapter Summary Measures

After the chapter review questions were coded, the assigned curriculum topic and performance expectation codes were used to create summary measures for each textbook chapter. The chapter summary measures were the total content page count and a set of weighted topic content page counts. The Center for Research in Mathematics and Science Education at MSU generated the chapter summary measures for all textbook chapters coded for use in the MCS.

The total content page count represents the total amount of instructional material in the chapter. For each mathematics topic, the number of times that topic was coded in the chapter review questions was summed and then divided by the number of chapter review questions. This proportion was then multiplied by the number of pages in the textbook chapter, creating a value that represents the number of content pages devoted to that topic in the chapter. These values were summed across all mathematics topics to produce a total content page count for the chapter.

Both the amount of instructional material covered, which is measured by total subject matter content, and the tasks attempted, which is measured by performance expectations, contribute to how a student learns a subject. Therefore, both are key components in a quality curriculum (Gamoran et al. 1997). A weighted topic content page count represents the amount of instructional material within the chapter devoted to a mathematics topic, as weighted by ratings of the performance expectation codes assigned to the chapter review questions concerning the topic. The weight represents the topic's cognitive challenge. Cognitive challenge categorizes the complexity of the student tasks required to answer the chapter review questions, as measured by the performance expectation codes. Each performance expectation code assigned to a chapter review question was ranked on a four-point scale. A rank of 1 indicated the lowest level of complexity, such as recalling mathematical definitions or performing routine procedures. A rank of 4 indicated the highest level of complexity, such as problem solving or proving theorems. Chart A1 lists all of the performance expectation codes in Section 2. Chart A3 shows the four-point scale on which the performance expectations were ranked. If a chapter review question was assigned more than one performance expectation code, then the question's rank equaled the highest rank among the performance expectation codes.

Calculating the weighted topic content page counts were similar to calculating the total content page count. For each chapter review question, a coded topic received a value equal to the highest rank of performance expectations coded for that question. If the topic was not coded for the question, indicating it was absent, it received a value of zero. These values were summed for all chapter review questions and divided by the number of chapter review questions. This proportion was then multiplied by the number of pages in the textbook chapter to create a weighted topic content page count for each topic in the chapter.

To streamline the creation of the weighted topic content page counts, the over 200 mathematics topics from the TIMSS framework were aggregated into 32 general topic groupings commonly associated with elementary and secondary education mathematics textbooks. These 32 groupings reflected the basic hierarchical structure of the mathematics curriculum framework as vetted by mathematics curriculum experts worldwide (Robitaille et al. 1993). These groupings are an expansion upon the 29 groupings used for the AHAA study conducted in 2004 (Schiller et al. 2008). The "Numbers and Arithmetic" grouping was replaced by groupings for its three main subtopics, while the "Validation and Structuring" groupings was split into separate groups. These groupings covered all mathematics topics, not just algebra I and geometry. Chart A1 lists all of the over 200 mathematics topics in section 1, Curriculum Topics. Chart A2 shows how the framework aggregated the mathematics topics into the 32 general topic groupings (in the third column labeled "Initial grouping label").

For example, the mathematics topic illustrated by Question 1 from the "Understanding textbook coding" section (found on pages 16 and 17 of this report) was "Linear equations and their formal (closed) solutions" (mathematics framework code 1.6.2.5). For the chapter summaries, this topic was among the five topics collapsed into the "Basic equation" grouping. For Question 4, both the mathematics topics of "Angles" and "Parallelism and perpendicularity" (mathematics framework codes 1.3.2.2 and 1.3.2.3) were collapsed into the "Two-dimensional geometry" grouping.

## Course Analyses

The second step of the study analyses focused on the courses. After the chapter review questions were coded and the information was summarized for chapters, course summary measures of the content of algebra I and geometry courses were created. A factor analysis was conducted to identify the patterns in the mathematics content coverage across courses. A discriminant analysis using the course summary measures and results of the factor analysis was conducted to classify the courses into four distinct content categories.

### Course Summary Measures

The chapter summary measures were used to create the course summary measures, which were the total content page count and weighted topic content page counts. The chapters that a course covered formed the basis for calculating the course summary measures. Each school reported the textbook chapters that the instruction in each algebra I or geometry course was expected to cover. Over 400 different variations of the course chapters covered were identified across the approximately 2,000 algebra I and geometry courses in the study for which information for at least one coded textbook was available.

The total content page count for each textbook chapter that was covered in a course was summed to create the course's total content page count. For each of the 32 general topic groupings, the weighted topic content page count for each textbook chapter that was covered in the course was summed to produce the course's weighted topic content page count. A weighted percentage distribution of the 32 general topic groupings for the course was calculated from these weighted topic content page counts. The overall cognitive challenge rating for the course was calculated by summing the weighted topic content page counts across all topic groupings and dividing the sum by the total content page count.

An examination of the weighted percentage distributions of the approximately 2,000 algebra I and geometry courses with coded textbook data showed that the content of the algebra I and geometry courses varied widely. For example, among algebra I courses, the percentage of course content devoted to basic equations varied from 1 percent to 46 percent, with the average being just under 27 percent. While some topic groupings tended to cluster together, the relationships among topics were complex. For example, algebra I courses with a greater percentage of basic equations content also tended to have greater percentages of content in pre-equations and basic number theory, as well as smaller percentages in basic functions content. However, the percentage of content devoted to basic equations was not related to the percentages in advanced equations or advanced functions. The inclusion of different nonfocal topics (i.e., those topics other than algebra for algebra I courses and other than geometry for geometry courses) varied greatly across courses and averaged approximately one-third of subject matter content in both algebra I and geometry courses.

### Factor Analysis: Identifying Patterns of Subject Matter Content Across Courses

The variation in the overall percentage distribution of the 32 general topic groupings and the clustering of some topic groupings revealed the complex relationships among the topics. Factor analysis was used to reduce the complex relationships among the mathematics topic groupings into a smaller number of algebra I and geometry course curriculum factors. A separate factor analysis was done for the two mathematics courses, and each analysis used the weighted percentage distribution of mathematics topic groupings. Weighted percentage distributions were used when conducting the factor analysis because they captured the relative content emphasis and no problems with the reliability of the factor scores were found. The factor analysis used principal component extraction with a varimax rotation procedure. The varimax rotation was used because it produced uncorrelated factors, which was necessary so that the factors generated from the factor analysis could be used in the subsequent discriminant analysis. Both the varimax orthogonal rotation and the direct oblimin oblique rotation methods were tested. While both approaches consistently identified the same number of correlated factors, the varimax orthogonal rotation method was used in the final factor analysis to address the multicollinearity among the topic groupings. The Kaiser criterion was used for factor selection, which determines the number of nontrivial latent dimensions in the input data by the number of eigenvalues from the input correlation matrix that are greater than 1.0. Factor analysis was used in this study solely for descriptive purposes—to mathematically summarize variation in topical content across courses. Both the Kaiser criterion and scree plots were selected as standard methods for determining the optimal number of factors that capture the vast majority of the underlying content variation. The Kaiser criterion and scree plots identified the same number of factors. The Kaiser criterion was used in the final analysis because it involved less subjectivity in determining the number of factors.

Initial factor analyses had problems of skewness due to the nonfocal mathematics topic groupings that rarely appear in algebra I and geometry textbooks. To address the skewness, the 32 general topic groupings used to calculate the chapter summary measures were aggregated to 17 topic groupings prior to the final factor analysis. Those topic groupings designated as algebra or geometry were not aggregated. Only nonfocal topic groupings (i.e., the groupings of mathematics topics that are not designated as algebra or geometry) were aggregated. There were two criteria required for aggregating the nonfocal topic groupings. First, topic groupings could be aggregated if they were taught at comparable grade levels internationally, as determined using the International Grade Placement (IGP) index from TIMSS (Schmidt et al. 1997). The IGP provides a composite among 40 international countries of at what grade levels mathematics topics are taught. What constituted "comparable" grade levels was in relation to high school algebra I and geometry courses. Topics normally taught before algebra I and geometry were considered comparable to one another, while topics normally taught after these courses were considered comparable to one another. For example, according to the IGP, both fractions and operations were primarily taught before high school, so they were considered taught at a comparable grade level, for the purposes of this analysis only. Second, topic groupings could be aggregated if the percentages of content in the categories were significantly positively related to each other, as determined by correlations across courses calculated in this study. For example, results from the MCS showed that courses with a greater percentage of content in fractions also tended to have a greater percentage of content in operations. Thus, the fractions topic grouping was aggregated with the operations topic grouping into a broader grouping called "Arithmetic." The goal was to have the combined topic groupings account for a sizeable amount of content without masking meaningful variation in topic coverage across courses. For example, if mathematics topics taught before algebra I and topics taught after geometry were combined, meaningful variations in topic coverage may not be evident. Chart A2 shows how the mathematics topic groupings were aggregated from 32 groupings (listed in the third column as "Initial grouping label") to 17 groupings (listed in the second column as "Factor analysis grouping label").

The five resulting algebra I factors constituted distinct patterns in mathematics course content. For example, the first algebra I factor focused on geometry content, as it was strongly associated with measures of vector and two- and three-dimensional geometry. The second factor concentrated less on introductory algebra, as it was negatively associated with measures of pre-equation and basic number theory. The third factor focused on higher level algebra, as it was associated with coverage of basic functions and data representation, but negatively associated with basic equations and coordinate geometry. The fourth factor concentrated more on higher level mathematics, as it was strongly associated with advanced mathematics and uncertainty and probability and negatively associated with arithmetic. The fifth factor was negatively associated with coverage of advanced equations and positively associated with advanced number theory. The five course curriculum factors accounted for 64 percent of the variation in content across algebra I courses. Table A1 lists the algebra I course factor analysis results.

