US-China Education Review A 6 (2012) 594-605
Earlier title: US-China Education Review, ISSN 1548-6613

Enhanced-Group Moore Method: Effects on van Hiele Levels of Geometric Understanding, Proof-Construction Performance and Beliefs

Douglas A. Salazar
AMA International University-Bahrain, Gudaibiya, Kingdom of Bahrain

This study aimed to improve the van Hiele levels of geometric understanding, proof-construction performance and beliefs about proofs of the research respondents: future mathematics teachers exposed to the traditional (instructor-based) method and the enhanced-group Moore method. By using the quasi-experimental method of research, the study employed qualitative and quantitative analysis relative to data generated by three instruments: the van Hiele geometry test, the proof-construction test, and proof beliefs questionnaire. Within the framework of the limitations of the study, the enhanced-group Moore method raised a higher van Hiele level, compared with the traditional method. The proof-construction performance of the future mathematics teachers has been improved. With regard to proofs, the future teachers believed that a theorem has no exception that the dual role of proof is to convince and explain, and that the validity of proof depends on its internal logic. Quantitative results revealed that there was a significant difference in the van Hiele levels and proof-construction performance of the future mathematics teachers before and after the study. In addition, there was a significant relationship between the proof-construction performance and van Hiele levels of the future teachers, and there was no noteworthy changes occurred in their beliefs about proofs. Qualitative assessments further showed that the Enhanced-Group Moore Method created "damay effect", developed self-confidence, encouraged effective communication and facilitated exchange of ideas towards a common goal. The future teachers from both groups were in favor of the sequence of the presentation of the lesson, especially with the incentives given.

Keywords: Enhanced-group Moore method, van Hiele levels, proof-construction and beliefs

Introduction

Mathematics holds a key position in the teacher education curriculum. It is also the main source of subject knowledge for school mathematics specialists at the university level. Proof is fundamental to the discipline of mathematics, because it is the convention that mathematicians and mathematics educators use to establish the validity of mathematical statements (Martin & McCrone, 2004). Thus, future mathematics teachers must come to terms with proofs, that is, they should be able to read, understand and write them. However, research (Jones, 2000) indicated that some students may complete their degrees with an incomplete picture of what constitutes a proof and how proof is developed.

The researcher, in his 20 years of teaching mathematics courses at the university level, found proof as the

Douglas A. Salazar, Ph.D., Center for General Education, AMA International University-Bahrain.

most hated part in mathematics. He agreed with Schoenfeld (1988) that in a typical mathematics classroom, students very rarely engage in a proof on their own. Mathematics is always presented to them as a neat package without struggles involved. Young (2003) stressed that the term “proof” causes such a stir among mathematics educators and mathematics students. College students will be likely to meet requests to do proof with groans and feelings of exasperation, and they have probably encountered proof in high school geometry where they were expected to follow a strict format, without much freedom to formulate proofs on their own. Most of the students are frustrated by the fact that they are asked to prove theorems that the book has done already and which have been proved to be true. Students have the perception that their mathematics instructors are just writing nonsense on the chalkboard, proving theorems with no practical use.

There must be a purpose of doing a proof. What is it that constitutes a proof? Why do we prove? Mingus and Grassl (1999) conducted a survey among future teachers on what constitute proofs. Results showed that a proof consists of the understanding of the question being asked or the statement being made to future teachers. Proofs are shown in a systematic process on how and why the statement being proved is true; each step in the proof relates directly to another. Dreyfus and Hadas (1987; as cited by Mar`tin & McCrone, 2001) articulated six principles that form the bases for understanding geometric proofs. The six principles are that: (1) A theorem has no exception; (2) The dual role of proof is to convince and to explain; (3) A proof must be general; (4) The validity of proof depends on its internal logic; (5) Statements are logically equivalent to their contra positive, but not necessarily to their converse and inverses; and (6) Diagrams that illustrate statements have benefits and limitations. These principles were the bases to classify prospective teachers’ beliefs about proofs in this study. Vistro-Yu (2001) pointed out that beliefs cannot be ignored, because of the impact they have on classroom teaching. Examining students’ beliefs about proofs can help teachers understand the reason why students do not perform well in proof tasks.

