COGNITIVE LEVEL OF DEVELOPMENT AND MATHEMATICAL FLUENCY OF FIRST GRADE CHILDREN

THESIS

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COGNITIVE LEVEL OF DEVELOPMENT AND MATHEMATICAL FLUENCY OF

FIRST GRADE CHILDREN

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DEDICATION

This thesis is dedicated to my grandfather.

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ABSTRACT

COGNITIVE LEVEL OF DEVELOPMENT AND MATHEMATICAL FLUENCY OF FIRST GRADE CHILDREN

By

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December 2010

SUPERVISING PROFESSOR: DR. BARBARA DAVIS

This study was designed to investigate the cognitive level of development and mathematical fluency of first grade children. A total of (N=100) 6- and 7-year-olds from two low socioeconomic level elementary schools participated in this study. Piaget's conservation-of-liquid task was administered to children to determine their cognitive level of development. The fixed factor was the between-subjects variable group, which included (n=50) conserving and (n=50) nonconserving children in the first grade. The research hypotheses were addressed by using a MANOVA with the two dependent variables addition fluency and subtraction fluency. A counterbalanced method was employed to administer two separate single-skill math fact probes for two minutes to measure addition and subtraction fluency. The results indicated a highly significant effect on addition fluency and subtraction fluency (both *p*'s < .001) as a factor of conservation ability with neither addition nor subtraction having a substantial advantage over the other. The covariate of age had a separate effect on mathematical fluency above and beyond cognitive level of development. The covariates race and gender had no effect on fluency. As indicated in this study, cognitive level of development was not a grade-level-based designation; levels of cognitive development were characterized by different abilities in mathematical fluency. Implications of this study point out that instruction and curriculum are not the sole factors in mathematics achievement. A teacher's use of curriculum through instruction should be considered subject to the cognitive abilities children. Developmentally appropriate teaching practices should be considered in the context of learning elementary school mathematics.

Keywords: Piagetian Theory; Conservation; Cognitive Development; Grade 1; Elementary School Mathematics; Addition; Subtraction; Mathematics Achievement

CHAPTER I

INTRODUCTION

Education in the twenty-first century must focus upon increased skills of children related to success in the technological revolution (Chapin & Johnson, 2006). Stigler et al. (1990) stated that as early as first grade the mathematics achievement of children in the United States compared unfavorably with that of other countries (as cited in Sarama & Clements, 2009). Charles (2005) stated that in mathematics the teaching practices were fundamentally different than those of higher achieving countries. For example, effective teachers asked appropriate and timely questions and were able to assess the thinking and understanding of children during instruction. These developmentally appropriate teaching practices have increased attention towards mathematics education for elementary age children. Addition and subtraction were the most prominent topics in elementary mathematics education (Sarama & Clements, 2009).

In the past there have been contrary arguments by scholars pertaining to the effects of cognition and academic achievement of children. Jerome Bruner, who was a scholar in the United States, believed that acceleration in learning could be achieved at any age if appropriately presented. Jean Piaget, who was a Swiss developmental psychologist, often argued that educators in the United States who tried to accelerate learning violated the theory of operational structures of cognition (Pulaski, 1969). Copeland (1974) indicated that Piaget argued that the learning abilities of children were subsequent to the operational structures of development. Bruner's perspective maintained importance of an educator's pedagogical techniques and the design of curriculum. Learning and development were not mutually exclusive, they both contributed to the ability of children to learn mathematics. Piaget's perspective focused on cognition as a factor that influenced the ability to learn. Piagetian theory was developed as a framework that considered the abilities of children according to their cognitive level of development (Ojose, 2008). Children in their first years of elementary school transitioned from the preoperational to the concrete operational level of cognitive development. These two invariant levels are differentiated by the ability of children to conserve. In part, logical thinking and reversibility characterize conservation ability and affect the learning of mathematics (Jarsild, 1968; Lemlech, 2010). Children who have difficulty in mathematics often have not become fluent in the necessary skills. Sarama and Clements (2010) argue that intensive work in achieving fluency should be reserved for core subject areas, such as addition and subtraction. The fluency of children contributed to greater achievement in mathematics (Ramos-Christian, Schleser, & Varn, 2008). In elementary grades, learning operations was one of the most researched domains in mathematics (Sarama & Clements, 2010). However, there is relatively little empirical knowledge about the development of fluency and the direct relationship between cognition and fluency for children in elementary school (Ramos-Christian et al., 2008).

Cooper and Schleser (2006) and Ramose-Christian et al. (2008) examined cognitive level of development and mathematical fluency. These studies measured mathematical fluency as a result of the combined factors of addition, subtraction, and a minimal amount of multiplication problems. Cooper and Schleser (2006) found that conserving children had greater mathematical fluency than nonconserving children. However, this finding considered the ethnicities of children to be a contributing factor in mathematical achievement. Ramos-Christian et al. (2008) supported the finding that conservation ability had an effect on mathematical fluency. In addition, the authors examined mathematical fluency subject to the components of speed and accuracy. The authors concluded that conservation ability had a comparable effect on accuracy, but an incomparable effect on speed. There was a limitation in these studies that needed to be examined. These studies did not separately address addition fluency and subtraction fluency subject to conservation ability. As a result of this aperture of knowledge, the problem of this study was formulated.

Statement of Problem

The problem of this study was to investigate the effect of cognitive level of development on mathematical fluency of first grade children. The designation of cognitive level of development was partitioned by conserving and nonconserving ability. The factors inclusive to mathematical fluency were addition and subtraction. The addition fluency and subtraction fluency was separately measured subject to the conserving and nonconserving abilities of children in first grade. The problem of this study led to the development of several research questions.

Statement of Research Questions

This study was designed to investigate nonconserving and conserving children in first grade and their addition fluency and subtraction fluency. Three research questions guided this study.

1. Is there a difference in addition fluency and subtraction fluency among first grade children who were conserving and nonconserving?
2. Is there a difference in addition fluency among first grade children who were conserving and nonconserving?
3. Is there a difference in subtraction fluency among first grade children who were conserving and nonconserving?

Statement of Hypotheses

The three research questions were posed in the form of three non-directional null and alternative hypotheses. The symbols: H_0 represented the null hypothesis, μ_1 represented nonconserving children in first grade, and μ_2 represented conserving children in first grade. The level of significance for this study was set at the .05 alpha level.

Addition Fluency and Subtraction Fluency

Null Hypothesis. It is hypothesized that there will not be a significant difference in addition fluency and subtraction fluency, at the .05 alpha level, between first grade children who were nonconserving and conserving.

$$H_0 : \mu_1 = \mu_2$$

Alternative Hypothesis. It is hypothesized that first grade children who were nonconserving and conserving will differ significantly, at the .05 alpha level, in addition fluency and subtraction fluency.

$$H_1 : \mu_1 \neq \mu_2$$

Addition Fluency

Null Hypothesis. It is hypothesized that there will not be a significant difference in addition fluency, at the .05 alpha level, between first grade children who were nonconserving and conserving.

$$H_0 : \mu_1 = \mu_2$$

Alternative Hypothesis. It is hypothesized that first grade children who were nonconserving and conserving will differ significantly, at the .05 alpha level, in addition fluency.

$$H_1 : \mu_1 \neq \mu_2$$

Subtraction Fluency

Null Hypothesis. It is hypothesized that there will not be a significant difference in subtraction fluency, at the .05 alpha level, between first grade children who were nonconserving and conserving.

$$H_0 : \mu_1 = \mu_2$$

Alternative Hypothesis. It was hypothesized that first grade children who were nonconserving and conserving will differ significantly, at the .05 alpha level, in subtraction fluency.

$$H_1 : \mu_1 \neq \mu_2$$

Statement of Purpose

The purpose of this study was a result of limited research on the effects of cognitive level of development and mathematical fluency. Ramos-Christian et al. (2008) stated that few studies focused on the development of fluency and that no studies examined the direct relationship between cognition and fluency. This study was aimed at examining the aperture of empirical knowledge that existed on this topic. The contributions to expanding the knowledge base on cognition and fluency led to the validation of prior studies, further research in elementary school mathematics, and teaching practices that considered cognition and learning. This study was significant in many ways.

Statement of Significance

The significance of this study was for stakeholders in mathematics education to be cognizant that the ages of children were often a categorical measure of grade level, however cognitive level of development was not a grade level designation. The conservation ability of first grade children leads to different abilities in mathematics. Tomlinson (2009) argues that the flexibility of curriculum and instruction should meet the needs of all children within a grade level. Research that validated and expanded the knowledge in elementary school mathematics contributed to future success of learning mathematics based on teaching practices that were developmentally appropriate.

Statement of Terminology

The terms related to cognitive level of development and mathematical fluency were defined especially for use in this study.

- *Accommodation*, referred to the ability to make changes to the cognitive structure for information that does not fit into already existing schemes (Crain, 2005).
- *Addition*, referred to the addends that are joined together to create the sum or group (Copeland, 1974).

- *Assimilation*, referred to new information acquired into the existing cognitive structure (Crain, 2005).
- *Centration*, referred to one-dimensional thought that was focused on the most compelling part perceived and the exclusion of other parts or the whole that were of equal importance (Pulaski, 1971).
- *Concrete Operational*, referred to children who have the ability to conserve and is associated with concrete objects, ordering, arrangement, classification, reversibility, and decentration (Pulaski, 1971).
- *Cognition*, referred to the intellective activities of the mind, such as thinking, knowing, remembering, perceiving, recognizing, and generalizing (Pulaski, 1971).
- *Conservation*, referred to the ability to think logically and have reversibility of thought where changes in appearance occur while the object remains the same (Jarsild, 1968).
- *Decentration*, referred to consideration for the parts that are incorporated into the whole (Pulaski, 1971).
- *Mathematical Fluency*, referred to the combination of accuracy and speed (Ramos-Christian et al., 2008).
- *Level*, referred to children who are either conserving or nonconserving (Copeland, 1973).
- *Math Fact Probe*, referred to an instrument of single-skill addition or subtraction (AIMSweb, 2010).
- *Nonconserving*, referred to children in the preoperational level of cognitive development who maintained illogical thought, lacked reversibility, were perceptually

driven, and focused on one-dimension of a whole and while not considering the parts (Ginsburg & Opper, 1969).