The six resulting geometry factors also portrayed distinct patterns in mathematics course content. For example, the first factor indicated algebraic content within a geometry course. This factor was strongly associated with coverage of basic functions, advanced functions, advanced number theory, and data representation, and negatively associated with two-dimensional geometry. The second factor also focused on algebraic content, being strongly associated with the measures of basic number theory and advanced equations and negatively associated with pre-geometry. Factors 3, 4, and 5 had strong (but reversed) associations with at least one type of advanced geometry content, such as three-dimensional geometry or coordinate geometry. The sixth factor was strongly associated with advanced mathematics. The six factor-based indicators accounted for 77 percent of the variation in content across geometry courses. Table A1 lists the geometry course factor analysis results.

### Discriminant Analysis: Classifying Courses Into Course Categories

When the analysis plan for this study was developed, there were no universally established criteria for what defined the curriculum or the rigor for algebra I and geometry courses. Therefore, information from the 2005 NAEP HSTS was used to develop empirically derived categorizations of algebra I and geometry courses based on patterns in subject matter content and degree of cognitive challenge in textbooks adopted by schools. The classification process was done separately for algebra I and geometry. Discriminant function analyses were conducted to create distinct course categories. The process behind the discriminant function analyses was to identify school courses by category (low, medium, high, or integrated), using the assumption that there were identifiable differences in curriculum content across the nation. In broad terms, low courses indicate mathematics courses that generally cover basic topics and offer simple or repetitive student exercises, while high courses indicate mathematics courses that generally include more advanced topics and pose more challenging exercises to the students. Medium courses resemble the content and challenge found in regular mathematics courses. The Classification of Secondary School Courses (CSSC) codes assigned to algebra I and geometry courses were used to develop criteria for classifying courses based on observed trends in curriculum content in different types of courses across the country. As part of the HSTS, each course was assigned a CSSC code by matching the course description from the high school course catalog to the course descriptions on the CSSC code list.

There were four algebra I courses distinguished by the CSSC: year one of two-year algebra I, year two of two-year algebra I, regular algebra I, and integrated (or unified) mathematics I. A two-year algebra I course is an algebra course designed to be taught in a two-year sequence. Year one reviews pre-algebra topics and teaches students to solve first-degree equations and inequalities, while the year two covers topics such as polynomial and quadratic equations with an emphasis on formal problem solving. A first-year integrated mathematics course interweaves algebra, geometry, trigonometry, analysis, statistics, and other mathematics topics into a single course that is generally taken at the same time most students take algebra I courses. There were also four geometry courses distinguished by the CSSC: informal geometry, regular geometry, honors geometry, and integrated (or unified) mathematics II. An informal geometry course is a simplified geometry course that focuses more on practical applications and less on proving theorems. An honors geometry course covers such topics as three-dimensional and coordinate geometry and incorporates formal proofs. A second-year integrated mathematics course interweaves algebra, geometry, trigonometry, analysis, statistics, and other mathematics topics into a single course that is generally taken at the same time most students take geometry courses.

A discriminant function analysis for algebra I courses generated predictions of classification into one of the four defined algebra I course categories. Classification of first-year integrated mathematics courses was based on how closely they matched a course with an integrated curriculum. Classification of the nonintegrated algebra I courses was based on how closely the course content mirrored either year one or year two of a two-year algebra I class or a regular algebra I class. The input data for this discriminant function analysis included the course summary measures of total content page count and overall cognitive challenge rating, as well as the five algebra I subject-matter content factors from the algebra I course factor analysis.

A discriminant function analysis for geometry courses generated predictions of classification into one of the four defined geometry course categories. Classification of second-year integrated mathematics courses was based on how closely the course content matched a course with an integrated curriculum. Classification of the nonintegrated geometry courses was based on how closely the course content mirrored either informal geometry, regular geometry, or honors geometry. The input data for this discriminant function analysis included the course summary measures of total content page count and overall cognitive challenge rating, as well as the six geometry subject-matter content factors from the geometry course factor analysis.

Courses were classified into course categories based on the probabilities calculated from the discriminant functions. If a course best matched an integrated curriculum, it was classified into the integrated category. Otherwise, the probability of a course being each CSSC classification was rank ordered and divided into thirds. These cutpoints in the range of probabilities were used to classify the courses into three categories—low, medium, and high. Considerations used to set the cutpoints and determine the number of course categories included: (a) that groups of courses were large enough that statistics generated for major student subgroups, such as student race/ethnicity, would meet the minimum reporting size (i.e., 62 or more observations); (b) that groups of courses were still relatively similar in subject matter content; and (c) that the interpretation of course types described in the study would apply for both algebra I and geometry. Initially, five groups of nonintegrated courses were considered in producing distinctive profiles in subject matter content, but it resulted in two of the groups being too small to meet statistical reporting standards. Based on the discriminant analysis results, approximately 67 percent of geometry courses and 71 percent of algebra I courses had predicted probabilities of approximately 0.4 (i.e., 40 percent) or greater of belonging to the CSSC code on which its course category was derived.

There were four different course categories that algebra I and geometry courses could be classified: low, medium, high, and integrated. The definitions of the course categories follow, including the minimum probabilities needed to be classified for each category. Table A3 shows the subject matter percentage breakdown for both integrated and nonintegrated algebra I and geometry courses.

**Integrated Course Category:** Courses that had a unified or integrated approach to mathematics were readily identified because they had extremely high similarities (e.g., probabilities greater than 0.7) to the distinctive characteristics of courses assigned an integrated course CSSC code. The probability of being an integrated course was also strongly negatively associated with the probability of being any of the other algebra I and geometry course types. An algebra I course that had a probability of 0.422 or higher of being an integrated course was assigned the integrated course category, while a geometry course that had a probability of 0.900 or higher of being an integrated course was assigned the integrated course category.

**Low Course Category:** A low course is defined as a mathematics course that covers the basic topics of the subject and/or has students gain knowledge of the subject through simple and repetitive exercises. For this study, a low course would most resemble year one of a two-year algebra I course or an informal geometry course. Low algebra I courses had higher probabilities of being associated with year one of a two-year algebra I course than being associated with year two of a two-year algebra I course. If an algebra I course had a probability of 0.489 or greater of being year one of a two-year algebra I course and a probability of 0.410 or less of being year two of a two-year course, then it was assigned the low algebra I course category. Low geometry courses had higher probabilities of being associated with an informal geometry course than being associated with an honors geometry course. If a geometry course had a probability of 0.158 or greater of being an informal geometry course, a probability of 0.208 or less of being an honors geometry course, and a higher probability of being an informal geometry course than an honors geometry course, then it was assigned the low geometry course category.

**High Course Category:** A high course is defined as a mathematics course that delves into advanced topics within the subject and/or challenges students' knowledge of the subject through exercises such as multistage problems and theorem proofs. For this study, a high course would most resemble year two of a two-year algebra I course or an honors geometry course. High algebra I courses had higher probabilities of being associated with year two of a two-year algebra I course than being associated with year one of a two-year algebra I course. If an algebra I course had a probability of 0.548 or greater of being year two of a two-year algebra I course and a probability of 0.321 or less of being year one of a two-year algebra I course, then it was assigned the high algebra I course category. High geometry courses had higher probabilities of being associated with an honors geometry course than being associated with an informal geometry course. If a geometry course had a probability of 0.294 or greater of being an honors geometry course and a probability of 0.059 or less of being an informal geometry course, then it was assigned a high geometry course category.

**Medium Course Category:** A medium course is defined as a course that covers both basic and advanced topics, and the exercises used to measure students' knowledge of the subject ranges from easy to challenging. For this study, a medium course would most resemble a regular algebra I or geometry course. Algebra I and geometry courses not classified as integrated, low, or high courses were assigned to the medium course category.

## Student Analyses

The third step of the study analyses focused on the students who took algebra I and geometry courses. In this final step, summaries of the data for reporting purposes were developed that described the students who completed the courses. To develop the summaries, the course summaries were linked to the students, patterns in topic coverage were identified through factor analysis, and discriminant analysis was used to classify students' coursework into course levels.

### Student Summary Measures

High schools frequently offer multiple algebra I and geometry courses, and students often take more than one such course. Approximately 50 percent of the MCS schools identified more than one algebra I course in their catalogs or transcripts, while 56 percent had more than one geometry course. The course summary measures were matched to students through the algebra I and geometry courses on students' transcripts. Because of the different ways schools record students' coursetaking on their transcripts, a large number of high school students in this study had multiple entries for algebra I or geometry on their high school transcripts. All cases were examined to determine whether the multiple entries represented a single or multiple courses.

Creating the student summary measures involved processing the course summary measures (i.e., the total content page count, the overall cognitive challenge rating, and the weighted content page counts). How the student summary measurements were created depended on the number of algebra I or geometry course entries recorded on the student's transcript. If there was a single entry on the transcript, or there were multiple entries for the same course (e.g., when students had separate transcript records for the first and second semesters of the same algebra I course), then the student summary measures were equal to the course summary measures. If there were multiple entries on the transcript that represented different courses (e.g., a student repeated an algebra I course that used a different textbook, or a student switched midyear from a regular geometry course to an honors geometry course), then each set of course summary measures was weighted by the course credits earned for the course and then standardized to a year-long (i.e., one credit) course.

In situations where there were multiple entries on the transcript that represented different courses, the total content page count and the weighted content page counts for each course were weighted by the course credits the student earned for the course. If a student did not pass the course and earned no course credits, then it was weighted by the course credits the student would have earned for passing the course. These counts were summed across all courses and then divided by the total number of course credits earned by the student. These calculations resulted in the student's total content page count and weighted content page counts. These weighted content page counts were then summed across all topics, and the sum was used to calculate a weighted percentage distribution of the mathematics topic groupings for the student. The student's overall cognitive challenge rating was also calculated by dividing the sum of its weighted topic content page counts by its total content page count.

