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# Examining the Differences of the 8th-Graders' Estimation Performance Between Contextual and Numerical Problems*

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Two 12-question estimation instruments were designed to compare the differences of estimating strategies used by the 8th-graders when solving contextual and numerical problems. Both instruments are parallel, meaning that the numbers used in both instruments are the same; however, they were presented differently. One hundred and ninety-eight 8th-graders in southern Taiwan were selected to participate in this study. The correct percentage for the numerical problems and contextual problems were 45% and 33%, respectively. The *t*-test result shows a significant difference for the estimation performance between contextual and numerical problems at $\alpha = 0.05$. This implies that these 8th-graders perform significantly better on numerical problems than that of contextual problems. In addition, the *t*-test results show that there is a significant difference in the performance of reformulation, transformation, and compensation dimensions for numerical and contextual problems at $\alpha = 0.05$, respectively. The results show that students performed better on numerical problems than on contextual problems for each dimension. In addition, data show that students tended to use the written method to solve numerical and contextual problems instead of estimation. Implications are discussed.

*Keywords:* the 8th-graders, contextual problems, numerical problems

## Introduction

Estimation has been considered as an important topic of mathematics education (Hogan & Brezinski, 2003; NCTM (National Council of Teachers of Mathematics), 2000; Mullis, Martin, Beaton, Gonzalez, Kelly, & Smith, 1997; Sowder, 1992). There is an especially strong relationship between estimation and number sense (Sowder, 1992; Yang, 1997). In addition, estimation is closely linked to life, such as with estimating money amounts, estimating length measurements, intuition of area measurements, and concept of volume. Sowder (1992) stated that estimation is better than actuarial when the authors encounter numerical problems involved in daily-life. Estimation ability, indeed, directly provides help in solving daily-life math problem. Therefore, estimation not only raises practicability of math in real life, but also enhances one's convenience in handling daily life. The main characteristic of estimation is to provide learners the knowledge to tackle problems they encounter in life.

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Estimation is a useful tool in daily-life situations. The authors want to know whether children have differences in estimation performance when solving contextual and numerical problems. Therefore, this study aims to examine differences of estimating strategies used by the 8th-graders when solving contextual and numerical problems. The research questions are as follows:

(1) Is there any significant difference in estimation performance between contextual and numerical problems?

(2) Is there any significant difference in estimation performance between contextual and numerical problems for each estimation strategy?

## Background

### What Is Estimation?

Sowder (1992) considered estimation as a problem-solving method that combines mental computation and problem-solving strategies to find the value closest to the real value. So, instead of precise calculation, the characters of estimation are to get the answer, a reasonable estimate, quickly, without using paper-and-pencil. In sum, this study defines "estimation" as a strategy to find an approximate number which is close to the original number, and then, according to the description of problem to estimate the appropriate answer.

### Estimation Strategies

The study by Reys, Bestgen, Rybolt, and Wyatt (1982) of estimation performance examined the strategies students used; they found three problem-solving strategies mostly used by estimators:

(1) Reformation: It is a process for calculating easily by rounding or truncating to change the original value. For example, to solve 0.528 × 987.6, it will get a rounding number 0.5 from 0.528, and convert 987.6 to 1,000. The estimated value will be 0.5 × 1,000 = 500;

(2) Translation: It implies that an individual changes the structure and operation order from the original math problem to make estimation easy. For example, people convert 964 + 965 + 966 + 967 + 968 to 966 × 5 or (352 × 6) ÷ 43 to 352 × (6 ÷ 43) by estimation;

(3) Compensation: Compensation means that after reformulation and translation, students use meta-cognition to rejudge the answer in order to reduce the deviation. For example, using reformation to calculate 2,005 × 0.4987, it gets 2,005 × 0.5 = 1,002.5. When using compensation, students will discover 0.4987 is smaller than 0.5, so the actual value will be smaller than the estimated value.

Based on the finding of Reys et al. (1982), this study defines three estimation strategies described as above to analyze the estimation strategy used by 8th-grade students in Taiwan.

