Algebra I California Content Standards

Standards Deconstruction Project

2008

Version 2.0

Version 2.0 produced by Eric Preibisius, Bonnie Boswell, Nemie Capacia, Dolores Dreher, Jim Hartman, Peg Hovde, Terrie Nichols, Michael Orr, Shahrokh Parvini, Terrie Teegarden, Mailei Vargas

A note to the reader: This project was coordinated and funded by the California Partnership for Achieving Student Success (Cal-PASS). Cal-PASS is a data sharing system linking all segments of education. Its purpose is to improve student transition and success from one educational segment to the next.

The original version, Version 1.0, of this document was first published in 2005. This current version, Version 2.0, includes updates based on feedback from faculty across the state. Also a listing of the Algebra I California Content Standards organized into clusters has been added in Appendix #1.

Cal-PASS is unique in that it is the only data collection system that spans and links student performance and course-taking behavior throughout the education system— K–12, community college, and university levels. Data are collected from multiple local and state sources and shared, within regions, with faculty, researchers, and educational administrators to use in identifying both barriers to successful transitions and strategies that are working for students. These data are then used regionally by discipline-specific faculty groups, called "Intersegmental Councils," to better align curriculum.

This Algebra 1 deconstruction project was initiated by the faculty serving on the math intersegmental councils after reviewing data on student transition. A deconstruction process was devised by the participating faculty with suggestions from the San Bernardino County Unified School District math faculty (Chuck Schindler and Carol Cronk) and included adaptations of the work of Dr. Richard Stiggins of the Assessment Training Institute and Bloom's *Taxonomy of Educational Objectives* (Bloom, B. S., 1984, Boston: Allyn and Bacon). The following document represents a comprehensive review by K–16 faculty to deconstruct and align Algebra 1 (Elementary Algebra) standards.

In order to continue the collaboration on these standards, thus improving on the current work, we invite and encourage the reader to provide feedback to us. Please contact Dr. Shelly Valdez at: **svaldez@calpass.org.**

Table of Contents

Standards 1–23 are presented in numerical order; each includes the following:
➢ Standard as written in the California DOE Algebra I Content Standards
➢ Deconstructed standard
➢ Prior knowledge needed by the student to begin learning the standard
➢ New knowledge student will need in order to achieve mastery of the standard
➢ Categorization of educational outcomes
➢ Necessary physical skills required
➢ Assessable result of the standard

    Assessment Items
➢ Computational and Procedural Skills
➢ Conceptual Understanding
➢ Problem Solving/Application

Algebra I California Content Standards by Clusters

Developing Learning Targets for Algebra Standards
(Deconstruction instructions)

Categorization of Educational Outcomes
(Explanation and instructions)

Cal-PASS Algebra I California Content Standards Deconstruction Project Version 2.0

California Algebra I Content Standards

CALIFORNIA CONTENT STANDARDS: ALGEBRA I
Standard Set 1.0: Students identify and use the arithmetic properties of subsets of integers and rational, irrational, and real numbers, including closure properties, for the four basic arithmetic operations where applicable.   1.1: Students use properties of numbers to demonstrate whether assertions are true or false.
2.0*: Students understand and use such operations as taking the opposite, finding the reciprocal, taking a root, and raising to a fractional power. They understand and use the rules of exponents.
3.0: Students solve equations and inequalities involving absolute values.
4.0*: Students simplify expressions prior to solving linear equations and inequalities in one variable, such as $3(2x–5) + 4(x–2) = 12$.
5.0*: Students solve multi-step problems, including word problems, involving linear equations and linear inequalities in one variable and provide justification for each step.
6.0*: Students graph a linear equation and compute the x- and y- intercepts (e.g., graph $2x+ 6y= 4$). They are also able to sketch the region defined by linear inequality (e.g., they sketch the region defined by $2x+ 6y< 4$).
7.0*: Students verify that a point lies on a line, given an equation of the line. Students are able to derive linear equations using the point-slope formula.
8.0: Students understand the concepts of parallel lines and perpendicular lines and how those slopes are related. Students are able to find the equation of a line perpendicular to a given line that passes through a given point.
9.0*: Students solve a system of two linear equations in two variables algebraically and are able to interpret the answer graphically. Students are able to solve a system of two linear inequalities in two variables and to sketch the solution sets.
10.0*: Students add, subtract, multiply, and divide monomials and polynomials. Students solve multi-step problems, including word problems, by using these techniques.
11.0: Students apply basic factoring techniques to second- and simple third-degree polynomials. These techniques include finding a common factor for all terms in a polynomial, recognizing the difference of two squares, and recognizing perfect squares of binomials.
12.0*: Students simplify fractions with polynomials in the numerator and denominator by factoring both and reducing them to the lowest terms.
13.0*: Students add, subtract, multiply, and divide rational expressions and functions. Students solve both computationally and conceptually challenging problems by using these techniques.
14.0*: Students solve a quadratic equation by factoring or completing the square.
15.0*: Students apply algebraic techniques to solve rate problems, work problems, and percent mixture problems.

1

16.0: Students understand the concepts of a relation and a function, determine whether a given relation defines a function, and give pertinent information about given relations and functions.
17.0: Students determine the domain of independent variables and the range of dependent variables defined by a graph, a set of ordered pairs, or a symbolic expression.
18.0: Students determine whether a relation defined by a graph, a set of ordered pairs, or a symbolic expression is a function and justify the conclusion.
19.0*: Students know the quadratic formula and are familiar with its proof by completing the square.
20.0*: Students use the quadratic formula to find the roots of a second-degree polynomial and to solve quadratic equations.
21.0*: Students graph quadratic functions and know that their roots are the *x*-intercepts.
22.0: Students use the quadratic formula or factoring techniques or both to determine whether the graph of a quadratic function will intersect the *x*-axis in zero, one, or two points.
23.0*: Students apply quadratic equations to physical problems, such as the motion of an object under the force of gravity.
24.0: Students use and know simple aspects of a logical argument. 24.1: Students explain the difference between inductive and deductive reasoning and identify and provide examples of each. 24.2: Students identify the hypothesis and conclusion in logical deduction. 24.3: Students use counterexamples to show that an assertion is false and recognize that a single counterexample is sufficient to refute an assertion.
25.0: Students use properties of the number system to judge the validity of results, to justify each step of a procedure, and to prove or disprove statements. 25.1: Students use properties of numbers to construct simple, valid arguments (direct and indirect) for, or formulate counterexamples to, claimed assertions. 25.2: Students judge the validity of an argument according to whether the properties of the real number system and the order of operations have been applied correctly at each step. 25.3: Given a specific algebraic statement involving linear, quadratic, or absolute value expressions or equations or inequalities, students determine whether the statement is true sometimes, always, or never.

Standard #1

1.0 Students identify and use the arithmetic properties of subsets and integers and rational, irrational, and real numbers, including closure properties for the four basic arithmetic operations where applicable.

1.1 Students use properties of numbers to demonstrate whether assertions are true or false.

Deconstructed Standard

1. Students identify arithmetic properties of subsets of the real number system including closure for the four basic operations.
2. Students use arithmetic properties of subsets of the real number system including closure for the four basic operations.
3. Students use arithmetic properties of numbers to demonstrate whether assertions are true or false.

Prior Knowledge Necessary

Students should know how to:
- ➢ identify the subsets of the real numbers system.
- ➢ use the commutative property.
- ➢ use the associative property.
- ➢ use the distributive property.
- ➢ informally use the addition property of equality.
- ➢ informally use the multiplication property of equality.
- ➢ informally use additive inverses.
- ➢ informally use multiplicative inverses.

New Knowledge

Students will need to learn to:
- ➢ apply arithmetic properties of the real number system when simplifying algebraic expressions.
- ➢ use arithmetic properties to justify each step in the simplification process.
- ➢ apply arithmetic properties of the real number system when solving algebraic equations and inequalities.
- ➢ use arithmetic properties to justify each step in solving equations and inequalities.
- ➢ identify which arithmetic property of a subset of the real numbers has been applied.
- ➢ identify whether or not an arithmetic property of a subset of the real number system has been properly applied.
- ➢ use the closure property.

Categorization of Educational Outcomes
Competence Level: Application
1. Students will identify arithmetic properties of subsets of the real number system including closure for the four basic operations.
2. Students use arithmetic properties of subsets of the real number system including closure for the four basic operations.
3. Students use arithmetic properties of numbers to justify whether assertions are true or false.

Necessary New Physical Skills
None

Assessable Result of the Standard
1. Students will provide examples and counterexamples to support or disprove assertions about arithmetic properties of subsets of the real number system.
2. Students will use arithmetic properties of subsets of the real number system to justify simplification of algebraic expressions.
3. Students will use arithmetic properties of subsets of the real number system to justify steps in solving algebraic equations.

Standard #1 Model Assessment Items

(Much of this standard is embedded in problems that are parts of other standards. Some of the examples below are problems that are from other standards that also include components of this standard.)

Computational and Procedural Skills
1. State the error made in the following distribution. Then complete the distribution correctly.
    $$-4(x+2) = -4x + 2$$

2. Solve the following equation and state the properties you used in each step.
    $$3(x-2)-(x+5) = -22$$

3. Which of the following sets of numbers are not closed under addition?
    - A. the set of real numbers
    - B. the set of irrational numbers
    - C. the set of rational numbers
    - D. the set of negative integers

4. Name the property illustrated in each statement, then find the missing number that makes it correct:
    - A. $-3+(5+1.5) = (-3+\_\_\_\_)+1.5$
    - B. $\dfrac{1}{8} \cdot 7 = 7 \cdot \_\_\_\_\_$

5. Name the property illustrated in each statement:
    - A. $9 \cdot \dfrac{1}{9} = 1$
    - B. $8+0 = 8$
    - C. $6(4+x) = 24 + 6x$

Conceptual Understanding
1. Show the following is true. If not true, give an example that shows it is not true.
    If two rational numbers are averaged, the result is a rational number.

2. Prove or give a counterexample:
    $x + x = x^2$ for all real numbers *x*.

Problem Solving/Application
1. The sum of three consecutive even integers is –66. Find the three integers.

Cal-PASS Algebra I California Content Standards Deconstruction Project Version 2.0

Standard #2

Standard Set 2.0
Students understand and use such operations as taking the opposite, finding the reciprocal, taking a root, and raising to a fractional power. They understand and use the rules of exponents.

Deconstructed Standard
1. Students understand the operation of taking the opposite.
2. Students use the operation of taking the opposite.
3. Students understand the operation of finding the reciprocal.
4. Students use the operation of finding the reciprocal.
5. Students understand the operation of taking a root.
6. Students use the operation of taking a root.
7. Students understand the operation of raising to a fractional power.
8. Students use the operation of raising to a fractional power.
9. Students understand the rules of exponents.
10. Students use the rules of exponents.

Prior Knowledge Necessary
Students should know how to:
   ➢ complete computations with rational numbers.
   ➢ identify the opposite of a number.
   ➢ find the reciprocal of a number.
   ➢ take the square root of a perfect square.
   ➢ estimate the value of the square root of a non-perfect square.
   ➢ simplify arithmetic expressions using the exponent rules.
   ➢ simplify simple algebraic expressions using the exponent rules.

New Knowledge
Students will need to learn to:
   ➢ use opposites with variable terms: such as $(-x) + (x) = 0$.
   ➢ use the reciprocal as a tool for solving any multiplication or division component of an equation such as:

Old Method	New Method
$3x = -9$	$3x = -9$
$\dfrac{3x}{3} = \dfrac{-9}{3}$	$\dfrac{1}{3}(3x) = \dfrac{1}{3}(-9)$
$x = -3$	$x = -3$

   ➢ extend current knowledge of square roots to a deeper understanding of the concept of square roots and the rational number system.
   ➢ simplify square roots that are not perfect squares.
   ➢ evaluate expressions with fractional exponents.
   ➢ convert between fractional exponents and radical notation.
   ➢ use the rules of exponents.

➢ simplify complex computations with monomials.

Categorization of Educational Outcomes
Competence Level: Comprehension and Application
1. Students will be able to demonstrate their understanding of and ability to use:
   A. taking the opposite.
   B. finding the reciprocal.
   C. taking a root.
   D. raising an expression to a fractional power.
   E. applying the rules of exponents.

Necessary New Physical Skills
None

Assessable Result of the Standard
1. Students will simplify arithmetic expressions by using the rules of exponents and the operations of taking a root and raising to a fractional power.
2. Students will simplify algebraic expressions by using the rules of exponents and the operations of taking an opposite, finding a reciprocal, taking a root, and raising to a fractional power.
3. Students will solve algebraic equations by using the rules of exponents and the operations of taking an opposite, finding a reciprocal, taking a root, and raising to a fractional power.

Standard #2 Model Assessment Items

(Most of the problems students will complete in relation to this standard will be embedded in problems that are parts of other standards. For example, problems that involve solving multi-step linear equations, quadratics, and rational expressions all require the components of this standard to complete.)

Computational and Procedural Skills
1. Released item from CAHSEE:
    If $x = -7$, then $-x =$

        A. –7   C. 1/7

        B. –1/7   D. 7

2. Released item from the *Algebra I CST*:
    Simplify the following: $\sqrt{16} + \sqrt[3]{8}$

        A. 4   C. 9

        B. 6   D. 10

3. Problem from the *Mathematics Framework for California Public Schools* :

    Write as a power of x: $\dfrac{\sqrt{x}}{x\sqrt[3]{x}}$

4. Solve the following: $(x+1)^2 = 25$

5. Solve the following: (Leave answer in exact form) $(x-3)^2 = 27$

6. Simplify: $\left(2x^3\right)^4$

7. Simplify: $3x^{-2}$

8. Simplify: $\left(\dfrac{4x}{12x^2 y}\right)^{-2}$

9. Write the following in simplified radical form: $x^{\frac{2}{3}}$

Conceptual Understanding
1. Problem from the *Mathematics Framework for California Public Schools*:

    What must be true about the real number x if $x = \sqrt{x^2}$?

2. For what value of x is the following true: $x = -x$?

Cal-PASS Algebra I California Content Standards Deconstruction Project Version 2.0

Problem Solving/Application

1. Released item from CAHSEE:

   The perimeter, P, of a square may be found by using the formula $\left(\dfrac{1}{4}\right)P = \sqrt{A}$, where A is the area of the square. What is the perimeter of the square with an area of 36 square inches?

