Algebra II California Content Standards

Standards Deconstruction Project

2008

Version 2.0

Version 2.0 produced by Bonnie Boswell, Nemie Capacia, Dolores Dreher, Jim Hartman, Peg Hovde, Michael Orr, Shahrokh Parvini, Eric Preibisius, Kindra Samuelson, Trysta Schwenker, Sarah Thomson, Mailei Vargas

For information please contact
Dr. Shelly Valdez
Director of Regional Collaboration
svaldez@calpass.org
619-219-9855

A note to the reader: This project was coordinated and funded by the California Partnership for Achieving Student Success (Cal-PASS) and Girard Foundation. Cal-PASS is a data sharing system linking all segments of education. Its purpose is to improve student transition and success from one educational segment to the next.

The original version, Version 1.0, of this document was first published in 2006. This current version, Version 2.0, includes updates based on feedback from faculty across the state. Also a listing of the Algebra I California Content Standards organized into clusters has been added in Appendix #1.

Cal-PASS is unique in that it is the only data collection system that spans and links student performance and course-taking behavior throughout the education system—K–12, community college, and university levels. Data are collected from multiple local and state sources and shared, within regions, with faculty, researchers, and educational administrators to use in identifying both barriers to successful transitions and strategies that are working for students. These data are then used regionally by discipline-specific faculty groups, called "Intersegmental Councils," to better align curriculum.

Cal-PASS' Algebra I deconstruction project was initiated by the faculty serving on the math intersegmental councils after reviewing data on student transition. A deconstruction process was devised by the participating faculty with suggestions from the San Bernardino County Unified School District math faculty (Chuck Schindler and Carol Cronk) and included adaptations of the work of Dr. Richard Stiggins of the Assessment Training Institute and Bloom's *Taxonomy of Educational Objectives* (B. S. Bloom, 1984,. Boston: Allyn and Bacon).

The Algebra II deconstruction project followed using the same procedure that was used for deconstructing Algebra I standards and has been supported by a generous grant from Girard Foundation. The following document represents a comprehensive review by K–16 faculty to deconstruct and align Algebra II (Intermediate Algebra) standards.

In order to continue the collaboration on these standards, thus improving on the current work, we invite and encourage the reader to provide feedback to us. Please contact Dr. Shelly Valdez at: **svaldez@calpass.org.**

Table of Contents

California Algebra II Content Standards

Retrieved from CDE website (4-03-06): http://www.cde.ca.gov/be/st/ss/mthalgebra2.asp

ALGEBRA II **Grades Eight Through Twelve - Mathematics Content Standards.**
This discipline complements and expands the mathematical content and concepts of Algebra I and Geometry. Students who master Algebra II will gain experience with algebraic solutions of problems in various content areas, including the solution of systems of quadratic equations, logarithmic and exponential functions, the binomial theorem, and the complex number system.
Standard Set 1.0: Students solve equations and inequalities involving absolute value.
2.0: Students solve systems of linear equations and inequalities (in two or three variables) by substitution, with graphs, or with matrices.
3.0: Students are adept at operations on polynomials, including long division.
4.0: Students factor polynomials representing the difference of squares, perfect square trinomials, and the sum and difference of two cubes.
5.0: Students demonstrate knowledge of how real and complex numbers are related both arithmetically and graphically. In particular, they can plot complex numbers as points in the plane.
6.0: Students add, subtract, multiply, and divide complex numbers.
7.0: Students add, subtract, multiply, divide, reduce, and evaluate rational expressions with monomial and polynomial denominators and simplify complicated rational expressions, including those with negative exponents in the denominator.
8.0: Students solve and graph quadratic equations by factoring, completing the square, or using the quadratic formula. Students apply these techniques in solving word problems. They also solve quadratic equations in the complex number system.
9.0: Students demonstrate and explain the effect that changing a coefficient has on the graph of quadratic functions; that is, students can determine how the graph of a parabola changes as *a, b,* and *c* vary in the equation $y = a(x-b)^2 + c$.
10.0: Students graph quadratic functions and determine the maxima, minima, and zeros of the function.
11.0: Students prove simple laws of logarithms. 11.1: Students understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents. 11.2: Students judge the validity of an argument according to whether the properties of real numbers, exponents, and logarithms have been applied correctly at each step.
12.0: Students know the laws of fractional exponents, understand exponential functions, and use these functions in problems involving exponential growth and decay.
13.0: Students use the definition of logarithms to translate between logarithms in any base.

14.0: Students understand and use the properties of logarithms to simplify logarithmic numeric expressions and to identify their approximate values.
15.0: Students determine whether a specific algebraic statement involving rational expressions, radical expressions, or logarithmic or exponential functions is sometimes true, always true, or never true.
16.0: Students demonstrate and explain how the geometry of the graph of a conic section (e.g., asymptotes, foci, eccentricity) depends on the coefficients of the quadratic equation representing it.
17.0: Given a quadratic equation of the form $ax^2 + by^2 + cx + dy + e = 0$, students can use the method for completing the square to put the equation into standard form and can recognize whether the graph of the equation is a circle, ellipse, parabola, or hyperbola. Students can then graph the equation.
18.0: Students use fundamental counting principles to compute combinations and permutations.
19.0: Students use combinations and permutations to compute probabilities.
20.0: Students know the binomial theorem and use it to expand binomial expressions that are raised to positive integer powers.
21.0: Students apply the method of mathematical induction to prove general statements about the positive integers.
22.0: Students find the general term and the sums of arithmetic series and of both finite and infinite geometric series.
23.0: Students derive the summation formulas for arithmetic series and for both finite and infinite geometric series.
24.0: Students solve problems involving functional concepts, such as composition, defining the inverse function, and performing arithmetic operations on functions.
25.0: Students use properties from number systems to justify steps in combining and simplifying functions.
Probability and Statistics
1.0: Students know the definition of the notion of *independent events* and can use the rules for addition, multiplication, and complementation to solve for probabilities of particular events in finite sample spaces.
2.0: Students know the definition of *conditional probabilit*y and use it to solve for probabilities in finite sample spaces.
7.0: Students compute the variance and the standard deviation of a distribution of data.

Standard #1

Standard Set 1.0
Students solve equations and inequalities involving absolute value.

Deconstructed Standard
1. Students solve equations involving absolute value.
2. Students solve inequalities involving absolute value.

Prior Knowledge Necessary
Students should know how to:
- solve algebraic equations.
- solve algebraic inequalities.
- evaluate absolute value expressions for a given value of the variable.
- graph solutions to an inequality on the real number line.
- interpret the absolute value of a number as a distance from zero on a number line.
- represent solution sets using inequalities.
- convert solution sets written in interval notation to inequality notation and vice versa.
- solve compound inequalities.
- translate absolute value inequalities into compound inequalities.
- identify absolute value equations that have no solution (e.g., $|x-3| = -9$).
- identify absolute value inequalities that have no solution (e.g., $|x+2| < 0$).

New Knowledge
Students will need to learn to:
- solve more advanced problems involving equations with absolute value.
- solve more advanced problems involving inequalities with absolute value.
- solve more advanced problems involving inequalities with unions, intersections, or no solution.

Categorization of Educational Outcomes
Competence Level: Application
1. Students will classify solution sets to absolute value equations/inequalities as the union or intersection of two sets. (Or/And)
2. Students will solve absolute value equations/inequalities and graph the solution set.
3. Students will identify if an absolute value equation/inequality will have no solution.

Necessary New Physical Skills
None

Assessable Result of the Standard

1. Students will solve absolute value equations and graph the solution set.
2. Students will translate an absolute value inequality into a compound inequality and solve, and graph, the solution set.
3. Students will identify if an absolute value equation/inequality will have no solution.
4. Students will find solutions to real-world problems.

Standard #1 Model Assessment Items

Computational and Procedural Skills

1. Solve each equation/inequality and graph the solution set. If there is no solution, state the reason.

A. $|x+2|=4$

B. $10=|7-3x|$

C. $|2x-5|=3$

D. $x+4=|x-2|$

E. $|x-4|>1$

F. $|2x|>12$

G. $|-2x-3|\le 11$

H. $|4x+6|\ge 14$

I. $|2x-1|>-4$

J. $-|x+3|>4$

K. $4-3|x+2|\le 5$

L. $2|3x-1|-4>8$

Conceptual Understanding

1. Solve and graph on a number line:

A. $|x-3|=6$

B. $|3x-4|=10$

C. $|x+8|>12$

D. $|9x+4|\le 10$

E. $|3x+5|<14$

F. $|2x-3|>-7$

Problem Solving/Application

1. A recent survey reported that 72% of teenage boys prefer burritos to tacos. The margin of error for the poll was 5%. What are the minimum and maximum possible percents according to the survey?

2. An instrument measures a wind speed of 40 feet per second. The true wind speed is within 3 feet per second of the measured speed. What range is possible?

Standard #2

Standard Set 2.0
Students solve systems of linear equations and inequalities (in two or three variables) by substitution, with graphs, or with matrices[1].

Deconstructed Standard
1. Students solve systems of linear equations in two variables by graphing.
2. Students solve systems of linear inequalities in two variables by graphing.
3. Students solve systems of linear equations in two variables by substitution.
4. Students solve systems of linear equations in two variables by matrices (Elimination).
5. Students solve systems of linear equations in three variables by substitution.
6. Students solve systems of linear equations in three variables by matrices (Elimination).

Prior Knowledge Necessary
NOTE: This standard is a review and extension of Standard #9 from Algebra I which reads as follows: "Students solve a system of two linear equations in two variables algebraically and are able to interpret the answer graphically. Students are able to solve a system of two linear inequalities in two variables and to sketch the solution sets."

Students should know how to:
- solve and graph a linear equation in two variables.
- solve and graph a linear inequality in two variables.
- substitute a rational number or expression for a variable.
- identify the coefficients from an equation in standard form.

New Knowledge
NOTE: This standard is a review and extension of Standard #9 from Algebra I which reads as follows: "Students solve a system of two linear equations in two variables algebraically and are able to interpret the answer graphically. Students are able to solve a system of two linear inequalities in two variables and to sketch the solution sets."

Students will need to learn to:
- identify the solution from a graph, given a system of linear equations in two variables.
- identify and shade the region of a graph that contains the solutions to a system of linear inequalities in two variables.
- determine the solution to a linear system in two variables through matrices (elimination).
- determine the solution to a linear system in three variables through substitution and/or matrices (elimination).

---
[1] Matrices are interpreted as Elimination (Addition, Subtraction, and/or Multiplication) Methods.

- recognize when a system of linear equations has exactly one solution, an infinite number of solutions, or no solution.
- recognize when a system of linear inequalities in two variables has an infinite number of solutions or no solution.

Categorization of Educational Outcomes

Competence Level: Application

1. Students will use methods they have learned to solve systems of linear equations and inequalities.
2. Students will demonstrate their ability to find a solution to a system of linear equations or inequalities by graphing.
3. Students will calculate the intersection point, if one exists, through solving for x, y and/or z by substitution or matrices (Elimination).

Necessary New Physical Skills

1. Use of a straight edge for graphing straight lines.

Assessable Result of the Standard

1. Students will produce the graph and interpret the solution of a system of linear equations in two variables.
2. Students will produce the graph and interpret the solution of a system of linear inequalities in two variables.
3. Students will produce the solution to a system of two linear equations in two variables.
4. Students will produce the solution to a system of three linear equations in three variables.

Standard #2 Model Assessment Items

Computational and Procedural Skills

1. Solve the system by graphing.

A. $\begin{cases} -3x+2y=-6 \\ 2x-y=4 \end{cases}$

B. $\begin{cases} x \geq 0 \\ y \geq 0 \\ -x-y \geq -5 \\ -2x+3y \geq -3 \end{cases}$

2. Solve the system by substitution:

A. $\begin{cases} -x+2y=11 \\ 3x-2y=-13 \end{cases}$

B. $\begin{cases} -x-3y=-2 \\ 3x+9y=9 \end{cases}$

3. Solve the system by matrices (Elimination):

A. $\begin{cases} 4x+4y=44 \\ -2x-2y=-22 \end{cases}$

B. $\begin{cases} y=-\frac{1}{8}x-\frac{5}{8} \\ y=\frac{2}{5}x-\frac{11}{5} \end{cases}$

C. $\begin{cases} -5x+4y-z=19 \\ -10y-8z=-14 \\ z=-2 \end{cases}$

D. $\begin{cases} x-2y+3z=2 \\ -x-5y+2z=-11 \\ 2x-y-4z=0 \end{cases}$

Conceptual Understanding

1. If you graph two lines in the same coordinate plane, what are the possible outcomes?

2. A system of linear equations in two or three variables may have infinitely many solutions. Explain how this is possible in each case.

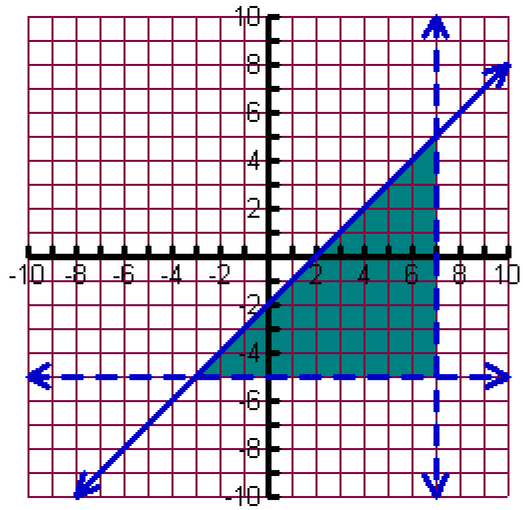
3. Does every system of linear equations in two or three variables have a solution? Explain for each case.

4. After a solution to a system of linear equations is found, why should the solution be checked in the system?

5. If the solution of a system of linear inequalities exists, describe what the solution looks like.

6. When is it advantageous to use the substitution method? The matrix (Elimination) method? Give an example to illustrate your answers to both parts of this question.

7. Write the system of linear inequalities that determine the solution graphed below, given the vertices are as follows: (7 , 5) , (7 , -5) and (-3 , -5) *(Inside the triangle is shaded):*



<u>Problem Solving/Application</u>

1. Your family receives basic cable and two movie channels for $32.30 a month. Your neighbor receives basic cable and four movie channels for $43.30 a month. What is the monthly charge for just the basic cable? (Assume that the movie channels have the same monthly cost.) What is the monthly charge for one movie channel?
    A. Write a system of two equations using two variables.

    B. Solve and put answer in a complete sentence.

2. The senior class has a carnival to raise money for a senior trip. Student tickets are $6 each and adult tickets are $11 each. With 324 people in attendance, the senior class raised $2,359. How many of the people in attendance were adults?

3. Sally has a combination of nickels, dimes, and quarters. She has three more dimes than quarters. If she has 16 coins totaling $2.20, how many of each coin does she have?

4. Jonathan has $10,000 to invest in three different accounts. He invests some into ~~a~~ an account that earns 3% interest, some into ~~a~~ an account that earns 4.2% interest, and some into ~~a~~ an account that earns 5.1% interest. The amount he invested into the account earning 5.1% interest is three times more than the amount he invested into the account earning 4.2% interest. If he earned an annual income of $450 in interest, how much did he invest into each of the accounts?

Standard #3

Standard Set 3.0
Students are adept at operations on polynomials, including long division.

Deconstructed Standard
1. Students are adept at adding polynomials.
2. Students are adept at subtracting polynomials.
3. Students are adept at multiplying polynomials.
4. Students are adept at dividing polynomials.

Prior Knowledge Necessary
This standard is an extension of Algebra I, Standard #10 and students should have the computational and conceptual knowledge outlined in that standard.
Students should know how to:
- add and subtract polynomials.
- use the distributive property to multiply polynomials.
- apply the appropriate exponential rules to simplify algebraic expressions.
- divide polynomials by a monomial.
- divide polynomials by binomials using long division.
- translate and solve multi-step word problems involving polynomials.

New Knowledge
This standard is an extension of Standard #10 in Algebra I. The following knowledge reflects the extension beyond Standard #10 in Algebra I.

Students will need to learn to:
- divide a higher order polynomial by another polynomial containing 2 or more terms using long division.
- *(optional)* divide a polynomial by a binomial using synthetic division.

Categorization of Educational Outcomes
Competence Level: Application
1. Students will use the different operations to simplify polynomials (adding, subtracting, multiplying, and dividing).

Necessary New Physical Skills
None

Assessable Result of the Standard
1. Students will produce the sum, difference, product, and quotient of polynomials.