### Estimation-Related Studies

Estimation has been studied in different countries for many years. Sowder (1992) categorized five aspects for estimation: (1) how to estimate; (2) what kind of ability was related to estimation; (3) how to develop the concept of estimation; (4) the influence of education on estimation; and (5) the influence of affection on estimation.

R. E. Reys, B. J. Reys, Nohda, Ishida, Yoshikawa, and Shimizu (1991) addressed that students in Japan and Mexico did not like estimation, and they preferred to calculate and find an exact answer. Even when they were participating in the experiment, they refused to use estimation. Several studies all detected that students in middle schools and elementary schools, in fact, even in college thought estimation was quite difficult and felt uncomfortable during the process of estimation (Hanson & Hogan, 2000; R. J. Reys, R. E. Reys, & Penafiel, 1991; R. E. Reys et al., 1991; Sowder & Wheeler, 1989; Threadgill-Sowder, 1984).

Hanson and Hogan (2000) stated that even though they told college students to use estimation before their experiment and not to find the exact answer, participants still insisted on finding the exact answers. Therefore, students' performance on calculating accurately was better than that of estimation. Generally, students could not keep from calculating accurately. Reys and Yang (1998) and Yang (2003) also acquired the same result in that most of interviewees were used to finding actual answers or obtaining real values and only then would they estimate one; this result shows that students lacked of estimation concept or at least were affected deeply by the need to complete operations.

In different countries, selecting subjects for estimation research has included adults (Dowker, 1997; Dowker, Flood, Griffiths, Harriss, & Hook, 1996; Levine, 1982), teenagers, and children (Dowker, 1997; LeFevre, Greenham, & Waheed, 1993; Lemaire, Lecacheur, & Farioli, 2000; Reys et al., 1982). In Taiwan, however, scholars take elementary students (Yang, 2003) and pre-serve teachers as the main subjects for estimation research. It still lacks results taken from junior high students as research subjects; therefore, the authors select the 8th-graders as the subjects for this study. This study tried to compare the differences of estimating strategies used by the 8th-graders when solving contextual and numerical problems.

## Method

### Sample

Nine classes with 198 8th-graders from one junior high school in southern Taiwan were selected to participate in this study. The school, which is comprised of about 1,200 junior high school students (from Grade 7 to Grade 9), serves students from diverse areas. This implies that the students in the study come from families with a wide range of occupations, incomes, and educational levels.

### Instrument

According to relevant literature reviewed, this study defined estimation as including three different strategies: reformulation, transformation, and compensation (Hanson & Hogan, 2000; McIntosh, B. Reys, & R. Reys, 1997; R. E. Reys, B. J. Reys et al., 1991; Sowder, 1992). The instrument includes contextual problems and numerical problems. Both instruments are parallel, which implies that the numbers used in both instruments are the same; however, the presentation types are different. Three questions for each of the three strategies were constructed, giving a grand total of 12 questions for each instrument. The Cronbach $\alpha$ for contextual and numerical problems is 0.77 and 0.80, respectively.

### Data Collection and Analysis

Table 1

*Coding Standard for the Examination of Estimation*

| Answer selection | Use of estimation strategy | | | |
|---|---|---|---|---|
| | Never | Seldom | Usually | Always |
| Correct | 1 | 3 | 5 | 8 |
| Incorrect | 0 | 2 | 4 | 6 |

*Note.* Eight points per problem.

Two instruments were used to collect the data and each test required about 45 minutes to complete. All students were administered the contextual problem test at the same time in the first week. Two weeks later, all students were administered the numerical problem test at the same time. During the tests, students were required to write down their procedures and answers on scratch paper.

The coding standard for the examination of estimation is shown in Table 1.

