



Center for the Study of Mathematics Curriculum

**K-12 Mathematics:
What Should Students Learn and When Should They Learn it?**

**February 5-6, 2007
Arlington, Virginia**

CONFERENCE HIGHLIGHTS

Conference Cosponsors:

Achieve, Inc.
American Statistical Association
College Board
Mathematical Association of America
National Council of Teachers of Mathematics

with

Center for the Study of Mathematics Curriculum

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TABLE OF CONTENTS

INTRODUCTION	1
CONFERENCE AGENDA	2
OPENING REMARKS Dr. Cora Marrett, Assistant Director of Education and Human Resources, National Science Foundation	6
OVERVIEW OF CURRICULUM DOCUMENTS Achieve, Inc. (<i>Secondary Mathematics Expectations</i>) College Board (<i>Standards for Success: Mathematics and Statistics</i>) American Statistical Association (<i>Guidelines for Assessment and Instruction in Statistics Education</i>) National Council of Teachers of Mathematics (<i>Curriculum Focal Points: A Quest for Coherence</i>)	8
KEYNOTE ADDRESS <i>Tracing the Evolution of Mathematics Content Standards in the United States: Looking Back and Projecting Forward towards National Standards</i> Dr. Jere Confrey, Washington University in St. Louis	10
ANALYSIS OF CURRICULUM RECOMMENDATIONS <i>Some Common Themes and Notable Differences across Recent National Mathematics Curriculum Documents</i> Dr. Chris Hirsch, Co-Director of CSMC, Western Michigan University	40
CLOSING COMMENTS Dr. Joan Ferrini-Mundy, Division Director, Division of Elementary, Secondary, and Informal Education, National Science Foundation	52

INTRODUCTION

In the fall of 2006 several influential national groups (Achieve, American Statistical Association, College Board, and the National Council of Teachers of Mathematics) developed and released documents that recommended curriculum standards or focal points for K-12 mathematics. The timing of the release of these documents provided a unique and important window of opportunity to promote and stimulate collaboration among producers and users of standards. In response, the Center for the Study of Mathematics Curriculum (CSMC) organized several meetings of representatives of each group to discuss the nature of their work and possible collaboration. As a result of those meetings, the groups agreed to cosponsor a national conference to highlight the recommendations and to engage “users” of standards (state and district curriculum specialists, textbook and assessment publishers, K-12 district and teacher leaders, and representatives from higher education and business) in discussions about implications for their work.

The conference was held on February 5-6, 2007 at the Rural Electric Cooperative Association Conference Center in Arlington, Virginia. Participants included about 200 representatives from schools, state departments of education, institutions of higher education, textbook and assessment publishers, professional organizations, and major research and development centers. The conference format allowed the cosponsoring organizations to explain the rationale, process and product of their work. It also provided opportunities for participants to engage with each other and with the cosponsors in conversations about the implications of the work.

The conference was also webcast and archived sessions are available at: <http://cltnet.org/cltnet/misc/csmcmath07/> This document provides another record of the conference. It includes a commissioned paper that served as the basis for the opening keynote session by Dr. Jere Confrey.

A conference planning committee worked for over 6 months to plan the event. The success of the conference is due, in large part, to their efforts, commitment and willingness to collaborate. They include:

Achieve, Inc
Laura Slover,
Kaye Forgione

American Statistical Association
Henry Kranendonk
Martha Aligia

College Board
Arthur VanderVeen
Robyn O’Callaghan

Mathematical Association of America
Michael Pearson

National Council of Teachers of Mathematics
Francis (Skip) Fennell
Jim Rubillo
Ken Krehbiel

Center for the Study of Mathematics Curriculum
Glenda Lappan
Chris Hirsch
Barbara Reys

For information on the mathematics curriculum recommendations showcased during the conference, see the documents produced by each co-sponsor (see page 9). For more information about the work of the Center for the Study of Mathematics Curriculum, see: <http://mathcurriculumcenter.org/>

CONFERENCE AGENDA



Center for the Study of Mathematics Curriculum

***K-12 Mathematics:
What should students learn and when should
they learn it?***

A National Conference

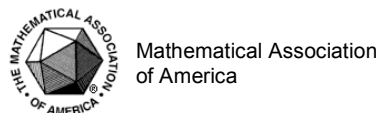
Organized by the Center for the Study of Mathematics Curriculum

February 5-6, 2007

National Rural Electric Cooperative Conference Center

4301 Wilson Blvd. Arlington, VA

Conference Cosponsors:



The Center for the Study of Mathematics Curriculum (CSMC) is funded by the National Science Foundation under Grant No. ESI-0333879.

AGENDA

MONDAY, FEBRUARY 5, 2007

Time	Event	Room
8:30-8:45	Welcome: Glenda Lappan, Center for the Study of Mathematics Curriculum (CSMC) Cora Marrett, Assistant Director of Education and Human Resources, National Science Foundation	CCI
8:45-9:45	Plenary Session: <i>Mathematics Curriculum Standards: A Path Toward Coherence</i> Jere Confrey, Washington University - St. Louis	CCI
9:45-10:15	Break	
10:15-11:45	Concurrent Breakout Sessions	
	<p><i>Secondary Mathematics Expectations</i> (Achieve), Laura Slover and Kaye Forgione</p> <p><i>College Board Standards for College Success: Mathematics and Statistics</i> (College Board), John Dossey and Katherine Halvorsen</p> <p>Session Presider: Ira Papick, University of Missouri and CSMC</p>	CC1 RED group
	<p><i>Curriculum Focal Points: A Quest for Coherence</i> (NCTM), Francis (Skip) Fennell and Janie Schielack</p> <p><i>Guidelines for Assessment and Instruction in Statistics Education</i> (GAISE) Report (American Statistical Association), Christine Franklin and Gary Kader</p> <p>Session Presider: Kathryn Chval, University of Missouri and CSMC</p>	CC2 GREEN group
12:00-12:45	Lunch	CC1
1:00-2:30	Repeat of Concurrent Breakout Sessions	
	<p><i>Secondary Mathematics Expectations</i> (Achieve), Laura Slover and Kaye Forgione</p> <p><i>College Board Standards for College Success: Mathematics and Statistics</i> (College Board), John Dossey and Katherine Halvorsen</p> <p>Session Presider: Ira Papick, University of Missouri and CSMC</p>	CC2 GREEN Group
	<p><i>Curriculum Focal Points: A Quest for Coherence</i> (NCTM), Francis (Skip) Fennell and Janie Schielack</p> <p><i>Guidelines for Assessment and Instruction in Statistics Education</i> (GAISE) Report (American Statistical Association), Christine Franklin and Gary Kader</p> <p>Session Presider: Kathryn Chval, University of Missouri and CSMC</p>	CC1 RED group
2:30-3:00	Break	
3:00-4:00	Plenary Session: <i>Common Themes and Notable Differences across Mathematics Curriculum Documents</i> Chris Hirsch, Western Michigan University and CSMC	CC1
4:15-5:15	Panel: <i>Discussion of Curriculum Recommendations by Achieve, ASA, the College Board, and NCTM</i> Panelists: Roxy Peck (ASA), Skip Fennell (NCTM), John Dossey (College Board), Laura Slover (Achieve) Moderator: Iris Weiss, Horizon Research, Inc. and CSMC	CC1
5:15-6:30	Reception Welcome by Co-sponsors	CC1