### Factor Analysis: Identifying Patterns of Subject Matter Content Across Students

After the student summary measures were created, the information was used in factor analyses to identify the patterns of the mathematics topics across students who took algebra I and geometry courses. Separate factor analyses were done for the two mathematics courses, and each analysis used the weighted percentage distribution of mathematics topic groupings calculated for students. Factor analyses were run for students who took the courses, using the same procedures as were done for the course factor analyses (i.e., a principal component analysis with a varimax rotation procedure, and using the Kaiser criterion for factor selection). The uncorrelated factors produced were then used in the discriminant function analyses.

The 17 mathematics topic groupings that were aggregated during the course analyses from the 32 general mathematics topic groupings were also used in this factor analysis. The results from the factor analyses showed similar patterns of subject matter groupings across students as were found across courses. For students who took one or more algebra I courses, factors 2 and 3 corresponded to factors 3 and 2, respectively, from the algebra I course factor analysis. The other three factors showed quite similar loadings. The five underlying factors of cumulative algebra I course content accounted for 67 percent of the variation in the 17 mathematics topic groupings. For students who took one or more geometry courses, factors 1, 4, and 5 had similar factor loadings to factors 1, 5, and 4, respectively, from the geometry course factor analysis. Factors 2, 3 and 6 showed different patterns from the course factor analyses. The six underlying factors of cumulative geometry content indicators accounted for nearly 80 percent of the variation in the 17 mathematics topic groupings. Table A2 lists the student factor analysis results.

### Discriminant Analysis: Classifying Student Coursework Into Course Levels

The purpose of the discriminant function analyses was to create distinct categories of students' algebra I and geometry coursework.

These categories of student coursework are identified as "course levels" within this report. The classification process was done separately for algebra I and geometry coursework. For these discriminant function analyses, students' coursework was not characterized using CSSC codes, because a given student could enroll in more than one course with different codes (e.g., first and second year of algebra I or honors and regular geometry). The algebra I or geometry curricula taken by students throughout high school were instead classified based on the previously defined low, medium, high, or integrated course categories, which served as the dependent variable for the student discriminant analyses. The input data for the discriminant function analyses included the student summary measures of the total content page count and the overall cognitive challenge ranking, along with the subject-matter content indicators from the student factor analysis (five factors for algebra I and six factors for geometry).

The estimated discriminant functions generated predictions of classification into the three course categories in each subject. Student coursework could also be categorized as integrated courses. For cases in which the course categories defined by the course discriminant analysis could be compared to the student discriminant analysis results, approximately 95 percent of students' geometry coursework and 91 percent of students' algebra I coursework had probabilities of approximately 0.4 or greater of being classified as their given course category. The final course levels were based on the highest classification probability generated from the student discriminant analysis. Students who had enrolled in courses with different course categorizations (e.g., both low and high because they had enrolled in both years of two-year algebra I) were then classified into the course level that most closely matched their overall curriculum profile (i.e., had the highest probability).

If a student took different types of algebra I courses, such as a regular algebra I course and honors algebra I course, then the student was classified as having mixed algebra I course types and was not included in the results presented in the Comparison of School Courses section of the report. Students who took different types of geometry courses were treated in the same manner. Approximately 3.0 percent of the students in the algebra I sample had mixed algebra I course types, while approximately 2.7 percent of the students in the geometry sample had mixed geometry course types.

## Course Levels

Students' algebra I and geometry coursework could be classified into one of three course levels—beginner, intermediate, and rigorous—or as an integrated mathematics course. Each student was assigned a course level based on the highest probability, as generated by the student discriminant function analysis, that the student's coursework could be categorized as a beginner, intermediate, rigorous, or integrated course. The beginner, intermediate, and rigorous course levels are analogous to the low, medium, and high course categories, respectively. The difference between the two sets of measures is the level of the analysis. The course categories are generated at the course level and measure the content and challenge of a single algebra I or geometry course that was listed on the student's transcript, while the course levels are generated at the student level and measure the overall content and challenge of a student's coursework in all algebra I or geometry courses that were listed on the student's transcript. The definitions of the course levels follow. Table A4 shows the subject matter percentage breakdown for both integrated and nonintegrated algebra I and geometry coursework among students.

**Integrated Mathematics Courses:** If a student's coursework most associated with an integrated or unified approach to mathematics, then the student was classified as having taken an integrated mathematics course. The probability of being an integrated mathematics course must be higher than the probabilities of being any of the other algebra I and geometry course levels. As noted in the report, integrated courses were not ranked by course level due to the small number of high school graduates who took integrated mathematics courses.

**Beginner Level Courses:** If a student's coursework most associated with the low course category, then the student was classified as having taken a beginner level course. The probability of being a beginner level course must be higher than the probabilities of being intermediate level, rigorous level, or integrated mathematics courses. A beginner level algebra I course most associated with year one of a two-year algebra I course, while a beginner level geometry course most associated with an informal geometry course.

**Intermediate Level Courses:** If a student's coursework most associated with the medium course category, then the student was classified as having taken an intermediate level course. The probability of being an intermediate level course must be higher than the probabilities of being beginner level, rigorous level, or integrated mathematics courses. An intermediate level algebra I course most associated with a regular algebra I course, while an intermediate level geometry course most associated with a regular geometry course.

**Rigorous Level Courses:** If a student's coursework most associated with the high course category, then the student was classified as having taken a rigorous level course. The probability of being a rigorous level course must be higher than the probabilities of being beginner level, intermediate level, or integrated mathematics courses. A rigorous level algebra I course most associated with year two of a two-year algebra I course, while a rigorous level geometry course most associated with an honors geometry course

The labels that were assigned to the three course level measures and the integrated course category describe the overall difficulty of each level, as defined by curriculum content and cognitive challenge. The beginner, intermediate, and rigorous labels reflect the hierarchical nature of the course levels, from the least amount of difficulty to the most amount of difficulty. The labels were also deliberately chosen because they contrast with the names that schools assigned their mathematics courses. While the rigorous course level is the highest level, it was not labeled "advanced" as to differentiate the level from the higher level courses that schools often label "advanced" (e.g., Advanced Algebra/ Algebra III and Advanced Geometry). These advanced courses include advanced mathematics topics that are generally not covered in algebra I and geometry courses.

## Curriculum Topics

The percentage distribution of students' algebra I and geometry courses by curriculum topics were reported in the Mathematics Course Profiles section of the report. The 17 mathematics topic groupings used in the course and student factor analyses provided too much detail to concisely characterize the differences in students' algebra I and geometry courses for the report. Therefore, the 17 mathematics topic groupings were collapsed to six broad categories. Both the algebra and geometry topic groupings were aggregated into introductory and advanced categories. The remaining mathematics topic groupings were assigned to categories that indicated whether the topics were traditionally taught before an algebra I course or after a geometry course. The six resulting mathematics topic categories are identified as the "curriculum topic categories" in table A of the Executive Summary and "curriculum topic groups" in figures 1, 3, and 5 and tables 2, 3, and 4 in the Mathematics Course Profiles section. Chart A2 shows how the 17 mathematics topic groupings (in the second column marked "Factor analysis grouping label") were collapsed into the six mathematics topic categories used for reporting (in the first column marked "Main curriculum category").

The definitions of the six main curriculum topic categories are as follows:

**Elementary and Middle School Mathematics:** This category includes mathematics topics that are traditionally taught before a student takes an algebra I course. These topics include elements of basic arithmetic (e.g., addition, subtraction, fractions, and rounding) and pre-geometry (e.g., patterns, perimeter, area, and proportion).

**Introductory Algebra:** This category includes mathematics topics needed to understand the basics of algebra and provide the foundation for learning advanced algebra. These topics include pre-algebra, basic algebraic equations (e.g., algebraic expression, simple linear equations, and simple inequalities), and the basic elements of number theory (e.g., integers, absolute value, and rational numbers).

**Advanced Algebra:** This category includes mathematics topics that cover the more complex elements of algebra. These topics include advanced equations (e.g., quadratic equations, polynomial equations, and matrix solutions), basic functions (e.g., representation of relationships and functions, and graphing functions), advanced functions (e.g., functions of several variables and quadratic functions), and advanced number theory (e.g., real numbers, exponents, roots, radicals, and matrices).

**Two-Dimensional Geometry:** This category includes mathematics topics that focus on basic linear and planar geometric concepts. Examples of topics in this category include basic geometric concepts (e.g., points, angles, parallelism, and perpendicularity) and the properties of shapes.

**Advanced Geometry:** This category includes mathematics topics that cover advanced geometric concepts such as three-dimensional geometry (e.g., three-dimensional shapes, conic sections), coordinate geometry (e.g., equations of lines, planes, and surfaces in space), and vector geometry (e.g., vectors, transformation, congruence, and similarity).

**Other High School Mathematics:** This category includes mathematics topics that are traditionally taught in courses taken after geometry and algebra II. Examples of topics in this category include trigonometry, pre-calculus, statistics (e.g., data representation and analysis, uncertainty and probability), validation and structuring (e.g., logic, set theory, and axioms), discrete mathematics (e.g., tree diagrams and binary arithmetic), finite mathematics, and calculus.

## Weighting and Variance Estimation

In the same way that schools and students participating in the HSTS were chosen to be nationally representative of public and private high school graduates, the schools and students participating in the MCS were selected to be representative of those same graduates. The results from the NAEP twelfth-grade mathematics assessment were included to provide accurate estimates of overall student performance. Results are weighted to take into account the fact that schools and students represent different proportions of the overall populations. All estimates were weighted to provide unbiased estimates of the national population.