Standard #3 Model Assessment Items

Computational and Procedural Skills

1. Perform the indicated operations and simplify completely:

    A. $\left(-9xy^2 - xy + 6x^2y\right) + \left(-5x^2y - xy + 4xy^2\right)$

    B. $3x\left(2x^2 - 3x + 2\right) - 5x\left(6x^2 - 11x - 2\right)$

    C. $\left(8x^2 - 4xy + y^2\right) - \left(2x^2 + 3xy - 2y^2\right)$

    D. $\left(x - 5\right)\left(x^2 + 5x + 25\right)$

    E. $\left(x - 2y\right)\left(x + 2y\right)$

    F. $\left(2x^2 - 3x + 2\right)^2$

    G. $\left(2x - 5\right)^2$

    H. $\left(x^3y^2 - x^3y^3 - x^4y^2\right) \div \left(x^2y^2\right)$

    I. Use long division: $\left(2x^4 - x^3 - 5x^2 + x - 6\right) \div \left(x^2 + 2\right)$

    J. *(Optional)* Use synthetic division: $\left(y^3 - 3y + 10\right) \div \left(y - 2\right)$

Conceptual Understanding

1. Do addition, subtraction, and multiplication of polynomials always result in a polynomial? Does division? Explain why or why not.

2. Jesse insists that $\left(x + 2\right)^2 = x^2 + 2^2 = x^2 + 4$ and that $\left(x + 2\right)^3 = x^3 + 2^3 = x^3 + 8$. What is wrong with this and how can you convince her that this simplification is incorrect?

3. Without performing any long division, how could you show that this division is incorrect? $\left(x^3 + 9x^2 - 6\right) \div \left(x^2 - 1\right) = x + 9 + \dfrac{x + 4}{x^2 - 1}$

4. When performing synthetic division *(optional) or* long division if there are missing terms in the dividend, why is it necessary to either write them with 0 coefficients or leave space for them?
    A. *(Optional)* Can you always use synthetic division in place of long division on any type of polynomial?

    B. Why or why not?

5. Find "k" such that when x^3+2x^2-kx+5 is divided by $x-2$, the remainder is 3.

6. When dividing one polynomial by another, when do you stop the division process?

Problem Solving/Application

1. The volume of a rectangular solid box is given by the following: $x^3+6x^2-7x-60$. The length of this box is given by $(x+5)$ and the width is given by $(x+4)$. Find the expression that represents the height.

2. The sides of a square are lengthened so that the new sides are 5 more than twice the original side:
    A. Find the expression that represents the perimeter.

    B. Find the expression that represents the area.

Standard #4

Standard Set 4.0
Students factor polynomials representing the difference of two squares, perfect square trinomials, and the sum and difference of two cubes.

Deconstructed standard
1. Students factor the difference of two squares.
2. Students factor perfect square trinomials.
3. Students factor sum of two cubes.
4. Students factor difference of two cubes.

Prior Knowledge Necessary
Knowledge of Algebra I, Standard #11: Students apply basic factoring techniques to simple second- and simple third-degree polynomials. These techniques include finding a common factor for all terms in a polynomial, recognizing the difference of two squares, and recognizing perfect squares of binomials.

Students should know how to:
- apply the Laws of Exponents to algebraic expressions.
- recognize numbers as being perfect squares or perfect cubes.
- recognize binomials as the difference of two square terms.
- factor a second degree polynomial as the product of two binomials.
- factor out a common monomial from a polynomial.
- recognize that a polynomial is a perfect square trinomial.

New knowledge
Students will need to learn to:
- recognize that a binomial is the difference of two cubes.
- recognize that a binomial is the sum of two cubes.
- factor a binomial that is the difference of two cubes.
- factor a binomial that is the sum of two cubes.
- factor a polynomial that includes more than one of the following: common monomial, a non-prime trinomial, the difference of squares, a perfect square trinomial, a difference of cubes, or the sum of cubes.

Categorization of Educational Outcomes
Competence Level: Knowledge:
1. Students will identify a binomial as the difference of squares.
2. Students will identify a trinomial as a perfect square of a binomial.
3. Students will identify a binomial as the sum of two cubes.
4. Students will identify a binomial as the difference of two cubes.

Competence Level: Application

1. Students will use the methods of grouping, reverse FOIL, or other methods to factor a second-degree polynomial.
2. Students will demonstrate knowledge of factoring the difference of two squares.
3. Students will demonstrate knowledge of perfect square binomials to factor a trinomial.
4. Students will use a combination of one or more of the above methods to factor simple third-degree polynomials.
5. Students will demonstrate knowledge of factoring the difference of two cubes.
6. Students will demonstrate knowledge of factoring the sum of two cubes.

Necessary New Physical Skills

None

Assessable Results of the Standard

1. Students will completely factor polynomials that include more than one of the following: common monomial, a non-prime trinomial, the difference of squares, a perfect square trinomial, a difference of cubes, or the sum of cubes.

Standard #4 Model Assessment Items

Computational and Procedural Skills

1. Factor completely:

A. $121m^2 - 25$

B. $16z^2 + 24z + 9$

C. $4x^2 - 20x + 25$

D. $8c^3 + 27a^3$

E. $125y^3 - 27$

F. $54x^6y - 2y$

G. $3x^4 + 3x^3 - 3x - 3$

H. $5x^4 - 5$

I. $x^4 - x^3 + 8x - 8$

J. $8x^3 - 8$

Conceptual Understanding

1. How can you check your answer when you factor a polynomial?
2. How could you use factoring to convince someone that $x^3 + y^3 \neq (x + y)^3$?
3. How could you use factoring to convince someone that $x^2 + y^2 \neq (x + y)^2$?
4. Given: $(x^3 + 8) = (x + 2)(x^2 + 2x + 4)$. Is this true? Explain.

Problem Solving/Application

1. Given a square sheet of posterboard with 3 inch squares cut from each corner used to form a box, find the polynomial expression that represents the volume of the box.
2. Find *a* so that $ay^2 - 12y + 4 = (3y - 2)^2$.
3. The volume of a rectangular prism is $2x^3 + 3x^2 - 8x - 12$. The length of the box is given by $2x + 3$ and the width is given by $x + 2$. Find the expression representing the height.

Standard #5

Standard Set 5.0
Students demonstrate knowledge of how real and complex numbers are related both arithmetically and graphically. In particular, they can plot complex numbers as points in the plane.

Deconstructed Standard
1. Students demonstrate knowledge of how real numbers and complex numbers are related arithmetically.
2. Students demonstrate knowledge of how real numbers and complex numbers are related graphically.
3. Students can plot complex numbers as points on the complex plane.

Prior Knowledge Necessary
Students should know how to:
- find the absolute values of numerical expressions.
- illustrate the definition of absolute value geometrically.
- use the distance formula to determine the distance between two points on a plane.
- identify the relationships among the subsets of the real number system.
- plot points on a coordinate plane.
- simplify radical expressions.

New Knowledge
Students will need to learn to:
- identify the real part and imaginary part of a complex number.
- identify the real axis and imaginary axis of a complex plane.
- graph a complex number on a complex plane.
- differentiate between a real number, imaginary number, and a complex number.
- recognize the square root of a negative number as an imaginary number.
- identify the square root of –1 as i ($i = \sqrt{-1}$.)
- identify the square of i as –1 ($i^2 = -1$).
- simplify the square root of a negative number.
- calculate the absolute value of a complex number.
- find the conjugate of a complex number.

Categorization of Educational Outcomes
Competence Level: Knowledge
1. Students will identify the real and imaginary parts of a complex number.

Competence Level: Application
1. Students will demonstrate their ability to graph a complex number in a complex plane.
2. Students will demonstrate their ability to work with imaginary numbers when computing square roots of negative numbers.

3. Students will simplify square roots of negative numbers.

Competence Level: Analysis

1. Students will explain the process for finding the conjugate of a complex number.

Necessary New Physical Skills

None

Assessable Result of the Standard

1. Students will determine the ordered pairs representing real and imaginary parts of complex numbers.
2. Students will plot the graph of a complex number on a complex plane.

Standard #5 Model Assessment Items

Computational and Procedural Skills

1. Identify the real and complex parts of each:
   A. $-3+6i$

   B. $7i$

   C. 12

2. Simplify:
   A. $-\sqrt{-25}$

   B. $\sqrt{-24}$

3. Calculate the absolute value of the following
   A. $1+2i$

   B. $3i$ D. $-4-5i$

   C. $-2+3i$ E. $3-5i$

4. Write the conjugate of each complex number:
   A. 6 C. $2-5i$ E. $4+8i$

   B. $4i$ D. $-6-3i$ F. $-5+9i$

Conceptual Understanding

1. Determine the conjugate of each term. Graph each number and its conjugate in the complex plane. Explain how they are the same and how they are different.
   A. 2 D. $-4i$ G. $4-6i$

   B. -3 E. $3+6i$ H. $-5-i$

   C. $2i$ F. $-2+3i$

2. Sketch a diagram that shows the absolute value of the following:
   A. $1+2i$ D. $-4-5i$

   B. $3i$ E. $3-5i$

   C. $-2+3i$

3. From the standard form of a complex number, $a+bi$, explain what a and b represent.

Standard #6

Standard Set 6.0
Students add, subtract, multiply, and divide complex numbers.

Deconstructed Standard
1. Students add complex numbers.
2. Students subtract complex numbers.
3. Students multiply complex numbers.
4. Students divide complex numbers.

Prior Knowledge Necessary
Students should know how to:
- add and subtract algebraic expressions involving variables.
- use the distributive property in algebraic expressions involving variables.
- simplify algebraic expressions involving positive integer exponents.
- simplify square roots of integers.
- multiply binomials.
- rationalize a monomial or binomial denominator.
- identify conjugates and calculate their products.
- multiply two numbers with exponents having the same base.

New Knowledge
Students will need to learn to:
- calculate the sum of two complex numbers.
- calculate the difference of two complex numbers.
- calculate the product of two complex numbers.
- calculate the quotient resulting from division of two complex numbers.
- calculate powers of *i* (e.g.,) i^2, i^3, i^4.
- use $i = \sqrt{-1}$ and $i^2 = -1$ to compute higher powers of *i*.
- use $i^2 = -1$ to further simplify complex numbers.
- write complex numbers in standard form: $a + bi$.
- use conjugates of complex numbers to rationalize a denominator that is a complex number.

Categorization of Educational Outcomes
Competence Level: Application
1. Students will use methods they have learned to add, subtract, multiply, and divide complex numbers.
2. Students will show that they can further simplify complex numbers involving *i* raised to higher powers.

Necessary New Physical Skills
None

<u>Assessable Result of the Standard</u>

1. Students will produce a simplified complex number in standard form.

Standard #6 Model Assessment Items

Computational and Procedural Skills

1. Simplify using imaginary numbers:
    A. i^{57}

2. Simplify the expression:
    A. $(5i)^2$
    B. $i\left(\sqrt{-7}\right)^2$

3. Perform the indicated operation:
    A. $(8+2i)+(4-i)$
    B. $(6-3i)-(-3+i)$
    C. $(-9-i)(3+5i)$
    D. $i(7-3i)(2+10i)$
    E. $(8+6i)-2i^2(2-7i)$
    F. $\dfrac{2i(1+4i)}{6i-10i^2}$

Conceptual Understanding

1. Compute and record the first 12 powers of an imaginary unit, *i*. What pattern is emerging?

2. Use the above pattern to find: i^{142}

3. Which properties of real numbers also hold for complex numbers?

Problem Solving/Application

1. Fractals: Complex numbers are usually symbolized by the variable, *z*. Let $f(z)=4iz$ represent a complex function. Beginning with the initial value $z=2+5i$, determine the first two iterations of the function (hint: substitute $2+5i$ for *z* in the initial function. Then take the result and plug it into the initial function again).

2. Engineering: In an electrical circuit, the voltage *E* in volts, the current *I* in amps, and the opposition to the flow of current, called impedance (*Z* in ohms), are related by the equation $E=IZ$. A circuit has a current of $3+6i$ amps and an impedance of $-2+4i$ ohms. Determine the voltage and $|Z|$, the magnitude of the impedance.

Standard #7

<u>Standard Set 7.0</u>

Students add, subtract, multiply, reduce, and evaluate rational expressions with monomial and polynomial denominators and simplify complicated rational expressions, including those with negative exponents in the denominator.

<u>Deconstructed Standard</u>

1. Students add rational expressions with monomial and polynomial denominators.
2. Students subtract rational expressions with monomial and polynomial denominators.
3. Students multiply rational expressions with monomial and polynomial denominators.
4. Students reduce rational expressions with monomial and polynomial denominators.
5. Students evaluate rational expressions with monomial and polynomial denominators.
6. Students simplify complicated rational expressions, including those with negative exponents in the denominator.

<u>Prior Knowledge Necessary</u>

Students should have the computational and conceptual knowledge outlined in Standards #12 and #13 for Algebra I.

Students should know how to:

- use the properties of exponents.
- add, subtract, multiply, divide, and simplify rational expressions.
- add, subtract, multiply, and divide polynomials.
- factor polynomials.
- identify when a rational expression is undefined.
- simplify a rational expression by canceling common factors in the numerator and the denominator.
- multiply and divide rational expressions with monomial and polynomial denominators.
- find the LCD between two or more monomial or polynomial denominators.
- add and subtract rational expressions.
- identify complex rational expressions.

<u>New Knowledge</u>

This standard is an extension of Standards #12 and #13 in Algebra I. The following knowledge reflects the extension beyond Standards #12 and #13 in Algebra I.

Students will need to learn to:

- simplify a complex rational expression by writing its numerator and its denominator as single fractions and then dividing by multiplying with the reciprocal of the denominator.
- simplify complex rational expressions by multiplying the numerator and the denominator by the LCD of the numerator and the denominator.
- simplify rational expressions involving variables with negative exponents.

- evaluate application problems involving rational expressions.
- solve equations involving rational expressions, paying particular attention to values for which the rational equation is undefined (restrictions on *x*).
- solve application problems involving rational equations (e.g., number problems, work problems, distance/rate/time problems).

Categorization of Educational Outcomes

Competence Level: Knowledge:

1. Students will identify complex rational expressions.

Competence Level: Comprehension:

1. Students will understand and distinguish between the different uses of the Least Common Denominator (LCD) in simplifying rational expressions and solving rational equations.
2. Students will understand and differentiate between the two methods of simplifying complex rational expressions.

Competence Level: Application

1. Students will use the different operations on rational expressions (adding, subtracting, multiplying, and dividing) to solve rational equations.
2. Students will use methods of solving rational equations to solve application problems involving rational expressions.

Necessary New Physical Skills

None

Assessable Result of the Standard

1. Students will state the sum, difference, product, and quotient of rational expressions in simplified form.
2. Students will find solutions to rational equations and application problems involving rational equations.

Standard #7 Model Assessment Items

<u>Computational and Procedural Skills</u>

1. Simplify the following:

A. $\dfrac{1+\dfrac{3}{x}}{2-\dfrac{5}{x^2}}$

B. $\dfrac{\dfrac{1}{x+1}+\dfrac{1}{x-1}}{\dfrac{x}{x+1}}$

C. $\dfrac{x^{-1}+y^{-1}}{\dfrac{x^2-y^2}{xy}}$

2. Perform the indicated operation(s) and simplify the result:

A. $\dfrac{x^2-3x}{4x^2-8x}\cdot\left(4x^2-16\right)$

D. $\dfrac{1}{4x^2y^3}+\dfrac{5}{6xy^5}$

B. $\dfrac{6x+18}{3x-5}\div\dfrac{x^2-9}{x^2-25}$

E. $\dfrac{4}{x^2-5x+4}-\dfrac{5}{x^2-1}$

C. $\dfrac{2a^2b^4c^{-3}}{\left(a^{-2}b^3\right)^2}\cdot\dfrac{\left(2a^{-3}b\right)^3}{3a^2c^{-4}}$

F. $\dfrac{x^2}{x-2}+\dfrac{4}{2-x}$

3. Evaluate the following rational expressions:

A. $\dfrac{3x^4y}{4xy^3}$, where $x=-2$ and $y=3$.

B. $\dfrac{12-4x}{x+6}$, where $x=3$.

C. $\dfrac{4}{x^2-5x+4}$, where $x=1$.

4. Solve the following rational equations:

A. $\dfrac{2}{m+5}+\dfrac{1}{m-5}=\dfrac{16}{m^2-25}$

B. $\dfrac{x-1}{x-5}=\dfrac{4}{x-5}$

Conceptual Understanding

1. Is the sum of two rational expressions always a rational expression?

2. Kevin incorrectly simplifies $\frac{x+3}{x}$ as $\frac{x+3}{x}=\frac{\cancel{x}+3}{\cancel{x}}=1+3=4$. He insists that this is correct because when he replaces *x* by 1, he gets 4. What is he doing incorrectly and what can you do to convince him that this simplification is incorrect?

3. Kyle is adding two rational expressions with unlike denominators as follows:

$$\frac{3}{x+1}-\frac{4}{x}$$
$$\Rightarrow\frac{3}{x+1}\cdot\frac{x}{x}-\frac{4}{x}\cdot\frac{(x+1)}{(x+1)}$$
$$\Rightarrow\frac{3\cancel{x}}{\cancel{x}(x+1)}-\frac{4\cancel{(x+1)}}{x\cancel{(x+1)}}$$

   What is wrong with the last step? What should he do at this step?