The proving process requires higher thinking level skills of the adult learner, specifically, mathematical reasoning skills. Mathematical reasoning is necessary to geometric reasoning. That is, if mathematical reasoning is successfully developed, geometric reasoning follows. Mathematical reasoning should be developed long before the students are asked to construct proofs (Battista & Clements, 1995). There are two predominant theories about the development of geometric reasoning in students: Jean Piaget’s theory and Dina and Pierre van Hiele’s theory.

Piaget structured his cognitive development theories in several domains around four stages of cognitive development. These stages are: sensorimotor (infancy); pre-operational (early childhood through pre-school); concrete operational (childhood through adolescence); and formal operational (early adulthood). Piaget claimed that these stages were physiological in nature, in that a child progresses through each stage at certain points of their biological development. One major focus of Piaget’s work examined how children organize and construct ideas about geometry (Piaget & Inhelder, 1967; as cited in Battista & Clements, 2000).

Van Hiele’s theory claimed that there are five levels of geometric thought, through which students progress in learning geometry: level 1: visualization; level 2: description; level 3: abstraction; level 4: deduction; and level 5: rigor. Students go through these levels as they progress from merely recognizing a figure to being able to construct a formal geometric proof. This theory partially explains the reason why many students encountered difficulties in their geometry course, especially with formal proofs (Mason, 1997).

However, Piaget and Inhelder (1967; as cited in Battista & Clements, 2000) added level 0: pre-recognition, for students who do not qualify for van Hiele level 1. This new level was characterized as a level of total cluelessness. The six levels of geometric understanding were used to characterize the mathematics cognitive development of the future mathematics teachers in this study. The van Hiele levels of geometric understanding are widely used as indicators of students' geometry readiness (Battista & Clements, 1995).

The research of Piaget and the research of the van Hiele presented implications on how students learn the proof. Specifically, Piaget claims that students' progress through four levels in the development of their justification and proof skills coincide with their biological development. Progression through the van Hiele levels, however, is dependent on the ability to consistently reason successfully in the preceding levels. Stage 4 (formal operation stage) of Piaget is parallel to van Hiele level 4 (deduction), at which students can construct formal proofs in geometry. This is the level which is expected by future mathematics teachers after a course in college geometry.

The study of Erfe (1996) on the validation of the van Hiele levels of thinking in learning high school geometry among Filipino students from different levels of curriculum year showed that Filipino students' rate is only at level 2 (descriptive). At this level, students can recognize and name properties of geometric figures, but do not see relationships between these properties. Thus, Erfe (1996) recommended that raising the levels of understanding of geometric concepts entails raising classroom instruction to a higher level of thinking. Further, Caluya (2000) found that students' van Hiele levels of geometric thinking are significantly related to teachers' transformational practices (i.e., lesson plans, test questions, syllabi and teaching strategies) in geometry classes, in particular, their teaching strategies. Her study revealed that activity-oriented strategies of teaching geometry raised students' van Hiele levels to level 4 (deduction). Weber (2003) recommended the Moore Method of teaching proof with some modifications to suit the needs of students and instructional strategies of the teacher. Moreover, Dancis and Davidson (1970) modestly suggested that a mathematics major should have an opportunity to take at least one course taught, using the Moore Method during his/her undergraduate career.

The Moore method (Texas method) of mathematics instruction is a teaching/learning style propagated by Dr. Robert Lee Moore (1882-1974). In this method, the baseline is that students enrolled are homogenously ignorant as possible (O'Connor & Robinson, 1992). The students are given a set of notes on the first day of classes and told to come back the next day to present their proofs of some theorems. In the interim, they are to discuss the proofs to the class and not given class time to fix their mistakes, over one minute thinking time. If they are unable to answer the question in one minute, then they are asked to try another theorem, or sit down and try the same theorem at a later time. Other students are not allowed to make helpful suggestions to the presenter, nor are books allowed to the latter (Taylor, 2004). This method, however, which tends to develop ambition, competitive spirit and individualism (Dancis & Davidson, 1970), is commonly used only in graduate mathematics classes.