The school weights for the 2005 NAEP HSTS participating schools served as the basis for applying textbook nonresponse adjustments to compensate for textbook nonresponse. The 2005 NAEP HSTS sampling weights for schools and students in the textbook study were adjusted to compensate for the loss of 2005 NAEP HSTS participating schools that offered algebra I and geometry classes but did not provide textbook data for this study. The technical details of the original 2005 NAEP HSTS sampling weights are described in The 2005 High School Transcript Study User's Guide and Technical Report (Shettle et al. 2008).

The school weights of the nonresponding schools were distributed to those responding schools within weighting classes. The weighting classes were defined following the classification criteria adopted for the 2005 NAEP HSTS. The adjustment factor for each class was prorated by total student enrollment among the responding and nonresponding schools. Both linked and unlinked samples were weighted to represent the national population. Two sets of adjustment factors were defined—for all 2005 NAEP HSTS participating schools (unlinked weights) and for the 2005 NAEP HSTS schools that also participated in 2005 NAEP (linked weights). The school weights were used in analyzing the variation in curriculum content across courses during the development of course-level indicators to adjust for differences in probabilities that schools were selected for and participated in the 2005 NAEP HSTS.

The student weights were computed using the textbook-compensated school weights; that is, the school weights that were revised for textbook nonresponse. The sequence of steps for student weighting was the same as those used for the 2005 NAEP HSTS students. For the student base weights, the textbook-compensated school weights were used to replace the school nonresponse adjusted weights. All adjustments for nonresponse and weight trimming were conducted in the same manner, again producing two sets of student weights—for students in all 2005 NAEP HSTS participating schools (unlinked weights) and the students in 2005 NAEP HSTS that were also 2005 NAEP participating schools (linked weights). The student weights were used to adjust for differences in sampling probabilities of the 12,500 students who had taken algebra I and the 15,900 students who had taken geometry during high school. All results generated for the MCS were at the student-level and incorporated either the unlinked or linked weights. The replicate weights for variance estimation were calculated along with the full sample weights, repeating each adjustment in the same manner as the full sample weights.

High school student estimates for the MCS were subject to sampling error because they were derived from a sample, rather than the whole population. Sampling error was measured by the sampling variance, which indicated how much the population estimate for a given statistic was likely to change if it had been based on another equivalent sample of individuals drawn in exactly the same manner as the actual sample. Variances were estimated using jackknife replication methods (Krewski and Rao 1981). This estimation involved measuring the variability among subsamples (replicates) to generate an accurate estimate of variance for the full sample.

## Interpreting Statistical Significance

Comparisons over time or between groups are based on statistical tests that consider both the estimated size of the difference and the standard error of that estimated difference. When a difference—such as the difference between the average scores of two groups—has a large standard error, a numerical difference that seems large may not be statistically significant (i.e., a null hypothesis of no difference cannot be rejected with sufficient confidence). Differences of the same estimated size may be statistically significant in some cases but not others, depending on the sizes of the standard errors involved. For this report, only those differences that are found to be statistically significant are discussed as higher or lower. In conducting the statistical significance tests used in this report, no adjustments were made for multiple comparisons.

## CHART A1. Mathematics framework curriculum topics and performance expectations

- 1. Curriculum Topics
  - 1.1. Numbers
    - 1.1.1. Whole numbers
      - 1.1.1.1. Meaning
        - 1.1.1.1.1. The uses of numbers
        - 1.1.1.1.2. Place value and numeration
        - 1.1.1.1.3. Ordering and comparing numbers
      - 1.1.1.2. Operations
        - 1.1.1.2.1. Addition
        - 1.1.1.2.2. Subtraction
        - 1.1.1.2.3. Multiplication
        - 1.1.1.2.4. Division
        - 1.1.1.2.5. Mixed operations
      - 1.1.1.3. Properties of operations
        - 1.1.1.3.1. Associative properties
        - 1.1.1.3.2. Commutative properties
        - 1.1.1.3.3. Identity properties
        - 1.1.1.3.4. Distributive properties
        - 1.1.1.3.5. Other number properties
    - 1.1.2. Fractions and decimals
      - 1.1.2.1. Common fractions
        - 1.1.2.1.1. Meaning and representation of common fractions
        - 1.1.2.1.2. Computations with common fractions and mixed numbers
      - 1.1.2.2. Decimal fractions
        - 1.1.2.2.1. Meaning and representation of decimals
        - 1.1.2.2.2. Computations with decimals
      - 1.1.2.3. Relationships of common and decimal fractions
        - 1.1.2.3.1. Conversion to equivalent forms
        - 1.1.2.3.2. Ordering of fractions and decimals
      - 1.1.2.4. Percentages
        - 1.1.2.4.1. Percent computations
        - 1.1.2.4.2. Various types of percent problems
      - 1.1.2.5. Properties of common and decimal fractions
        - 1.1.2.5.1. Associative properties
        - 1.1.2.5.2. Commutative properties
        - 1.1.2.5.3. Identity properties
        - 1.1.2.5.4. Inverse properties
        - 1.1.2.5.5. Distributive properties
        - 1.1.2.5.6. Cancellation properties
        - 1.1.2.5.7. Other number properties
    - 1.1.3. Integers, rational and real numbers
      - 1.1.3.1. Negative numbers, integers, and their properties
        - 1.1.3.1.1. Concept of integers
        - 1.1.3.1.2. Operations with integers
        - 1.1.3.1.3. Concept of absolute value
        - 1.1.3.1.4. Properties of integers

**CHART A1.** Mathematics framework curriculum topics and performance expectations (continued)

- 1.1.3.2. Rational numbers and their properties
  - 1.1.3.2.1. Concept of rational numbers
  - 1.1.3.2.2. Operations with rational numbers
  - 1.1.3.2.3. Properties of rational numbers
  - 1.1.3.2.4. Equivalence of differing forms of rational numbers
  - 1.1.3.2.5. Relation of rational numbers to terminating and recurring decimals
- 1.1.3.3. Real numbers, their subsets, and properties
  - 1.1.3.3.1. Concept of real numbers (including concept of irrationals)
  - 1.1.3.3.2. Subsets of real numbers (integers, rational numbers, etc.)
  - 1.1.3.3.3. Operations with real numbers and absolute value
  - 1.1.3.3.4. Properties of real numbers (including density, order, properties of absolute value, completeness, etc.)

1.1.4. Other numbers and number concepts
- 1.1.4.1. Binary arithmetic and other number bases
- 1.1.4.2. Exponents, roots, and radicals
  - 1.1.4.2.1. Integer exponents and their properties
  - 1.1.4.2.2. Rational exponents and their properties
  - 1.1.4.2.3. Roots and radicals and their relation to rational exponents
  - 1.1.4.2.4. Real exponents
- 1.1.4.3. Complex numbers and their properties
  - 1.1.4.3.1. Concept of complex numbers
  - 1.1.4.3.2. Algebraic form of complex numbers and their properties
  - 1.1.4.3.3. Trigonometric form of complex numbers and their properties
  - 1.1.4.3.4. Relation of algebraic and trigonometric form of complex numbers
- 1.1.4.4. Number theory
  - 1.1.4.4.1. Primes and factorization
  - 1.1.4.4.2. Elementary number theory,
- 1.1.4.5. Systematic counting
  - 1.1.4.5.1. Tree diagrams and other forms of systematic counting
  - 1.1.4.5.2. Permutations, combinations
- 1.1.4.6. Matrices
  - 1.1.4.6.1. Concept of a matrix
  - 1.1.4.6.2. Operations with matrices
  - 1.1.4.6.3. Properties of matrices

1.1.5. Estimation and number sense
- 1.1.5.1. Estimating quantity and size
- 1.1.5.2. Rounding and significant figures
- 1.1.5.3. Estimating computations
  - 1.1.5.3.1. Mental arithmetic
  - 1.1.5.3.2. Reasonableness of results
- 1.1.5.4. Exponents and orders of magnitude

1.2. Measurement

1.2.1. Units
- 1.2.1.1. Concept of measure (including nonstandard units)
- 1.2.1.2. Standard units (including metric system)
- 1.2.1.3. Use of appropriate instruments
- 1.2.1.4. Common measures (length, area, volume, time, calendar, money, temperature, mass, weight, angles)
- 1.2.1.5. Quotients and products of units (km/h, m/s, etc.)
- 1.2.1.6. Dimensional analysis

**CHART A1.** Mathematics framework curriculum topics and performance expectations (continued)

- 1.2.2. Computations and properties of length, perimeter, area, and volume
  - 1.2.2.1. Computations, formulas and properties of length and perimeter
  - 1.2.2.2. Computations, formulas and properties of area
  - 1.2.2.3. Computations, formulas and properties of surface area
  - 1.2.2.4. Computations, formulas and properties of volumes
- 1.2.3. Estimation and error
  - 1.2.3.1. Estimation of measurement and errors of measurement
  - 1.2.3.2. Precision and accuracy of measurement

1.3. Geometry: position, visualization, and shape

- 1.3.1. One- and two-dimensional coordinate geometry
  - 1.3.1.1. Line and coordinate graphs
  - 1.3.1.2. Equations of lines in a plane
  - 1.3.1.3. Conic sections and their equations
- 1.3.2. Two-dimensional geometry basics
  - 1.3.2.1. Points, lines, segments, half-lines, and rays
  - 1.3.2.2. Angles
  - 1.3.2.3. Parallelism and perpendicularity
- 1.3.3. Polygons and circles
  - 1.3.3.1. Triangles and quadrilaterals: their classification and properties
  - 1.3.3.2. Pythagorean Theorem and its applications
  - 1.3.3.3. Other polygons and their properties
  - 1.3.3.4. Circles and their properties
- 1.3.4. Three-dimensional geometry
  - 1.3.4.1. Three-dimensional shapes and surfaces and their properties
  - 1.3.4.2. Planes and lines in space
  - 1.3.4.3. Spatial perception and visualization
  - 1.3.4.4. Coordinate systems in three dimensions
  - 1.3.4.5. Equations of lines, planes and surfaces in space
- 1.3.5. Vectors
- 1.3.6. Simple topology