Problem Solving/Application

1. The reciprocal of 3 plus the reciprocal of 6 is the reciprocal of what number?

2. A local bus travels 7 mph slower than the express. The express travels 45 miles in the time it takes the local to travel 38 miles. Find the speed of each bus.

3. Ferdinand can deliver papers 3 times as fast as Ronnel can. If they work together, it takes them 1 hour. How long would it take each to deliver the papers alone?

4. Eileen’s bathtub can be filled in 10 minutes and drained in 8 minutes. How long will it take to empty a full tub if the water is left on?

Standard #8

Standard Set 8.0
Students solve and graph quadratic equations by factoring, completing the square, or using the quadratic formula. Students apply these techniques in solving word problems. They also solve quadratic equations in the complex number system.

Deconstructed Standard
1. Students solve quadratic equations by factoring.
2. Students solve quadratic equations by completing the square.
3. Students graph quadratic equations by completing the square to find the roots.
4. Students solve quadratic equations by using the quadratic formula.
5. Students solve word problems using the above techniques.
6. Students solve quadratic equations with complex number solutions.
7. Students graph quadratic functions using the roots found by the techniques indicated above

Prior Knowledge Necessary
Students should have the computational and conceptual knowledge outlined in Standards #14, 21, and 22 from Algebra 1.

Students should know how to:
- factor quadratic expressions.
- simplify radicals.
- solve a quadratic equation using factoring, completing the square or the quadratic formula.
- simplify a radical when the radicand is negative.
- find the roots of an equation.
- find the *x*- and *y*-intercepts of an equation.
- find the vertex of a parabola.
- determine if a parabola will open up or down.

New Knowledge
This standard is an extension of Standards #14, 21, and 22 from Algebra 1.

Students will need to learn to:
- solve quadratic equations by completing the square where $a \neq 1$.
- find imaginary roots.
- recognize that imaginary roots come in conjugate pairs and find them.
- convert quadratic functions in the form $f(x) = ax^2 + bx + c$ into vertex form [$f(x) = a(x-h)^2 + k$, where (h,k) is the vertex].

Categorization of Educational Outcomes

Competence Level: Application

1. Students will use methods they have learned to solve quadratic equations.
2. Students will use methods they have learned to graph quadratic equations/functions, and/or identify the x- and y-intercepts and the vertex for a given quadratic equations/functions.
3. Students will show they know the correct interpretation of x- and y-intercepts and the vertex of quadratics by solving word problems involving quadratics.

Necessary New Physical Skills

None

Assessable Result of the Standard

1. Students will solve a quadratic equation.
2. Students will produce the graph of a quadratic equation/function.
3. Students will solve a variety of word problems requiring the solving of quadratic equations.

Standard #8 Model Assessment Items

Computational and Procedural Skills

1. Solve the following equations by using the quadratic formula:

   A. $2x^2 - 7x = -15$ B. $3x^2 + 2x - 6 = 0$ C. $2x^2 + 4x + 3 = 0$

2. Solve the following equations by completing the square:

   A. $x^2 - 6x + 2 = 0$ B. $2x^2 = 8x - 1$ C. $x^2 - 5x + 2 = 0$

3. Given that $2 + 3i$ is a root of an equation, what is the other root?

4. Graph each of the following using a minimum of five points and identify the vertex, the x-intercepts and the *y*-intercept

   A. $y = x^2 - 2x + 3$

   B. $f(x) = (x + 3)^2 - 4$

Conceptual Understanding

1. Given the graph of a quadratic, identify the roots, the *y*-intercept, the vertex, and the axis of symmetry.

2. Sketch the graph of a quadratic that has imaginary roots.

3. Sketch the graph of a quadratic that has a double root.

4. Find a quadratic equation with 2 and –3 as roots.

Problem Solving/Application

1. A rectangle is twice as long as it is wide. If the length and width are both increased by 5 cm, the resulting rectangle has an area of 50 cm^2. Find the dimensions of the original rectangle.

2. A rectangular field with area 5,000 m^2 is enclosed by 300 m of fencing. Find the dimensions of the field.

3. A swimming pool 6 m wide and 10 m long is to be surrounded by a walk of uniform width. The area of the walk happens to equal the area of the pool. What is the width of the walk?

4. A ball is thrown vertically upward with an initial speed of 48 ft/second. Its height, in feet, after *t* seconds is given by $h = 48t - 16t^2$. When will the ball hit the ground?

Standard #9

Standard Set 9.0

Students demonstrate and explain the effect that changing a coefficient has on the graph of quadratic functions; that is, students can determine how the graph of a parabola changes as *a, b,* and *c* vary in the equation $y = a(x-b)^2 + c$.

Deconstructed Standard

Despite the fact that the standard is written $y = a(x - b)^2 + c$, the customary notation for the vertex form is given by $y = a(x - h)^2 + k$ where (h, k) is the vertex.

1. Students explain how the value of *a* affects the graph of a parabola.
2. Students explain how the value of *h* affects the graph of a parabola.
3. Students explain how the value of *k* affects the graph of a parabola.

Prior Knowledge Necessary

Students should know how to:

- recognize that the graph of any quadratic equation is a parabola.
- recognize what the graph of $y = x^2$ looks like.

New Knowledge

Students will need to learn to:

- identify how changing the value of *a* will affect the graph of $y = a(x-h)^2 + k$.
- identify how changing the value of *h* will affect the graph of $y = a(x-h)^2 + k$.
- identify how changing the value of *k* will affect the graph of $y = a(x-h)^2 + k$.

Categorization of Educational Outcomes

Competence Level: Application

1. Students will use the methods they have learned to explain how the graph of $y = a(x-h)^2 + k$ is obtained from the graph of $y = x^2$.

Necessary New Physical Skills

None

Assessable Result of the Standard

1. Students will explain how the *a*, *h,* and *k* values affect the graph of $y = x^2$.
2. Given an equation, students will demonstrate that they can match the appropriate graph to its equation.

Standard #9 Model Assessment Items

Computational and Procedural Skills

1. Explain how the graph of $y = \frac{1}{2}(x - 2)^2 + 1$ is obtained from the graph of $y = x^2$.

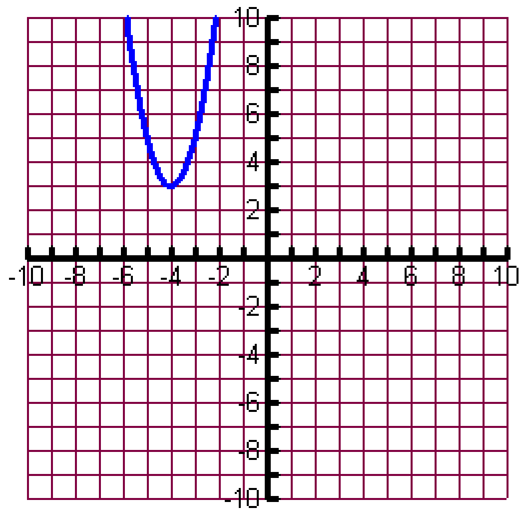
2. Explain how the graph of $y = (x-2)^2 + 1$ is obtained from the graph of $y = x^2$.

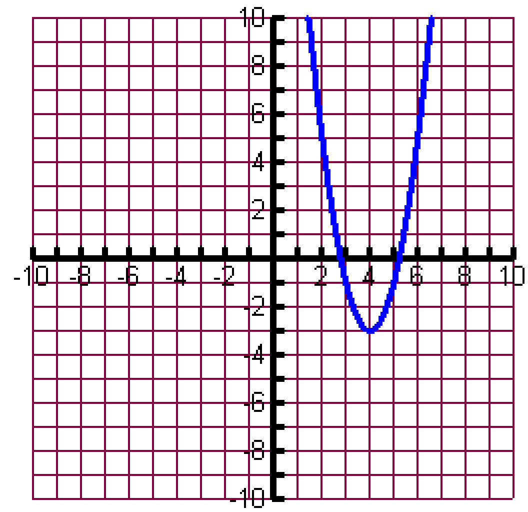
3. Explain how the graph of $y = (x+2)^2 - 1$ is obtained from the graph of $y = x^2$.

4. Explain how the graph of $y = -3(x+4)^2 + 3$ is obtained from the graph of $y = x^2$.

Conceptual Understanding

1. Given the 2 graphs shown below, determine which is the graph of $y = 2(x + 4)^2 + 3$. Explain.





2. Compare the vertices and shapes of the graphs for the following pairs of functions:

$f(x) = 2(x-3)^2 + 4$ and $g(x) = 2(x+3)^2 + 4$

$f(x) = 2(x-3)^2 + 4$ and $g(x) = -2(x-3)^2 + 4$

$f(x) = 2(x+3)^2 + 4$ and $g(x) = 2(x+3)^2 - 4$

Problem Solving/Application

None

Standard #10

Standard Set 10.0
Students graph quadratic functions and determine the maxima, minima, and zeros of the function.

Deconstructed Standard
1. Students graph quadratic functions.
2. Students determine the maximum or minimum of a quadratic function from its graph.
3. Students determine the zeros of a quadratic function from its graph.

Prior Knowledge Necessary
Students should know how to:
- solve a quadratic equation by factoring or completing the square.
- graph quadratic functions and know their roots are the x-intercepts.
- identify the zeroes of a function as the x-intercepts of the graph.
- recognize the relationship between a quadratic in vertex form and its graph.
- find the zeros from the graph of a quadratic function.

New Knowledge
Students will need to learn to:
- recognize when the graph of a quadratic has no real roots.
- find the maximum or minimum of the graph of a quadratic function.

Categorization of Educational Outcomes
Competence Level: Knowledge:
1. Students will identify the zeros of the graph of a quadratic function
2. Students will identify the maximum or minimum point of the graph of a quadratic function.
3. Students will show that the vertex of the graph of a quadratic equation/function is the maximum or minimum point of the graph.

Competence Level: Application
1. Students will use the graph of a quadratic to estimate the zeros of the function.
2. Students will use the graph of a quadratic to estimate the maximum or minimum of the function.
3. Students will demonstrate knowledge of the relationship between the x-intercepts of the graph of a quadratic function and the zeros of that function.
4. Students will demonstrate knowledge of the relationship between the vertex of the graph of a quadratic function and the maximum or minimum point of the function.

Necessary New Physical Skills
1. Graphing a quadratic function using paper and pencil and/or using graphing calculators.

Assessable Result of the Standard

1. Students will identify (find) the zeros of the graph of a quadratic function.
2. Students will identify the maximum or minimum of the graph of a quadratic function.

Standard #10 Model Assessment Items

Computational and Procedural Skills

1. Find the zeros and the maximum or minimum point of the function given by:
   A. $f(x) = x^2 - 11x + 28$

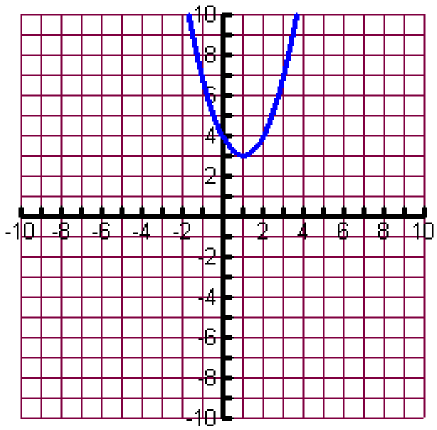
   B. $f(x) = (x-1)(x-4) - 10$

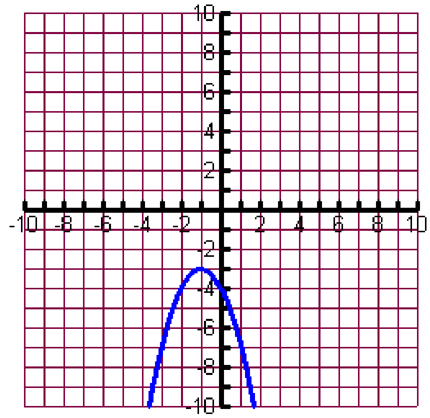
   C. $f(x) = -2x^2 + 11x + 40$

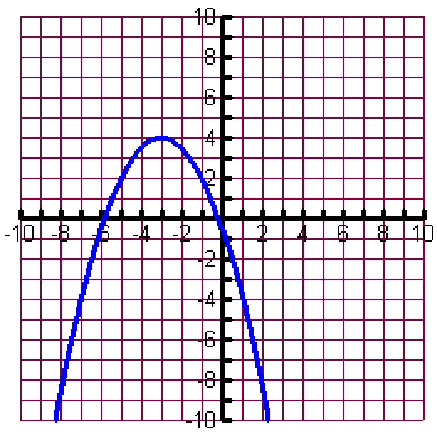
2. Graph $y = -2(x-1)^2 + 1$
   A. Identify the vertex. Is it a maximum or minimum point?

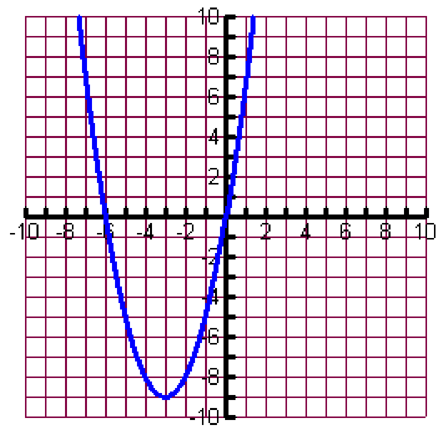
   B. Identify the *x*-intercepts.

3. For the following graphs, estimate the maximum or minimum value.









Conceptual Understanding

1. What do the zeros of the function represent?
2. What is the relationship between the maximum or minimum point and the vertex of the graph of a quadratic function?

Problem Solving/Application

1. To celebrate a town's centennial, fireworks are launched over a lake off a dam 36 ft above the water. The height of a display, *t* seconds after it has been launched, is given by $h(t) = -16t^2 + 64t + 36$. After how long will the shell from the fireworks reach the water? What is the maximum height the shell will obtain?

2. A ball is thrown vertically upward with an initial speed of 48 ft/second. Its height, in feet, after *t* seconds is given by $h\left(t\right) = 48t - 16t^2$. What is the maximum height of the ball?

Standard #11

Standard Set 11.0
11.0 Students prove simple laws of logarithms.

11.1 ***(items in grey are directly associated with 11.1)*****:** Students understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

11.2: Students judge the validity of an argument according to whether the properties of real numbers, exponents, and logarithms have been applied correctly at each step.

Deconstructed Standard
1. Students understand the inverse relationships between exponential and logarithmic functions.
2. Students use the exponential and logarithmic operations to solve problems.
3. Students use properties of real numbers to judge the validity of an argument.
4. Students use properties of exponents to judge the validity of an argument.
5. Students use properties of logarithms to judge the validity of an argument.

Prior Knowledge Necessary
Students should know how to:
- recognize inverse functions and their properties.*
- use properties of exponents. **
- use properties of real numbers.

New Knowledge
Students will need to learn to:
- translate between exponential and logarithmic notation utilizing the definition of logarithms (If $b > 0$ and $b \neq 1$, then $y = \log_b x$ means $x = b^y$, for every $x > 0$ and every real number *y*.
- use the Logarithm Property of Equality to find solutions to problems.
- identify the appropriate property of logarithms to utilize in a problem:
  If *x*, *y*, and *b* are positive real numbers, $b \neq 1$, and *r* is a real number, then
  1. $\log_b 1 = 0$ 4. Product property : $\log_b xy = \log_b x + \log_b y$
  2. $\log_b b^x = x$ 5. Quotient property :$\log_b \dfrac{x}{y} = \log_b x - \log_b y$
  3. $b^{\log_b x} = x$ 6. Power property: $\log_b x^r = r \log_b x$

* see Standard 24, Algebra II
** see Standard 12, Algebra II

Categorization of Educational Outcomes

Competence Level: Knowledge

1. Students will recall properties of exponents.
2. Students will recall properties of logarithms.

Competence Level: Comprehension

1. Students will convert between logarithmic and exponential notation.

Competence Level: Application

1. Students will apply properties of exponents to solve problems.
2. Students will apply properties of logarithms to solve problems.

Necessary New Physical Skills

None

Assessable Result of the Standard

1. Students will find solutions to problems by converting between exponential and logarithmic notation.
2. Students will find solutions to problems involving properties of exponents and logarithms.
3. Students will use properties of real numbers, exponents, and logarithms to judge the validity of an argument.