TUESDAY FEBRUARY 6, 2007

Time	Event	Room
7:30-8:15	Continental Breakfast	CCI
8:30-9:45	Panel: <i>How can/should/will the new curriculum recommendations be used?</i> Panelists: Everly Broadway (North Carolina Department of Public Instruction); Karen Usiskin (Pearson Scott Foresman Publishing Company); Alfinio Flores (Arizona State University); Sherri Miller (ACT) Moderator: Diane Briars (Pittsburgh, PA)	CCI
10:00-11:00	Panel: <i>What are the implications of the new curriculum recommendations for post-secondary education and employment?</i> Panelists: Bernard Madison (MAA); William Goldman (AMS); Susan Traiman (Business Roundtable) Moderator: Richard Scheaffer (Chair of CBMS, past-president of ASA)	CCI
11:00-11:30	Break with light snacks	
11:30-12:45	Panel: <i>Is consensus on national curriculum standards necessary for advancing student learning?</i> Panelists: Diane Schaefer (Rhode Island Department of Education and President of ASSM); Susan Jo Russell (TERC); Bob Borst (Columbia MO Public Schools); Bill Schmidt (Michigan State University) Moderator: Randy Charles (San Jose, CA)	CCI
12:45-1:30	Closing remarks: Joan Ferrini-Mundy (National Science Foundation), Glenda Lappan (Michigan State University and CSMC)	CCI

CLTNet, in collaboration with Elluminate, will host a live and archived webcast of the conference, including streaming video of sessions and access to the PowerPoint presentations and other supporting documents.

The archive of the conference webcast is available at:

<http://cltnet.org/cltnet/misc/csmcmath07/>



OPENING REMARKS

Cora Marrett

***Assistant Director for Education and Human Resources
National Science Foundation***

Opening Remarks by Dr. Cora Marrett

I'm delighted to be here. There's probably an advantage of coming in on your third day of work at NSF—there's so many things you don't know, you can say a lot of things. But it's also an advantage because early on I need to learn as much as I can from distinguished communities, such as you clearly represent.

A casual observer might think of this conference as indicative of the fragile state of K-12 mathematics curriculum. After all, there's been wide circulation of the observation that the mathematics curriculum is a mile wide and an inch deep. But in my estimation, the conference represents a stage aimed far beyond the mere listing of complaints. It illustrates the deep and thoughtful work that surrounds efforts to improve learning opportunities for all in mathematics education. It centers on what various stakeholders have talked about as coherence and, indeed, I might have used as a title for my brief remarks "A Quest for Coherence," if NCTM hadn't already appropriated that title or, "A Path Towards Coherence," if that were not the theme for the plenary session. But obviously coherence is what we're striving for here.

In thinking about coherence I was struck by this observation from Barbara Reys: "It will take strong leadership, cooperation, and collaboration to realize the goal of a coherent, rigorous mathematics curriculum for all students in the United States. There is no better time to begin this work."

I endorse that statement heartily. This is, in fact, an opportune time, given that several leading organizations, the cosponsors of this conference, have released, or will release soon, new or clarified standards in mathematics. It is an opportune time, for key organizations have agreed to ponder together the question, *What should students know, and when should they know it?*

The National Science Foundation is privileged to be a part of an effort to enhance coherence. As you know, NSF has long been involved in curricular and teacher education developments, with reference to mathematics education. Of particular interest to the Foundation are the paths towards coherence that are possible in what is a decentralized, or at least this state-and-district-centered system of education. How do we achieve coherence in that kind of environment?

This conference, the conversations here and beyond, undoubtedly will have profound implications for the future of NSF's investments. I don't intend now to elaborate for you on what NSF hopes to learn out of this occasion, but you will hear later from one of my colleagues, Joan Ferrini-Mundy, who will certainly have thought more about this than I've had time to do in three days.

Let me end, then, by noting there is a thread that brings together potentially disparate partners. As the College Board has stated, we share a common goal of providing all students with a rigorous education that will prepare them for success; success after the high school years, success in the workforce, success in civic life. Thus, it is because of the importance of this activity, it's because of the sense that people can come together for important outcomes, I'm delighted not just to say welcome, but to say thank you—thank you very much.

OVERVIEW OF CURRICULUM DOCUMENTS

New K-12 Mathematics Curriculum Recommendations



Achieve, Inc.

Achieve, Inc. – *Secondary Mathematics Expectations*. The cornerstone of Achieve’s work on standards is benchmarking, or clearly defining what students should know and be able to do at different grade levels. Building on mathematics expectations developed for grades K–8, Achieve is developing content expectations for high school that will seamlessly connect the expectations for the end of 8th grade with those identified by the American Diploma Project for the end of high school. These benchmarks will outline a progression of mathematics content through high school that, if followed, will ensure that students master the content they need to be successful in college and in the workforce. For more information, see:
<http://www.achieve.org/node/479>.



American Statistical Association (ASA) – *Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report*. This document provides a conceptual framework for K-12 statistics education. The foundation for this *Framework* rests on the NCTM *Principles and Standards for School Mathematics*. The *Framework* is intended to support and complement the objectives of the NCTM *Principles and Standards*, not to supplant them. This *Framework* provides a conceptual and developmental structure for statistics education that presents a coherent model for the overall curriculum. For more information see:
<http://www.amstat.org/education/gaise/>



CollegeBoard
connect to college success™

College Board – *College Board Standards for College Success in Mathematics and Statistics*. The College Board standards identify the critical thinking skills and knowledge in mathematics and statistics that all students need to succeed in college. The standards-based instructional framework begins with middle school and, through the grades, builds the academic skills students need to master for success in college-level work, including Advanced Placement Program (AP) courses. A final draft of the standards was released Fall 2006. For more information, see:
<http://www.collegeboard.com/about/association/academic/academic.html>



NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS

National Council of Teachers of Mathematics (NCTM)
– *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence*.

Curriculum focal points represent a set of important mathematical topics for each grade level, PreK-8. They serve as possible organizing structures for curriculum design and instruction at and across grade levels by identifying areas of instructional emphasis. The topics are central to mathematics and they provide the foundations for further mathematical learning. As organizing structures, curriculum focal points lay a conceptual foundation that can connect and bring coherence to multiple concepts and processes taught within and across grade levels by identifying core structures around which related content can be addressed. The document was released on September 12, 2006. For more information, see:
<http://www.nctm.org/focalpoints/>

KEYNOTE ADDRESS

Jere Confrey

Professor of Mathematics Education

Washington University

Tracing the Evolution of Mathematics Content Standards¹ in the United States: Looking Back and Projecting Forward towards National Standards

Jere Confrey
Washington University in St. Louis

A paper prepared for
the Conference on K-12 Mathematics Curriculum Standards
February 5-6, 2007
Sponsored by CSMC, NCTM, Achieve, College Board, MAA, ASA

This paper was prepared with support from the Center for the Study of Mathematics Curriculum. I would also like to acknowledge Barbara and Robert Reys for helpful comments on the earlier draft and for editing and technical assistance from Alan Maloney and Kenny Nguyen. I am responsible for all final content decisions.

¹ It is important here to offer a set of distinctions in language and definition. The *content standards* are negotiated consensus among experts of what students should know and be able to do; often these are broad and across grade span, and if so, they may be specified at grade level using terms such as *grade level expectations*, *benchmarks* or *learning or performance objectives*. The set of these expectations, benchmarks or learning objectives are often referred to as a *curriculum framework*. As a result, in some instances, there are content standards and a curriculum framework; in other cases with the specificity of grades, the content standards are the curriculum framework. When there is only one document, it may be called the *curriculum standards*. The content and curriculum standards are not the same as *assessment or test standards*, usually narrower, that are used to guide the construction of the test. Finally, these standards documents should not be confused with curricula or curriculum materials that are used in instruction with students and for the assistance of teachers. For the purposes of this paper, *content standards* refers to the articulation of both content and process standards whether it is at grade level or by grade band. *Curricula* will refer to the materials and tasks used in instruction.