- *One-dimensional*, referred to the ability to only focus on one part of perception (Crain, 2005).
- *Operations*, referred to reversibility and logical thinking (Pulaski, 1971).
- *Piagetian Theory*, referred to the developmental framework of cognition that included sensory-motor, preoperational, concrete operational, and formal operations (Ojose, 2008).
- *Preoperational,* referred to children who were nonconserving and maintained illogical thought and perceptual reasoning (Ramos-Christian et al., 2008).
- *Race*, referred to children who were Hispanic, Caucasian, Asian/Pacific Islander, and African American.
- *Reversibility*, referred to illogical thought that occurred in one way (Lemlech, 2010).
- *Subtraction*, referred to an action of separating objects to obtain a difference (Copeland, 1974).
- *Two-dimensional*, referred to children who can simultaneously consider two parts at the same time while changes occurred in one part while the other parts remained constant (Crain, 2005).

CHAPTER II

REVIEW OF LITERATURE

The following review of literature examined cognitive levels of development and mathematical fluency. The first section focused on the Piaget's different levels and characteristics of cognitive development. The second section focused on cognitive level of development combined with mathematical fluency. The cognitive levels of development and mathematical fluency were examined to build a theoretical foundation for this study.

Cognitive Level of Development

Piaget and Inhelder (1969) argued that there were four levels of cognitive development of mental actions classified into mental structures. These levels of cognitive development included the sensory-motor, preoperational, concrete operational, and formal operations. These levels were not a designation of the ages of children, but were representative of the different levels of understanding and ability as a result of the development of cognitive structures into operational ability. There were two cognitive levels of development that were pertinent to this study. The preoperational and concrete operational levels of cognitive development were examined to identify operational characteristics and abilities of children at each of these levels.

Ojose (2008) stated that the development of children continues through a series of continuous transformation that were attributed to different levels of cognitive functioning. Children remained in each of these levels for an amount of time that varied. As a result of changes in cognitive structure, children progress in their cognitive ability with the preceding cognitive structure serving as the foundation for the new level of cognitive development. However, the transition of children through these levels was not predetermined or a result of chronological age. It was rather a designation of maturity, experiences, culture, and ability. All children, however, do transition through these levels chronologically or in the same order without skipping a level.

Lemlech (2010) explained that if age is to be designated around the levels of development then children who are between the ages of two to seven can be represented by preoperational ability and children between the ages of seven to eleven can be represented by concrete operational ability. Children are most likely to transition from the preoperational level to the concrete operational level around six and seven years old. Consequently, this corresponds to children in first and second grade. There were several characteristics that encompassed children in the preoperational level of cognitive development.

Crain (2005) explained that children progress from the preoperational to the concrete operational level of cognitive development in an invariant sequence. The developed frameworks of the cognitive structures were fixed and there were particular abilities that were represented at each level. As a means of progressing to the next level, children undergo developmental change in their cognitive structures. This change was influenced through exploration, manipulation, and interaction with the external environment. This is explained through the characterization of common biological tendencies that include assimilation, accommodation, and organization. As mentioned by Piaget and Inhelder (1969) children maintain cognitive structures that represent their level of functioning through the invariant sequence of development. The process of assimilation occurs as children integrate external elements into their already existing internal cognitive structure. However, as the integration of external elements occurs though assimilation there becomes a point where the internal cognitive structures must accommodate new external elements. The accommodation of new external elements occurs when a change in the existing cognitive structure is required to continue the process of assimilation. The process of accommodating new external elements creates efficiency in the assimilation process. The integration between assimilation and accommodation occurs through the organization of newly formed cognitive structures that work in cooperation. These newly formed cognitive structures become realized through the different levels of cognitive development as represented by the ability of children.

Copeland (1974) discusses children in the preoperational level of cognitive development as being characterized by cognitive structures that were semi-logical and one-dimensional. This semi-logical and one-dimensional thought was indicated through the lack of understanding the reversibility of action. That is, one action aimed at obtaining a result can also be reversed to its original position. The mind builds up reversible sequences underneath perception, which eventually leads children to perform logical reversibility. The characteristics of preoperational thought can be observed through conservation tasks in which children in different levels respond differently to the occurrence of perceived actions. These perceived actions represent the development of the internal structure of cognitive functioning.

Schwebal and Raph (1973) explained that Piaget's most notable experiment, conservation-of-liquid, used two cups that have the same diameter and contain the same amount of liquid. The liquid in one of these cups was then poured into a cup with a smaller diameter. The liquid in the original cup that had a wider diameter and the liquid in the new cup that had a smaller diameter were compared to each other. Children failed to conserve in this conservation task by responding that the liquid in the to two cups were different. For example, the levels of liquid were the same in the original two cups, but when the liquid in one of these cups was transferred to a new cup with a smaller diameter the children were not able to realize that although the liquid levels appeared different, the liquid had not changed, but had been transferred. In this example, the children held in their mind that the original cups contained the same amount of liquid and this amount changed when the transfer occurred. The conservation ability of children was identified as being in the preoperational level of cognitive development. This level demonstrated an inability to compensate for perceptual discrepancies by only being capable of focusing on one part of a situation while simultaneously excluding other equally relevant parts. This preoperational capacity was reflected in a one-dimensional perspective. Children who were nonconserving were limited by one-dimensionality that was represented by a static perception of the events. The liquid in the cup was perceived to have changed after the transfer occurred even though no liquid had been added or taken away. Their perception of these events validated the absence of an underlying operational structure, which is maintained in conservation.

Ginsburg and Opper (1969) explained that preoperational children demonstrate *centration*, which was when children only consider one-dimension and then center on that dimension while neglecting other dimensions. Illogical thought causes children to fail in conservation tasks, because the reversibility of the action is not realized. These children are drawn in by the perceptual influences of the actions. However, the transition from nonconserving to conserving appears to occur spontaneously. The conservation ability of children was indicated when children expressed that although the two cups appeared different, the amount of liquid was unchanged. Children realized that no liquid was added or taken away, but only transferred. These children were able to realize that if the liquid were then poured back into the original container then it would again be equal. Children in the concrete operational level of cognitive development become *decentered* from one-dimensional thought, and instead demonstrated an ability to consider two-dimensions simultaneously. For example, the levels of liquid were the same in the original two cups, however when the liquid in one of these cups was transferred to a new cup with a smaller diameter the children were able to realize that although the liquid appeared different this action could be reversed. In this example, the children are capable of holding in their mind that the original cups contained the same amount of liquid and that after the transfer occurred they still contained an equal amount of liquid. The children who conserve are freed from perceptual influence. They demonstrated logical thought that underlies the perceptual interpretation of action. This represented that once actions have occurred they can also be reversed. This is the requisite for operational ability of conservation. These children are able to reject perception in favor of logical reasoning. This allows the mind to accept reversible sequences underneath perceptual stimulus. This action is indicated by the transition of children into the concrete operational level of cognitive development by their ability to conserve liquid quantities through logical and reversible thought. This ability was summarized by three characteristics that distinguish nonconserving and conserving children. The three characteristics included centration-decentration, static-dynamic, and irreversibility-reversibility. When nonconserving children focus on the static aspect of a situation then this affected their awareness of perceptual transformations. These transformations were critical to reversibility and dynamic thought. The awareness of transformations becomes realized through decentration of thought by the ability to consider more than one part that may be equally important to other parts not realized in centration. The decentration ability allowed children to become aware of the reversibility of action.

Webb (1980) applies the findings of Piaget's theory of cognitive development to mathematics abilities and tasks. The preoperational level of cognitive development in mathematics was characterized by actions that promote expression of the child's thought process. During this level of development children lacked the rationality of logical thought and reversibility of action. Children in this level of development contain elements of one-dimensionality and have difficulty focusing on multiple tasks and limit their cognitive capacity to process information. During instruction, teachers should elicit engaging and investigational techniques for introducing new concepts. Children should engage in socialization activities. Children should be allowed to manipulate concrete objects and encounter a multitude of experiences. The concrete operational level of cognitive development in mathematics is characterized by an increase in the cognitive capacity to process tasks simultaneously in a two-dimensional manner. A rational and logical thought process was a mark of children in this level of cognitive development. Children gain an understanding of reversibility and benefit from hands-on activities that foster the connection between concrete and abstract representation (Chapin & Johnson, 2006; Copeland, 1974; Lemlech, 2010; Ojose, 2008; Pulaski, 1971). Utilization of multiple representations of mathematical tasks provides meaningful insights into the in-depth conceptual understanding of mathematical concepts. Children understand that subtraction tasks are the inverse of addition tasks (Copeland, 1974). This appears to increase the need for conservation ability as a prerequisite for addition and subtraction task, because reversibility of thought must be maintained (Chapin & Johnson, 2006).

In order to study the effects of preoperational and concrete operational functioning on children's ability applied to mathematical fluency some objective means of determining the cognitive level of functioning of children must be utilized. The ability of children through the conservation-of-liquid task appears to be seen as objective evidence in being able to represent the abilities of conserving and nonconserving children.

Mathematical Fluency

Cooper and Schleser (2006) examined the effects of cognitive development on closing the achievement gap between African-American and Caucasian children. The sample consisted of a total of (N=56) kindergarten and first grade children. The children in kindergarten represented (n=26) of the sample and the children in first grade represented (n=30) of the sample. There were 16 African-American children and 40 Caucasian children. The mathematical abilities of these children were assessed using Woodcock Johnson III (WJ III) Test of Achievement subtest in Math Fluency. The results of this study suggested that children of African-American descent remained in the preoperational level of cognitive development for a longer duration of time than Caucasian children. This led to an increase in achievement scores in mathematics for children who were Caucasian. The Caucasian children scored significantly higher than the African-American children on the subtest of Math Fluency. However, when cognitive level of development was controlled for the differences in ethnicity were indistinguishable. The authors suggest that cognitive level of development may mediate the relationship between race and mathematical abilities as measured through formal assessments. However, two African-American children who qualified for free or reduced lunch were excluded from the results of this study. The authors still supported the findings after including the previously excluded children from the study based on their economically disadvantaged background. The authors in part, concluded that conserving children had greater mathematical fluency than nonconserving children.