## Result

Table 2 reports the *t*-test result of the students sample for contextual and numerical problems, which showed that the students' performance on numerical problems is better than that on contextual problems ($p = 0.000 < 0.05$). Students' average score was 32.5 points for the numerical problems with a correct rate of 45%; their average score was 23.5 points for the contextual problems with a correct rate of 33%. The difference in the correct rate was 12%, and the average difference was nine points, meaning the students performed better on numerical problems rather than contextual problems. This research shows that when situational descriptions are added to problems, the difficulty of estimation will be increased. Moreover, Table 2 shows that the *SD* (standard deviation) of numerical problems was larger than that of contextual problems, indicating that the students' difference on numerical problems was larger than that of contextual problem as well.

Table 2

*T-test Result of the 8th-Grade Students Taking Contextual and Numerical Estimation Exams (N = 198)*

| Problem type | Average | Number of problems | Correct rate (%) | *SD* | *t*-value (numerical-contextual) | Significant |
|---|---|---|---|---|---|---|
| Contextual | 23.52 | 12 | 32.7 | 9.38 | 11.39 | 0.000 |
| Numerical | 32.53 | 12 | 45.2 | 13.8 | | |

*Note*. $\alpha = 0.05$.

Because of the significant difference between numerical and contextual problems for students, the researchers further analyzed contextual and numerical estimation problems with a *t*-test to examine for any possible discrepancies among "reformation", "translation", and "compensation" dimensions.

The problems were divided into "reformation", "translation", and "compensation" dimensions. Each dimension contained four problems and each problem was worth six points; the total of 24 points could be obtained for each dimension. Table 3 includes detailed information regarding dimension coding.

Table 3

*Statistic Analysis and T-test Result of the 8th-Grade Students Taking the Contextual and Numerical Examinations for Three Dimensions (N = 198)*

| Three dimension | Problem type | Number of problem | Average score | Correct rate (%) | *SD* | *t*-value (numerical-contextual) | Significant (two-tails) |
|---|---|---|---|---|---|---|---|
| Reformation | Contextual | 4 | 6.58 | 27.4 | 3.11 | 13.106 | 0.000 |
| | Numerical | 4 | 11.68 | 48.7 | 5.86 | | |
| Translation | Contextual | 4 | 8.94 | 37.3 | 4.30 | 5.436 | 0.000 |
| | Numerical | 4 | 11.9 | 46.2 | 6.04 | | |
| Compensation | Contextual | 4 | 8.00 | 33.3 | 4.39 | 4.971 | 0.000 |
| | Numerical | 4 | 9.76 | 40.7 | 4.63 | | |

*Note.* The full marks of each dimension are 24 points (four problems with each problem being worth six points), $\alpha = 0.05$.

Data in Table 3 show that there was a significant difference ($p < 0.05$) between the three dimensions "reformation", "translation", and "compensation", with the average scores of the numerical problems all found to be higher than that of contextual problems. In other words, the *t*-test value showed that students' performance on numerical problems was superior to that of contextual problems; this study also shows that when situational descriptions are added, the difficulty of problems will be increased.

In addition, when the authors compared the average scores and correct rate for contextual problems, students' performance on "translation" was found to be superior to that of "compensation" and their performance on "compensation" was found to be superior to that of "reformation". Students tended to use "translation" to solve the context problems involving estimation. However, for the numerical problems, students' performance on "reformation" was superior to that of "translation" and their performance of "translation" was better than that of "compensation". Students tended to use "reformation" to solve the numerical problems involving estimation.

The *SD* of numerical problems was found to be larger than that of the contextual problems for three dimensions, which means there were bigger discrepancies among participants for the numerical problems (see Table 3). The biggest deviation (6.04) was found for "translation" of numerical problems; the smallest deviation was found for "reformation" of contextual problems.

