Geometry California Content Standards

Standards Deconstruction Project

2008

Version 1.0

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A note to the reader: This project was coordinated and funded by the California Partnership for Achieving Student Success (Cal-PASS). Cal-PASS is a data sharing system linking all segments of education. Its purpose is to improve student transition and success from one educational segment to the next.

Cal-PASS is unique in that it is the only data collection system that spans and links student performance and course-taking behavior throughout the education system—K–12, community college, and university levels. Data are collected from multiple local and state sources and shared, within regions, with faculty, researchers, and educational administrators to use in identifying both barriers to successful transitions and strategies that are working for students. These data are then used regionally by discipline-specific faculty groups, called "Professional Learning Councils," to better align curriculum.

Cal-PASS' math deconstruction projects were initiated by the faculty serving on the math Professional Learning Councils after reviewing data on student transition. An Algebra I deconstruction process was devised by the participating faculty with suggestions from the San Bernardino County Unified School District math faculty (Chuck Schindler and Carol Cronk) and included adaptations of the work of Dr. Richard Stiggins of the Assessment Training Institute and Bloom's *Taxonomy of Educational Objectives* (B. S. Bloom, 1984,. Boston: Allyn and Bacon). The Algebra II project followed the same procedure.

The Geometry deconstruction project continued with the procedure that was used for deconstructing Algebra I and II standards. The following document represents a comprehensive review by K–16 faculty to deconstruct and align Geometry standards.

In order to continue the collaboration on these standards, thus improving on the current work, we invite and encourage the reader to provide feedback to us. Please contact Dr. Shelly Valdez at: **svaldez@calpass.org.**

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California Geometry Content Standards

Retrieved from CDE Web site (4/17/07) http://www.cde.ca.gov/be/st/ss/mthgeometry.asp

GEOMETRY **Grades Eight Through Twelve - Mathematics Content Standards.**
The geometry skills and concepts developed in this discipline are useful to all students. Aside from learning these skills and concepts, students will develop their ability to construct formal, logical arguments and proofs in geometric settings and problems.
Standard Set 1.0: Students demonstrate understanding by identifying and giving examples of undefined terms, axioms, theorems, and inductive and deductive reasoning.
2.0: Students write geometric proofs, including proofs by contradiction.
3.0: Students construct and judge the validity of a logical argument and give counterexamples to disprove a statement.
4.0: Students prove basic theorems involving congruence and similarity.
5.0: Students prove that triangles are congruent or similar, and they are able to use the concept of corresponding parts of congruent triangles.
6.0: Students know and are able to use the triangle inequality theorem.
7.0: Students prove and use theorems involving the properties of parallel lines cut by a transversal, the properties of quadrilaterals, and the properties of circles.
8.0: Students know, derive, and solve problems involving the perimeter, circumference, area, volume, lateral area, and surface area of common geometric figures.
9.0: Students compute the volumes and surface areas of prisms, pyramids, cylinders, cones, and spheres; and students commit to memory the formulas for prisms, pyramids, and cylinders.
10.0: Students compute areas of polygons, including rectangles, scalene triangles, equilateral triangles, rhombi, parallelograms, and trapezoids.
11.0: Students determine how changes in dimensions affect the perimeter, area, and volume of common geometric figures and solids.
12.0: Students find and use measures of sides and of interior and exterior angles of triangles and polygons to classify figures and solve problems.
13.0: Students prove relationships between angles in polygons by using properties of complementary, supplementary, vertical, and exterior angles.
14.0: Students prove the Pythagorean theorem.
15.0: Students use the Pythagorean theorem to determine distance and find missing lengths of sides of right triangles.

16.0: Students perform basic constructions with a straightedge and compass, such as angle bisectors, perpendicular bisectors, and the line parallel to a given line through a point off the line.
17.0: Students prove theorems by using coordinate geometry, including the midpoint of a line segment, the distance formula, and various forms of equations of lines and circles.
18.0: Students know the definitions of the basic trigonometric functions defined by the angles of a right triangle. They also know and are able to use elementary relationships between them. For example, $\tan(x) = \sin(x)/\cos(x)$, $(\sin(x))^2 + (\cos(x))^2 = 1$.
19.0: Students use trigonometric functions to solve for an unknown length of a side of a right triangle, given an angle and a length of a side.
20.0: Students know and are able to use angle and side relationships in problems with special right triangles, such as 30°, 60°, and 90° triangles and 45°, 45°, and 90° triangles.
21.0: Students prove and solve problems regarding relationships among chords, secants, tangents, inscribed angles, and inscribed and circumscribed polygons of circles.
22.0: Students know the effect of rigid motions on figures in the coordinate plane and space, including rotations, translations, and reflections.

Standard #1

Standard Set 1.0

Students demonstrate understanding by identifying and giving examples of undefined terms, axioms, theorems, and inductive and deductive reasoning.

Deconstructed Standard

1. Students identify undefined terms.
2. Students identify axioms/postulates.
3. Students identify theorems.
4. Students identify inductive reasoning.
5. Students identify deductive reasoning.
6. Students give examples of undefined terms.
7. Students give examples of axioms/postulates.
8. Students give examples of theorems.
9. Students give examples of inductive reasoning.
10. Students give examples of deductive reasoning.

Prior Knowledge Necessary

Students should know how to:

- apply the concept of the coordinate plane.
- apply the concept of an ordered pair.
- apply the concept of infinity.
- apply the concept of a definition.

New Knowledge

Students will need to learn to:

- identify and give examples of the undefined term *point.*
- identify and give examples of the undefined term *line.*
- identify and give examples of the undefined term *plane.*
- identify the dimension of a point.
- identify the dimension of a line.
- identify the dimension of a plane.
- recall and produce examples of axioms/postulates.
- recall and produce examples of theorems.
- recall and produce examples of inductive reasoning.

Categorization of Educational Outcomes

Competence Level: Knowledge

1. Students will describe the undefined term *point* in different contexts.
2. Students will describe the undefined term *line* in different contexts.
3. Students will describe the undefined term *plane* in different contexts.
4. Students will identify whether a logical argument is inductive.
5. Students will identify whether a logical argument is deductive.
6. Students will describe an axiom/postulate.

7. Students will describe a theorem.

Competence Level: Comprehension

1. Students will give examples of inductive reasoning.
2. Students will give examples of deductive reasoning.
3. Students will give examples of points.
4. Students will give examples of lines.
5. Students will give examples of planes.

Competence Level: Application

1. Students will use inductive reasoning to construct a logical argument.
2. Students will use deductive reasoning to construct a logical argument.
3. Students will use axioms/postulates in the construction of logical arguments.
4. Students will use theorems in the construction of logical arguments.

Competence Level: Analysis

1. Students will differentiate between inductive and deductive reasoning.

Necessary New Physical Skills

None

Assessable Result of the Standard

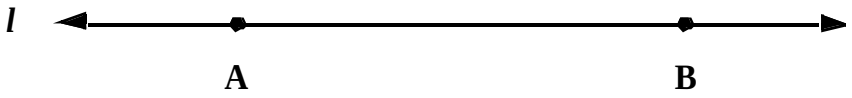
1. Students will identify points.
2. Students will identify lines.
3. Students will identify planes.
4. Students will state whether a logical argument is deductive.
5. Students will state whether a logical argument is inductive.

Standard #1 Model Assessment Items

Computational and Procedural Skills

1. What are the undefined terms of Geometry?

2. Name the following diagram in two different ways.



3. Given any three non-collinear points A, B, and C. How many different lines can be drawn through different pairs of points? Name the lines.

Conceptual Understanding

1. If you are on one side of a plane, can you get to the other side without going through the plane? Explain your reasoning.

2. Which of the following best describes deductive reasoning?
   A. Using logic to draw conclusions based on accepted statements
   B. Accepting the meaning of a term without definition
   C. Defining mathematical terms to correspond with physical objects
   D. Inferring a general truth by examining a number of specific examples

3. Jon noticed dark clouds above the mountains. Realizing that the wind was blowing toward him, he decided it was going to rain.
   This is an example of _____ reasoning.
   A. deductive
   B. inductive
   C. supportive
   D. declarative

Standard #2

Standard Set 2.0
Students write geometric proofs, including proofs by contradiction.

Deconstructed Standard
1. Students write geometric proofs.
2. Students write proofs by contradiction.

Prior Knowledge Necessary
Students should know how to:
- use and apply aspects of a logical argument.
- identify the hypothesis and conclusion in a logical deduction.
- recognize that a single counterexample is sufficient to refute an assertion.
- use the properties of numbers to construct simple, valid arguments.
- analyze problems by identifying relationships and distinguishing relevant information from irrelevant information.
- determine when and how to break a problem into simpler parts.

New Knowledge
Students will need to learn to:
- identify the parts (hypothesis and conclusion) of a conditional statement.
- write the related statements to a conditional statement (inverse, converse, and contrapositive).
- write the negation of a statement.
- apply the concept of a valid argument, including proof by contradiction.
- apply basic undefined terms, definition, initial postulates and theorems to justify each statement in a proof.
- apply properties of real numbers (commutative, associative, distributive); properties of equality (addition, multiplication); properties of a relation (reflexive, symmetric, and transitive), as justifying statements in a proof.
- analyze and organize the plan for a proof.
- write different types of proof (e.g., two-column, indirect, paragraph, flow chart).

Categorization of Educational Outcomes
Competence Level: Knowledge
1. Students will recall basic undefined terms, definitions, postulates, and theorems.
2. Students will state the negation of a statement.
3. Students will state the related statements of a conditional statement.

Competence Level: Comprehension
1. Students will identify the hypothesis and conclusion of a conditional statement.
2. Students will restate a conditional statement in “if–then” form.

Competence Level: Application

1. Students will apply properties of real numbers, equality, and of a relation to justify steps of proof statements.
2. Students will use basic undefined terms, definitions, postulates, and theorems to justify steps of proof statements.

Competence Level: Analysis

1. Students will analyze a statement to be proven prior to preparing a plan of proof.

Competence Level: Synthesis

1. Students will develop a plan of proof.
2. Students will arrange the steps of a proof in a logical sequence.

Competence Level: Evaluation

1. Students will support each statement in a proof.
2. Students will evaluate a completed proof.

Necessary New Physical Skills

None

Assessable Result of the Standard

1. Students will produce a proof given a conditional statement.
2. Students will produce an proof by contradiction.

Standard #2 Model Assessment Items

Computational and Procedural Skills

1. Write the negation of the statement: "A square has four sides."

2. For the statement "If two angles are congruent, these angles have equal measures," state the hypothesis and the conclusion.

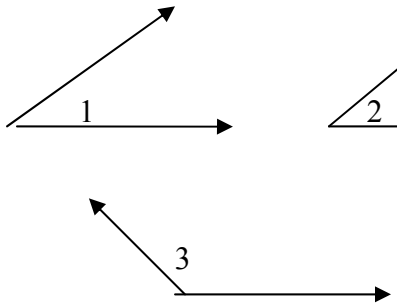
3. TRUE or FALSE: If B is the midpoint of $\overline{AC}$, then $AB = \frac{1}{2}(AC)$

4. Angles 1 and 2 are complementary. If $m\angle 1 = x$ and $m\angle 2 = y$, what equation relates *x* and *y*?

5. Given that $b = c$, name the algebraic property that allows you to state that $a + b = a + c$.

6. If M-N-P on $\overline{MP}$, you may conclude that MN + NP = __?__

7. Use the drawing provided for the theorem: "If two angles are supplementary to the same angle, then they are congruent."
   A. Complete the given information:



Given: $\angle 1$ is supplementary to $\angle 3$;

_____??_____

   B. Complete the prove statement: Prove: _____??_____

   C. In the proof of the theorem above are the statements $m\angle 1 + m\angle 2 = 180$ and $m\angle 2 + m\angle 3 = 180$. Write a conclusion that is based upon substitution.

Conceptual Understanding

1. Does the relation "is greater than" have a transitive property; that is, if $a > b$, and $b > c$, then is $a > c$?

2. Write the conclusion for the following argument: If I have more than $10 in my wallet, then I am going to see a movie today. I have $25 in my wallet.

3. Upon which part (hypothesis or conclusion) of a theorem is the drawing based?

4. Upon which part (hypothesis or conclusion) of a theorem is the prove statement based?

5. State the contrapositive of the following statement: "If an angle is a straight angle, then it measures 180° ."

6. Write the first statement of this indirect proof:
   Given: $\overline{AM} \cong \overline{MB}$
   Prove: M is the only midpoint of $\overline{AB}$

7. Write the first statement of this indirect proof:
   Given: $\overline{RS}$ is not perpendicular to $\overline{RT}$
   Prove: $\angle 1$ is not a right angle

Problem Solving/Application

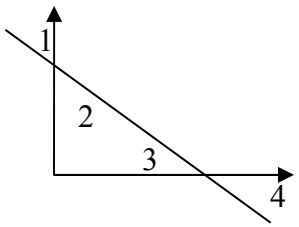
1. Fill in the missing reasons for the algebraic proof:
   Given: $3(x-5)=21$
   Prove: $x = 12$

Statements	Reasons
A. $3(x-5)=21$	A. ?
B. $3x-15=21$	B. ?
C. $3x=36$	C. ?
D. $x = 12$	D. ?

2. Fill in the missing reasons for the geometric proof:
   Given: E is the midpoint of $\overline{DF}$
   Prove: $DE = \frac{1}{2}(DF)$

Statements	Reasons
A. E is the midpoint of $\overline{DF}$	A. ?
B. DE = EF	B. ?
C. DE + EF = DF	C. ?
D. DE + DE = DF	D. ?
E. 2(DE) = DF	E. ?
F. $DE = \frac{1}{2}(DF)$	F. ?

3. Write a two-column proof of the following information:
   Given: $\angle s$ 1 and 3 are complementary
   $\angle s$ 2 and 3 are complementary
   Prove: $\angle 1 \cong \angle 2$



4. Which of the following statements would you prove by the indirect method?
   A. In triangle ABC, if $m\angle A > m\angle B$, then $AC \neq BC$.

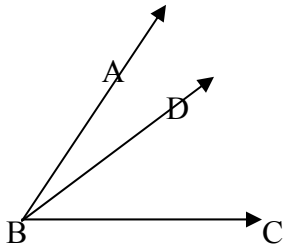
   B. If alternate exterior $\angle 1 \ncong$ alternate exterior $\angle 8$, then *l* is not parallel to *m*.

   C. If $(x+2)\cdot(x-3)=0$, then $x=-2$ or $x=3$.

   D. If two sides of a triangle are congruent, then the two angles opposite these sides are also congruent.

   E. The perpendicular bisector of a line segment is unique.

5. Write an indirect (paragraph) proof for the following:
   Given: $\angle ABD \ncong \angle DBC$
   Prove: $\overrightarrow{BD}$ does not bisect $\angle ABC$



6. Write a paragraph proof for the following statement:
   The midpoint of a line segment is unique.

Standard #3

Standard Set 3.0

Students construct and judge the validity of a logical argument and give counterexamples to disprove a statement.

Deconstructed Standard

1. Students construct a logical argument.
2. Students judge validity of logical argument.
3. Students give counterexamples to disprove a conditional statement.

Prior Knowledge Necessary

Students should have the computational and conceptual knowledge outlined in Algebra I, Standard #24.

Students should know how to:

- use strategies in finding solutions.
- use skills in finding solutions.
- use understanding of concepts in finding solutions.
- make and test conjectures by using both inductive and deductive reasoning.
- use a variety of methods to explain mathematical reasoning, including:
  i. words
  ii. symbols
  iii. charts
  iv. graphs
  v. tables
  vi. diagrams
  vii. models

New Knowledge

Students will need to learn to:

- rewrite statements in conditional form (if/then).
- write the converse of a conditional statement.
- write the inverse of a conditional statement.
- write the contrapositive of a conditional statement.
- apply bi-conditional statements to the definitions of terms.
- rewrite conditional statements in symbolic logic form ($p \rightarrow q, q \rightarrow p$).
- construct a logical argument with definitions, postulates, and theorems.
- justify a logical argument with definitions, postulates, and theorems.
- identify a counterexample as an argument.

Categorization of Educational Outcomes

Competence Level: Knowledge

1. Students will describe conditional statements.
2. Students will identify the different forms of a conditional statement.

Competence Level: Comprehension:

1. Students will give examples of reasoning to justify statements.
2. Students will classify the components of an argument.

Competence Level: Analysis:

1. Students will recognize the validity of an argument.
2. Students will differentiate the components of an argument.
3. Students will differentiate the different forms of a conditional statement.

Competence Level: Evaluation:

1. Students will judge the validity of an argument.
2. Students will justify the validity of an argument.

Necessary New Physical Skills

None

Assessable Result of the Standard

1. Students will rewrite a statement in conditional form.
2. Students will write an argument in conditional form.
3. Students will use counterexamples to prove a conditional statement is false.
4. Students will write the converse, inverse, and contrapositive of a conditional statement.

Standard #3 Model Assessment Items

Computational and Procedural Skills

1. Underline the hypothesis of the conditional statement with one line and the conclusion with two lines:
   If a figure is part of a line, then it is a segment

2. Determine whether the conditional statement is true or false:
   If today is July 4^{th}, then it is U.S. Independence Day

Conceptual Understanding

1. Write the converse of the conditional statement:
   If two plane figures are similar, then they are congruent.

2. Give a counterexample to show that the conditional statement is false:
   If a figure is part of a line, then it is a segment.

Problem Solving/Application

1. Justify the conditional statement: If $\angle A$ is a right angle, then $m\angle A = 90°$.

2. Justify the conditional statement: If M is the midpoint of $\overline{AB}$, then $AM = MB$.

Standard #4

Standard Set 4.0
Students prove basic theorems involving congruence and similarity.

Deconstructed Standard
1. Students prove theorems involving congruence.
2. Students prove theorems involving similarity.

Prior Knowledge Necessary
Students should have the computational and conceptual knowledge outlined in Algebra I Standard #24 and Geometry Standards #1, #2, and #3.

Students should know how to:
- construct and read drawings and models made to scale.
- demonstrate an understanding of conditions that indicate two elementary geometrical figures are congruent.
- recognize congruence of figures.
- recognize similarity of figures.
- use geometric notations and symbols (i.e. $\cong, \angle, \perp$).
- identify the corresponding parts of geometric figures.
- analyze problems by identifying relationships, distinguishing relevant from irrelevant information, sequencing and prioritizing information, and observing patterns.

New Knowledge
Students will need to learn to:
- write paragraph proofs.
- write two-column proofs.
- use postulates as necessary to justify their reasoning:
  - Postulate 1 (Unique Line Postulate). Through two distinct points, there is exactly one line.
  - Postulate 2. A line contains at least two distinct points.
  - Postulate 3 (Unique Plane Postulate). Through three non-collinear points, there is exactly one plane.
  - Postulate 4. A plane contains at least three non-collinear points.
  - Postulate 5. If two distinct points lie in a plane, then the line joining them lies in that plane.
  - Postulate 6. If two distinct planes intersect, then their intersection is a line.
  - Postulate 7. The points on a line can be paired, one-to-one with the real numbers so that any point is paired with 0 and any other point is paired with 1.
  - Postulate 8 (Segment Addition Postulate). If point C is between points A and B, then AC + CB = AB.

- Postulate 9 (Protractor Postulate). Let O be a point on $\overleftrightarrow{AB}$ such that O is between A and B. Consider $\overrightarrow{OA}$, $\overrightarrow{OB}$, and all the rays that can be drawn from O on one side of $\overleftrightarrow{AB}$. These rays can be paired with the real numbers from 0 to 180 so that
  i. $\overrightarrow{OA}$ is paired with 0 and $\overrightarrow{OB}$ is paired with 180.
  ii. If $\overrightarrow{OP}$ is paired with x and $\overrightarrow{OQ}$ is paired with y, then the number paired with $\angle POQ$ is $|x-y|$. This number is called the measure, or the degree measure, of $\angle POQ$.
- Postulate 10 (Angle Addition Postulate). If point B is in the interior of $\angle AOC$, then $m\angle AOB + m\angle BOC = m\angle AOC$.
- Postulate 11 (Linear Pair Postulate). If two angles form a linear pair, then they are supplementary.

- identify the appropriate postulates.
- logically organize an argument.
- construct congruent objects.
- construct similar objects.
- prove objects congruent.
- prove objects similar.
- apply basic algebraic properties applied to geometric figures (i.e., reflexive angles, congruent angles).

Categorization of Educational Outcomes

Competence Level: Knowledge

1. Students will identify the format of proofs.
2. Students will identify the appropriate postulates to use.

Competence Level: Application:

1. Students will construct two-column proofs.
2. Students will construct paragraph proofs.
3. Students will develop logical reasoning for proofs.
4. Students will use appropriate postulates in formulating proofs.
5. Students will use appropriate definitions in formulating proofs.

Competence Level: Analysis:

1. Students will break down an argument in developing a proof.
2. Students will recognize the next logical step in their reasoning.

Competence Level: Synthesis:

1. Students will structure their arguments in constructing a proof.
2. Students will combine postulates in constructing their arguments.

Necessary New Physical Skills

Students will use a straightedge and compass.

Assessable Result of the Standard

1. Students will apply their understanding to recognize postulates.
2. Students will solve problems with congruence.
3. Students will solve problems with similarity.
4. Students will generate two-column proofs.
5. Students will generate paragraph proofs.

Standard #4 Model Assessment Items

Computational and Procedural Skills

1. Name the property, definition, or postulate that justifies the conclusion:
   If points R, S, and T are non-collinear, then they determine a plane.

2. Copy and complete the following paragraph proof for the theorem:
   Supplements of congruent angles are congruent.

   Given: $\angle 1$ is supplementary to $\angle 3$.
   $\angle 2$ is supplementary to $\angle 4$.
   $m\angle 3 = m\angle 4$.

   Prove: $m\angle$ ____ $= m\angle$ ____

   It is given that _____ and _____. By the definition of _____, $m\angle 1 + m\angle 3 = 180°$ and $m\angle 2 + m\angle 4 = 180°$. Since both quantities are equal to $180°$, $m\angle 1 + m\angle 3 =$ _____. It is ____ that $m\angle 3 = m\angle 4$. Since you can subtract equal quantities from each side of an equation, it follows that $m\angle 1 = m\angle 2$.

Conceptual Understanding

1. Write a two-column proof of this theorem: All right angles are congruent.

2. Write a paragraph proof of this theorem: Complements of congruent angles are congruent.

Problem Solving/Application

1. In line segment $\overline{MP}$, point *N* is between points *M* and *P* and $MP = 104$, $MN = 2y$, and $NP = 3y - 16$. Find NP.

2. Two angles are supplementary. If the measure of one angle is $5m - 5$ and the measure of the other angle is $7m + 29$, solve for m. Then find the measure of each angle.

Standard #5

Standard Set 5.0

Students prove that triangles are congruent or similar, and they are able to use the concept of corresponding parts of congruent triangles.

Deconstructed Standard

1. Students prove that triangles are congruent.
2. Students prove that triangles are similar.
3. Students are able to use the concept of corresponding parts of congruent triangles.

Prior Knowledge Necessary

Students should know how to:

- write proofs of basic geometric relationships.
- work with ratios and proportions.
- identify various types of triangles and the relationships of the angles and sides of these triangles.
- work with and understand the angle relationships formed by parallel lines cut by a transversal (alternate interior angles and corresponding angles are congruent, etc.).
- apply and understand the vertical angle theorem.
- apply and understand the reflexive property for line segments and angles.
- apply and understand the properties of midpoints, angle bisectors, and perpendicular bisectors.
- use the fact that all right angles are congruent.

New Knowledge

Students will need to learn to:

- apply the SSS, SAS, ASA, and AAS congruence postulates.
- apply the fact that "Corresponding Parts of Congruent Triangles are Congruent" (CPCTC).
- apply the AA, SSS, and SAS Similarity Theorems.