      A   9 inches   C   24 inches

      B   12 inches   D   72 inches

2. Problem from the *Mathematics Framework for California Public Schools*:
   I start with a number and apply a four-step process: I add 13, multiply by 2, take the square root and then take the reciprocal. The result is ¼. What number did I start with?

Standard #3

Standard 3.0
Students solve equations and inequalities involving absolute values.

Deconstructed Standard
1. Students solve equations involving absolute.
2. Students solve inequalities involving absolute value.

Prior Knowledge Necessary
Students should know how to:
- ➤ solve basic algebraic equations.
- ➤ solve basic algebraic inequalities.
- ➤ interpret the absolute value of a number as a distance from zero on a number line.
- ➤ find the absolute value of a number.
- ➤ evaluate absolute value expressions for a given value of the variable.
- ➤ graph solutions to an inequality on the real number line.
- ➤ solve compound inequalities.

New Knowledge
Students will need to learn to:
- ➤ solve absolute value equations.
- ➤ solve absolute value inequalities.
- ➤ translate an absolute value inequality into a compound inequality.
- ➤ represent the solution set to an absolute value inequality by graphing the solution set on the real number line.
- ➤ represent the solution set to an absolute value inequality by writing an appropriate inequality (union or intersection).

Categorization of Educational Outcomes
Competence Level: Application
1. Students will classify solution sets to absolute value equations/inequalities as the union or intersection of two sets. (Or/And)
2. Students will solve absolute value equations/inequalities and graph the solution set.

Necessary New Physical Skills
None

Assessable Result of the Standard
1. Students will solve absolute value equations and graph the solution set.
2. Students will translate an absolute value inequality into a compound inequality and solve, and graph, the solution set.

Standard #3 Model Assessment Items

Computational and Procedural Skills
1. Solve each equation/inequality and graph the solution set:

    A. $|2x - 3| = 2$

    B. $|x - 3| = 2$

    C. $|x + 2| \leq 12$

    D. $4|x + 2| > 16$

    E. $\left|\dfrac{x - 2}{3}\right| \leq 2$

    F. $-|x + 4| \leq -2$

    G. $|2x - 3| > 2$

Conceptual Understanding
1. $|a - b|$ is the distance between *a* and *b* on the real number line. For any number x, what do $|x + 1|$ and $|x - 3|$ represent?

Problem Solving/Application
1. You work in the quality control department of a manufacturing company. The diameter of a drill bit must be between 0.74 inch and 0.75 inch.
   - A. Write an absolute value inequality which mathematically represents this requirement.
   - B. A drill bit has a diameter of 0.743 inch. Does this particular drill bit meet the stated requirement?

2. On Katie's recreational basketball team, the starting players' scoring averages are between 11 and 25 points per game. Write an absolute-value inequality which mathematically represents the scoring averages for the players on Katie's recreational basketball team.

3. The test scores in your class range from 63 to 97. Write an absolute-value inequality which describes the range of the test scores in your class.

Cal-PASS Algebra I California Content Standards Deconstruction Project Version 2.0

Standard #4

Standard Set 4.0

Students simplify expressions prior to solving linear equations and inequalities in one variable, such as $3(2x-5)+4(x-2)=12$.

Deconstructed standard

1. Students simplify expressions prior to solving linear equations in one variable. (Such as the left side of the sample equation in the standard given above)
2. Students simplify expressions prior to solving linear inequalities in one variable.

Prior knowledge necessary

Students should know how to:
  - ➢ add, subtract, multiply, and divide rational numbers (integers, fractions and terminating decimals).
  - ➢ use algebraic terminology (e.g., variable, equation, term, coefficient).
  - ➢ apply algebraic order of operations and arithmetic properties.
  - ➢ to simplify expressions.

New knowledge

Students will need to learn to:
  - ➢ use the distributive property and combine like terms to simplify algebraic expressions.

Categorization of Educational Outcomes

Competence Level: Application
  1. Students will use appropriate methods to combine like terms in an expression.
  2. Students will use the distributive property to multiply a rational number through a binomial.
  3. Students will use both combining like terms and the distributive property to simplify an algebraic expression.

Necessary New Physical Skills

None

Assessable Result of the Standard

  1. Students will produce a completely simplified expression.

Standard #4 Model Assessment Items

Computational and Procedural Skills
1. Simplify the following expressions:

    A. $a + 2 - 5a - 8$

    B. $-\dfrac{5}{2}x - 6 + \dfrac{1}{4}x - 5$

    C. $5(3x - 4)$

    D. $-3(2x - 5) + 3x$

    E. $-2(3x - 1) - 2(5x - 4)$

    F. $6\left(\dfrac{1}{3}x - \dfrac{1}{2}\right) + 4\left(\dfrac{1}{8}x + \dfrac{3}{4}\right)$

    G. $4 - (3x - 1) - 5(x + 5)$

Conceptual Understanding
1. Choose the equivalent equation to the following:
    $-4(8 + 3x) + 5(3x + 5) = 5$

    A. $3x + 57 = 5$

    B. $32x + 57 = 5$

    C. $3x - 7 = 5$

    D. $32x - 7 = 5$

2. Completely simplify the left side of the given inequality: (Do not solve!)
    $5x - 2(7x + 1) \geq 14x$

Problem solving/Application
None

Cal-PASS Algebra I California Content Standards Deconstruction Project Version 2.0

Standard #5

Standard Set 5.0
Students solve multi-step problems, including word problems, involving linear equations and linear inequalities in one variable and provide justification for each step.

Deconstructed standard
1. Students solve multi-step linear equations in one variable.
2. Students solve multi-step linear inequalities in one variable.
3. Students set-up word problems involving linear equations.
4. Students solve word problems involving linear equations.
5. Students set-up word problems involving linear inequalities.
6. Students solve word problems involving linear inequalities.
7. Students justify the use of the property at each step when solving a multi-step linear equation.
8. Students justify the use of the property at each step when solving a multi-step linear inequality.

Prior knowledge necessary
Students should know how to:
> ➢ simplify algebraic expressions using arithmetic properties (see Algebra I, Standard #4).
> ➢ correctly identify and use inequality symbols.
> ➢ translate verbal phrases into algebraic expressions.
> ➢ solve one-step algebraic equations and inequalities
> ➢ use basic geometric formulas such as perimeter of a square and/or rectangle.

New knowledge
Students will need to learn to:
> ➢ isolate the variable in multi-step linear equations and inequalities.
> ➢ apply the multiplicative property of inequalities.
> ➢ justify each step when solving a linear equation or inequality.
> ➢ translate word problems into multi-step linear equations or inequalities.
> ➢ interpret the results from a multi-step word problem.
> ➢ apply standard formulas such as distance, rate, time.

Categorization of Educational Outcomes
Competence Level: Application
1. Students will use methods they have learned to solve multi-step linear equations and inequalities.
2. Students will set up and solve a variety of multi-step word problems involving linear equations or inequalities.
3. Students will be able to correctly justify each step when solving multi-step liner equations or inequalities.

Necessary New Physical Skills
None

Assessable Result of the Standard
1. Students will find solutions to multi-step linear equations and inequalities.
2. Students will find solutions to a variety of word problems involving multi-step linear equations and inequalities.
3. Students will provide justification for each step in the solution of linear equations and inequalities.

Standard #5 Model Assessment Items

Computational and Procedural Skills
1. Solve the following equations and/or inequalities:

    A. $2x+3=15$ F. $-3(x+4)<9$

    B. $\dfrac{1}{4}x-3=-8$ G. $4x-7\leq5x+3$

    C. $-4(m+6)=-36$ H. $5(x-1)>\dfrac{5}{4}x$

    D. $\dfrac{x-8}{4}-3=-2$ I. $8-2y\geq4-y$

    E. $5x+2(1-x)=2(2x-1)$

Conceptual Understanding
1. Which is the first incorrect step in the solution of the following inequality?

$$-3(x+4)+2>5$$

    A. $-3(x+4)>3$

    B. $-3x-12>3$

    C. $-3x>15$

    D. $x>-5$

2. Justify each step used in solving the given equation:

$$4(x+3)-2(x-1)=30$$

    A. $4x+12-2x+2=30$ _____

    B. $2x+14=30$ _____

    C. $2x=16$ _____

    D. $x=8$ _____

3. Justify each step used in solving the given inequality:

$4 - (x + 3) \le 10$

    A. $4 - x - 3 \le 10$ _____

    B. $-x + 1 \le 10$ _____

    C. $-x \le 9$ _____

    D. $x \ge -9$ _____

Problem Solving/Application

1. The length of a rectangle is six less than twice the width. Its perimeter is 36 inches. Find the dimensions of the rectangle.

2. The greater of two consecutive integers is 15 more than twice the smaller. Find the integers.

3. Jan hiked up a hill at 4 mi/hr and back down at 6 mi/hr. Her total hiking time was 3 hours. How long did the trip up the hill take her?

4. The sum of two consecutive positive integers is at most 18. What are the integers?

5. For a particular phone company a long distance phone call costs $3.25 for the first three minutes plus $0.25 per minute for each minute or fractional part of a minute after the first three minutes. If "x" represents the number of minutes of the length of a call after the first three minutes, then 3.25 + 0.25x represents the cost of the call. If a particular customer of this phone company has $7.50 to spend on a call, what is the maximum total time he can use to make a long distance call?

6. The total cost of renting a bike at the beach for "n" hours is give by the following: $5.00 + 1.50n = C$ If the total cost for the rental is $11.00, how many hours did you rent the bike?

7. In the 2002 Winter Olympics, the United States won 10 more medals than Norway. The two countries won a total of 58 medals. How many medals did each country win? (Source: U.S. Olympic Committee)

Cal-PASS Algebra I California Content Standards Deconstruction Project Version 2.0

Standard #6

Standard Set 6.0
Students graph a linear equation and compute the *x*- and *y*-intercepts (e.g., $2x + 6y = 4$). They are also able to sketch the region defined by a linear inequality (e.g., they sketch the region defined by: $2x + 6y < 4$).

Deconstructed Standard
1. Students graph linear equations.
2. Students compute the *x*-intercept for a linear equation.
3. Students compute the *y*-intercept for a linear equation.
4. Students use intercepts to graph linear equations.
5. Students sketch the region defined by a linear inequality.

Prior Knowledge Necessary
Students should know how to:
  ➢ graph ordered pairs.
  ➢ graph a linear equation using a t-chart (x-y table).
  ➢ evaluate a linear equation for a given *x* or *y* value.
  ➢ solve one-variable linear inequalities.
  ➢ graph the solution set for a one-variable linear inequality.
  ➢ verify that any element in the solution set of a one-variable inequality satisfies the original inequality.

New Knowledge
Students will need to learn to:
  ➢ plot the *y*-intercept (given the slope/intercept form of a line, $y = mx + b$), and then use the slope to find a second point in order to complete the graph of the line.
  ➢ identify the graphical representation of (*a* , 0) as the *x*-intercept, and (0 , *b*) as the *y*-intercept.
  ➢ compute the *x*-intercept and *y*-intercept given a linear equation.
  ➢ identify that the linear equation implied by the linear inequality forms a boundary for the solution set and that this boundary may or may not be included in the final graph.
  ➢ interpret the inequality symbol to determine whether or not the boundary is solid or dashed.
  ➢ identify and shade the region of the graph that contains the solutions to the inequality.
  ➢ recognize that linear inequalities have multiple ordered-pair solutions.

Cal-PASS Algebra I California Content Standards Deconstruction Project Version 2.0

Categorization of Educational Outcomes
Competence Level: Application
1. Students will use methods they have learned to graph lines, solve inequalities, and to locate and/or identify the *x*- and *y*-intercepts for a given equation or graph.
2. Students will calculate *x*- and *y*-intercepts.
3. Students will demonstrate their ability to use *x*- and *y*-intercepts in the context of graphing.
4. Students will solve linear inequalities in two variables.
5. Students will show that they know the correct interpretation of the boundary line for the solution of a linear inequality by appropriately making the boundary solid or dashed.

Necessary New Physical Skills
1. Use of a straight edge.

Assessable Result of the Standard
1. Students will produce the graph of a line.
2. Students will produce the ordered pairs representing the *x*- and *y*-intercepts.
3. Students will produce a bounded and shaded region of the *x-y* plane representing the solution set of a linear inequality in two variables.
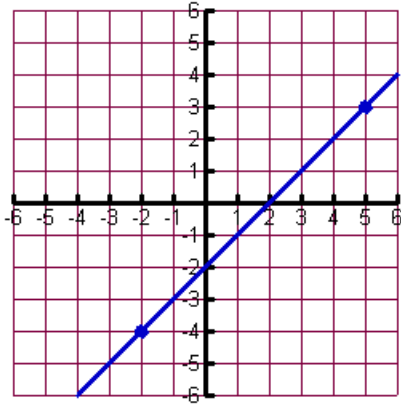
# Standard #6 Model Assessment Items

Computational and Procedural Skills

1. Find the *x*- and *y*-intercepts for the line defined by the following equation:
   $$2x + 3y = 9$$

2. Use the *x*- and *y*-intercepts to graph the line given by the equation above:
   $$2x - 3y = 6$$

3. Graph the following lines using the method of your choice. Identify and label the *x*- and *y*-intercepts for each graph if they exist:

   A. $3x - 5y = 10$

   B. $y = -\dfrac{2}{3}x + 4$

   C. $y = 2$

   D. $x = 3.5$

   E. $2x + 4y = 3$

   F. $\dfrac{1}{2}x - \dfrac{3}{4}y = 2$

4. Graph the solution set for the following inequalities:

   A. $2x - 3y < 6$

   B. $y \geq -\dfrac{3}{4}x + 2$

   C. $\dfrac{1}{2}x - \dfrac{2}{3}y \leq \dfrac{5}{6}$

   D. $y < 3$

   E. $x \geq -2$

Cal-PASS Algebra I California Content Standards Deconstruction Project Version 2.0

Conceptual Understanding

1. Sketch the graph of a line that has no *x*-intercept.

2. Identify the *x*- and *y*-intercepts from the graph of the given line.

3. Can a line have more than one *x*-intercept? Explain your answer using a diagram.

4. The solution to an inequality has been graphed correctly below. Insert the correct inequality symbol in the inequality below to match the graph of the solution. (Everything else about the inequality is correct—it just needs the correct symbol. Note: Assume that the line graphed is a solid line).
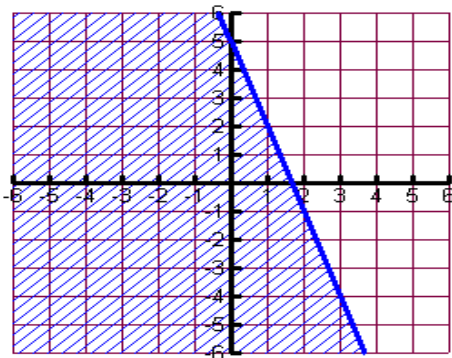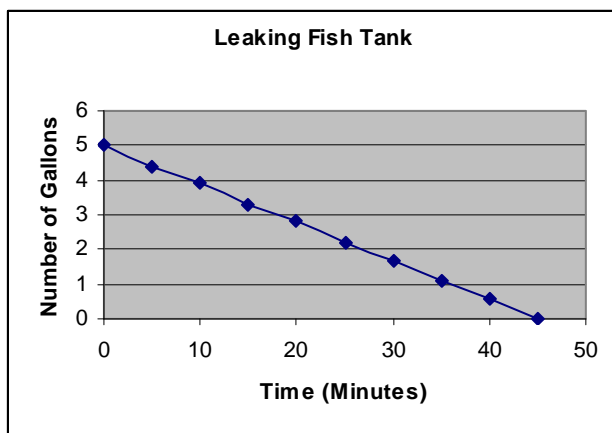

$y \ \boxed{} \ -3x+5$

Insert correct symbol in box.