Ramos-Christian, Schleser, and Varn (2008) examined math fluency as it related to cognitive development in preoperational and concrete operational children in first and second grade. Their sample (N=39) consisted of (n=19) children in preoperational and (n=20) children in concrete operational level of development from an upper-middle class background. Children involved in this study were of average to above average intelligence. The Woodcock Johnson III (WJ III) Test of Achievement subtest in Math Fluency was used as the testing instrument. The WJ III was a norm-referenced three-minute timed multi-skill instrument that combined addition, subtraction, and a few multiplication operations. The first and second grade children in the concrete operational level had greater mathematical fluency than children in the preoperational level of cognitive development. The authors concluded that preoperational children lacked the speed, but had comparable levels of accuracy to children in the concrete operational level of cognitive development. Overall, the children who had the ability to conserve had greater mathematical fluency than children who were nonconserving.

Poncy, Skinner, and Jaspers (2007) evaluate two methods that serve as an instructional intervention for math fluency (accuracy and speed). The two methods examined include Cover, Copy and Compare (CCC), and Taped Problems (TP). These methods where used on a 10 year old female with a low level of cognitive functioning and a Full Scale IQ of 44. These interventions where presented during 12 pullout sessions. Both of these interventions increased mathematical achievement. However, the mathematical interventions utilizing TP was found to be more efficient in its implementation although both interventions increased mathematical fluency by a significant amount. The authors supported that instructional methods may promote the development of mathematical fluency among children with low levels of cognitive functioning. Therefore, learning mathematical fluency may in part be affected by curriculum and instructional techniques.

Copper and Schlesser (2006) and Ramos-Christian et al. (2008) conducted two studies with similar results. These studies included children from mid-to-high socioeconomic level backgrounds in Kindergarten to second grade. However, the size of the sample in each of these studies was relatively small. The study conducted by Cooper and Schlesser (2006) included 56 children and they study by Ramos-Christian et al. (2008) included 39 children. These studies used a multi-skill instrument to measure mathematical fluency. This instrument combined addition, subtraction, and a minimal amount of multiplication problems. However, the use of an instrument that encompasses many different mathematical operations may run contrary to the characteristics of children who are nonconserving. These characteristics included illogical thinking and irreversibility. This may have affected their ability to interchange between different mathematical operations by the effect of centration. The use of an addition and subtraction instrument administered separately may have measured mathematical fluency in terms of one-dimensionality by focusing on each operation. This may have perspicuously delineated addition fluency, subtraction fluency, or both as the contributing factor to the difference in mathematical fluency among conservation level.

In summary, theories, which have been formulated, and experiments, which have been conducted with children regarding the cognitive levels of development, appear to have indicated that logical operations including the element of reversibility begin to occur in 6 and 7-year-olds. The mathematical fluency of children may be affected by their conservation ability in first grade.

CHAPTER III

PROCEDURE

This chapter examined the procedures involved in conducting this study. The sites and subjects were selected. Then the subjects took part in a conservation-of-liquid task to identify their level of conservation. Following this task, the subjects were administered two separate math fact probes to measure their addition fluency and subtraction fluency.

Site Selection

There were two public elementary schools selected to participate in this study. The two elementary schools were located in south Texas in a University community within the Austin-San Antonio corridor. The two elementary schools included children from kindergarten to fifth grade. According to the U.S. Department of Education, the two elementary schools were designated low-income, because more than 30% of the children enrolled were from low-income families (http://www.tea.state.tx.us/loan.aspx).

As shown in Figure 1, the demographic characteristics of the two elementary schools were quite similar. The student population at elementary school one consisted of 553 children. Sixty-one percent of the children were Economically disadvantaged, 33% At-risk, 6% Bilingual, 1% English as a Second Language (ESL), and 10% Special Education (Monica Weaver, personal communication, February 22, 2010). The children at this elementary school were the first to participate in this study. The children were tested during the spring semester at the end of April and beginning May. The student population at elementary school two consisted of 578 children. Seventy-four percent of the children were Economically disadvantaged, 45% At-risk, 15%, Bilingual, 1% English as a Second Language (ESL), and 8% Special Education (Monica Weaver, personal communication, February 22, 2010). Following the culmination of testing at elementary school one, children at elementary school two were tested during the spring semester in the middle of May.

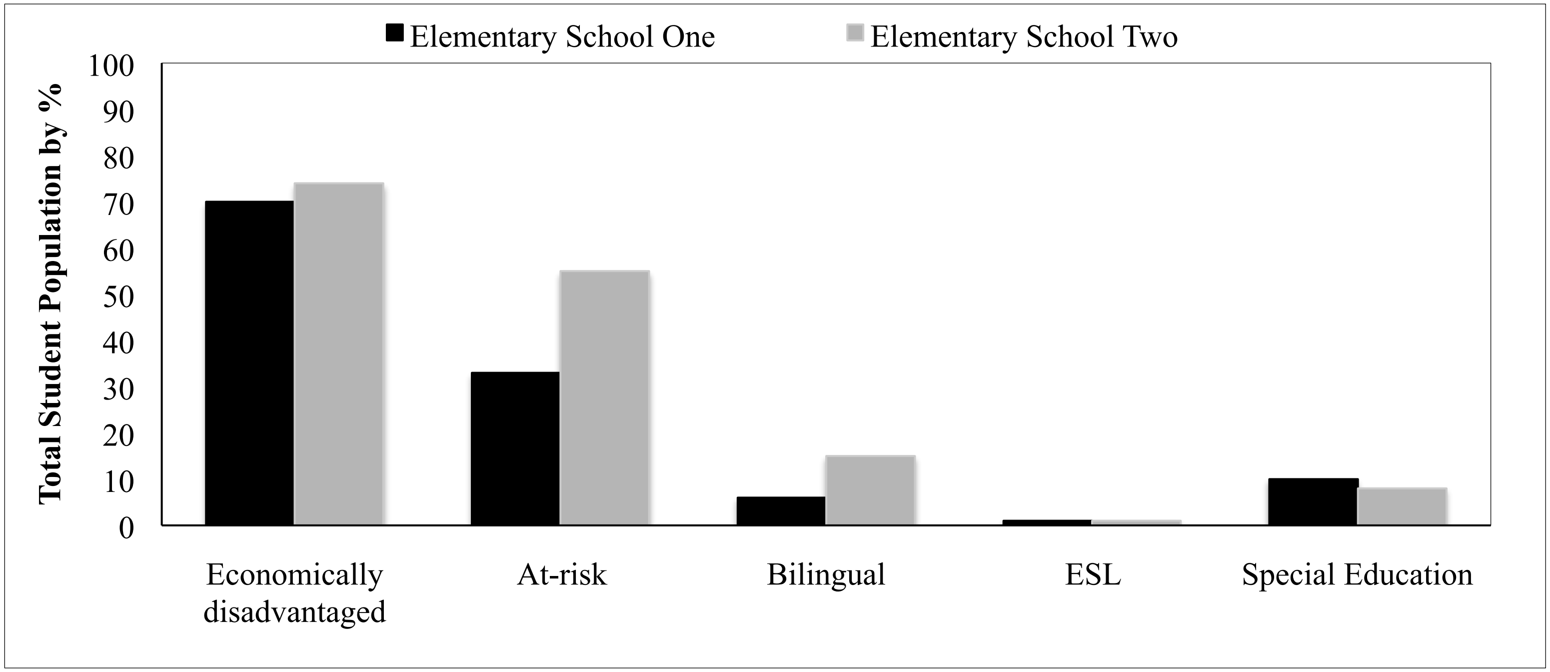

Figure 1. Demographic Characteristics of Elementary School One and Two.

Subjects

Subjects were obtained according to the Texas State University-San Marcos protection of human subjects practices. Subjects for this study were selected from among 195 children in first grade classrooms two elementary schools. There were 101 children in five first grade classrooms at elementary school one, and 94 children in first grade classrooms at elementary school two who were available as potential subjects in this study. The demographic covariates included gender, race, and age. Final selection of the subjects in this study was accomplished by the following procedures.

1. A consent form in English and Spanish was provided to all participants (see Appendix A & B) as partial requirement for adherence to the Institutional Review Board (IRB) approval to conduct research on human subjects (see Appendix C).
2. English and Spanish consent forms where distributed to children on three separate occasions over a 3-week period beginning each Monday and picked up from the classroom teacher on Friday.
3. Children who did not return a consent form, who returned a consent form and then moved, or who returned a consent form electing not to participate were excluded from participation in the study. There were a total of 81 children who did not return a consent form. Two children returned a consent form, but moved prior to the commencement of testing. There were two children who returned their consent form indicating they would not participate in the study.

Method

At elementary school one and two the children who brought back their signed consent form with permission to participate in the study were individually called upon to come and sit at a table positioned outside of their classroom. Testing occurred between classroom transition times to minimize possible distractions for the children. Most often the hallway was quiet with few children, staff, teachers, and administration occasionally passing. With minimal noise level and distractions the testing environment appeared to be ideal according to the researcher.

The researcher followed standardized directions when administering to children the conservation-of-liquid task (see Appendix D). There were three cups used in this experiment. Each child was shown two clear plastic cups with identical diameters containing an equal amount of water. The child was then asked whether both cups contained the same or whether one cup contained more water. If a child did not agree that the water levels in the two cups were equal they were allowed to manipulate the water until they believed the water levels were equivalent. When a child was in consensus with the equivalence of water level they proceeded to the next step. Water in one of the original cups was then transferred into a clear plastic cup with a smaller diameter (see Figure 2). The child was then asked whether the water levels between the wider diameter cup and the smaller diameter cup were the same or different. If the child said that the same amount of water was contained in each cup, then the child was presented with the cup from which the water had been poured and asked to point to the place on the empty glass where the water would rise to if poured from the taller glass. In every case those children who stated that the water was the same amount also estimated correctly the level to which the water would rise if poured back. These children were in the concrete operational level of cognitive development (Schwebel & Raph, 1973).