Standard #11 Model Assessment Items

Computational and Procedural Skills

1. Rewrite the logarithmic equation into exponential form: $\log_2 8 = 3$.

2. Rewrite the exponential equation into logarithmic form: $3^2 = 9$.

3. Utilize the properties of exponentials and logarithms to evaluate each of the following:
   A. $\log_7 1$

   B. $\log_5 5^3$

   C. $4^{\log_4 3}$

4. Use the properties of logarithms to write each expression as a single logarithm:
   A. $\log 12 + \log 3$

   B. $\log 15 - \log 3$

   C. $2\log_3 5 + \log_3 2$

   D. $3\log_5 x + 4\log_5 x - 2\log_5 (x+6)$

5. Use the properties of logarithms to match expressions on the left with equivalent expressions on the right.

$\log(3x)$	$\log 3 + \log x$
$\log\left(\dfrac{3}{x}\right)$	$3\log x$
$\log x^3$	$(\log x)(\log 3)$
$\log\left(\dfrac{x}{3}\right)$	$\log 3 - \log x$
	$\log x - \log 3$

6. Solve:
   A. $\log_3 (x+5) = 4$

   B. $4^{x+6} = 3$

Conceptual Understanding

1. Use a table of values to graph the function $f(x) = 2^x$, and identify the graph and corresponding table of values for its inverse.

2. Can the logarithm of a negative value be calculated?

3. Is the following true or false? Why?
   $(\log_3 6)(\log_3 4) = \log_3 24$

4. It is true that $\log 7 = \log(7 * 1) = \log 7 + \log 1$?
   Explain how $\log 7$ can equal $\log 7 + \log 1$.

5. Answer the following True or False. If False, explain in which way(s) the equality is untrue. $\dfrac{\log_7 10}{\log_7 5} = \log_7 2$

Problem Solving/Application

1. Graph each function and its inverse function on the same set of axes. Label any intercepts.

   A. $y = 3^x; y = \log_3 x$

   B. $y = \left(\dfrac{1}{3}\right)^x; y = \log_{1/3} x$

Standard #12

Standard Set 12.0
Students know the laws of fractional exponents, understand exponential functions, and use these functions in problems involving exponential growth and decay.

Deconstructed Standard
1. Students know the properties of exponents and how to apply them to fractional exponents.
2. Students understand exponential functions.
3. Students solve problems involving exponential growth.
4. Students solve problems involving exponential decay.

Prior Knowledge Necessary
Students should know how to:
- apply the properties of exponents.
- use the basic notation of fractional exponents.
- simplify radicals.

New Knowledge
Students will need to learn to:
- apply exponential properties to fractional exponents.
- recognize properties of exponential functions and their graphs.
- apply exponential functions to growth and decay problems.

Necessary New Physical Skills
None

Categorization of Educational Outcomes
Competence Level: Knowledge
1. Students will identify exponential functions.
2. Students will identify exponential functions as representations of growth or decay.
3. Students will graph exponential functions including intercepts and asymptotes.

Competence Level: Application
1. Students will apply properties of fractional exponents to simplify expressions.
2. Students will evaluate expressions involving exponential growth and decay.

Assessable Result of the Standard
1. Students will simplify expressions with fractional exponents.
2. Students will find solutions for problems involving exponential growth or decay functions.

Standard #12 Model Assessment Items

Computational and Procedural Skills

1. Evaluate the following expressions:

   A. $(-27)^{2/3}$

   B. $16^{-3/4}$

2. Use properties of exponents to simplify $\dfrac{\left(2x^{2/5}y^{-1/3}\right)^5}{x^2y}$

Conceptual Understanding

1. Explain what happens if a negative value is raised to a rational exponent in the case of an even denominator and an odd denominator.

2. Graph $y = 2^x$ using a table of values.

3. Using the graph of $y = 2^x$, in #2, identify any intercepts, asymptote(s), and determine if this is a graph of a function.

Problem Solving/Application

1. Basal metabolic rate (BMR) is the number of calories per day a person needs to maintain life. A person's basal metabolic rate $B(w)$ in calories per day can be estimated with the function $B(w) = 70w^{3/4}$, where w is the person's weight in kilograms. Use this information to calculate the BMR for a person who weighs 81 kilograms.

2. 50 grams of radioactive material has been found in a local pond and has led to the presence of radioactive debris decaying at a rate of 6% each week. Find how much debris still remains after 10 weeks. Use $\mathrm{y} = 50(2.7)^{-0.06\mathrm{t}}$ where *t* represents the number of weeks since the find.

Standard #13

Standard Set 13.0

Students use the definition of logarithms to translate between logarithms in any base.

Deconstructed Standard

1. Students use the definition of logarithms to translate between logarithms in any base including base *e* and base 10.

Prior Knowledge Necessary

Students should know how to:

- apply the properties of logarithms.
- apply the properties of exponents.

New Knowledge

Students will need to learn to:

- recognize the definition of *e*.
- use the definition of common and natural logarithms
- use the definition of Change of Base:
  If *a*, *b*, and *c* are positive real numbers and neither *b* nor *c* is 1, then
  $$\log_b a = \frac{\log_c a}{\log_c b}$$

Necessary New Physical Skills

None

Categorization of Educational Outcomes

Competence Level: Knowledge

1. Students will recognize problems requiring change of base to complete.
2. Students will recognize when to use the natural or common base.

Assessable Result of the Standard

1. Students will calculate estimated solutions for logarithms of varied bases.

Standard #13 Model Assessment Items

Computational and Procedural Skills

1. Change $\log_2 5$ into both natural and common logarithms.

2. Approximate each logarithm to four decimal places, by translating into log base ten:
   A. $\log_2 3$

   B. $\log_3 2$

3. Approximate each logarithm to four decimal places, by translating into the log base *e*:
   A. $\log_2 3$

   B. $\log_3 2$

Conceptual Understanding

1. Without using a calculator, explain which of $\log 40$ or $\ln 40$ must be larger.

Problem Solving/Application

1. The formula $R = \log\left(\frac{a}{T}\right) + 2.1$ is used to find the intensity *R* on the Richter scale of an earthquake where *a* is the amplitude in micrometers and *T* is the time between waves in seconds. Determine the intensity of an earthquake with an amplitude of 300 micrometers and 2.3 seconds between waves.

Standard #14

Standard Set

Students understand and use the properties of logarithms to simplify logarithmic numeric expressions and to identify their approximate values.

Deconstructed Standard

1. Students know the properties of logarithms.
2. Students use the properties of logarithms to simplify logarithmic numeric expressions.
3. Students use the definitions and properties of logarithms to-determine their approximate values.

Prior Knowledge Necessary

Students should know how to:

- apply the definition/properties of logarithms.
- apply the laws of exponents.
- factor numbers into their prime factorizations with appropriate exponentiation.
- recognize that a single logarithmic expression can be expanded into an equivalent expression.
- simplify or expand logarithmic expressions.

New Knowledge

Students will need to learn to:

- substitute known (given) values of logarithms into logarithmic expressions to get an approximate numeric answer.
- calculate the exact numeric value of a logarithmic expression where possible.

Categorization of Educational Outcomes

Competence Level: Knowledge:

1. Students will identify the exponential equivalent of a logarithmic expression in order to simplify the logarithmic expression.

Competence Level: Application

1. Students will calculate the approximate value of a more complex logarithmic numeric expression by substituting known values of simple logarithmic numeric expressions.

Necessary New Physical Skills

None

Assessable Result of the Standard

1. Students will approximate values of a logarithmic numeric expression where necessary.
2. Students will calculate the exact value of a logarithmic expression where possible.

Standard #14 Model Assessment Items

Computational and Procedural Skills

1. What is the value of $\log_3 27$?

2. What is the value of $\log_2 64$?

3. What is the value of $\log 1000$?

4. What is the value of $\log 0.00001$?

5. If $\log 2 \approx 0.301$ and $\log 3 \approx 0.477$, what is the approximate value of $\log 72$?

6. If $\log 2 \approx 0.301$ and $\log 3 \approx 0.477$, what is the approximate value of $\log_6 81$?

Conceptual Understanding

1. Using a calculator, James incorrectly says that $\log 81$ is between 4 and 5. How could you convince him, without using a calculator, that he is mistaken?

Problem Solving/Application

1. The measure of loudness L, in decibels (dB), of a sound is given by the following formula: $L = 10 \cdot \log \dfrac{I}{I_0}$ where I is the intensity of the sound, in watts per square meter (W/m^2) and $I_0 = 10^{-12}$ W/m^2. (I_0 is approximately the intensity of the softest sound that can be heard by the human ear.)
   At a rock concert the intensity of the sound may reach $10^{-1.2}$. How loud is this in decibels?

2. The pH of a liquid is a measure of its acidity and is given by the formula: $pH = -\log\left[H^+\right]$, where H^+ is the hydrogen ion concentration in moles per liter.
   If the hydrogen ion concentration of sea water is approximately 5.01×10^{-9}, find the pH level.

Standard #15

Standard Set 15.0
Students determine whether a specific algebraic statement involving rational expressions, radical expressions, or logarithmic or exponential functions is sometimes true, always true, or never true.

Deconstructed Standard
1. Students determine whether a specific algebraic statement involving rational expressions is sometimes true, always true, or never true.
2. Students determine whether a specific algebraic statement involving radical expressions is sometimes true, always true, or never true.
3. Students determine whether a specific algebraic statement involving logarithmic or exponential functions is sometimes true, always true, or never true.

Prior Knowledge Necessary
Students should know how to:
- apply the properties of rational expressions and how to apply them.
- apply the properties of radical expressions and how to apply them.
- apply the properties of logarithmic expressions and how to apply them.
- apply the properties of exponential expressions and how to apply them.
- determine the restrictions on the domain of rational, radical, logarithmic, and exponential functions.

New Knowledge
Students will need to learn to:
- derive counter examples to demonstrate that a statement is false.
- apply definitions or properties to demonstrate that a statement is sometimes true.
- apply definitions or properties to demonstrate that a statement is true.

Categorization of Educational Outcomes
Competence Level: Application
1. Students demonstrate that a statement is true or false.
2. Students apply definitions and properties to draw valid conclusions.

Necessary New Physical Skills
None

Assessable Result of the Standard
1. Students will be able to synthesize and evaluate all prior knowledge to determine whether a specific algebraic statement involving rational expressions, radical expressions, or logarithmic or exponential functions is sometimes true, always true, or never true.

Standard #15 Model Assessment Items

Conceptual Understanding

1. Find counterexamples to demonstrate that each of the following statements are false:

   A. $\log\left(\mathrm{a}/\mathrm{b}\right)=\dfrac{\log \mathrm{a}}{\log \mathrm{b}}$ is false for all *a* and *b*.

   B. $x^{-m} = -x^{m}$

   C. $\dfrac{x^2+4x+5}{x^2+4x}=5$

   D. $\dfrac{-b+\sqrt{b^2-4ac}}{2a}=-b+\dfrac{\sqrt{b^2-4ac}}{2a}$

2. Find all values of *x* which make the following statements true:

   A. $\sqrt{x^2} = x$

   B. $|x| = x$

   C. $\sqrt{(x+5)^2} = x+5$

   D. $\sqrt{a}.\sqrt{b} = \sqrt{ab}$

3. Apply properties to demonstrate that the following statements are true:

   A. $\log MN = \log M + \log N$

   B. $\sqrt{x^2+y^2} \neq x+y$

Standard #16

Standard Set 16.0

Students demonstrate and explain how the geometry of the graph of a conic section (e.g., asymptotes, foci, eccentricity) depends on the coefficients of the quadratic equation representing it.

Deconstructed Standard

1. Students demonstrate how the asymptotes of the graph of a conic section depend on the coefficients of the quadratic equation representing it.
2. Students demonstrate how the foci of the graph of a conic section depend on the coefficients of the quadratic equation representing it.
3. Students demonstrate how the eccentricity of the graph of a conic section depends on the coefficient of the quadratic equation representing it.
4. Students explain how the asymptotes of the graph of a conic section depend on the coefficients of the quadratic equation representing it.
5. Students explain how the foci of the graph of a conic section depend on the coefficients of the quadratic equation representing it.
6. Students explain how the eccentricity of the graph of a conic section depends on the coefficients of the quadratic equation representing it.

Prior Knowledge Necessary

Students should know how to:

- perform arithmetic computations with rational numbers.
- graph ordered pairs.
- graph a linear equation.
- graph a quadratic equation.
- convert between general quadratic equation and standard form.
- interpret the concept of asymptote.
- draw a rectangle through four given ordered pairs.
- draw the diagonals through the corners of a rectangle.
- use the Pythagorean Theorem.
- use the distance formula.

New Knowledge

Students will learn:

- to analyze the coefficients of the quadratic equation representing a conic, $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, to determine the value of $b^2 - 4ac$, and then to identify the geometry of the conic as determined by the following conditions:
    - Circle: $b = 0$, $a = c$, and $b^2 - 4ac < 0$.
    - Ellipse: Either $b = 0$, or $a \neq c$ and $b^2 - 4ac < 0$.
    - Parabola: $b^2 - 4ac = 0$.
    - Hyperbola: $b^2 - 4ac > 0$

- to interpret the meaning of coefficient $B = 0$ to mean that the axes of the conic are either vertical or horizontal, and to demonstrate the geometry by graphing a conic with horizontal or vertical axes.
- to interpret the coefficients of a parabola's standard quadratic equation to formulate the equation of the directrix, to determine the focus and axis of symmetry, and to demonstrate the geometry by plotting the focus, drawing the axes of symmetry and directrix, and then using these to graph the parabola.
- to interpret the coefficients of a circle's standard quadratic equation to determine the center of the circle and its radius, and to demonstrate the geometry by plotting the center and then using the radius to graph the circle.
- to interpret the coefficients of an ellipse's standard quadratic equation to determine the lengths and orientation of the major and minor axes, the vertices and co-vertices, the foci, and to demonstrate the geometry by plotting the foci, the vertices and co-vertices, drawing the axes and then using these guides to graph the ellipse.
- to interpret the coefficients of a hyperbola's standard quadratic equation to determine the vertices, the foci, the equations of the asymptotes, the vertices and the orientation of the transverse axis, and to demonstrate the geometry by drawing the asymptotes and transverse axis and plotting the vertices and foci, and then using these guides to graph the hyperbola.
- to interpret the coefficients of the quadratic equation of a conic to determine the eccentricity of the conic (ellipse: $e = c/a$, and $0 < e < 1 0$; hyperbola $e = c/a$, and $e > 1$; parabola: $e = 1$; circle: $e = 0$).

Categorization of Educational Outcomes

Competence Level: Application and Analysis

1. Students will interpret coefficients to determine the geometry of a conic.
2. Students will calculate the discriminate to determine the geometry of the conic.
3. Students will calculate eccentricity ratios.
4. Students will analyze coefficients and graphs to identify that graph is correct and explain their reasoning.
5. Students will produce equations from graphs of conic sections.
6. Students will produce graphs from equations of conic sections.

Necessary New Physical Skills

None

Assessable Result of the Standard

1. Students will produce the graph of a circle, ellipse, parabola, or hyperbola as determined by the value of $b^2 - 4ac$ and the interpretation of coefficients of the quadratic equation of the conic.
2. Students will produce the equation and graph of the directrix, plot the focus, draw the axis of symmetry, and graph a parabola to demonstrate interpretation of coefficients of a quadratic equation.
3. Students will plot the center, draw the radius, and graph a circle to demonstrate interpretation of coefficients of a quadratic equation.

4. Students will produce a drawing of major and minor axes, plot vertices, co-vertices, and foci, and graph an ellipse to demonstrate interpretation of coefficients of a quadratic equation.
5. Students will produce a drawing of asymptotes and transverse axis, plot vertices and foci, and graph a hyperbola to demonstrate interpretation of coefficients of a quadratic equation.
6. Students will produce ratios that represent the eccentricity of a parabola, a circle, an ellipse, and a hyperbola.

Standard #16 Model Assessment Items

Computational and Procedural Skills

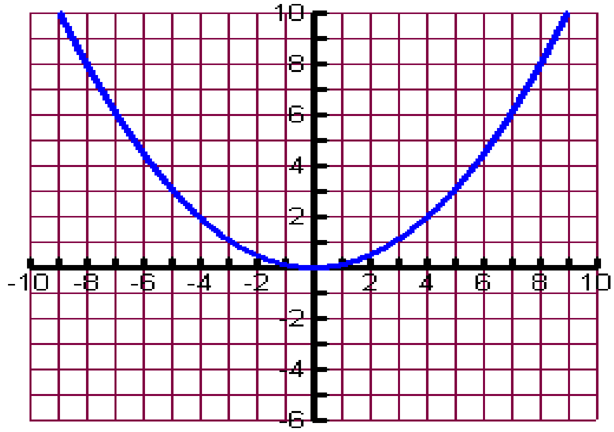
1. What is the graph of $x^2 + wy^2 - 4x + 10y - 26 = 0$ when $w = 1$? When $w = 4$? When $w = -4$?