5. When is it advantageous to use the *x*- and *y*-intercepts to graph the equation of a line? When would it perhaps be easier or better to use another graphing method? Give an example to illustrate your answers to both of these questions.

Cal-PASS Algebra I California Content Standards Deconstruction Project Version 2.0

Problem Solving/Application

1. The graph displayed below is the graph of the following equation: $y = \left(-\dfrac{1}{9}\right)x + 5$, where x represents the amount of time that has passed since a 5 gallon fish tank began to leak, and y represents the number of gallons of water in the tank after the leak began.
   - A. What is the significance of the x-intercept in this situation? What information is given to us by this point?
   - B. What is the significance of the y-intercept in this situation? What information is given to us by this point?

Leaking Fish Tank

2. The cost of a trash pickup service is given by the following formula:
   $y = 1.50x + 11$, where x represents the number of bags of trash the company picks up, and y represents the total cost to the customer for picking up the trash.
   - A. What is the y-intercept for this equation?
   - B. What is the significance of the y-intercept in this situation? What does it tell us about this trash pickup service?
   - C. Draw a sketch of the graph which represents this trash pickup service.

Standard #7

Standard Set 7.0
Students verify that a point lies on a line, given an equation of the line. Students are able to derive linear equations using the point-slope formula.

Deconstructed Standard
1. Given a point, students will determine if the point lies on a given line.
2. Students will derive the equation of a line given sufficient information about the line (i.e. two points, one point and a slope, etc).

Prior Knowledge Necessary
Note: Knowledge from Standard #6, Algebra I, is necessary.
Students should know how to:
   ➢ evaluate an algebraic expression in two variables for given values of the variables.
   ➢ simplify algebraic expressions - particularly by using the distributive property.
   ➢ define the slope of a line as the ratio of the change in y with respect to the change in x.
   ➢ calculate the slope of a line given two points on the line.

New Knowledge
Students will need to learn to:
   ➢ verify that an ordered pair of values satisfies the equation of a line in two variables.
   ➢ write an equation of the line in standard form, point-slope form, and/or slope-intercept form, given the slope of a line and one point on the line.
   ➢ write the equation of the line in standard form, point-slope form, and/or slope-intercept form, given the slope of a line and the y-intercept.
   ➢ write the equation of the line in standard form, point-slope form, and/or slope-intercept form, given two points on the line.
   ➢ write the equation of the line in standard form, point-slope form, and/or slope-intercept form, given the graph of a line.

Categorization of Educational Outcomes
Competence Level: Application
   1. Students will choose the appropriate form of the equation of a line, given certain data, to write the equation of that line.
   2. Students will apply previously acquired knowledge and skills to write the equation of a line.
   3. Students will verify whether or not a point lies on a line.

Necessary New Physical Skills
None

Assessable Result of the Standard

Cal-PASS Algebra I California Content Standards Deconstruction Project Version 2.0

1. Students will produce the equation of a line in standard form, point-slope form, and/or slope-intercept form.
2. Students will produce a linear model (equation) representing data from an application problem.

Standard #7 Model Assessment Items

Computational and Procedural Skills

1. Given the equation of a line, $y = \dfrac{3}{2}x - 1$, determine whether or not the following points lie on

   the line: $(4, 7)$, $(-6, -10)$, and $\left(\dfrac{1}{3}, -\dfrac{1}{2}\right)$.

2. Choose the most efficient form of the line (either point-slope or slope-intercept) to write the

   equation of the line with slope of $\dfrac{2}{3}$ and *y*-intercept of –2.

3. Choose the most efficient form of the line (either point-slope or slope-intercept) to write the
   equation of the line with a slope of 4 and passing through the point $(-2, 5)$.

4. Choose the most efficient form of the line (either point-slope or slope-intercept) to write the
   equation of the line passing through the points $(-3, 7)$ and $(4, -7)$.

5. Write the equation of the line graphed below.

Cal-PASS Algebra I California Content Standards Deconstruction Project Version 2.0
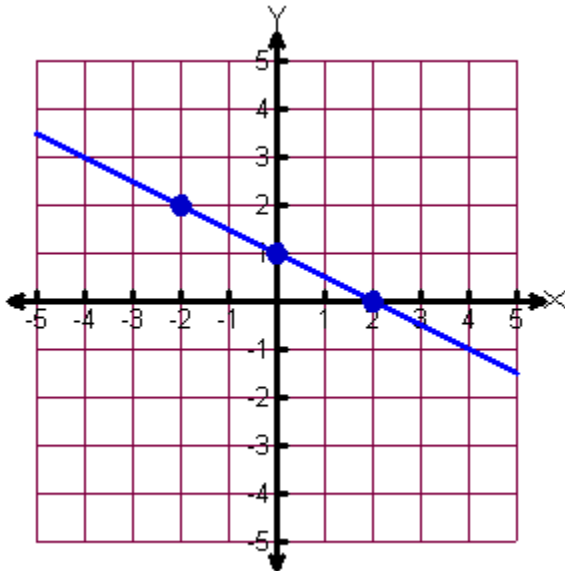
Conceptual Understanding

1. Consider the table of values below:

Episode Number	Number of Survivors
1	25
2	23
3	21

    A. Identify the input and output variables.
    B. Verify that the data is linear.
    C. Write the equation of the line representing the data.

Problem Solving/Application

1. A taxi driver charges a flat fee of $2.00 plus $3 for each mile traveled.
    A. Identify the input and output variables.
    B. Make a table for inputs 0, 1, and 2.
    C. Identify and interpret the meaning of the slope.
    D. Identify and interpret the meaning of the y-intercept.
    E. Write an equation describing the output as a function of the input.
    F. Use your equation to determine the cost of a 12-mile taxi ride.

Cal-PASS Algebra I California Content Standards Deconstruction Project Version 2.0

Standard #8

Standard Set

Students understand the concepts of parallel lines and perpendicular lines and how those slopes are related. Students are able to find the equation of a line perpendicular to a given line that passes through a given point.

Deconstructed Standard

1. Students understand concepts of parallel lines.
2. Students understand concepts of perpendicular lines.
3. Students understand the relationship between the slopes of parallel lines.
4. Students understand the relationship between the slopes of perpendicular lines.
5. Students determine the equation of a line perpendicular to a given line that passes through a given point.

Prior Knowledge Necessary

Students should know how to:
- ➢ identify the slope and *y*-intercept from the equation of a line.
- ➢ identify the slope and *y*-intercept from the graph of a line.
- ➢ identify the slope of a line given the equation of a line in various forms.
- ➢ determine the opposite reciprocal of a rational number.
- ➢ determine the equation of a line given a point and a slope.
- ➢ differentiate between parallel and perpendicular lines.

New Knowledge

Students will need to learn to:
- ➢ identify from a graph parallel lines as having the same slope and different *y*-intercepts.
- ➢ determine whether two lines are parallel based on their slopes and *y*-intercepts.
- ➢ determine whether two lines are perpendicular based on their slopes.
- ➢ determine the equation of a line parallel to a given line and passing through a point not on the line.
- ➢ determine the equation of a line perpendicular to a given line and passing through a point not on the line.

Categorization of Educational Outcomes

Competence Level: Knowledge
1. Students will identify pairs of lines as parallel, perpendicular, or neither by their slopes and/or *y*-intercepts.

Competence Level: Comprehension
1. Students will compare and contrast the relationships between parallel and perpendicular lines.

Competence Level: Application

1. Given an equation of a line and a point not on the line, a student will be able to calculate the equation of a parallel line through the given point.
2. Given an equation of a line and a point not on the line, a student will be able to calculate the equation of a line perpendicular to the given line passing through the given point.

Necessary New Physical Skills
None

Assessable Result of the Standard
1. Students will calculate the equation of a line parallel to a given line and passing through a given point not on the line.
2. Students will calculate the equation of a line perpendicular to a given line and passing through a given point not on the line.

Standard #8 Model Assessment Items

Computational and Procedural Skills

1. Write the equation of a line, in slope-intercept form, that passes through the point $P(3,-2)$ and is parallel to $5x+3y=7$.

2. Write the equation of a line, in slope-intercept form, that passes through the point $P(-12,4)$ and is perpendicular to $-6x+y=3$.

3. Given the equation $-5x+2y=10$, what is the slope of a line parallel to this line?

4. Given the equation $-5x+2y=10$, what is the slope of a line perpendicular to this line?

Conceptual Understanding

1. For the following pairs of lines, determine if each pair is parallel, perpendicular, or neither. Justify your answer algebraically!
    - A. $y=2x+4$ and $y-2x=1$
    - B. $2y-4x=9$ and $y=2x-5$
    - C. $y+x=5$ and $y-x=0$
    - D. $4y-3x=7$ and $3y+4x=15$
    - E. $y=5$ and $x=-4$

2. Given the points $(3,-5)$, $(8,4)$ lie on a line, and the points $(8,-9)$, $(13,0)$ lie on a second line. Are these two lines parallel, perpendicular, or neither? Justify your answer algebraically.

Problem Solving/Application
None

Standard #9

Standard Set 9.0
Students solve a system of two linear equations in two variables algebraically and are able to interpret the answer graphically. Students are able to solve a system of two linear inequalities in two variables and to sketch the solution sets.

Deconstructed Standard
1. Students solve systems of two linear equations in two variables algebraically.
2. Students interpret solutions to systems of two linear equations in two variables graphically.
3. Students solve systems of two linear inequalities in two variables by sketching the solution set.

Prior Knowledge Necessary
Students should know how to:
- identify points on a Cartesian coordinate system.
- solve linear equations in one variable.
- solve linear inequalities in one variable.
- graph linear equations in two variables.
- graph linear inequalities in two variables.
- isolate a variable in an equation of two variables.

New Knowledge
Students will need to learn to:
- solve systems of two linear equations in two variables by graphing.
- solve systems of two linear equations in two variables using substitution.
- solve systems of two linear equations in two variables using elimination/addition method.
- recognize when a system of linear equations has exactly one solution, an infinite number of solutions, or no solution.
- interpret the solution as representing parallel lines, the intersection of two lines or the same line.
- identify the intersection of two shaded areas created by the graphs of linear inequalities in two variables as the solution to the system.

Categorization of Educational Outcomes
Competence Level: Application
1. Students will use appropriate methods to solve systems of two equations in two variables.
2. Students will determine the solution of a linear system of two equations in two variables graphically.
3. Students will determine the solution set from the graph of a system of linear inequalities in two variables.

Necessary New Physical Skills
1. Use of a straight edge.

Assessable Result of the Standard

1. Students will find solutions to systems of linear equations in two variables.
2. Students will interpret the solution to a system of linear equations in two variables as no solution, one solution, or an infinite number of solutions.
3. Students will find solutions to systems of linear inequalities in two variables by sketching a graph.

Cal-PASS Algebra I California Content Standards Deconstruction Project Version 2.0

Standard #9 Model Assessment Items

Computational and Procedural Skills

1. Solve the system: $\begin{cases} 2x + y = 6 \\ -3x - 2y = -15 \end{cases}$ by the substitution method.

2. Solve the system: $\begin{cases} 3x + 5y = 18 \\ x - 2y = -5 \end{cases}$ by the elimination method.

3. Solve the system: $\begin{cases} 2x - 3 = y \\ y - 2x = 1 \end{cases}$ by the method of your choice.

4. Solve the system: $\begin{cases} x - 2y = 3 \\ 3x = 6y + 9 \end{cases}$ by the method of your choice.

5. What is the approximate solution to the system of linear equations graphed below?
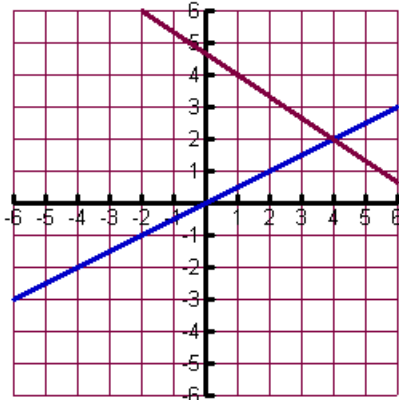

6. Solve the system of inequalities $\begin{cases} x + y < 7 \\ 2x - 3y \geq 12 \end{cases}$ graphically.

7. Solve the system of inequalities $\begin{cases} x > 4 \\ y < 2 \end{cases}$ graphically

Cal-PASS Algebra I California Content Standards Deconstruction Project Version 2.0

Conceptual Understanding

1. Suppose a system of two linear equations in two variables has a solution of $(3,5)$. What would the graph look like?

2. Suppose a system of two linear equations in two variables has infinite solutions in the form (x, y). When graphing these two equations on the same Cartesian coordinate system, what will the graph of the two lines to look like?