1.4. Geometry: symmetry, congruence, and similarity

- 1.4.1. Transformations
  - 1.4.1.1. Patterns, tessellations, friezes, stencils
  - 1.4.1.2. Symmetry
  - 1.4.1.3. Transformations
- 1.4.2. Congruence and similarity
  - 1.4.2.1. Congruence
  - 1.4.2.2. Similarities (similar triangles and their properties, other similar figures and properties)
- 1.4.3. Constructions with straight-edge and compass

1.5. Proportionality

- 1.5.1. Proportionality concepts
  - 1.5.1.1. Meaning of ratio and proportion
  - 1.5.1.2. Direct and inverse proportion
- 1.5.2. Proportionality problems
  - 1.5.2.1. Solving proportional equations
  - 1.5.2.2. Solving practical problems with proportionality
  - 1.5.2.3. Scales (maps and plans)
  - 1.5.2.4. Proportion based on similarity

**CHART A1.** Mathematics framework curriculum topics and performance expectations (continued)

- 1.5.3. Slope and simple trigonometry
  - 1.5.3.1. Slope and gradient in straight line graphs
  - 1.5.3.2. Trigonometry of right triangles
- 1.5.4. Linear interpolation and extrapolation

1.6. Functions, relations, and equations

- 1.6.1. Patterns, relations, and functions
  - 1.6.1.1. Number patterns
  - 1.6.1.2. Relations and their properties
  - 1.6.1.3. Functions and their properties
  - 1.6.1.4. Representation of relations and functions
  - 1.6.1.5. Families of functions (graphs and properties)
  - 1.6.1.6. Operations on functions
  - 1.6.1.7. Related functions (inverse, derivative, etc.)
  - 1.6.1.8. Relationship of functions and equations (e.g., zeroes of functions as roots of equations)
  - 1.6.1.9. Interpretation of function graphs
  - 1.6.1.10. Functions of several variables
  - 1.6.1.11. Recursion
  - 1.6.1.12. Linear functions
  - 1.6.1.13. Quadratic functions
  - 1.6.1.14. Logarithmic and exponential functions
  - 1.6.1.15. Trigonometric functions
- 1.6.2. Equations and formulas
  - 1.6.2.1. Representation of numerical situations by equations
  - 1.6.2.2. Informal solution of simple equations
  - 1.6.2.3. Operations with expressions and evaluating expressions
  - 1.6.2.4. Equivalent expressions (including factorization and simplification)
  - 1.6.2.5. Linear equations and their formal (closed) solutions
  - 1.6.2.6. Quadratic equations and their formal (closed) solutions
  - 1.6.2.7. Polynomial equations and their solutions
  - 1.6.2.8. Trigonometrical equations and identities
  - 1.6.2.9. Logarithmic and exponential equations and their solutions
  - 1.6.2.10. Solution of equations reducing to quadratics, radical equations, absolute value equations, etc.
  - 1.6.2.11. Other solution methods for equations (e.g., successive approximation)
  - 1.6.2.12. Inequalities and their graphical representation
  - 1.6.2.13. Systems of equations and their solutions (including matrix solutions)
  - 1.6.2.14. Systems of inequalities
  - 1.6.2.15. Substituting into or rearranging formulas
  - 1.6.2.16. General equation of the second degree and its interpretation
- 1.6.3. Trigonometry and analytic geometry
  - 1.6.3.1. Angle measures: radians and degrees
  - 1.6.3.2. Law of sines and cosines
  - 1.6.3.3. Unit circle and trigonometric functions
  - 1.6.3.4. Parametric equations
  - 1.6.3.5. Polar coordinates
  - 1.6.3.6. Polar equations and their graphs

**CHART A1.** Mathematics framework curriculum topics and performance expectations (continued)

- 1.7. Data representation, probability, and statistics
  - 1.7.1. Data representation and analysis
    - 1.7.1.1. Collecting data from experiments and simple surveys
    - 1.7.1.2. Representing data
    - 1.7.1.3. Interpreting tables, charts, plots and graphs
    - 1.7.1.4. Kinds of scales (nominal, ordinal, interval, ratio)
    - 1.7.1.5. Measures of central tendency
    - 1.7.1.6. Measures of dispersion
    - 1.7.1.7. Sampling, randomness, and bias related to data samples
    - 1.7.1.8. Prediction and inferences from data
    - 1.7.1.9. Fitting lines and curves to data
    - 1.7.1.10. Correlations and other measures of relations
    - 1.7.1.11. Use and misuse of statistics
  - 1.7.2. Uncertainty and probability
    - 1.7.2.1. Informal likelihoods and the vocabulary of likelihoods
    - 1.7.2.2. Numerical probability and probability models
    - 1.7.2.3. Counting principles
    - 1.7.2.4. Mutually exclusive events
    - 1.7.2.5. Conditional probability and independent events
    - 1.7.2.6. Bayes' Theorem
    - 1.7.2.7. Contingency tables
    - 1.7.2.8. Probability distributions for discrete random variables
    - 1.7.2.9. Probability distributions for continuous random variables
    - 1.7.2.10. Expectation and the algebra of expectations
    - 1.7.2.11. Sampling (distributions and populations)
    - 1.7.2.12. Estimation of population parameters
    - 1.7.2.13. Hypothesis testing
    - 1.7.2.14. Confidence intervals
    - 1.7.2.15. Bivariate distributions
    - 1.7.2.16. Markov processes
    - 1.7.2.17. Monte Carlo methods and computer simulations
- 1.8. Elementary analysis
  - 1.8.1. Infinite processes
    - 1.8.1.1. Arithmetic and geometric sequences
    - 1.8.1.2. Arithmetic and geometric series
    - 1.8.1.3. Binomial Theorem
    - 1.8.1.4. Other sequences and series
    - 1.8.1.5. Limits and convergence of series
    - 1.8.1.6. Limits and convergence of functions
    - 1.8.1.7. Continuity
  - 1.8.2. Change
    - 1.8.2.1. Growth and decay
    - 1.8.2.2. Differentiation
    - 1.8.2.3. Integration
    - 1.8.2.4. Differential equations
    - 1.8.2.5. Partial differentiation

**CHART A1.** Mathematics framework curriculum topics and performance expectations (continued)

- 1.9. Validation and structure
  - 1.9.1. Validation and justification
    - 1.9.1.1. Logical connectives
    - 1.9.1.2. Quantifiers ("for all," "there exists")
    - 1.9.1.3. Boolean algebra and truth tables
    - 1.9.1.4. Conditional statements, equivalence of statements (including converse, contrapositive, and inverse)
    - 1.9.1.5. Inference schemes (e.g., modus ponens, modus tollens)
    - 1.9.1.6. Direct deductive proofs
    - 1.9.1.7. Indirect proofs and proof by contradiction
    - 1.9.1.8. Proof by mathematical induction
    - 1.9.1.9. Consistency and independence of axiom systems
  - 1.9.2. Structuring and abstracting
    - 1.9.2.1. Sets, set notation and set combinations
    - 1.9.2.2. Equivalence relations, partitions and classes
    - 1.9.2.3. Groups
    - 1.9.2.4. Fields
    - 1.9.2.5. Linear (vector) spaces
    - 1.9.2.6. Subgroups, subspaces, etc.
    - 1.9.2.7. Other axiomatic systems
    - 1.9.2.8. Isomorphism
    - 1.9.2.9. Homomorphism
- 1.10. Other content
  - 1.10.1. Informatics (operation of computers, flow charts, learning a programming language, programs, algorithms with applications to the computer, complexity)
  - 1.10.2. History and nature of mathematics
  - 1.10.3. Special applications of mathematics (kinematics, Newtonian mechanics, population growth, networks, linear programming, critical path analysis, economics examples)
  - 1.10.4. Problem solving heuristics
  - 1.10.5. Nonmathematical science content
  - 1.10.6 Nonmathematical content other than science

2. Performance Expectations
- 2.1. Knowing
  - 2.1.1. Representing
  - 2.1.2. Reorganizing equivalents
  - 2.1.3. Recalling mathematical objects and properties
- 2.2. Using routine procedures
  - 2.2.1. Using equipment
  - 2.2.2. Performing routine procedures
  - 2.2.3. Using more complex procedures
- 2.3. Investigating and problem solving
  - 2.3.1. Formulating and clarifying problems and situations
  - 2.3.2. Developing strategy
  - 2.3.3. Solving
  - 2.3.4. Predicting
  - 2.3.5. Verifying

**CHART A1.** Mathematics framework curriculum topics and performance expectations (continued)

- 2.4. Mathematical reasoning
  - 2.4.1. Developing notion and vocabulary
  - 2.4.2. Developing algorithms
  - 2.4.3. Generalizing
  - 2.4.4. Conjecturing
  - 2.4.5. Justifying and proving
  - 2.4.6. Axiomatizing
- 2.5. Communicating
  - 2.5.1. Using vocabulary and notation
  - 2.5.2. Relating representations
  - 2.5.3. Describing/discussing
  - 2.5.4. Critiquing

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, High School Transcript Study (HSTS), Mathematics Curriculum Study, 2005.