2. Calculate the value of the determinant, $b^2 - 4ac$, for each of the following equations. What conic is represented by each quadratic equation?
   A. $x^2 + y^2 - 2x - 4y - 14 = 0$

   B. $4x^2 - 9y^2 + 32x - 144y - 548 = 0$

3. In the sketch below, determine the equation of the directrix. Draw the directrix and the axis of symmetry. Plot and label the focus.

$$y = \frac{x^2}{8}$$

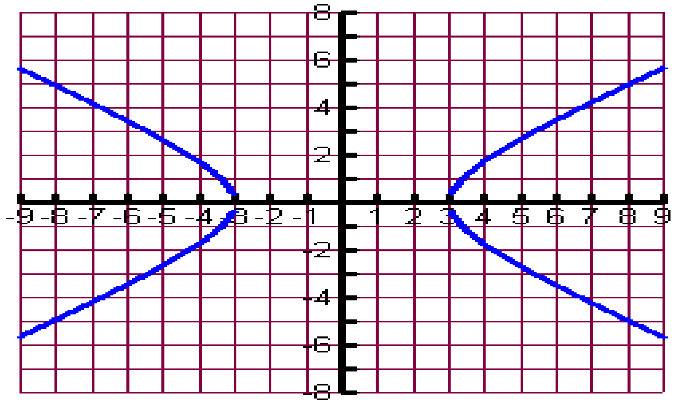


4. What is the eccentricity of a parabola? Of a circle?

5. Calculate the eccentricity of the conic described by the equation $25(x+2)^2 - 36(y-1)^2 = 900$.

6. In the sketch below, determine the equations of the asymptotes. Draw the asymptotes. Plot and label the center, the vertices, and the foci.

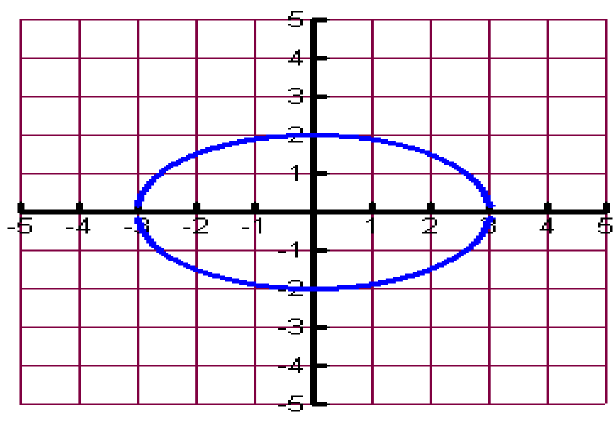
$$\frac{x^2}{9}-\frac{y^2}{4}=1$$



Conceptual Understanding

1. Given the conic sections $9x^2-4y^2-36=0$ and $-4x^2+9y^2-36=0$. Predict the behavior of the graph of the conic when the coefficients for x^2 and y^2 are interchanged.

2. John was asked to write an equation of the ellipse shown below. He wrote the following equation: $\frac{x^2}{4}+\frac{y^2}{9}=1$. Explain what John did wrong. What is the correct equation?



3. Determine the equation of the ellipse with vertex at (–4, 0) and focus (2, 0). Explain how to determine the equation. Sketch the graph of the ellipse represented by this equation.

4. Determine the equation of the hyperbola with center (3,–5), vertex (9, –5), and eccentricity equal to 2. Explain how to decide which coefficients to use in the equation.

5. Kyler and Erin were asked to use eccentricity to determine the type of conic represented by the quadratic equation $3x^2 - 5x + y + 20 = 0$. Kyler interpreted the coefficients of the equation and determined the graph was a parabola so the eccentricity must be 1. Aaron calculated the answer using $e = c/a = 0$ and decided the conic was a circle. Which student is correct? What did the other student do wrong?

6. Two students are given the following assignment by their teacher: A six-by-eight rectangle is centered at the origin. Sketch the rectangle and its diagonals. If you extend the diagonals, you will have the asymptotes for a hyperbola. Sketch the graph of the hyperbola and determine its equation. The next day, the two students returned with two different sketches and equations. Is it possible that they are both correct? Explain your reasoning.

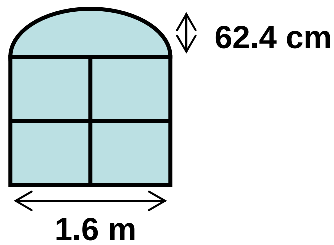
7. In the definition of eccentricity, the values of e for the conics are implied as 0 for a circle, 1 for a parabola, $0 < e < 1$ for an ellipse, and $e > 1$ for a hyperbola. Use the geometry of the conics to support how these values make sense.

8. Erin and Alexis are working together to solve a homework problem. The assignment requires that they solve for the two points of intersection of a line with the circle. The circle is centered at the origin and the line passes through the origin and intersects the circle. Erin solves a system of equations to determine one of the points of intersection. Alexis is not fond of solving systems of equations. How can she use geometry to determine the second point of intersection?

Problem Solving/Application

1. A portion of the White House lawn is called the Ellipse. It is 1,060 feet long and 890 feet wide. The First Lady has requested that the lawn be planted with a winter grass. In order to determine the amount of grass needed, the resident gardener needs to calculate the area of the ellipse. The resident gardener finds an Algebra 2 reference book in the White House Library where he reads that the area of an ellipse is given by Area = π ab where $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$. Does the gardener have enough information to solve for the area? If yes, explain and find the area. If not, explain what other information he needs.

2. The top of a window is designed so that it is half of an ellipse. The width of the window is 1.6 m and the height of the half-ellipse is 62.4 cm. A piece of tinted glass is to be cut to fit the half-ellipse. The workers want to trace the half-ellipse on a piece of cardboard to use as a pattern. They have two pins and a piece of string. Determine how far apart they should place the compass. Determine how long the tracing string should be.



Standard #17

Standard Set 17.0

Given a quadratic equation of the form $Ax^2 + By^2 + Cx + Dy + E = 0$, students can use the method for completing the square to put the equation into standard form and can recognize whether the graph of the equation is a circle, ellipse, parabola, or hyperbola. Students can then graph the equation.

Deconstructed Standard

1. Given a quadratic equation of the form $Ax^2 + By^2 + Cx + Dy + E = 0$, students can use the method for completing the square to put the equation into standard form.
2. Given a quadratic equation in standard form, students can recognize if the graph of the equation is a circle.
3. Given a quadratic equation in standard form, students can recognize if the graph of the equation is an ellipse.
4. Given a quadratic equation in standard form, students can recognize if the graph of the equation is a parabola.
5. Given a quadratic equation in standard form, students can recognize if the graph of the equation is a hyperbola.
6. Given a quadratic equation of a circle in standard form, students can graph the equation.
7. Given a quadratic equation of an ellipse in standard form, students can graph the equation.
8. Given a quadratic equation of a parabola in standard form, students can graph the equation.
9. Given a quadratic equation of a hyperbola in standard form, students can graph the equation.

Prior Knowledge Necessary

Students should know how to:

- calculate the slope of the line through two given points.
- write the equation of a line in slope-intercept form given a point and a slope.
- plot points on a coordinate plane.
- calculate the distance between the two given points.
- calculate the midpoint of two points.
- solve equations with a single variable.
- solve systems of equations with two variables.
- complete the square of a given polynomial.
- rewrite an equation in standard form.
- identify whether the graph opens up or down given a quadratic equation in the form $y = ax^2 + bx + c$.
- calculate the axis of symmetry of the graph of the quadratic equation $y = ax^2 + bx + c$.

- calculate the vertex of the graph the quadratic equation $y = ax^2 + bx + c$.
- identify the axis of symmetry and vertex given the graph of a parabola.

New Knowledge

Students will need to learn to:

- convert a quadratic equation into the standard form of a parabola with its vertex at the origin.
- convert a quadratic equation into the standard form of a parabola with its vertex not at the origin by completing the square.
- graph the parabola and identify the focus and directrix given the standard form of a parabola with its vertex at the origin.
- graph the parabola and identify the focus and directrix given the standard form of a parabola with its vertex not at the origin.
- convert a quadratic equation into the standard form of a circle with its center at the origin.
- convert a quadratic equation into the standard form of a circle with its center not at the origin by completing the square.
- graph the circle and identify the center and radius given the standard equation of a circle with its center at the origin.
- convert a quadratic equation into the standard equation of a circle with its center not at the origin by completing the square.
- graph the circle and identify the center and radius given the standard equation of a circle with its center not at the origin.
- convert a quadratic equation into the standard equation of an ellipse that is centered at the origin.
- convert a quadratic equation into the standard equation of an ellipse that is not centered at the origin by completing the square.
- graph the ellipse and identify the center, vertices, co-vertices, and foci given the standard equation of an ellipse that is centered at the origin.
- convert a quadratic equation into the standard equation of an ellipse that is centered at the origin.
- convert a quadratic equation into the standard equation of an ellipse that is not centered at the origin.

Categorization of Educational Outcomes

Competence Level: Knowledge

1. Students will identify the equations of conic sections.
2. Students will identify graphs of conic sections.
3. Students will convert an equation to standard form.
4. Students will identify the difference between a conic section centered at (0, 0) and not centered at (0, 0).

Competence Level: Application

1. Students will plot and label all important information for a circle, an ellipse, a parabola and a hyperbola.

Competence Level: Analysis

1. Students will explain how to write the equation of a conic section in standard form.

<u>Necessary New Physical Skills</u>

None

<u>Assessable Result of the Standard</u>

1. Students will produce the graph of a line.
2. Students will produce the graph of a parabola.
3. Students will produce the graph of a circle.
4. Students will produce the graph of an ellipse.
5. Students will produce the graph of a hyperbola.

Standard #17 Model Assessment Items

Computational and Procedural Skills

1. Identify the vertex, focus, and directrix of the parabola:
   A. $y = \frac{1}{4}x^2$

   B. $y - 4 = \frac{1}{2}(x-3)^2$

2. Write the parabola equation in standard form:
   A. $x^2 - 6x + 10y = 1$

   B. $2x + y^2 - 4y = 9$

3. Identify the center and radius of the circle equation:
   A. $x^2 + y^2 = 16$

   B. $(x+3)^2 + (y-1)^2 = 4$

4. Write the circle equation in standard form:
   A. $x^2 + y^2 + 4y = 12$

   B. $x^2 + 4x + y^2 + 4y = 8$

5. Find and identify the center, vertices, co-vertices, and foci of the ellipse equation:
   A. $\frac{x^2}{25} + \frac{y^2}{4} = 1$

   B. $\frac{(x-3)^2}{4} + \frac{(y-1)^2}{9} = 1$

   C. $\frac{(x-3)^2}{9} + \frac{(y-1)^2}{4} = 1$

6. Write the ellipse equation in standard form:
   A. $3x^2 + 12y^2 = 12$

   B. $x^2 + 4y^2 + 6x - 8y = 3$

7. Find and identify the center, vertices, co-vertices, foci, conjugate axis, transverse axis, and asymptotes of the hyperbola:
   A. $x^2 - y^2 = 1$

   B. $\dfrac{y^2}{3^2} - \dfrac{x^2}{4^2} = 1$

8. Write the hyperbola equation in standard form:
   A. $4x^2 - 9y^2 - 8x + 54y = 113$

   B. $y^2 - 9x^2 - 6y = 36 + 36x$

Conceptual Understanding

1. Graph each equation and identify the conic section:
   A. $y - 2 = \dfrac{1}{4}(x-5)^2$

   B. $\dfrac{(x-2)^2}{25} + \dfrac{(y-3)^2}{9} = 1$

   C. $x^2 + y^2 = 36$

   D. $y = 2x^2$

   E. $\dfrac{x^2}{16} + \dfrac{y^2}{49} = 1$

   F. $\dfrac{x^2}{49} + \dfrac{y^2}{16} = 1$

   G. $\dfrac{(x-4)^2}{25} - \dfrac{(y+3)^2}{16} = 1$

   H. $(x-3)^2 + (y+5)^2 = 64$

   I. $\dfrac{x^2}{16} - \dfrac{y^2}{4} = 1$

   J. $\dfrac{(x-4)^2}{9} + \dfrac{(y-5)^2}{25} = 1$

   K. $\dfrac{y^2}{4} - \dfrac{x^2}{9} = 1$

   L. $\dfrac{(y-3)^2}{4} - \dfrac{(x-5)^2}{9} = 1$

Problem Solving/Application

1. A satellite orbits at an altitude of 21,000 miles above the earth. The earth's diameter is 7,900 miles. Assuming that the earth's center is at the origin and that satellite orbits are circular, write an equation of the satellite's orbit.

2. A satellite is launched into an elliptical orbit with Earth at one focus. The major axis is 30,000 miles long. The minor axis is 20,000 miles long. Produce and graph the equation of the orbit, showing the position of the Earth.

Standard #18

Standard Set 18.0
Students use fundamental counting principles to compute combinations and permutations.

Deconstructed Standard
1. Students use fundamental counting principles to compute combinations.
2. Students use fundamental counting principles to compute permutations.

Prior Knowledge Necessary
Students should know how to:
- correctly apply the order of operations.
- accurately use properties of exponents.
- use a calculator to work with large numbers and exponents.

New Knowledge
Students will need to learn to:
- use the fundamental counting principle.
- define a "factorial".
- use a factorial in computing combinations and permutations.
- distinguish the difference between a permutation and a combination.
- use the formula for computing permutations.
- use the formula for computing distinguishable permutations.
- use the formula for computing combinations.

Categorization of Educational Outcomes
Competence Level: Application
1. Students will compute permutations and combinations.
2. Students will determine if a counting problem requires the use of a permutation or a combination and then apply the appropriate strategy to calculate the correct result.

Necessary New Physical Skills
None

Assessable Results of the Standard
1. Students will identify the correct number of ways an event can occur when dealing with a permutation.
2. Students will identify the correct number of ways an event can occur when dealing with a combination.

Standard #18 Model Assessment Items

Computational and Procedural Skills

1. Give the definition of the Fundamental Counting Principle.

2. Evaluate the following:
   A. $7!$

   B. $\dfrac{15!}{5!\cdot 10!}$

   C. $\dfrac{100!}{98!}$

3. What is the formula for finding the number of permutations of *n* objects taken *r* at a time?

4. What is the formula for finding the number of combinations of *n* objects taken *r* at a time?

5. Evaluate the following: ${}_{12}C_7$

6. Evaluate the following: ${}_{10}P_4$

7. Evaluate the following: 0!

8. How many different ways can the letters AMATYC be arranged?

Conceptual Understanding

1. What is the difference between a permutation and a combination?

2. What is meant by "distinguishable permutations"?

3. Create a counting problem that would justify the use of a permutation.

4. Create a counting problem that would justify the use of a combination.

5. Create a counting problem where distinguishable permutations would play a role.

Problem Solving/Application

1. How many ways can a four-member committee be chosen from 10 people?

2. How many different batting lineups are possible for the starting 9 players on a softball team?

3. How many ways can the letters A, B, C, D, and E be arranged for a 5-letter security code?

4. At a certain restaurant, customers can choose a main course, a vegetable, a beverage, and a desert. If there are 10 choices for the main course, 4 vegetable choices, 5 beverage choices and 3 choices of desert, how many different meals are possible?

5. How many ways could someone answer a True or False quiz with 5 questions, given that no answers are left blank?

6. A lottery has 52 numbers to choose from. In how many different ways can 6 of those numbers be selected?

7. In a certain state, license plates must have 3 letters followed by 4 numbers. How many different license plates are possible if none of the letters and none of the numbers can be repeated?

8. A jury is to be selected from a pool of 45 people. In how many ways can a jury of 12 people be selected from this pool of 45 people?

9. In how many distinguishable ways can the letters in the word MISSISSIPPI be arranged?

10. A landscaper wants to plant 5 oak trees, 7 maple trees, and 6 poplar trees along a certain street. In how many distinguishable ways can they be planted?

11. From a pool of 10 candidates, the offices of president, vice-president, and secretary are to be filled. In how many different ways can the offices be filled?

12. A jar contains 5 red marbles, 7 green marbles, and 8 blue marbles. If a person randomly selected 7 marbles, in how many ways could that person select 2 red, 3 green, and 2 blue marbles?

13. A bin contains a total of 25 electronic timers, 3 of which are defective. If a person randomly selects 4 of these electronic timers from this bin, in how many ways could that person select 2 good timers and 2 defective timers?

Standard #19

Standard Set 19.0
Students use combinations and permutations to compute probabilities.

Deconstructed Standard
1. Students use combinations to compute probabilities.
2. Students use permutations to compute probabilities.

Prior Knowledge Necessary
Students should know how to:
- define and use of a factorial.
- use the formula for finding permutations.
- use the formula for finding combinations.
- recognize the difference between a permutation and a combination.
- represent very large or very small numbers in scientific notation.
- convert numbers written in scientific notation back into standard form.

New Knowledge
Students will need to learn to:
- use the definition of statistical probability of an event occurring as the following: $P(\mathrm{E}) = \dfrac{\text{Frequency of Event "E"}}{\text{Total Frequency}}$.
- find basic probabilities using the definition of statistical probability given above.
- find probabilities requiring the use of permutations.
- find probabilities requiring the use of combinations.

Categorization of Educational Outcomes
Competence Level: Application
1. Students will solve probability problems that require the use of permutations or combinations.
2. Students will determine which counting method should be applied to calculate various probability problems.

