Abstract Title Page

Title: Effectiveness of Small-Group Tutoring Interventions for Improving the Mathematical Problem-Solving Performance of Third-Grade Students with Mathematics Difficulties**:** A Randomized Experiment

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Abstract Body

Background/Context

Results of the National Assessment of Educational Progress (NAEP; National Center for Education Statistics [NCES], 2011) in mathematics document the persistently low achievement of students with disabilities, English language learners, students eligible for free/reduced price school lunch, and minority students. Among fourth-graders, for example, 45% of students with disabilities contrasted with 15% of students without disabilities scored below the basic level on the 2011 NAEP (NCES, 2011). Although the focus has been on improving mathematics instruction so that all students meet the high standards as measured by state-administered achievement tests, it is crucial that students at risk for mathematics difficulties (MD), who vary considerably in ability, achievement, and motivation, develop the necessary mathematical knowledge to meet grade level benchmarks.

One approach that is designed to promote positive mathematics achievement outcomes for all students is the use of a multitiered service delivery system, the response to intervention (RTI) model (Newman-Gonchar, Clarke, & Gersten, 2009). Typically, RTI mathematics models include three levels of support; with Tier 1 comprising high quality mathematics instruction provided to all students in general education classrooms by the classroom teacher, Tier 2 comprising targeted supplemental instruction provided in small groups to at-risk students, and Tier 3 consisting of intensive individualized instruction for students who demonstrate inadequate response to Tier 2 interventions. The focus of this study is on Tier 2 level of support provided to at-risk third-grade students for understanding and solving arithmetic word problems involving whole number concepts and operations.

Arithmetic word problems play a prominent role in elementary school mathematics curricula. Word problems serve as “a vehicle for developing students’ general problem-solving skills” ... [and] “can promote a thorough and broad understanding of, and proficiency with, whole-number arithmetic” (Verschaffel, Greer, & De Corte, 2007, p. 583). However, problem solving is challenging for students with MD (e.g., Fuchs, Fuchs, & Prentice, 2004), who evidence deficiencies in domain-general abilities such as working memory, language, attentive behavior, and simultaneous storage and processing speed of information (Andersson, 2008; Andersson & Lyxell, 2007; Fuchs et al., 2010; Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007; Zheng, Swanson, Marcoulides, 2010).

Based on findings of a research synthesis related to mathematics interventions for struggling students, the What Works Clearinghouse (Gersten et al., 2009) recommended that Tier 2 interventions in elementary grades should include “instruction on solving word problems that is based on common underlying structures” (p. 6). In addition, this instruction should be “explicit and systematic” such as “providing models of proficient problem solving, verbalization of thought processes, guided practice, corrective feedback, and frequent cumulative review” (p. 6). Further, instructional materials should support student “use of visual representations of mathematical ideas” and provide opportunities for developing fluent retrieval of basic arithmetic facts” (p. 6). These recommendations are consistent with the findings of several meta-analyses of mathematics intervention research for at-risk students and students with learning disabilities that have identified explicit instruction and teaching students to use heuristics as enhancing computation and word problem solving performance (Baker, Gersten, & Lee, 2002; Gersten, Chard, Jayanthi, Baker, Morphy, & Flojo, 2009; National Mathematics Advisory Panel, 2008).

Purpose / Objective / Research Question / Focus of Study

Therefore, the purpose of this study was to compare the effects of delivering a supplemental, Tier 2 small-group intervention on the mathematics outcomes of at-risk third-grade students randomly assigned to either a schema-based instruction (SBI) group or a control group using a standards-based, school-provided curriculum. SBI and standards-based curricula are similar in their theoretical underpinnings (i.e., emphasize meaningful learning to develop conceptual understanding); however, they differ in terms of their instructional practices. Standards-based instruction is characterized by an inquiry based, student-directed approach, whereas SBI incorporates an explicit, teacher-mediated approach. Our SBI approach is a multicomponent intervention based on schema theories of cognitive psychology, research on expert problem solvers, and research regarding effective instructional practices for at-risk students. We posed the following research questions. 1. Does SBI improve the mathematical word problem-solving performance of third-grade students with mathematics difficulties immediately following the intervention (posttest) and 8-weeks after the termination of the intervention (delayed posttest)? 2. Does SBI impact transfer from whole number instruction involving word problem solving to overall mathematics achievement for third-grade students with mathematics difficulties? 3. Does SBI improve the metacognitive strategy knowledge of third-grade students with mathematics difficulties?