**CHART A2.** Aggregation of the Trends in International Mathematics and Science Study (TIMSS) mathematics curriculum framework topics to produce the six main curriculum categories for the Mathematics Curriculum Study: 2005

| Main curriculum category | Factor analysis grouping label | Initial grouping label | Original framework codes |
|---|---|---|---|
| Elementary and middle school mathematics | Arithmetic | Meaning | 1.1.1.1.1 1.1.1.1.2 1.1.1.1.3 |
| | | Operations | 1.1.1.2.1 1.1.1.2.2 1.1.1.2.3 1.1.1.2.4 1.1.1.2.5 |
| | | Properties of operations | 1.1.1.3.1 1.1.1.3.2 1.1.1.3.3 1.1.1.3.4 1.1.1.3.5 |
| | | Fractions | 1.1.2.1.1 1.1.2.1.2 1.1.2.2.1 1.1.2.2.2 1.1.2.3.1 1.1.2.3.2 1.1.2.4.1 1.1.2.4.2 1.1.2.5.1 1.1.2.5.2 1.1.2.5.3 1.1.2.5.4 1.1.2.5.5 1.1.2.5.6 1.1.2.5.7 |
| | | Number theory | 1.1.4.4.1 1.1.4.4.2 |
| | | Estimation | 1.1.5.1 1.1.5.2 1.1.5.3.1 1.1.5.3.2 1.1.5.4 |
| | | Measurement | 1.2.1.1 1.2.1.2 1.2.1.3 1.2.1.4 1.2.1.5 1.2.1.6 1.2.3.1 1.2.3.2 |
| | | Proportionality concepts | 1.5.1.1 1.5.1.2 |
| | Pre-geometry | Patterns | 1.6.1.1 |
| | | Perimeter, area, volume | 1.2.2.1 1.2.2.2 1.2.2.3 1.2.2.4 |
| | | Proportionality problems | 1.5.2.1 1.5.2.2 1.5.2.3 1.5.2.4 |
| Introductory algebra | Pre-equation | Pre-equation | 1.6.2.1 1.6.2.2 |
| | Basic number theory | Basic number theory | 1.1.3.1.1 1.1.3.1.2 1.1.3.1.3 1.1.3.1.4 1.1.3.2.1 1.1.3.2.2 1.1.3.2.3 1.1.3.2.4 1.1.3.2.5 |
| | Basic equations | Basic equations | 1.6.2.3 1.6.2.4 1.6.2.5 1.6.2.12 1.6.2.15 |
| Advanced algebra | Advanced equations | Advanced equations | 1.6.2.6 1.6.2.7 1.6.2.8 1.6.2.9 1.6.2.10 1.6.2.11 1.6.2.13 1.6.2.14 1.6.2.16 |
| | Basic functions | Basic functions | 1.6.1.2 1.6.1.3 1.6.1.4 1.6.1.5 1.6.1.9 1.6.1.12 |
| | Advanced functions | Advanced functions | 1.6.1.6 1.6.1.7 1.6.1.8 1.6.1.10 1.6.1.11 1.6.1.13 1.6.1.14 1.6.1.15 |
| | Advanced number theory | Advanced number theory | 1.1.3.3.1 1.1.3.3.2 1.1.3.3.3 1.1.3.3.4 1.1.4.2.1 1.1.4.2.2 1.1.4.2.3 1.1.4.2.4 1.1.4.3.1 1.1.4.3.2 1.1.4.3.3 1.1.4.3.4 1.1.4.6.1 1.1.4.6.2 1.1.4.6.3 |

**CHART A2.** Aggregation of the Trends in International Mathematics and Science Study (TIMSS) mathematics curriculum framework topics to produce the six main curriculum categories for the Mathematics Curriculum Study: 2005 (continued)

| Main curriculum category | Factor analysis grouping label | Initial grouping label | Original framework codes |
|---|---|---|---|
| Two-dimensional geometry | Two-dimensional geometry | Two-dimensional geometry | 1.3.2.1 1.3.2.2 1.3.2.3 1.3.3.1 1.3.3.2 1.3.3.3 1.3.3.4 |
| Advanced geometry | Three-dimensional geometry | Three-dimensional geometry | 1.3.4.1 1.3.4.2 1.3.4.3 1.3.4.4 1.3.4.5 |
| | Coordinate geometry | Coordinate geometry | 1.3.1.1 1.3.1.2 1.3.1.3 1.5.3.1 |
| | Vectors, transformation, congruence and similarity | Vectors, transformation, congruence and similarity | 1.3.5 1.3.6 1.4.1.1 1.4.1.2 1.4.1.3 1.4.2.1 1.4.2.2 1.4.3 |
| Other high school mathematics | Data representation and analysis | Data representation and analysis | 1.7.1.1 1.7.1.2 1.7.1.3 1.7.1.4 1.7.1.5 1.7.1.6 1.7.1.7 1.7.1.8 1.7.1.9 1.7.1.10 1.7.1.11 |
| | Uncertainty and probability | Uncertainty and probability | 1.7.2.1 1.7.2.2 1.7.2.3 1.7.2.4 1.7.2.5 1.7.2.6 1.7.2.7 1.7.2.8 1.7.2.9 1.7.2.10 1.7.2.11 1.7.2.12 1.7.2.13 1.7.2.14 1.7.2.15 1.7.2.16 1.7.2.17 |
| | Other high school topics | Discrete math | 1.1.4.1 1.1.4.5.1 1.1.4.5.2 |
| | | Linear interpolation and extrapolation | 1.5.4 |
| | | Trigonometry | 1.5.3.2 1.6.3.1 1.6.3.2 1.6.3.3 1.6.3.4 1.6.3.5 1.6.3.6 |
| | | Infinite process | 1.8.1.1 1.8.1.2 1.8.1.3 1.8.1.4 1.8.1.5 1.8.1.6 1.8.1.7 |
| | | Change | 1.8.2.1 1.8.2.2 1.8.2.3 1.8.2.4 1.8.2.5 |
| | | Validation | 1.9.1.1 1.9.1.2 1.9.1.3 1.9.1.4 1.9.1.5 1.9.1.6 1.9.1.7 1.9.1.8 1.9.1.9 |
| | | Structuring | 1.9.2.1 1.9.2.2 1.9.2.3 1.9.2.4 1.9.2.5 1.9.2.6 1.9.2.7 1.9.2.8 1.9.2.9 |
| | Other topics | Other topics | 1.10.1 1.10.2 1.10.3 1.10.4 1.10.5 1.10.6 |

NOTE: The header "Original framework codes" refers to the codes assigned to the more than 200 mathematics topics listed in the TIMSS mathematics curriculum framework.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, High School Transcript Study (HSTS), Mathematics Curriculum Study, 2005.

**CHART A3.** Ranked aggregation of the Trends in International Mathematics and Science Study (TIMSS) mathematics curriculum framework performance expectations codes for the Mathematics Curriculum Study: 2005

| Group rank | Group label | Original framework codes |
|---|---|---|
| 1 (Lowest) | Definition and computation | 2.1.1 2.1.2 2.1.3 2.2.1 2.2.2 2.5.1 |
| 2 | Estimating, using and representing data | 2.2.3 2.5.2 2.5.3 |
| 3 | Formulating problems and critiquing | 2.3.1 2.3.5 2.4.1 2.5.4 |
| 4 (Highest) | Problem solving, advanced reasoning, justifying, and proving | 2.3.2 2.3.3 2.3.4 2.4.2 2.4.3 2.4.4 2.4.5 2.4.6 |

NOTE: The header "Original framework codes" refers to the codes assigned to the more than 20 performance expectations listed in the TIMSS mathematics curriculum framework.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, High School Transcript Study (HSTS), Mathematics Curriculum Study, 2005.

**TABLE A1.** Course-level factor loadings for algebra I and geometry mathematics topics, by mathematics topic grouping: 2005

| Mathematics topic groupings | Algebra I | | | | | Geometry | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Factor 1 | Factor 2 | Factor 3 | Factor 4 | Factor 5 | Factor 1 | Factor 2 | Factor 3 | Factor 4 | Factor 5 | Factor 6 |
| Arithmetic | -0.015 | 0.063 | 0.011 | -0.733 | 0.207 | -0.007 | 0.447 | 0.673 | 0.378 | -0.209 | -0.063 |
| Pre-equation | -0.194 | -0.755 | -0.017 | 0.015 | 0.176 | 0.096 | -0.064 | -0.078 | 0.291 | 0.842 | -0.153 |
| Basic number theory | -0.013 | -0.779 | -0.023 | -0.100 | 0.143 | -0.088 | 0.754 | 0.341 | 0.183 | 0.014 | 0.085 |
| Basic equations | -0.303 | -0.277 | -0.688 | -0.157 | -0.248 | 0.138 | 0.126 | 0.378 | 0.840 | 0.104 | -0.037 |
| Advanced equations | -0.247 | 0.088 | 0.025 | 0.044 | -0.784 | 0.340 | 0.511 | -0.106 | -0.132 | 0.217 | 0.535 |
| Basic functions | -0.184 | 0.045 | 0.773 | 0.169 | -0.194 | 0.668 | 0.082 | 0.099 | 0.259 | -0.469 | 0.092 |
| Advanced functions | -0.110 | 0.258 | 0.078 | 0.146 | -0.515 | 0.723 | 0.283 | -0.104 | -0.061 | 0.334 | 0.068 |
| Advanced number theory | -0.433 | 0.050 | -0.117 | 0.390 | 0.503 | 0.815 | 0.098 | 0.033 | 0.198 | 0.027 | -0.235 |
| Pre-geometry | 0.500 | -0.419 | -0.108 | -0.188 | 0.213 | -0.277 | -0.712 | 0.109 | -0.111 | 0.028 | 0.276 |
| Two-dimensional geometry | 0.681 | 0.110 | 0.210 | -0.077 | 0.197 | -0.779 | -0.021 | 0.064 | -0.190 | 0.090 | -0.486 |
| Three-dimensional geometry | 0.786 | 0.143 | -0.030 | 0.218 | 0.120 | -0.189 | -0.108 | 0.185 | -0.672 | -0.191 | 0.315 |
| Coordinate geometry | -0.314 | 0.417 | -0.606 | -0.093 | 0.178 | -0.074 | 0.070 | -0.822 | 0.061 | 0.033 | 0.149 |
| Vectors, transformation, congruence, and similarity | 0.766 | 0.125 | 0.114 | 0.203 | -0.018 | -0.211 | -0.231 | -0.573 | -0.363 | -0.506 | -0.193 |
| Data representation and analysis | 0.264 | 0.583 | 0.418 | -0.172 | 0.326 | 0.880 | 0.077 | 0.115 | 0.135 | 0.064 | -0.071 |
| Uncertainty and probability | -0.016 | 0.022 | 0.277 | 0.621 | 0.449 | 0.570 | 0.395 | 0.049 | -0.257 | 0.462 | -0.079 |
| Advanced mathematics[1] | 0.294 | 0.095 | 0.106 | 0.671 | -0.036 | -0.035 | -0.155 | -0.119 | -0.183 | -0.168 | 0.805 |
| Other topics | 0.245 | 0.511 | 0.594 | -0.366 | -0.026 | 0.658 | -0.352 | 0.204 | -0.073 | 0.235 | 0.226 |
| Percentage of topic variance explained | 15.6 | 13.8 | 12.6 | 11.3 | 10.4 | 24.0 | 12.0 | 11.0 | 10.9 | 10.3 | 9.2 |