Jensen modified the Moore method—now popularly known as the modified Moore method (Taylor, 2004)—so that it can be used in undergraduate mathematics classes. In addition, Neil Davidson, modifying the modified Moore method into what is known as the Small group discovery method, changed the social environment to render the idea workable for a much larger number of students in undergraduate courses (Davidson, 1973).

The current study adapted Davidson's small group discovery method, with variants using the researcher's creative way of grouping. That is, the first stage is individual competition, where those who succeed are then allowed to form a group by choosing one classmate at each stage of the proving activity, until the group is composed of three members. Individual competition begins again after a round. This manner of grouping was used by the researcher throughout the semester.

Moreover, to enhance the competitive atmosphere, additional five points were awarded as an incentive to students/groups who successfully proved and defended theorem(s). The top three students who earned the highest accumulated additional points were exempted from the final examination. This method is named by the researcher as the enhanced-group Moore method.

Research Questions

This study used the enhanced-group Moore method of instruction that aimed to raise the van Hiele level of future mathematics teachers up to level 4 (deduction). It also aimed to show that by enhancing the proof-construction performance and beliefs about proof of the future teachers, their mathematical and geometric reasoning skills will likewise to be improved.

Specifically, this study sought to answer the following questions:

(1) What is the van Hiele level of geometric understanding of the future mathematics teachers?

(2) Is there a significant difference in the van Hiele levels of the future mathematics teachers exposed to the enhanced-group Moore method and the traditional (instructor-based) method?

(3) How well can the future mathematics teachers construct proofs?

(4) Is there a significant difference between the proof-construction performance of the future teachers before and after teaching them using the enhanced-group Moore method and the traditional (instructor-based) method?

(5) What are the future mathematics teachers' beliefs about proofs?

(6) Is there a change in the future teachers' beliefs about proofs after the intervention?

(7) Is there a significant relationship between the future mathematics teachers' proof-construction performance and their van Hiele levels of geometric understanding?

(8) What insights with respect to strategies, difficulties and kinds of proofs can be drawn from the prospective teachers' journal, quizzes, seat work, interview results, and observations of teaching and learning episodes?

Methodology

The main objective of this study was to raise the van Hiele levels of the future mathematics teachers by enhancing their proof-construction performance. It also aimed to determine their beliefs about proofs based on the six principles of Dreyfus and Hadas. The van Hiele levels, proof-construction performance, and beliefs about proofs of the future mathematics teachers—second-year students of BSEd-2A (Bachelor of Secondary Education major in mathematics) at a state university in Eastern Visayas, Philippines—were assessed using the van Hiele geometry test, the proof-construction test, and the proof beliefs questionnaire before and after exposing them to two teaching methods: the enhanced-group Moore method and the traditional (instructor-based) method. Insights were drawn from videotaped teaching and learning episodes, journal entries, interview results, seat work, and quizzes from the experimental and control groups.

The BSEd-2A mathematics major section's 20 students, who were officially enrolled in Math 233 (plane and solid geometry) were involved in the study. The students were alternately distributed between two groups (control and experimental group), based on their ranked mean grade in their prerequisite subjects (basic mathematics and college algebra).

The researcher handled both classes in the same room assignment. List of terms defined, postulates/axioms and theorems to be proven, and syllabus copies were provided before the start of the classes. No textbook and other references were allowed in class.

Prior to the conduct of the study, a try out was done for four meetings to familiarize the students, as much as possible put the students at ease with the new method, and make necessary adjustments.

The two methods of teaching differed only in the seatwork stage. The following sequence of activities defined the classes for each group: (1) morning prayer; (2) preparation; (3) introduction/motivation; (4) lesson proper; (5) math jokes; (6) seat work; (7) summary; (8) journal writing; (9) math trivia; and (10) quiz.

In the control group (Traditional Method), students did their seatwork (proved theorems) individually. The teacher moved around to assist/give hints and suggestions on how to prove the theorem. Sometimes, the students approached the teacher, showed their work and requested for hints. The first student to present to the teacher a correct proof defended his/her proof in class. If the proof was successfully defended, the student earned additional five points.