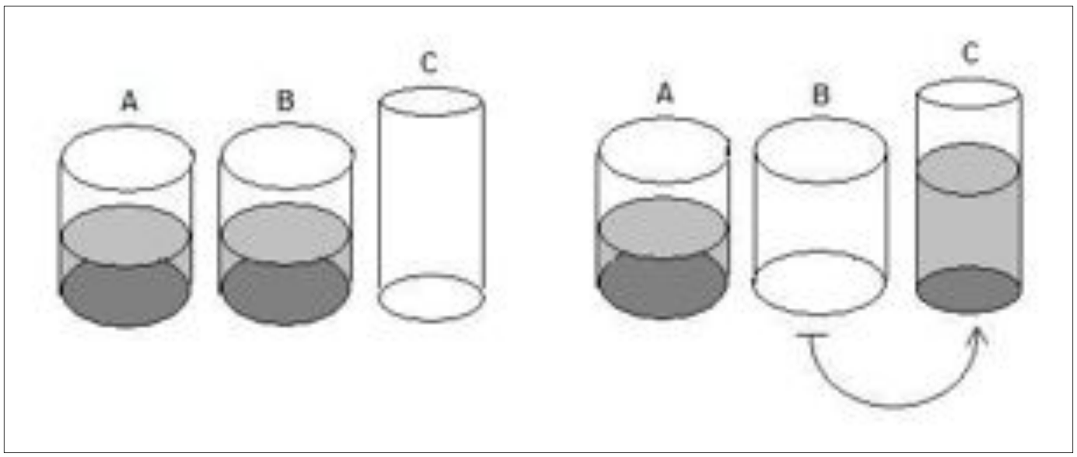

Figure 2. Conservation-of-Liquid Task. The first set of cups A and B have a wider diameter and contain an equal amount of liquid. The second set includes cup A, which has a wider diameter and cup C, which has a smaller diameter, however both cups contain an equal amount of liquid.

Fifty children identified the water levels as unchanged when the transfer of water occurred. These children were considered to be in the concrete operational level of cognitive development by their ability to conserve. Sixty children indicated the water levels were different after the transfer occurred. These children were considered to be in the preoperational level of cognitive development by their inability to conserve.

There were a total of 50 conserving and 60 nonconserving first grade children. To create a balanced research design, the 60 nonconserving children were each assigned a number and using Microsoft Excel the numbers associated with the 60 children were randomized to include a set of 50 children. These randomly chosen 50 nonconserving children were selected to participate in this study. The remaining 10 children after the randomization occurred (3, 8, 12, 28, 43, 47, 48, 49, 50, & 54) were excluded from participating in the study. This method was used to create a balanced research design to make the analyses of data more robust in the face of assumption violations and to increase validity. However, the original sample sizes of 60 nonconserving and 50 conserving children yielded a ratio of 1.2. The ratio of 1.2 fell within the parameters of 1.0 and 1.5 allowing the differences in sample sizes, 60 nonconserving and 50 conserving, to be within an acceptable range had the randomization not occurred.

As seen in Table 1, final testing was administered to (N=100) first grade children. Fifty-two percent were female and 48% were male. Sixty-three percent were Hispanic, 34% were Caucasian, 2% were Asian/Pacific Islander, and 1% were African American. Thirty-four percent of subjects were 6-year-olds and 66% were 7-year-olds. The (n=50) nonconserving children included 56% female and 44% male. Seventy percent were Hispanic, 28% were Caucasian, and 2% were African American. Fifty-four percent were six-year-olds and 46% were 7-year-olds. The (n=50) conserving children included 48% female and 52% male. Fifty-six percent were Hispanic, 40% were Caucasian, and 4% were Asian/Pacific Islander. Fourteen percent were 6-year-olds and 86% were 7-year-olds.

Table 1

Frequency of Demographic Covariates

Covariates	Demographics	Children (*N*=100)	Nonconserving (*n*=50)	Conserving (*n*=50)
Gender	Female	52	28	26
	Male	48	22	24
Race	Hispanic	63	35	28
	Caucasian	34	14	20
	Asian/Pacific Islander	2	0	2
	African American	1	1	0
Age	6	34	27	7
	7	66	23	43

Instrumentation

The instrumentation used in this study included two separate single-skill math fact probes from AIMSweb. The math fact probes were used to measure addition fluency and subtraction fluency. According to Shinn (2005) the math fact probes were created based on the expected computational abilities at each grade level. The primary grade level math fact probes were used to control for acquisition deficits in addition and subtraction. Shinn (2005) cited Tindel, Martson, and Deno (1983) to confirm reliability and validity of the math fact probes of addition fluency and subtraction fluency. The authors indicated that the math fact probe for addition fluency yielded reliability of 0.87, 0.72, and 0.98 with a mean reliability of 0.86. The math fact probe for subtraction fluency yielded reliability of 0.89, 0.70, and 0.99 with a mean reliability of 0.86. A comprehensive review of other single-skill math fact probes, which could have been used to measure mathematical fluency, was conducted, however, they compared unfavorably to the AIMSweb math fact probes used in this study.

There were two math fact probes, one included addition facts and the other included subtraction facts (see Appendix E & F). The mathematical facts in each math fact probe were arranged in a columnar format. The layout of each math fact probe consisted of 5 rows with 6 mathematical facts in each row. The math fact probes consisted of whole numbers that included addition facts 0-12 (0+0 to 12+12) and subtraction facts 0-12 (0-0 to 24-12). When these two math fact probes were compared to each other the mathematical facts in each probe were in the same location, but were in an inverse position (AIMSweb, 2010).

Test Procedure

Each child was tested individually at a table and chair positioned outside of the classroom. The researcher called upon the children individually and walked with the child to a seat and chair located at the examination table. The child was provided two number two pencils. The researcher stood next to the child and read the standardized directions at which point the child began either the math fact probe for addition fluency or subtraction fluency at the start of a two-minute digital timer (Thurber, Shinn, & Smolkowski, 2002).

The standardized testing procedures were used in accordance with the Shinn (2005) AIMSweb instructions manual (see Appendix G). The math fact probes of addition fluency and subtraction fluency were administered to the children using a counter-balanced method. For example, the odd numbered children were administered the math fact probe of addition fluency for two-minutes. When the time was up a one-minute break was given and then the children were administered the math fact probe of subtraction fluency. The even numbered children were administered the math fact probe of subtraction fluency for two minutes. When the time was up a one-minute break was given and then the children were administered the math fact probe of addition fluency. The counter-balanced method was used to control for sequencing effect, order effect, and carry-over effect to avoid the influence of confounding the scores obtained from the administration of the math fact probes. After children completed the math fact probes of addition fluency and subtraction fluency the child was then given a sticker of choice on the hand and walked back into their classroom.

Scoring of each math fact probe took place by a method called Correct Digits (CD). This method included counting the number of correct digits in each row and then summing the total of all CD that were completed within two minutes. Each CD was underlined with a red pen. If the child did not complete the entire problem then the CD of that problem was counted. If a child marked a problem with an X and then continued, but later returned to that problem and wrote the CD then the CD on that former problem was scored and the X was dismissed. There were three rules to address the legibility and reversed or rotated numbers when the scorer tried to figure out what number was written. The first rule stated that, "if it is difficult to determine what the number is at all, count it wrong. The second rule stated that, "If the reversed number is obvious, but correct, count it as a correct digit." The third rule stated that, "If the numbers 6 or 9 are potentially rotated and the digit is currently incorrect, count it as an incorrect digit" (Shinn, 2005, p. 20).

Analysis of Data

The data obtained from the children in this study were input into the program Statistical Package for the Social Sciences (SPSS) software. This created a data set that included the variables grade level, cognitive level of development, gender, race, age, addition fluency scores, and subtraction fluency scores (see Appendix H & I). The independent variables were nonconserving and conserving children. The dependent variables were addition fluency and subtraction fluency. The variable grade level was controlled for by the inclusion of only first grade children. The covariates were gender, race, and age. These data were then subjected to statistical analysis by using a Multivariate Analysis of Variance (MANOVA) and a repeated-measures ANOVA.

CHAPTER IV

RESULTS AND DISCUSSION

Statistical Analyses

Results

The result of the data obtained from the addition fluency and subtraction fluency scores from the administration of the conservation-of-liquid task given to 100 children was subjected to statistical analyses. The addition fluency scores and subtraction fluency scores were tested using a Multivariate Analysis of Variance (MANOVA) with gender, race, and age as the covariates. Additionally, for post-hoc analysis, the addition fluency scores and subtraction fluency scores were converted into z-scores for a repeated-measures Analysis of Variance (ANOVA).

Statistical assumptions. Addition fluency scores ranged from 3 to 53; subtraction fluency scores ranged from 0 to 52. Mean addition fluency score was 23.32 ($SD=11.79$) and mean subtraction fluency score was 15.07 ($SD=11.22$). Computation of skewness and kurtosis statistics revealed an unacceptable degree of positive skew for subtraction fluency (ratio of statistic to standard error > 2); consequently, subtraction fluency was transformed using a square root transformation, which reduced skew to an acceptable level. The untransformed and transformed subtraction fluency scores were analyzed.

MANOVA. The relationship between cognitive development level (nonconserving, conserving) and addition fluency and subtraction fluency scores were analyzed using a Multivariate Analysis of Variance (MANOVA). Using the untransformed subtraction fluency scores, the cognitive level of development effect was highly significant, $F(2,97) = 72.78$, $p < .001$. Between-subjects effects tests revealed that cognitive level of development predicted both addition fluency, $F(1,98) = 96.00$, $p < .001$, and subtraction fluency, $F(1,98) = 131.72$, $p < .001$. As seen in Table 2, conserving subjects had higher mean addition fluency scores ($M = 31.46$) than did nonconserving subjects ($M = 14.96$). In addition, conserving subjects had higher mean subtraction fluency scores ($M = 23.52$) than did nonconserving subjects ($M = 6.62$).