Setting/Participants

Participants were drawn from 28 classrooms across nine elementary schools in a large urban school district in the Midwest. Participants were third-grade students receiving mathematics instruction in general education classrooms and considered in need of supplemental mathematics support. Mathematics and reading scores on the school-administered Measures of Academic Progress (MAP; Northwest Evaluation Association, 2010) from the beginning of third-grade were examined to select students who scored at or below the 40^{th} percentile on the mathematics subtest and demonstrated a reading score of at least a beginning second-grade level based on school district norms. Because students at or below the 40^{th} percentile represent a fairly heterogeneous population of students, we created a dichotomous variable (1^{st}-25^{th} percentile vs. 26-40^{th} percentile) representing students' at-risk status to test whether SBI was equally effective for students from high and low at-risk subgroups.

A total of 127 students met the above screening criteria and 125 provided informed parent consent. Of this group, 115 (92%) participants completed all pretest and posttest assessments. Ten students did not complete the study for various reasons. To determine if there were significant differences between the original sample (125) and the final sample (115), we conducted a series of chi-square analyses. Results showed no significant differences between the students who provided data for the final sample and the 10 students who left the study on condition, gender, ethnicity, eligibility for free or reduced priced lunch, English language learner status and special education status (all $p > .40$). Demographic information for participating students is presented in Table 1.

A total of 18 tutors (17 females, 1 male) recruited from the community received training and provided all instruction in the study. Fourteen tutors were Caucasian, two were Asian American, one was African-American, and one was biracial. The majority of the tutors ($n = 16$) had bachelor's degrees, two had master's degrees and one tutor had a two-year college degree. Two tutors also had undergraduate degrees in mathematics.

Intervention

Tutoring sessions were conducted following core mathematics instruction in various locations (e.g., library, cafeteria, hallways, another classroom) in the schools students attended. In these sessions, students received 30 min of supplemental mathematics instruction using the assigned tutoring program (SBI or control) five times a week for 12 weeks. Students in the control group received instruction related to place value, whole number addition and subtraction computation and strategies, and word problem solving in the number and operations strand of their textbook. We developed an instructional package from *Investigations in Number, Data, and Space* for both tutors and students. The tutor package focused on instructional activities from four units (*Trading Stickers; Combining Coins; Collections and Travel Stories; Stories, Tables, and Graphs;* and *How Many Hundreds? How Many Miles?)*. The student package included materials from the *Student Activity Book* that corresponded to daily lessons within each unit.

For the SBI tutoring, we used the SBI curriculum designed to provide intensive instruction on word problem solving instruction. Given that students with MD struggle with whole number combinations (Chong & Siegel, 2008; Geary, 2011) and calculation skills (e.g., Jordan et al., 2002), we included a pre-unit that comprised four lessons on addition and subtraction number combinations, addition categories, inverse relation between addition and subtraction using fact families, and place value concept. The core instructional content of the SBI intervention focused on enhancing students' reasoning skills within the context of solving one-step and two-step word problems involving *Change, Group,* and *Compare* problem types (see Table 2). Tutors used the following four instructional practices of (a) priming the mathematical structure of problems, (b) visually mapping information in the problem using schematic diagrams, (c) providing explicit instruction on a problem-solving heuristic, and (d) promoting metacognition of strategy knowledge. Tutors explicitly modeled and scaffolded instruction by gradually shifting responsibility for problem solving to the students.

Research Design

We conducted a randomized control trial with a pretest–intervention–posttest–retention test design. Students within the same classroom, identified as having MD, were randomly assigned to either schema-based instruction (SBI) or the control group. Then, students within each condition were randomly assigned to instructional groups. Finally, we randomly assigned tutors to the instructional groups. Classroom teachers administered the MAP tests in fall and spring of the academic year as part of the district-wide evaluation program. We collected pretest data on word problem solving (WPS) and metacognitive strategy knowledge two weeks before the intervention started. Posttest data on the same variables were collected at the end of the intervention as well as eight weeks later on the WPS test to measure retention of problem solving skills. Research assistants and tutors were trained to administer these measures in small-group arrangements. We gathered tutoring implementation data via observations.