In the experimental group (enhanced-group Moore method), the first stage was the same as that in the control group, but different (i.e., with enhanced-groupings) in the next stages. That is, as soon as the student successfully defended his/her proof in class and earned additional five points, he/she was allowed to select a group member and proceed to the next theorem/exercise. When the group (with two members) successfully defended their proof in class, each group member earned additional five points. They were then allowed to select a third group member of their choice and move to the next theorem/exercise. If again, the group (with three members) successfully defended their proof, each group member earned additional five points, and then returned to stage 1. This cycle continued, until the end of the semester. However, group membership was never the same in the next rounds. In cases where all the students finally belonged to a group of two or three, they started proving the theorem individually, all over again.

Quantitative and qualitative data were gathered to answer the research questions. Quantitative data were gathered from the students' or future teachers' scores from the three tests. The Statistical Package for Social Sciences and VassarStats: Website for Statistical Computations (retrieved from http://faculty.vassar.edu) was used in the statistical computations. Qualitative data were gathered through videotaped teaching and learning episodes, audio taped interview, journals, seat work and quizzes of the future teachers from the experimental and control groups.

The proof-construction test was adapted from McCrone and Martin's study (2004), with modifications that suited the present study. The test consisted of filling in statement and/or reason in a two-column and paragraph form of proof, providing strategies used in proving, restating the theorem in an if-and-then form, and proving a theorem using a two-column and a paragraph form of proof. The proof-construction test consisted of eight problems, with 40 as the highest possible score. Part I of the proof beliefs questionnaire, which was based on each of the description of the six principles of Dreyfus and Hadas (1987; as cited by Martin & Pulley, 2000), consisted of fourteen distributed statements. Part II of the questionnaire, adapted from Martin and McCrone's (2004) study, consisted of twelve questions. The van Hiele geometry test was a 25-question, multiple-choice test adapted from Frykholm's (1994) study. The questions were arranged sequentially in blocks of each five questions.

The three instruments were validated by three experts: a Ph.D. in education (mathematics) graduate, a

M.A.T mathematics graduate who had been teaching geometry for the past 15 years and the researcher's adviser. A dry run of the three instruments was conducted on June 21, 2006 on the second- and third- year BSED/BEED (Bachelor of Secondary Education/Bachelor of Elementary Education) math major students in a teacher education university in Tacloban City.

The study was carried out from July 31, 2006 to October 4, 2006, except for the second and fourth weeks of August which were devoted to intramurals and mid-term examination in the university, respectively.

Findings

The findings of this study are presented as follows.

Results from the van Hiele geometry test (see Table 1) indicated that before the study both groups were at van Hiele level 1, interpreted as visual. That is, the future teachers judged figures according to their appearance. After the intervention, the control group's van Hiele level improved from level 1 to level 2 (visual to descriptive), while that of the experimental group improved from level 1 to level 3 (visual to abstract).

At van Hiele level 2 (descriptive/analytic), the future teachers of the control group could recognize and characterize shapes by their properties, but were unable to identify the properties of a rectangle that some parallelograms do not have. They also were not able to draw statements from given statements and were not convinced that a square is a rectangle.

At van Hiele level 3 (abstract/relational), the future teachers of the experimental group could classify figures hierarchally and could give informal arguments to justify their classifications. They were convinced that a square is a rectangle. However, they had difficulty in identifying relationships that are true for all squares and properties, but which are not true for rhombuses. That is, they still could not manipulate the intrinsic characteristics of relations.

Results of non-parametric statistical tests (see Table l) showed that there were significant difference in the van Hiele levels of the future mathematics teachers in the experimental and control groups before and after the intervention.