3. Solve the system: $\begin{cases} y - 3x = -3 \\ 3x + 2y = 12 \end{cases}$ by graphical method.

4. Solve the system: $\begin{cases} y = 3x + 6 \\ -6x + 2y = 12 \end{cases}$ by graphical method.

5. Solve the system: $\begin{cases} x + 2y = 8 \\ x = 4 - 2y \end{cases}$ by graphical method.

6. Select the system of inequalities that matches the given graph.
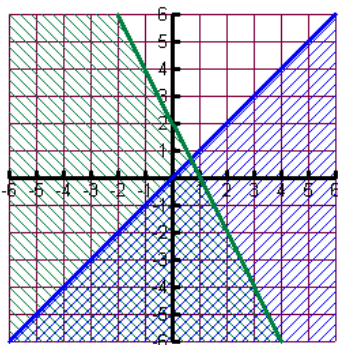
A. $y \leq x$
   $y \geq -2x + 2$

B. $y \geq x$
   $y \geq -2x + 2$

C. $y \geq x$
   $y \leq -2x + 2$

D. $y \leq x$
   $y \leq -2x + 2$

Problem Solving/Application

1. The sum of two numbers is 67. The difference of these same two numbers is 17. Find these two numbers.

2. Jose has 21 coins which are made up of only dimes and quarters. All together the coins are worth $4.05. How many of each coin does Jose have?

3. A certain theater charges $6.00 for adults to see a movie, and charges children only $3.00. On a certain day a total of $1554 was collected from 394 admissions. How many adults and how many children went to the movie on that day?

Cal-PASS Algebra I California Content Standards Deconstruction Project Version 2.0

Standard #10

Standard Set 10.0
Students add, subtract, multiply, and divide monomials and polynomials. Students solve multi-step problems, including word problems, by using these techniques.

Deconstructed Standard
1. Students add monomials and polynomials.
2. Students subtract monomials and polynomials.
3. Students multiply monomials and polynomials.
4. Students divide monomials and polynomials.
5. Students solve multi-step problems involving monomials and polynomials.
6. Students solve multi-step word problems involving monomials and polynomials.

Prior Knowledge Necessary
Students should know how to:
  - combine like terms.
  - use the distributive property to simplify expressions.
  - add, subtract, multiply, and divide rational numbers.
  - translate multi-step application problems to algebraic equations.
  - solve multi-step equations.

New Knowledge
Students will need to learn to:
  - add and subtract polynomials.
  - use the distributive property to multiply polynomials.
  - apply the appropriate exponential rules to simplify algebraic expressions.
  - divide polynomials by a monomial.
  - divide polynomials by binomials using long division.
  - translate and solve multi-step word problems involving polynomials.

Categorization of Educational Outcomes
Competence Level: Application
1. Extend the rules of arithmetic to monomials and polynomials.
2. Demonstrate the ability to translate word problems into polynomial equations, and solve the resulting equations algebraically.
3. Interpret the solutions to word problems.

Necessary New Physical Skills
None

Assessable Result of the Standard
1. Students will completely simplify polynomial expressions.
2. Students will find solutions to a variety of multi-step problems requiring the use of polynomials.

3. Students will find solutions to a variety of word problems requiring the use of polynomials.

Standard #10 Model Assessment Items

Computational and Procedural Skills

1. Perform the indicated operations and simplify completely:

A. $2x^3y^2 + 3x^3y^2 - 7x^3y^2$

B. $4x^5y^4\left(\dfrac{3}{8}x^2y\right)$

C. $3x^2(7x-2)$

D. $(3x+7)(2x-6)$

E. $(x-7)(x+7)$

F. $\dfrac{1}{2}xy(2y-4x^2)$

G. $(x-y)(x^2+xy+y^2)$

H. $(2x+1)^2$

I. $3x(2x-1)-(x-3)$

J. $5x^2(3x^2-2x-2)-4(x+7)$

K. $(x^2+3x+4)-(3x^2-5x-6)$

L. $(3x^2-2x+7)+(5x^2-6x-5)$

M. $(x+5)(x^2-x+3)$

N. $\dfrac{12x^3y^2}{4x^5y}$

O. $\dfrac{18x^2+12x+6}{6x}$

P. $\dfrac{2x^3-3x^2+7}{x-2}$

Q. $\dfrac{x^3+3x^2-x-12}{x+4}$

Conceptual Understanding

1. Is $(x+y)^2 = x^2+y^2$? Why or why not?

2. Explain the difference between "subtract x^2-2x+5 from $6x^2+11x-7$" and "subtract $6x^2+11x-7$ from x^2-2x+5". Is there a difference in the answers to these two problems? If so, what is it?

3. Is $x-y$ the same as $y-x$? Why or why not?

Cal-PASS Algebra I California Content Standards Deconstruction Project Version 2.0

Problem Solving/Application

1. The sum of two integers in 6. The second number is 14 more than the first. Find the smaller of the two integers.

2. The perimeter of a triangle is 23 cm. The medium side is three more than the shorter side. The longest side is twice the shorter side. Find the length of all three sides of this triangle.

3. The length of a square has been increased by 3 meters, and the width was decreased by 8 meters. The new area is 12 square meters. Find the dimensions of the original square.

Standard #11

Standard Set 11.0
Students apply basic factoring techniques to second- and simple third-degree polynomials. These techniques include finding a common factor for all terms in a polynomial, recognizing the difference of two squares, and recognizing perfect squares of binomials.

Deconstructed Standard
1. Students find a common factor for all terms in a polynomial.
2. Students apply basic factoring techniques to second-degree polynomials.
3. Students apply basic factoring techniques to simple third-degree polynomials.
4. Students recognize the difference of two squares.
5. Students recognize perfect squares of binomials.

Prior Knowledge Necessary
Students should know how to:
- ➢ identify the greatest common factor (GCF) of two or more numbers.
- ➢ identify and apply the distributive property of multiplication.
- ➢ factor a constant from a simple linear expression.
- ➢ multiply polynomials.
- ➢ identify the degree of a polynomial.

New Knowledge
Students will need to learn to:
- ➢ identify the greatest common factor (GCF) of a polynomial.
- ➢ factor a polynomial by factoring out the GCF.
- ➢ factor a second-degree polynomial as the product of two binomials. (Reverse FOIL method, Master Product method or AC method).
- ➢ factor a second or third-degree polynomial using grouping.
- ➢ recognize that a binomial is the difference of two squares.
- ➢ recognize that a polynomial is a perfect square trinomial.
- ➢ use GCF, difference of squares, and perfect squares of binomials to factor simple third-degree polynomials.

Categorization of Educational Outcomes
Competence Level: Knowledge
1. Students will identify the GCF of a polynomial.
2. Students will identify a second-degree polynomial as the product of two binomials.
3. Students will identify a polynomial as the difference of two squares.
4. Students will identify a polynomial as the square of a binomial.

Cal-PASS Algebra I California Content Standards Deconstruction Project Version 2.0

Competence Level: Application
1. Students will use methods of factoring the GCF from a polynomial.
2. Students will use the methods of grouping, Reverse FOIL, or other methods to factor a second-degree polynomial.
3. Students will demonstrate knowledge of the difference of two squares to factor a binomial.
4. Students will demonstrate knowledge of perfect square binomials to factor a polynomial.
5. Students will use a combination of one or more of the above methods to factor simple third-degree polynomials completely.

Necessary New Physical Skills
None

Assessable Result of the Standard
1. Students will generate factors of a polynomial.

Standard #11 Model Assessment Items

Computational and Procedural Skills

1. Factor the following completely. If not possible, write "Prime".

    A. $x^2 + 2x$

    B. $x^2 + 6x + 7$

    C. $m^2 - 25$

    D. $4z^2 + 24z - 13$

    E. $x^3 + 2x^2 - 3x$

    F. $3x^2 - 15x + 18$

    G. $x^2 + 5x + 2$

    H. $x^3 + 2x^2 - x - 2$

    I. $3x^2 - 27$

Conceptual Understanding

1. How can you check your answer when you factor a polynomial?

2. Given the expression $5y(2x - 3) + 8(2x - 3)$, is the expression completely factored? If not, what is the next step in factoring this expression.

3. If you are asked to completely factor the polynomial $3x^2 + 9x - 12$, why would it be incorrect to give $(x - 1)(3x + 12)$ as your answer?

Problem Solving/Applications

1. Find a value of b so that $x^2 + bx + 25 = (x + 5)^2$.

2. Find a so that $ay^2 - 12y + 4 = (3y - 2)^2$.

Standard #12

Standard Set 12.0
Students simplify fractions with polynomials in the numerator and denominator by factoring both and reducing them to the lowest terms.

Deconstructed standard
1. Students apply basic factoring techniques to factor completely the numerator and denominator of rational expressions.
2. Students simplify rational expressions by identifying common factors in the numerator and denominator and reducing them.

Prior knowledge necessary
Students should know how to:
  ➢ simplify numeric fractions by reducing common factors in the numerator and denominator.
  ➢ factor polynomials of the form $ax^2 + bx + c$ for $a = 1$ and $a \neq 1$.
  ➢ factor third degree polynomials by factoring the greatest common factor to reduce the polynomial to a second degree polynomial.
  ➢ recognize and factor the difference of perfect squares.
  ➢ recognize and factor trinomials as the square of a binomial.

New knowledge
Students will need to learn to:
  ➢ apply factoring techniques to the numerator and denominator of rational expressions.
  ➢ recognize when it is appropriate to factor out a negative one.
  ➢ recognize that $\dfrac{a}{a}$, for any expression a ($a \neq 0$) is equivalent to 1.
  ➢ identify common factors in the numerator and denominator of a rational expression.
  ➢ simplify rational expressions by canceling common factors in the numerator and denominator.
  ➢ recognize when a rational expression is undefined.

Categorization of Educational Outcomes
Competence Level: Knowledge
  1. Students will identify for what values a rational expression/function is undefined.
  2. Students will identify common factors in the numerator and denominator of a rational function.
  3. Students will recognize that the occurrence of a common factor in the numerator and denominator is equivalent to multiplying by 1 and hence cancel them.
  4. Students will be able to recognize when a rational expression has been reduced to its lowest terms.

Cal-PASS Algebra I California Content Standards Deconstruction Project Version 2.0

Competence Level: Application
1. Students will use methods of factoring polynomials to reduce rational expressions to their lowest terms.

Necessary New Physical Skills
None

Assessable Result of the Standard
1. Students will reduce rational expressions to their lowest terms,

Standard #12 Model Assessment Items

Computational and Procedural Skills

1. Applying factoring techniques to the numerator and the denominator of rational expressions.

   A. $\dfrac{x^3 - 9x}{x^4 - 81}$

   B. $\dfrac{x^2 + 10x + 25}{x^2 - 25}$

2. Identifying common factors in the numerator and denominator of a rational expression as equivalent to one. Identify the following as being equal to 1, -1 or neither.

   A. $\dfrac{x^2 + 8x + 15}{15 + x^2 + 8x}$

   B. $\dfrac{x^2 - 1}{1 - x^2}$

   C. $\dfrac{x - 1}{x + 1}$

3. Simplify the rational expression:

   A. $\dfrac{(x - 2)(x + 2)}{(x + 2)(x - 1)}$

   E. $\dfrac{x^3 - 9x}{x^4 - 81}$

   B. $\dfrac{x^2 + 10x + 25}{x^2 - 25}$

   F. $\dfrac{4x^2 - 12x + 9}{4x - 6}$

   C. $\dfrac{7x^3 + 13x^2 - 2x}{49x^2 - 14x + 1}$

   G. $\dfrac{16x^2 + 24x + 9}{4x^2 - x - 3}$

   D. $\dfrac{x + 3}{x^2 + 5x + 6}$

4. Identify those values (if any) for which the following rational expressions are undefined.

   A. $\dfrac{x + 2}{4 - x}$

   B. $\dfrac{x^2 - 4}{x^2 + 1}$

Cal-PASS Algebra I California Content Standards Deconstruction Project Version 2.0

Standard #13

Standard Set 13.0
Students add, subtract, multiply, and divide rational expressions and functions. Students solve both computationally and conceptually challenging problems by using these techniques.

Deconstructed standard
1. Students add rational expressions/functions.
2. Students subtract rational expressions/functions.
3. Students multiply rational expressions/functions.
4. Students divide rational expressions/functions.
5. Students solve computationally challenging problems involving equations with rational expressions.
6. Students solve conceptually challenging problems involving equations with rational expressions.

Prior knowledge necessary
Students should know how to:
- identify the least common denominator (LCD) of two or more fractions.
- simplify rational expressions.
- add, subtract, multiply, and divide fractions.
- identify when a rational expression is undefined.
- factor second-degree and simple third-degree polynomials.
- add, subtract, and multiply polynomials.
- solve linear and quadratic equations.
- identify when it is appropriate to factor out a -1.

New knowledge
Students will need to learn to:
- multiply rational expressions, simplifying the resulting rational expression into lowest terms.
- divide rational expressions, including complex fractions and simplifying the resulting expression to its lowest terms.
- determine the LCD for two or more rational expressions
- add rational expressions, using the LCD and simplify the resulting expression into lowest terms.
- subtract rational expressions, using the LCD and simplify the resulting expression into lowest terms.
- apply the above techniques/knowledge to rational equations.

Cal-PASS Algebra I California Content Standards Deconstruction Project Version 2.0

Categorization of Educational Outcomes
Competence Level: Application
1. Students will use methods of factoring polynomials to identify the LCD of rational expressions involving addition and subtraction.
2. Students will add, subtract, multiply, and divide rational expressions and simply the answers.
3. Students will use methods of factoring polynomials to simplify rational expressions when multiplying and dividing.

Necessary New Physical Skills
None

Assessable Result of the Standard
1. Students will generate reduced expressions for operations involving rational expressions.
2. Students will generate solutions for rational equations.

Standard #13 Model Assessment Items

Computational and Procedural Skills

1. Multiply. Write the answer in lowest terms.

    A. $\dfrac{3(p-q)}{p} \bullet \dfrac{q}{2(p-q)}$

    B. $\dfrac{x^2+3x}{x^2-3x-4} \bullet \dfrac{x^2-5x+4}{x^2+2x-3}$

2. Divide. Write the answer in lowest terms.

    A. $\dfrac{9p^2}{3p+4} \div \dfrac{6p^3}{3p+4}$

    B. $\dfrac{x^2-4}{x^2+x-6} \div \dfrac{x^2+5x+6}{-2x}$

    C.