Necessary New Physical Skills
None

Assessable Results of the Standard
1. Students will solve probability problems that require the use of permutations.
2. Students will solve probability problems that require the use of combinations.

Standard #19 Model Assessment Items

Computational and Procedural Skills

1. One card is randomly selected from a standard deck of playing cards. Find the probability that this card is an ace.

2. A jar contains 5 red marbles, 7 blue marbles, and 10 green marbles. One marble is randomly selected from this jar. What is the probability that this marble is blue?

3. Evaluate the following:

$$P(E)=\frac{\left(\frac{4!}{2!\cdot 2!}\right)\left(\frac{4!}{3!\cdot 1!}\right)}{\left(\frac{52!}{5!\cdot 47!}\right)}$$

4. Evaluate the following:

$$P(E)=\frac{39}{6!}$$

Conceptual Understanding

1. A friend of yours calculates the probability of an event and comes up with an answer of 1.06 for the probability. Explain to your friend why this cannot be the correct answer to this probability problem.

2. A friend of yours answers the following problem with the given answer:
$(5!)(3!)(0!)=0$ Do you agree or disagree with this solution? Why?

3. The following 11 letters are randomly selected one at a time and put in order from left to right. (M, I, I, I, I, P, P, S, S, S, S). (this is computation) Will the following calculation give the correct result? Why or why not?

$$P(E)=\frac{1}{11!}$$

4. A jar contains 5 red marbles, 7 green marbles, and 8 blue marbles. If a person randomly selects 7 marbles, what is the probability that this person will not select 2 red, 3 green, and 2 blue marbles?

Problem Solving/Application

1. A jar contains 5 red marbles, 7 green marbles, and 8 blue marbles. If a person randomly selects 7 marbles, what is the probability that this person will select 2 red, 3 green, and 2 blue marbles?

2. A lottery is played in a certain state where players must correctly select 6 numbers from a total of 52 numbers. The order in which the player selects the numbers is not

important. What is the probability that a person will successfully select all 6 numbers and win the lottery?

3. A password consists of 3 letters followed by 3 digits in the correct order. What is the probability of correctly guessing the password on the first try, given that in the password no letter or number can be used twice?

4. A speech club consists of 15 members, 7 of which are male. Four members will be chosen at random to travel to a special activity. What is the probability that none of the males will be chosen?

Standard #20

Standard Set 20.0

Students know the binomial theorem and use it to expand binomial expressions that are raised to positive integer powers.

Deconstructed Standard

1. Students know the binomial theorem.
2. Students apply the binomial theorem to expand binomial expressions to positive integer powers.

Prior Knowledge Necessary

Students should know how to:

- put an expression in descending or ascending order.
- multiply binomials and polynomials.
- combine like terms.
- apply properties of exponents.
- compute combinations.

New Knowledge

Students will need to learn to:

- identify the pattern for the exponents of each term in the expansion.
- calculate the exponents for the variables in each term.
- identify and calculate the binomial coefficient for each term in the expansion.

Categorization of Educational Outcomes

Competence Level: Knowledge

1. Students recall the binomial theorem.

Competence Level: Application

1. Students apply the binomial theorem to the expansion of binomials raised to positive integer powers.

Necessary New Physical Skills

None

Assessable Result of the Standard

1. Students will produce appropriate expansions for binomials raised to positive integer powers.

Standard #20 Model Assessment Items

Conceptual Understanding

1. Identify the correct pattern for the exponents of each term in the expansion.

   A. Insert the exponents for each term in the expansion of $(3m+2n)^4$.

$$(3m+2n)^4 = \binom{4}{0}(3m)^{\square}(2n)^{\square} + \binom{4}{1}(3m)^{\square}(2n)^{\square} + \binom{4}{2}(3m)^{\square}(2n)^{\square} + \binom{4}{3}(3m)^{\square}(2n)^{\square} + \binom{4}{4}(3m)^{\square}(2n)^{\square}$$

   B. Without calculating the binomial coefficients, simplify each term in the expansion of expansion from part A.

$$(3m+2n)^4 = \binom{4}{0}\square m^{\square}n^{\square} + \binom{4}{1}\square m^{\square}n^{\square} + \binom{4}{2}\square m^{\square}n^{\square} + \binom{4}{3}\square m^{\square}n^{\square} + \binom{4}{4}\square m^{\square}n^{\square}$$

   C. Calculate the binomial coefficients in the expansion from part B and simplify into their final form.

$$(3m+2n)^4 = \square m^{\square}n^{\square} + \square m^{\square}n^{\square} + \square m^{\square}n^{\square} + \square m^{\square}n^{\square} + \square m^{\square}n^{\square}$$

2. Use the binomial theorem to expand and simplify: $(x-4y)^3$.

3. Find the 18th term in $(x+y)^{21}$.

Standard #21

Standard Set 21.0
Students apply the method of mathematical induction to prove general statements about the positive integers.

Deconstructed Standard
1. Students know the process of mathematical induction.
2. Students apply the method of mathematical induction to prove general statements about the sum of a series.

Prior Knowledge Necessary
Students should know how to:
- combine like terms.
- simplify rational expressions.
- recognize and use basic sequence notation.
- recognize and use basic series notation.

New Knowledge
Students will need to learn to:
- identify an induction hypothesis.
- demonstrate that a statement is true for $n = 1$.
- use appropriate algebraic manipulation to demonstrate that if a statement is true for $n = k$, the statement is true for $n = k + 1$.
- state the conclusion of the mathematical induction proof in appropriate language.

Categorization of Educational Outcomes
Competence Level: Application
1. Student will apply the method of mathematical induction to prove the validity of generalized statements about the sum of a series.

Necessary New Physical Skills
None

Assessable Result of the Standard
1. Students will produce proofs of mathematical statements about positive integers using appropriate mathematical induction language and processes.

Standard #21 Model Assessment Items

Conceptual Understanding

1. Given the assertion: $1+4+9+16+25+\cdots+n^2 = \dfrac{n(n+1)(2n+1)}{6}$, identify the induction hypothesis, P_k.

2. Given the assertion: $5+9+13+17+\cdots+(4n+1) = n(2n+3)$, show the assertion is true for n=1.

3. Given the assertion: $5+10+15+20+25+\cdots+5n = \dfrac{5n(n+1)}{2}$, assuming P_k is true, prove the statement is true for P_{k+1}.

4. Prove the assertion: $1^3+2^3+3^3+\cdots+n^3 = \left[\dfrac{n(n+1)}{2}\right]^2$. State the conclusion.

Standard #22

Standard Set 22.0
Students find the general term and the sums of arithmetic series and of both finite and infinite geometric series.

Deconstructed Standard
1. Students find the general term of a finite arithmetic series.
2. Students find the general term of a finite geometric series.
3. Students find the sum of a finite arithmetic series.
4. Students find the sum of a finite geometric series.
5. Students find the sum of an infinite geometric series.

Prior Knowledge Necessary
Students should know how to:
- evaluate algebraic expressions.
- simplify expressions prior to solving linear equations.
- solve multi-step problems, including word problems involving linear and non-linear equations.
- use notation involving sub-scripts and exponents (for example: a_{n-1}, a_n, r^n).
- differentiate between a finite set of numbers vs. infinite set of numbers.

New Knowledge
Students will need to learn to:
- identify that an arithmetic sequence is a sequence of terms (numbers) in which a term is found by adding a constant (common difference = *d*) to the previous term.
- find consecutive terms of an arithmetic sequence.
- find the *n*th term of an arithmetic sequence by using the formula $a_n = a_1 + (n-1)d$, where a_1 = first term, a_n = n$^{\text{th}}$ term, *n* = number of terms, and *d* = common difference.
- find missing terms in an arithmetic sequence (arithmetic means).
- identify that an arithmetic series is the sum of the terms of a finite arithmetic sequence.
- find the sum of an arithmetic series using the following formulas, where S_n is the sum of the series:
  - $S_n = \frac{n}{2}(a_1 + a_n)$.
  - $S_n = \frac{n}{2}\left[2a_1 + (n-1)d\right]$, when the n$^{\text{th}}$ term is not known.
- use sigma or summation notation to express arithmetic series (an enrichment opportunity).

- recognize that a geometric sequence is a sequence of terms (numbers) in which a term is found by multiplying a constant (common ratio = *r*) with the previous term: $a_n = a_{n-1}r$.
- find consecutive terms of a geometric sequence.
- find the *n*th term of a geometric sequence by using the formula $a_n = a_1 r^{n-1}$.
- find missing terms in a geometric sequence (geometric means.)
- recognize that a finite geometric series is the sum of a finite number of terms of a geometric sequence.
- find the sum of a finite geometric series using the following formulas, where S_n is the sum of the series:
    - $S_n = \frac{a_1 - a_1 r^n}{1-r}, \quad r \neq 1$.
    - $S_n = \frac{a_1\left(1-r^n\right)}{1-r} \quad r \neq 1$, a modification of the first formula.
- use sigma or summation notation to express finite geometric series (an enrichment opportunity).
- recognize that an infinite geometric series with a common ratio between -1 and 1 will have a sum given by the formula $S = \frac{a_1}{1-r}$.
- recognize that an infinite geometric series with a common ratio *r* such that $|r| \geq 1$ does not have a sum.
- use sigma or summation notation to express finite geometric series (an enrichment opportunity).

Categorization of Educational Outcomes

Competence Level: Comprehension

1. Students will know the formulas used.
2. Students will show understanding of the sums of infinite geometric series.

Competence Level: Application

1. Students will apply the knowledge they have of sequences and series in a variety of situations.

Competence Level: Synthesis

1. Students will synthesize their understanding of infinity and sums to predict the limit of an infinite series.

Necessary New Physical Skills

None

Assessable Result of the Standard

1. Students will solve problems involving arithmetic series and sequences and both finite and infinite geometric series and sequences.

Standard #22 Model Assessment Items

Computational and Procedural Skills

1. Find the ninth term in the arithmetic sequence where $a_1 = 22,\ a_2 = 18$.

2. Find the sum of the arithmetic sequence 16 + 21 + 26 + …+ 56.

3. Evaluate the arithmetic series $\sum_{n=4}^{10}(2n-1)$.

4. Find the twelfth term in the geometric sequence where $a_1 = 64$ and $r = \frac{1}{2}$.

5. Find the sum of the geometric series for which $a_1 = 12$, $r = 12$, and $n = 5$.

6. Find the sum of the infinite geometric series for which $a_1 = -4$ and $r = 0.5$.

7. Find the sum of the infinite geometric series for which $a_1 = -4$ and $r = -0.5$.

Conceptual Understanding

1. Find *y* in the following arithmetic sequence, $2,\ y+4,\ 16,\ 4y+3$, then rewrite the sequence.

2. Find the missing terms of the following arithmetic sequence 2, ___, ___, ___, –22.

3. Find the missing terms of the arithmetic series with five terms, a second term equal to 11, and a series sum equal to 0.

4. Find the value(s) of *y* in the geometric sequence $4,\ 2y,\ 6y+40$.

5. Why is there not a sum for an infinite series where $|r| \geq 1$?

Problem Solving/Application

1. A certain automobile loses 25% of its value each year. If the starting value of the automobile is $26,500, what will its value be after 10 years?

2. A free-falling object falls 16 feet in the 1st second, 48 feet in the 2nd second. If it continues to fall at this rate, how far will it have fallen in 15 seconds (ignore air resistance)?

3. A rumor is spreading fast on campus. If I started the rumor by telling two friends, and one minute later they had told two other friends, then those two friends told two more friends one minute later, and so on, how many people would know the rumor within a half hour (assuming no one was told the rumor by more than one person)?

Standard #23

Standard Set 23.0
Students derive the summation formulas for arithmetic series and for both finite and infinite geometric series.

Deconstructed Standard
1. Students will derive the summation formula for arithmetic series.
2. Students will derive the summation formula for finite geometric series.
3. Students will derive the summation formula for infinite geometric series.

Prior Knowledge Necessary
Students should know how to:
- identify the domain of a function.
- manipulate rational expressions.
- manipulate formulas with few if any digits.
- identify an arithmetic series.
- identify a finite geometric series how to identify an infinite geometric series.
- write the general form of an arithmetic series.
- write the general form of a finite geometric series.
- write the general form of an infinite geometric series.

New Knowledge
Students will need to learn to:
- understand what a summation formula represents.
- manipulate the general form of a series to produce the summation formula.

Categorization of Educational Outcomes
Competence Level: Synthesis
1. Students derive the summation formulas for arithmetic series and for both finite and infinite geometric series.

Necessary New Physical Skills
None

Assessable Result of the Standard
1. Students will derive the summation formula for arithmetic series.
2. Students will derive the summation formula for finite geometric series.
3. Students will derive the summation formula for infinite geometric series.

Standard #23 Model Assessment Items

Computational and Procedural Skills

1. Find the general formula for finding the partial sum of a geometric series:
   $S_n = a_1 r + a_1 r^2 + ... + a_1 r^{n-1}$.

Standard #24

Standard Set 24.0
Students solve problems involving functional concepts such as composition, defining the inverse function, and performing arithmetic operations on functions.

Deconstructed Standard
1. Students solve problems involving composition of functions.
2. Students know the definition of an inverse of a function.
3. Students solve problems using the inverse of a function.
4. Students solve problems involving arithmetic operations on functions.

Prior Knowledge Necessary
Students should know how to:
- define a function.
- use function notation.
- add, subtract, multiply, and divide polynomials.
- to graph functions.
- recognize the concept of reflection in graphical terms.

New Knowledge
Students will need to learn to:
- understand the concept of composition of functions.
- solve problems of function composition.
- use composition of functions to solve problems.
- define the inverse of a function.
- find the inverse of a function.
- verify that a function is the inverse of another function.
- perform arithmetic operations such as addition, subtraction, multiplication, and division involving functions.

Categorization of Educational Outcomes
Competence Level: Knowledge
1. Students will define the inverse of functions.
2. Students will identify the inverse of simple functions.

Competence Level: Application
1. Students will solve function composition problems.
2. Students will compute the inverse of a function.
3. Students will verify that a function is the inverse of another function.
4. Students will perform arithmetic operations with functions.

Necessary New Physical Skills
None

Assessable Result of the Standard

1. Students will solve composition of function problems.
2. Students will find the inverse of a function.
3. Students will verify that a function is the inverse of another function.
4. Students will perform arithmetic operations with functions.

Standard #24 Model Assessment Items

Computational and Procedural Skills

1. Let $f(x) = 3x + 2$ and $g(x) = 4x$. Find the following:

   A. $g\left(f(2)\right)$

   B. $(f + g)(x)$

   C. $(f - g)(x)$

   D. $(fg)(x)$

   E. $(f / g)(x)$

2. What is the inverse of $f(x) = 4x - 6$?

Conceptual Understanding

1. Verify that $f(x) = 4x + 9$ and $g(x) = \dfrac{x-9}{4}$ are inverse functions.

Problem Solving/Application

1. A department store is having a "20% - off everything" sale. You also have a \$10 coupon for any purchase.

   A. Write the function *M* that represents the sale price after the 20% discount, and a function *K* that represents the price of an item after the \$10 coupon.

   B. Determine which is the best deal when buying an item costing \$25: discount then coupon, or coupon then discount.

   C. What would be the initial price of an item for which the values of *M* and *K* would be the same?

Standard #25

Standard Set 25.0
Students use properties from number systems to justify steps in combining and simplifying functions.

Deconstructed Standard
1. Students use properties of number systems to justify steps in combining functions.
2. Students use properties of number systems to justify steps in simplifying functions.

Prior Knowledge Necessary
Students should know how to:
- use the basic number system properties: commutative, associative, distributive, identities inverses and zero.
- use symmetric, transitive, and reflexive properties of equality.
- use the laws of exponents and logarithms.

New Knowledge
Students will need to learn to:
- recognize the properties of the number system used in expressions.
- deconstruct an expression into component steps.

Categorization of Educational Outcomes
Competence Level: Evaluation
1. Students use properties from number systems to justify steps in combining and simplifying functions.

Necessary New Physical Skills
None

Assessable Result of the Standard
1. Students will deconstruct a function and describe the rationale for each step according to the properties of the number system.

Standard #25 Model Assessment Items

Computational and Procedural Skills

1. What property of real numbers enables you to simplify $f(x) = \dfrac{x^2 + 3x + 2}{x + 1}$ to $g(x) = x + 2$?

2. What properties of real numbers enable you to simplify $f(x) = \dfrac{x^2 + 3x - 18}{x - 3}$ to $g(x) = x + 6$?

Conceptual Understanding

1. Given $f(x) = x^2 + 3x + 2$ and $g(x) = 2(x-1)^2$. Identify the properties used in creating the equivalent form of $2f(x) + g(x)$ (i.e. distributive property, addition property, and multiplicative property).