The untransformed subtraction fluency scores were used as a result of the unacceptable degree of positive skew. However, using the transformed subtraction fluency scores, the cognitive level of development effect was still highly significant, $F(2,97) = 74.77$, $p < .001$. Between-subjects effects tests revealed that cognitive level of development predicted both addition fluency, $F(1,98) = 96.00$, $p < .001$, and subtraction fluency, $F(1,98) = 133.61$, $p < .001$. As noted earlier, conserving subjects had higher mean addition fluency scores than did nonconserving subjects; in addition, conserving subjects had higher mean (transformed) subtraction fluency scores ($M = 4.76$) than did nonconserving subjects ($M = 2.30$). The untransformed and transformed subtraction fluency scores yielded identical results.

Table 2

Mean Fluency of Addition and Subtraction

Operation	*n*	Level	*M*	*SE*
Addition Fluency	50	Nonconserving	14.96	1.19
	50	Conserving	31.46	1.14
Subtraction Fluency	50	Nonconserving	6.62	1.04
	50	Conserving	23.52	1.04
Total Subjects	*N*=100			

The covariates were analyzed using the transformed subtraction scores, addition scores, and the covariates gender (male = 1, female = 2), age (6 = 1, 7 = 2), and race (Asian = 1, African American = 2, Hispanic = 3, Caucasian = 4), the cognitive level of development effect was still highly significant, $F(2,94) = 56.74$, $p < .001$. The age effect was also highly significant, $F(2,94) = 26.96$, $p < .001$. The gender effect was not significant ($p = .16$), nor was the race effect ($p = .67$). Between-subjects effects tests revealed that cognitive level of development predicted both addition fluency, $F(1,95) = 59.54$, $p < .001$, and subtraction fluency, $F(1,95) = 95.41$, $p < .001$. Conserving subjects had higher mean addition fluency scores (adjusted $M = 29.61$) than did nonconserving subjects (adjusted $M = 16.81$); in addition, conserving subjects had higher mean (transformed) subtraction fluency scores (adjusted $M = 4.47$) than did nonconserving subjects (adjusted $M = 2.58$).

The effect of the between-subjects tests revealed that age, separately from cognitive level of development, predicted both addition fluency, $F(1,95) = 25.27$, $p < .001$, and subtraction fluency, $F(1,95) = 47.48$, $p < .001$. Seven-year-olds had higher mean addition fluency scores ($M = 28.18$) than did 6-year-olds ($M = 13.56$); in addition, 7-year-olds had higher mean (transformed) subtraction fluency scores ($M = 4.29$) than did 6-year-olds ($M = 2.05$). Thus, cognitive level of development and age were demonstrated to be two separate factors that each related to addition fluency and subtraction fluency. That is, the cognitive level of development variable was not simply a proxy for age—it had its own effect apart from age—but age had an additional effect on fluency above and beyond cognitive level of development.

Post-hoc Analysis

The data were analyzed in an additional way by converting the addition fluency and transformed subtraction fluency scores into z-scores, as well as using the covariates gender, age, and race to run a repeated-measures ANOVA.

The analysis of the addition z-scores and transformed subtraction z-scores, along with the demographic covariates, using a repeated-measures ANOVA, with cognitive level of development as a between-participants effect and skill (addition, subtraction) as a within-participants effect, revealed a highly significant cognitive level of development effect, $F(1,95) = 108.78$, $p < .001$, and a highly significant age effect, $F(1,95) = 50.204$, $p < .001$. The gender effect approached significance, $F(1,95) = 3.78$, $p = .06$, such that males scored slightly higher overall than females; the race effect was not significant ($p = .52$). The skill effect was not significant ($p = .49$), and neither was the Skill x Cognitive Level of Development interaction ($p = .61$). The latter result suggests that neither addition fluency nor subtraction fluency demonstrated a substantial advantage over the other by cognitive level of development.

Discussion

The statistical analyses tested three non-directional null and alternative hypotheses. All significance tests were measured at the .05 alpha level. The fixed factor was the between-subjects variable group, which included conserving and nonconserving children in first grade. The dependent variables were addition fluency and subtraction fluency. The covariates considered in this study were gender, race, and age.

Addition Fluency and Subtraction Fluency

The null hypothesis, that there was no significant difference between nonconserving and conserving children in addition fluency and subtraction fluency together, was rejected, and the alternative hypothesis that there was a significant difference in addition fluency and subtraction fluency was accepted. Conserving children scored higher on addition fluency and subtraction fluency together than nonconserving children. The effect of conservation ability was considered highly significant $p < .001$ on the combination of addition fluency and subtraction fluency.

Addition Fluency

The null hypothesis, that there was no significant difference between nonconserving and conserving children in addition fluency, was rejected, and the alternative hypothesis that there was a significant difference in addition fluency was accepted. Conserving children scored higher on addition fluency than nonconserving children. The effect of conservation ability was considered highly significant $p < .001$ on addition fluency.

Subtraction Fluency

The null hypothesis, that there was no significant difference between nonconserving and conserving children in subtraction fluency, was rejected, and the alternative hypothesis that there is a significant difference in subtraction fluency was accepted. Conserving children scored higher on subtraction fluency than nonconserving children. The effect of conservation ability was considered highly significant $p < .001$ on subtraction fluency.

In summary, the overall results of this study indicated that conserving children had greater addition fluency and subtraction fluency both together and separate than nonconserving children. Pulaski (1971) argued that the operational ability of conserving children allowed dynamic thinking that was capable of being transposed. This speeds up cognitive processing which results in greater mobility and freedom. Ramos-Christian et al. (2008) indicated that an increase in speed contributed to conserving children having greater mathematical fluency than nonconserving children. Cooper and Schleser (2006) also confirmed that conserving children had greater mathematical fluency than nonconserving children. Therefore, this study highlighted that the conservation ability of children contributed to addition fluency and subtraction fluency being acquired at the same time.

Covariates

The results of the covariate analyses revealed the covariates gender and race had no statistically significant effect on addition fluency and subtraction fluency. The covariate age had a separate, but additional effect on mathematical fluency above and beyond cognitive level of development. The ability to conserve improved along with age

(Bisanz, 1995). Conservation ability was not a designation of age (Ginsberg & Opper, 1969). Children pass though the cognitive levels of development at different rates and as a result the ages associated with the different levels holds little importance (Crain, 2005). The order at which children pass through the invariant levels of cognitive development remains identical in most cases (Pulaski, 1971).

Post-hoc Analysis

A repeated-measures ANOVA was used for statistical analysis that considered addition z-scores, subtraction z-scores, demographic covariates, and cognitive level of development to determine if a substantial advantage existed between addition fluency and subtraction fluency. Copeland (1974) argued that addition and subtraction appear to be two distinct mathematical operations. However, psychologically children interpret addition and subtraction to be one mathematical operation in which reversibility exists in both. Nonconserving children have not yet acquired reversibility or invariance of thought. This affects their ability to realize that addition and subtraction can be used to compensate each other when performing mathematical operations. The results of the post-hoc analysis indicated that no substantial advantage between addition fluency and subtraction fluency as it related to different abilities in conservation existed. Therefore, this study highlighted that addition and subtraction was acquired at a rate that correlates to the conservation ability of children.

CHAPTER V

SUMMARY, CONCLUSIONS, LIMITATIONS,

AND IMPLICATIONS

Summary

The preoperational and concrete operational levels of cognitive development were central to the theme of this research. Piaget identified that children proceed through these invariant levels sequentially based on the ability of conservation (Crain, 2005). Cooper and Schleser (2006) and Ramose-Christian et al. (2009) indicated that conservation ability affected mathematical fluency. However, the authors' research focused on a combination of mathematical operations and not separately as they related to conservation ability. This study was designed to separately measure the addition fluency and subtraction fluency of conserving and nonconserving children in first grade.

There were a total of 100 children who participated in this study. Conservation ability was measured using Piaget's conservation-liquid task. The two groups of 50 conserving and 50 nonconserving children were then tested using two separate math fact probes in addition fluency and subtraction fluency. The data were analyzed using a MANOVA and considered the covariates gender, age and race to test for the posed hypotheses. Post-hoc analysis was also conducted to determine if a substantial advantage existed between addition and subtraction as it related to conservation ability.

The results of this study indicated that cognitive level of development was not a grade level designation. The children in first grade were conserving and nonconserving. These invariant levels were characterized by different abilities in mathematical fluency. Educators' pedagogical techniques and the design of curriculum should reflect these differences. Developmentally appropriate teaching practices should be used in the classroom to increase mathematics achievement for first grade children.

Conclusions

This study examined the cognitive level of development and mathematical fluency of first grade children. The results of the MANOVA indicated that conserving children had significantly greater addition fluency, subtraction fluency, and addition and subtraction fluency together than nonconserving children. The null was rejected in all three hypotheses and the alternative hypotheses were accepted. Gender and race were considered to be insignificant covariates. Age was not seen as a proxy for cognitive level of development, but as a covariate that had an additional effect on mathematical fluency above and beyond cognitive level of development. The results of the post-hoc analysis indicated that no substantial advantage between addition fluency and subtraction fluency as it related to different abilities in conservation existed. Therefore, conservation ability of children appeared subject to addition and subtraction as being acquired at a rate that correlates to conservation ability. Children may be in the same grade, but in different levels of cognitive development. This had an effect on mathematical fluency that should be a consideration when teaching children in first grade. Overall, conserving children had significantly greater addition fluency and subtraction fluency both together and separate than nonconserving children.

Limitations

There are several limitations that should be considered when interpreting the results of this study. These limitations included internal threats and external threats to the validity of the results that may have confounding effect.

The potential threat to internal validity considered that the participants in this study did not undergo psychometric testing for intelligence. Piaget did not disregard that differences in intelligence exist (Ginsburg & Opper, 1969). Intelligence was considered a fixed factor based on randomization of the population (Copeland, 1974). Piaget considered the nonrandom factors of the genetic determination of intelligence, in which children moved sequentially through a series of invariant levels where new mental structures were created and resulted in optimum capabilities at each level (Ginsburg & Opper, 1969). Kingma (1983) found the conservation experiment to be a better predictor of addition and subtraction achievement than an intelligence test. Wubbena (1977) used a psychometric task to control for intelligence as a covariate. Intelligence was considered insignificant between nonconserving and conserving first grade children. The intelligence of children was not measured as a covariate in this study. This may have confounded the results and interpretation of study should consider this as a factor.