## Discussion and Conclusion

Results showed students' performance on numerical problems was better than that of contextual problems. Furthermore, the *t*-test value was found to be significantly different for numerical and contextual problems for the three dimensions of "reformation", "translation", and "compensation". The results were similar to Verschffel, Greer, and De Corte (2000) in which numerical problems with situational descriptions affected students' problem-solving performance. In addition, Sowder (1992) and Reys et al. (1982) have argued that for contextual problems with uncommon words, students will feel it is difficult to solve such problems; however, if contextual problems contain verbal descriptions, students will feel it easy to understand the meaning of the problems. Therefore, from their finding, verbal contextual problems were solved more easily by students than numerical problems. However, this study showed that due to the clear operation of the numerical problems, students could successfully solve such problems, and it even helped them develop their estimation ability. On the other hand, to solve such problems students had to read the situated problems, list the operations, and then solve the problems. They needed to transform verbal words into number symbols in order to use estimation to solve problems easily and use the wrong operations. These were the reasons why students' performance on numerical problems was better than that of contextual problems.

However, the key points that influenced students' usage of the problem-solving strategy were presentation of problems (contextual or numerical form): How they thought about the problems, and whether they needed to convert contextual descriptions to mathematics operations. Because numerical problems were composed of number and operations, students could solve them directly without representation conversion. On the contrary, when solving contextual problems, students had to list the operations and then solve the problems. This was the reason why participants' performance on numerical problems was superior to that of contextual problems.

Besides, it is discovered that when students were taking the exams, they would follow the numbers by their order of appearance in problem to calculate, which may cause them to be unable to observe the relationship among numbers or possibly misunderstand the meaning of problems.

From the low correct rate (less than 50%), it can be concluded that the percentage of using estimation to solve problems was still low, meaning students still had room for improvement in their estimation abilities. Past research has shown that students in middle schools considered estimation difficult and felt uncomfortable when asked to use estimation; this result has also been shown at the college level (R. J. Reys, R. E. Reys et al., 1991; R. E. Reys, B. J. Reys et al., 1991; Sowder & Wheeler, 1989; Threadgill-Sowder, 1984). Moreover, students are accustomed to using written computation methods to solve problems in every dimension including "compensation", "translation", and "reformation". Research indicates that most students favor written computation methods; therefore, their performances for actuarial methods have been shown to be better than estimation (Hanson & Hogan, 2000). Even though the researchers of this study instructed students to use estimation and not find the actual value, they still insisted on finding the actual value.

## Implications

Two implications are discussed below:

(1) Estimation should be edited into math curriculum. This study results indicate that the percentage of students using estimation to solve problems is very low. Students took estimation as only rounding up or rounding off and thus could not develop an advanced estimation strategy. Although Grades 1−9 Curriculum Guidelines mention the importance of estimation, it has still not been applied in mathematics textbooks in Taiwan. Mathematics learning constitutes not only proficiency of computation, but also an appropriate estimation strategy. After students have learned the mathematic concept of estimation, they can judge the reasonableness of an answer. However, it is still observed that mathematics curriculum is short of estimation related syllabi, and mathematics curriculum guides do not present how to teach estimation in classes. Besides, Trafton (1994) pointed out that estimation promotes students to flexibly manipulate and apply numbers. Therefore, it is important and necessary that estimation should be integrated into mathematics curriculum in Taiwan;

(2) Estimation should be taught in mathematics classrooms. This study showed that most students depended on written computation, and they aimed to find the exact answer. These students lacked the ability of using estimation to solve the problems because traditionally Taiwanese teachers emphasize accuracy without taking into consideration estimation, which will restrain students from developing estimation ability (Reys & Yang, 1998; Yang, Hsu, & Huang, 2004). In addition, teachers have had few experiences in leading students in estimation activities in class. It is surprising that students used written methods to solve problems no matter the numbers were complex or simple. To help children develop and use estimation strategies, teachers should be trained to know what estimation is and know how to teach estimation in mathematics classes.

In addition, estimation involves meta-cognition, having the ability to choose the appropriate round number to simplify calculation, determining whether the error value is in an acceptable range, knowing how to adjust the estimated value size, and knowing how to check the answer reasonably using estimation will help one's math learning tremendously. Therefore, teachers are suggested to allow students the opportunity to increase their estimation skills instead of only emphasizing accuracy in their teaching.

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