Part I: The Historical Evolution of “Standards”

The American Dialect Society selected its 2006 word of the year, to replace Stephen Colbert’s “truthiness,” and the word is “plutoed.” It means, “to demote or devalue someone or something, as happened to the former planet Pluto when the General Assembly of the International Astronomical Union decided Pluto no longer met its definition of a planet” (American Dialect Society, 2007). The fact that society at large assimilated this term, capturing a significant scientific conflagration over a definition, illustrates a number of salient points about the debates surrounding national content standards in mathematics K-12.

The debate among scientists regarding whether or not to demote Pluto to the status of a “dwarf planet” (which is not considered a planet), was fierce by scientific standards. During a week-long period in August 2005, the 26th General Assembly of the International Astronomical Union discussed the problem of whether to add three bodies comparable to Pluto ([Quaoar](#), [Sedna](#) and [Eris](#)) to the list of planets, or to eliminate them all by revising the definition of a planet. They contested whether to use roundness or differentiation into sedimentary layers in their definition, to apply human-stipulated restrictions (such as size) vs. determinations by natural limits (hydro-static balance), and whether to exclude “brown dwarfs” from being categorized as planets while including rogue or interstellar planets which “drift off” rather than circling the sun. In addition, broad issues of culture and society, such as the affection of the public for the planet, the cost of changing resource and educational material, the importance of tradition and emotion, and concern about alienating the public influenced the debate. The final definition passed but did not resolve the controversy, with over 300 astronomers currently refusing to support it.

This definition, which applies only to the Solar System, states that a planet is a body that orbits the [Sun](#), is large enough for its own gravity to make it round (hydrostatic balance), and has “[cleared its neighbourhood](#)” of smaller objects (Overbye, 2006). Apparently neither the term “round” nor the term “cleared the neighbourhood” are fully defined, in that some of the current planets including Earth could fail the second criterion. Further dissatisfaction remains due to the fact that Pluto and the other three mentioned previously are now called dwarf planets, which are not planets at all, catalyzing the ire of many linguists as well.

The case is instructive in that if the denotation of a planet nearly six billion kilometers away (on average, that is) from the Sun, invisible to the naked eye, and lacking influence on the daily lives of the vast majority of the world’s population, can spawn substantial dispute and attract international attention, is there any wonder that the development of standards to guide the mathematics education of America’s children would generate controversy and debate? Secondly, the fact that an object as distinctive as a planet needed redefinition reminds us that all knowledge is evolving. Finally, the example reminds us that the boundaries between “scientific” and “social/cultural” issues are porous—principles, definitions, human loyalties and interactions, tradition, linguistics, technologies, and new discoveries all play into the mix.

Standards inherently involve tensions. They are goal statements about which different people, even different experts, will have varied opinions. They require negotiations and

represent compromises among varied legitimate participants and groups. However, the original use of the term “standard” was that of the king’s standard; waving above the battlefield, representing the king’s authority; it was in a real sense fought for as though it were the king himself (Oxford English Dictionary, n.d.). Thus the earliest standards were essentially authoritarian—proclamations of religious and political leaders, accompanied by no explanation and no justification other than the authority as warranted by the full and unchallenged status vested in the proclaimer.

The term “standard” subsequently evolved, becoming closely woven into the emergence and subsequent maturation of fields of science and technology. As science flowered during the 1600s, there were increasing needs to coordinate results and findings. The *second* aspect of the term “standards” rested on coordinating measurement from one place to another so that the accuracy of results could be secured by a common metric, that is, by “a standard of measurement.” Metrics were often first shaped to fit the circumstance, sized for human convenience—the meter or yard as the length of an arm, the pace with the foot, and so on—taking into account such qualities as convenience, portability, relative size, and reliability. While human needs and preferences were expressed in the metrics, an outside world simultaneously pushed back on the measures, demanding standardization across place and time. Standardization constrains variations, and permits other discoveries of regularities and new inventions often at a higher level of understanding.

As an illustration, Hasok Chang (2004), in *Inventing Temperature: Measurement and Scientific Progress*, described the challenges, now taken for granted, of inventing thermometers and, in particular, in establishing the fixed points of freezing and boiling. The temperature of boiling water, in turns out, varies by 3-8° F depending on the rate of the boil, the ambient atmospheric pressure, and the depth of immersion of the thermometer. Chang reveals not only what the invention made possible, but how the invention of standard instrumentation affected our understanding of heat itself. Standards represent intellectual accomplishments, often tied to the creation of landmarks in that environment, in this example, in the form of measurements.

A *third* development in the meaning of the term “standards” emerged as societies became more closely linked via transportation and communication. Standards became *a means to modulate or direct change while constraining variation* in complex systems, and were the result of negotiations among authorized individuals with relevant expertise often in relation to measurement. Close ties thus exist between standardization and stabilization. Determining when and to what degree we need to act in accordance with each other, over what unit of coherence (school, district, state or nation), over what period of time (frequency of change) and to what degree to tolerate and support variation, become increasingly essential in globalized societies. Authorized standard-setting bodies must technically and empirically determine what produces the best outcome—delivers the clearest, most reliable signal to the audience, guides practice most effectively, or can be learned and mastered by practitioners, in a reliable, fair, and valid way. Invariably, setting standards to regulate and guide complex systems requires a bootstrapping process of setting initial standards and making appropriate adjustments in response to feedback.

Adjustment of standards recurs regularly. Recently the Environmental Protection Agency (EPA) announced changes in their standards for calculations of gasoline mileage ratings in response to changes in driving patterns. Previously cars were tested

at 48 miles per hour in ambient temperatures of 75 degrees, yet beginning in 2008, the standards for conducting mileage tests will accommodate to more aggressive driving (greater acceleration) and the frequent use of air conditioning. Changes in standards can affect the ratings and rankings of elements in the system. While the effects of these changes will be to depress gasoline mileage predictions across the board, they will also have differential impacts on particular vehicles. Overall mileage ratings will drop around 11%, but hybrids' ratings will drop by 30% because the new standards eliminate all electric driving while diesel engines will drop only 7% based on the adjustment.

If standards fail to consider technical and empirical data, the effects on a system can be serious. For instance, in *No Child Left Behind*, the concept of Annual Yearly Progress (AYP) lacked an empirical base, both for how long it should take to reach full proficiency, and for a clearly defined model of change. As a result, most states predicted linear progress towards full compliance. As a result, compliance with the law is exceedingly difficult, and districts are demanding reconsideration of AYP as the reauthorization proceeds.

In summary, the concept of standards has evolved from (a) authoritarian proclamation, to (b) agreement within the context of measurement, to (c) a negotiated and political consensus which permits other kinds of innovation and progress to proceed often closely affiliated with issues of systemic implementation. A key rationale for standards is to modulate change and to restrict variation to reasonable bounds.

Some people would define *content* standards simply as a specification of “what a person should know or be able to do” (National Research Council, 2002a, p. 2). I propose here a more complex definition that acknowledges more fully the varied uses of the term: *Content standards consist of (a) a negotiated settlement among authorized experts concerning the specification of what a person should know or be able to do, (b) with consideration of how that is to be measured and/or documented, and (c) as a means of modulating or effecting change within the system of education and restricting excessive variation.* In the remainder of the paper, I will demonstrate historically that the debates within the mathematics education community have often failed to consider the second and third components adequately, leading to more acrimonious debates than necessary, while the broader policy community has situated the mathematics content standards in such a three-part framework to some extent. I will show that the evolution of standards is nonetheless moving in the direction of such a framework. In the final section of the paper, I argue for what next steps are required to move in this direction more aggressively and urgently as required by international pressures.