Categorization of Educational Outcomes

Competence Level: Application

1. Students will analyze given information about a geometric relationship.
2. Students will utilize their understanding and knowledge of the properties and relationships of geometry to write proofs.
3. Students will determine which theorems and/or postulates may be used in a given situation to help in writing proofs.
4. Students will arrange a series of geometric arguments, appropriately supported, for the purpose of proving a theorem or argument from geometry.

Necessary New Physical Skills

None

Assessable Result of the Standard

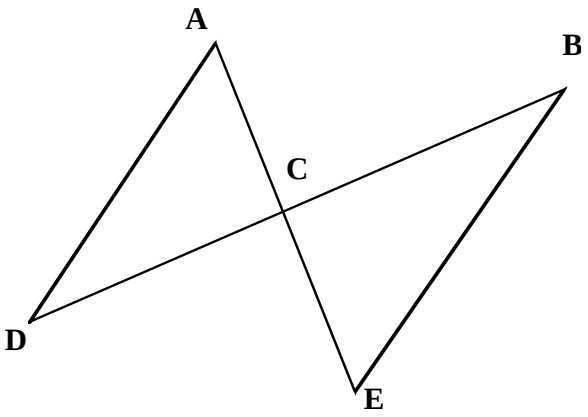
1. Students will produce well organized and well supported proofs dealing with triangle congruence.
2. Students will produce well organized and well supported proofs dealing with triangle similarity.
3. Students will produce proofs in which the concept of "Corresponding Parts of Congruent Triangles are Congruent" is used appropriately and effectively.
4. Students will produce justification as to why two triangles may or may not be congruent or similar.

Standard #5 Model Assessment Items

Computational and Procedural Skills

1. Is it possible to prove that the given triangles, under the stated conditions, are congruent? If so, what postulate or theorem supports your conclusion?

A.

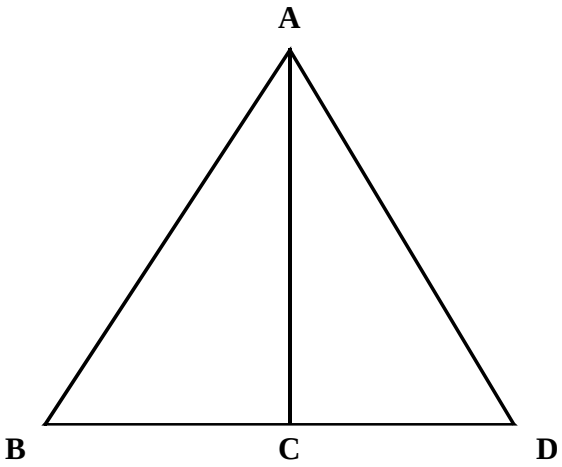


Given:

a. $\overline{AD} \cong \overline{BE}$

b. $\overline{AD}$ is parallel to $\overline{BE}$

B.

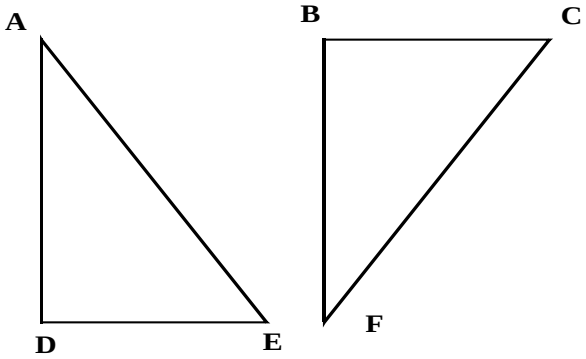


Given:

a. C is the midpoint of $\overline{BD}$

b. $\overline{AB} \cong \overline{AD}$

C.



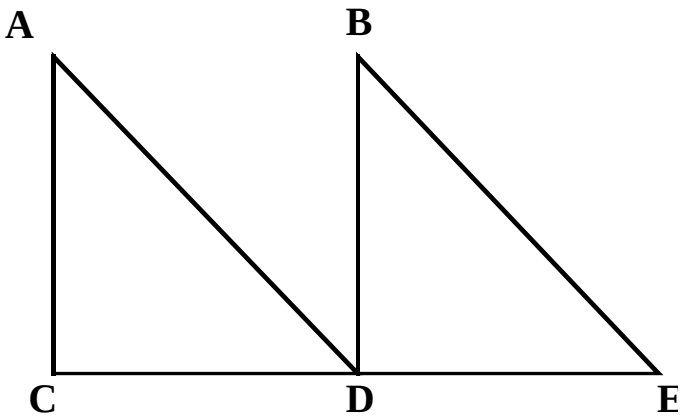
Given:

a. $\angle E \cong \angle C$

b. $\overline{AD} \cong \overline{BF}$

c. $\overline{DE} \cong \overline{BC}$

D.



Given:

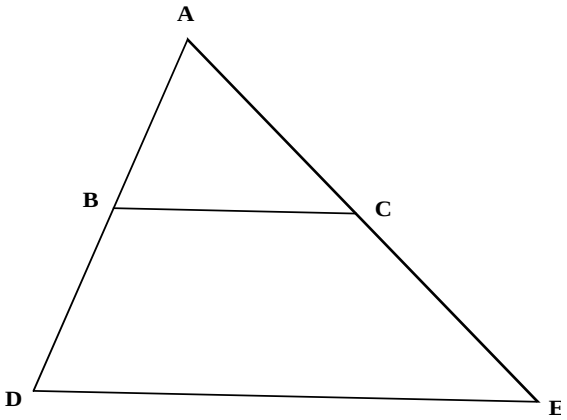
a. $\overline{AC} \perp \overline{CE}$ and $\overline{BD} \perp \overline{CE}$

b. $\overline{AD}$ is parallel to $\overline{BE}$

c. D is the midpoint of $\overline{CE}$

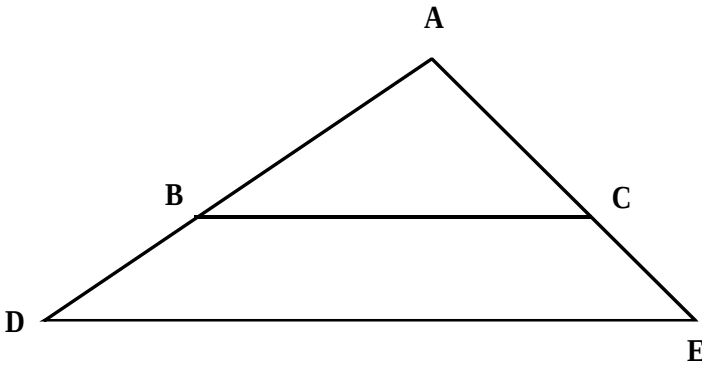
2. Is it possible to prove that the given triangles, under the stated conditions, are similar? If so, what postulate or theorem supports your conclusion?

A.



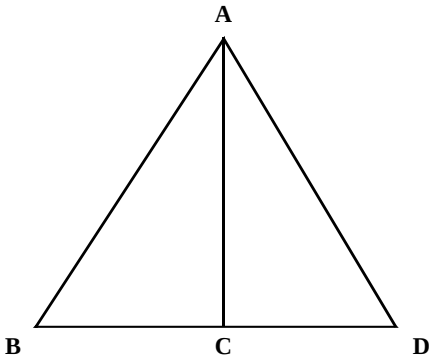
Given:
AB = 4, BD = 12, AC = 5, CE = 15

B.



Given:
$\overline{BC}$ is parallel to $\overline{DE}$

3. What theorem or postulate can be used to prove that $\Delta ABC \cong \Delta ADC$?



Given:
$\overline{AC}$ is the perpendicular bisector of $\overline{BD}$

4. If $\triangle PQR$ and $\triangle DEF$ are two triangles such that $\frac{PQ}{DE} = \frac{PR}{DF}$, which of the following would be sufficient to prove that $\Delta PQR \sim \Delta DEF$?

   A. $\angle P \cong \angle D$

   B. $\angle Q \cong \angle E$

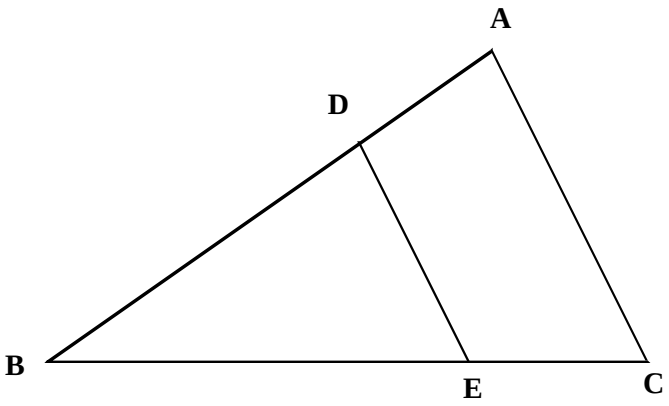
   C. $\angle R \cong \angle F$

   D. $\angle P \cong \angle R$

Conceptual Understanding

1. Complete the following proofs based on the given information:

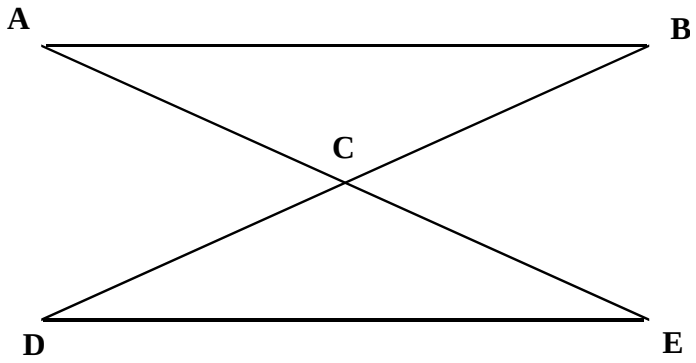
   A.



Given:
$\overline{AC}$ is parallel to $\overline{DE}$

Prove:
$\Delta ABC \sim \Delta DBE$

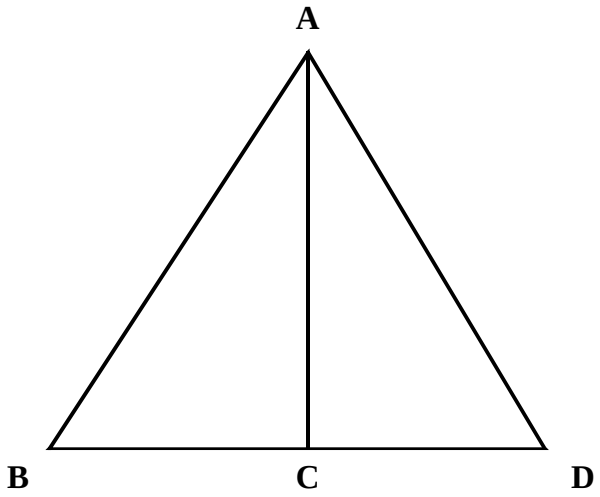
B.



Given:
- A. C is the midpoint of $\overline{BD}$
- B. $\overline{AB}$ is parallel to $\overline{DE}$

Prove:
$\overline{AB} \cong \overline{DE}$

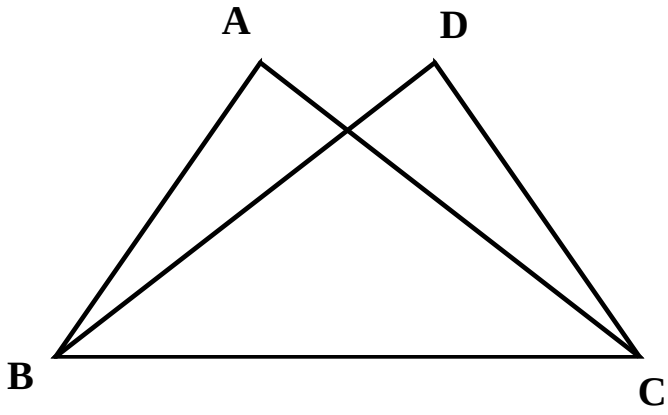
C.



Given:
- A. $\overline{AC}$ bisects $\angle BAD$
- B. $\overline{AB} \cong \overline{AC}$

Prove:
C is the midpoint of $\overline{BD}$

D.



Given:
- A. $\angle A \cong \angle D$
- B. $\angle DBC \cong \angle ACB$

Prove:
$\overline{AB} \cong \overline{DC}$

Problem Solving/Application

Not applicable.

Standard # 6

Standard Set 6.0
Students know and are able to use the triangle inequality theorem.

Deconstructed Standard
1. Students know the triangle inequality theorem.
2. Students use the triangle inequality theorem.

Prior Knowledge Necessary
Students should have the computational and conceptual knowledge outlined in Geometry Standards #4 and #5.

Students should know how to:
- perform basic calculations in the rational number system.
- identify and name the parts of a triangle.

New Knowledge
Students will need to learn to:
- reproduce the triangle inequality theorem.
- determine if three given measures form a triangle.
- write an algebraic inequality, given two sides of a triangle, that identifies the range of possible values for the third side.

Categorization of Educational Outcomes
Competence Level: Knowledge
1. Students will describe the triangle inequality theorem.
2. Students will write the triangle inequality theorem.

Competence Level: Comprehension
1. Students will classify the measures of three line segments to determine if the segments form a triangle.
2. Students will restate the triangle inequality theorem in their own words.

Competence Level: Application
1. Students will apply the triangle inequality theorem to determine if three segments form a triangle.
2. Students will write an inequality that identifies the range of possible values for creating a triangle given two side lengths.

Necessary New Physical Skills
1. Students will construct three given line segments using a straightedge and compass to determine if the segments form a triangle.

Assessable Result of the Standard

1. Students will determine if three line segments can form a triangle.
2. Students will determine the range of values possible for the third side of a triangle given two side lengths.

Standard #6 Model Assessment Items

Computational and Procedural Skills

1. Determine whether the three given measures could be the lengths of the sides of a triangle.
   A. 2, 3, 5

   B. 4, 4, 6

   C. 7, 8, 14

   D. 6, 9, 17

   E. $5\sqrt{2}$, $10\sqrt{2}$, $18\sqrt{2}$

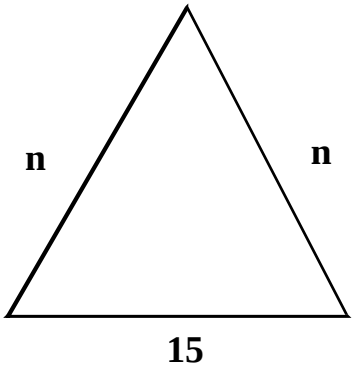
   F. 2.7, 3.9, 6.6

2. Using a compass and straightedge, construct a triangle with sides of 4 cm, 5 cm and 8 cm.

Conceptual Understanding

1. Write an algebraic inequality that identifies the range of possible values for the third side of the triangle given two sides measure 12 cm and 9 cm.

2. In the figure, *n* is a whole number. What is the smallest possible value for *n*?



   A. 1

   B. 7

   C. 8

   D. 14

3. Using a compass and straightedge, construct two different triangles that have side measures of 6 cm and 9 cm. Measure and label the length of the third side on each construction.

Problem Solving/Application

1. Britt has two sides of fencing that each measure 6 feet and 8 feet. What is the minimum length of fencing that she needs to create the third side in order to have a triangle-shaped dog run?

2. If Britt wants to create the largest triangle possible for the dog run, what length of fence does she need?

Standard #7

Standard Set 7.0

Students prove and use theorems involving the properties of parallel lines cut by a transversal, the properties of quadrilaterals, and the properties of circles.

Deconstructed Standard

1. Students prove theorems involving the properties of parallel lines cut by a transversal.
2. Students prove theorems involving the properties of quadrilaterals.
3. Students prove theorems involving the properties of circles.
4. Students use theorems involving the properties of parallel lines cut by a transversal.
5. Students use theorems involving the properties of quadrilaterals.
6. Students use theorems involving the properties of circles.

Prior Knowledge Necessary

Students should have the computational and conceptual knowledge outlined in Geometry Standards #2 and #4.

Students should know how to:

- identify the properties of triangles (i.e., definition of isosceles and equilateral triangles, sum of interior angles of a triangle equals 180^{o}).
- recognize supplementary angles.
- recognize complementary angles.
- recognize vertical angles.
- recognize exterior angles.
- identify the properties of bisectors of line segments.
- identify the properties of perpendicular bisectors of lines segments.
- apply the definition of supplementary angles in solving problems.
- apply the definition of complementary angles in solving problems.
- apply the definition of vertical angles in solving problems.
- apply the definition of exterior angles in solving problems.
- recall the definition of a quadrilateral.
- recall the distance formula between two points.
- recall the definition of a circle.
- recall the definition of the radius of a circle.
- recall the definition of the diameter of a circle.
- recall the definition of a chord of a circle.
- recall the definition of a tangent line to a circle.
- recall the definition of central angles as it applies to circles.
- recall the definition of an arc as it applies to circles.
- recall the definition of arc length as it applies to circles.

New Knowledge

Students will need to learn to:

- recognize alternate interior angles as relating to parallel lines cut by a transversal.
- recognize alternate exterior angles as relating to parallel lines cut by a transversal.
- recognize corresponding angles as relating to parallel lines cut by a transversal.
- recognize symmetry as it exists for quadrilaterals.
- apply knowledge of the relationships in 1-3 above as related to quadrilaterals.
- recognize how mid-segments of a triangle apply to quadrilaterals.
- identify the definition of a secant.
- identify the definition of a secant angle.

Categorization of Educational Outcomes

Competence Level: Knowledge:

1. Students will identify alternate interior angles formed by two parallel lines cut by a transversal.
2. Students will identify alternate exterior angles formed by two parallel lines cut by a transversal.
3. Students will identify corresponding angles formed by two parallel lines cut by a transversal.
4. Students will identify a polygon as a quadrilateral.
5. Students will identify a quadrilateral as a special type of quadrilateral (i.e. trapezoid, kite, parallelogram, rectangle, square, rhombus).
6. Students will identify the associated properties of the different types of quadrilaterals.
7. Students will identify the radius and diameter of a circle.
8. Students will identify chords, secants, and tangent lines of a circle.
9. Students will identify arcs of a circle.
10. Students will identify arc lengths of a circle.

Competence Level: Application

1. Students will use the methods of inductive reasoning to prove the properties of two parallel lines cut by a transversal.
2. Students will use the methods of deductive reasoning to prove the properties of two parallel lines cut by a transversal.
3. Students will demonstrate knowledge of properties of parallel lines cut by a transversal to prove other theorems, corollaries, and axioms related to parallel lines cut by a transversal.
4. Students will demonstrate knowledge of properties of parallel lines cut by a transversal to prove other theorems related to parallel lines cut by a transversal.
5. Students will demonstrate knowledge of properties of parallel lines cut by a transversal to prove other corollaries related to parallel lines cut by a transversal.
6. Students will demonstrate knowledge of properties of parallel lines cut by a transversal to prove other axioms related to parallel lines cut by a transversal.
7. Students will use the methods of inductive reasoning to prove the properties of quadrilaterals.
8. Students will use the methods of deductive reasoning to prove the properties of quadrilaterals.

9. Students will demonstrate knowledge of properties of parallel lines cut by a transversal to prove properties of quadrilaterals.
10. Students will demonstrate knowledge of properties of triangles to prove properties of quadrilaterals.
11. Students will use algebraic methods to determine characteristics of a specified quadrilateral given some of its characteristics.
12. Students will use the methods of inductive reasoning to prove the properties of circles.
13. Students will use the methods of deductive reasoning to prove the properties of circles.
14. Students will use algebraic methods to determine characteristics of a specified circle given some of its characteristics.

Necessary New Physical Skills

1. Students will use a compass to construct parallel lines.
2. Students will use a compass to construct circles.

Assessable Result of the Standard

1. Students will generate proofs of theorems involving the properties of parallel lines cut by a transversal.
2. Students will solve for unknown angle measures using properties of parallel lines cut by a transversal.
3. Students will generate proofs of theorems involving the properties of quadrilaterals.
4. Students will solve for unknown angle measures of quadrilaterals using properties of parallel lines cut by a transversal.
5. Students will solve for unknown segment lengths using properties of quadrilaterals.
6. Students will generate proofs of theorems involving the properties of circles.
7. Students will solve for unknown central angles using the properties of circles.
8. Students will solve for unknown arcs using the properties of circles.
9. Students will solve for unknown arc lengths using the properties of circles.

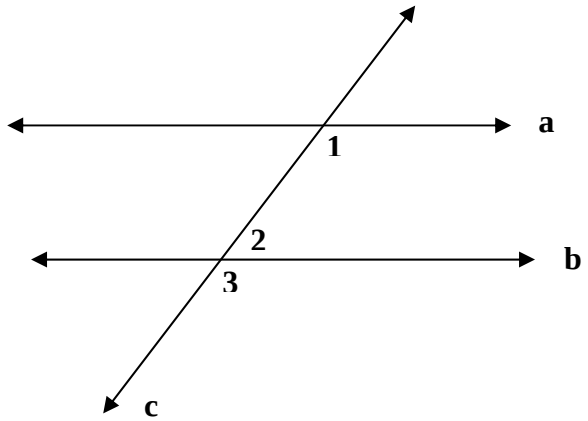
Standard #7 Model Assessment Items

Computational and Procedural Skills

1. Proofs:

Given: $a \parallel b$

Prove: $\angle 1$ *and* $\angle 2$ are supplementary



Proof Statements *Reasons*

A. $a \parallel b$

B. $\angle 1 = \angle 3$

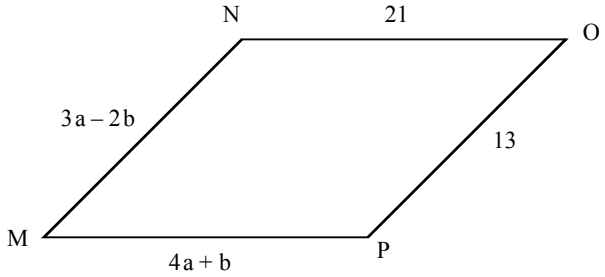
C. $\angle 2$ *and* $\angle 3$ are supplementary.