Data Analysis

We fitted a two-way analysis of covariance (ANCOVA) model to each outcome of interest, where the treatment variable and the at-risk status variable were the effects of primary interest and the corresponding pretest measure and demographic variables (gender, ethnicity, and eligibility for free or reduced priced lunch [FRL]) were included as covariates. At-risk status was coded so that students between the 26^{th}-40^{th} percentiles = 1, and students between the 1^{st}-25^{th} percentiles = 0. Gender was coded so that males = 0 and females = 1, ethnicity was coded so that minority students (African American, Hispanic, Asian American) = 0 and Caucasian students = 1, and FRL was coded so that students eligible for free or reduced priced lunch = 1 and students not eligible = 0. We calculated effect sizes by dividing the covariate adjusted mean differences on the posttests by the square root of the mean square error (Glass, McGaw, & Smith, 1981).

Results

On the WPS posttest, results indicated a statistically significant treatment effect, $F(1, 104) = 6.66$, $p = .011$. As hypothesized, SBI students outperformed control students ($d = 0.48$). Results also indicated a statistically significant effect for at-risk status, $F(1, 104) = 4.75$, $p = .03$, with low at-risk students outperforming high at-risk students ($d = 0.45$). The interaction between the treatment effect and students' at-risk status was not significant, $F(1, 104) = 0.88$, $p = .35$, indicating that SBI was effective regardless of students' at-risk status. On the WPS delayed posttest, the treatment effect was not statistically significant, $F(1, 103) = 1.09$, $p = .30$. Likewise, there was no significant effect for students' at-risk status, $F(1, 103) = 2.48$, $p = .12$, or the interaction between the treatment effect and students' at-risk status, $F(1, 103) = 0.20$, $p = .66$.

For the MAP mathematics achievement test, there was a statistically significant treatment effect, $F(1, 101) = 4.29$, $p = .041$. As hypothesized, SBI students outperformed control students ($d = 0.40$). Results showed no significant effect for students' at-risk status, $F(1, 101) = 0.001$, $p = .98$ or the interaction between the treatment effect and students' at-risk status, $F(1, 101) = 0.32$, $p = .57$, the latter indicating that SBI was equally effective for students regardless of their at-risk status. For metacognitive strategy knowledge, results indicated a statistically significant treatment effect, $F(1, 100) = 12.80$, $p = .001$. SBI students outperformed control students ($d = 0.68$). The effect of students' at-risk status was non-significant, $F(1, 100) = 3.18$, $p = .08$ as was the interaction between the treatment effect and students' at-risk status, $F(1, 100) = 0.16$, p = .693.

Five research assistants observed and audiotaped a total of 54 tutoring sessions in both conditions during the 12-week tutoring intervention. Tutor fidelity of implementation was evaluated using a 3-point scale (1 = *low adherence*, 3 = *high adherence*) for each item on the observation forms (SBI and control). SBI tutors averaged 2.93 ($SD = 0.16$) and control tutors averaged 2.79 ($SD = 0.25$). A second rater listened to the audiotape and rated one randomly selected audiotape of a tutor. Inter-rater agreement calculated using kappa (K) averaged .98 for SBI and .78 for the control condition.

Conclusions

The present research assessed the efficacy of two tutoring protocols for improving the mathematics outcomes of at-risk third-grade students. Results indicated that students in the SBI group outperformed students in the control group on word problem solving performance after 30 hours of problem-solving experience, but the problem-solving performance was not maintained 8 weeks following the tutoring intervention. However, the SBI condition led to superior performance on a standardized mathematics achievement test compared with the control group. With respect to metacognitive strategy knowledge, the SBI condition outscored the control condition. Further, results suggested that SBI was equally effective for students from various at-risk subgroups. These findings demonstrate the efficacy of SBI tutoring conducted by trained tutors.