[1] Advanced mathematics is a combination of mathematics topics generally not associated with algebra and geometry. They are generally taught in later mathematics courses, although elements may be taught in algebra and geometry courses. These topics include discrete mathematics, linear interpolation and extrapolation, trigonometry, calculus, and validation and structuring.

NOTE: This table provides course-level factor loadings for algebra I and geometry mathematics topics by mathematics topic groupings. Principal components analyses were used to extract the factor loadings, with a varimax rotation utilizing Kaiser normalization. After 11 rotations, a five factor solution explained 64 percent of the variance for the algebra I topics. After 29 rotations, a six factor solution explained 77 percent of the variance for the geometry topics.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, High School Transcript Study (HSTS), Mathematics Curriculum Study, 2005.

**TABLE A2.** Student-level factor loadings for algebra I and geometry mathematics topics, by mathematics topic grouping: 2005

| Mathematics topic groupings | Algebra I | | | | | Geometry | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Factor 1 | Factor 2 | Factor 3 | Factor 4 | Factor 5 | Factor 1 | Factor 2 | Factor 3 | Factor 4 | Factor 5 | Factor 6 |
| Arithmetic | -0.134 | 0.163 | -0.011 | -0.792 | 0.096 | 0.109 | 0.867 | -0.048 | -0.176 | -0.167 | 0.230 |
| Pre-equation | -0.035 | -0.217 | -0.802 | 0.147 | 0.100 | -0.051 | -0.082 | 0.119 | 0.899 | -0.139 | 0.000 |
| Basic number theory | -0.066 | -0.116 | -0.836 | -0.091 | 0.105 | -0.109 | 0.739 | 0.376 | -0.055 | -0.153 | -0.019 |
| Basic equations | -0.261 | -0.764 | -0.187 | -0.176 | -0.213 | 0.330 | 0.738 | -0.272 | 0.446 | -0.035 | -0.001 |
| Advanced equations | -0.331 | 0.055 | 0.188 | 0.030 | -0.763 | 0.084 | 0.136 | 0.787 | -0.052 | 0.335 | -0.072 |
| Basic functions | -0.215 | 0.749 | 0.055 | 0.239 | -0.292 | 0.812 | 0.224 | -0.114 | -0.295 | -0.028 | 0.066 |
| Advanced functions | -0.130 | -0.126 | 0.384 | -0.025 | -0.430 | 0.558 | -0.138 | 0.570 | 0.293 | -0.123 | 0.052 |
| Advanced number theory | -0.316 | -0.161 | 0.132 | 0.232 | 0.723 | 0.816 | 0.046 | 0.237 | 0.177 | -0.256 | 0.043 |
| Pre-geometry | 0.538 | -0.213 | -0.331 | -0.071 | 0.057 | -0.188 | -0.340 | -0.447 | 0.093 | 0.498 | 0.268 |
| Two-dimensional geometry | 0.678 | 0.372 | -0.064 | -0.091 | 0.144 | -0.801 | -0.043 | -0.158 | 0.000 | -0.502 | 0.112 |
| Three-dimensional geometry | 0.823 | 0.017 | 0.112 | 0.210 | 0.021 | -0.310 | -0.276 | 0.101 | -0.486 | 0.308 | 0.329 |
| Coordinate geometry | -0.129 | -0.484 | 0.498 | -0.135 | 0.294 | -0.069 | -0.302 | 0.039 | 0.073 | 0.063 | -0.834 |
| Vectors, transformation, congruence, and similarity | 0.849 | 0.168 | 0.179 | 0.167 | -0.059 | -0.114 | -0.598 | -0.286 | -0.508 | -0.128 | -0.358 |
| Data representation and analysis | 0.344 | 0.480 | 0.449 | -0.156 | 0.263 | 0.887 | 0.096 | 0.257 | 0.110 | -0.074 | 0.085 |
| Uncertainty and probability | -0.072 | 0.253 | -0.014 | 0.687 | 0.439 | 0.258 | 0.015 | 0.795 | 0.157 | -0.174 | 0.117 |
| Advanced mathematics[1] | 0.117 | 0.199 | -0.084 | 0.675 | 0.061 | -0.024 | -0.143 | 0.039 | -0.193 | 0.844 | -0.030 |
| Other topics | 0.257 | 0.653 | 0.345 | -0.438 | -0.086 | 0.511 | -0.207 | 0.175 | 0.310 | 0.251 | 0.482 |
| Percentage of topic variance explained | 16.1 | 14.6 | 13.6 | 12.0 | 10.8 | 21.7 | 15.6 | 13.5 | 11.3 | 9.9 | 7.9 |

[1] Advanced mathematics is a combination of mathematics topics generally not associated with algebra and geometry. They are generally taught in later mathematics courses, although elements may be taught in algebra and geometry courses. These topics include discrete mathematics, linear interpolation and extrapolation, trigonometry, calculus, and validation and structuring.

NOTE: This table provides student-level factor loadings for algebra I and geometry mathematics topics by mathematics topic groupings. Principal components analyses were used to extract the factor loadings, with a varimax rotation utilizing Kaiser normalization. After 40 rotations, a five factor solution explained 67 percent of the variance for the algebra I topics. After 13 rotations, a six factor solution explained 80 percent of the variance for the geometry topics.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, High School Transcript Study (HSTS), Mathematics Curriculum Study, 2005.

**TABLE A3.** Percentage of subject matter content in algebra I and geometry courses, by course category and mathematics topic grouping: 2005

| Mathematics topic groupings | Algebra I course category | | | | Geometry course category | | | |
|---|---|---|---|---|---|---|---|---|
| | Low | Medium | High | Integrated | Low | Medium | High | Integrated |
| Arithmetic | 10.57 | 9.07 | 7.45 | 11.99 | 4.02 | 3.29 | 1.69 | 4.29 |
| Pre-equation | 10.12 | 5.82 | 3.97 | 2.36 | 1.65 | 1.07 | 2.11 | 2.00 |
| Basic number theory | 7.57 | 3.14 | 2.22 | 1.53 | 0.30 | 0.43 | 0.07 | 0.66 |
| Basic equations | 28.67 | 30.67 | 21.40 | 13.02 | 8.29 | 7.16 | 6.63 | 15.78 |
| Advanced equations | 10.70 | 14.63 | 17.55 | 4.18 | 0.68 | 0.76 | 0.27 | 7.41 |
| Basic functions | 2.58 | 2.66 | 6.80 | 3.09 | 0.16 | 0.33 | 0.00 | 3.15 |
| Advanced functions | 0.73 | 2.36 | 2.07 | 1.52 | 0.12 | 0.08 | 0.02 | 2.22 |
| Advanced number theory | 6.16 | 7.03 | 9.47 | 3.61 | 1.16 | 0.95 | 0.65 | 8.94 |
| Pre-geometry | 6.81 | 3.90 | 1.99 | 5.72 | 10.99 | 9.59 | 10.06 | 5.18 |
| Two-dimensional geometry | 3.78 | 2.25 | 3.31 | 12.69 | 43.61 | 42.02 | 43.52 | 11.74 |
| Three-dimensional geometry | 0.23 | 0.10 | 0.23 | 2.47 | 6.54 | 5.63 | 5.03 | 2.82 |
| Coordinate geometry | 5.54 | 7.61 | 5.89 | 5.05 | 2.74 | 4.03 | 6.32 | 4.17 |
| Vectors, transformation, congruence, and similarity | 0.47 | 0.63 | 0.79 | 3.91 | 10.24 | 13.99 | 15.29 | 4.18 |
| Data representation and analysis | 3.31 | 4.34 | 6.18 | 15.59 | 0.32 | 0.36 | 0.05 | 12.81 |
| Uncertainty and probability | 1.55 | 2.24 | 5.30 | 3.00 | 0.53 | 0.59 | 0.89 | 4.06 |
| Advanced mathematics[1] | 1.09 | 2.96 | 4.33 | 4.54 | 8.08 | 9.00 | 6.93 | 7.77 |
| Other | 0.12 | 0.60 | 1.04 | 5.72 | 0.57 | 0.73 | 0.46 | 2.83 |
| Number of courses | 173 | 527 | 197 | 93 | 138 | 581 | 213 | 63 |

[1] Advanced mathematics is a combination of mathematics topics generally not associated with algebra and geometry. They are generally taught in later mathematics courses, although elements may be taught in algebra and geometry courses. These topics include discrete mathematics, linear interpolation and extrapolation, trigonometry, calculus, and validation and structuring.