D. $\angle 2 + \angle 3 = 180^o$

E. $\angle 2 + \angle 1 = 180^o$

F. $\angle 1$ *and* $\angle 2$ are supplementary.

2. What values of *a* and *b* make quadrilateral *MNOP* a parallelogram?



A. $a = 1, b = 5$

B. $a = 5, b = 1$

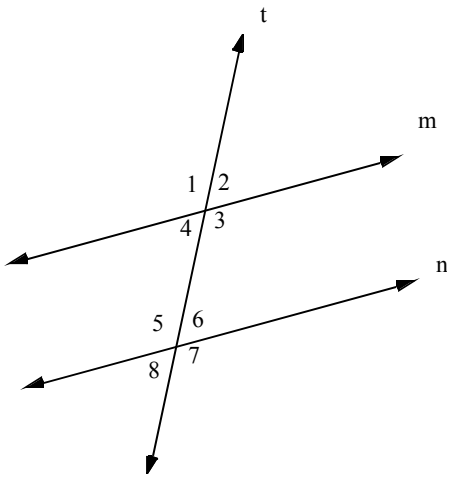
C. $a = \frac{11}{7}, b = \frac{34}{7}$

D. $a = \frac{34}{7}, b = \frac{11}{7}$

3. If an arc of a circle is 135° and the length is 9 cm, what is the length of the radius of the circle?

Conceptual Understanding

1. Given the figure below with m||n, justify each conclusion:



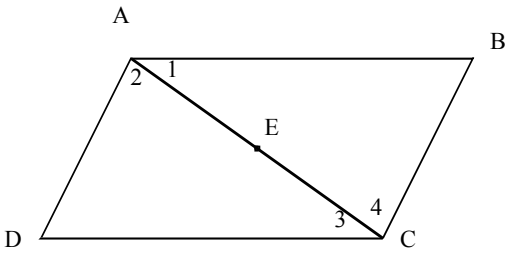
A. $\angle 1 \cong \angle 5$

B. $\angle 4 \cong \angle 6$

C. $\angle 5 \cong \angle 7$

D. This is a two-step proof: $\angle 2 \cong \angle 8$

2. Given ABCD is a parallelogram and E is the midpoint of $\overline{AC}$, justify the following conclusions:



A. $\overline{AB} \cong \overline{CD}$

B. $m\angle 1 = m\angle 3$

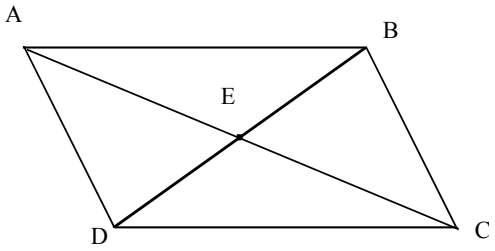
C. $\overline{AD} \cong \overline{BC}$

D. $m\angle 2 = m\angle 4$

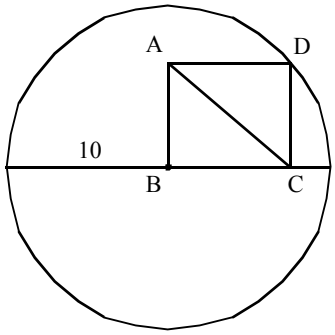
E. $\triangle ADC \cong \triangle CBA$

F. $AB = CD$

3. Given parallelogram ABCD and $m\angle BAD = 70$, find as many other angle measures as you can:



4. The figure below appeared in a problem on an SAT test. B is in the center of the circle with radius 10 and ABCD is a rectangle.



What is the length of diagonal AC?

Problem Solving/Application

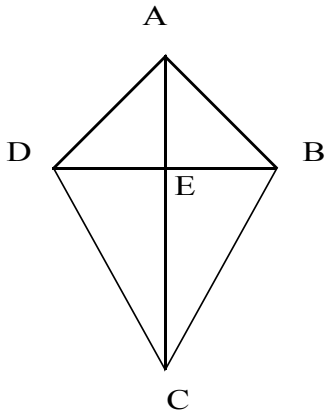
1. Given ABCD is a kite, $m\angle BCA = 42^o$, and $m\angle DAE = 55^o$, find:

A. $m\angle BAC =$ _____

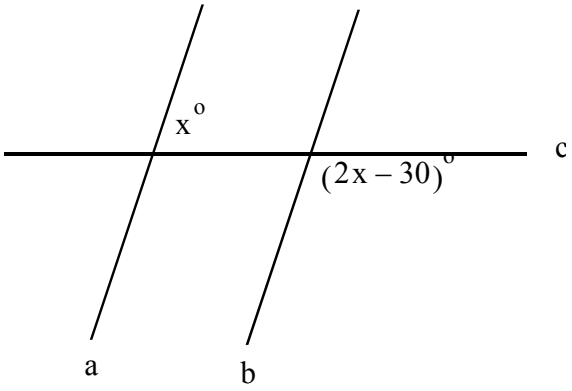
B. $m\angle DCA =$ _____

C. $m\angle DEC =$ _____

D. $\overline{ED} \cong$ _____



2. In the figure below, $a \parallel b$:

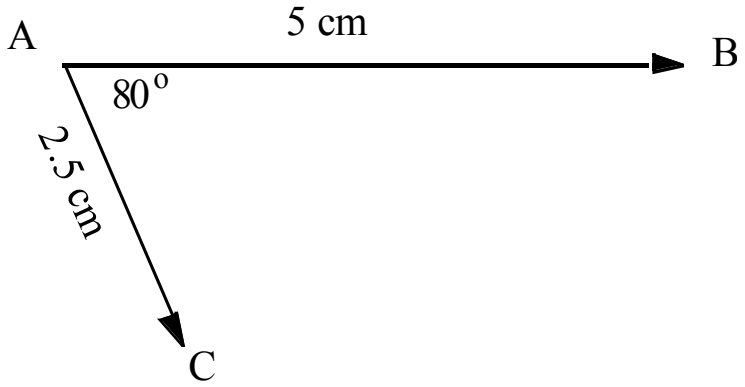


A. Write an equation relating the measures of the two indicated angles.

B. Find the measure of the acute angle.

C. Find the measure of the obtuse angle.

3. Given the figure below, suppose the force represented by vector AB is 50 pounds. How many pounds does 1 cm on your figure represent? How many pounds is force AC?

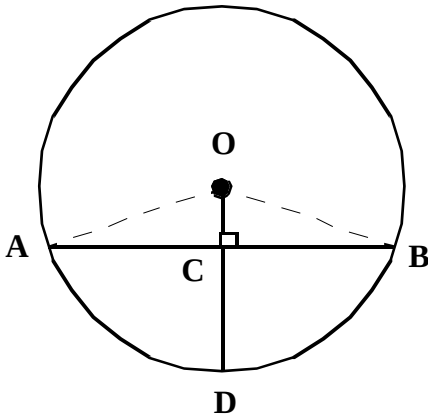


4. A Black and Decker circular saw is used to cut into a board as pictured below. The radius OD of the blade is perpendicular to the board. The diameter of the blade is 7.75 inches and the cutting depth, CD, is 2.375 inches.

A. Find the length of the cut from A to B.

B. Find the measure of $\angle DOA$.

C. Find $m\angle ADB$.



Standard # 8

Standard Set 8.0

Students know, derive, and solve problems involving the perimeter, circumference, area, volume, lateral area, and surface area of common geometric figures*.

**Note: Common geometric figures include triangles, quadrilaterals, prisms, circles, cones, cubes, and cylinders.*

Deconstructed Standard

1. Students know the definition of perimeter of polygons.
2. Students know the definition of circumference of circles.
3. Students know the definition of area of common geometric figures.
4. Students know the definition of volume of common geometric figures.
5. Students know the definition of lateral area of common geometric figures.
6. Students know the definition of surface area of common geometric figures.
7. Students derive formulas for perimeter of polygons.
8. Students derive the formula for the circumference of a circle.
9. Students derive formulas for area of common geometric figures.
10. Students derive formulas for volume of common geometric figures.
11. Students derive formulas for lateral area of common geometric figures.
12. Students derive formulas for surface area of common geometric figures.
13. Students solve problems involving perimeter of common geometric figures.
14. Students solve problems involving the circumference of a circle.
15. Students solve problems involving area of common geometric figures.
16. Students solve problems involving volume of common geometric figures.
17. Students solve problems involving lateral area of common geometric figures.
18. Students solve problems involving surface area of common geometric figures.

Prior Knowledge Necessary

Students should know how to:

- identify triangles, quadrilaterals, prisms, circles, cones, cubes and cylinders.
- identify the basic parts of quadrilaterals, prisms, circles, cones, cubes and cylinders.
- measure angles.
- sketch and label triangles, quadrilaterals, prisms, circles, cones, cubes and cylinders.

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New Knowledge

Students will need to learn to:

- apply formulas for perimeter of triangles and quadrilaterals.
- apply formulas for circumference of circle.
- apply formulas for area of triangles, quadrilaterals and circle.
- apply formulas for the volume of cones, cubes, and cylinders.
- apply formulas for lateral area and surface area of prisms, cones, cubes and cylinders.
- calculate measures of angles.
- make connections between common geometric figures and their perimeter, circumference, area, volume, lateral area, and surface area.

- solve real world problems involving perimeter, circumference, area, volume, lateral area, and surface area.

Categorization of Education Outcome

Competence Level: Knowledge

1. Students will recall formulas for perimeter, circumference, area, volume, lateral area, and surface area of common geometric figures.

Competence Level: Comprehension

1. Students will derive formulas for perimeter, circumference, area, volume, lateral area, and surface area of common geometric figures.

Competence Level: Application

1. Students will apply perimeter, circumference, area, volume, lateral area and surface area relationships of common geometric figures to solve real world problems.

Necessary New Physical Skills

None

Assessable Result of the Standard

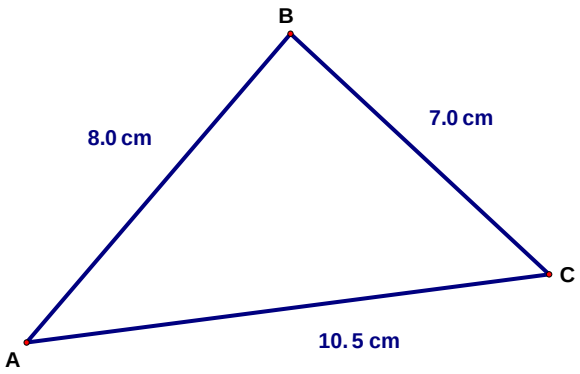
1. Students will derive the formula for the perimeter of a triangle.
2. Students will derive the formula for the circumference of a circle.
3. Students will derive the formulas for area of a triangle, quadrilateral, and a circle.
4. Students will derive formulas for the volume of a prism, cone, cube and cylinder.
5. Students will derive formulas for the lateral area of a prism, cone and cylinder.
6. Students will derive formulas for the surface area of a prism, cone, cube and cylinder.
7. Students will solve problems involving the perimeter triangles and quadrilaterals.
8. Students will solve problems involving the circumference of a circle.
9. Students will solve problems involving the area of triangles, quadrilaterals, and circles.
10. Students will solve problems involving the circumference of circles.
11. Students will solve problems involving the lateral area of prisms, cones and cylinders.
12. Students will solve problems involving the volume of prisms, cones, cubes and cylinders.

Standard #8 Model Assessment Items

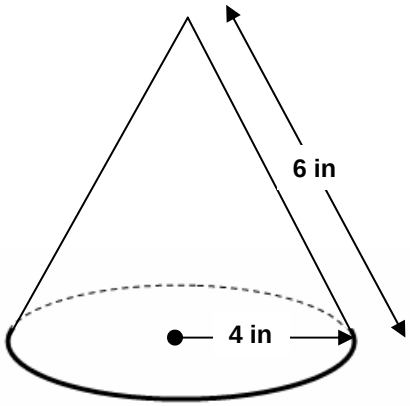
Computational and Procedural Skills

1. Find the perimeter and area of a rectangle that has a length of 15 inches and a width of 7 inches.

2. Find the perimeter of the following triangle.

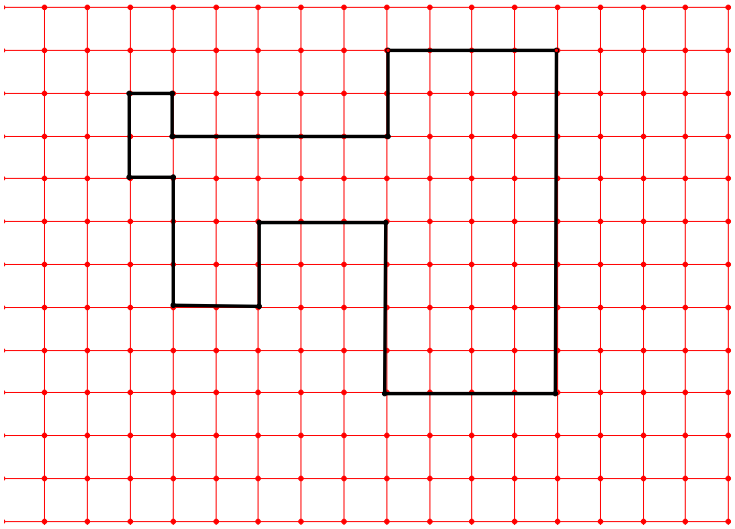


3. Find the lateral area of the right cone shown below.

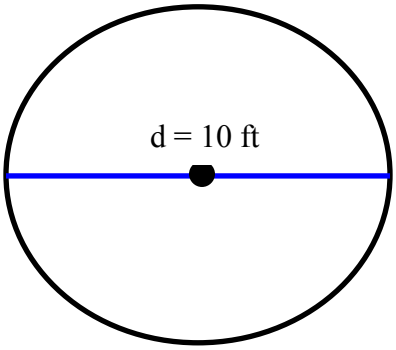


4. What is the area of a circle with a radius of 7.5 m?

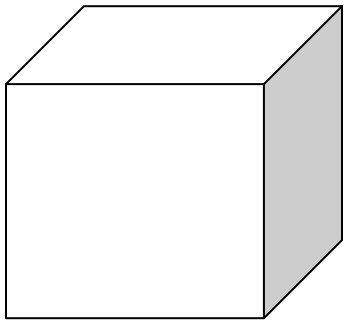
5. Find the perimeter and area of the following figure (each square is 1 unit).



6. Find the circumference of the circle below.



7. Find the volume of the cube below.



4 mm

8. Find the surface area of the figure below.

10. Fill the blank using the proper geometric term that best describes the following:

A. _____ the distance around a building

B. _____ the distance around a ball

C. _____ the amount of land in a field

D. _____ the amount of space inside any 3D object

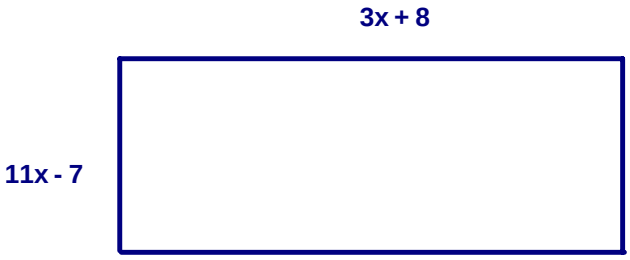
E. _____ the amount of wood needed to make a dog house

Conceptual Understanding

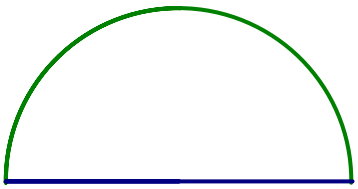
1. The area of a square is 144 cm^2. What are the lengths of the sides of the square?

2. The area of a rectangle is 72 in^2. If the length is twice the width, what are the dimensions of the rectangle?

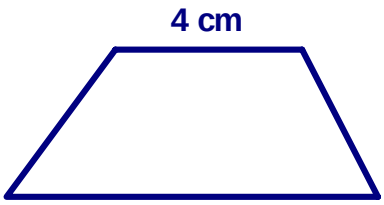
3. The perimeter of the following rectangle is 58 cm. Find the length of each side and the area of the rectangle.



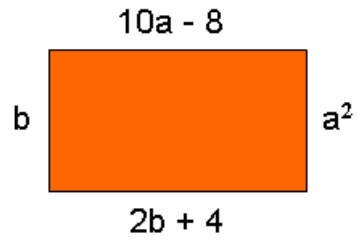
4. The area of the following semicircle is 32π cm^2. What is the radius of the whole circle?



5. The area The length of the longer base of a trapezoid is twice as long as the shorter base. The shorter base is 4 cm. The area of the trapezoid is 84 cm^2. What is the height of the trapezoid?

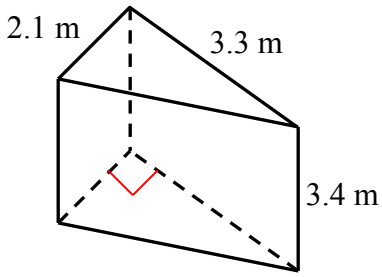


6. A rectangle has the following sides. One of the lengths is an odd integer. Find the perimeter of the rectangle.

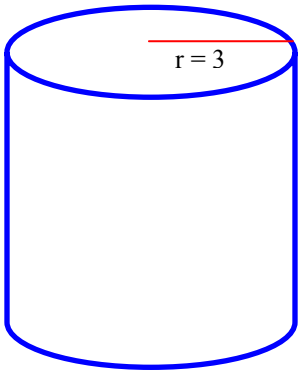


7. What is the circumference of a circle if the area is 28.26 cm^2? (Use the approximation $\pi \approx 3.14$ for all calculations.)

8. What is the volume of the right prism below with the given side lengths.

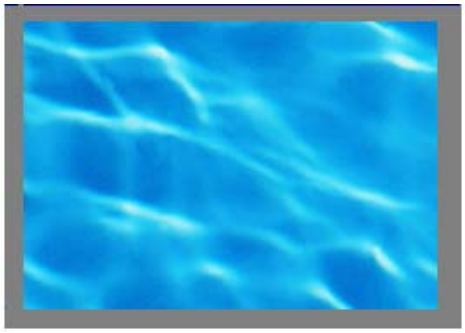


9. Find the height of the cylinder below given that the surface area is 42π in^2 and radius is 3 in. ($SA = 4\pi r + 2\pi rh$).

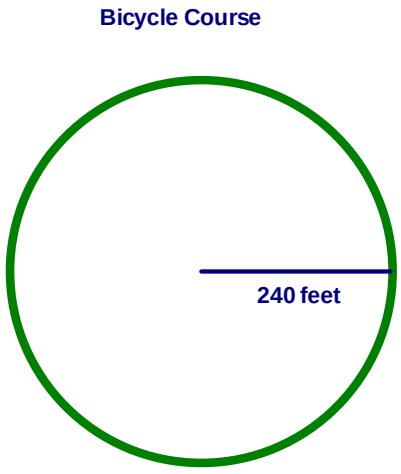


Problem Solving/Application

1. Alexis has a 24 feet by 18 feet pool that she wants to frame with tile. She plans to put a 2-foot tile border around the pool as shown below. Tile costs $ 2.50 per square foot. She receives a bill for $520 for the tile. Was she billed correctly? Justify your answer.



2. Kyler enters a bike riding event. The bicycle course is circular as illustrated below. His bicycle spokes are 11 inches long from the center of the tire to the edge of the rubber and the width of the tire rubber is 1 inch, as shown below. How many times will each wheel turn at the end of one complete lap around the track?

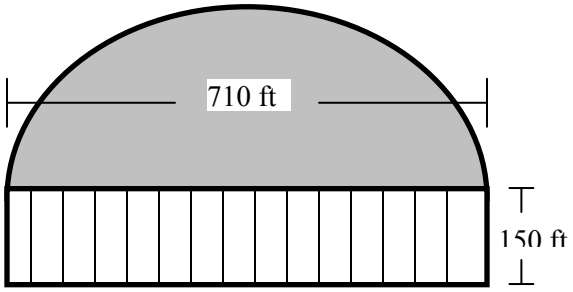


3. K & J Development Company has purchased 140 acres of vacant land to build a new housing community. The company plans to include four home options ranging from a fairly large home with an additional storage building to a smaller home with no additional building. Because of the geographic layout of the land, the following models have been considered:

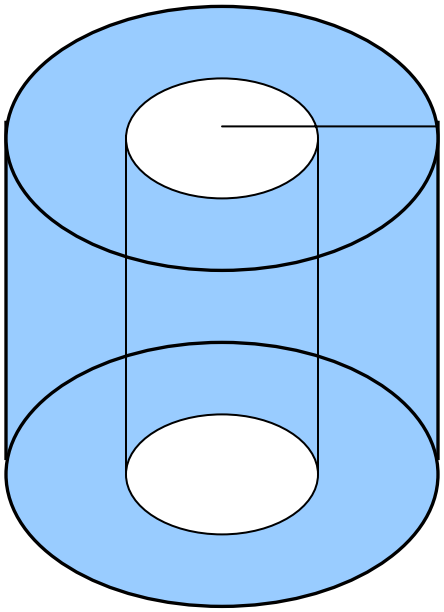
Plan	Number of this model	Basic Dimensions of House	Additional Building
A	85	70 feet by 60 feet	20 feet by 15 feet
B	90	60 feet by 60 feet	15 feet by 12 feet
C	100	60 feet by 55 feet	15 feet by 10 feet
D	120	50 feet by 55 feet	None

There are two county ordinances that are of concern for K & J. The first ordinance requires that new homes have twice as much yard area as structure area.
The second ordinance requires the establishment of a park when a company develops more than 100 acres of land. The park must be one fourth the size of the total community. If K & J proceeds with the numbers presented in the table above, will the company be able to satisfy both county ordinances? Justify your answer. (One acre = approximately 43,560 square feet.)

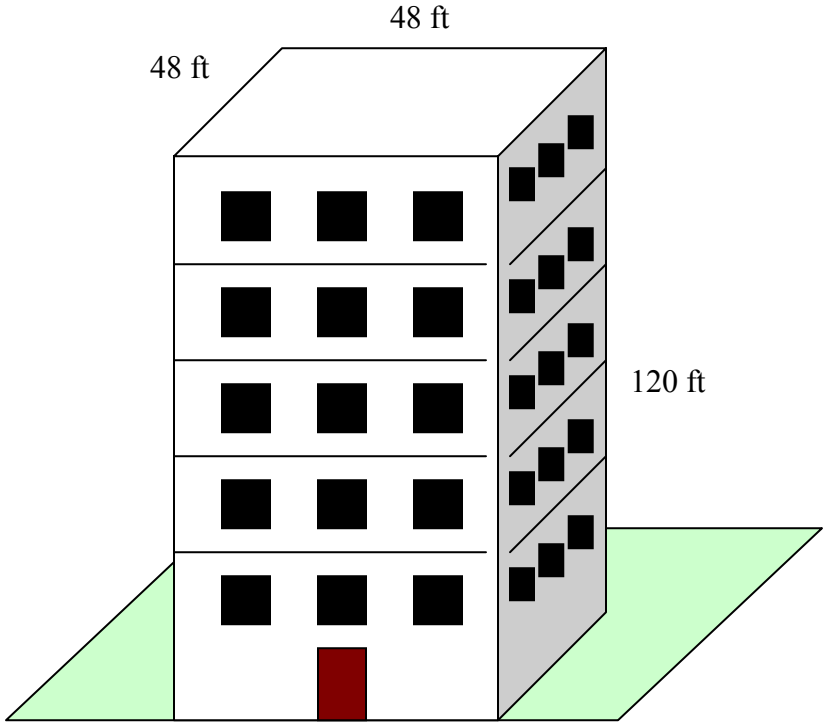
4. If the Astro Dome in Houston, TX, has a diameter of 710 ft, what is the circumference of the outer shell? Suppose the height of the rectangular building was 150 ft: What would be the perimeter of the entire building as it is shown below?



5. How much water can a cylinder tank with a radius of 25 ft hold given that it is missing the internal cylinder section with a radius of 10 ft?



6. What is the surface area of the concrete portion of a five story building with three 6 ft x 6 ft windows on each floor side?



Standard #9

Standard Set 9.0

Students compute the volumes and surface areas of prisms, pyramids, cylinders, cones and spheres; and students commit to memory the formulas for prisms, pyramids, and cylinders.

Deconstructed Standard

1. Students compute the volume and surface area of a prism.
2. Students compute the volume and surface area of a pyramid.
3. Students compute the volume and surface area of a cylinder.
4. Students compute the volume and surface area of a cone.
5. Students compute the volume and surface area of a sphere.
6. Students commit to memory the formulas for prisms.
7. Students commit to memory the formulas for pyramids.
8. Students commit to memory the formulas for cylinders.

Prior Knowledge Necessary

Student should know how to:

- substitute values into variables.
- identify a height from a shape.
- solve equations using squares and square roots.
- find a radius from a diameter.
- use π to find an approximate value.
- calculate the area and perimeter of a rectangle.
- calculate the area and perimeter of a triangle.
- calculate the area and circumference of a circle.
- use the Pythagorean theorem to find a missing side.
- identify parallel and perpendicular lines.
- identify intersecting planes.

New Knowledge

Students will need to learn to:

- recognize and use vocabulary such as lateral edge, lateral face, base, base edge, vertex, height (altitude), and slant height.
- recognize the difference between the lateral area and the total surface area.
- apply the formulas for calculating the surface area and volumes for prisms, pyramids, cylinders, cones and spheres.