2. Explain when $\dfrac{f(x)}{g(x)}$ does not exist and why.

Probability and Statistics Standard # 1

Standard Set 1.0 Probability and Statistics
Students know the definition of *independent events* and can use the rules for addition, multiplication, and complementation to compute probabilities of particular events in finite sample spaces.

Deconstructed Standard
1. Students know the definition of "*independent events.*"
2. Students can use the rules for addition to compute probabilities of particular events in finite sample spaces.
3. Students can use the rules for multiplication to compute probabilities of particular events in finite sample spaces.
4. Students can use the rules for complementation to compute probabilities of particular events in finite sample spaces.

Prior Knowledge Necessary
Students should know how to:
- perform arithmetic computations with rational numbers.
- calculate the probability of an event.
- calculate probabilities of events involving combinations and permutations.
- calculate probabilities involving compound and mutually exclusive events.
- produce tree diagrams and Venn diagrams.

New Knowledge
Students will need to learn to:
- recognize when an event is an "independent event".
- recognize when events are "mutually exclusive events".
- compute probabilities of independent events using the rules for addition of probabilities.
- compute probabilities of independent events using the rules for multiplication of probabilities.
- compute probabilities of events using the rules for complementation.

Categorization of Educational Outcomes
Competence Level: Application and Analysis
1. Students will calculate probability for outcomes of independent events.
2. Students will calculate probability using the rules of addition, multiplication and complementation.
3. Students will create examples of probability problems involving independent events.
4. Students will explain the relationship between the outcome of an event and its complement.
5. Students will compare outcomes of events to determine if they are independent or dependent events.

6. Students will interpret the solution of probability problems in the context of real world problems.

<u>Necessary New Physical Skills</u>

None

<u>Assessable Result of the Standard</u>

1. Students will solve probability problems involving independent events.
2. Students will be able to create examples of problems involving independent events that require use of the probability rules for addition, multiplication, and complementation.
3. Students will be able to create an illustration that depicts probabilities of complementation.

Probability and Statistics Standard #1 Model Assessment Items

Computational and Procedural Skills

1. Given events A and B.
   A. Events A and B are mutually exclusive. If $P(A) = 2/10$ and $P(B) = 2/3$, calculate $P(A \text{ or } B)$.

   B. If $P(A) = 2/5$, $P(B) = 2/5$, and $P(A \text{ and } B) = 1/5$, calculate $P(A \text{ or } B)$.

2. Events A and B are independent. If $P(A) = 1/3$ and $P(B) = 3/4$, calculate $P(A \text{ and } B)$.

3. Given event A.
   A. For Event A, if $P(A) = 2/5$, calculate $P(A')$.

   B. For Event A, if $P(A') = 1/3$, calculate $P(A)$.

Conceptual Understanding

1. Consider a standard 52-card deck.
   A. Produce an example of two events that are mutually exclusive.

   B. Produce an example of two events that are not mutually exclusive.

2. Is it possible that $P(A) = P(A')$? If yes, give an example. If no, explain why not.

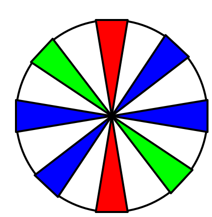
3. A five is rolled on a six-sided dice and a one is rolled on a four-sided die. Determine if the outcome is an independent outcome or a dependent outcome. Support your answer.

4. For events A and B, $P(A \text{ or } B) = P(A) + P(B) – P(A \text{ and } B)$. If A and B are mutually exclusive, determine $P(A \text{ or } B)$. Explain your answer and illustrate it with a Venn diagram.

Problem Solving/Application

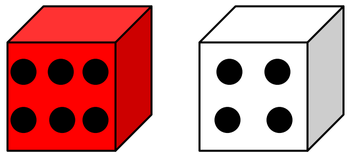
1. You have just entered a high stakes game of Black Jack. The object of the game is to get 21, or as close as possible to 21, and beat the dealer. Determine the probability that the first card pulled is an ace or a face card from a standard deck of 52 Cards?

2. Determine the probability of spinning a red color followed by a white color from a spinning wheel.



Spinning Wheel

3. You are rolling a red die followed by a white die.
   Determine the probability of not rolling a total of 7 with both dice.



4. You arrive at math class and find that the teacher is giving a true-false quiz for which you are totally unprepared. You decide to guess randomly at the answers. There are four questions. Find the probabilities described below. Explain your reasoning and use a diagram to illustrate your answers.
   A. P(none correct)

   B. P(exactly one correct)

   C. P(exactly two correct)

   D. P(exactly three correct)

   E. P(all four correct)

   F. Predict the sum of the probabilities in A-E.

   G. In order to pass the quiz, you must get at least three correct answers. What is the probability of passing the quiz?

Probability and Statistics Standard #2

Standard Set 2.0 Probability and Statistics
Students know the definition of "conditional probability" and use it to solve for probabilities in finite space.

Deconstructed Standard
1. Students know the definition of "conditional probability."
2. Students can solve for probabilities using the *conditional probability* of particular events in finite sample spaces.

Prior Knowledge Necessary
Students should have the computational and conceptual knowledge outlined in Probability and Statistics Standard #1.

Students should know how to:

- calculate probabilities of events involving combinations and permutations.
- calculate probabilities involving compound and mutually exclusive events.
- calculate probabilities involving independent events produce tree diagrams and Venn diagrams.

New Knowledge
Students will need to learn to:
- recognize the definition of "conditional probability".
- calculate conditional probabilities.
- determine whether events are dependent.
- recognize the notation $P(A|B)$ and its meaning.

Categorization of Educational Outcomes
Competence Level: Application and Analysis
1. Students will compute probabilities.
2. Students will produce examples of conditional events.
3. Students will illustrate probabilities of conditional events.
4. Students will explain outcomes.
5. Students will interpret conditional probability in the context of real world problems.

Necessary New Physical Skills

None

Assessable Result of the Standard
1. Students will solve probability problems dealing with conditional events.
2. Students will be able to produce examples of conditional events.
3. Students will be able to create an illustration that depicts probabilities of conditional events.

Probability and Statistics Standard #2
Model Assessment Items

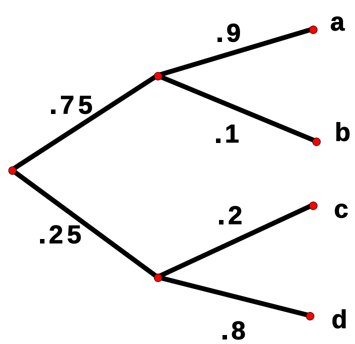
Computational and Procedural Skills

1. A and B are dependent events. If $P(A) = 1/10$ and $P(B|A) = 8/10$, calculate $P(A \text{ and } B)$.

2. A and B are dependent events. If P(A) = 4/5 and P(A and B) = 1/5, calculate P(B|A).

3. Given the tree diagram shown below.
   A. Calculate the probability of each path a – d.

   B. Calculate the sum of probabilities *a*, *b*, *c*, and *d*.



Conceptual Understanding

1. Produce a simple example of a probability problem involving dependent events using a standard 52-card deck. Solve your example and explain each step.

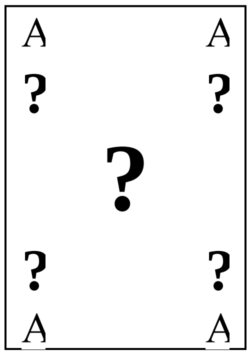
2. Mr. Thometz teaches three classes. Each class has 20 students. His first class has 12 sophomores, his second class has 8 sophomores, and his third class has 10 sophomores. He randomly chooses one student from each class to participate in a competition. Scenarios A and B are stated below. Compare the two scenarios. Decide which describes dependent events and which describes independent events. Explain your answer and illustrate with a diagram.
   A. That he selects three sophomores to participate in the competition.

   B. That he selects only one sophomore to participate in the competition.

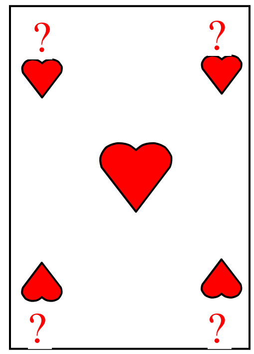
Problem Solving/Application

1. You will randomly select 2 cards from a standard 52-card deck. What is the probability that the first card you select is an ace and the second is a non-face card?

1st Card



2nd Card



2. A box contains 3 red marbles, 4 white marbles and 3 blue marbles. If 3 marbles are removed from the box without replacement, what is the probability that the third marble is white if the first two marbles are red?

3. If a pair of fair dice are rolled and the first die is a 3, what is the probability that the sum on the two dice is greater than 7?

4. A random sample of 100 men and 150 women showed that 53 men and 70 women are going to vote "Yes" on Proposition 27. If one person is randomly chosen from this sample, what is the probability that:
   A. a female is chosen?

   B. someone who voted "Yes" on the proposition is chosen?

   C. a female is chosen, given that someone who voted "Yes" on the proposition was chosen?

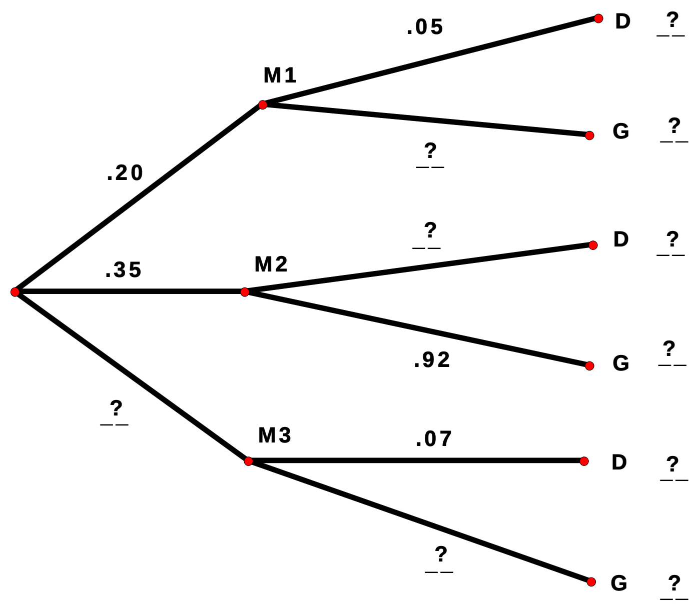
   D. someone who voted "Yes" on the proposition was chosen, given a female was chosen?

4. The ratios of the number of phones manufactured at three sites, M1, M2, and M3, are 20%, 35%, and 45%, respectively. The diagram below shows some of the ratios of the numbers of defective (D) and good (G) phones manufactured at each site. The top branch indicates a 0.20 probability that a phone made by this manufacturer was manufactured at site M1. The ratio of these phones that are defective is 0.05. Therefore, 0.95 of these phones are good. The probability that a randomly selected phone is both from site M1 and defective is (0.20)(0.05), or 0.01.
   A. Copy the diagram and determine the missing probabilities.

   B. Determine P (a phone from site M2 is defective).

   C. Determine P (a randomly chosen phone is defective).

   D. Determine P (a phone was manufactured at site M2 if you already know it is defective).



Probability and Statistics Standard #7

Standard Set 7.0 Probability and Statistics
Students compute the "variance" and the "standard deviation" of a distribution of data.

Deconstructed Standard
1. Students compute the "variance" of a distribution of data.
2. Students compute the "standard deviation" of a distribution of data.

Prior Knowledge Necessary
Students should know how to:
- perform arithmetic computations with rational numbers.
- calculate the mean of a data set.

New Knowledge
Students will need to learn to:
- calculate the variance of a data set.
- calculate the standard deviation of a data set.
- explain the meaning of the standard deviation and the variance of a given data set.

Categorization of Educational Outcomes
Competence Level: Application and Analysis
1. Students will calculate variance for a given set of data.
2. Students will calculate the standard deviation for a given set of data.
3. Students will create examples of data sets.
4. Students will explain relationships between mean, standard deviation, and variance.
5. Students will compare units of standard deviation and data.
6. Students will interpret standard deviation and variance in the context of real world problems.

Necessary New Physical Skills
None

Assessable Result of the Standard
1. Students will be able to calculate the standard deviation and the variance for a given set of data.
2. Students will be able to create examples of data sets and calculate the standard deviation and variance for the data.

Probability and Statistics Standard #7
Model Assessment Items

Computational and Procedural Skills

1. Calculate the standard deviation of the data set: $\{20, 30, 50, 60\}$.

2. Calculate the variance of the data set: $\{12.5, 14.5\ 19.5, 13.5\}$.

Conceptual Understanding

1. Create a simple set of numbers between 1 and 10. Calculate the mean and standard deviation of your data set. Explain the meaning of the standard deviation with respect to the mean of your data set.

2. Explain why the sum of the deviations in a data set is always equal to zero.

3. When calculating the variance, explain why you divide by one less (i.e., $n-1$) than the number of values in the data set?

4. Compare the units of the standard deviation with the units of the data. Explain your findings.

5. Explain the connection between the variance, the standard deviation and the mean for any given data set.

Problem Solving/Application

1. You have just entered the real estate market in your area. The first eight homes you sold and their prices are given in the table below. Calculate the average selling price and the standard deviation.

Number of Homes	Selling Price
1	$340,000
2	$260,000
2	$280,000
3	$300,000

2. For the same table above, find the variance.

3. The mean diameter of a Cardinal Best Compact Disc is 12.0 cm, with a standard deviation of 0.012 cm. CDs that are more than one standard deviation from the mean cannot be shipped. How would those statistics be useful to a quality control engineer of Cardinal Best Company?

Appendix #1

Algebra II California Content Standards by Clusters

California Standards Testing organizes related standards for Algebra II into four reporting clusters. Student performance is reported by these clusters. The clusters for Algebra II are:

- Polynomials and Rational Expressions
- Quadratics, Conics and Complex Numbers
- Exponents and Logarithms
- Series, Combinatorics, and Probability and Statistics

The following table, compiled by the San Diego County Office of Education, organizes the standards by cluster and shows the number of items testing each standard on the Algebra II CST and High School Summative (HSS) CST. The actual questions change each year, but the distribution of questions, indicated in the table, below remains the same.

Mathematics Content Standards: Algebra II		Algebra II CST	Summative High School CST
		65 items	**28 items**
Cluster 1: Polynomials and Rational Expressions			
1.0*	Students solve equations and inequalities involving absolute value.	1	1
2.0*	Students solve systems of linear equations and inequalities (in two or three variables) by substitution, with graphs, or with matrices.	5	3
3.0	Students are adept at operations on polynomials, including long division.	4	1
4.0	Students factor polynomials representing the difference of squares, perfect square trinomials, and the sum and difference of two cubes.	3	1
7.0	Students add, subtract, multiply, divide, reduce, and evaluate rational expressions with monomial and polynomial denominators and simplify complicated rational expressions, including those with negative exponents in the denominator.	6	2
Cluster 2: Quadratics, Conics and Complex Numbers			
5.0*	Students demonstrate knowledge of how real and complex numbers are related both arithmetically and graphically. In particular, they can plot complex numbers as points in the plane.	2	
6.0*	Students add, subtract, multiply, and divide complex numbers.	3	1
8.0*	Students solve and graph quadratic equations by factoring, completing the square, or using the quadratic formula. Students apply these techniques in solving word problems. They also solve quadratic equations in the complex number system.	4	3
9.0*	Students demonstrate and explain the effect that changing a coefficient has on the graph of quadratic functions; that is, students can determine how the graph of a parabola changes as a, b, and c vary in the equation $y = a(x-b)\leq + c$.	2	
10.0*	Students graph quadratic functions and determine the maxima, minima, and zeros of the function.	4	2
16.0	Students demonstrate and explain how the geometry of the graph of a conic section (e.g., asymptotes, foci, eccentricity) depends on the coefficients of the quadratic equation representing it.	1/3	

17.0	Given a quadratic equation of the form $ax^2 + by^2 + cx + dy + e = 0$, students can use the method for completing the square to put the equation into standard form and can recognize whether the graph of the equation is a circle, ellipse, parabola, or hyperbola. Students can then graph the equation.	1	
Cluster 3: Exponents and Logarithms			
11.0	**Students prove simple laws of logarithms.**		
11.1*	Students understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.	3	1
11.2*	Students judge the validity of an argument according to whether the properties of real numbers, exponents, and logarithms have been applied correctly at each step.	2 1/2	
12.0*	Students know the laws of fractional exponents, understand exponential functions, and use these functions in problems involving exponential growth and decay.	3	2
13.0	Students use the definition of logarithms to translate between logarithms in any base.	1	
14.0	Students understand and use the properties of logarithms to simplify logarithmic numeric expressions and to identify their approximate values.	2	1
15.0*	Students determine whether a specific algebraic statement involving rational expressions, radical expressions, or logarithmic or exponential functions is sometimes true, always true, or never true.	4	1
Cluster 4: Series, Combinatorics, and Probability and Statistics			
18.0*	Students use fundamental counting principles to compute combinations and permutations.	2	1
19.0*	Students use combinations and permutations to compute probabilities.	2	1
20.0*	Students know the binomial theorem and use it to expand binomial expressions that are raised to positive integer powers.	2	
21.0	Students apply the method of mathematical induction to prove general statements about the positive integers.	1/3	
22.0	Students find the general term and the sums of arithmetic series and of both finite and infinite geometric series.	2	1/2
23.0	Students derive the summation formulas for arithmetic series and for both finite and infinite geometric series.	N/A	1/2
24.0	Students solve problems involving functional concepts, such as composition, defining the inverse function and performing arithmetic operations on functions.	1/2	1
25.0	Students use properties from number systems to justify steps in combining and simplifying functions.	1/3	
Probability and Statistics			
PS1.0	Students know the definition of the notion of independent events and can use the rules for addition, multiplication, and complementation to solve for probabilities of particular events in finite sample spaces.	1	2
PS2.0	Students know the definition of conditional probability and use it to solve for probabilities in finite sample spaces.	2	2
PS7.0	Students compute the variance and the standard deviation of a distribution of data.	2	1

Appendix #2

Developing Learning Targets for Algebra Standards
(Instructions given to teachers involved in the project)

Please note the following terms and/or definitions which have been agreed upon for this deconstruction project:

- ***Prior Knowledge:*** *Prior knowledge is defined as acquired knowledge that has been mastered in a previous standard.*
- ***New Knowledge:*** *New knowledge is defined as knowledge that students need to acquire and apply to the components in step #2 of this deconstruction process to create the products listed in step #7 of this process.*
- ***Introduced:*** *When a standard mentions that a concept or idea has been "introduced," this does not mean that it has been mastered.*
- ***Familiar With:*** *When a standard mentions that students should be "familiar with" certain concepts or ideas, this does not mean that students have actually mastered these ideas or concepts.*

Sample:
Deconstruction of Algebra I Standard #6

Step #1: Underline noun phrases, and box or circle the verbs.
Standard 6.0: Students graph a linear equation and compute the *x*- and *y*-intercepts. (e.g., graph $2x+6y=4$). They are also able to sketch the region defined by a linear inequality (e.g., they sketch the region defined by $2x+6y<4$).