I want to be sure to emphasize that my revised definition of content standards does not function independently of other key elements in the system: curriculum, instructional processes, teaching capacity, professional development and community, classroom assessment. As described by NRC (2002a), these additional elements can be viewed as the “channels to the investigation of the influence of standards.” My definition of standards, however, draws assessment and accountability explicitly into content standards, which should help ensure that the role of standards in scaffolding the educational system is brought to the forefront.

Part II: History of Standards in Mathematics Education

In mathematics education, debate about whether mathematics is for building mental muscle or for mastering practical tasks has a long history (David Klein, 2003). The story of content standards, however, is usually traced to 1957, when the United States, responded to the U.S.S.R.'s technological advantage shown by the launch of the Sputnik satellite by focusing on mathematics to produce the "new math," primarily targeted towards "high quality mathematics for college-capable students, particularly those heading for technical or scientific careers" (National Advisory Committee on Mathematics Education, 1975). Led largely by mathematicians (Beberman, Begle, Gleason and Pollack), the new math took set-theoretic foundations of mathematics and used them to create curricula dominated by attention to formal structure, properties, deductive proof, and building numeric systems, relying heavily on the ideas of set, relation, and function. Plane and solid geometry were combined and trigonometry was accelerated into the second year of algebra (National Advisory Committee on Mathematics Education, 1975, p. 1). Psychologically, the approach was influenced by research by Piaget, as interpreted in the U.S. (Duckworth, 1987), and by Jerome Bruner's (2006) *The Process of Education* as providing a means to transition children towards more abstract thinking at an earlier age. The National Science Foundation played a key role by supporting six-week summer institutes with stipends to increase the instructional capacity for some portion of the teaching population. Controversy ensued as parents complained about the content, some scores fell, and students lacked practical understanding (National Advisory Committee on Mathematics Education, 1975). Arguably, at the same time, the reform movement stumbled as implementation faltered at the classroom door in many locations (Goodlad, 1984).

In 1975, a committee known as NACOME (National Committee on Mathematics Education) under the auspices of the Conference Board of the Mathematical Sciences reexamined the new math and made a number of recommendations, hauntingly relevant today, including:

Anti-dichotomy: In the creation, introduction and support of mathematics programs, neither teachers, educational administrators, parents, or the general public allow themselves to be manipulated into false choices between:

1. the old and the new in mathematics
2. skills and concepts
3. the concrete and the abstract
4. intuition and formalism
5. structure and problem-solving
6. induction and deduction

The core of every mathematics program should contain a judicious combination of both elements of each pair with the balance, proportion and emphasis between the two being determined by the goals of the program, and by the nature, capabilities, and circumstances of the students and teachers in the program. (p. 136-137)

As to content, the committee recommended that (a) the logical structure of mathematics be maintained, (b) concrete experiences be an integral part, (c) applications be included

to a wide realm, (d) symbols and formalities be fostered, (e) by eighth grade, a calculator be made available, (f) metric system be used, and (g) statistical ideas be included (pp. 136-139).

Despite this call for balanced change, the “back to basics movement” ensued, beginning in the mid- to late seventies to respond to the concern that “new math” was leaving too many students behind unprepared to solve everyday word problems and applications. The growth of “back to basics” was simultaneously fueled by the complaints from the business community that social promotion was producing graduates lacking simple skills. Porter et al. (1991), described this movement as “Guaranteeing basic skills became the agenda; easy content for all students” (p. 12). Back to basics content was accompanied by the emergence of an “accountability” movement demanding a “bottom line” in “results,” akin to the end of year profits in business. Tom Luce’s *Now or Never* (1995) captured its pulse with his clear articulation of principles and measures. Standards, in this context, referred to the need to set explicit and measurable expectations of all graduates and accountability gauged the extent to which progress was made. Raizen (1998) refers to this use of standards as “a mechanism by which to hold schools accountable for what students learn” (p. 73).

In these first two reforms, we witness pendulum swings in emphasis rather than integration. The central focus switched from concerns for harvesting the “gifted” students to ensure national competitiveness at the highest level, to creating sanctions to compel compliance with expectations by the weaker students, for the purpose of ensuring a sufficiently skilled work base. Both reforms, however, targeted the full spectrum of students for intervention. This first oscillation was clearly reactive, an indication of the immaturity of the system.

During the eighties, two national reports emerged and catalyzed another surge in activity. In 1980, NCTM put out *An Agenda for Action* and in 1983, the Committee on Excellence on Education released *A Nation at Risk*. The latter document declared, “... the educational foundations of our society are presently being eroded by a rising tide of mediocrity that threatens our very future as a Nation and a people” (Gardiner, 1983, p. 5). The directors of the National Council of Teachers responded by creating a “Commission on Standards for School Mathematics,” charging the commission to:

create a coherent vision of what it means to be mathematically literate both in a world that relies on calculators and computers to carry out mathematical procedures and in a world where mathematics is rapidly growing and extensively being applied in diverse fields, and create a set of standards to guide the revision of the school mathematics curriculum and its associated evaluation towards this vision. (National Council of Teachers of Mathematics, 1989, p. 1)

In 1989, after three years of work led by Thomas Romberg together with mathematics teachers, researchers, and administrators, the National Council of Teachers of Mathematics produced the *NCTM Curriculum and Evaluation Standards*, with a set of *Professional Standards for Teaching Mathematics* (1991) and *Assessment Standards for School Mathematics* (1995). Only the *Curriculum and Evaluation Standards* exerted considerable influence on other factors in reform, curriculum, state changes, and assessment. The standards were intended to ensure quality, identify explicit goals, and promote change. They were purposed to create mathematically literate workers,

encourage lifelong learning, provide opportunities for all, and support an informed electorate. They were structured by grade bands (K-4, 5-8 and 9-12) and each addressed standards of problem solving, communication, reasoning, connections, and estimation. They addressed the content strands of number and numeration, geometry, measurement, statistics and probability, algebra, trigonometry, and discrete mathematics. Issues of pedagogy were integrated into issues of content, emphasizing the importance of active participation in learning by students. The Standards drew heavily on research on student thinking, student misconceptions, and how students learned particular ideas as they encountered challenging tasks. They warned against relying too heavily on memorization and procedural understanding, based on numerous studies documenting disintegration of students' apparent knowledge when asked for reasons and explanations, and stressed conceptual understanding (Erlwanger, 1973; Ginsburg, 1991; Kamii, 1985).

By any fair and objective assessment, the mathematics standards were pioneering. They spurred the development of standards in other areas, including science and technology (AAAS, 1993; International Technology Education Association, 2000; National Research Council, 1995). NCTM launched a massive national effort to reach teachers across the country, sending out prepared materials, video tapes, and the like to assist in dissemination. As a teachers' organization, NCTM built a strategy for reform based on recruiting teachers to become agents of change, a strategy subsequently known as "empowerment-oriented" (Darling-Hammond & McLaughlin, 1995).

Porter et al. (1991), as they studied reform in Texas, Florida, California, and New York, identified potential tensions between these two standards-based policy approaches to reform, control, and empowerment. They classified control strategies in relation to levels of prescriptiveness, power (the amount of reward or sanction attached to compliance), and authority (which derives from placement in the system). Empowerment was defined as centrally involving the "professionalization of teachers" (Darling-Hammond, 1988) by their inclusion in decision-making, more demanding certification requirements, and support for professional community and peer review. The accountability movement embraces a control strategy, while the NCTM approach relied on empowerment. Porter et al. (1991) recognized the need for both strategies but predicted that control strategies would be more readily sustained, as they have political capital and are relatively inexpensive. However, they asserted that the control strategy would fail if not also accompanied by investments in teacher education and materials development as called for in the empowerment strategy. Their predictions are proving eerily accurate as one judges the effects of current law (NCLB) *sans* concomitant capacity building.