Table 1

Comparison of the van Hiele Geometry Test Results

Group and test compared		*n*	Mean rank	Sum of ranks	Test used	*p*-value (two-tailed)	Group mean level	Group interpretations
Between groups	Pre-test (control) *vs.* pre-test (experimental)	10	10.80	118.00	Wilcoxon-Mann-Whitney test	0.8493N.S.	1.2	Level 1: Visual
		10	10.20	92.00			1.1	Level 1:Visual
	Post-test (control) *vs.* post-test (experimental)	10	8.00	75.00		0.0643N.S.	2.5	Level 2: Descriptive
		10	13.00	135.00			3.2	Level 3: Abstract
Within groups	Pre-test *vs.* post-test (control)	10	Negative/ positive difference	Ties	Sign test	0.004^{*}	1.2	Level 1: Visual
			0/9	1			2.5	Level 2: Descriptive
	Pre-test *vs.* post-test (experimental)	10	0/10	0		0.002^{*}	1.1	Level 1: Visual
							3.2	Level 3: Abstract

Note. * significant: $p < 0.05$ N.S. = not significant: $p > 0.05$.

However, there was no significant difference between the mean scores (pre-test and post-test) of the two groups. Although the future teachers who exposed to the enhanced-group Moore method obtained a van Hiele level higher (level 3) than the teachers who were exposed to the traditional (instructor-based) method (level 2), the score difference failed to reach the significant level.

The pre-test results of the two groups in the proof-construction test (see Table 2) showed that there is only a slight difference in their mean score: 5.8 and 5.4 for the control group and experimental groups, respectively. Both scores fall under the category of clueless. That is, the future teachers do not have any idea of what to do.

However, the proof-construction mean score in the control group has been improved after the study: from clueless to novice (5.8 to 13.6), because the future teachers understood what they were expected, but confused, to attempt. A significant increase of 7.8 in the mean score was achieved. The experimental group’s proof-construction test mean score, on the other hand, also has been improved significantly: from clueless to intermediate (5.4 to 20.5). That is, the future teachers showed understanding of the process of proving but appeared to be missing some knowledge in key concepts. A significant increase of 15.1 in the mean score was achieved.

The future mathematics teachers in the experimental and control groups did not differ significantly in their pre-test mean scores in the proof-construction test, even if the mean score in the control group (5.8) was numerically higher than the mean score of the experimental group (5.4). However, there was a significant difference in the post-test mean scores (20.5 and 13.6) in the proof-construction test between the groups (see Table 2).

In terms of gain scores, there was also a significant difference in the proof-construction test mean scores (5.8 and 13.6) of the future teachers in the control group, as well as in the experimental group (5.4 and 20.5).

Table 2

Comparison of Proof-Constructions Test Results

Groups and test compared		*n*	Mean rank	Sum of ranks	Test used	*p*-value (two-tailed)	Group mean score	Group interpretation
Between groups	Pre-test (control) *vs.* pre-test (experimental)	10	11.80	118.00	Wilcoxon-Mann-Whitney test	0.3472N.S.	5.8	Clueless
		10	9.20	92.00			5.4	Clueless
	Post-test (control) *vs.* post-test (experimental)	10	7.50	75.00		0.0257*	13.6	Novice
		10	13.50	135.00			20.5	Intermediate
Within groups	Pre-test *vs.* post-test (control)	10	5.50	55.00	Wilcoxon Sign Rank test	0.0054*	5.8	Clueless
							13.6	Novice
	Pre-test *vs.* post-test (experimental)	10	5.50	55.00		0.0054*	5.4	Clueless
							20.5	Intermediate

Note. * significant: $p < 0.05$ N.S. = not significant: $p > 0.05$.

While groups in the pre-test results of the proof beliefs questionnaire believed that a theorem has no exception and that the dual role of proof is to convince and to explain, but they disagreed that a proof must be general and were undecided on the statements that diagrams have both benefits and limitations.

However, after the intervention, both groups believed that a theorem has no exception that the dual role of proof is to convince and to explain, and that the validity of a proof depends on internal logic. Nevertheless, they disagreed with the statement that diagrams that illustrate statements have benefits and limitations, and were undecided on statements as logically equivalent to their contra-positive statements but not necessarily to their converse and inverse statements. That is, they tended to believe in the nature of the theorems and of proof and its purpose as well.

A change in beliefs about proofs between and within groups did not occur. No noteworthy changes occurred on the students' or future teachers' beliefs about proofs before and after the study (see Table 3).