Appendix A. References

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Appendix B. Tables and Figures

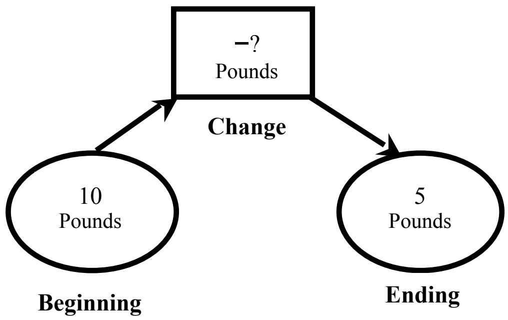
Table 1. *Student Descriptive Statistics by Condition*

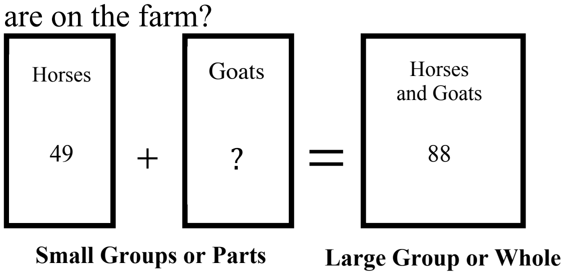
		Control (n = 58)		SBI (n = 57)				
		N	%	N	%	χ^2/F	*df*	*p*
Gender	Male	27	46.5	28	49.1	0.08	1	.783
	Female	31	53.5	29	50.9			
Ethnicity	Asian/Pacific Islander	3	5.2	2	3.5	4.64	5	.462
	Black	18	31.0	11	19.3			
	Hispanic	16	27.6	20	35.1			
	American Indian	2	3.5	5	8.8			
	White	19	31.0	19	33.3			
Eligible for free/reduced priced lunch		39	67.2	38	70.4	0.13	1	.721
English language learner		17	29.3	17	31.5	0.06	1	.803
Special education status		8	13.8	5	9.3	0.56	1	.454
Age (in months): *M (SD)*		100.90 (4.02)		100.91 (3.84)		0.00	1, 110	.988

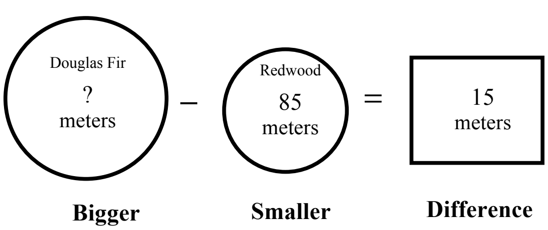
Note: SBI = schema-based instruction

Table 2

Solving One-Step and Two-Step Problems Involving *Change, Group, and Compare* Schemata

<table>
<tr><th>Problem Representation</th><th>Problem Solution[a]</th></tr>
<tr><td>*Change:* Jose and his father have gathered 10 pounds of wood from a sheep. So far, some of the wool has been used to make a sweater. Now there are 5 pounds of wool left. How many pounds of wool have been used?

</td><td>Number sentence: 10 – ? = 5 *(We can solve this problem by thinking about the fact or number families. We know the greater number in this subtraction problem. So, we can subtract the lesser number from the greater number [10 – 5 = ?] to solve for ?. Alternatively, we can think addition to solve this problem.)*

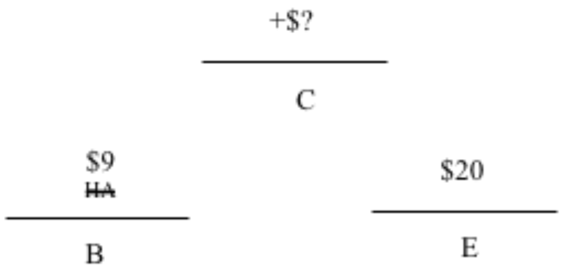
Answer: 5 pounds of wool have been used.</td></tr>
<tr><td>*Group:* Farmer Jake has 88 animals on his farm. He only has horses and goats. There are 49 horses on the farm. How many goats are on the farm?

</td><td>**Number sentence: 49 + ? = 88 *(We can solve this problem by thinking about how addition and subtraction are related. We can undo the addition problem, write a subtraction problem: 88 – 49 = ?, and solve for ?)***

Answer: There are 39 goats on the farm.</td></tr>
<tr><td>*Compare:* A redwood tree can grow to be 85 meters tall. A Douglas fir can grow to be 15 meters taller. How tall can the Douglas fir grow?