Step #2: Rewrite standard into short components.
1. Students graph linear equations.
2. Students compute *x*-intercepts.
3. Students compute *y*-intercepts.
4. Students use intercepts to graph linear equations.
5. Students sketch the region defined by a linear inequality.

Step #3: Identify prior knowledge students should know. *(See note above)*
1. Students must be able to perform arithmetic computations with rational numbers.
2. Students must be able to graph ordered pairs.
3. Students must be able to compute slope from the graph of a line.
4. Students must be able to compute slope when given two points.
5. Students must be able to recognize slope as a rate of change of *y* in relation to *x*.
6. Students must be able to graph a linear equation using a "t-chart".
7. Students must be able to evaluate a linear equation for a given *x* or *y* value.
8. Students must be able to solve one-variable linear inequalities.
9. Students must be able to graph the solution set for a one-variable linear inequality.

10. Students must be able to verify that any element in the solution set of a one-variable inequality satisfies the original inequality.

Step #4: Identify what new knowledge students will need to learn. *(See note above)*
1. Given the slope/intercept form of a line, $y = mx + b$, students will plot the *y*-intercept, and then use the slope to find a second point in order to complete the graph of the line.
2. Students will identify the graphical representation of $(a, 0)$ as the *x*-intercept, and $(0, b)$ as the *y*-intercept.
3. Students will be able to compute the *x*-intercept and *y*-intercept given a linear equation.
4. Students will identify that the linear equation implied by the linear inequality forms a boundary for the solution set and that this boundary may or may not be included in the final graph.
5. Students will interpret the inequality symbol to determine whether or not the boundary is solid or dashed.
6. Students will identify and shade the region of the graph that contains the solutions to the inequality.
7. Students will recognize that linear inequalities have multiple ordered-pair solutions.

Step #5: Identify patterns of reasoning using *Bloom's Taxonomy*.
Use the *Bloom's Taxonomy* handouts provided to describe the overall competence level expected of students with respect to these topics. Then highlight the "skills demonstrated" using as many of the key words and phrases provided on the handouts. See example below for Standard #6. Box in the key words or phrases taken from *Bloom's Taxonomy*.

Competence Level: Application
1. Students will $\boxed{\text{use methods}}$ they have learned to graph lines, solve inequalities, and to locate and/or identify the *x*- and *y*-intercepts for a given equation or graph.
2. Students will $\boxed{\text{demonstrate}}$ their ability to find and use *x*- and *y*-intercepts in the context of graphing.
3. Students will $\boxed{\text{calculate}}$ *x*- and *y*-intercepts.
4. Students will $\boxed{\text{solve}}$ inequalities in two variables.
5. Students will $\boxed{\text{use information}}$ they have learned to graph lines, $\boxed{\text{solve}}$ inequalities, and find *x*- and *y*-intercepts.
6. Students will $\boxed{\text{show}}$ that they know the correct interpretation of the boundary line for the solution of an inequality by appropriately making the boundary solid or dashed.

Step #6: Identify required physical skills.
In this section we are looking for physical skills such as: use of a calculator, protractor, ruler, compass, etc.
1. Use of a ruler

Step #7: Identify assessable results of the standard.

1. Students will produce the graph of a line.
2. Students will produce the ordered pairs representing the *x*- and *y*-intercepts.
3. Students will produce a bounded and shaded region of the *x*-*y* plane representing the solution set of a linear inequality in two variables.

Model Assessment Items

In this section you will write model, or exemplar, assessment items that will serve to demonstrate the level and depth of instruction for these particular topics. For example, you would expect a lower level and depth for a topic in Basic Algebra than you would for the same topic in Intermediate Algebra.

Please be sure to include assessment items that measure abilities in the following three categories:

1. Computational and Procedural Skills
2. Conceptual Understanding
3. Problem Solving/Application

Category #1: Computational and Procedural Skills

1. Find the *x*- and *y*-intercepts for the line defined by the following equation: $2x+3y=9$.
2. Use the *x*- and *y*-intercepts to graph the line given by the equation: $2x+3y=6$.
3. Graph the following lines using the method of your choice. Identify and label the *x*- and *y*-intercepts for each graph if they exist:
   A. $3x-5y=10$

   B. $y=-\frac{2}{3}x+4$

   C. $y=2$

   D. $x=3.5$

   E. $2x+4y=3$

   F. $\frac{1}{2}x-\frac{3}{4}y=2$

4. Graph the solution set for the following inequalities:
   A. $2x-3y<6$

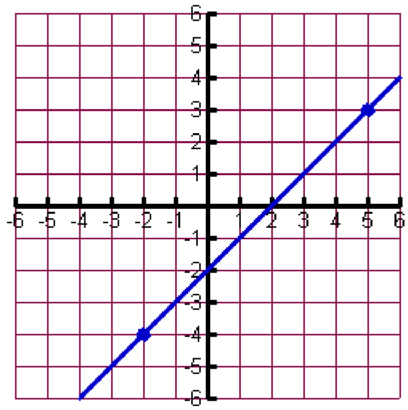
   B. $y\ge-\frac{3}{4}x+2$

C. $\frac{1}{2}x - \frac{2}{3}y \le \frac{5}{6}$

Category #2: Conceptual Understanding

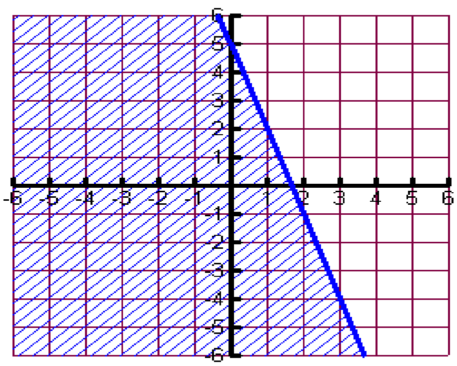
1. Sketch the graph of a line that has no *x*-intercept.

2. Identify the *x*- and *y*- intercepts from the graph of the given line.



3. Can a line have more than one *x*-intercept? Explain your answer using a diagram.

4. The solution to an inequality has been graphed correctly below. Insert the correct inequality symbol in the inequality below to match the graph of the solution. (Everything else about the inequality is correct—it just needs the correct symbol).



$y \;\boxed{}\; -3x + 5$

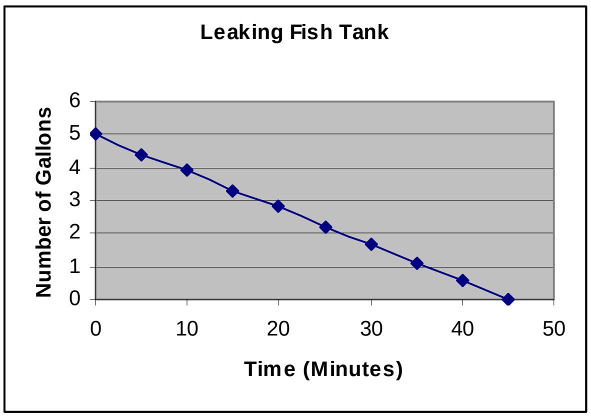
Insert correct symbol in box.

5. When is it advantageous to use the *x*- and *y*-intercepts to graph the equation of a line? When would it perhaps be easier or better to use another graphing method? Give an example to illustrate your answers to both of these questions.

Category #3: Problem Solving/Application

1. The graph displayed below is the graph of the following equation: $y = \left(\frac{-1}{9}\right)x + 5$, where x represents the amount of time that has passed since a 5 gal. fish tank sprung a leak, and y represents the number of gallons of water in the tank after the leak.
   A. What is the significance of the x-intercept in this situation? What information is given to us by this point?

   B. What is the significance of the y-intercept in this situation? What information is given to us by this point?



2. The cost of a trash pickup service is given by the following formula: $y = 1.50x + 11$, where x represents the number of bags of trash the company picks up, and y represents the total cost to the customer for picking up the trash.
   A. What is the y-intercept for this equation?

   B. What is the significance of the y-intercept in this situation? What does it tell us about this trash pickup service?

   C. Draw a sketch of the graph which represents this trash pickup service.

<u>Exemplar Teaching Methods</u>

Please record an example of "Lesson Plans" that demonstrate an excellent method of how the concepts in this standard could be presented to students. Be sure to give examples or illustrate any unique or creative methods that you have used that bring these concepts to life. Use as much detail as needed to communicate your ideas.

Appendix #3
Categorization of Educational Outcomes

Identifies the type of reasoning students will use to learn the skills necessary to master each standard. Teachers were asked to use *Bloom's Taxonomy* to describe the overall competence level expected of students with respect to these topics and highlight the skills demonstrated.

Major Categories in the* Taxonomy of Educational Objectives, *Bloom 1956*
Categories in the Cognitive Domain: (with Outcome-Illustrating Verbs)

Knowledge—of terminology; specific facts; ways and means of dealing with specifics (conventions, trends, and sequences; classifications and categories; criteria, methodology); universals and abstractions in a field (principles and generalizations, theories and structures)—**The remembering (recalling) of appropriate, previously learned information.**

- defines; describes; enumerates; identifies; labels; lists; matches; names; reads; records; reproduces; selects; states; views.

Comprehension: **Grasping (understanding) the meaning of informational materials.**

- classifies; cites; converts; describes; discusses; estimates; explains; generalizes; gives examples; makes sense out of; paraphrases; restates (in own words); summarizes; traces; understands.

Application: **The use of previously learned information in new and concrete situations to solve problems that have single or best answers.**

- acts; administers; articulates; assesses; charts; collects; computes; constructs; contributes; controls; determines; develops; discovers; establishes; extends; implements; includes; informs; instructs; operationalizes; participates; predicts; prepares; preserves; produces; projects; provides; relates; reports; shows; solves; teaches; transfers; uses; utilizes.

Analysis: **The breaking down of informational materials into their component parts, examining (and trying to understand the organizational structure of) such information to develop divergent conclusions by identifying motives or causes, making inferences, and/or finding evidence to support generalizations.**

- breaks down; correlates; diagrams; differentiates; discriminates; distinguishes; focuses; illustrates; infers; limits; outlines; points out; prioritizes; recognizes; separates; subdivides.

Synthesis: **Creatively or divergently applying prior knowledge and skills to produce a new or original whole.**

- adapts; anticipates; categorizes; collaborates; combines; communicates; compares; compiles; composes; contrasts; creates; designs; devises; expresses; facilitates; formulates; generates; incorporates; individualizes; initiates; integrates; intervenes;

models; modifies; negotiates; plans; progresses; rearranges; reconstructs; reinforces; reorganizes; revises; structures; substitutes; validates.

Evaluation: **Judging the value of material based on personal values/opinions, resulting in an end product, with a given purpose, without real right or wrong answers.**

- appraises; compares & contrasts; concludes; criticizes; critiques; decides; defends; interprets; judges; justifies; reframes; supports.

** http://faculty.washington.edu/krumme/guides/bloom.html*

Appendix #4
Sample Teaching Item for Standard #7

Below is a chart that can be used by students. It summarizes the different ways we simplify and solve rational expressions and equations. It also helps them see when and how to use or not to use the LCD.

SIMPLIFYING RATIONAL EXPRESSIONS VS. SOLVING RATIONAL EQUATIONS

<table>
<tr><td colspan="2">

SIMPLIFYING RATIONAL EXPRESSIONS

- Goal is to reduce a complicated rational expression to a single, much simpler rational expression.
- Answer is a rational expression.

</td></tr>
<tr><td>

I. Simplifying Simple Rationals

- Factor numerator & denominator and then cancel factors of 1. You can only cancel between the numerator and the denominator.
- Doesn't use the LCD.

$$\frac{x^2-4}{x^2+3x+2} \Rightarrow \frac{\cancel{(x+2)}(x-2)}{\cancel{(x+2)}(x+1)} \Rightarrow \frac{x-2}{x+1}$$

</td><td>

II. Simplifying Complex Rationals

- Find the LCD
- Multiply each term by the LCD and then simplify if you can.

LCD= xy - used to multiply each term of expression

$$\frac{\frac{1}{x}+\frac{2}{y}}{\frac{2}{x}-\frac{3}{y}} \Rightarrow \frac{\frac{1}{\cancel{x}}\cdot\cancel{x}y+\frac{2}{\cancel{y}}\cdot x\cancel{y}}{\frac{2}{\cancel{x}}\cdot\cancel{x}y-\frac{3}{\cancel{y}}\cdot x\cancel{y}} \Rightarrow \frac{y+2x}{2y-3x}$$

</td></tr>
<tr><td>

III. Multiplying/Dividing

- When multiplying, factor each numerator & denominator and then cancel factors of 1.
- When dividing, multiply the first rational with the reciprocal of the second rational.
- Doesn't use the LCD.

$$\frac{x^2-4}{x+2} \div \frac{x^2+4x+3}{x+3} \Rightarrow \frac{x^2-4}{x+2}\cdot\frac{x+3}{x^2+4x+3}$$
$$\Rightarrow \frac{\cancel{(x+2)}(x-2)}{\cancel{(x+2)}}\cdot\frac{\cancel{x+3}}{\cancel{(x+3)}(x+1)} \Rightarrow \frac{x-2}{x+1}$$

</td><td>

IV. Adding/Subtracting

- If rationals have same denominators, keep same denominator & combine numerators.
- If rationals have different denominators, first find LCD,
- Rewrite each rational with LCD as the new denominator & proceed adding/subtracting rationals with same denominators.

LCD= $m(m+1)$ - used as the new denominator

$$\frac{3}{m+1}-\frac{4}{m} \Rightarrow \frac{3}{m+1}\cdot\frac{m}{m}-\frac{4}{m}\cdot\frac{(m+1)}{(m+1)}$$
$$\Rightarrow \frac{3m}{m(m+1)}-\frac{4(m+1)}{m(m+1)} \Rightarrow \frac{3m-4(m+1)}{m(m+1)}$$
$$\Rightarrow \frac{3m-4m-4}{m(m+1)} \Rightarrow \frac{-m-4}{m(m+1)}$$

</td></tr>
</table>

SOLVING RATIONAL EQUATIONS

- Goal is to solve for a variable
- Answer is a value for *x*: a number, multiple numbers, or no solution at all, depending on restrictions on *x*.

- Find the LCD
- Multiply each term by the LCD (all denominators should disappear).
- Solve the much simpler equation.

LCD= $4(x-1)$ - used to multiply each term of the equation to get rid of fractions

$$\frac{3}{x-1}+\frac{2}{4x-4}=\frac{7}{4}$$

$$\frac{3}{\cancel{x-1}}\cdot 4\cancel{(x-1)}+\frac{2}{\cancel{4x-4}}\cdot \cancel{4(x-1)}=\frac{7}{\cancel{4}}\cdot \cancel{4}(x-1)$$

$$3\cdot 4+2=7(x-1)$$

$$12+2=7x-7$$

$$14=7x-7$$

$$21=7x$$

$$x=3$$