In contrast, O'Day and Smith (1993) argued for the desirability of integrating the two strategies into content-driven systemic reform, and later called "standards-based reform." O'Day and Smith (1993) called for (a) curriculum frameworks that would upgrade the quality of the content and instruction by emphasizing "challenging and important material" (centralized vision); (b) alignment of state education policies so as to provide a coherent structure to support school reform, including professional development, teacher licensure, curriculum materials, and state assessments; and (c) a restructured governance system with flexibility and control at the school site (top-down and bottom-up reform). As they wrote:

When fully implemented, this model of content-driven systemic reform would be a uniquely American adaptation of the educational policies and structures of many of the world's highly developed nations. It would marry the vision and guidance provided by coherent, integrated and centralized education policies common in many nations with the high degree of local responsibility and control demanded by U.S. tradition. (ibid., p. 252)

Others echoed this call for "hard content" defined by Newmann and colleagues as "to engage in disciplined inquiry, to produce knowledge that has value in their lives beyond simply proving their competence in schools" (Newman, 1991, p. 4). While pressing for harder content, the systemic reform advocates were particularly concerned that this approach should address issues of equity:

Our core premise is that a systemic state approach for providing a more challenging content of all children and greater local professional responsibility for schools *could* provide the structure necessary to extend the reforms to all schools and all children. Under these conditions, it *could* raise the general level of achievement while also helping to reduce educational inequalities substantially. (ibid., p. 252)

Policy groups flocked to the notion that one could successfully wed "hard content" with a demand for equity, carrying along "standards" in their wake, but the challenges were immense (Hurd, 1997; Porter et al., 1991). The National Science Foundation launched its systemic initiatives, state, urban and rural, which required that states present evidence of six drivers, the first of which required the development of high-challenge standards. Fifty-nine sites received funding. All of them developed sets of standards, most of which were closely aligned with the NCTM standards. Then, in 1994, in *Improving America's Schools* (following Goals 2000), the Congress reauthorized the Elementary and Secondary Education Act (ESEA) under the Clinton administration. Previously Title 1 focused on identifying and serving individual students using pull-out programs designed to teach low level, skills. The law switched to identifying whole schools for funding if 50% or more students qualified for Title 1, and insisted on interventions dependent on high standards. Instead of measuring assessment on norm-based testing, they switched to standards-based assessment (Fuhrman, 1994). This law extended the application of the standards across the socio-economic boundaries.

The influence of Standards was broadened further with the development of thirteen curricula that NSF required to be consistent with the NCTM Standards, to use technology broadly, to integrate mathematical content, and to address equity. Some later criticized these materials for conforming to the NCTM Standards (Mathematically Correct, 2000). Later, a compilation of evaluations of these thirteen curricula and a set of commercially-generated materials found that while the overall effectiveness of the materials could not be determined due to weaknesses in the design of the evaluations. The report did note, however, a serious dearth of quality evaluations of commercially-generated materials, and further, that the reports on the effects of the NSF-supported materials were positive enough overall to merit further serious study (National Research Council, 2004).

The 1990s were a turbulent decade for mathematics education. While systemic reform evolved into the "standards-based reform movement," several splinter groups attacked the NCTM Standards from a variety of perspectives, and still others offered new

versions. Scholars concerned with the results of United States in international comparisons began to raise concerns that United States practices were insufficiently competitive. This trend began during the previous decade, with the publication of *The Underachieving Curriculum: Assessing U.S. School Mathematics from an International Perspective* (McKnight et al., 1987) reporting on the Second International Mathematics and Science Study (SIMSS, 1982). Criticisms resurfaced in the nineties, with Harold Stevenson, an international scholar on Asian instruction, insisting that the NCTM Standards were too vague, that the grade bands were too obscure and broad, combined too many vision statements, and had too much focus on pedagogy with content, and lacked clear measurable criteria (Stevenson, n.d.). When the Third International Mathematics and Science Study (TIMSS) showed even further erosion in American mathematics education performance, including that of our most advanced students, the national debates increased in intensity (Beaton et al., 1996). Publication of Liping Ma's (1999) *Knowing and Teaching Elementary Mathematics*, comparing Chinese and American teachers, fueled questions about teacher knowledge. Using problems from notable mathematics education research studies on topics such as area and perimeter, multiplication of decimals, division of fractions and the like, she demonstrated convincingly that Chinese elementary teachers, compared with American teachers, possessed a depth of knowledge that permitted them to be more precise in speaking about mathematics ideas, establish more connections among ideas, and anticipate and sequence instructional moves to assist students better. For example, while American teachers tended to use the language of borrowing for subtraction with regrouping, Chinese teachers discussed "decomposing a high value unit" (ibid., p. 7), a more precise phrasing that anticipates the need to decompose not only the next unit, changing tens to ones to subtract, but beyond to hundreds (ibid., pp. 204-237). Because the Chinese teachers had less formal education, the study was used by some to critique the quality of mathematics education teacher preparation in the U.S., although it should be noted that the Chinese teachers she interviewed also taught for only three to four 45-minute classes per day, and taught only mathematics (ibid., p. 129). The study was adopted by both detractors and supporters of the Standards movement as support for their positions. What was most important about Ma's contribution, however was that she captured ways of talking about content that demonstrated both a deep understanding of mathematics and an anticipation of and reflection on how students learn.

The group "Achieve" also developed an active role in mathematics education in response to the poor international showing. They worked in the educational system at the state level, assisting states with policy issues of alignment, with benchmarking standards to grade levels and investing in capacity building. In addition, they enlisted advisors who helped them identify focused state and district level targets that would benefit from the expertise of an outside group. Achieve created a resource-based role for advising and consulting, along with a national role for introducing exemplary materials. For example, in assessment, they amassed examples of high quality items demonstrating that one could combine high quality mathematics and careful scoring rubrics. The Mathematics Achievement Partnership (MAP) produced a K-8 Mathematics Expectations (December 2004) and are currently ready to release their "Secondary Mathematics Expectations" (Achieve, January 2007). By keeping their work focused on meeting international performances, they agnostically worked with advocates from NCTM as well as with mathematicians and psychometricians to advocate for stronger and more internationally aggressive content bases for standards (Achieve, Inc., 2004).

A second source of controversy came from a group subsequently naming themselves “Mathematically Correct” (<http://www.mathematicallycorrect.com/>). They initially criticized curricular designers who were funded by NSF to develop new models of textbooks, reflective of the NCTM Standards. Criticism increased when the DoE Expert Panel released its report identifying several curricula as exemplary or promising.

Mathematically Correct conducted its own reviews of major curricular programs and raised concerns about the lack of early introduction of formal algorithms, neglect of fluency in basic facts, overuse of the calculator, and the tendency for instructors to move too slowly for advanced learners and to fail to accomplish aggressive agendas for learning. They also attacked colleges of education, mathematics educators, and the NCTM Standards, blaming them for poor performances by students. Using a highly successful guerilla-like tactic, Mathematically Correct proponents, seeking more political capital, used the Web as a means to connect to parents’ concerns, particularly to recruit to their position highly educated parents seeking assurance that their children would compete successfully at top research universities and fare well on entrance exams. The message that their children should be taught as the parents were taught resonated well with the Mathematically Correct adherents, who wanted their own successes mirrored in their children. As their work matured, they linked to some conservative foundations,² drew on savvy media connections, and linked their agenda to general critiques of the poor quality of public education. Gradually, they addressed achievement gaps, by advocating that more explicit instruction would serve impoverished children’s needs better (Kameenui et al., 2001; D. Klein, 1999). Other groups concerned for children of poverty chose instead to help parents learn to demand standards-based education (The Education Trust, 2002).