The potential threat to external validity included the ability to generalize the results from the sample of children in this study to children in other settings and the populations at large. This study contained 100 children at two low socioeconomic level elementary schools in the same city. This study was limited in location to two elementary schools. The covariate race was found to be an insignificant factor on mathematical fluency, however race was not inclusive to all possible ethnicity groups. The children in this study were a majority Hispanic followed by Caucasian. There were Asian/Pacific Islander and African American children included, however they were greatly underrepresented totaling only 3% of sample. The covariate gender was considered balanced, which included 52 females and 48 males. This yielded a ratio of 1.1, which is within the acceptable parameters of 1.0 to 1.5 to indicate that males and females were for the most part equally represented. The 6- and 7-year-olds do appear to be representative of children in first grade. However, cognitive level of development is not a grade level designation. Children of other ages and grades were not built into the design of this study. The ability to generalize the results of this study may be limited by several of these factors.

Implications

The implications for this study include recommendations for practical application of this study's findings in the classroom and recommendation for further research. This study discovered that a relationship does exist between cognitive levels of development and mathematical fluency of first grade children. Cognitive level of development was not a grade level designation. Children in first grade may be in different levels of cognitive development. Understanding the characteristics of learning associated with the operational structures of nonconserving and conserving children will direct educators to practice developmentally appropriate pedagogical techniques.

Curriculum should be designed and implemented to reinforce pedagogical techniques that focus on active learning experiences and participation. Concrete objects or manipulatives should be used to build meaning prior to being introduced to abstract symbols (Chapin 2006; Lemlech 2010; Pulaski, 1971; Sarama & Clements, 2010). The facilitation of learning should be differentiated, individualized, and curriculum should be scaffolded to build the skills necessary for mathematical fluency. "Curriculum and instruction must be flexible enough to address the broad range of needs within a grade level" (Tomlinson, et al., 2009, p.18). Educators who address the dyad of cognition for children to learn in first grade can ameliorate the mathematical needs of nonconserving children. The increased awareness of the importance of a child's cognitive level of development led to insight into the process of becoming fluent in addition and subtraction operations. This understanding led to the separation of learning and development, and in doing so allowed learning as a result of teaching to be a dynamic variable that must adapt to a child's cognitive level of development.

The findings of this study indicate several implications for further research. Replication through further research should be conducted to increase the validity of this study. Further research should be conducted that encompasses children from different socioeconomic level backgrounds, ethnicities, and controls for intelligence through psychometric testing. Finally, research that breaks down addition and subtraction fluency into the elements of speed and accuracy may lead to an understanding of the incremental divisions that serve in the acquisition of mathematical fluency.

APPENDICES

Appendix A

Consent Form: English

Consent Form

Introduction
Your child is invited to participate in a master's thesis study conducted by Zane Wubbena, a graduate student in the Department of Curriculum and Instruction at Texas State University-San Marcos. You may contact him at (512) 826-2110 or zane.wubbena@txstate.edu. You may also contact the supervising faculty member of this study, Dr. Barbara Davis at (512) 245-8196 or bd02@txstate.edu.

Purpose
The purpose of this study is to compare children in different levels of cognitive development with math fluency. It is expected that the results of this study will contribute to curriculum and instructional practices in math among children in the same grade, but in different levels of cognitive development.

Participants
Your child has been chosen to be a participant in this study because he/she is in the first grade in San Marcos CISD. It is estimated that 100 children will participate in this study.

Experimental Procedures
Your child will first be asked to evaluate the volume of water in a series of 3 plastic cups. The estimated time your child will spend on this is 2-5 minutes.

Your child will then complete two math assessments in addition and subtraction. Each assessment will last for 2 minute. The assessment used is a Math Facts Probe, which is used by your child's school on a regular basis. The estimated time your child will spend on this is 5-10 minutes.

Risks
There are no known risks to your child's participation in this study. The procedures followed in this study are similar assessment practices followed by your child's classroom teacher.

Benefits
The benefits to your child's participation in this study include verbal positive reinforcement (e.g. "Good job, Way to go, etc."). Your child will receive a sticker after their participation.

Confidentiality and Privacy Protection
The records of this study will be kept confidential and will be stored securely in either a locked file cabinet or digitally using encryption software for 3 years from the date of study closure.

The data resulting from your child's participation may be made available to other researchers in the future for research purposes not detailed within this consent form. In these cases, the data will contain no identifying information that could associate you with it, or with your child's participation in this study. Authorized persons from Texas State, and members of the Institutional Review Board (IRB) have the legal right to review research records and will protect the confidentiality of those records to the extent permitted by law. All publications will exclude any information that will make it possible to identify your child as a subject. Throughout the study, the researcher will notify you of new information that may become available and that might affect your decision to remain in the study.

Contacts and Questions
A summary of the findings will be provided to you upon completion of the study, if requested. Please contact Zane Wubbena at (512) 826-2110 or zane.wubbena@txstate.edu.

The Institutional Review Board (IRB) at Texas State University has approved this study. Pertinent questions about the research and research participants' rights, should be directed to IRB chair, Dr. Jon Lasser by phone at (512) 245-3413 or e-mail at lasser@txstate.edu, or Compliance Specialist, Ms. Becky Northcut by phone at (512) 245-2102.

Statement of Consent
Your signature indicates that you have read and understand the information provided above, that you willingly agree to allow your child to participate, that you may withdraw your consent at any time without prejudice or jeopardy to you or your child's standing with the University and any other relevant organization/entity with which you or your child is associated. Your child may choose not to answer any question(s) for any reason. You will receive a copy of this consent form, and by signing below you are not waiving any legal rights or releasing the researcher from liability.

Your child's participation in this study would be greatly appreciated.

Signatures
Please Print Your Child's Name: ____

Parent/Guardian: ___________________________________ Date: ________________

Printed Name: ____

Researcher: _______________________________________ Date: ________________

Printed Name: ____

THANK YOU

Appendix B

Consent Form: Spanish

Formulario de Consentimiento

Introducción

Su hijo(a) está invitado a participar en el estudio de investigación de tesis de maestría realizada por Zane Wubbena, quien es un estudiante graduado del Departamento de Currículo e Instrucción de la Universidad de Texas en San Marcos. Puede ponerse en contacto con él (512) 826-2110 o zane.wubbena@txstate.edu. También puede comunicarse con el miembro de la facultad que supervisa este estudio de investigación, la Dra. Barbara Davis al (512) 245-8196 o bd02@txstate.edu.

Propósito

El objetivo de este estudio es comparar los niños en las diferentes etapas del desarrollo cognitivo con la fluidez de Matemáticas. Se espera que los resultados de este estudio contribuirán al plan de estudios y prácticas de enseñanza en Matemáticas entre los niños en el mismo grado, pero en diferentes etapas de desarrollo cognitivo.

Participantes

Su hijo(a) ha sido escogido para que participe en este estudio porque él / ella está en el primer grado en la el Distrito Escolar de San Marcos. Se estima que 100 niños participarán en este estudio.

Procedimientos experimentales

Primeramente, se le pedirá a su hijo (a) que evalué el volumen del agua en una serie de 3 envases de plástico. El tiempo estimado que su hijo(a) tomara en esto es de 2-5 minutos.

Su hijo(a) va a realizar dos evaluaciones de Matemáticas de suma y resta. Cada evaluación tendrá una duración de 2 minutos. La prueba que se utilizara es una medida de un currículo basado en Matemáticas, que es utilizado regularmente por la escuela de su hijo. El tiempo estimado que su hijo(a) tomara en esta prueba es de 5-10 minutos.

Riesgos

No se conoce ningún riesgo por la participación de su hijo(a) en este estudio. Los procedimientos que se llevaran a cabo en este estudio son similares a las prácticas de evaluación dadas por el maestro de su hijo(a).

Beneficios

Los beneficios por la participación de su hijo(a) en este estudio incluyen el refuerzo positivo verbal (por ejemplo: "Buen trabajo, así se hace, etc.). Su hijo(a) recibirá una calcomanía después de su participación.

Confidencialidad y Protección de la Privacidad

Los registros de este estudio se mantendrán confidenciales durante 3 años a partir de la fecha que se termino el estudio y se guardaran de forma segura en un gabinete para archivos ya sea un archivo bloqueado o digital utilizando la programación de cifrado. Los datos y resultados de la participación de su hijo(a) estará a disposición de otros investigadores en el futuro para fines de investigación que no están detallados en este formulario de consentimiento. En estos casos, los datos no tendrán ninguna información de identificación que podrían asociarse con usted o con la participación de su hijo(a) en este estudio. Personas autorizadas del Estado de Texas, y los miembros de la Junta de Revisión Institucional (IRB), tienen el derecho legal de revisar los registros de la investigación y protegerán la confidencialidad de los datos y registros en la medida permitida por la ley. Todas las publicaciones excluyen cualquier información que permita identificar a su hijo(a). A lo largo del estudio, el investigador le notificará de nueva información que pueda estar disponible y que pueda afectar su decisión de permanecer en el estudio.

Contactos y Preguntas
Un resumen de los resultados serán proporcionados a usted al finalizar el estudio, si es que usted lo solicita. Por favor póngase en contacto con Zane Wubbena al (512) 826-2110 o zane.wubbena@txstate.edu.

La Junta de Revisión Institucional (IRB) de la Universidad Estatal de Texas ha aprobado este estudio de investigación. Cualquier pregunta pertinente acerca de la investigación y los derechos de los participantes en la investigación, deberá ser dirigida al Presidente del IRB, el Dr. Jon Lasser por teléfono al (512) 245-3413 o e-mail a lasser@txstate.edu, o al Especialista de Conformidad, la Sra. Becky Northcut al (512) 245-2102.