Table 3

Summary of the Proof Beliefs Questionnaire Results

Group		Principle number (group mean score)					
		1	2	3	4	5	6
Control	Pre-test	1	1	0	0.5	0	0.5
	Post-test	1	1	0	1	0.5	0
Experimental	Pre-test	1	1	0	1	0.5	0.5
	Post-test	1	1	0.5	1	0.5	0
Overall agreement		1	1	0	1	0.5	0

Notes. Legend: 1 = agree; 0.5 = undecided; 0 = disagree.

The Spearman Rank Order Correlation Coefficient (see Table 4) revealed a significant positive correlation between the proof-construction performance and van Hiele levels of the future mathematics teachers.

Table 4

Spearman Rho Test Results

Variable	Post-test results by group	n	r_s	p-value (two-tailed)
Proof-construction Performance *vs.* van Hiele levels	Control	10	0.4256*	0.221085
	Experimental	10	0.6699*	0.034175

Note. *The correlation is significant: $r_s > p$ at the level of 0.01.

The following insights were drawn from the teaching and learning episodes, interview results, journals, seat work and quizzes.

The kinds of proof made by the future teachers from both groups were direct proof and proof by contradiction. Direct proof is associated with two-column format, and proof by contradiction with paragraph format. "Compare and change variable method" of proof was also popular among them, and they found it easier to prove theorems similar to the examples. They were also looking forward to a formula in proving a theorem, wherein they can just substitute and then finally prove the theorem. They preferred to use the two-column format using direct method, rather than the paragraph format using proof by contradiction. Some of them did their proofs before they came to class and submitted them as their seatwork or quiz.

The future teachers listed the following steps in proving a theorem: (1) read and understand the theorem; (2) re-state the theorem into if-and-then form and identify the given and what to prove; (3) draw a diagram that illustrates the theorem; (4) decide what method of proof and format to use; (5) gather definitions, axioms/postulates and theorems needed to reach the conclusion desired; (6) write the statements and corresponding reasons in logical order; and (7) check the validity of the statements and consult the teacher.

The future teachers felt that proving was difficult. They had poor background in geometry concepts and knowledge in logic and deductive reasoning, which were prerequisite skills in proving.

The future teachers from both groups were in favor of the sequence of presentation of the lesson used in this study.

The future teachers from the control group were not in favor of the grouping method used in the experimental group, and thought that it was unfair to them (there was "damay syndrome" in the enhanced-group Moore method; that is, each group member received the additional five points, even if a group member did not or had less contribution to the work). In addition, they did not like the presentation of ready-made proofs by the teacher written on the manila paper and they wanted to be involved in proving the theorem being discussed.

The future teachers in the experimental group were in favor of the enhanced-group Moore method of instruction. The method of grouping encouraged them to make proofs by themselves and later on to share and help others in the next stages. The incentive (plus five points and exemption from the final examination) served as a driving force to do the proofs, as seen from the rank of the accumulated additional points. Two of the top three students who were exempted from the final examination belonged to the experimental group (first and third rank). However, the grouping method created an atmosphere of isolation, inferiority complex and self-pity among the students with low performance, when none of them was chosen by their classmates to join a group.

Conclusions

The future mathematics teachers' van Hiele levels increased after a focused study on proofs. However, those who were exposed to the traditional method had a smaller increase (level 1: visual to level 2: descriptive), compared with those exposed to the enhanced-group Moore method (level 1- visual to level 3- abstract). Those who studied proofs by using the enhanced-group Moore method obtained a significant increase of two levels, compared with the increase of one level by the future teachers who were exposed to the traditional method.

Although the result is below the target level (level 4: deduction), this study still deserves attention. An increase of two steps in the van Hiele hierarchy was achieved, in spite of the fact that the subjects were adult learners who were not usually flexible in learning new ideas.

The enhanced-group Moore method is thus deemed effective in raising the van Hiele level of geometric understanding of the second year BSEd math major students in a State University in the Eastern Visayas, given some limitations explained in the first part of the study. Hopefully, this method could also be effective in raising the van Hiele levels of future teachers globally all the way to level 4 (deduction) if used for a longer period of time.