The use of the Internet in communicating positions illustrates the way these controversies spread as a result of the distributed nature of governance of United States education. Mathematically Correct targeted their interventions at district level governance structures such as parent groups and school boards. (For an historical account of the activities from an insider’s perspective, see David Klein’s version of developments at <http://www.csun.edu/~vcnth00m/AHistory.html>.) As a result, teacher groups, school boards, and parent groups were frequently at odds with each other. A range of opinions was evident in all groups. There are teachers who agreed with the criticisms of the NCTM Standards and pedagogical approaches, and first-rate mathematicians who were some of the NCTM Standards’ strongest advocates (Bass, 2001; McCallum, 2003). Arguably, the largest group fell into neither camp; most teachers continued to use their textbooks to teach as they were taught, and most mathematicians stayed out of the fray, concentrating on their work at the post-secondary level.

It was clear that in contrast to the pendulum swings of the seventies and eighties, controversies about mathematics education had become more sophisticated and more contentious in many ways, employing media (the press and the Web), private sources of funding, and political channels. These debates were not just about what should be in the standards but who should possess the authority to make the decisions, which

² For example, David Klein chaired the Fordham Foundation’s review of mathematics standards and Ramii and Braden’s “The State of Mathematics Standards” (<http://mathematicallycorrect.com/links.htm>).

stakeholders could be heard the most audibly, and the role of outside organizations, foundations, and the media. Finally, a flat set of national scores on NAEP, decreases in performance on the SAT, and decreases in the numbers of students entering scientific and mathematical fields fueled the debates, but the absence of solid empirical evidence to support causal claims exacerbated the acrimony (National Research Council, 2004).

In the mid-1990s, NCTM, responding to some of the criticism of their 1989 Standards, specifically around issues such as the role of basic facts and memorization, when and how to use technology, and what forms of pedagogy to support, decided to revise their Standards. This effort, led by Joan Ferrini-Mundy, which included mathematicians, mathematics education researchers, classroom teachers, policymakers, curriculum developers, and teacher educators (Ferrini-Mundy & Martin, 2003), produced *Principles and Standards for School Mathematics* (NCTM, 2000). They devised a “consultative process” intentionally designed to reach out to all constituencies through Association Review Groups (ARGs), created ways to collect and analyze systematic feedback on the draft document, and commissioned 25 reviewers. Analysis of the feedback led to the identification of five categories of major issues: overarching issues, structure of the document issues, content issues, issues related to learning, and issues related to equity (Ferrini-Mundy, 2000; Ferrini-Mundy & Martin, 2003). The authors explicitly communicated with the field on how the writers considered the feedback and the rationale for decisions made. Furthermore, in response to criticisms that the previous Standards lacked a solid foundation in research, they produced an accompanying volume (Kilpatrick et al., 2003).

While PSSM did successfully rebalance the Standards, criticism on various fronts continued, much of it concerning interesting and fundamental questions about standards: should standards incorporate grade level rather than grade band specificity? How many standards should there be, and at what level of detail? Should pedagogy be included, or left to the practitioners to decide? How aggressive should standards be, in relation to those of other countries? Should the strands of content, algebra, geometry, trigonometry, probability and statistics be integrated or kept distinct? Are calculators undercutting mathematical maturity or are they obligate tools of the trade?

In the past two years, increasing attention has been paid to the transition to college. For years, increases in remediation on colleges campuses in mathematics have been reported (Lindholm et al., 2005, as cited in Steen, 2007). An NRC Committee in a report entitled “*Rising Above the Gathering Storm*” (COSEPUP, 2005) identified ten priority actions required to enhance science and technology in the age of globalization. The Committee’s first suggested action was to “increase America’s talent pool by vastly improving K-12 science and mathematics education” (ibid., p. 3), specifically calling for scholarships to attract first-rate teachers, summer teacher professional development, and to get more students passing AP and IB science and mathematics courses.³

³ The view that AP and IB (Advanced Placement and International Baccalaureate) programs are the best practices to university study is not held universally; see the NRC Report (National Research Council, 2002b) for a discussion of the strengths and weaknesses of each, emphasizing the priority of a curricular focus rather than a test focus as the basis for advanced study at the end of high school [2002-series of reports analyzing AP and IB].

Recognizing the importance of ensuring smooth and successful transition for students from high school to college, beginning in 2003, the College Board⁴ decided to conduct its own review of the needs of students and to produce another set of standards, “College Board Standards for College Success: Mathematics and Statistics” (The College Board, 2006). They reviewed assessment frameworks for AP, SAT, and PSAT tests, conducted content analyses of college entry courses, surveyed introductory college professors and high school mathematics teachers, built case studies, and reviewed other standards to produce their “standards for college readiness” which they describe as “articulate[ing] a developmental progression of student performance expectations that would lead students to being prepared for college-level work” (ibid., p. x).

The final chapter in the NCTM Standards history has become public during the last six months. In 2005, NCTM launched a new initiative to respond to concerns that a largely mobile population was encountering difficulties in moving from place to place. It further responded to research by Reys et al. (2006) that documented the variation among states concerning the grade level selected to teach a variety of topics including basic operations, fraction computation, the role of calculators, and algebra. They wrote, “Findings from this study confirm that state mathematics curriculum documents vary along several dimensions including grain size (level of specificity of learning expectations), language used to convey learning outcomes (understand, explore, memorize, and so on) and the grade placement of particular learning expectations” (Reys et al., 2006, p. 9).

The new NCTM report, *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics* (National Council of Teachers of Mathematics, 2006), identifies three topics at each grade and describes the connections made possible by these foci. A focal point “had to pass three rigorous tests:

Is it mathematically important, both for further study in mathematics and for use in applications in and outside of school?
Does it “fit” with what is known about learning mathematics?
Does it connect logically with the mathematics in earlier and later grade levels?”
(ibid., p. 5)

Even if one questions whether all the focal points are precisely placed, the importance of the document is twofold. First, it argues for more attention to fewer topics. Secondly, it moves to articulation of standards at specific grades at the K-8 level. In addition, its emphasis on linking numeration and geometry often through measurement, and its developmental approach to the formation of algorithms (conceptual, structural, fluency to applications) are also hallmarks of the document.

While the Wall Street Journal (Hechinger, 2006, September 12) heralded these as a reversal in NCTM’s direction, the New York Times author Tamar Lewin (2006, November 14) followed their lead and reported that:

For the second time in a generation, education officials are rethinking the teaching of math in American schools. The changes are being driven by students’ lagging performance on international tests and mathematicians’

⁴ The Advanced College Testing (ACT) has also conducted studies and produced benchmarks for secondary school mathematics. See ACT, 2005.

warnings that more than a decade of so-called reform math--critics call it fuzzy math--has crippled students with its de-emphasizing of basic drills and memorization in favor of allowing children to find their own ways to solve problems (ibid., p. A2)

Indeed, practically every major American press outlet reported that NCTM had capitulated to “the old math.” The Chicago Sun Times concluded “Fuzzy teaching ideas never added up” (Chicago Sun Times, 2006, September 13), the Star Tribune’s headline read “Teachers group takes lint-remover to ‘fuzzy math’” (Kersten, 2006, October 5), and the Boston Herald reported that the math wars were over—but without the article’s author ever having read the report himself, by his own admission (Boston Herald, 2006, September 17).

These analyses were in error. While it is true that 5 of the 24 topics target quick recall and fluency of basic operations, the other 19 stress the importance of deep conceptual understanding and relationships among key ideas. The report remains committed, moreover, to the organization’s current standards, found in *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics, 2000). It cites that document repeatedly in its introduction. As one of the *Focal Points* authors, University of Georgia mathematician Sybilla Beckmann explained that, despite the media’s preference for conflict, in math “there’s a lot of agreement about what students need to know” (Lewin, 2006, November 14, p. 1).