Categorization of Educational Outcomes

Competence Level: Application

1. Students will use methods they have learned to identify the necessary parts of the solid.
2. Students will use methods they have learned to find the necessary missing parts of the solid.

3. Students will demonstrate their ability to use the necessary parts to find the surface areas and volumes of various prisms.
4. Students will demonstrate their ability to use the necessary parts to find the surface areas and volumes of various pyramids.
5. Students will demonstrate their ability to use the necessary parts to find the surface areas and volumes of various cylinders.
6. Students will demonstrate their ability to use the necessary parts to find the surface areas and volumes of various cones.
7. Students will demonstrate their ability to use the necessary parts to find the surface areas and volumes of various spheres.
8. Students will show that they can recall from memory the accurate formulas for surface areas and volumes of prisms, pyramids, and cylinders.

Necessary New Physical Skills

None

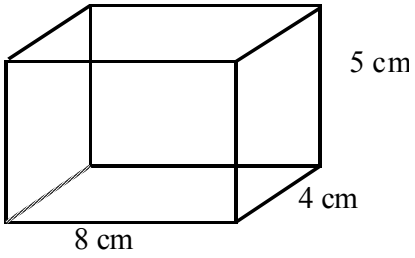
Assessable Result of the Standard

1. Students will determine the surface area and volume of a prism.
2. Students will determine the surface area and volume of a pyramid.
3. Students will determine the surface area and volume of a cylinder.
4. Students will determine the surface area and volume of a cone.
5. Students will determine the surface area and volume of a sphere.
6. Students will recite from memory the formulas for surface area and volume of prisms, pyramids, and cylinders.

Standard #9 Model Assessment Items

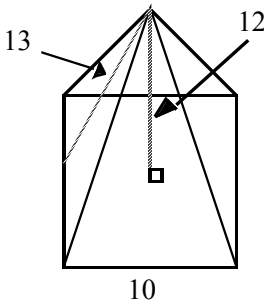
Computational and Procedural Skills

1. Find the indicated values for the prism shown.



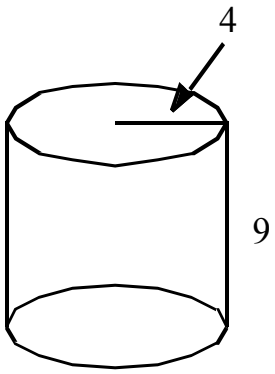
A. Base area
B. Height
C. Lateral area
D. Surface area
E. Volume

2. Find the indicated values for the pyramid shown



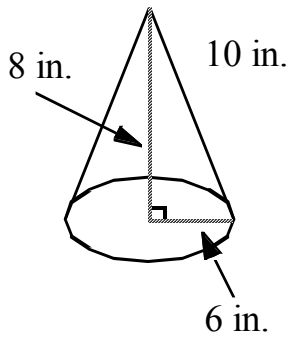
A. Height
B. Slant height
C. Base area
D. Lateral area
E. Surface area

3. Find the indicated values for the cylinder shown (leave answers in terms of π).



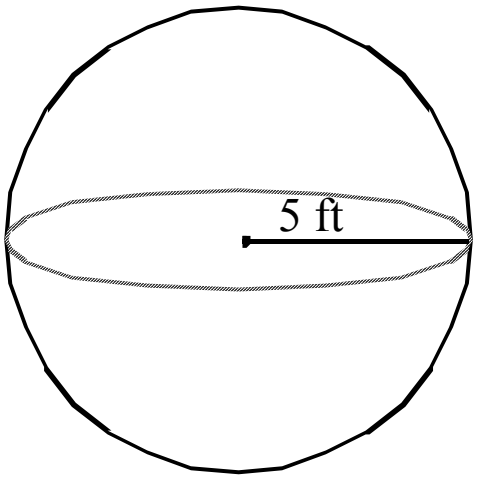
A. Height
B. Circumference of the base
C. Area of the base
D. Lateral area
E. Surface area
F. Volume

4. Find the indicated values for the cone shown (leave answers in terms of π).



A. Height
B. Slant height
C. Circumference of the base
D. Area of the base
E. Lateral area
F. Surface area
G. Volume

5. Find the indicated values for the sphere shown (leave answers in terms of π).

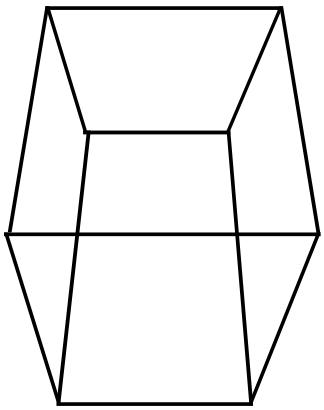


A. Surface area
B. Volume

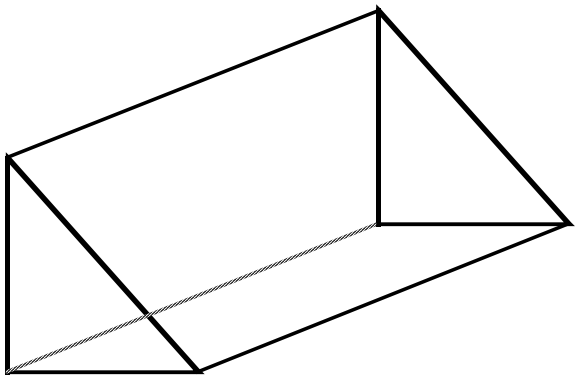
Conceptual Understanding

1. Are the following right prisms?

A.

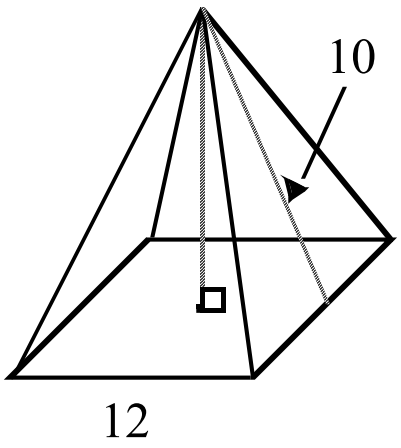


B.

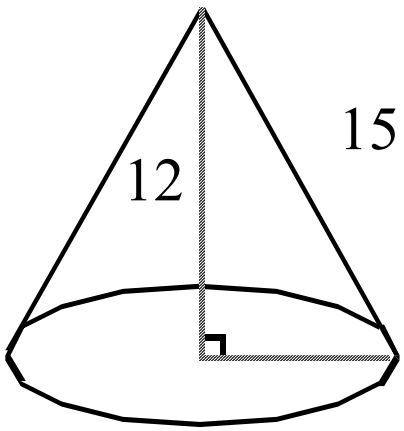


2. Find the volume of each shape.

A.



B.



3. The volume of a cylinder is 36π and the height is 4. Find the radius of the base.

4. The surface area of a cone is 96π and the radius is 6. Find the slant height.

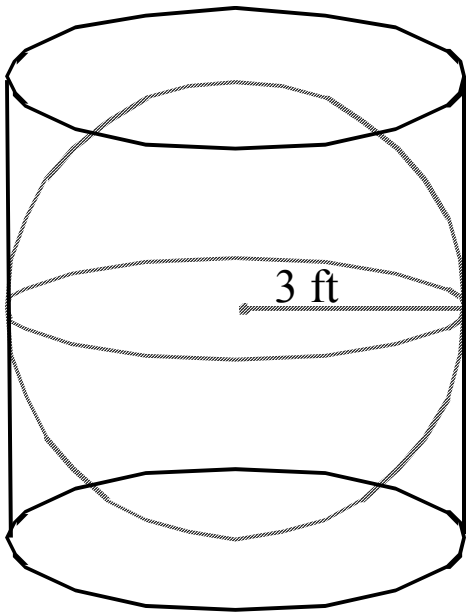
5. The surface area of a sphere is 144π. Find the volume.

6. The diameter of a sphere is $2\sqrt{3}$. Find the surface area and the volume.

7. A sphere sits inside a cylinder as shown. The top and bottom of the sphere touch the top and bottom of the cylinder.

   A. Show that the area of the sphere equals the lateral area of the cylinder.

   B. Find the ratio of the volume of the sphere to the volume of the cylinder

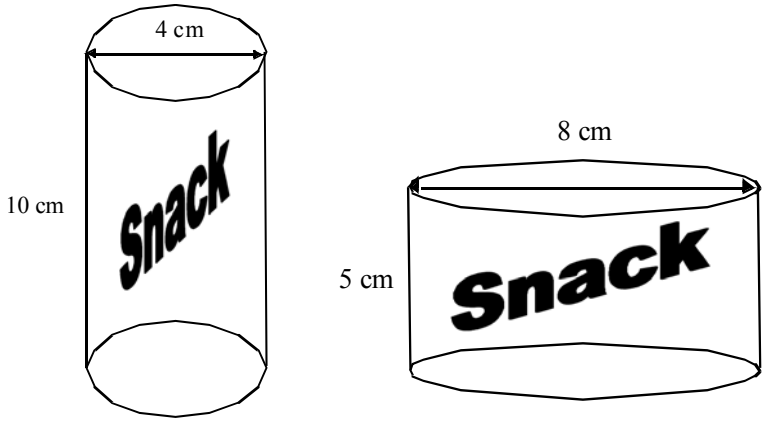


Problem Solving/Application

1. Use the cans at the right to answer the following:

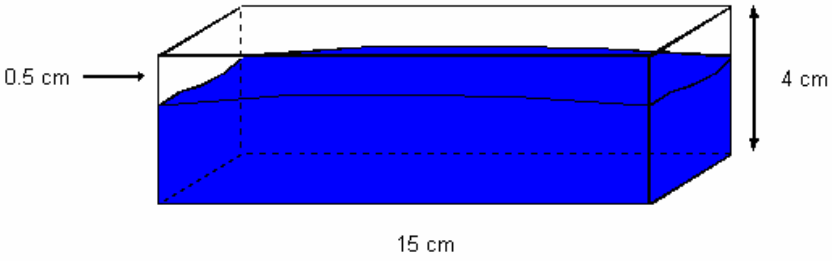
   A. One can of a snack is twice as tall as another, but only half as wide. Which can holds more of the snack?

   B. If the can on the left costs $2.99 and the one on the right costs $5.89, which is the better deal?



2. Use the pictures below to answer the following:

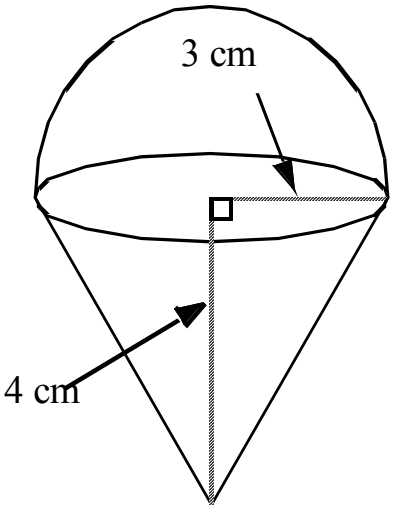
   A. How many liters of water is needed to fill the container shown within 0.5cm of the



   B. An ice cream store owner is trying to figure out the best profit for an ice cream cone, but he needs to know the volume. Find the volume of the ice cream cone at the right.

   HINT: Find the volume of the hemisphere.

   (Picture not to scale)



Standard #10

Standard Set 10.0

Students compute areas of polygons, including rectangles, scalene triangles, equilateral triangles, rhombi, parallelograms, and trapezoids.

Deconstructed Standard

1. Students will compute the area of regular polygons.
2. Students will compute the area of rectangles.
3. Students will compute the area of scalene triangles.
4. Students will compute the area of equilateral triangles.
5. Students will compute the area of rhombi.
6. Students will compute the area of parallelograms.
7. Students will compute the area of trapezoids.

Prior Knowledge Necessary

Students should know how to:

- perform computations with rational numbers.
- apply the order of operations.
- apply formulas to solve problems.
- use the distance formula.
- solve problems involving squares and square roots.
- recognize a coordinate pair.
- use the coordinate plane.
- find the length of the side of a polygon that is graphed on a coordinate plane.
- use the Pythagorean theorem.
- identify a rectangle and its properties.
- identify a scalene triangle and its properties.
- identify an equilateral triangle and its properties.
- identify a rhombus and its properties.
- identify a parallelogram and its properties.
- identify a trapezoid and its properties.
- identify the properties of special right triangles.

New Knowledge

Students will need to learn to:

- define a regular polygon.
- recall the formulas for finding the area of a polygon, including rectangles, scalene triangles, equilateral triangles, rhombi, parallelograms, and trapezoids, and be able to use the formulas to compute the area.
- use dissection of regular polygons to compute the area.
- find the missing length of a side or the height of a polygon given the area.
- find the missing length of a side or the height of a polygon using the definition of the polygon, the distance formula, or the Pythagorean theorem.

Categorization of Educational Outcomes

Competence Level: Knowledge

1. Students will define a polygon.
2. Students will know the formulas for finding the area of polygons.

Competence Level: Comprehension

1. Students will classify polygons as regular or non-regular polygons.

Competence Level: Application

1. Students will apply previously acquired knowledge and skills to compute the areas of polygons.
2. Students will demonstrate their ability to use formulas to compute the areas of polygons.
3. Students will demonstrate their ability to find missing measures of a polygon given the area.

Competence Level: Analysis

1. Students will examine how to break down regular polygons by dissection.

Necessary New Physical Skills

None

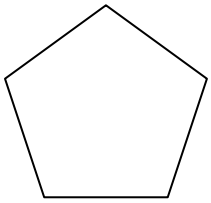
Assessable Result of the Standard

1. Students will be able to identify polygons.
2. Students will be able to compute the area of polygons using different methods including formulas or dissection.

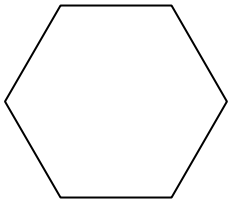
Standard #10 Model Assessment Items

Computational and Procedural Skills

1. Find the area of a regular pentagon with a radius of approximately 10.2 in. and each side is 12 in. using two different methods and compare the results.



2. Find the area of a regular hexagon with a perimeter of 36 in. using three different methods and compare the results.



Conceptual Understanding

1. If the short diagonal of a rhombus is 16 cm and its area is $136\,cm^2$, what is the length of the longer diagonal?

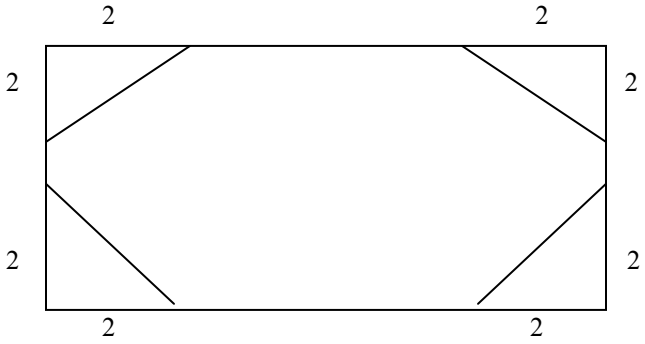
2. The area of a trapezoid is 60 ft^2. Its bases measure 7 ft. and 13 ft. What is the measure of its height?

3. A triangle has an area of $45\,m^2$ and a base of 7 m. Find the height of the triangle.

4. What is the area of a trapezoid ABCD with vertices A(2, 5), B(8, 4), C(11, -1), and D(8, -5)?

Problem Solving/Application

1. The rectangle shown below has a length of 10 ft. and a width of 5 ft. If the four triangles are removed, what would be the area of the remaining figure?



Standard #11

Standard Set 11.0

Students determine how changes in dimensions affect the perimeter, area, and volume of common geometric figures and solids.

Deconstructed Standard

1. Students determine how changes in geometric dimensions affect the perimeter of a triangle.
2. Students determine how changes in geometric dimensions affect the perimeter of a square.
3. Students determine how changes in geometric dimensions affect the perimeter of a rhombus.
4. Students determine how changes in geometric dimensions affect the perimeter of a rectangle.
5. Students determine how changes in geometric dimensions affect the perimeter of a parallelogram.
6. Students determine how changes in geometric dimensions affect the perimeter of a trapezoid.
7. Students determine how changes in geometric dimensions affect the perimeter of a regular pentagon.
8. Students determine how changes in geometric dimensions affect the perimeter of a regular hexagon.
9. Students determine how changes in geometric dimensions affect the circumference of a circle.
10. Students determine how changes in geometric dimensions affect the area of a triangle.
11. Students determine how changes in geometric dimensions affect the area of a square.
12. Students determine how changes in geometric dimensions affect the area of a rhombus.
13. Students determine how changes in geometric dimensions affect the area of a rectangle.
14. Students determine how changes in geometric dimensions affect the area of a parallelogram.
15. Students determine how changes in geometric dimensions affect the area of a trapezoid.
16. Students determine how changes in geometric dimensions affect the area of a circle.
17. Students determine how changes in geometric dimensions affect the surface area of a rectangular right prism.
18. Students determine how changes in geometric dimensions affect the surface area of a square right prism.
19. Students determine how changes in geometric dimensions affect the surface area of a triangular right prism.
20. Students determine how changes in geometric dimensions affect the surface area of a cylinder.

21. Students determine how changes in geometric dimensions affect the surface area of a rectangular pyramid.
22. Students determine how changes in geometric dimensions affect the surface area of a square pyramid.
23. Students determine how changes in geometric dimensions affect the surface area of a triangular pyramid.
24. Students determine how changes in geometric dimensions affect the surface area of a cone.
25. Students determine how changes in geometric dimensions affect the surface area of a sphere.
26. Students determine how changes in geometric dimensions affect the volume of a rectangular right prism.
27. Students determine how changes in geometric dimensions affect the volume of a square right prism.
28. Students determine how changes in geometric dimensions affect the volume of a triangular right prism.
29. Students determine how changes in geometric dimensions affect the volume of a cylinder.
30. Students determine how changes in geometric dimensions affect the volume of a rectangular pyramid.
31. Students determine how changes in geometric dimensions affect the volume of a square pyramid.
32. Students determine how changes in geometric dimensions affect the volume of a triangular pyramid.
33. Students determine how changes in geometric dimensions affect the volume of a cone.
34. Students determine how changes in geometric dimensions affect the volume of a sphere.

Prior Knowledge Necessary

Students should have the computational and conceptual knowledge outlined in Seventh Grade Measurement and Geometry Standard 2.0, Algebra I Standards #5, #13, and #15 and Geometry Standard #9.

Students should know how to:

- compute the perimeter, area, and volume of common geometric objects and use the results to find measures of less common objects.
- solve multi-step problems.
- calculate the volumes and surface areas of common geometric objects.

New Knowledge

Students will need to learn to:

- use ratios to solve for missing dimensions of similar figures.
- use ratios to solve for missing perimeters of similar figures.
- use ratios to solve for missing areas of similar figures.
- use ratios to solve for missing volumes of similar figures.

- identify the ratio of the corresponding sides (scale factor).
- identify the ratio of the perimeters.
- identify the ratio of the areas.
- identify the ratio of the volumes.
- understand the relationship between scale factor, perimeter ratio, area ratio, and volume ratio.
- identify the ratio to use for problem solving.
- find the ratio to use for problem solving.

Categorization of Educational Outcomes

Competence Level: Knowledge

1. Students will identify the scale factor of two similar common geometric figures.
2. Students will identify the ratio of the perimeters of two similar common geometric figures.
3. Students will identify the ratio of the areas of two similar common geometric figures.
4. Students will identify the scale factor of two similar common geometric solids.
5. Students will identify the ratio of the surface areas of two similar common geometric solids.
6. Students will identify the ratio of the volumes of two similar common geometric solids.
7. Students will select the ratio to be used for problem solving.

Competence Level: Comprehension

1. Students will give examples of each type of ratio.
2. Students will make sense out of the relationship between the different ratios.

Competence Level: Application

1. Students will compute the scale factor.
2. Students will compute the ratio of the perimeters.
3. Students will compute the ratio of the areas.
4. Students will compute the ratio of the surface areas.
5. Students will compute the ratio of the volumes.
6. Students will implement the scale factor in problem solving.
7. Students will implement the ratio of the perimeters in problem solving.
8. Students will implement the ratio of the areas in problem solving.
9. Students will implement the ratio of the surface areas in problem solving.
10. Students will implement the ratio of the volumes in problem solving.

Competence Level: Analysis

1. Students will recognize which ratio to use in problem solving.

Necessary New Physical Skills

None

Assessable Result of the Standard

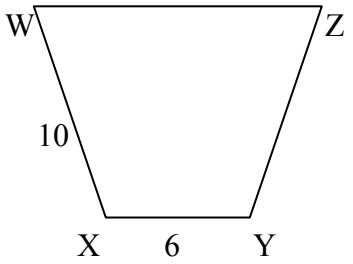
1. Students will find the scale factor of two similar triangles.
2. Students will find the ratio of the perimeters of two similar triangles.
3. Students will find the ratio of the areas of two similar triangles.
4. Students will find the scale factor of two squares.
5. Students will find the ratio of the perimeters of two squares.
6. Students will find the ratio of the areas of two squares.
7. Students will find the scale factor of two similar rhombi.
8. Students will find the ratio of the perimeters of two similar rhombi.
9. Students will find the ratio of the areas of two similar rhombi.
10. Students will find the scale factor of two similar rectangles.
11. Students will find the ratio of the perimeters of two similar rectangles.
12. Students will find the ratio of the areas of two similar rectangles.
13. Students will find the scale factor of two similar parallelograms.
14. Students will find the ratio of the perimeters of two similar parallelograms.
15. Students will find the ratio of the areas of two similar parallelograms.
16. Students will find the scale factor of two similar trapezoids.
17. Students will find the ratio of the perimeters of two similar trapezoids.
18. Students will find the ratio of the areas of two similar trapezoids.
19. Students will find the scale factor of two regular pentagons.
20. Students will find the ratio of the perimeters of two regular pentagons.
21. Students will find the ratio of the areas of two regular pentagons.
22. Students will find the scale factor of two regular hexagons.
23. Students will find the ratio of the perimeters of two regular hexagons.
24. Students will find the ratio of the areas of two regular hexagons.
25. Students will find the scale factor of two circles.
26. Students will find the ratio of the circumferences of two circles.
27. Students will find the ratio of the areas of two circles.
28. Students will find the scale factor of two similar rectangular right prisms.
29. Students will find the ratio of the surface areas of two similar rectangular right prisms.
30. Students will find the ratio of the volumes of two similar rectangular right prisms.
31. Students will find the scale factor of two similar square right prisms.
32. Students will find the ratio of the surface areas of two similar square right prisms.
33. Students will find the ratio of the volumes of two similar square right prisms.
34. Students will find the scale factor of two similar triangular right prisms.
35. Students will find the ratio of the surface areas of two similar triangular right prisms.
36. Students will find the ratio of the volumes of two similar triangular right prisms.
37. Students will find the scale factor of two similar cylinders.
38. Students will find the ratio of the surface areas of two similar cylinders.
39. Students will find the ratio of the volumes of two similar cylinders.
40. Students will find the scale factor of two similar rectangular pyramids.
41. Students will find the ratio of the surface areas of two similar rectangular pyramids.
42. Students will find the ratio of the volumes of two similar rectangular pyramids.
43. Students will find the scale factor of two similar square pyramids.
44. Students will find the ratio of the surface areas of two similar square pyramids.

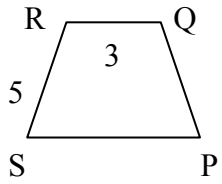
45. Students will find the ratio of the volumes of two similar square pyramids.
46. Students will find the scale factor of two similar triangular pyramids.
47. Students will find the ratio of the surface areas of two similar square pyramids.
48. Students will find the ratio of the volume of two similar square pyramids.
49. Students will find the scale factor of two similar cones.
50. Students will find the ratio of the surface areas of two similar cones.
51. Students will find the ratio of the volumes of two similar cones.
52. Students will find the scale factor of two spheres.
53. Students will find the ratio of the surface areas of two spheres.
54. Students will find the ratio of the volume of two spheres.
55. Students will find solutions to problems involving similar figures or solids and the corresponding dimension, perimeter, area, or volume with appropriate units.