3. Add. Write the answer in lowest terms.

    A. $\dfrac{2x}{x^2-1} + \dfrac{1}{x+1}$

    B. $\dfrac{2x}{x^2+5x+6} + \dfrac{x+1}{x^2+2x-3}$

4. Subtract. Write the answer in lowest terms.

    A. $\dfrac{2m}{m-n} - \dfrac{5m+n}{2m-2n}$

    B. $\dfrac{5}{x^2-9} - \dfrac{x+2}{x^2+4x+3}$

5. Simplify the complex fraction. Write the answer in lowest terms.

    $$\dfrac{\dfrac{4a^4b^3}{3a}}{\dfrac{2ab^4}{b^2}}$$

6. Solve the following equations.

    A. $\dfrac{3}{x}=5$

    C. $\dfrac{x+2}{x-4}=\dfrac{2}{3}$

    B. $\dfrac{5}{x+3}=\dfrac{8}{x-2}$

    D. $\dfrac{x^2-3x-4}{x+1}=0$

Conceptual Understanding

1. Why can't the denominator of a rational expression equal 0?

2. Explain how the LCD can be used in a different way when adding and subtracting rational expressions compared to solving equations with rational expressions.

3. Suppose that $(2x-5)^2$ is the LCD for two fractions. Is $(5-2x)^2$ also acceptable as an LCD? Why or why not?

Cal-PASS Algebra I California Content Standards Deconstruction Project Version 2.0

Problem Solving/Application

1. The Tickfaw River has a current of 3 mph. A motorboat takes as long to go 12 miles downstream as to go 8 miles upstream. What is the speed of the boat in still water?

2. In a certain fraction, the denominator is 6 more than the numerator. If 3 is added to both the numerator and the denominator, the resulting fraction is equivalent to $\frac{5}{7}$.
   What is the original fraction?

Standard #14

Standard Set 14.0
Students solve a quadratic equation by factoring or completing the square.

Deconstructed standard
1. Students solve a quadratic equation by factoring.
2. Students solve a quadratic equation by completing the square.

Prior knowledge necessary
Students should know how to:
➤ verify that a solution to an equation satisfies the original equation.
➤ multiply binomials.
➤ identify an equation as a quadratic equation.
➤ write a quadratic equation in standard form, $ax^2 + bx + c$.
➤ factor polynomials simple second- and third-degree polynomials using different techniques including factoring the greatest common factor (GCF), grouping, Reverse FOIL method, Master Product method or AC method, difference of squares, and perfect squares of binomials.
➤ identify a perfect square trinomial.
➤ evaluate the square roots of a perfect square.
➤ simplify a radical.

New knowledge
Students will need to learn to:
➤ apply the Principle of Zero Products in obtaining the solutions to a quadratic equation.
➤ identify quadratic equations that have no real solutions.
➤ use the Square Root Property to solve equations involving a squared binomial set equal to a constant (e.g. $(x+5)^2 = 9$).
➤ manipulate a quadratic equation in the form $ax^2 + bx + c = 0$ to produce a perfect square trinomial on one side of the equation.

Categorization of Educational Outcomes
Competency Level: Application
1. Students will use methods of factoring polynomials to solve quadratic equations.
2. Students will demonstrate the ability to use the technique of completing the square.
3. Students will use the method of completing the square of a quadratic polynomial to solve a quadratic equation.
4. Students will show that they know the correct interpretation of the roots or zeros of a polynomial equation.
5. Students will demonstrate the ability to identify the most efficient technique to solve a quadratic equation.

Necessary New Physical Skills
None

Assessable Result of the Standard
1. Students will generate solutions to a quadratic equation.

Standard #14 Model Assessment Items

Computational and Procedural Skills

1. Solve by factoring:

    A. $x^2 + 5x + 6 = 0$

    B. $4p^2 + 40 = 26p$

    C. $16m^2 - 25 = 0$

2. Solve by completing the square:

    A. $x^2 + 6x + 7 = 0$

    B. $2z^2 + 24z - 12 = 0$

    C. $(x+3)(x-4) = 2$

Conceptual Understanding

1. Factoring:

    A. In order to solve a quadratic equation by factoring, in what form must the equation be written?

    B. If the product of two binomial expressions (e.g. $(2x-1)(x+3)$) is set equal to zero, what does the Principle of Zero Products (or Zero-Factor Property) say about the two binomials?

2. Completing the square:

    A. In using the method of completing the square to solve $2x^2 - 10x = -8$, a student began by adding the square of half the coefficient of x (that is $\left[\frac{1}{2}(-10)\right]^2 = 25$) to both sides of the equation. He then encountered difficulty in his later steps. What was his error? Explain the steps needed to solve the problem, and give the solution set.

Problem Solving/Application

None

Standard #15

Students apply algebraic techniques to solve rate problems, work problems, and percent mixture problems.

Deconstructed standard
1. Students apply algebraic techniques to solve rate problems.
2. Students apply algebraic techniques to solve work problems.
3. Students apply algebraic techniques to solve percent mixture problems.

Prior knowledge necessary
Students should know how to:
- solve linear equations.
- solve rational equations.
- translate word problems into algebraic equations.

New knowledge
Students will need to learn to:
- set up and solve a rate problem using appropriate units.
- set up and solve a work problem using appropriate units.
- set up and solve a percent mixture problem using appropriate units.

Categorization of Educational Outcomes
Competence Level: Application
1. Students will write the appropriate equations and solve word problems, related to rate with appropriate units.
2. Students will write the appropriate equations and solve word problems, related to work with appropriate units.
3. Students will write the appropriate equations and solve word problems, related to percent mixture with appropriate units.

Competence Level: Analysis
1. Students will organize information given in a rate problem.
2. Students will organize information given in a work problem.
3. Students will organize information given in a percent mixture problem.

Necessary New Physical Skills
None

Assessable Result of the Standard
1. Students will find solutions to rate, work, and percent-mixture problems with appropriate units.

Standard #15 Model Assessment Items

Problem Solving/Applications

1. Setting up and solving a rate problem.

   A passenger train leaves Houston at a speed of 60 km/h. Two hours later a second passenger train leaves Houston going in the same direction as the first train on parallel tracks at 90 km/hr. How many hours later will the two train meet at the same spot?

2. Setting up and solving a work problem.

   Julie can wax her car in 2 hours. When she works together with Martha, they can wax the car in 45 minutes. How long would it take Martha working by herself, to wax the same car?

3. Setting up and solving a percent mixture problem.

   To make 30 gallons of fertilizer that is 11% potassium, a chemist mixes a 5% potassium solution with a 15% potassium solution. How much of each type of potassium solution is needed?

Standard #16

Standard Set 16.0
Students understand the concepts of a relation and a function, determine whether a given relation defines a function, and give pertinent information about given relations and functions.

Deconstructed standard
1. Students understand the definition of a relation.
2. Students understand the concept of a function.
3. Students determine whether or not a given relation is a function.
4. Students give pertinent information (the definition of domain and range) about a relation.
5. Students give pertinent information (the definition of domain and range) about a function.

Prior knowledge necessary
Students should know how to:
- ➤ identify an ordered pair.
- ➤ graph sets of ordered pairs.
- ➤ graph linear equations.

New knowledge
Students will need to learn to:
- ➤ identify a set of ordered pairs as a relation.
- ➤ identify the domain of a relation.
- ➤ identify the range of a relation.
- ➤ define a function.
- ➤ identify if a relation is a function.
- ➤ identify if an equation is a function.
- ➤ identify the domain of a function.
- ➤ identify the range of a function.

Categorization of Educational Outcomes
Competence Level: Knowledge
1. Students will identify the domain of a relation.
2. Students will identify the range of a relation.
3. Students will identify if a relation is a function.

Competence Level: Comprehension
1. Students will differentiate between sets of ordered pairs as those that are a function and those that are just a relation.
2. Students will differentiate between equations that are functions and those that are relations.

Competence Level: Application
1. Students will show that set of ordered pairs (a relation) satisfies or does not satisfy the definition of a function.
2. Students will show that an equation satisfies or does not satisfy the definition of a function.

Necessary New Physical Skills
Students should be capable of graphing a linear equation.

Assessable Result of the Standard
1. Students will identify if a relation is a function.
2. Students will identify if an equation is a function.
3. Students will identify the domain and range of a given function.

Cal-PASS Algebra I California Content Standards Deconstruction Project Version 2.0
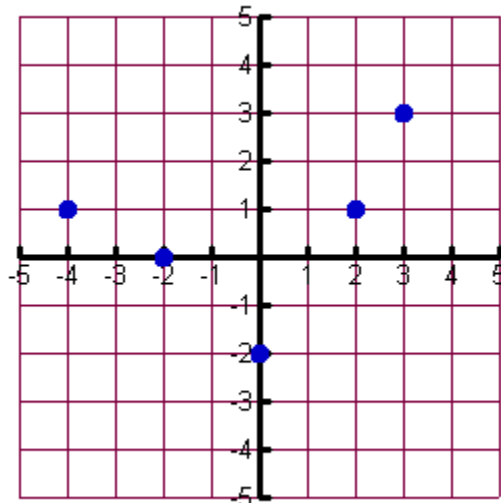
Standard #16 Model Assessment Items

Conceptual Understanding

1. Determine whether the relation *is* or *is not* a function.
    - A. {(-4,3), (-2,1), (0,5), (-2,-8)}
    - B. {(3,7), (1,4), (0,-2), (-1,-1), (-2,5)}

2. Given the following figures, determine whether the relation *is* or *is not* a function.

    A.

    B.
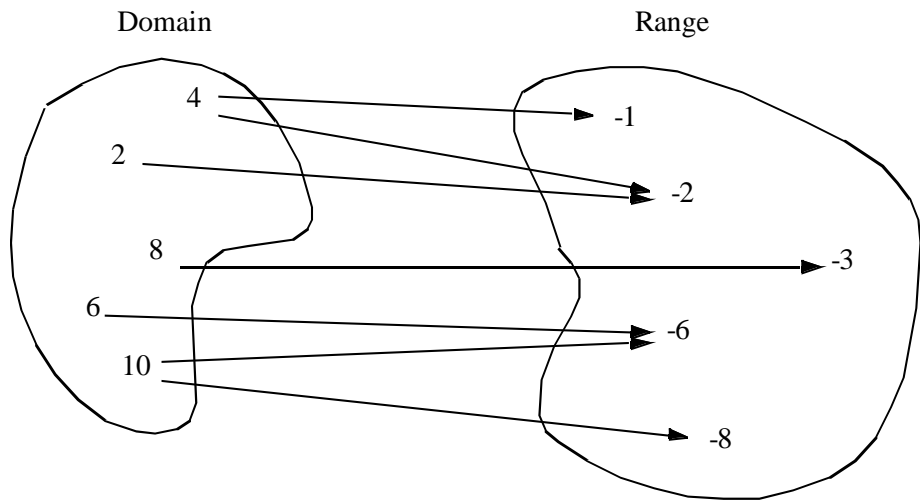
3. Define *relation* and *function*. Compare the two definitions. How are they alike? How are they different?

Cal-PASS Algebra I California Content Standards Deconstruction Project Version 2.0

Problem Solving/Applications

1. Decide whether each equation defines y as a function of x. *(Remember that to be a function, every value of x must give one and only one value of y.)*

    A. $y = 5x + 3$

    B. $x = y^2$

Standard #17

Standard Set 17.0
Students determine the domain of independent variables and the range of dependent variables defined by a graph, a set of ordered pairs, or a symbolic expression.

Deconstructed standard
1. Students determine the domain of a relation from its graph.
2. Students determine the domain of independent a set of ordered pairs.
3. Students determine the domain of a relation given as a symbolic expression.
4. Students determine the range of a relation from a graph.
5. Students determine the range of a set of ordered pairs.
6. Students determine the range of a symbolic expression.

Prior Knowledge Necessary
Students should know how to:
- ➢ identify ordered pairs of data from a graph.
- ➢ graph points on a coordinate plane.
- ➢ write ordered pairs correctly.
- ➢ use a variety of methods such as symbols, charts, graphs, tables, and diagrams to explain mathematical reasoning.

New Knowledge
Students will need to learn to:
- ➢ identify the domain in each symbolic expression, set of ordered pairs, or graph.
- ➢ identify the range in each symbolic expression, set of ordered pairs, or graph.
- ➢ the definition of independent variable.
- ➢ the definition of dependent variable.

Categorization of Educational Outcomes
Competence Level: Knowledge
1. Students will identify the domain of independent variables defined by a graph, a set of ordered pairs, or a symbolic expression.
2. Students will identify the range of dependent variables defined by a graph, a set of ordered pairs, or a symbolic expression.

Competence Level: Application
1. Students will examine data from a graph, a set of ordered pairs, or a symbolic expression and classify it into the domain of independent variables or range of dependent variables.

Necessary New Physical Skills
None

Cal-PASS Algebra I California Content Standards Deconstruction Project Version 2.0

Assessable Result of the Standard

1. Given a symbolic expression, a set of ordered pairs or a graph, the student will identify the domain and the range

Standard #17 Model Assessment Items

Computational and Procedural Skills

1. Complete each ordered pair so that it is a set of ordered pairs. Generate and identify the domain and range of the ordered pairs:
    A. (1, ?)
    B. (2, ?)
    C. (?, 4)
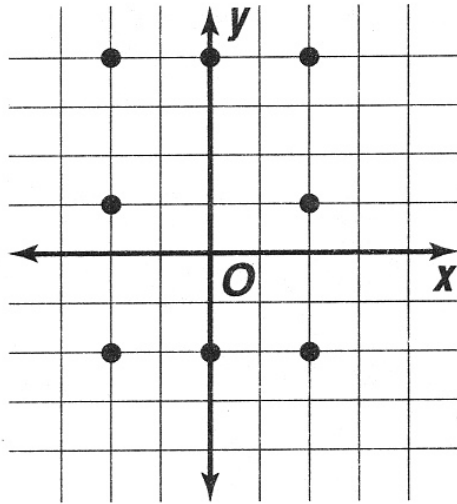   D. (3, ?)
    E. (0, ?)