The progress made in creating and refining national standards for mathematics is undeniable. In less than twenty years, as a nation we have progressed from no standards to multiple sets of standards, each created with clear attention to salient features affecting student learning. In a distributed system of education, we have crafted a means to advise diverse constituencies on what students should know and do and have seen many of the states—the national unit of educational change—adopt or adapt these for local consumption. It is also clear that we have embraced the complexity involved in acknowledging diverse groups of experts and their critical roles in establishing standards. All efforts now include mathematicians, mathematics educators, statisticians, and teachers, and most follow the lead of PSSM to create “consultative” processes.

Points of consensus about content are emerging. Statistics has taken its place as a significant part of mathematics education, an emphasis that is unlikely to fundamentally diminish. Technology, especially with regard to graphing calculators, plays a key role in secondary education, and as statistics becomes more central, the door will be opened to more diverse and robust technologies (computer algebra systems, dynamic geometry). Algebra is broadly distributed across the K-12 curriculum, though whether its meaning is limited to number sentences or expanded to a broader definition of functions and rates of change remains to be seen. And agreement that automaticity in number facts remains critical to mathematical proficiency and cognitive ease has been reached. While PSSM targets narrower grade bands than the 1989 Standards, with *Focal Points* and the other standards there is emerging consensus that they need the specificity of grades and/or courses. What this means at the high school level in relation to integrated mathematics is not yet resolved.

Furthermore, we have as a nation committed, at least rhetorically, to standards that demand “important and challenging” for all students. As stated by the College Board,

“all students need and deserve an academically rigorous high school program of study to succeed, whether they choose to attend college or seek a well-paying entry level job with opportunity for advancement in today’s knowledge-based global economy” (The College Board, 2006, p. ix). Not only do these standards recognize the economic value of mathematics competence, but they widely recognize the civic imperatives connected with proficiency. While the goal of “important and challenging mathematics for all students” is extraordinarily difficult to accomplish empirically, at least stabilizing it as a goal has reached consensus.

Nonetheless, the controversy underlying this history is far from over. Nor should it be, for three reasons: key targets of controversy remain unresolved, the implications of equity for standards are inadequately understood, and the Standards are not yet properly governed to permit adequate consideration of issues of measurement, modulation of change, and restriction of variation. The first two of these issues are addressed in the next section. The final issue will be discussed in the final section.

Part III: Unresolved Issues Around Content Priorities, Equity, and the Role of Pedagogy

Content Priorities. Disagreements about content priorities are often cast as debates about protecting or enhancing the level of rigor to be expected of students. While “rigor” is an important aspect of mathematics, use of this term in content debates is frequently reductive and dismissive. It can be used to disenfranchise certain legitimate parties from the debates, and it can conceal or obscure significant issues over which well-educated experts might disagree. Resolving debates over content does matter, and I would predict that a resolution will have a major impact on whether we solve the national challenges to increase the numbers of our young people in math and science and whether we will meet the challenges of hard content for all students. A tolerant climate for these debates is essential.

Key debates concern whether high school coursework should be integrated across algebra, geometry, trigonometry, statistics and probability or separated into distinct courses. Another debate centers around how to balance and sequence the introduction of formal definitions and theorems in relation to the use of a variety of contextual problems and applications. Lyn Steen (2007), professor of mathematics, former President of MAA, ex-chair of CBMS, and former executive director of MSEB, recently published an article that provides an alternative perspective on these issues which could help generate a resolution to these debates. He acknowledges the breadth of mathematical power in diverse fields:

Because of the extraordinary power, mathematics at the postsecondary level is used and taught in programs as diverse as farming and linguistics, forensics and genomics, finance and epidemiology. The number of such programs is growing rapidly as the applications of mathematics radiate outward from the physical sciences through the biological, social, behavioral and applied sciences, to the more distant humanities and fine arts. Courses in which students learn how mathematics is used constitute a stealth curriculum that thrives outside the confining boundaries of college and university mathematics departments. . . . A different but equally important perspective on what higher education expects of high school graduates can

be found under the amorphous label of numeracy or quantitative literacy. . . . [N]umeracy addresses issues such as investments, energy, health, taxes, global warming, and potential pandemics that confront citizens in the daily news or in their ordinary lives. (p. 92)

Because of the breadth of fields requiring quantitative competence, he argues that our current design of secondary mathematics, aiming all students towards calculus, is flawed and unnecessarily narrow. He suggests that we should reexamine the resulting overemphasis on algebraic manipulation in support of calculus readiness. He challenges colleagues who would characterize the path to calculus as the only means to ensure rigor in secondary school:

All areas of mathematics should be used to advance students' rigorous thinking and their capacity to create compelling arguments. It is this capacity, not the particular topics studied, that will serve students well in the postsecondary world." (ibid., p. 93)

This proposal opens the door to an *array of paths* through secondary school and, in so doing, invites students to experience mathematics multiple ways: as embedded in various contexts, as primarily a formal deductive system, or a combination of elements of both. While this proposal makes the specification of standards more difficult, it recognizes that some level of variability of course design and student selection should be supported, not to sort students, but to sort all students' pursuits of mathematics-related interests.

Thus, Steen calls for a broader participation by faculty from the partner disciplines in guiding the design of K-12 mathematics sequences, pointing to the need for other types of mathematicians, such as the "entrepreneurial mathematicians" of the Internet, or "Quants," who focus on algorithms rather than equations, and data rather than theorems.

Steen recognizes an element too often missing in the debates about the secondary school required curricula and the K-12 content standards: how to attract and retain students in mathematics and mathematics-dependent sciences at the post-secondary level. Stanching the departure of students from mathematics has to begin early, and must be accomplished through attracting students to the ideas and developing students' satisfaction in learning, instead of merely prodding them with requirements (the regulation and control strategies of standards). The solution he proposes is that "the secondary school curriculum should offer a coherent, balanced introduction to the most widely used parts of mathematical sciences in a manner that regularly connects each part with several others" (ibid., p. 92). He does not endorse integration of content topics as such but rather calls for "breadth, balance, utility, coherence and connectedness" (ibid., p. 92).

Equity. Steen's call is a novel combination of elements. He incorporates an explicit commitment to equity, the issue in mathematics and science education that is perhaps most difficult to articulate. A few facts are clear at this point. Mathematics education as practiced in schools continues to be a means of sorting and filtering students. Teachers face a vast range and distribution of student talent, preparation, interest, and effort. Current data continue to demonstrate large and persistent achievement gaps among students of different races and levels of socio-economic advantage (Grissmer & Ross, 2000; National Assessment of Educational Progress, 2005).

Gender differences in mathematics achievement have diminished due to attention to the mathematics gender gap. Race and SES differences diminished somewhat in the 1980s to a plateau in the nineties, and then crept upward again, especially evident in middle school and more so in high school (Thernstrom & Thernstrom, 2003).

Steen expresses a clear commitment to serving all students by outlining a path to civic literacy as well as advanced study. By calling for sequences of courses that attract and engage students, he goes beyond the rhetoric of “hard content for all students,” acknowledging the need for attracting and retaining students.

The question is: how do issues of equity relate to content standards? This question is in need of more attention and it unpacks to reveal the following related questions. Does cultural and socio-economic sensitivity apply as much to mathematics standards as anywhere else? Are there ways in which perspectives of the dominant majority overwhelm the interests of the non-dominant minorities when it comes to the determination of standards? Are standards agnostic with regard to diversity? Should there be different standards for different groups of students? If so, how would standards be matched with groups? Is it racist or classist to suggest that mathematics standards should be altered to meet the needs or reflect the interests of particular groups (David Klein, in press)?