Standard #11 Model Assessment Items

Computational and Procedural Skills

1. Given the following two figures.





WXYZ ~ PQRS

   A. Find the scale factor of the two figures.

   B. Find the ratio of the perimeters of the two figures.

   C. Find the ratio of the areas of the two figures.

Conceptual Understanding

1. If the ratio of the areas of two similar figures is $\frac{4}{9}$, what is the ratio of their perimeters?

   A. $\frac{16}{81}$ B. $\frac{2}{3}$ C. $\frac{9}{4}$ D. 1

2. The scale factor of two cylinders is 2:3. What is the ratio of their volumes?

   A. $\frac{2}{3}$ B. $\frac{4}{6}$ C. $\frac{6}{9}$ D. $\frac{8}{27}$

Problem Solving/Application

1. The areas of two similar triangles are 225 sq. cm and 16 sq. cm. If the length of a side of the larger triangle is 20 cm, find the length of the corresponding side of the smaller triangle.

2. A toy manufacturer makes spherical rubber balls in two sizes. The ratio of the radius of the small ball to the radius of the large ball is 3:4. The amount of rubber on the surface of the small ball is 400 sq. cm. What amount of rubber is on the surface of the large ball?

Standard #12

Standard Set 12.0

Students find and use measures of sides and interior and exterior angles of triangles and polygons to classify figures and solve problems.

Deconstructed Standard

1. Students will find measures of sides of triangles and polygons.
2. Students will find measures of interior angles of triangles and polygons.
3. Students will find measures of exterior angles of triangles and polygons.
4. Students will use measures of sides to classify triangles and polygons.
5. Students will use measures of interior angles to classify triangles and polygons.
6. Students will use measures of exterior angles to classify triangles and polygons.
7. Students will use measures of sides of triangles and polygons to solve problems.
8. Students will use measures of interior angles of triangles and polygons to solve problems.
9. Students will use measures of exterior angles of triangles and polygons to solve problems.

Prior Knowledge Necessary

Students should know how to:

- recognize and define triangles and polygons.
- recognize complementary and supplementary angles.
- identify a straight angle.
- identify a regular polygon.
- name polygons in relation to number of their sides.
- identify the relationship of the sum of interior angles and number of sides of a polygon.
- define a circle.
- identify the different types of angles.

New Knowledge

Students will need to learn to:

- classify polygons by their sides and angles.
- classify triangles by their angles and by sides.
- use formulas to find the measure of interior and exterior angles of regular and irregular polygons.
- identify what remote interior angles are and how to use them to solve problems involving triangles.
- use side and angle relations in polygons to solve problems.

Categorization of Educational Outcomes

Competence Level: Knowledge

1. Students will define polygons and triangles.
2. Student will define exterior and interior angles.

3. Students will state the relationship of sides to sum of interior angles.
4. Students will describe triangles by angle and side.
5. Students will identify the difference between regular and irregular polygons.

Competence Level: Application

1. Students will use definition of polygons to find sides and or angles.
2. Students will use measures of sides to define polygons.
3. Students will calculate from formulas the sum of interior angles.
4. Students will use definition of supplementary angles to find exterior and interior angles when given one or the other.
5. Students will use interior and exterior angle measure to classify polygons.

Necessary New Physical Skills

None

Assessable Result of the Standard

1. Students will name a given polygon by side relationships.
2. Students will name a polygon by angle relationships.
3. Students will classify a triangle by sides and angles.
4. Students will use remote interior angles of triangles to find other angles.
5. Students will state the relationship of number of sides of a polygon to sum of its interior angles.
6. Students will calculate a specific interior/exterior angle of a regular polygon.

Standard #12 Model Assessment Items

Computational and Procedural Skills

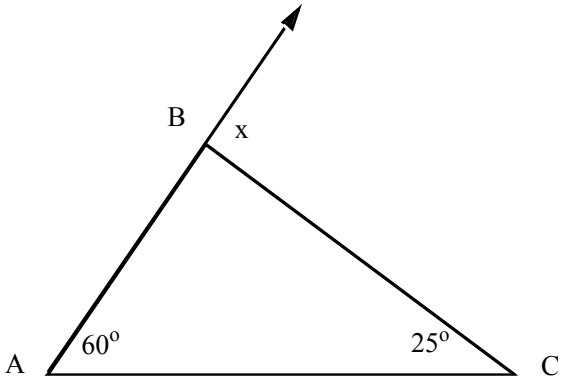
1. Name all of the polygons with four sides and the relationships of their sides.

2. Compute the sum of the interior angles of a decagon.

3. Draw an isosceles right triangle. Label the sides.

4. Classify a triangle with sides of 6, 15, and 23.

5. What is the measure of *x*:



Conceptual Understanding

1. Is there an equilateral right triangle? Why or why not?

2. Show why the formula for computing the sum of interior angles works.

3. What is the difference between squares and rectangles? Are they interchangeable? Why or why not?

4. Find the number of sides of a convex polygon if the measures of its interior angles have a sum of 1800º.

5. Two angles of a triangle have measures of 55º and 65º. Which of the following could *not* be a measure of an exterior angle of the triangle?

A. 115^{o} B. 120^{o} C. 125^{o} D. 130^{o}

Problem Solving/Application

1. An advertising executive was using her computer to design a graphic in the shape of a convex regular polygon. If one of the exterior angles has a measure of 36º, what is the name of the polygon?

2. In a person's heptagonal yard, when interior angles are put in increasing order, each differs from the next by 10º. Find the measure of the largest interior angle of the field to the nearest 0.1 degree.

Standard #13

Standard Set 13.0

Students prove relationships between angles in polygons by using properties of complementary, supplementary, vertical, and exterior angles.

Deconstruction of Standard

1. Students prove relationships between angles in polygons.
2. Students use properties of complementary angles.
3. Students use properties of supplementary angles.
4. Students use properties of vertical angles.
5. Students use properties of exterior angles.

Prior Knowledge Necessary

Students should know how to:

- identify the different types of line relationships that create complementary, supplementary, vertical angles.
- recognize that the sum of complementary angles equals 90°, the sum of supplementary angles equals 180°, and vertical angles are congruent.
- recognize that an exterior angle forms a linear pair with an interior angle.
- use the remote interior angle theorem.
- identify the properties of lines and angles of polygons.
- use deductive reasoning.
- apply deductive reasoning with angle relationships.
- write a proof: two-column, paragraph, or flow-proof.

New Knowledge

Students will need to learn to:

- identify complementary, supplementary, vertical, and exterior angles in polygons.
- identify relationships between angles in polygons.

Categorization of Educational Outcomes

Competence Level: Application

1. Students will use the properties of polygons to identify parallel and intersecting lines.
2. Students will use the properties of parallel and intersecting lines to identify complementary, supplementary, vertical, and exterior angle relationships.

Competence Level: Analysis

1. Students will distinguish between complementary, supplementary, vertical, and exterior angles in polygons.
2. Students will illustrate complementary, supplementary, vertical and exterior angles in polygons.

Competence Level: Synthesis

1. Students will devise a plan for proving relationships of angles in polygons using complementary, supplementary, vertical, and exterior angles.

Necessary New Physical Skills

None

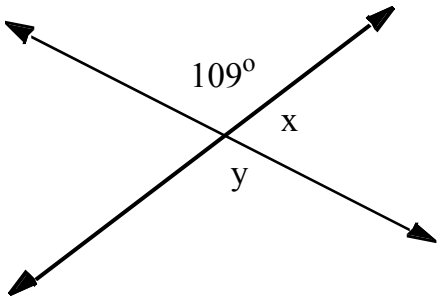
Assessable Result of the Standard

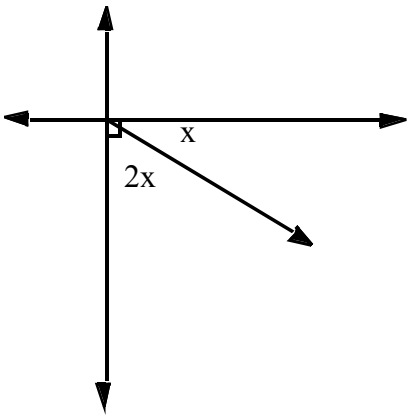
1. Students will illustrate polygons that contain complementary, supplementary, vertical, and exterior angles.
2. Students will use properties of complementary, supplementary, vertical, and exterior angles in polygons to find missing angle measures in a polygon.
3. Students will write or complete a paragraph, two-column, or flow proof.

Standard #13 Model Assessment Items

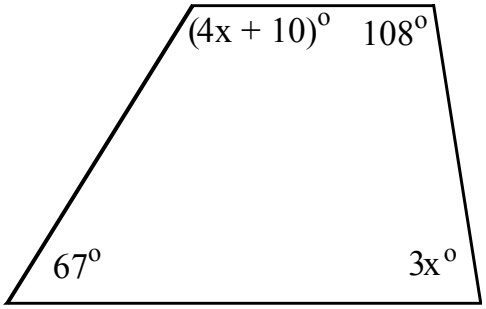
Computational and Procedural Skill

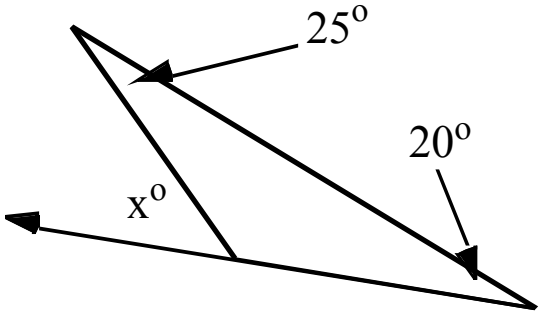
1. Solve for the missing angle measures.





2. Use the information in each diagram to solve for *x*.





Conceptual Understanding

1. In the diagram below, $\angle 1 \cong \angle 2$. Which of the following is not true?

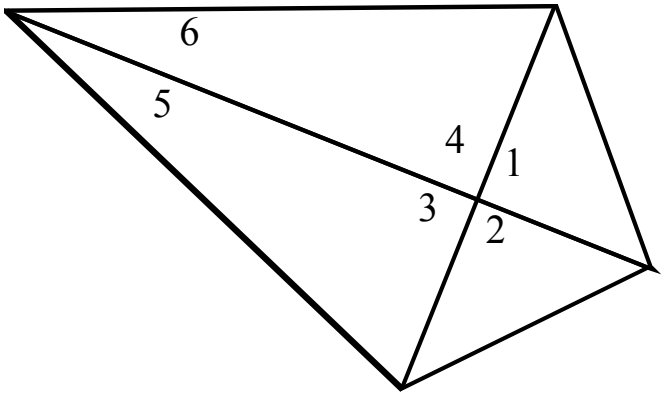
A. $\angle 1 \cong \angle 4$

B. $m\angle 1 + m\angle 2 = 180^o$

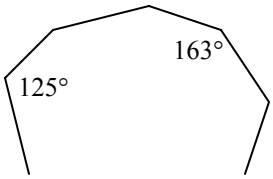
C. $\angle 1 \cong \angle 3$

D. $\angle 2$ and $\angle 4$ are supplementary.

E. $m\angle 6 \cong m\angle 4$

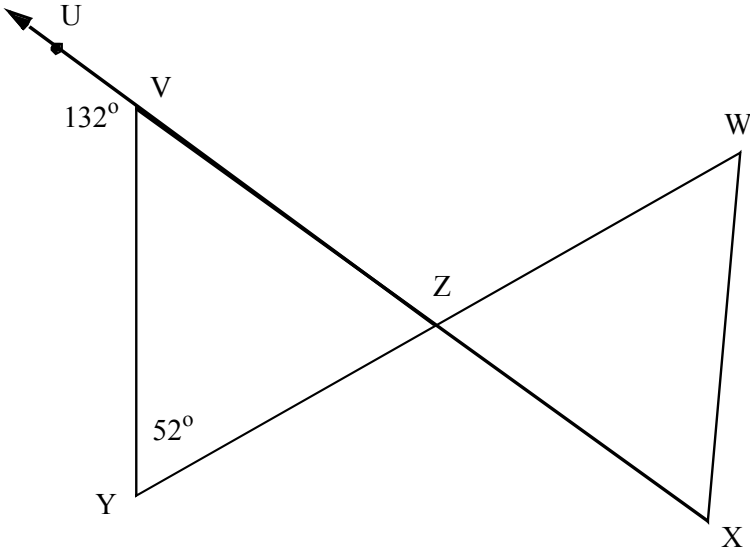


2. You are shown part of a convex *n*-gon. A pattern of alternating congruent angles continues around the polygon. Find the value of *n*.



Problem Solving and Application

1. Given: m∠UVY = 132°
   Prove: m∠WZX = 80°



2. Prove that if the consecutive angles of a quadrilateral are supplementary, then the quadrilateral is a parallelogram.

3. Prove that if the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

4. Prove that if the diagonals of a quadrilateral are perpendicular and bisect each other, then the quadrilateral is a rhombus.

5. Prove that if a parallelogram has one right angle, then it has four right angles.

Standard #14

Standard Set 14.0
Students prove the Pythagorean theorem.

Deconstructed Standard
1. Students prove the Pythagorean theorem using sum of areas.
2. Students prove the Pythagorean theorem using similar triangles.

(NOTE: The standard does not mandate one method of proof over the other.)

Prior Knowledge Necessary
Students should have the computational and conceptual knowledge outlined in Geometry Standards #2, #4, #5, #8, and #10.

Students should know how to:
- write geometric proofs, including proofs by contradiction.
- prove basic theorems involving congruence and similarity.
- prove that triangles are congruent or similar, and they are able to use the concept of corresponding parts of congruent triangles.
- solve problems involving perimeter, circumference, area, volume, lateral area, and surface area of common geometric figures.
- compute areas of polygons, including rectangles, scalene triangles, equilateral triangles, rhombi, parallelograms, and trapezoids.

New Knowledge
Students will need to learn to:
- recognize the association between a square inscribed in a second square and the created right triangles.
- translate a problem from the geometric perspective to the represented algebraic sums.
- apply the ratios associated with congruent triangles.
- translate a problem involving congruence from the geometric perspective, associated ratios to the represented algebraic sums.

Categorization of Educational Outcomes
Competence Level: Application
1. Students will use their knowledge of geometric figures to determine the areas involved.
2. Students will apply their knowledge of equality to sum the areas.
3. Students solve the equation using substitution and addition property steps.
4. Students will apply their knowledge of similar triangles to identify associated ratios.
5. Students will apply their knowledge of equality and solving equations to rearrange the ratios and sum total sides of triangles.

Necessary New Physical Skills
None

Assessable Result of the Standard
The following apply to the method of the sum of areas:
1. Students will identify the right triangles created by a square inscribed in a second square.
2. Students will build an equation which sums appropriate areas of the identified geometric figures.
3. Students will derive the Pythagorean theorem from the equation.

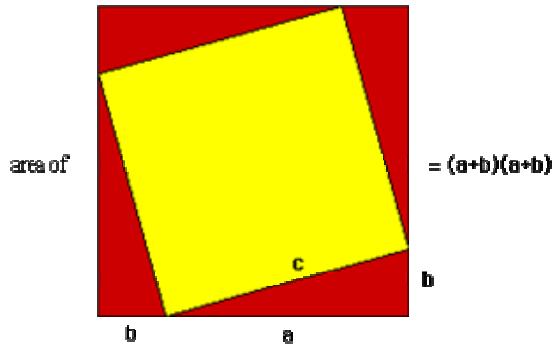
The following apply to the method of similar triangles:
1. Students will construct an altitude to form a total of three right triangles from the initial one.
2. Students will identify equivalent ratios from similar triangles and build an equation from the sums of the lengths of the triangles.
3. Students will derive the Pythagorean theorem from the equation.

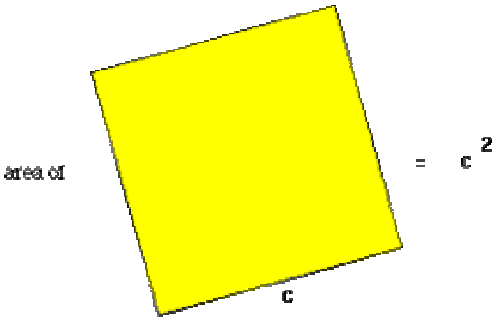
Standard #14 Model Assessment Items

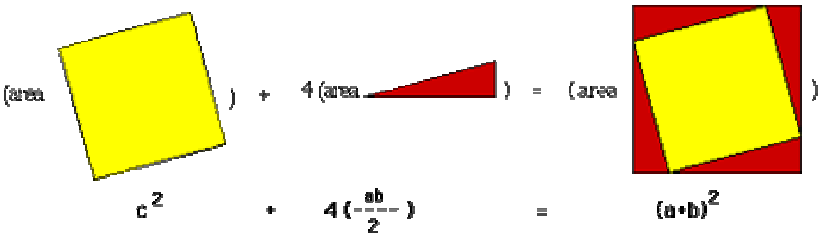
Computational and Procedural Skills

Sum of Areas:

1. Given a square inscribed in a second square, derive the Pythagorean theorem. The Indian astronomer Bhaskara (1114-1185) developed this proof:







$$c^2+4\left(\frac{ab}{2}\right)=(a+b)^2$$

$$c^2+2ab=a^2+2ab+b^2$$

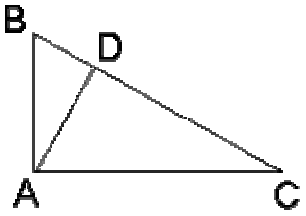
$$c^2=a^2+b^2$$

Information from http://mathforum.org/isaac/problems/pythagthm.html

Similar triangles:

1. Given a right triangle, construct an altitude to total three triangles. From these triangles, form ratios and then an equation of the sum of the triangle sides to derive the Pythagorean theorem.

We start with the original triangle, now denoted ABC, and need only one additional construct—the altitude AD. The triangles ABC, BDA and ADC are similar, which leads to two ratios:



$$AB/BC = BD/AB \text{ and } AC/BC = DC/AC.$$

Written another way these become

$$AB \cdot AB = BD \cdot BC \text{ and } AC \cdot AC = DC \cdot BC$$

Summing up we get

$$\begin{aligned} AB \cdot AB + AC \cdot AC &= BD \cdot BC + DC \cdot BC \\ &= (BD+DC) \cdot BC = BC \cdot BC. \end{aligned}$$

Information from http://www.cut-the-knot.org/pythagoras/index.shtml

Conceptual Understanding

None

Problem Solving/Application

1. Gail purchased a 12-foot flag pole to install at her ranch. In order to get the pole home, she will rent a moving truck. The rental truck available has dimensions 6 ft wide by 7 ft high by 10 ft long. Will Gail be able to fit the flag pole into the truck to get it home?

Standard # 15

Standard Set # 15.0

Students use the Pythagorean theorem to determine distance and find the missing lengths of sides of right triangles.

Deconstructed Standard

1. Students use the Pythagorean theorem to determine distance between points.
2. Students use the Pythagorean theorem to find the unknown lengths of legs of right triangles.
3. Students use the Pythagorean theorem to find the unknown length of the hypotenuse of right triangles.

Prior Knowledge Necessary

Students should know how to:

- recognize right angles.
- recognize right triangles.
- recognize legs of right triangles.
- recognize hypotenuse of right triangles.
- compute squares of a number.
- compute square roots of a number.

New Knowledge

Students will need to learn to:

- apply the Pythagorean theorem.
- use $leg^2 + leg^2 = hypotenuese^2$ to find the length of the hypotenuse when the length of both legs of a right triangle is known.
- find the length of an unknown leg of a right triangle given the length of the other leg and the length of the hypotenuse.
- recognize that only the positive value of the square root is used to determine the length of sides.
- determine when triangles are right triangles.

Categorization of Educational Outcomes

Competence Level: Knowledge

1. Students will identity the legs and the hypotenuse of right triangles.
2. Students will identify when it is appropriate to use the Pythagorean theorem.

Competence Level: Application

1. Students will demonstrate their ability to solve multi-step equations to find unknown lengths of sides of right triangles.
2. Students will demonstrate their ability to determine the distance between two points using the Pythagorean theorem.

Competence Level: Analysis

1. Students use the Pythagorean theorem to determine if a triangle is a right triangle.

Necessary New Physical Skills

None

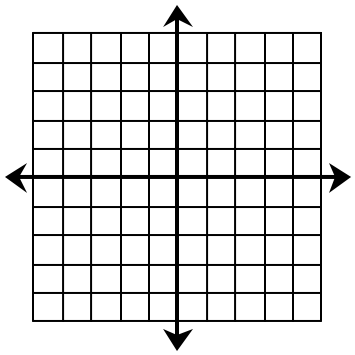
Assessable Results of the Standard

1. Students use the Pythagorean theorem to find the distance between two points.
2. Students calculate the length of one leg of a right triangle.
3. Students calculate the length of the hypotenuse of a right triangle.
4. Students determine when triangles are right triangles.

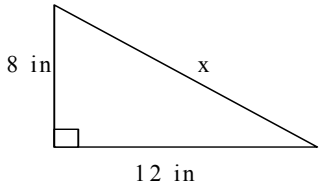
Standard # 15 Model Assessment Items

Computational and Procedural Skills

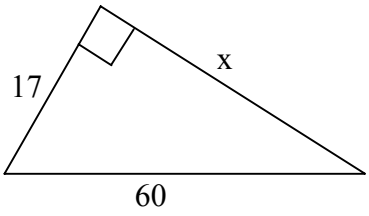
1. Plot the points A (-3, 5) and B (3, -3). Use the Pythagorean theorem to find the distance between the points. Show your work.



2. Use the Pythagorean theorem to solve for *x*.

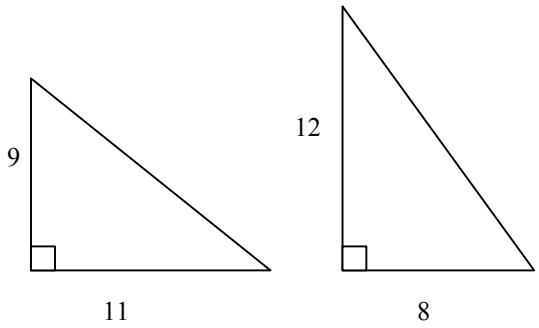


3. Use the Pythagorean theorem to solve for *x*.



Conceptual Understanding

1. Which triangle has a longer hypotenuse? Why? Show your work and explain your answer.



2. Use the Pythagorean theorem to determine if a triangle with sides of 6, 8, and 9 is a right triangle. Explain your answer.

Problem Solving/Application

1. A 25-foot ladder is placed against the side of a house. To be safe, the ladder must be at least 7 feet from the base of the house. Will the ladder be long enough to be used to reach the cat stuck on the window sill 26 feet high? Think twice. Draw a diagram, solve the problem, and explain your answer.

2. Plot the points A (5, 1) and B (8, 5) on a coordinate graph.
   A. Find the distance between the 2 points.

   B. Use this segment as the hypotenuse of a right triangle. Draw the squares on each side of the triangle to demonstrate that the Pythagorean theorem works.

   C. Explain your answer.

Standard #16

Standard Set 16.0

Students perform basic constructions with a straightedge and compass, such as angle bisectors, perpendicular bisectors, and the line parallel to a given line through a point off line.

Deconstructed Standard

1. Students will construct angle bisectors with a straightedge and compass.
2. Students will construct perpendicular bisectors with a straightedge and compass.
3. Students will construct a line parallel to a given line through a point off line with a straightedge and compass.
4. (implied) Students will construct figures as required with a straightedge and compass.