NOTE: This table provides the percentage of subject matter content, as defined by the mathematics topic groupings, for algebra I and geometry courses by course category. The percentages were generated by calculating the mean for each of the 17 mathematics topic groupings by the course categories generated from the discriminant analysis. School weights were used to adjust for sampling differences. Details may not sum to total because of rounding.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, High School Transcript Study (HSTS), Mathematics Curriculum Study, 2005.

**TABLE A4.** Percentage of subject matter content in students' algebra I and geometry courses, by course level and mathematics topic grouping: 2005

| Mathematics topic groupings | Algebra I course level | | | | Geometry course level | | | |
|---|---|---|---|---|---|---|---|---|
| | Beginner | Intermediate | Advanced | Integrated | Beginner | Intermediate | Advanced | Integrated |
| Arithmetic | 10.16 | 9.40 | 8.18 | 9.89 | 3.16 | 3.33 | 1.66 | 4.49 |
| Pre-equation | 6.66 | 3.87 | 2.19 | 5.58 | 10.99 | 9.73 | 9.72 | 5.25 |
| Basic number theory | 10.64 | 5.71 | 4.19 | 2.89 | 1.88 | 1.08 | 1.98 | 1.39 |
| Basic equations | 7.53 | 2.98 | 2.41 | 0.76 | 0.27 | 0.44 | 0.07 | 0.80 |
| Advanced equations | 28.09 | 31.35 | 20.62 | 11.02 | 7.56 | 7.50 | 6.34 | 16.59 |
| Basic functions | 11.70 | 15.17 | 16.23 | 4.47 | 1.05 | 0.80 | 0.20 | 7.05 |
| Advanced functions | 2.31 | 2.45 | 6.46 | 4.22 | 0.13 | 0.35 | 0.00 | 3.17 |
| Advanced number theory | 0.43 | 2.51 | 1.74 | 1.62 | 0.25 | 0.09 | 0.02 | 1.50 |
| Pre-geometry | 6.10 | 6.02 | 10.73 | 2.42 | 1.11 | 0.95 | 0.65 | 9.41 |
| Two-dimensional geometry | 4.49 | 2.16 | 3.59 | 13.81 | 41.56 | 41.19 | 43.91 | 13.54 |
| Three-dimensional geometry | 0.25 | 0.14 | 0.12 | 2.12 | 7.28 | 5.78 | 5.21 | 2.57 |
| Coordinate geometry | 4.96 | 7.54 | 6.81 | 5.03 | 3.31 | 3.94 | 6.76 | 3.61 |
| Vectors, transformation, congruence, and similarity | 0.34 | 0.53 | 0.54 | 5.02 | 10.30 | 13.90 | 15.95 | 3.62 |
| Data representation and analysis | 3.08 | 4.55 | 6.12 | 18.08 | 0.47 | 0.34 | 0.06 | 12.50 |
| Uncertainty and probability | 1.48 | 2.04 | 5.33 | 2.12 | 0.70 | 0.61 | 0.80 | 4.05 |
| Advanced mathematics[1] | 1.67 | 2.86 | 3.87 | 3.41 | 9.21 | 9.21 | 6.33 | 7.54 |
| Other | 0.11 | 0.73 | 0.88 | 7.56 | 0.79 | 0.77 | 0.37 | 2.93 |

[1] Advanced mathematics is a combination of mathematics topics generally not associated with algebra and geometry. They are generally taught in later mathematics courses, although elements may be taught in algebra and geometry courses. These topics include discrete mathematics, linear interpolation and extrapolation, trigonometry, calculus, and validation and structuring.

NOTE: This table provides the percentage of subject matter content, as defined by the mathematics topic groupings, for algebra I and geometry courses by course level. The percentages were generated by calculating the mean for each of the 17 mathematics topic groupings by the course levels generated from the discriminant analysis. School weights were used to adjust for sampling differences. Details may not sum to total because of rounding.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, High School Transcript Study (HSTS), Mathematics Curriculum Study, 2005.

# Glossary

### Advanced algebra
A broad curriculum topic category that is used in the 2005 Mathematics Curriculum Study to describe mathematics topics that cover the more complex elements of algebra. These topics include advanced equations, basic function, advanced functions, and advanced number theory.

### Advanced geometry
A broad curriculum topic category that is used in the 2005 Mathematics Curriculum Study and includes mathematics topics that cover advanced geometric concepts. These topics include three-dimensional geometry, coordinate geometry, and vector geometry.

### Beginner course level
The course level that indicates a student's coursework covers more introductory material and less advanced material than intermediate level courses.

### Chapter summary measures
Statistical measures used to summarize the subject matter content and cognitive challenge of a chapter in a textbook. The measures include the total content page count and a set of weighted topic content page counts.

### Cognitive challenge
The complexity of student tasks that are required to answer the chapter review questions in a textbook. It is measured by the performance expectations.

### Course categories
Empirically derived categories of algebra I or geometry courses based on the combination of content and challenge of the course, as determined by the textbooks used. There are four course categories: low, medium, high, and integrated.

### Course level
A ranking of a student's algebra I or geometry courses based on the combination of curriculum topics covered and the level of challenge of the courses, as determined by the content of their textbooks. There are three course levels: beginner, intermediate, and rigorous.

### Course summary measures
Statistical measures used to summarize the subject matter content and cognitive challenge of an algebra I or geometry course. The measures include the total content page count, the overall cognitive challenge rating, and a set of weighted topic content page counts.

### Curriculum topics
Six broad categories of mathematics topics used to present results of the 2005 Mathematics Curriculum Study. The six curriculum topic categories that are covered in algebra I and geometry courses are elementary and middle school mathematics, introductory algebra, advanced algebra, two-dimensional geometry, advanced geometry, and other high school mathematics.

### Elementary and middle school mathematics
A broad curriculum topic category that is used in the 2005 Mathematics Curriculum Study to describe mathematics topics that are traditionally taught before a student takes an algebra I course. These topics include elements of basic arithmetic and pre-geometry.

### High course category
A course category that indicates that the content and challenge of an algebra I or geometry course most closely resembled year two of a two-year algebra I course or an honors geometry course, respectively.

### "Honors" algebra I course
An algebra I course described by the school as covering more advanced algebra topics and/or more in-depth analysis of algebra topics than a "regular" algebra I course, including courses labeled honors, gifted and talented, and college preparatory.

### "Honors" geometry course
A geometry course described by the school as covering more advanced geometry topics and/or more in-depth analysis of geometry topics than a "regular" geometry course, including courses labeled honors, gifted and talented, and college preparatory.

### "Informal" geometry course
A geometry course described by the school as de-emphasizing the need for proofs.

### Integrated mathematics course
A mathematics course that covers several mathematics topics or strands, such as algebra, geometry, trigonometry, statistics, and analysis, in one course.

### Intermediate course level
The course level that indicates a student's coursework contains a balance of both introductory and advanced material.

### Introductory algebra
A broad curriculum topic category that is used in the 2005 Mathematics Curriculum Study and includes mathematics topics needed to understand the basics of algebra and provide the foundation for learning advanced algebra. These topics include pre-algebra, basic algebraic equations, and basic number theory.

### Low course category
A course category that indicates that the content and challenge of an algebra I or geometry course most closely resembled year one of a two-year algebra I course or an informal geometry course, respectively.

### Medium content category
A course category that indicates that the content and challenge of an algebra I or geometry course most closely resembled a regular algebra I or geometry course, respectively.

### Other high school mathematics
A broad curriculum topic category that is used in the 2005 Mathematics Curriculum Study and includes mathematics topics that are traditionally taught in courses taken after geometry and algebra II. These topics include trigonometry, pre-calculus, statistics, validation and structuring, discrete mathematics, finite mathematics, and calculus.

### Overall cognitive challenge rating
A summary measure that indicates the overall complexity of the student tasks that are required to answer the chapter review questions in a textbook. It is calculated by summing the weighted topic content page counts across all topic groupings and dividing the sum by the total content page count.

### Performance expectations
The activities or skills a student is expected to use to correctly answer a chapter review question.

### "Regular" algebra I course
An algebra I course described by the school as the course students take when progressing through the school's standard mathematics sequence.

### "Regular" geometry course
A geometry course described by the school as the course students takes when progressing through the school's standard mathematics sequence.

### Rigorous course level
The course level that indicates a student's coursework covers more advanced material and less introductory material than intermediate level courses.

### Student summary measures
Statistical measures used to summarize the subject matter content and cognitive challenge of a student's coursework in algebra I or geometry. The measures include the total content page count, the overall cognitive challenge rating, and a set of weighted topic content page counts.

### Total content page count
A summary measure that represents the total amount of instructional material in a textbook chapter, algebra I or geometry course, or student coursework.

### Two-dimensional geometry
A broad curriculum topic that is used in the 2005 Mathematics Curriculum Study and includes mathematics topics that focus on basic linear and planar geometric concepts. These topics include basic geometric concepts (e.g., points, angles, parallelism, and perpendicularity) and the properties of shapes.

### "Two-year" algebra I course
An algebra I course described by the school as taking two school years to complete.

### Weighted topic content page count
A summary measure that represents the amount of instructional material within a textbook chapter, algebra I or geometry course, or student coursework devoted to a mathematics topic, as weighted by ratings of the performance expectation codes assigned to the chapter review questions for that mathematics topic.

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