Conceptual Understanding

1. State the domain and range of the relation:

    { (2,4), (2,5), (4,6), (7,2), (5,10), (8,4), (3,6) }

2. Express the relation in each figure, table, or graph as a set of ordered pairs and then state the domain and range of each:

    A.
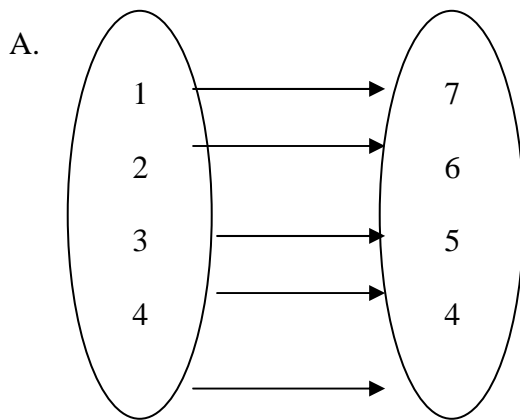
    B.

X	Y
3	4
5	6
7	8
7	6
5	4
3	3

C.

Problem Solving/Application

1. The table below shows the amount that a company charges for a bike rental. Identify the domain and range. Write a set of ordered pairs for the function.

Time (hrs)	1	2	3	4	5	6
Cost ($)	20	24	28	32	36	40

2. The table below shows the per minute rate for a cell phone. Identify the domain and range. Write a set of ordered pairs for the function.

Minutes	1	2	3	4	5	6
Cost ($)	2.00	2..25	2.50	2.75	3.00	3.25

3. The table below shows the distance that a car travels over time. Identify the domain and range. Write a set of ordered pairs for the function.

Time (hrs)	1	2	3	4	5	6
Distance (miles)	50	100	150	200	250	300

Cal-PASS Algebra I California Content Standards Deconstruction Project Version 2.0

Standard #18

Standard Set 18.0
Students determine whether a relation defined by a graph, a set of ordered pairs, or a symbolic expression is a function and justify the conclusion.

Deconstructed Standard
1. Given a graph, students can determine if the relation graphed is also a function.
2. Students can justify why the graph of a relation is or is not a function.
3. Students can determine if the relation defined by a set of ordered pairs is also a function.
4. Students can justify why the set of ordered pairs is or is not a function.
5. Given an algebraic equation, students can determine whether or not it represents a function.
6. Students can justify why the algebraic equation is or is not a function.

Prior Knowledge Necessary
Students should know how to:
- define a relation.
- define a function.
- define the "domain" of a relation.
- define the "range" of a relation.
- determine the domain of a relation from a graph, a set of ordered pairs, or a symbolic expression (see Algebra I, Standard #17).
- determine the range of a relation from a graph, a set of ordered pairs, or a symbolic expression (see Algebra I, Standard #17).
- determine whether a given relation defines a function (see Algebra I, Standard #16).
- state exclusions on the variable when given a rational expression.
- state exclusions on the variable when given a radical expression.
- give the ordered pairs plotted on a Cartesian plane.

New Knowledge
Students will need to learn to:
- use the vertical line test to determine if a given graph represents a function.
- justify their conclusion as to whether a given graph, set of ordered pairs or symbolic expression is a function.

Categorization of Educational Outcomes
Competence Level: Knowledge
1. Students will identify whether or not a given graph, set of ordered pairs, or symbolic expression is a function.
2. Students will use the definition of a function to justify their response.

Cal-PASS Algebra I California Content Standards Deconstruction Project Version 2.0

Necessary New Physical Skills
None

Assessable Result of the Standard
1. Students will state whether or not a given graph, set of ordered pairs or algebraic equation is a function and justify their answer.
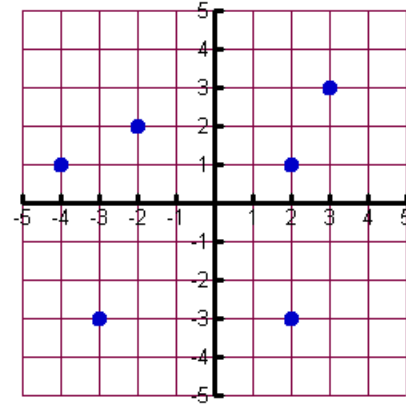
Standard #18 Model Assessment Items

Computational and Procedural Skills

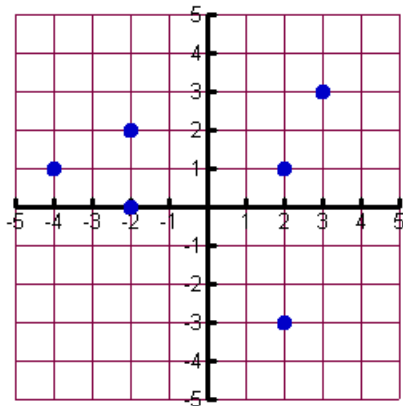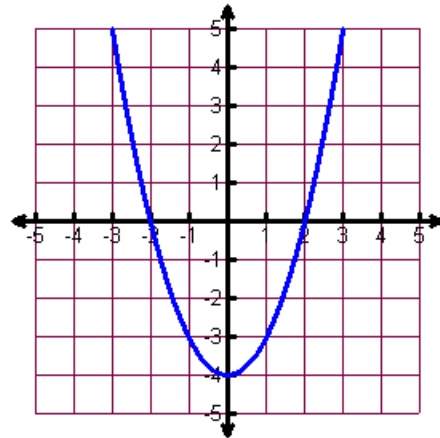1. Determine whether or not the relations below are functions. Justify your answer.

A.

C.

B.

D.

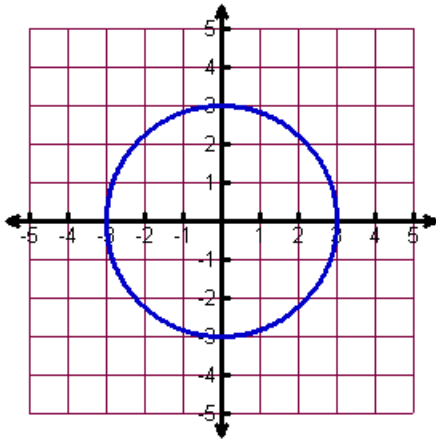*Cal-PASS Algebra I California Content Standards Deconstruction Project Version 2.0*
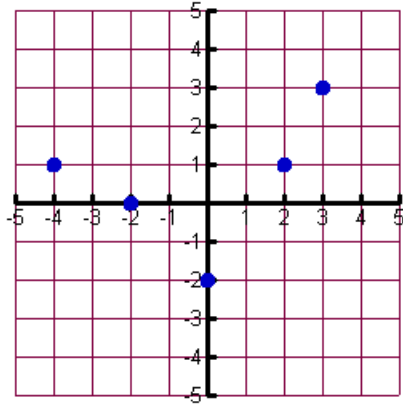
E.

F. $\{(4,5),(-3,6),(5,6),(-2,4)\}$

G. $\{(7,-3),(-3,6),(7,3),(-6,5)\}$

H. $y = 3x - 1$

I. $y = x^2 - 5$

J. $x = y^2 - 3$

K. $x^2 + y^2 = 16$

Conceptual Understanding
1. If you are looking at a graph, how do you determine whether or not it is the graph of a function?

2. Write a set consisting of three ordered pairs that is a relation, but not a function. Explain why the set of ordered pairs you wrote is not a function.

3. Draw a graph that is both a relation and a function. Write a sentence that states why the graph you drew represents both a relation and a function.

Standard #19

Standard Set 19.0

Students know the quadratic formula and are familiar with its proof by completing the square.

Deconstructed Standard

1. Students know the quadratic formula.
2. Students are familiar with the proof, or derivation, of the quadratic formula by completing the square.

Prior Knowledge Necessary

Students should know how to:

➢ solve a quadratic equation by factoring.
➢ use the Property of Zero in solving quadratic equations using factoring methods.
➢ identify the standard form of a quadratic equation with regards to the quadratic formula: $ax^2 + bx + c = 0$.
➢ solve linear equations.
➢ evaluate the square root of a number that is a perfect square.
➢ simplify the square root of a number that is not a perfect square.
➢ solve a quadratic equation by completing the square.

New Knowledge

Students will need to learn to:

➢ memorize the quadratic formula.
➢ become familiar with the derivation of the quadratic formula by completing the square.

Categorization of Educational Outcomes

Competence Level: Knowledge

1. Students will be able to recall the quadratic formula.

Necessary New Physical Skills

None

Assessable Result of the Standard

1. Students will state the quadratic formula.
2. Students will be able to fill in steps in the proof of the quadratic formula.

Standard #19 Model Assessment Items

Computational and Procedural Skills

1. State the quadratic formula.

2. Fill in the missing value to complete the step in the derivation of the quadratic formula:

$$ax^2 + bx + c = 0$$

$$ax^2 + bc = -c$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

$$x^2 + \frac{b}{a}x + \underline{\quad\quad} = -\frac{c}{a} + \underline{\quad\quad}$$

3. Fill in the missing value to complete the step in the derivation of the quadratic formula:

$$ax^2 + bx + c = 0$$

$$ax^2 + bc = -c$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$$

$$\left(x + \frac{b}{2a}\right)^2 = \underline{\quad\quad}$$

Conceptual Understanding

1. Without actually proving/deriving the quadratic formula, explain in your own words how the quadratic formula can be derived by using the technique known as Completing the Square.

Problem Solving/Application

None

Cal-PASS Algebra I California Content Standards Deconstruction Project Version 2.0

Standard #20

Standard Set 20.0
Students use the quadratic formula to find the roots of a second-degree polynomial and to solve quadratic equations.

Deconstructed Standard
1. Students use the quadratic formula.
2. Students solve quadratic (second-degree polynomial) equations.

Prior Knowledge Necessary
Students should know how to:
- apply algebraic order of operations and the commutative, associative, and distributive properties to evaluate expressions.
- simplify expressions manually by using the correct order of operations or on a scientific calculator.
- differentiate between linear and quadratic functions.
- evaluate formulas for given formula values.
- simplify square roots.

New Knowledge
Students will need to learn to:
- identify the appropriate values for a, b and c from a quadratic equation.
- substitute appropriate a, b, and c values into the quadratic formula.
- simplify the resulting expressions.
- identify when it is appropriate to use the quadratic formula vs. other techniques to solve a quadratic equation.
- solve a quadratic equation by using the quadratic formula.

Categorization of Educational Outcomes
Competence Level: Knowledge
1. Students will identify the appropriate values for a, b and c in a given quadratic equation.

Competence Level: Application
1. Students will simplify the rational expression resulting from the quadratic formula to obtain the solutions for the given quadratic equation.

Necessary New Physical Skills
None

Assessable Result of the Standard
1. Students will produce the roots of a quadratic equation using the quadratic formula.

Standard #20 Model Assessment Items

Computational and Procedural Skills

1. Identify the appropriate values for *a, b,* and *c* in the following:
   A. $2x^2 + 7x - 15 = 0$

   B. $3x^2 = -4x + 1$

2. Solve using the quadratic formula:
   A. $8x^2 + 18x - 5 = 0$

   B. $3x^2 = -4x + 1$

   C. $x^2 + (x + 2)^2 = 7$

3. Identify the most appropriate method for solving each of the following quadratic equations (square root property, factoring, completing the square, quadratic formula)
   A. $3x^2 + 6x = 0$

   B. $x^2 - 8 = 0$

   C. $x^2 + 3x + 1 = 0$

   D. $x^2 + 4x - 3 = 0$

Conceptual Understanding

1. In what situations would it be advantageous to use the quadratic formula to solve a quadratic equation?

2. List three techniques, other than the quadratic formula, for solving quadratic equations. Create an example to illustrate each technique.

Problem Solving/Application

1. The area of a rectangle is 44 inches. The perimeter of the rectangle is 30 inches. Find the length and width.

2. A company produces DVDs. The function for the profit of the company is:
   $P(x) = 2x^2 - 16x - 66$. Find the break-even points (the selling prices for which the profit is 0).

3. The length of a rectangle is 6 feet longer than its width. If the area is 50 square feet, find the length and width of the rectangle.

Cal-PASS Algebra I California Content Standards Deconstruction Project Version 2.0

4. A right triangle is such that one leg is 3 feet longer than the other leg. The length of the hypotenuse is 17 feet. Find the lengths of the legs of this right triangle (round your answers to the nearest hundredth if necessary).

Standard #21

Standard Set 21.0
Students graph quadratic functions and know that their roots are the *x*-intercepts.

Deconstructed Standard
1. Students graph quadratic functions.
2. Students know that the roots of a quadratic equation are the *x*-intercepts on the graph of the function.

Prior Knowledge Necessary
Students should know how to:
- differentiate between linear and quadratic functions.
- graph linear functions and vertical lines.
- plot points on a coordinate system.
- solve quadratic equations.
- identify *x*- and *y*-intercepts from a graph.

New Knowledge
Students will need to learn to:
- using the formula $x = -b/2a$, determine the axis of symmetry of a quadratic function.
- determine the *y* value of the vertex of a quadratic function by substituting the *x* value into the quadratic equation and solving for *y*.
- graph the axis of symmetry using the equation $x = -b/2a$.
- identify whether the parabola opens upward or downward.
- graph the vertex.
- graph the *x*- and *y*–intercepts.
- draw the graph of the parabola using a minimum of 5 points including the vertex and intercepts.
- choose appropriate *x* values close to the vertex, determine the corresponding *y* value, and plot the points.

Categorization of Educational Outcomes
Competence Level: Knowledge
1. Students will identify the vertex of a quadratic function.
2. Students will identify the axis of symmetry of a quadratic function.
3. Students will identify the *x*-intercepts on the graph of a quadratic function.

Competence Level: Comprehension
1. Students will describe how to locate the vertex of a quadratic function
2. Students will describe how to locate the axis of symmetry of a quadratic function.

Cal-PASS Algebra I California Content Standards Deconstruction Project Version 2.0

Competence Level: Application
1. Students will calculate the vertex of a quadratic function.
2. Students will calculate the axis of symmetry of a quadratic function.
3. Students will calculate the *x*-intercepts of a quadratic function.

Competence Level: Analysis
1. Students will explain the process of graphing a quadratic function.