Prior Knowledge Necessary

Students should know how to:

- compare measurements.

New Knowledge

Students will need to learn to:

- identify the procedures of construction.
- use the techniques of construction.
- read a protractor.

Categorization of Educational Outcomes

Competence Level: Application

1. Students will understand the order in which to proceed in construction.
2. Students will demonstrate how to read a protractor.
3. Students will demonstrate how to use a compass.
4. Students construct an angle bisector
5. Students will construct a perpendicular bisector.
6. Students will construct a line parallel to a given line through a non-collinear point.

Necessary New Physical Skills

1. Students will use a compass.

Assessable Results of the Standard

1. Students will construct an angle bisector.
2. Students will construct a perpendicular bisector.
3. Students will construct a line parallel to a given line through a non-collinear point.

Standard #16 Model Assessment Items

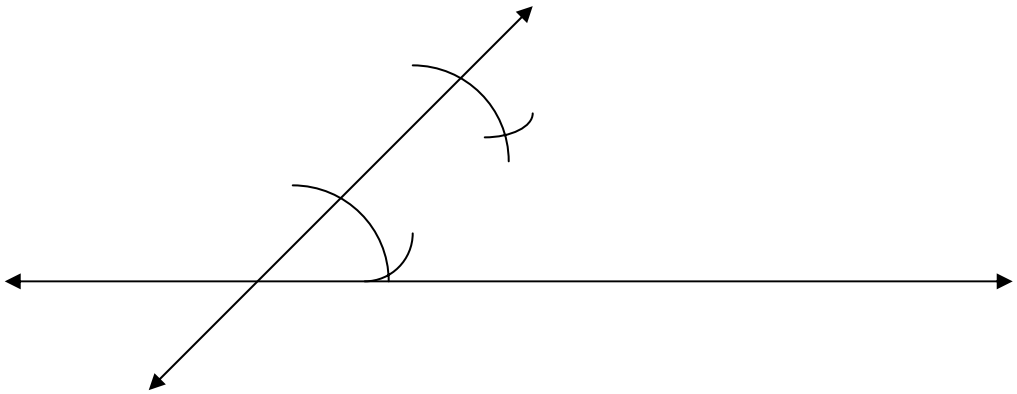
Computational and Procedural Skills

1. Construct an angle bisector.

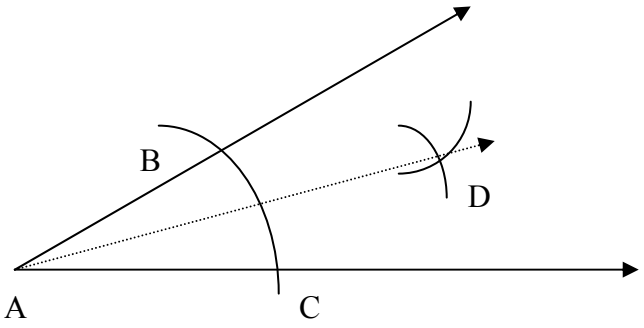
2. Construct a perpendicular bisector.

Conceptual Understanding

1. Marsha is using a straightedge and compass to do the construction shown. Identify the construction.



2. What is the first step in constructing the angle bisector of angle A?



Problem Solving

There are no problems to be solved at this level using construction.

Standard 17

Standard Set 17.0

Students prove theorems by using coordinate geometry, including the midpoint of a line segment, the distance formula, and various forms of equations of lines and circles.

Deconstruction of Standard

1. Students prove theorems using coordinate geometry.
2. Students prove theorems using the midpoint of a line segment.
3. Students prove theorems using the distance formula.
4. Students prove theorems using various forms of equations of lines.
5. Students prove theorems using equations of circles.

Prior Knowledge Necessary

Students should know how to:

- plot and identify points in a coordinate plane.
- find the midpoint of a line segment using the midpoint formula.
- use the distance formula to determine the length of a segment.
- recognize and use various forms of equations of lines (e.g., slope intercept form, point-slope form, and standard form).
- recognize and use the equation of a circle.
- follow the logic of a proof.

New Knowledge

Students will need to learn to:

- use the process of a coordinate geometry proof.
- prove theorems of polygons using coordinate geometry.

Categorization of Educational Outcomes

Competence Level: Application

1. Students will use the midpoint formula.
2. Students will use the distance formula.
3. Students will write the equation of a line.
4. Students will use the equations of lines to compare slopes.

Competence Level: Analysis

1. Students will distinguish different types of quadrilaterals with data from applied formulas.
2. Students will distinguish between polygons with data from applied formulas.

Competence Level: Synthesis

1. Students will compare data and evaluate formula calculations on polygons to determine if a theorem has been supported.

Necessary New Physical Skills

None

Assessable Result of the Standard

1. Students will use the midpoint formula to locate coordinates for determining bisectors.
2. Students will use the distance formula to prove segments are bisectors.
3. Students will use the distance formula to find the length of the segments in polygons to determine congruency.
4. Students will use the equations of lines to compare slopes of lines to distinguish between parallel and perpendicular relationships.
5. Students will identify a quadrilateral in the coordinate plane using the midpoint, and distance formulas and slopes of lines to determine the side and diagonal relationships.
6. Students will use the midpoint, distance and slope formulas of lines to determine segment relationships in polygons (i.e., the mid-segments of a triangle, an altitude of a triangle, an apothem of a polygon, etc.).

Standard #17 Model Assessment Items

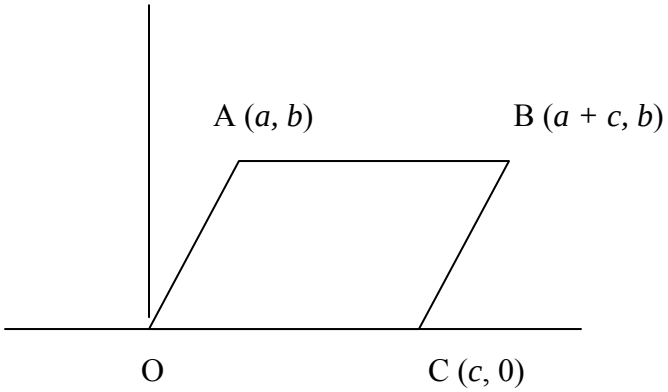
Computational and Procedural Skill

1. Quadrilateral ABCD has vertex points of A(3, 1), B(3,8), C(1,3), D(1,10). Calculate the slopes of each side and find the midpoint of the diagonals. Name the figure.

2. Find the center and radius length of a circle with diameter end points (-4,8) and (3,2). Use the data to write the equation of the circle.

Conceptual Understanding

1. Figure ABCO is a parallelogram.



What are the coordinates of the intersection of the diagonals?

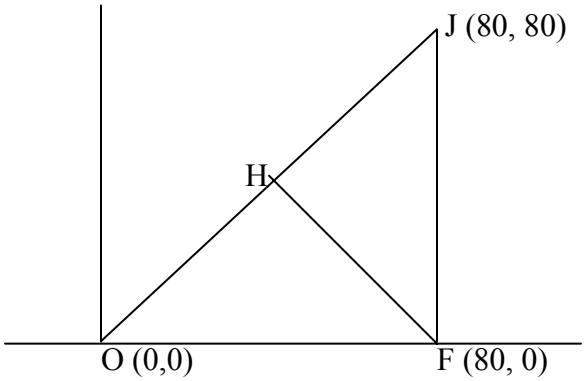
A. $\left(\frac{a}{2},\frac{b}{2}\right)$

B. $\left(\frac{c}{2},\frac{b}{2}\right)$

C. $\left(\frac{a+c}{2},\frac{b}{2}\right)$

D. $\left(\frac{a+c}{2},\frac{a+b}{2}\right)$

2. Use the given information and the diagram to find the coordinates of H.



Given: Δ FOH $\cong \Delta$ FJH

Problem Solving and Application

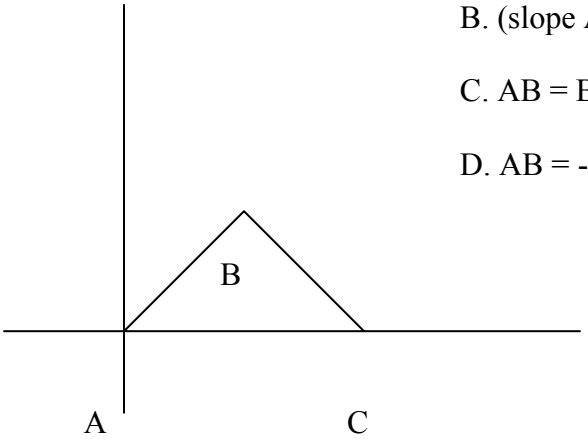
1. Which statement would prove that ΔABC is a right triangle?

A. (slope AB) (slope BC) = 1

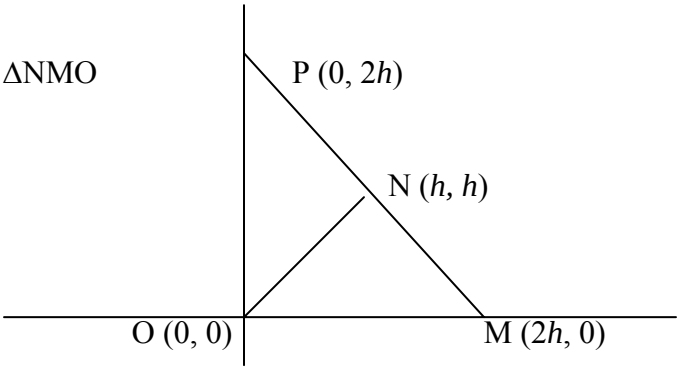
B. (slope AB) (slope BC) = -1

C. AB = BC

D. AB = - BC



2. Write a coordinate proof.
   Given: coordinates of ΔNPO and ΔNMO
   Prove: ΔNPO ≅ ΔNMO



Standard #18

Standard Set 18.0

Students know the definitions of the basic trigonometric functions defined by the angles of a right triangle. They also know and are able to use elementary relationships between them. For example, $\tan x = \sin x / \cos x$, $\sin^2 x + \cos^2 x = 1$.

Deconstructed Standard

1. Students know the definition of the cosine of an angle defined by the angles of a right triangle.
2. Students know the definition of the sine of an angle defined by the angles of a right triangle.
3. Students know the definition of the tangent of an angle defined by the angles of a right triangle.
4. Students know the elementary relationships between cosine, sine, and tangent.
5. Students use the elementary relationships between cosine, sine, and tangent.

Prior Knowledge Necessary

Students should have the computational and conceptual knowledge outlined in Geometry Standards #2, #4, #5, #14, and #15.

Students should know how to:

- write geometric proofs, including proofs by contradiction.
- prove basic theorems involving congruence and similarity.
- prove that triangles are congruent or similar, and they are able to use the concept of corresponding parts of congruent triangles.
- prove the Pythagorean theorem.
- use the Pythagorean theorem to determine distance and find missing lengths of sides of right triangles.

New Knowledge

Students will need to learn to:

- apply the definition of the cosine of an angle.
- apply the definition of the sine of an angle.
- apply the definition of the tangent of an angle.
- recognize the relationship between the tangent of an angle and the sine and cosine of that angle.
- recognize the relationship between the sine and cosine of an angle (i.e., the basic identity: $\cos^2(x) + \sin^2(x) = 1$).

Categorization of Educational Outcomes

Competence Level: Knowledge

1. Students will identify a triangle as a right triangle.
2. Students will identify the opposite and adjacent sides of a triangle along with the hypotenuse relative to an identified angle.

Competence Level: Application

1. Students will demonstrate knowledge of the sine of an angle.
2. Students will demonstrate knowledge of the cosine of an angle.
3. Students will demonstrate knowledge of the tangent of an angle.
4. Students will demonstrate knowledge of the trigonometric identity $\cos^2(x) + \sin^2(x) = 1$.

Necessary New Physical Skills

None

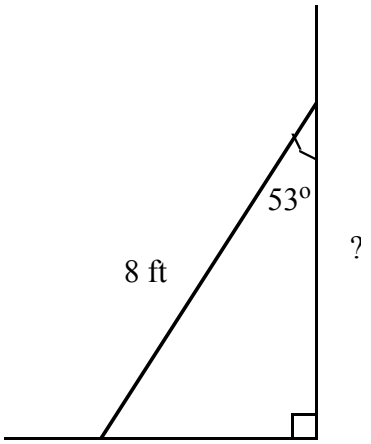
Assessable Result of the Standard

1. Students will identify the sine of a given angle of a right triangle.
2. Students will identify the cosine of a given angle of a right triangle.
3. Students will identify the tangent of a given angle of a right triangle.

Standard #18 Model Assessment Items

Computational and Procedural Skills

1. The diagram shows an 8-foot ladder leaning against a wall. The ladder makes a 53^o angle with the wall. Which is the closest to the distance up the wall the ladder reaches?



A. 3.2 ft

B. 4.8 ft

C. 6.4 ft

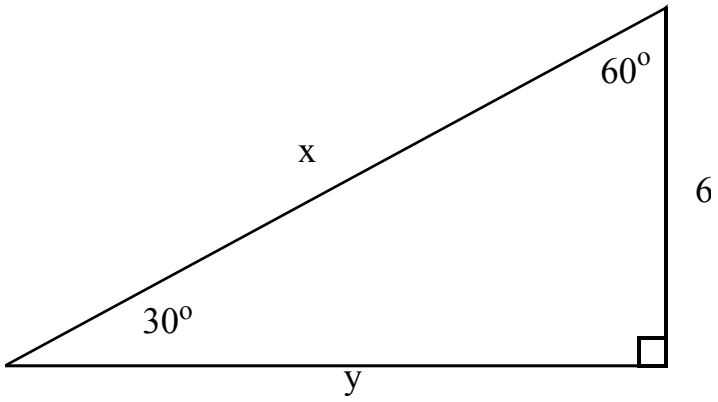
D. 9.6 ft

$\sin 53^o \approx 0.80$

$\cos 53^o \approx 0.60$

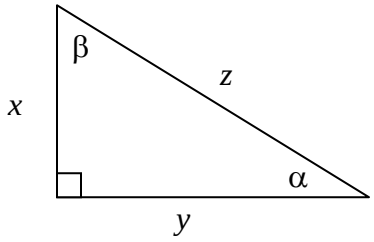
$\tan 53^o \approx 1.33$

2. Find *x* and *y* in the following figure.



Conceptual Understanding

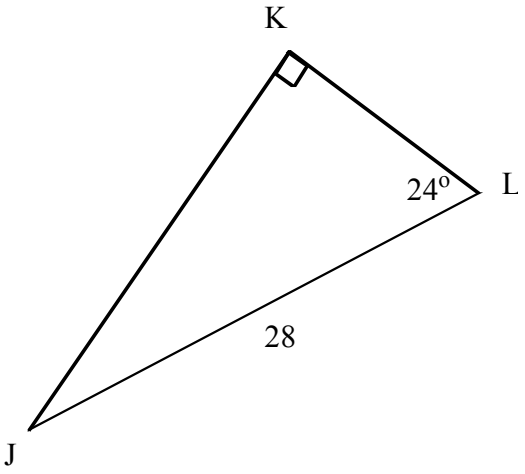
1. Given the triangle below, and that $\sin\alpha = \frac{3}{5}$, find $\cos\alpha$, $\tan\alpha$,and prove that the trigonometric identity $\cos^2(x) + \sin^2(x) = 1$ holds.



2. Given the triangle above, what is the relationship between $\sin\alpha$ and $\cos\beta$, $\cos\alpha$ and $\sin\beta$?

3. Given the triangle above, what is the relationship between $\tan\alpha$ and $\tan\beta$?

4. Triangle *JKL* is shown below. Which equation should be used to find the length of $\overline{JK}$?



A. $\sin 24^\circ = \frac{JK}{28}$

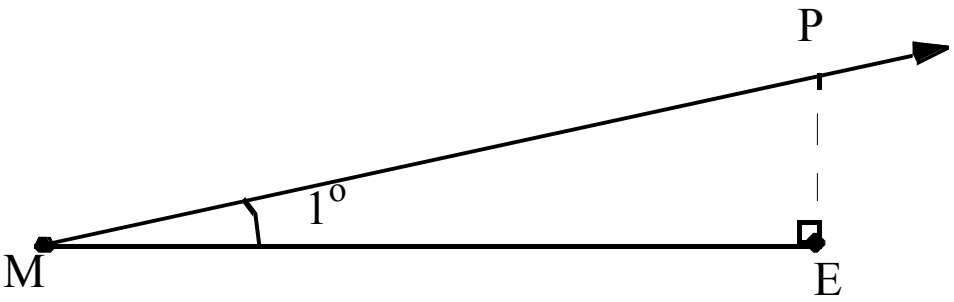
B. $\sin 24^\circ = \frac{28}{JK}$

C. $\cos 24^\circ = \frac{JK}{28}$

D. $\cos 24^\circ = \frac{28}{JK}$

Problem Solving/Application

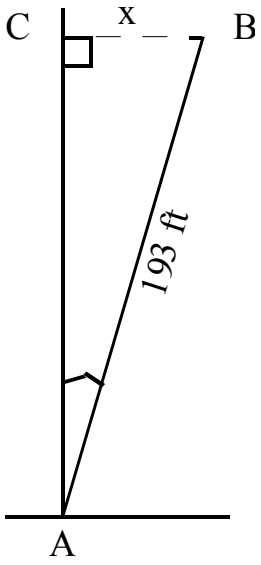
1. Because of the great distances in space, small navigational errors can have serious consequences. In the figure below (not to scale), ME represents the intended path of a space flight from the moon back to Earth, and ray MP represents the path resulting from an error of 1 degree; $ME \approx 239,000$ miles. Find PE, the distance by which path MP misses the Earth.



2. Ever since it was built, the Leaning Tower of Pisa has been leaning more and more. In the figure below, AB represents the tower and CB represents the distance that the top leans over the base.
   A. In 1400 the tower leaned 3^o to one side. How long was CB at that time?

   B. In 1500, the distance CB had increased to 13.5 feet. At what angle did the tower lean then?

   C. Now the tower leans 5.5^o. How long is CB now?



Standard #19

Standard Set 19.0
Students use trigonometric functions to solve for an unknown length of a side of a right triangle, given an angle and a length of a side.

Deconstructed Standard
1. Students use trigonometric functions:
   A. sine function
   B. cosine function
   C. tangent function
2. Students solve for an unknown length of a right triangle, given an angle and side length.

Prior Knowledge Necessary
Students should know how to:
- apply ratios and how to solve proportions and equations.
- identify the sides of a right triangle, with respect to an acute angle, as the opposite leg, the adjacent leg, and the hypotenuse.
- identify the side ratios that correspond to each trigonometric function with respect to an acute angle.

New Knowledge
Students will need to learn to:
- compute the trigonometric value of an acute angle with a calculator or a table of values.
- determine the appropriate trigonometric function to use given an acute angle, a side length, and an unknown measure.
- calculate the length of a side of a right triangle given an acute angle and a side length.

Categorization of Educational Outcomes
Competence Level: Application
1. Students show the different trigonometric ratios with respect to a given angle.
2. Students determine the decimal value of an angle evaluated by a trigonometric function.
3. Students will use the correct trigonometric function given a side length and angle to solve for an unknown measure.

Necessary New Physical Skill
Students will use the trigonometric keys on a scientific calculator.

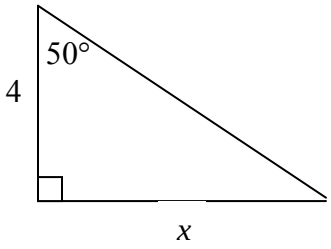
Assessable Result of the Standard

1. Students will write an equation using the appropriate trigonometric function given an angle and a length of a side of a right triangle.
2. Students will solve for the length of a side of a right triangle with the cosine function.
3. Students will solve for the length of a side of a right triangle with the sine function.
4. Students will solve for the length of a side of a right triangle with the tangent function.

Standard #19 Model Assessment Items

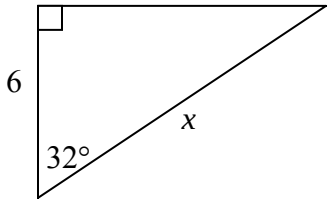
Computational and Procedural Skills

1. Given that the $\tan 50^0 = 1.19175$, find the value of x.



Conceptual Understanding

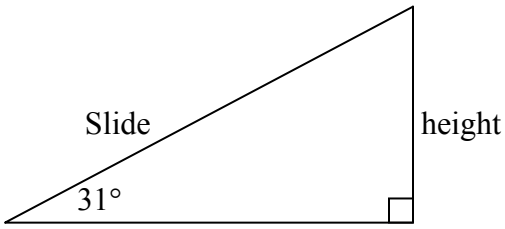
1. Write an equation that could be used to find the unknown side length.



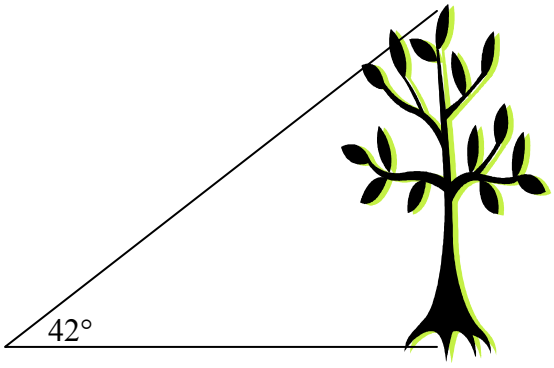
2. Solve the equation in #1.

Problem Solving/Application

1. A slide 3.8 meters long makes an angle of 31° with the ground. How high is the top of the slide above the ground?



2. A tree is 20 feet tall. The straight line connecting the top of the tree to the tip of the tree's shadow makes an angle of 42° with the ground. How long is the tree's shadow to the nearest hundredth?



3. A salvage ship uses sonar to determine that the angle of depression to a wreck on the ocean floor is 13.25°. The depth chart shows that the ocean floor is 40 meters below the surface. How far must a diver, lowered from the salvage ship, walk along the ocean floor to reach the wreck?

Standard #20

Standard Set 20.0

Students know and are able to use angle and side relationships in problems with special right triangles, such as 30°, 60°, 90° triangles, and 45°, 45°, 90° triangles.

Deconstructed Standard

1. Students know the special angle and side relationship of 30°, 60°, 90° triangles.
2. Students know the special angle and side relationship of 45°, 45°, 90° triangles.
3. Students use the special angle and side relationship of 30°, 60°, 90° triangles.
4. Students use the special angle and side relationship of 45°, 45°, 90° triangles.

Prior Knowledge Necessary

Students should know how to:

- identify parts of a right triangle: hypotenuse and legs.
- use the Pythagorean theorem.
- recall that the sum of the interior angles of a triangle total 180°.
- recall the properties of an isosceles triangle including that the base angles are also equal.
- simplify square roots.
- rationalize a denominator.
- identify that the smallest angle is opposite the smallest leg.
- identify that the hypotenuse is opposite the right angle (largest angle—90°) and that it is the largest side.
- identify that the second largest angle is opposite the larger leg.
- identify that the acute angles of a right triangle are complementary.

New Knowledge

Students will need to learn to:

- recognize that the hypotenuse is twice as long as the shorter leg in a 30°, 60°, 90° triangle.
- recognize that the longer leg is $\sqrt{3}$ times as long as the shorter leg in a 30°, 60°, 90° triangle.
- recognize that the legs are the same length in a 45°, 45°, 90° triangle.
- recognize that the hypotenuse is $\sqrt{2}$ times as long as a leg in a 45°, 45°, 90° triangle.

Categorization of Educational Outcomes

Competence Level: Application

1. Students will use methods they have learned to find the missing sides of a 30°, 60°, 90° triangle.
2. Students will use methods they have learned to find the missing sides of a 45°, 45°, 90° triangle.
3. Students will show that they know the difference between the two triangles.