</td><td>**Number sentence: ? – 85 = 15 *(We can solve this problem by thinking about how addition and subtraction are related. We don't know the greater number in this subtraction problem. So, we can undo the subtraction problem, write an addition problem, 85 + 15 = ?, and solve for ?)***

Answer: The Douglas fir can grow 100 meters.</td></tr>
<tr><td>[b]Two-step problem: Alesha has $15. Arlen</td><td></td></tr>
</table>

has $6 less than Alesha. How much more money does Arlen need to buy a theme park admission ticket that costs $20.

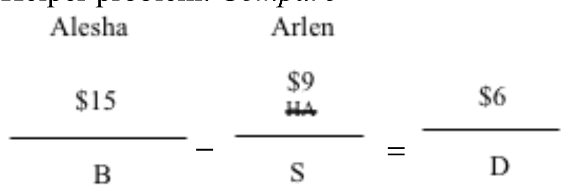
Main problem: *Change*



Number sentence: $9 + $? = $20 *(We can solve this problem by asking ourselves 9 + ? = 20, or we can undo the addition problem, write a subtraction problem [$20 – $9 = $?], and solve for ?)*

Final Answer: Arlen needs $11 more to buy a theme park admission ticket.

Helper problem: *Compare*



Number sentence: 15 – HA = 6 *(We can solve this problem by thinking about the fact or number families. We know the greater number [15] in this subtraction problem. So, we can subtract the lesser number from the greater number [15 – 6 = HA] to solve for HA.)*

Helper Answer (HA): Arlen has $9.

Note. [a]Students also were encouraged to use previously learned solution strategies (see Verschaffel et al., 2007) to solve the number sentence; [b]Diagrams in the *two-step* problem are faded versions.

Table 3

Means and Standard Deviations for Measures by Condition and At-Risk Status

	Condition					
	Control			SBI		
	Mean	SD	N	Mean	SD	N
WPS						
Pretest						
High at-risk[a]	13.76	5.50	27	13.45	5.32	30
Low at-risk[b]	15.79	5.35	31	18.44	6.44	24
Combined at-risk	14.84	5.47	58	15.46	6.38	57
Posttest						
High at-risk[a]	21.21	6.68	27	24.58	4.91	30
Low at-risk[b]	25.52	7.09	31	28.60	5.76	24
Combined at-risk	23.51	7.18	58	26.39	5.56	57
Adjusted Posttest						
High at-risk[a]	22.31			25.85		
Low at-risk[b]	25.59			27.23		
Combined at-risk	23.95			26.54		
Delayed Posttest						
High at-risk[a]	22.34	7.17	31	22.94	6.90	30
Low at-risk[b]	25.05	6.58	30	28.26	7.05	24
Combined at-risk	23.77	6.94	57	25.32	7.31	57
Adjusted Delayed Posttest						
High at-risk[a]	22.99			23.72		
Low at-risk[b]	24.49			26.30		
Combined at-risk	23.74			25.01		
MAP Mathematics						
Pretest						
High at-risk[a]	179.00	3.22	27	177.77	4.77	30
Low at-risk[b]	186.35	1.52	31	186.67	1.69	24
Combined at-risk	182.93	4.43	58	181.72	5.80	54
Posttest						
High at-risk[a]	194.62	9.37	26	195.97	9.76	29
Low at-risk[b]	200.37	5.94	30	204.17	6.44	24
Combined at-risk	197.70	8.18	56	199.68	9.31	53
Adjusted Posttest						
High at-risk[a]	198.09			200.27		
Low at-risk[b]	197.33			201.16		
Combined at-risk	197.71			200.71		

Metacognitive Strategy Knowledge						
Pretest						
High at-risk[a]	9.56	1.63	27	9.11	1.77	28
Low at-risk[b]	8.87	1.98	31	9.13	1.48	24
Combined at-risk	9.19	1.84	58	9.22	1.65	55
Posttest						
High at-risk[a]	8.81	1.65	26	10.03	1.78	29
Low at-risk[b]	9.19	1.99	31	10.21	1.79	24
Combined at-risk	9.02	1.84	57	10.20	1.76	56
Adjusted Posttest						
High at-risk[a]	8.44			9.66		
Low at-risk[b]	9.13			10.10		
Combined at-risk	8.78			9.88		

Note: [a]scored < 26th percentile in mathematics; [b]scored ≥ 26th percentile and <40th percentile in mathematics; SBI = schema-based instruction; WPS = word problem solving; MAP = Measures of Academic Progress.