Declaración de Consentimiento
Su firma indica que usted ha leído y comprendido la información proporcionada anteriormente, y que voluntariamente está de acuerdo en permitir que su hijo(a) participe, y que usted en cualquier momento puede retirar su consentimiento sin que haya algún perjuicio o riesgo para su reputación o la de su hijo(a) con la Universidad y cualquier otra organización o entidad pertinente con la que usted o su hijo(a) está asociado. Su hijo(a) puede optar por no contestar a ninguna pregunta(s) por cualquier razón determinada. Usted recibirá una copia de este formulario de consentimiento, y mediante la firma que aparece a continuación, usted no renuncia a ningún derecho legal ni esta absolviendo al investigador de la responsabilidad.

La participación de su hijo(a) en este estudio será grandemente apreciada.

Firmas
Por favor escriba el nombre de su hijo(a): ____

Padre / Guardián: ____________________________________ Fecha: _________________

Nombre: ____

Investigador: ____ Fecha: _________________

Nombre: ____

GRACIAS

Appendix C

Institutional Review Board Approval

TEXAS STATE UNIVERSITY
SAN MARCOS
The rising STAR of Texas

Institutional Review Board Application

Certificate of Approval

Applicant: Zane Wubbena

Application Number : 2009W4645

Project Title: Cognitive Level of Development and Mathematical Fluency of First Grade Children

Date of Approval: 08/30/10 07:32:00

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Appendix D

Conservation-of-Liquid Task: Standard Directions

Conservation-of-Liquid Task
Standard Directions

1. Do this step before the child is present-Use the smaller diameter cup to measure the water poured into each of the two wider diameter cups.

2. Point to each cup of water that equally contains water.

> ***"Look at these two cups of water"***
>
> ***"Do these two cups have the "same" amount of water, OR does one cup have a "different" amount water?"***

3. The child responds by saying either **"YES,"** they have the same amount of water, or ***"NO,"*** one cup has more water. If the child says, "No," one cup has more water then pour out that water until the child agrees each cup has the same amount of water.

4. Point to the one of the wider diameter cups that contains water then to the empty cup with a smaller diameter.

> ***"I'm going to pour the water from this cups (wider diameter cup) into this cup (smaller diameter)***

5. Pour the water from the wider diameter cup into the smaller diameter cup and then position the smaller diameter cup filled with water next to the wider diameter cup that is filled with water.

> ***"Do these two cups have the same amount of water, or does one cup have more water?"***

6. Point to each cup: wider diameter cup and smaller diameter cup.

7. The child should respond by saying they have the same amount of water, which is an indication of conservation, or that one cup has more water which an indication of the child being nonconserving.

8. Pour the water from the smaller diameter cup back into the original cup. There should be two cups with the same diameter that contain the same amount of water.

> **"Do the two cups contain the same amount of water, or does one cup contain more water."**

Appendix E

Math Fact Probe: Addition

AIMSweb® Basic Addition Facts #1 - Primary

Student Name: ________________ Grade: ________ Teacher Name: ________________

$2 + 2$	$7 + 8$	$8 + 9$	$1 + 0$	$5 + 5$	$4 + 0$
$12 + 7$	$3 + 10$	$1 + 6$	$8 + 3$	$9 + 6$	$8 + 9$
$0 + 8$	$12 + 0$	$7 + 8$	$11 + 2$	$2 + 4$	$0 + 9$
$6 + 3$	$5 + 3$	$2 + 1$	$12 + 11$	$11 + 8$	$5 + 9$
$2 + 10$	$1 + 12$	$7 + 3$	$9 + 8$	$6 + 8$	$8 + 0$

AIMSweb® Basic Addition Facts #1 - Primary

Student Name: ______________ Grade: ________ Teacher Name: ______________

5 + 8	4 + 3	5 + 0	0 + 0	4 + 8	7 + 4
12 + 0	2 + 6	10 + 1	9 + 4	5 + 2	3 + 1
9 + 5	4 + 8	1 + 7	11 + 11	10 + 10	6 + 7
6 + 0	6 + 12	8 + 9	7 + 4	12 + 9	8 + 8
6 + 0	8 + 0	2 + 8	3 + 0	8 + 7	7 + 5

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AIMSweb® Basic Addition Facts #1 - Primary Answer Key

2 + 2 = **4** (1)	7 + 8 = **15** (2)	8 + 9 = **17** (2)	1 + 0 = **1** (1)	5 + 5 = **10** (2)	4 + 0 = **4** (1)	9 (9)
12 + 7 = **19** (2)	3 + 10 = **13** (2)	1 + 6 = **7** (1)	8 + 3 = **11** (2)	9 + 6 = **15** (2)	8 + 9 = **17** (2)	11 (20)
0 + 8 = **8** (1)	12 + 0 = **12** (2)	7 + 8 = **15** (2)	11 + 2 = **13** (2)	2 + 4 = **6** (1)	0 + 9 = **9** (1)	9 (29)
6 + 3 = **9** (1)	5 + 3 = **8** (1)	2 + 1 = **3** (1)	12 + 11 = **23** (2)	11 + 8 = **19** (2)	5 + 9 = **14** (2)	9 (38)
2 + 10 = **12** (2)	1 + 12 = **13** (2)	7 + 3 = **10** (2)	9 + 8 = **17** (2)	6 + 8 = **14** (2)	8 + 0 = **8** (1)	11 (49)

AIMSweb® Basic Addition Facts #1 - Primary Answer Key

$\begin{array}{r} 5 \\ +8 \\ \hline \mathbf{13} \end{array}$ (2)	$\begin{array}{r} 4 \\ +3 \\ \hline \mathbf{7} \end{array}$ (1)	$\begin{array}{r} 5 \\ +0 \\ \hline \mathbf{5} \end{array}$ (1)	$\begin{array}{r} 0 \\ +0 \\ \hline \mathbf{0} \end{array}$ (1)	$\begin{array}{r} 4 \\ +8 \\ \hline \mathbf{12} \end{array}$ (2)	$\begin{array}{r} 7 \\ +4 \\ \hline \mathbf{11} \end{array}$ (2)	9 (58)
$\begin{array}{r} 12 \\ +0 \\ \hline \mathbf{12} \end{array}$ (2)	$\begin{array}{r} 2 \\ +6 \\ \hline \mathbf{8} \end{array}$ (1)	$\begin{array}{r} 10 \\ +1 \\ \hline \mathbf{11} \end{array}$ (2)	$\begin{array}{r} 9 \\ +4 \\ \hline \mathbf{13} \end{array}$ (2)	$\begin{array}{r} 5 \\ +2 \\ \hline \mathbf{7} \end{array}$ (1)	$\begin{array}{r} 3 \\ +1 \\ \hline \mathbf{4} \end{array}$ (1)	9 (67)
$\begin{array}{r} 9 \\ +5 \\ \hline \mathbf{14} \end{array}$ (2)	$\begin{array}{r} 4 \\ +8 \\ \hline \mathbf{12} \end{array}$ (2)	$\begin{array}{r} 1 \\ +7 \\ \hline \mathbf{8} \end{array}$ (1)	$\begin{array}{r} 11 \\ +11 \\ \hline \mathbf{22} \end{array}$ (2)	$\begin{array}{r} 10 \\ +10 \\ \hline \mathbf{20} \end{array}$ (2)	$\begin{array}{r} 6 \\ +7 \\ \hline \mathbf{13} \end{array}$ (2)	11 (78)
$\begin{array}{r} 6 \\ +0 \\ \hline \mathbf{6} \end{array}$ (1)	$\begin{array}{r} 6 \\ +12 \\ \hline \mathbf{18} \end{array}$ (2)	$\begin{array}{r} 8 \\ +9 \\ \hline \mathbf{17} \end{array}$ (2)	$\begin{array}{r} 7 \\ +4 \\ \hline \mathbf{11} \end{array}$ (2)	$\begin{array}{r} 12 \\ +9 \\ \hline \mathbf{21} \end{array}$ (2)	$\begin{array}{r} 8 \\ +8 \\ \hline \mathbf{16} \end{array}$ (2)	11 (89)
$\begin{array}{r} 6 \\ +0 \\ \hline \mathbf{6} \end{array}$ (1)	$\begin{array}{r} 8 \\ +0 \\ \hline \mathbf{8} \end{array}$ (1)	$\begin{array}{r} 2 \\ +8 \\ \hline \mathbf{10} \end{array}$ (2)	$\begin{array}{r} 3 \\ +0 \\ \hline \mathbf{3} \end{array}$ (1)	$\begin{array}{r} 8 \\ +7 \\ \hline \mathbf{15} \end{array}$ (2)	$\begin{array}{r} 7 \\ +5 \\ \hline \mathbf{12} \end{array}$ (2)	9 (98)

Appendix F

Math Fact Probe: Subtraction

AIMSweb® Basic Subtraction Facts #1 - Primary

Student Name: ____________ Grade: ________ Teacher Name: ____________

2	8	9	1	5	4
- 2	- 7	- 8	- 0	- 5	- 0
12	10	6	8	9	9
- 7	- 3	- 1	- 3	- 6	- 8
8	12	8	11	4	9
- 0	- 0	- 7	- 2	- 2	- 0
6	5	2	12	11	9
- 3	- 3	- 1	- 11	- 8	- 5
10	12	7	9	8	8
- 2	- 1	- 3	- 8	- 6	- 0

AIMSweb® Basic Subtraction Facts #1 - Primary

Student Name: ______________ **Grade:** ________ **Teacher Name:** ______________

8	4	5	0	8	7
- 5	- 3	- 0	- 0	- 4	- 4
12	6	10	9	5	3
- 0	- 2	- 1	- 4	- 2	- 1
9	8	7	11	10	7
- 5	- 4	- 1	- 11	- 10	- 6
6	12	9	7	12	8
- 0	- 6	- 8	- 4	- 9	- 8
6	8	8	3	8	7
- 0	- 0	- 2	- 0	- 7	- 5