__Necessary New Physical Skills__
None

__Assessable Result of the Standard__
1. Students will graph a quadratic function with the roots, axis of symmetry, and vertex.
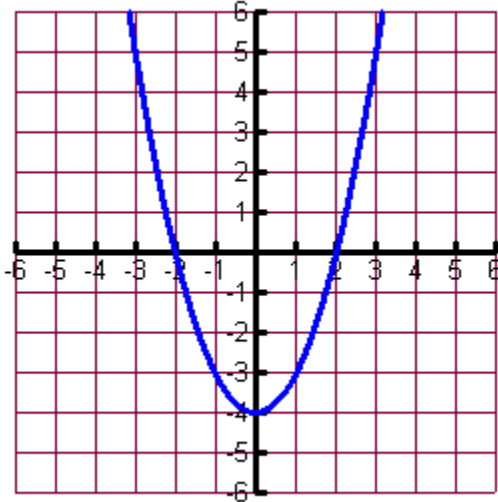
Standard #21 Model Assessment Items

Computational and Procedural Skills

1. Write the equation of the axis of symmetry:

   A. $y = 4x^2$

   B. $y = -x^2 + 4x - 1$

   C. $y = x^2 - 3x - 10$

   D. $y = 2x^2 + 16$

2. Find the coordinates of the vertex.

   A. $y = x^2 + 2$

   B. $y = -x^2 - 6x + 9$

   C. $y = 2x^2 + 4x + 7$

   D. $y = x^2 - 14x + 13$

3. Graph the following using a minimum of 5 points including the vertex and *x*- and *y*-intercepts.

   A. $y = x^2 - 3x - 10$

   B. $y = x^2 - 9$

   C. $y = -2x^2 + 8$

Cal-PASS Algebra I California Content Standards Deconstruction Project Version 2.0
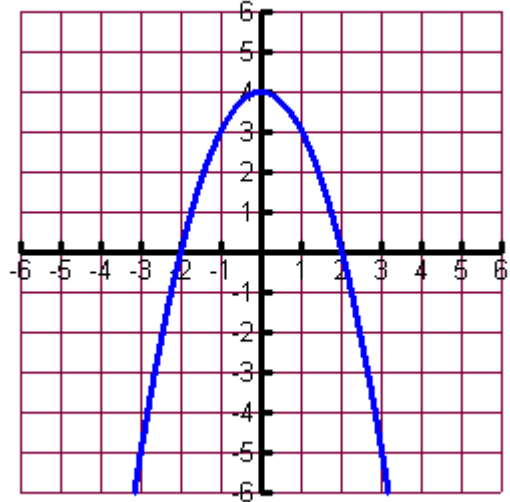
Conceptual Understanding

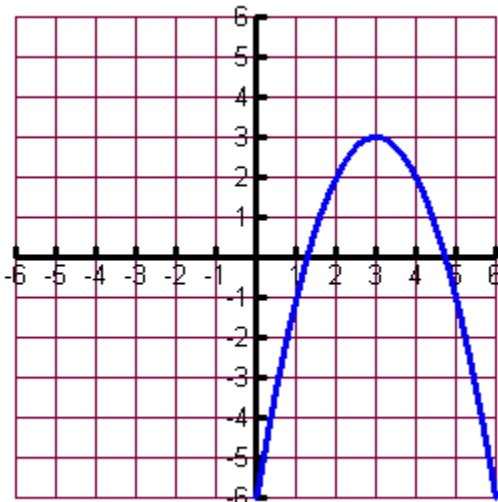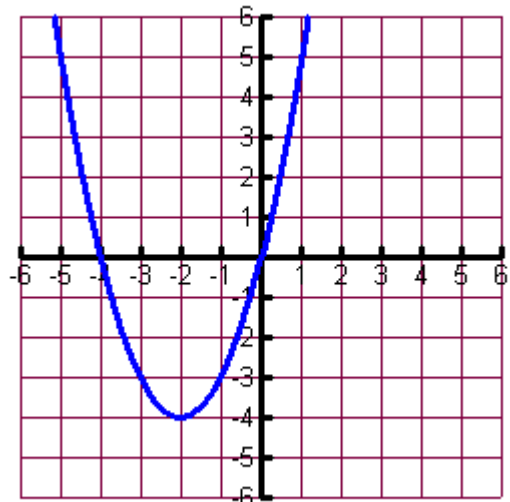1. Estimate the roots from the following graphs:

A.

B.

C.

D.

Problem Solving/Application

1. An object is thrown straight upward with an initial velocity of 30 feet per second. Its height in feet, y, after t seconds, is represented by: $y = -16t^2 + 32t$. After how many seconds does the object hit the ground?

Cal-PASS Algebra I California Content Standards Deconstruction Project Version 2.0

Standard #22

Standard Set 22.0
Students use the quadratic formula or factoring techniques or both to determine whether the graph of a quadratic function will intersect the *x*-axis in zero, one, or two points.

Deconstructed Standard
1. Students use the discriminant from the quadratic formula to determine the number of real solutions to quadratic equations.
2. Students use factoring techniques to determine the number of real solutions to quadratic equations.
3. Students determine the number of points that the graph of a quadratic function intersects the *x*-axis.

Prior Knowledge Necessary
Students should know how to:
> ➢ identify an equation as a quadratic equation.
> ➢ write a quadratic equation in standard form.
> ➢ factor the greatest common factor (GCF) from a polynomial equation.
> ➢ factor a polynomial using grouping.
> ➢ factor quadratic expressions into the product of two binomials.
> ➢ identify a perfect square trinomial.
> ➢ recognize the relationship between the solutions or roots of a quadratic equation and the zeroes of a quadratic function.

New Knowledge
Students will need to learn to:
> ➢ use the definition of the discriminant to calculate its value for a given quadratic function.
> ➢ use the discriminant to identify whether the quadratic equation has zero, one, or two real solutions.
> ➢ use factoring techniques to determine whether the quadratic equation has zero, one, or two real solutions.
> ➢ use the results from the statements above to identify on the graph of the quadratic equation the number of times that the graph of the equation crosses the *x*-axis.

Categorization of Educational Outcomes
Competence Level: Knowledge
   1. Students will define the discriminant.

Competence Level: Application
   1. Students will compute the value of the discriminant for a given quadratic equation

Cal-PASS Algebra I California Content Standards Deconstruction Project Version 2.0

Competence Level: Comprehension
1. Students will show that they know the correct interpretation of the *roots* or *zeroes* of a quadratic equation.
2. Students will demonstrate the relationship between the discriminate and the number of roots/zeroes of a quadratic equation.
3. Students will demonstrate knowledge of the relationship between the roots/zeroes of a quadratic equation and how many times the graph of the equation intersects the *x*-axis.

Necessary New Physical Skills
None

Assessable Result of the Standard
1. Students will identify the number of solutions to a quadratic equation and how many times the graph of the equation intersects the *x*-axis.

Cal-PASS Algebra I California Content Standards Deconstruction Project Version 2.0

Standard #22 Model Assessment Items

Computational and Procedural Skills

1. By factoring, determine the number of times the graph of the equation intersects the x-axis:

   A. $x^2 + 5x + 6 = 0$

   B. $4p^2 + 40 = 26p$

   C. $16m^2 - 25 = 0$

   D. $x^2 + 6x + 9 = 0$

2. Using the discriminate, determine the number of times the graph of the equation intersects the x-axis:

   A. $2x^2 - 7x - 9 = 0$

   B. $3x^2 + 4x - 8 = 0$

   C. $\dfrac{1}{2}x^2 + \dfrac{1}{6}x = 1$

Conceptual Understanding

1. Factoring:

   A. In solving a quadratic equation that is a perfect square trinomial, how many times does the graph of the equation intersect the x-axis?

   B. In factoring a quadratic equation and there are no solutions, what does that mean for the graph of the quadratic equation?

2. Quadratic Formula:

   A. What is the relationship between the discriminate and the number of times the graph of the quadratic equation intersects the x-axis?

   B. If the quadratic formula results in only one solution, what does the graph look like in relationship to the x-axis?

Problem Solving/Application
None

Standard #23

Standard Set 23.0
Students apply quadratic equations to physical problems, such as the motion of an object under the force of gravity.

Deconstructed Standard
1. Students use quadratic equations to model physical problems.
2. Students use the appropriate techniques to solve quadratic equations describing physical problems.
3. Students will interpret the solutions of the quadratic equation in terms of the physical situation.

Prior Knowledge Necessary
Students should know how to:
- ➤ translate a word problem describing a physical situation into a linear equation, identifying the unknown quantity and assigning a variable to it.
- ➤ find the solutions to a quadratic equation.

New Knowledge
Students will need to learn to:
- ➤ translate a word problem describing a physical situation into a quadratic equation, identifying the unknown quantity and assigning a variable to it.
- ➤ use the solutions from the quadratic equation and knowledge of the problem to obtain meaningful solutions to physical problems (i.e., discarding unrealistic solutions).

Categorization of Educational Outcomes
Competence Level: Application
1. Students will write a quadratic equation that models a physical problem.
2. Students will identify correct solutions by rejecting physically impossible solutions to quadratic equations describing physical problems.

Necessary New Physical Skills
None

Assessable Result of the Standard
1. Students will identify the meaningful solutions to a quadratic equation describing physical problems.

Standard #23 Model Assessment Items

Conceptual Understanding

1. The solutions to a quadratic equation involving the length of an object include a negative solution. Is this a meaningful solution? Why or why not?

Problem Solving/Application

1. The formula for freely falling objects is given as $d = 16t^2$. The distance d in feet that an object falls depends on the time elapsed (t) in seconds.

    A. Use the formula and complete the following table (*Hint:* substitute each given value into the formula and solve for the unknown value):

t in seconds	0	1		6
d in feet	0		144	

      i.   When *t = 0, d = 0*. Explain this in the context of the problem.
     ii. When you substituted 144 for *d* and solved for *t*, you should have found two solutions. Are they both meaningful? Why or why not?

    B. Pam's high school is due south of home. Her brother, Pete's middle school is due west. They leave for school at the same time. By the time Pam is 6 miles from home, the distance between them is`2 miles more than Pete's distance from home. How far from home is Pete? (*Hint:* draw a picture.)

Problem Solving/Application

1. Factoring:
    A. The O'Connors want to plant a flower bed in a triangular area in a corner of their garden. One leg of the right-triangular flower bed will be 2 m shorter than the other leg, and they want it to have an area of 24 m². Find the lengths of the legs.

    B. The product of two consecutive integers is 11 more than their sum. Find the integers.

2. Completing the square:
    A. If an object is propelled upward on the surface of Mars from ground level with an initial velocity of 104 ft per sec, its height s (in feet) in t sec is given by the formula $s = -13t^2 + 104t$. How long will it take for the object to be at a height of 195 ft? How long will it take the object to return to the surface (hint: s = 0)?

    B. A farmer has a rectangular cattle pen with a perimeter of 350 feet and an area of 7500 ft². What are the dimensions of the pen?

Cal-PASS Algebra I California Content Standards Deconstruction Project Version 2.0

Appendix #1
Algebra I California Content Standards by Clusters

The California Standards Test organizes related standards for Algebra I into four reporting clusters. Student performance is reported by these clusters. The clusters for Algebra I are:
- ➤ Number Properties, Operations, and Linear Equations
- ➤ Graphing and Systems of Linear Equations
- ➤ Quadratics and Polynomials
- ➤ Functions and Rational Expressions

The questions change each year, but the distribution of questions remains the same as listed in the table below.

The following table, compiled by the San Diego County Office of Education, organizes the standards by cluster and shows the number of items testing each standard on the California High School Exit Exam (CAHSEE), Algebra I CST, and High School Summative (HSS) CST

Mathematics Content Standards: Algebra I	CAHSEE Items	Algebra I CST	Summative High School CST
	12 total	65 items	18 items
Cluster 1: Number Properties, Operations, and Linear Equations			
1.0 Students identify and use the arithmetic properties of subsets of integers and rational, irrational, and real numbers, including closure properties for the four basic arithmetic operations where applicable:			
1.1 Students use properties of numbers to demonstrate whether assertions are true or false.	0	1/2	
2.0* Students understand and use such operations as taking the opposite, finding the reciprocal, taking a root, and raising to a fractional power. They understand and use the rules of exponents.	1	4	
3.0 Students solve equations and inequalities involving absolute values.	1	1	
4.0* Students simplify expressions before solving linear equations and inequalities in one variable, such as $3(2x\text{-}5) + 4(x\text{-}2) = 12$.	2	3	1
5.0* Students solve multi-step problems, including word problems, involving linear equations and linear inequalities in one variable and provide justification for each step.	1	6	3
24.0 Students use and know simple aspects of a logical argument:			
24.1 Students explain the difference between inductive and deductive reasoning and identify and provide examples of each.	0	1/3	
24.2 Students identify the hypothesis and conclusion in logical deduction.	0	1/3	
24.3 Students use counterexamples to show that an assertion is false and recognize that a single counterexample is sufficient to refute an assertion.	0	1/3	
25.0 Students use properties of the number system to judge the validity of results, to justify each step of a procedure, and to prove or disprove statements:			
25.1 Students use properties of numbers to construct simple, valid arguments (direct and indirect) for, or formulate counterexamples to, claimed assertions.	0	1/2	

Cal-PASS Algebra I California Content Standards Deconstruction Project Version 2.0

25.2	Students judge the validity of an argument according to whether the properties of the real number system and the order of operations have been applied correctly at each step.	0	1/2	
25.3	Given a specific algebraic statement involving linear, quadratic, or absolute value expressions or equations or inequalities, students determine whether the statement is true sometimes, always, or never.	0	1/2	
Cluster 2: Graphing and Systems of Linear Equations				
6.0*	Students graph a linear equation and compute the x-and y-intercepts (e.g., graph 2x + 6y = 4). They are also able to sketch the region defined by linear inequality (e.g., they sketch the region defined by 2x + 6y < 4).	2	4	2
7.0*	Students verify that a point lies on a line, given an equation of the line. Students are able to derive linear equations by using the point-slope formula.	1	4	1
8.0	Students understand the concepts of parallel lines and perpendicular lines and how those slopes are related. Students are able to find the equation of a line perpendicular to a given line that passes through a given point.	1	1	1
9.0*	Students solve a system of two linear equations in two variables algebraically and are able to interpret the answer graphically. Students are able to solve a system of two linear inequalities in two variables and to sketch the solution sets.	1	5	
Cluster 3: Quadratics and Polynomials				
10.0*	Students add, subtract, multiply, and divide monomials and polynomials. Students solve multi-step problems, including word problems, by using these techniques.	1	4	3
11.0	Students apply basic factoring techniques to second-and simple third-degree polynomials. These techniques include finding a common factor for all terms in a polynomial, recognizing the difference of two squares, and recognizing perfect squares of binomials.	0	2	1
14.0*	Students solve a quadratic equation by factoring or completing the square.	0	3	1
19.0*	Students know the quadratic formula and are familiar with its proof by completing the square.	0	2	
20.0*	Students use the quadratic formula to find the roots of a second-degree polynomial and to solve quadratic equations.	0	3	1
21.0*	Students graph quadratic functions and know that their roots are the *x*-intercepts.	0	3	
22.0	Students use the quadratic formula or factoring techniques or both to determine whether the graph of a quadratic function will intersect the x-axis in zero, one, or two points.	0	1	
23.0*	Students apply quadratic equations to physical problems, such as the motion of an object under the force of gravity.	0	3	1
Cluster 4: Functions and Rational Expressions				
12.0*	Students simplify fractions with polynomials in the numerator and denominator by factoring both and reducing them to the lowest terms.	0	3	1
13.0*	Students add, subtract, multiply, and divide rational expressions and functions. Students solve both computationally and conceptually challenging problems by using these techniques.	0	4	
15.0*	Students apply algebraic techniques to solve rate problems, work problems, and percent mixture problems.	1	4	2
16.0	Students understand the concepts of a relation and a function, determine whether a given relation defines a function, and give pertinent information about given relations and functions.	0	1/2	
17.0	Students determine the domain of independent variables and the range of dependent variables defined by a graph, a set of ordered pairs, or a symbolic expression.	0	1	
18.0	Students determine whether a relation defined by a graph, a set of ordered pairs, or a symbolic expression is a function and justify the conclusion.	0	1/2	

CST: Fractional values indicate noted standards (e.g., 1/2 = rotated every two years: 1/3 = rotated every 3 years).
*Key Standard: Key Standards make up 70% of the test.