An increase in the proof-construction performance of the future teachers was also realized in this study. While the future teachers in the control group, have been improved from initially the category of clueless to the category of novice, those in the experimental group, also have been improved from initially the category of clueless to the category of intermediate. The proof-construction mean score of the future teachers exposed to the enhanced-group Moore method is significantly better than that of those exposed to the traditional (instructor-based) method. Moreover, it was found out that there is a significant positive correlation between the proof-construction performance and the van Hiele levels of the future teachers. Therefore, an increased performance in the proof-construction test affected a raise in the van Hiele levels of the future mathematics teachers.

The future teachers' agreement with the six principles on beliefs about proofs which provided useful explanations for the difficulties encountered in doing proofs. Proving in geometry for the future teachers was difficult, even after intervention, because of inadequate prerequisite skills. They preferred the two-column format rather than the paragraph format, and they were mechanical in the way they proved. This study showed that there is a way that helps future mathematics teachers improve their mathematical and geometric reasoning skills.

The future mathematics teachers in this study who enrolled in College geometry had no idea on how to prove a theorem, as revealed in their proof-construction test result—which is parallel to the low van Hiele entry level (level 1), interpreted as visual: the expected level for an elementary graduate. This implies that the future teachers in this study had poor preparation in proving. Poor preparation in proving of the future teachers in this study implies poor teaching in elementary and high school geometry.

The future mathematics teachers in this study held certain beliefs that hampered their cognitive mathematical development, specifically in doing proofs. This implies that beliefs are somehow connected to how future mathematics teachers do proofs.

Recommendations

On Poor Preparation in Proving

It is not that future teachers "do not get proof" but that they are "not yet ready for proof." Thus, it is recommended that students who are not yet at van Hiele level 2 (Philippine settings) be placed in Algebra 2 (an algebra proof-oriented course with logic, i.e., Fundamental Concepts in Math), before enrolling in a College Geometry course.

On Poor Geometry Teaching

It is recommended that mathematical and geometric reasoning skills of future mathematics teachers must be developed in the elementary and high school years by using appropriate approaches. Moreover, teachers in geometry must stop avoiding topics that are difficult. Future mathematics teachers must experience the struggle to do proofs by themselves and learn to appreciate the importance and beauty of this struggle in establishing truth.

Mathematics teachers must promote the development of proof skills among students, which is make proof meaningful to their students by: (1) promoting the development of communication skills; and (2) promoting justification. With respect to the former, teachers must first elicit the students' own explanation in their own words, and then lead them to developing the language formal proof. Teachers can use questioning techniques and open problems to promote good communication skills. With regard to the latter, teachers must modify instructions to accommodate a variety of different proof schemes or methods of proving as much as possible. That is, teachers must not confine students to the two-column format by using direct method and paragraph format using proof by contradiction.

On Proof Beliefs

Considering the importance of beliefs about proofs in understanding students' success in proving, a year of implementation intervention of the enhanced-group Moore method is recommended. This study showed a positive effect of the method on enhancing future teachers' beliefs about proofs. A longer implementation of the method could further enhance their beliefs.

On Future Researches

Proofs and geometry are the most feared topics in mathematics and are the areas of lowest performance among our students (as shown by TIMSS (third international mathematics and science study) and the pre-test of this study). Unless interventions are implemented to stop the decline in the performance of the students, this trend may take decades to recover. This study ought to serve as a wake-up call to teacher educators. Hopefully, there will be a renewed interest in teaching proving and develop mathematical geometric reasoning skills among students and future mathematics teachers.

Future research studies need to investigate or look into:

(1) Mathematical and geometric reasoning skills, proof writing ability, students conceptions about proofs, as well as Van Hiele levels using enhanced-group Moore method. The following could be considered: use of homogeneous or heterogeneous small grouping according to ability levels, gender type, reverse order of grouping used in this study, or other types of grouping within the context of Filipino values (i.e., damayan, bayanihan, and utang na loob); use of large samples (two sections with 40 students for each group); and use of a teacher-made instrument to measure mathematical and geometric reasoning skills, a proof writing test which focuses on the different types of proof schemes or methods, a beliefs questionnaire based on a survey of beliefs among high school students after a course in high school geometry, a van Hiele level of geometric understanding (after relating it with SOLO (structure of observed learning outcomes) Taxonomy), as well as other possible methods of teaching proofs (i.e., inquiry-based or activity oriented approach);