AIMSweb® Basic Subtraction Facts #1 - Primary Answer Key

2	8	9	1	5	4	
- 2	- 7	- 8	- 0	- 5	- 0	
0	**1**	**1**	**1**	**0**	**4**	6 (6)
(1)	(1)	(1)	(1)	(1)	(1)	
12	10	6	8	9	9	
- 7	- 3	- 1	- 3	- 6	- 8	
5	**7**	**5**	**5**	**3**	**1**	6 (12)
(1)	(1)	(1)	(1)	(1)	(1)	
8	12	8	11	4	9	
- 0	- 0	- 7	- 2	- 2	- 0	
8	**12**	**1**	**9**	**2**	**9**	7 (19)
(1)	(2)	(1)	(1)	(1)	(1)	
6	5	2	12	11	9	
- 3	- 3	- 1	- 11	- 8	- 5	
3	**2**	**1**	**1**	**3**	**4**	6 (25)
(1)	(1)	(1)	(1)	(1)	(1)	
10	12	7	9	8	8	
- 2	- 1	- 3	- 8	- 6	- 0	
8	**11**	**4**	**1**	**2**	**8**	7 (32)
(1)	(2)	(1)	(1)	(1)	(1)	

AIMSweb® Basic Subtraction Facts #1 - Primary Answer Key

8 - 5 **3** (1)	4 - 3 **1** (1)	5 - 0 **5** (1)	0 - 0 **0** (1)	8 - 4 **4** (1)	7 - 4 **3** (1)	6 (38)
12 - 0 **12** (2)	6 - 2 **4** (1)	10 - 1 **9** (1)	9 - 4 **5** (1)	5 - 2 **3** (1)	3 - 1 **2** (1)	7 (45)
9 - 5 **4** (1)	8 - 4 **4** (1)	7 - 1 **6** (1)	11 - 11 **0** (1)	10 - 10 **0** (1)	7 - 6 **1** (1)	6 (51)
6 - 0 **6** (1)	12 - 6 **6** (1)	9 - 8 **1** (1)	7 - 4 **3** (1)	12 - 9 **3** (1)	8 - 8 **0** (1)	6 (57)
6 - 0 **6** (1)	8 - 0 **8** (1)	8 - 2 **6** (1)	3 - 0 **3** (1)	8 - 7 **1** (1)	7 - 5 **2** (1)	6 (63)

Appendix G

Single-Skill Math Fact Probe: Standard Directions

Single-Skill Math Fact Probe:
Standard Directions

1. Students have a Math Fact Probe and a pencil.

2. Say to the student(s):

   We're going to take a 2-minute Math Fact Probes test. I want you to write your answers to these

   <Addition>

   <Subtraction>

   problems. Look at each problem carefully before you answer it."

   When I say 'BEGIN,' write your answer to the FIRST problem (demonstrate by pointing) ***and work ACROSS the page. Then go to the next row.***

   Try to work EACH problem. If you come to one YOU REALLY DON'T KNOW HOW TO DO, put an 'X' through it and go to the next one.

   If you finish the first side, turn it over and continue working. Are there any questions? (Pause)***"***

3. Say, ***"BEGIN"*** and start your stopwatch/timer.

4. Monitor student to ensure they are not skipping problems, are working across the page, and continue to write answers to the problems during the test time.

If a student is excessively skipping problems they should know how to do, say to the student:

"Try to work EACH problem. You can do this kind of problem so don't skip or put an 'X' over it."

If a student is not working across the page, say to the student.
"Work across the page. Try to work each problem in the row."

If a student stops working before the test is done, say to the student.
"Keep doing the best work you can."

5. At the end of 2 minutes, say, ***"Stop. Put your pencils down."*** Monitor to ensure student stops working.

Appendix H

Nonconserving Subjects

Nonconserving Subjects (*n*=50)							
Subject	Grade	Level	Gender	Race	Age	Addition Fluency	Subtraction Fluency
30	first grade	nonconserving	male	Caucasian	7	26	14
04	first grade	nonconserving	female	Hispanic	7	20	7
33	first grade	nonconserving	male	Hispanic	7	22	13
23	first grade	nonconserving	male	Hispanic	6	16	13
39	first grade	nonconserving	female	Caucasian	6	19	1
42	first grade	nonconserving	female	Hispanic	6	9	7
24	first grade	nonconserving	female	Hispanic	7	24	11
58	first grade	nonconserving	female	Hispanic	6	17	7
41	first grade	nonconserving	female	Caucasian	7	24	13
37	first grade	nonconserving	male	Hispanic	6	10	2
32	first grade	nonconserving	male	Caucasian	6	11	1
46	first grade	nonconserving	female	Hispanic	7	16	9
53	first grade	nonconserving	female	Caucasian	6	7	2
06	first grade	nonconserving	male	Hispanic	7	14	10
51	first grade	nonconserving	female	Hispanic	6	6	4
20	first grade	nonconserving	male	Hispanic	7	17	14
07	first grade	nonconserving	female	Caucasian	6	5	2
26	first grade	nonconserving	male	Hispanic	7	15	15
25	first grade	nonconserving	female	Hispanic	7	23	5
15	first grade	nonconserving	female	Hispanic	6	13	1
59	first grade	nonconserving	male	Caucasian	7	22	14
45	first grade	nonconserving	male	Caucasian	7	14	7
18	first grade	nonconserving	female	Hispanic	6	9	2
44	first grade	nonconserving	male	Hispanic	6	18	2
35	first grade	nonconserving	male	Caucasian	7	13	10
52	first grade	nonconserving	female	Hispanic	6	7	10
31	first grade	nonconserving	male	Caucasian	6	3	0
29	first grade	nonconserving	female	Hispanic	7	25	9
02	first grade	nonconserving	male	Hispanic	6	14	7
13	first grade	nonconserving	female	Hispanic	6	8	0
27	first grade	nonconserving	male	Hispanic	6	11	5
38	first grade	nonconserving	female	Caucasian	7	24	14
22	first grade	nonconserving	female	Hispanic	6	3	0
57	first grade	nonconserving	female	Caucasian	7	8	3
21	first grade	nonconserving	male	Hispanic	7	15	7
11	first grade	nonconserving	male	Hispanic	6	23	8
60	first grade	nonconserving	male	Caucasian	6	27	0
14	first grade	nonconserving	male	Caucasian	6	3	0
05	first grade	nonconserving	female	African American	7	19	3

Nonconserving Subjects (*n*=50)—Continued							
Subject	Grade	Level	Gender	Race	Age	Addition Fluency	Subtraction Fluency
19	first grade	nonconserving	female	Hispanic	6	13	2
17	first grade	nonconserving	female	Hispanic	6	13	3
10	first grade	nonconserving	male	Hispanic	7	18	17
40	first grade	nonconserving	female	Hispanic	6	20	5
56	first grade	nonconserving	female	Hispanic	7	10	12
36	first grade	nonconserving	male	Hispanic	7	30	5
16	first grade	nonconserving	male	Hispanic	7	14	8
55	first grade	nonconserving	female	Hispanic	6	11	5
01	first grade	nonconserving	female	Hispanic	6	8	5
09	first grade	nonconserving	female	Hispanic	6	5	2
34	first grade	nonconserving	female	Hispanic	7	26	15

Appendix I

Conserving Subjects

Conserving Subjects (*n*=50)							
Subject	Grade	Level	Gender	Race	Age	Addition Fluency	Subtraction Fluency
1	first grade	Conserving	female	Caucasian	7	35	25
2	first grade	Conserving	male	Caucasian	7	34	22
3	first grade	Conserving	male	Hispanic	7	53	31
4	first grade	Conserving	female	Caucasian	7	33	33
5	first grade	Conserving	male	Hispanic	7	37	28
6	first grade	Conserving	female	Caucasian	7	42	32
7	first grade	Conserving	female	Hispanic	7	36	26
8	first grade	Conserving	male	Caucasian	7	23	23
9	first grade	Conserving	male	Caucasian	7	53	39
10	first grade	Conserving	male	Hispanic	6	20	21
11	first grade	Conserving	female	Caucasian	7	27	22
12	first grade	Conserving	female	Hispanic	7	36	20
13	first grade	Conserving	male	Caucasian	7	18	20
14	first grade	Conserving	male	Hispanic	7	30	23
15	first grade	Conserving	male	Caucasian	7	38	25
16	first grade	Conserving	male	Caucasian	6	36	13
17	first grade	Conserving	male	Hispanic	7	30	23
18	first grade	Conserving	female	Hispanic	7	23	16
19	first grade	Conserving	female	Hispanic	7	23	16
20	first grade	Conserving	male	Hispanic	7	31	22
21	first grade	Conserving	male	Hispanic	6	19	22
22	first grade	Conserving	male	Caucasian	7	53	52
23	first grade	Conserving	male	Hispanic	7	40	25
24	first grade	Conserving	female	Hispanic	7	29	22
25	first grade	Conserving	female	Hispanic	7	32	24
26	first grade	Conserving	male	Caucasian	7	39	34
27	first grade	Conserving	male	Hispanic	7	38	29
28	first grade	Conserving	female	Caucasian	7	26	46
29	first grade	Conserving	female	Hispanic	7	38	24
30	first grade	Conserving	male	Hispanic	7	34	30
31	first grade	Conserving	female	Hispanic	7	36	29
32	first grade	Conserving	female	Hispanic	7	42	20
33	first grade	Conserving	female	Caucasian	7	31	22
34	first grade	Conserving	female	Caucasian	7	25	19
35	first grade	Conserving	male	Caucasian	7	23	17
36	first grade	Conserving	female	Hispanic	7	40	40
37	first grade	Conserving	male	Hispanic	6	24	11
38	first grade	Conserving	female	Hispanic	6	6	6

Conserving Subjects (*n*=50)—Continued							
Subject	Grade	Level	Gender	Race	Age	Addition Fluency	Subtraction Fluency
39	first grade	Conserving	female	Caucasian	7	28	25
40	first grade	Conserving	male	Asian	7	39	34
41	first grade	Conserving	male	Hispanic	7	27	24
42	first grade	Conserving	female	Caucasian	7	30	13
43	first grade	Conserving	male	Hispanic	6	35	12
44	first grade	Conserving	male	Caucasian	7	35	28
45	first grade	Conserving	female	Hispanic	7	20	9
46	first grade	Conserving	male	Caucasian	7	21	9
47	first grade	Conserving	female	Hispanic	7	39	11
48	first grade	Conserving	female	Hispanic	6	12	19
49	first grade	Conserving	male	Hispanic	7	28	21
50	first grade	Conserving	female	Asian	7	26	19

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