Necessary New Physical Skills
None

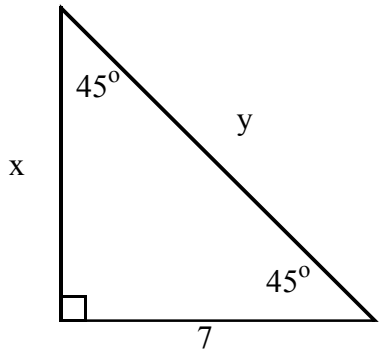
Assessable Result of the Standard
1. Students will produce the missing sides of a 30°, 60°, 90° triangle.
2. Students will produce the missing sides of a 45°, 45°, 90° triangle.
3. Students will identify the type of right triangle given all the sides.

Standard #20 Model Assessment Items

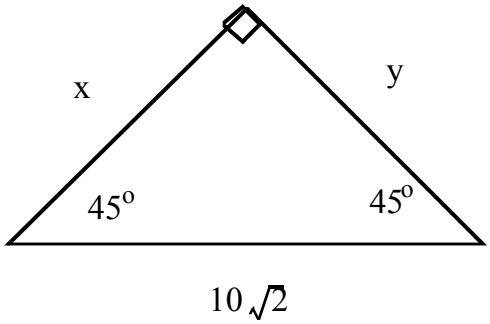
Computational and Procedural Skills

1. Find the missing sides (x and y) of each triangle below:

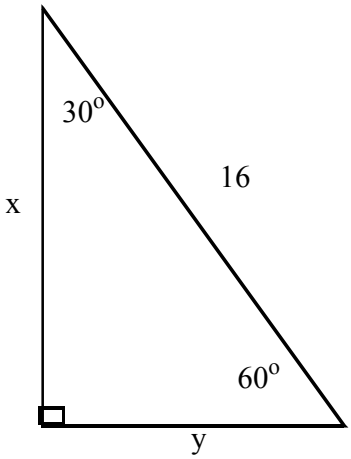
A.



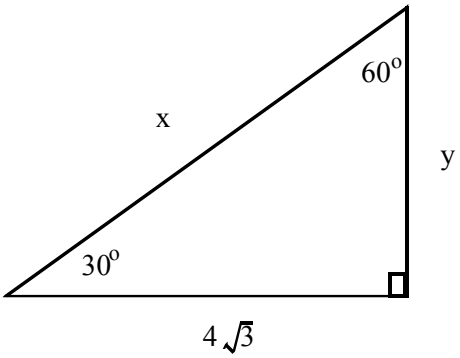
B.



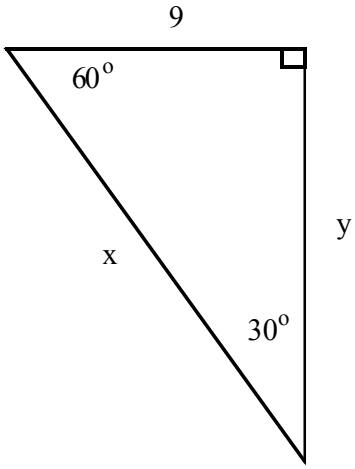
C.



D.

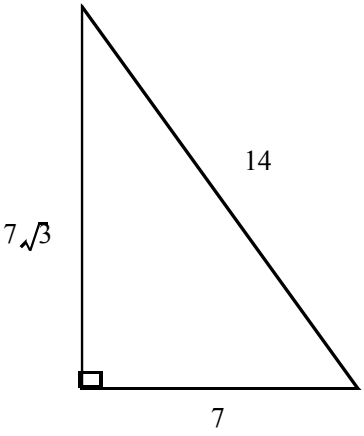


E.

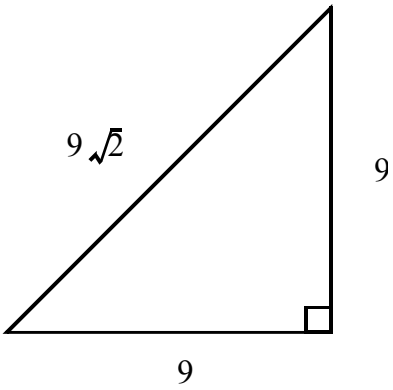


2. Determine the type of triangle (30°, 60°, 90° or 45°, 45°, 90°) given all the sides.

A.

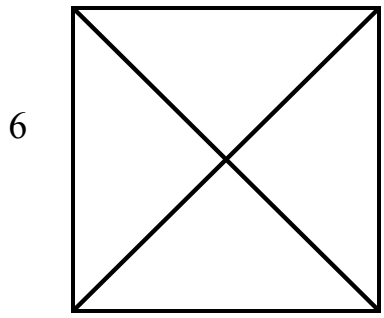


B.



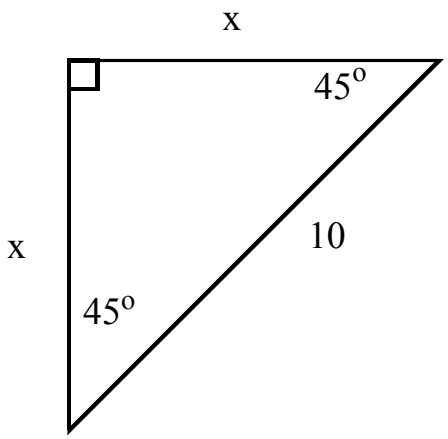
Conceptual Understanding

1. Find the length of the diagonal of the square.

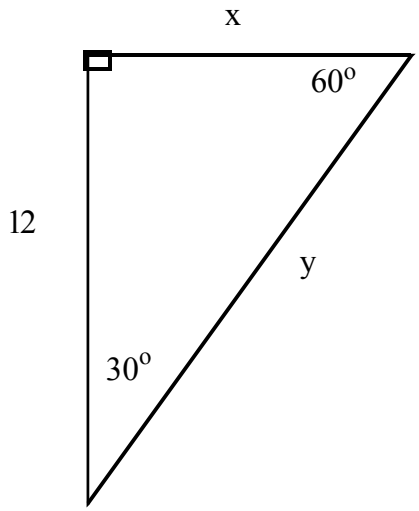


2. Find the exact lengths (no decimals) of *x* and *y*.

A.



B.

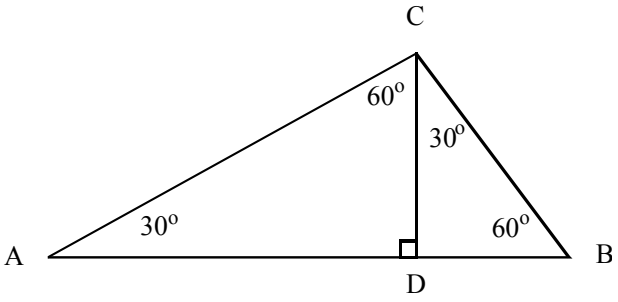


3. Use the triangle below to find the exact missing lengths.

   A. If BC = 10, then find all other lengths (AB, AC, CD, AD, DB).

   B. If AC = $4\sqrt{3}$, then find the missing lengths (AB, BC, CD, AD, DB).

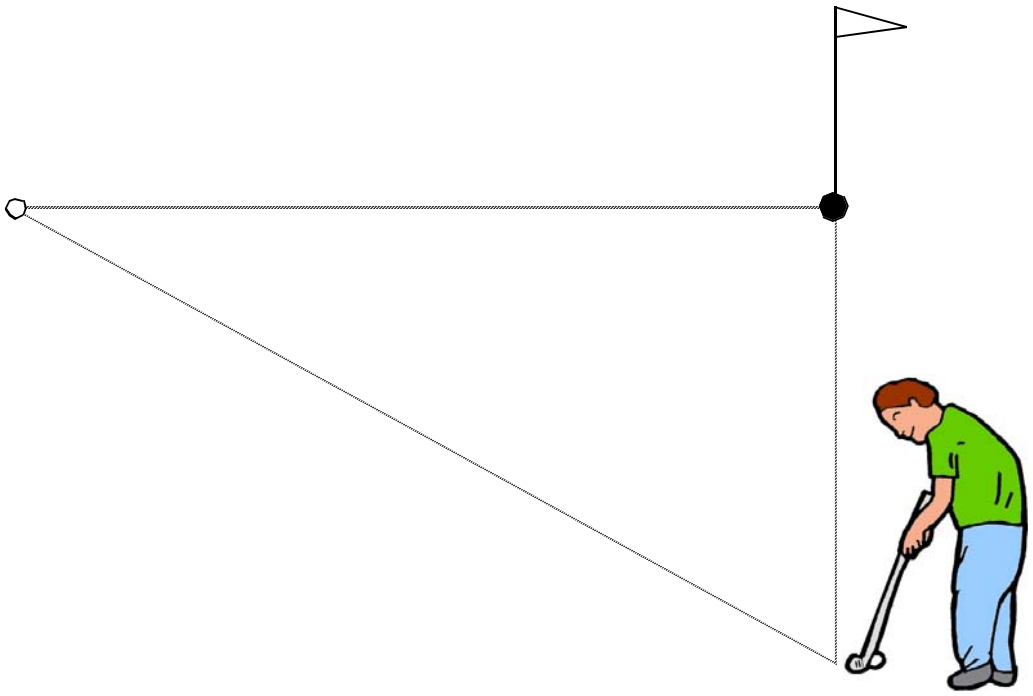
   C. If CD = $7\sqrt{3}$, then find the missing lengths (AB, BC, AC, AD, DB).



Problem Solving/Application

1. A tree trimmer has a ladder that is 12 feet long. He leans the ladder against a tree at a 45° angle with the ground. How high up the tree did the ladder reach?

2. A golfer is standing about 20 m from the 9th hole. He putts the ball and it sadly goes 60° off course and lands directly left of the hole. Use the 30°, 60°, 90° triangle angle and side relationship to find how far the ball is from the hole now.



Standard #21

Standard Set 21.0
Students prove and solve problems regarding relationships among chords, secants, tangents, inscribed angles, and inscribed and circumscribed polygons of circles.

Deconstructed Standard
1. Students prove relationships of chords and circles.
2. Students prove relationships between secants and circles.
3. Students prove relationships between tangents and circles.
4. Students prove relationships between inscribed angles and circles.
5. Students prove relationships between inscribed polygons and circles.
6. Students prove relationships between circumscribed polygons and circles.
7. Students solve problems involving chords and circles.
8. Students solve problems involving secants and circles.
9. Students solve problems involving tangents and circles.
10. Students solve problems involving inscribed angles and circles.
11. Students solve problems involving inscribed polygons and circles.
12. Students solve problems involving circumscribed polygons and circles.

Prior Knowledge Necessary
Students should have the computational and conceptual knowledge outlined in Geometry Standards #2, #4, #5, #14, #15, #16, and #17.

Students should know how to:
- define a circle.
- write geometric proofs, including proofs by contradiction.
- prove basic theorems involving congruence and similarity.
- prove that triangles are congruent or similar, and they are able to use the concept of corresponding parts of congruent triangles.
- prove the Pythagorean theorem.
- use the Pythagorean theorem to determine distance and find missing lengths of sides of right triangles.
- perform basic constructions with a straightedge and compass, such as angle bisectors, perpendicular bisectors, and the line parallel to a given line through a point off the line.
- prove theorems by using coordinate geometry, including the midpoint of a line segment, the distance formula, and various forms of equations of lines and circles.

New Knowledge
Students will need to learn to:
- recognize the definition of a chord of a circle.
- recognize the definition of a secant line of a circle.
- recognize the definition of a tangent line of a circle.
- recognize the definition of an inscribed angle.

- recognize the definition of a polygon inscribed in a circle.
- recognize the definition of a circle circumscribed about a polygon.

Categorization of Educational Outcomes

Competence Level: Knowledge

1. Students will identify a polygon as inscribed in a circle.
2. Students will identify a circle as circumscribed about a polygon.
3. Students will identify a line segment as a chord of a circle.
4. Students will identify a particular chord as the diameter of a circle.
5. Students will identify a secant line of a circle.
6. Students will identify a line/segment as tangent to a circle.

Competence Level: Application

1. Students will utilize knowledge to solve problems involving chords of a circle.
2. Students will utilize knowledge to solve problems involving secants of a circle.
3. Students will utilize knowledge to solve problems involving tangents of a circle.
4. Students will utilize knowledge to solve problems involving inscribed angels of a circle.
5. Students will utilize knowledge to solve problems involving inscribed polygons of circumscribed circles.

Competence Level: Analysis:

1. Students will distinguish between a chord and a radius.
2. Students will distinguish between a secant line/segment and a tangent line/segment.
3. Students will recognize an inscribed angle and its relationships to corresponding chords of the circle.

Necessary New Physical Skills

None

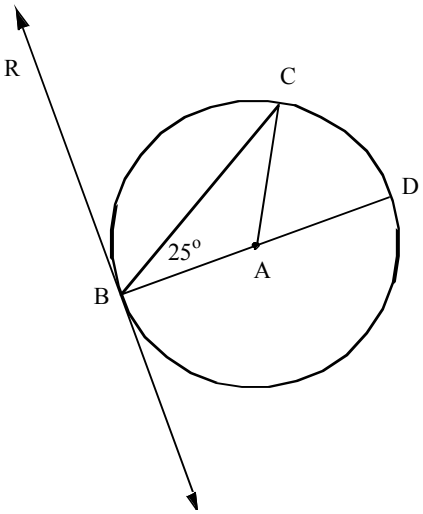
Assessable Result of the Standard

1. Students will generate proofs of properties of chords.
2. Students will generate proofs of properties of secant lines/segments.
3. Students will generate proofs of tangent lines/segments.
4. Students will generate proofs of properties of inscribed angles of a circle.
5. Students will generate proofs of properties of polygons inscribed in a circle.
6. Students will solve problems involving lengths of chords.
7. Students will solve problems involving lengths of chords and measures of inscribed angles.
8. Students will solve problems involving inscribed polygons in circumscribed circles.

Standard #21 Model Assessment Items

Computational and Procedural Skills

1. $\overleftrightarrow{RB}$ is tangent to a circle, whose center is A, at point B. $\overline{BD}$ is a diameter. What is $m\angle CBR$?



A. 50^o

B. 65^o

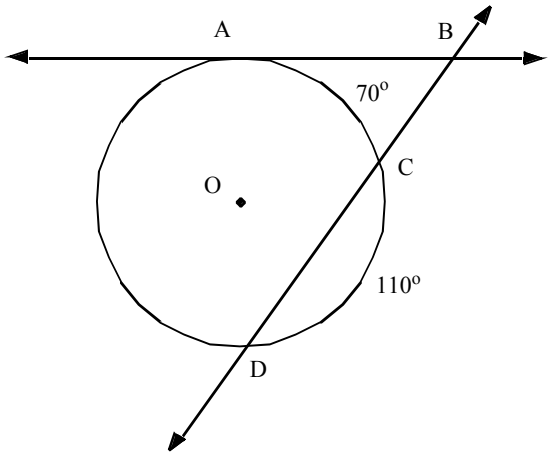
C. 90^o

D. 130^o

2. Given the same diagram above, what is $m\angle CAD$?

Conceptual Understanding

1. In the figure below, $\overleftrightarrow{AB}$ is tangent to circle O at point A, secant $\overleftrightarrow{BD}$ intersects circle O at points C and D, $m\overset{\frown}{AC} = 70^\circ$, and $m\overset{\frown}{CD} = 110^\circ$. What is $m\angle ABC$?



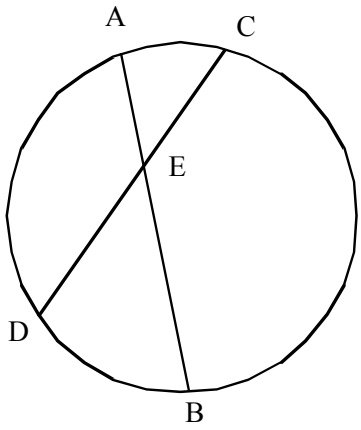
A. 20^o

B. 40^o

C. 55^o

D. 70^o

2. In the circle below, $\overline{AB}$ and $\overline{CD}$ are chords intersecting at E. If $AE = 5$, $BE = 12$, and $CE = 6$, what is the length of $\overline{DE}$?



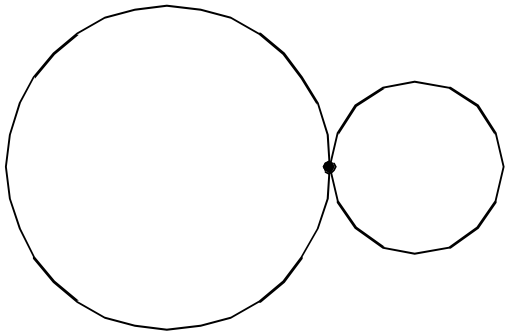
A. 7

B. 9

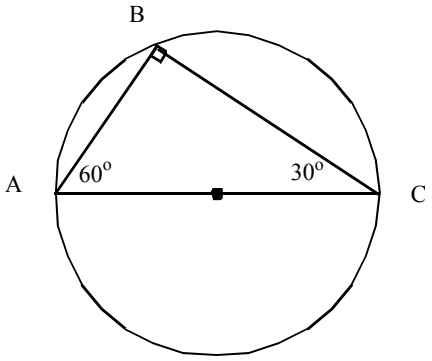
C. 10

D. 13

3. The two circles in the figure below have one point in common. Draw the lines that are tangent to both circles.



4. In the figure below, $\triangle ABC$ is a $30^\circ - 60^\circ$ right triangle whose sides are chords of circle *O*.



A. Why is $m\overset{\frown}{AB} + m\overset{\frown}{BC} = m\overset{\frown}{ABC}$?

B. Is $AB + BC = AC$? Explain.

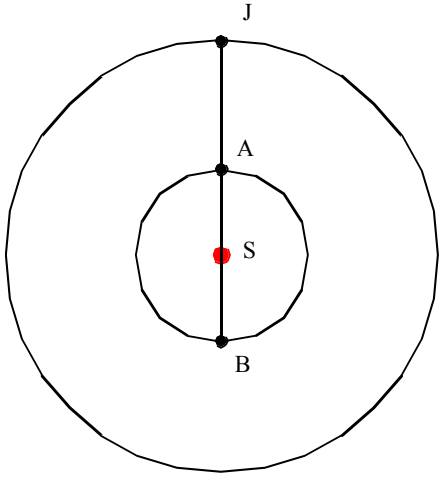
C. Find $m\overset{\frown}{AB}$, $m\overset{\frown}{BC}$, and $m\overset{\frown}{ABC}$.

D. $AC = 2AB$. How are $m\overset{\frown}{ABC}$ and $m\overset{\frown}{AB}$ related?

E. $m\overset{\frown}{BC} = 2m\overset{\frown}{AB}$. How are BC and AB related?

Problem Solving/Application

1. **Speed of Light:** How the speed of light was first calculated is shown by the figure below. Point S represents the sun, and the two circles represent the orbits of Earth and Jupiter.



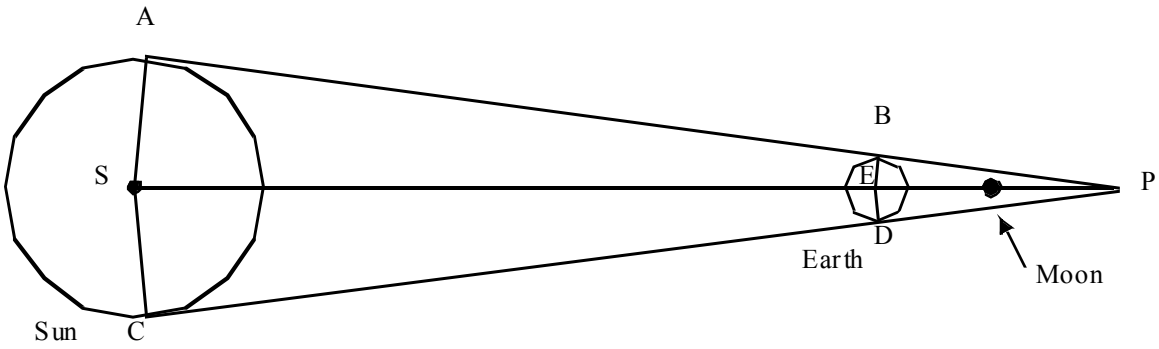
The distance from the Earth to the sun is 93,000,000 miles. Light from Jupiter takes about 16 minutes, 40 seconds longer to reach Earth when Earth is at B than when it is at A. Use this information to find each of the following measures.

A. The diameter of Earth's orbit around the sun.

B. The time it takes light to travel across the Earth's orbit in seconds.

C. The speed of light in miles per second.

2. **Lunar Eclipse:** Given the figure below and the radii of the sun and the Earth are 432,000 miles and 3,960 miles, respectively, and that the distance between their centers is 93,000,000 miles, find EP, the length of the Earth's shadow.



Standard # 22

Standard Set # 22.0

Students know the effects of rigid motions on figures in the coordinate plane and space, including rotations, translations, and reflections.

Deconstructed Standards

1. Students know the effect of rotations in the coordinate plane.
2. Students know the effect of translations in the coordinate plane.
3. Students know the effect of reflections in the coordinate plane.
4. Students know the effect of rotations in space.
5. Students know the effect of translations in space.
6. Students know the effect of reflections in space.

Prior Knowledge Necessary

Students should know how to:

- recognize basic geometric shapes and their properties.
- locate points in a coordinate plane.
- identify congruent parts of figures.
- locate points in space.

New Knowledge

Students will need to learn to:

- recognize that transformations are movements of geometric figures.
- recognize that reflections can be described as a "flip" over a line.
- recognize that translations can be described as a "slide" vertically, horizontally, or both.
- recognize that rotations can be described as a turn around a point.
- recognize the definition of lines of symmetry for given geometric figures.
- differentiate between clockwise rotations and counter-clockwise rotations.
- perform three dimensional rotations.
- perform three dimensional translations.
- perform three dimensional reflections.

Categorization of Educational Outcomes

Competence Level: Knowledge

1. Students will identify lines of symmetry.

Competence Level: Application

1. Students will use lines of symmetry to separate the figure into two congruent parts.
2. Students will classify a transformation as a translation.
3. Students will classify a transformation as a reflection.
4. Students will classify a transformation as a rotation.

Competence Level: Analysis

1. Students will differentiate between transformations.
2. Students will differentiate between combinations of different transformations.

Necessary New Physical Skills

1. Students will use a ruler to draw lines of symmetry.
2. Students will plot and connect points to draw figures.
3. Students will use mirrors to draw reflections.
4. Students will use tracing paper to draw rotations.

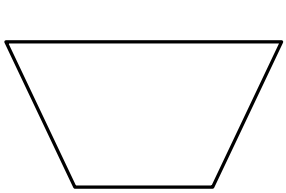
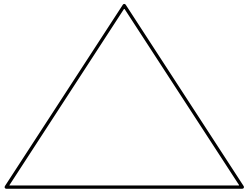
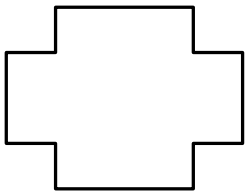
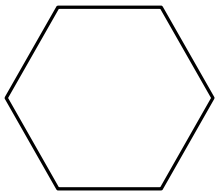
Assessable Results of the Standard

1. Students draw lines of symmetry.
2. Students identify all lines of symmetry.
3. Students draw figures and graph translations.
4. Students draw figures and graph rotations.
5. Students draw figures and graph reflections.