(2) Implementation of the enhanced-group Moore method in other basic mathematics courses in the tertiary level, such as College Algebra, Trigonometry, Analytic Geometry, and Calculus;

(3) Replication of the experiment in graduate education courses in mathematics, specifically, modern geometries;

(4) Research in using dynamic geometry software to improve future mathematics teachers' mathematical and geometric skills, proof-construction performance, and van Hiele levels;

(5) Development of a training program to teach teacher educators the enhanced-group Moore method, including other innovations that could be more effective.

References

Battista, M., & Clements, D. (1995). Geometry and proof. *Mathematics Teacher, 88*(1), 48-54. National Council of Teachers of Mathematics.

Battista, M., & Clements, D. (2000). Geometry and spatial reasoning. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning*. New York: McMillan.

Burton, J. (1997). The Moore method. *American Mathematical Monthly*, *84*, 273-277

Caluya, D. (2000). Student's level of geometry thinking in relation to teacher's transformational practices in geometry classes (Unpublished doctoral dissertation, University of the Philippines).

Dancis, J. S., & Davidson, N. (1970*). The Texas method and small group discovery method*. Retrieved 2006, from http://www. discovery.utexas.edu/rlm.reference/dancisdavidson

Davidson, N. (1973). The small group-discovery method of mathematics introduction as applied in calculus (Unpublished doctoral dissertation, University of Wisconsin-Madison).

Erfe, A. (1996). *A validation of the van Hiele levels of thinking in learning high school geometry* (Unpublished doctoral dissertation, University of the Philippines).

Frykholm, J. A. (1994). *External variables as predictors of the van Hiele levels in and geometry students* (Unpublished dissertation, University of Wisconsin-Madison).

Griffiths, P. A. (2000). Mathematics at the turn of millennium. *American Mathematical Montly, 107*(1), 1-14

Jones, K. (2000). The student experience of mathematical proof at the university level. *International Journal of Mathematics in Science and Technology*, *31*(1), 53-60.

Martin, T., & McCrone, S. S. M. (2004). Assessing high school students' understanding of geometric proof. *Canadian Journal of Science, Mathematics and Technology Education, 4*(2).

Martin, T., & Pulley, C. (2000). *What our students really know about proof and reasoning geometry: A look at classroom-based research data.* Retrieved 2006, from http://www.math.ilstu.edu/tsmartin/NCTM2000

Mason, M. (1997). The van Hiele levels of geometric understanding. *Professional Handbook for Teachers, Geometry, Explorations and Applications*. McDougal Little Inc..

Mingus, T., & Grassl, R. M. (1999). Preservice beliefs about proofs. *School Science and Mathematics, 99*(8), 38-44. Retrieved from http://www.eric.ed

O'Connor, J., & Robinson, F. (1992). *Robert lee moore.* Retrieved from http://www-groups.dcs

Schoenfeld, A. H. (1988). When good teaching leads to bad results: The disasters of "well taught" mathematics courses. *Educational Psychologist, 23*(2), 1-22. Retrieved from http://www.ithaca.edu/compass/pdf/schoenfeld.pdf

Taylor. (2004). *The Moore method.* Retrieved 2006 from http://www.discovery.utexas.edu/rim/reference/moorefilm.html

Vistro-Yu, C. P. (2001). Implications of teachers beliefs about mathematics for classroom and teacher education reforms. In Cujegkeng, Ma. Assunta (Ed.), *The Loyola schools review.* Ateneo de Manila University: School of Science and Engineering, Office of s and Publications.

Weber, K. (2003). *Student's difficulties with proofs.* Research Sampler. Retrieved 2006, from http://www.maa.ort.t_and1/sampler /rs_8html

Young, L. (2003). *Proof introduction: Final paper in math 791* (pp. 1-13). Retrieved from http://www.math.ksu.edu/math791/ finalpaper/laneeterm.pdf