Cal-PASS Algebra I California Content Standards Deconstruction Project Version 2.0

Appendix #2
Developing Learning Targets for Algebra Standards
(Instructions given to teachers involved in the project)

Please note the following terms and/or definitions which have been agreed upon for this deconstruction project:

➢ ***Prior Knowledge:*** *Prior knowledge is defined as acquired knowledge that has been mastered in a previous standard.*
➢ ***New Knowledge:*** *New knowledge is defined as knowledge that students need to acquire and apply to the components in step #2 of this deconstruction process to create the products listed in step #7 of this process.*
➢ ***Introduced:*** *When a standard mentions that a concept or idea has been "introduced," this does not mean that it has been mastered.*
➢ ***Familiar With:*** *When a standard mentions that students should be "familiar with" certain concepts or ideas, this does not mean that students have actually mastered these ideas or concepts.*

Sample:
Deconstruction of Standard #6

Step #1: Underline noun phrases, and box or circle the verbs.
Standard 6.0: Students graph a linear equation and compute the *x*- and *y*-intercepts. (e.g., graph $2x + 6y = 4$). They are also able to sketch the region defined by a linear inequality (e.g., they sketch the region defined by $2x + 6y < 4$).

Step #2: Rewrite standard into short components.
1. Students graph linear equations.
2. Students compute *x*-intercepts.
3. Students compute *y*-intercepts.
4. Students use intercepts to graph linear equations.
5. Students sketch the region defined by a linear inequality.

Step #3: Identify prior knowledge students should know. *(See note above)*
1. Students must be able to perform arithmetic computations with rational numbers.
2. Students must be able to graph ordered pairs.
3. Students must be able to compute slope from the graph of a line.
4. Students must be able to compute slope when given two points.
5. Students must be able to recognize slope as a rate of change of *y* in relation to *x*.
6. Students must be able to graph a linear equation using a "t-chart".
7. Students must be able to evaluate a linear equation for a given *x* or *y* value.
8. Students must be able to solve one-variable linear inequalities.
9. Students must be able to graph the solution set for a one-variable linear inequality.

Cal-PASS Algebra I California Content Standards Deconstruction Project Version 2.0

10. Students must be able to verify that any element in the solution set of a one-variable inequality satisfies the original inequality.

Step #4: Identify what new knowledge students will need to learn. *(See note above)*
1. Given the slope/intercept form of a line, $y = mx + b$, students will plot the y-intercept, and then use the slope to find a second point in order to complete the graph of the line.
2. Students will identify the graphical representation of $(a, 0)$ as the x-intercept, and $(0, b)$ as the y-intercept.
3. Students will be able to compute the x-intercept and y-intercept given a linear equation.
4. Students will identify that the linear equation implied by the linear inequality forms a boundary for the solution set and that this boundary may or may not be included in the final graph.
5. Students will interpret the inequality symbol to determine whether or not the boundary is solid or dashed.
6. Students will identify and shade the region of the graph that contains the solutions to the inequality.
7. Students will recognize that linear inequalities have multiple ordered-pair solutions.

Step #5: Identify patterns of reasoning using *Bloom's Taxonomy*.
Use the *Bloom's Taxonomy* handouts provided to describe the overall competence level expected of students with respect to these topics. Then highlight the "skills demonstrated" using as many of the key words and phrases provided on the handouts. See example below for Standard #6. Box in the key words or phrases taken from *Bloom's Taxonomy*.

Competence Level: Application
1. Students will use methods they have learned to graph lines, solve inequalities, and to locate and/or identify the x- and y-intercepts for a given equation or graph.
2. Students will demonstrate their ability to find and use x- and y-intercepts in the context of graphing.
3. Students will calculate x- and y-intercepts.
4. Students will solve inequalities in two variables.
5. Students will use information they have learned to graph lines, solve inequalities, and find x- and y-intercepts.
6. Students will show that they know the correct interpretation of the boundary line for the solution of an inequality by appropriately making the boundary solid or dashed.

Step #6: Identify required physical skills.
In this section we are looking for physical skills such as: use of a calculator, protractor, ruler, compass, etc.
1. Use of a ruler

Cal-PASS Algebra I California Content Standards Deconstruction Project Version 2.0

Step #7: Identify assessable results of the standard.
1. Students will produce the graph of a line.
2. Students will produce the ordered pairs representing the *x*- and *y*-intercepts.
3. Students will produce a bounded and shaded region of the *x-y* plane representing the solution set of a linear inequality in two variables.

Model Assessment Items
In this section you will write model, or exemplar, assessment items that will serve to demonstrate the level and depth of instruction for these particular topics. For example, you would expect a lower level and depth for a topic in Basic Algebra than you would for the same topic in Intermediate Algebra.

Please be sure to include assessment items that measure abilities in the following three categories:
1. Computational and Procedural Skills
2. Conceptual Understanding
3. Problem Solving/Application

Category #1: Computational and Procedural Skills
1. Find the *x*- and *y*-intercepts for the line defined by the following equation: $2x + 3y = 9$.
2. Use the *x*- and *y*-intercepts to graph the line given by the equation: $2x + 3y = 6$.
3. Graph the following lines using the method of your choice. Identify and label the *x*- and *y*-intercepts for each graph if they exist:
    A. $3x - 5y = 10$

    B. $y = -\dfrac{2}{3}x + 4$

    C. $y = 2$

    D. $x = 3.5$

    E. $2x + 4y = 3$

    F. $\dfrac{1}{2}x - \dfrac{3}{4}y = 2$

4. Graph the solution set for the following inequalities:
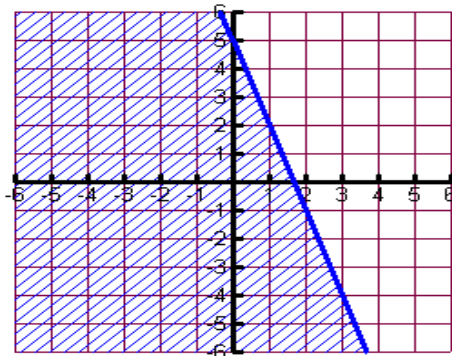   A. $2x - 3y < 6$

    B. $y \geq -\dfrac{3}{4}x + 2$

Cal-PASS Algebra I California Content Standards Deconstruction Project Version 2.0

C. $\dfrac{1}{2}x - \dfrac{2}{3}y \le \dfrac{5}{6}$

Category #2: Conceptual Understanding
1. Sketch the graph of a line that has no x-intercept.
2. Identify the x- and y- intercepts from the graph of the given line.
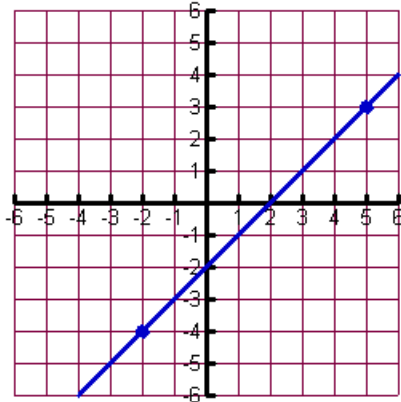
3. Can a line have more than one x-intercept? Explain your answer using a diagram.
4. The solution to an inequality has been graphed correctly below. Insert the correct inequality symbol in the inequality below to match the graph of the solution. (Everything else about the inequality is correct—it just needs the correct symbol).
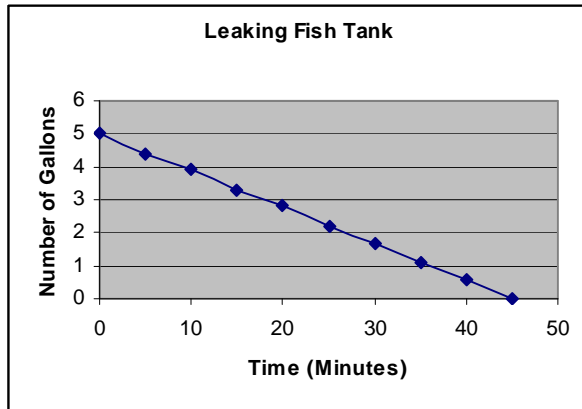

$y \boxed{} \; -3x + 5$

Insert correct symbol in box.

5. When is it advantageous to use the x- and y-intercepts to graph the equation of a line? When would it perhaps be easier or better to use another graphing method? Give an example to illustrate your answers to both of these questions.

Category #3: Problem Solving/Application

1. The graph displayed below is the graph of the following equation: $y = \left(-\dfrac{1}{9}\right)x + 5$,

   where x represents the amount of time that has passed since a 5 gal. fish tank sprung a leak, and y represents the number of gallons of water in the tank after the leak.

Cal-PASS Algebra I California Content Standards Deconstruction Project Version 2.0

A. What is the significance of the *x*-intercept in this situation? What information is given to us by this point?
B. What is the significance of the *y*-intercept in this situation? What information is given to us by this point?

Leaking Fish Tank

2. The cost of a trash pickup service is given by the following formula: $y = 1.50x + 11$, where *x* represents the number of bags of trash the company picks up, and *y* represents the total cost to the customer for picking up the trash.
A. What is the *y*-intercept for this equation?
B. What is the significance of the *y*-intercept in this situation? What does it tell us about this trash pickup service?
C. Draw a sketch of the graph which represents this trash pickup service.

Exemplar Teaching Methods
Please record an example of "Lesson Plans" that demonstrate an excellent method of how the concepts in this standard could be presented to students. Be sure to give examples or illustrate any unique or creative methods that you have used that bring these concepts to life. Use as much detail as needed to communicate your ideas.

Appendix #3
Categorization of Educational Outcomes

Identifies the type of reasoning students will use to learn the skills necessary to master each standard. Teachers were asked to use *Bloom's Taxonomy* to describe the overall competence level expected of students with respect to these topics and highlight the skills demonstrated.

Major Categories in the* Taxonomy of Educational Objectives*, Bloom 1956*
Categories in the Cognitive Domain: (with Outcome-Illustrating Verbs)

Knowledge—of terminology; specific facts; ways and means of dealing with specifics (conventions, trends, and sequences; classifications and categories; criteria, methodology); universals and abstractions in a field (principles and generalizations, theories and structures)—**The remembering (recalling) of appropriate, previously learned information.**
➢ defines; describes; enumerates; identifies; labels; lists; matches; names; reads; records; reproduces; selects; states; views.

Comprehension: **Grasping (understanding) the meaning of informational materials.**
➢ classifies; cites; converts; describes; discusses; estimates; explains; generalizes; gives examples; makes sense out of; paraphrases; restates (in own words); summarizes; traces; understands.

Application: **The use of previously learned information in new and concrete situations to solve problems that have single or best answers.**
➢ acts; administers; articulates; assesses; charts; collects; computes; constructs; contributes; controls; determines; develops; discovers; establishes; extends; implements; includes; informs; instructs; operationalizes; participates; predicts; prepares; preserves; produces; projects; provides; relates; reports; shows; solves; teaches; transfers; uses; utilizes.

Analysis: **The breaking down of informational materials into their component parts, examining (and trying to understand the organizational structure of) such information to develop divergent conclusions by identifying motives or causes, making inferences, and/or finding evidence to support generalizations.**
➢ breaks down; correlates; diagrams; differentiates; discriminates; distinguishes; focuses; illustrates; infers; limits; outlines; points out; prioritizes; recognizes; separates; subdivides.

Synthesis: **Creatively or divergently applying prior knowledge and skills to produce a new or original whole.**
➢ adapts; anticipates; categorizes; collaborates; combines; communicates; compares; compiles; composes; contrasts; creates; designs; devises; expresses; facilitates; formulates; generates; incorporates; individualizes; initiates; integrates; intervenes;

Cal-PASS Algebra I California Content Standards Deconstruction Project Version 2.0

models; modifies; negotiates; plans; progresses; rearranges; reconstructs; reinforces; reorganizes; revises; structures; substitutes; validates.

Evaluation: **Judging the value of material based on personal values/opinions, resulting in an end product, with a given purpose, without real right or wrong answers.**

➢ appraises; compares & contrasts; concludes; criticizes; critiques; decides; defends; interprets; judges; justifies; reframes; supports.

* http://faculty.washington.edu/krumme/guides/bloom.html

Cal-PASS Algebra I California Content Standards Deconstruction Project Version 2.0