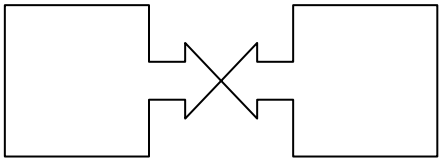
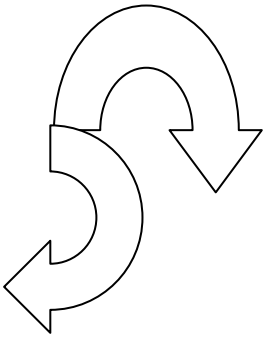
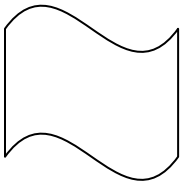
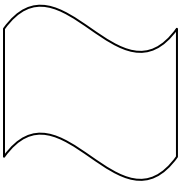
Standard # 22 Model Assessment Items

Computational and Procedural Skills

1. Draw all lines of symmetry.

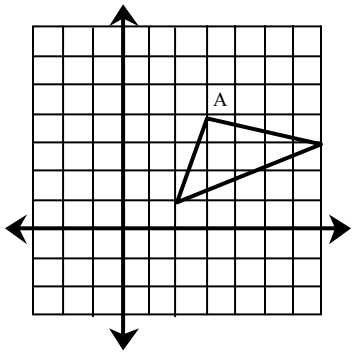





2. Identify each transformation as a translation, a reflection, or a rotation.

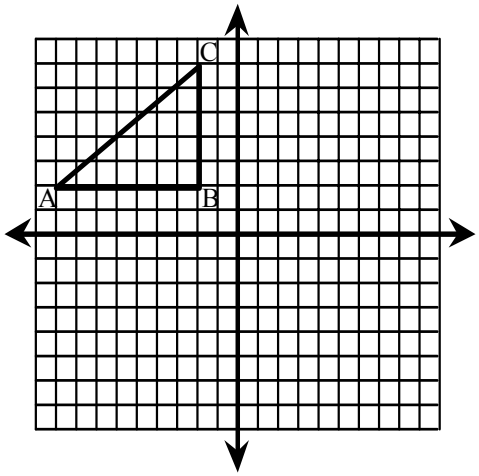





Conceptual Understanding

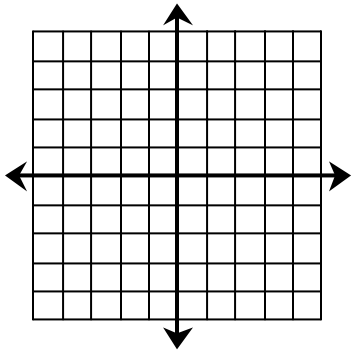
1. Translate (slide) this figure so that point A is at (-1, 6). Label the coordinates of the new figure.



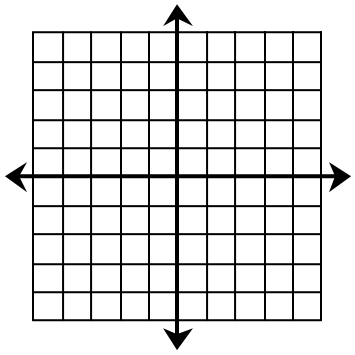
2. Draw the triangle that results when this figure is rotated 90° clockwise around the origin, and when it is rotated 180° clockwise around the origin.



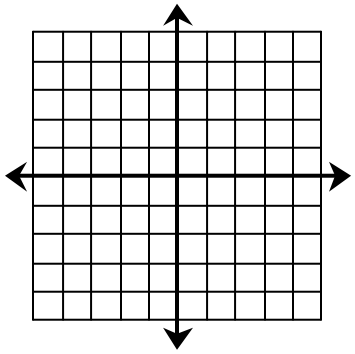
3. Plot and graph A (-1, 4), B (4, 5), and C (2, 2) on a coordinate plane. Draw this triangle and then graph its reflection across the *x*-axis.



4. Plot and graph A (-1, 4), B (4, 5), and C (2, 2) on a coordinate plane. Draw this triangle and then graph its translation five units to the left and six units down.



5. Graph A (2, 4), B (4, 5), and C (2, 2) on a coordinate plane. Draw this triangle, then graph its rotation 180° counter-clockwise around the origin.



Problem Solving/Application

1. Design an object such as a rug, wallpaper, or some pottery using one basic shape and rotations, reflections, and/or translations. Draw your pattern(s) and discuss which translations were used. Can two different methods or combinations of methods be used to produce the same pattern? Why?

2. Research uses of transformations as decorative art. Consider M. C. Escher paintings, the Iroquois or Navajo Indian tribes' designs, Hopi pottery, Arabian tiles, or African Adumbral cloth. Create your own pattern. Explain the transformation used.

Appendix #1

Geometry California Content Standards by Clusters

California Standards Testing organizes related standards for Geometry into four reporting clusters. Student performance is reported by these clusters. The clusters for Geometry are:

- Logic and Geometric Proofs
- Volume and Area Formulas
- Angle Relationships, Constructions, and Lines
- Trigonometry

The following table, compiled by the San Diego County Office of Education, organizes the standards by cluster and shows the number of items testing each standard on the Geometry CST and High School Summative (HSS) CST. The actual questions change each year, but the distribution of questions, indicated in the table, below remains the same.

Mathematics Content Standards: Geometry		Geometry CST	Summative High School CST
Cluster 1: Logic and Geometric Proofs		65 items 100%	19 items 29%
1.0*	Students demonstrate understanding by identifying and giving examples of undefined terms, axioms, theorems, and inductive and deductive reasoning.	2	
2.0*	Students write geometric proofs, including proofs by contradiction.	3	
3.0*	Students construct and judge the validity of a logical argument and give counterexamples to disprove a statement.	4	1
4.0*	Students prove basic theorems involving congruence and similarity.	5	3
5.0	Students prove that triangles are congruent or similar, and they are able to use the concept of corresponding parts of congruent triangles.	2	2
6.0	Students know and are able to use the triangle inequality theorem.	1	
7.0*	Students prove and use theorems involving the properties of parallel lines cut by a transversal, the properties of quadrilaterals, and the properties of circles.	5 2/3	2
Cluster 2: Volume and Area Formulas			
8.0*	Students know, derive, and solve problems involving the perimeter, circumference, area, volume, lateral area, and surface area of common geometric figures.	4	1
9.0	Students compute the volumes and surface areas of prisms, pyramids, cylinders, cones, and spheres; and students commit to memory the formulas for prisms, pyramids, and cylinders.	2	1
10.0*	Students compute areas of polygons, including rectangles, scalene triangles, equilateral triangles, rhombi, parallelograms, and trapezoids.	4	1
11.0	Students determine how changes in dimensions affect the perimeter, area, and volume of common geometric figures and solids.	1	1
Cluster 3: Angle Relationships, Constructions, and Lines			
12.0*	Students find and use measures of sides and of interior and exterior angles of triangles and polygons to classify figures and solve problems.	5	

13.0	Students prove relationships between angles in polygons by using properties of complementary, supplementary, vertical, and exterior angles.	2	
14.0*	Students prove the Pythagorean theorem.	1/3	
15.0	Students use the Pythagorean theorem to determine distance and find missing lengths of sides of right triangles.	2	2
16.0*	Students perform basic constructions with a straightedge and compass, such as angle bisectors, perpendicular bisectors, and the line parallel to a given line through a point off the line.	4	
17.0*	Students prove theorems by using coordinate geometry, including the midpoint of a line segment, the distance formula, and various forms of equations of lines and circles.	3	
Cluster 4: Trigonometry			
18.0*	Students know the definitions of the basic trigonometric functions defined by the angles of a right triangle. They also know and are able to use elementary relationships between them. For example, $\tan(x) = \sin(x)/\cos(x)$, $(\sin(x))^2 + (\cos(x))^2 = 1$.	3	2
19.0*	Students use trigonometric functions to solve for an unknown length of a side of a right triangle, given an angle and a length of a side.	3	1
20.0	Students know and are able to use angle and side relationships in problems with special right triangles, such as 30°, 60°, and 90° triangles and 45°, 45°, and 90° triangles.	1	
21.0*	Students prove and solve problems regarding relationships among chords, secants, tangents, inscribed angles, and inscribed and circumscribed polygons of circles.	5	2
22.0*	Students know the effect of rigid motions on figures in the coordinate plane and space, including rotations, translations, and reflections.	3	

CST: Fractional values indicate rotated standards (e.g., 1/2 = rotated every two years; 1/3 = rotated every three years).
CST: N/A: Not assessable in a multiple-choice format. Embedded: Content of standard is embedded within items in other strands.

Appendix #2

Developing Learning Targets for Geometry Standards (Instructions given to teachers involved in the project—same as the instructions used in Algebra deconstruction projects)

Please note the following terms and/or definitions which have been agreed upon for this deconstruction project:

- ***Prior Knowledge:*** *Prior knowledge is defined as acquired knowledge that has been mastered in a previous standard.*
- ***New Knowledge:*** *New knowledge is defined as knowledge that students need to acquire and apply to the components in step #2 of this deconstruction process to create the products listed in step #7 of this process.*
- ***Introduced:*** *When a standard mentions that a concept or idea has been "introduced," this does not mean that it has been mastered.*
- ***Familiar With:*** *When a standard mentions that students should be "familiar with" certain concepts or ideas, this does not mean that students have actually mastered these ideas or concepts.*

Sample:
Deconstruction of Algebra I Standard #6

Step #1: Underline noun phrases, and box or circle the verbs.
Standard 6.0: Students graph a linear equation and compute the *x*- and *y*-intercepts. (e.g., graph $2x + 6y = 4$). They are also able to sketch the region defined by a linear inequality (e.g., they sketch the region defined by $2x + 6y < 4$).

Step #2: Rewrite standard into short components.
1. Students graph linear equations.
2. Students compute *x*-intercepts.
3. Students compute *y*-intercepts.
4. Students use intercepts to graph linear equations.
5. Students sketch the region defined by a linear inequality.

Step #3: Identify prior knowledge students should know. *(See note above)*
1. Students must be able to perform arithmetic computations with rational numbers.
2. Students must be able to graph ordered pairs.
3. Students must be able to compute slope from the graph of a line.
4. Students must be able to compute slope when given two points.
5. Students must be able to recognize slope as a rate of change of *y* in relation to *x*.
6. Students must be able to graph a linear equation using a "t-chart."
7. Students must be able to evaluate a linear equation for a given *x* or *y* value.
8. Students must be able to solve one-variable linear inequalities.
9. Students must be able to graph the solution set for a one-variable linear inequality.

10. Students must be able to verify that any element in the solution set of a one-variable inequality satisfies the original inequality.

Step #4: Identify what new knowledge students will need to learn. *(See note above)*

1. Given the slope/intercept form of a line, $y = mx + b$, students will plot the *y*-intercept, and then use the slope to find a second point in order to complete the graph of the line.
2. Students will identify the graphical representation of $(a, 0)$ as the *x*-intercept, and $(0, b)$ as the *y*-intercept.
3. Students will be able to compute the *x*-intercept and *y*-intercept given a linear equation.
4. Students will identify that the linear equation implied by the linear inequality forms a boundary for the solution set and that this boundary may or may not be included in the final graph.
5. Students will interpret the inequality symbol to determine whether or not the boundary is solid or dashed.
6. Students will identify and shade the region of the graph that contains the solutions to the inequality.
7. Students will recognize that linear inequalities have multiple ordered-pair solutions.

Step #5: Identify patterns of reasoning using *Bloom's Taxonomy*.

Use the *Bloom's Taxonomy* handouts provided to describe the overall competence level expected of students with respect to these topics. Then highlight the "skills demonstrated" using as many of the key words and phrases provided on the handouts. See example below for Standard #6. Box in the key words or phrases taken from *Bloom's Taxonomy*.

Competence Level: Application

1. Students will use methods they have learned to graph lines, solve inequalities, and to locate and/or identify the *x*- and *y*-intercepts for a given equation or graph.
2. Students will demonstrate their ability to find and use *x*- and *y*-intercepts in the context of graphing.
3. Students will calculate *x*- and *y*-intercepts.
4. Students will solve inequalities in two variables.
5. Students will use information they have learned to graph lines, solve inequalities, and find *x*- and *y*-intercepts.
6. Students will show that they know the correct interpretation of the boundary line for the solution of an inequality by appropriately making the boundary solid or dashed.

Step #6: Identify required physical skills.

In this section we are looking for physical skills such as: use of a calculator, protractor, ruler, compass, etc.

1. Use of a ruler

Step #7: Identify assessable results of the standard.

1. Students will produce the graph of a line.
2. Students will produce the ordered pairs representing the *x*- and *y*-intercepts.
3. Students will produce a bounded and shaded region of the *x*-*y* plane representing the solution set of a linear inequality in two variables.

Model Assessment Items

In this section you will write model, or exemplar, assessment items that will serve to demonstrate the level and depth of instruction for these particular topics. For example, you would expect a lower level and depth for a topic in Basic Algebra than you would for the same topic in Intermediate Algebra.

Please be sure to include assessment items that measure abilities in the following three categories:

1. Computational and Procedural Skills
2. Conceptual Understanding
3. Problem Solving/Application

Category #1: Computational and Procedural Skills

1. Find the *x*- and *y*-intercepts for the line defined by the following equation: $2x+3y=9$.
2. Use the *x*- and *y*-intercepts to graph the line given by the equation: $2x+3y=6$.
3. Graph the following lines using the method of your choice. Identify and label the *x*- and *y*-intercepts for each graph if they exist:
   A. $3x-5y=10$

   B. $y=-\frac{2}{3}x+4$

   C. $y=2$

   D. $x=3.5$

   E. $2x+4y=3$

   F. $\frac{1}{2}x-\frac{3}{4}y=2$

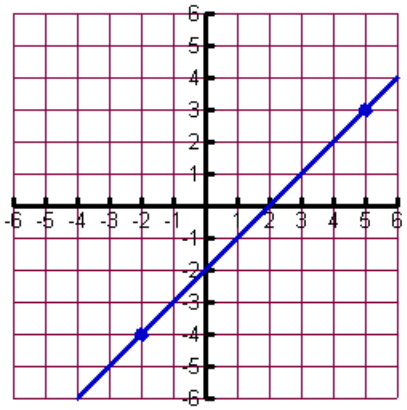
4. Graph the solution set for the following inequalities:
   A. $2x-3y<6$

   B. $y\geq-\frac{3}{4}x+2$

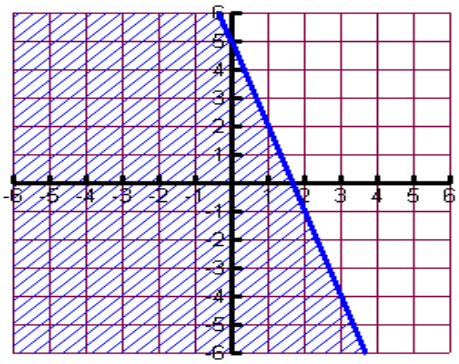
C. $\frac{1}{2}x - \frac{2}{3}y \le \frac{5}{6}$

Category #2: Conceptual Understanding

1. Sketch the graph of a line that has no *x*-intercept.
2. Identify the *x*- and *y*- intercepts from the graph of the given line.



3. Can a line have more than one *x*-intercept? Explain your answer using a diagram.
4. The solution to an inequality has been graphed correctly below. Insert the correct inequality symbol in the inequality below to match the graph of the solution. (Everything else about the inequality is correct—it just needs the correct symbol).



$y \square -3x+5$

Insert correct symbol in box.

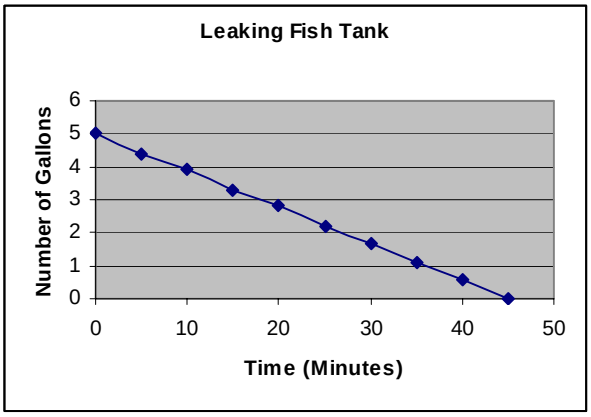
5. When is it advantageous to use the *x*- and *y*-intercepts to graph the equation of a line? When would it perhaps be easier or better to use another graphing method? Give an example to illustrate your answers to both of these questions.

Category #3: Problem Solving/Application

1. The graph displayed below is the graph of the following equation: $y = \left(\frac{-1}{9}\right)x + 5$,

   where *x* represents the amount of time that has passed since a 5 gal. fish tank sprung a leak, and *y* represents the number of gallons of water in the tank after the leak.

A. What is the significance of the *x*-intercept in this situation? What information is given to us by this point?
B. What is the significance of the *y*-intercept in this situation? What information is given to us by this point?



2. The cost of a trash pickup service is given by the following formula: $y = 1.50x + 11$, where *x* represents the number of bags of trash the company picks up, and *y* represents the total cost to the customer for picking up the trash.
   A. What is the *y*-intercept for this equation?
   B. What is the significance of the *y*-intercept in this situation? What does it tell us about this trash pickup service?
   C. Draw a sketch of the graph which represents this trash pickup service.

Exemplar Teaching Methods

Please record an example of "Lesson Plans" that demonstrate an excellent method of how the concepts in this standard could be presented to students. Be sure to give examples or illustrate any unique or creative methods that you have used that bring these concepts to life. Use as much detail as needed to communicate your ideas.

Appendix #3
Categorization of Educational Outcomes

Identifies the type of reasoning students will use to learn the skills necessary to master each standard. Teachers were asked to use *Bloom's Taxonomy* to describe the overall competence level expected of students with respect to these topics and highlight the skills demonstrated.

Major Categories in the* Taxonomy of Educational Objectives, *Bloom 1956*
Categories in the Cognitive Domain: (with Outcome-Illustrating Verbs)

Knowledge—of terminology; specific facts; ways and means of dealing with specifics (conventions, trends, and sequences; classifications and categories; criteria, methodology); universals and abstractions in a field (principles and generalizations, theories and structures)—**The remembering (recalling) of appropriate, previously learned information.**

- defines; describes; enumerates; identifies; labels; lists; matches; names; reads; records; reproduces; selects; states; views.

***Comprehension:* Grasping (understanding) the meaning of informational materials.**

- classifies; cites; converts; describes; discusses; estimates; explains; generalizes; gives examples; makes sense out of; paraphrases; restates (in own words); summarizes; traces; understands.

***Application:* The use of previously learned information in new and concrete situations to solve problems that have single or best answers.**

- acts; administers; articulates; assesses; charts; collects; computes; constructs; contributes; controls; determines; develops; discovers; establishes; extends; implements; includes; informs; instructs; operationalizes; participates; predicts; prepares; preserves; produces; projects; provides; relates; reports; shows; solves; teaches; transfers; uses; utilizes.

***Analysis:* The breaking down of informational materials into their component parts, examining (and trying to understand the organizational structure of) such information to develop divergent conclusions by identifying motives or causes, making inferences, and/or finding evidence to support generalizations.**

- breaks down; correlates; diagrams; differentiates; discriminates; distinguishes; focuses; illustrates; infers; limits; outlines; points out; prioritizes; recognizes; separates; subdivides.

***Synthesis:* Creatively or divergently applying prior knowledge and skills to produce a new or original whole.**

- adapts; anticipates; categorizes; collaborates; combines; communicates; compares; compiles; composes; contrasts; creates; designs; devises; expresses; facilitates; formulates; generates; incorporates; individualizes; initiates; integrates; intervenes;

models; modifies; negotiates; plans; progresses; rearranges; reconstructs; reinforces; reorganizes; revises; structures; substitutes; validates.

Evaluation: **Judging the value of material based on personal values/opinions, resulting in an end product, with a given purpose, without real right or wrong answers.**

- appraises; compares & contrasts; concludes; criticizes; critiques; decides; defends; interprets; judges; justifies; reframes; supports.

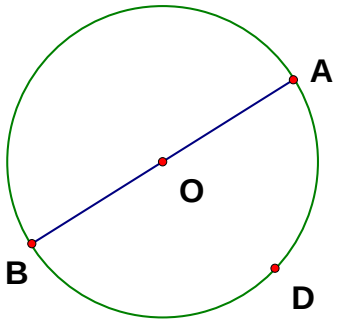
Appendix #4
Sample Teaching Items

Standard #8

GEOMETER'S SKETCHPAD LESSON

EXPLORING RELATIONSHIP BETWEEN DIAMETER AND CIRCUMFERENCE OF A CIRCLE

1. Sketch a Circle with Cente O.
2. Label Point D, the dragging point of Circle O.
3. Place Point A anywhere on the circumference of Circle O.
4. Construct a line through Points A and O.
5. Label Point B, the point of intersection of $\overline{AO}$ with the opposite side of the circle from Point A.
7. Hide line $\overleftrightarrow{AB}$, but leave diameter $\overline{AB}$ on display.
8. Measure diameter $\overline{AB}$.
9. Measure circumference of Circle O.
10. Calculate the measure of circumference Circle O divided by measure of diameter $\overline{AB}$.
11. Drag Point D in and out and observe the effect on the quotient in step 10.
12. Make a conjecture about the relationship between the diameter and the circumference of Circle O.
13. Make a conjecture about the relationship between the radius and the circumference of Circle O.
14. Make a conjecture about the formulas for finding the circumference of a circle when you are given:
    a) the diameter.
    b) the radius.
15. Compare your answers to (a) and (b). Explain the similarities and differences.



$m\overline{AB}$ = ___

Circumference ⊙O D = ___

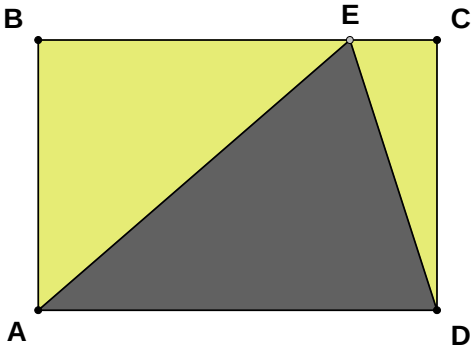
$$\frac{\text{(Circumference } \odot O D)}{m\overline{AB}} = ___$$

Standard #8

GEOMETER'S SKETCHPAD LESSON

EXPLORING RELATIONSHIP BETWEEN AREA OF A TRIANGLE AND AREA OF A RECTANGLE

1. **Construct Rectangle ABCD.**
2. **Construct Quadrilateral Interior for Rectangle ABCD.**
3. **Measure area of Rectangle ABCD.**
4. **Place point E anywhere on $\overline{AB}$.**
5. **Construct segments AE and DE to create $\triangle AED$.**
6. **Construct Triangle Interior for $\triangle AED$ and display interior in different color from interior of Rectangle ABCD.**
7. **Measure Area $\triangle AED$.**
8. **Calculate the measure of Rectangle Area ABCD divided by the Area of $\triangle AED$.**
9. **Drag Point E along $\overline{AB}$ and observe the effect on Area $\triangle AED$ and the quotient from Step 8.**
10. **Make a conjecture about the relationship between the Area of Rectangle ABCD and the Area of $\triangle AED$.**
11. **Make a conjecture about how you might find the Area of $\triangle AED$.**
12. **Make a conjecture about how you might find the Area of any triangle.**



Area ABCD = ____ cm^2

Area $\triangle AED$ = ____ cm^2

$$\frac{(\text{Area ABCD})}{(\text{Area } \triangle AED)} = ____$$

Standard #11

1. Students use actual similar rectangles to calculate scale factors, perimeters, and areas. Then, through the inquiry method, students discover the relationships of the various ratios.

2. Student can do the same with volume using cereal to fill similar prisms or cones.

Standard #14

1. Using graph paper, draw a right triangle to scale. Create squares with lengths of sides corresponding to the sides of each leg and hypotenuse. Cut and paste these squares with the side matching the length of the side of the triangle in contact with that side. Compare the areas of the squares and determine the relationship between the squares.