It must be noted here, though, that even the best high schools tolerate the loss of too many students from advanced study in mathematics. This is a problem that crosses boundaries of race, SES, gender, and language, though its effects are felt more keenly by underrepresented or non-dominant groups. Based on Steen’s proposal, it seems at least plausible that an unnecessarily constrained, narrow path to calculus, demanding extensive algebraic manipulation, is itself a source of inequity and a constraint to accomplishment. Many students who endure, persist, and succeed with this path do so due in large measure to parental encouragement or mandate, access to suitable external resources, and clear understandings of the implications of failure on college and career choice. Others, lacking these resources, drop out or are counseled out. I believe that a diverse student body would fare better in a system with more alternative paths and with careful attention to stimulating interest and engagement.

Furthermore, in considering the development of standards and their evaluation, I propose that we should hold schools not only accountable by race, SES, gender, and language group, but by performance level, and they should be accountable for continued participation within these levels. Schools certainly must serve those very few students who are likely to be *research* mathematicians, but also those who will be “*resourceful mathematicians*,” drawing on the ideas of mathematics in pursuing related fields. Furthermore, schools should be required to assist students who demonstrate a need for *remediation*. Finally, we all know that the study of mathematics is a one-way turnstyle: many people leave, and very few return. Therefore, schools should show evidence of full programs for *recovery* of students who drop out of the pursuit of advanced mathematics into mathematics by soliciting students into the advanced courses, not simply allow narrow curricular paths or assessment systems to sort them out of mathematical competence. This suggestion is not a call for more “tracking.” In many states, to stem the temptation to lose students in order to meet performance standards, the law requires them to report drop-out rates and keep them low (although their calculation algorithm is hotly disputed). Likewise, schools should not be tempted to lose

students in mathematics and, in fact, should be rewarded for their recovery and retention. We must have incentives for inclusion and continuation if we are to broaden the pipeline and increase fairness.

Steen's call for a mathematics with "breadth, balance, utility, coherence and connectedness" (Steen, 1997, p. 92) has the potential to meet the needs of these four groups (research, resource, recovering, and remedial), and as such should be carefully considered, especially in light of the implications for building content standards at the secondary level.

Steen's proposal leads to a variety of policy consequences concerning content standards. First, we must ensure that the set of mathematicians authorized to engage in standard-setting includes the full breadth of disciplines described by Steen. Secondly, while his approach does not support nor reject what is called "integration" in secondary mathematics, it does require that as we assemble the standards, they are flexible enough to allow multiple configurations of instructional pathways. One does not want to achieve a narrow set of standards that does a disservice to the entirety of the mathematical enterprise.

The Role of Pedagogy. A third unresolved issue concerns the relationship between content and pedagogy in content standards for K-12 mathematics education. Some would prefer to see only propositional content statements included in standards. They do so because they prefer direct instruction over the constructivist approaches. Others believe that pedagogical decisions should be made only locally by teachers. Both groups suggest that the negotiated settlements about content standards should focus on content alone.

Their concerns are understandable when one examines practices that are supposedly in support of the standards by the many elementary and some middle school and high school teachers who exhibit weak content preparation. When standards incorporate pedagogy, these teachers can and often do enact solely the pedagogical practices as techniques (what some of us call the "trappings of constructivism," the use of manipulatives, small groups, and opportunities for student expression), and fail to use these practices to bring forth critical mathematical ideas. Critics cite this as evidence of the failure of standards, when it may be rather a failure of professional development.

Alternatively, restricting standards to propositional content statements renders invisible that which led to the development of concepts. For example, consider the contrast between angular speed and linear speed in the case of rotational motion. Formally, we can define angular speed as $\omega = \theta/t$ and linear speed as $v = r\theta/t$, where a point, on a circle of radius r , traverses θ radians in time t . Written as a standard it might read, "Students will learn the concepts of angular and linear speed." Most students memorize these formulae, mechanically apply them to word problems, and never encounter the richness in the ideas implicit in the formulae. Contrast this with the following: imagine a record spinning (if you recall what a record looks like), with two dots located at different distances along a single radius, one closer to the center, and one farther. Ask a group of people if the dots are moving at the same speed or different speeds. A spirited debate will follow in which some will claim they are moving at the same speed and others will claim different speeds. In the end, it depends on whether you define speed or rate as the distance traveled per unit time or as the angular rotation per unit time; and the demonstration explains why one formula (linear speed) depends on the radius and

the other (angular speed) does not. If students don't encounter the tension between these two concepts in the same frame of reference, even though they can in fact repeat the two definitions back, have they achieved this standard? Could the standard be written rather, "Students will relate and contrast the multiple meanings of speed in relation to the concepts of linear and angular speed in rotational motion and their dependence on/independence of the distance to the center, and learn when and why one or the other is useful"?

In studying student learning, mathematics educators have come to recognize the importance of creating conceptual contexts, and sequenced obstacles and challenges for students using a variety of tasks. These are not simply instructional techniques, but they are essential to making mathematics a "reasoned discipline," just as inquiry is essential in science. Reverting to propositionally logical standards, absent the insights about meaning that derive from careful study of learning, would be unfortunate and unwise. Success in mathematics would depend too heavily on high quality instructors, who are not as likely to be serving at-risk populations. As experts, we would have failed to utilize our best knowledge in writing the standards.

As standards evolve, are debated, and revised, we will increasingly move towards a better resolution of this issue of content and pedagogy if we keep in mind the reasons for negotiating a consensus among a diverse set of experts. We are increasingly able to craft language of mathematics learning that incorporates both content and pedagogy, in a more integrated way. Mathematics, we can all agree, is not solely about exhibiting performances to produce correct answers; it is a language of explanation, justification, and sense-making. Ma's (1999) notion of "profound understanding of fundamental mathematics," the examples in the PSSM standards (National Council of Teachers of Mathematics, 2000), and the discussion of research behind the standards are all contributions to a better articulation of mathematics education standards.

The work in statistics education of the past ten years represents how discipline-based academicians can work effectively with their educational counterparts to craft shared language and description. The report by the American Statistical Association presented here today, *A Curriculum Framework for PreK-12 Statistics Education* (Franklin et al., 2005), illustrates the benefits of deep and sustained cooperation. Statisticians and statistics educators have formed long-term collaborative and mutually respectful efforts. These include an organization hosting annual meetings, journals, and now the crafting of a set of educational standards. How their standards represent the fusion of content and pedagogy can be seen in their expressed learning trajectory towards understanding variation and distribution. They recognize that students' understanding of central tendency must be accompanied by a transition from attention to cases towards attention to aggregation—a process that begins with shape, and evolves to measures of spread. True, the group still faces the question of how to link their standards to mathematics standards, especially as they must rely on teachers in core disciplines to enact their proposals. Nonetheless, their work stands as an exemplar of a positive collaboration between researchers in statistics education and statisticians.

If the new generation of standards is going to adequately address the three critical issues of content priorities, equity, and a fusion of content and pedagogy, it is essential that membership on the expert panels involve a broader view of the mathematical sciences and statistics, and both pedagogical and mathematical experts. Furthermore, the committee must be charged to ensure a broad and flexible view of secondary

mathematics that will attract, retain, and assist students in recovery. The standards must lend themselves to being assembled in multiple ways in their curricular treatments.

Part IV: Next Steps in Standards: Obtaining Coherence and Permeation through New Policy Initiatives in Relation to Content Standards

Progress is occurring in the treatment of content standards in mathematics. The oscillations between the approaches are decreasing, as more and more mathematicians and mathematics educators communicate effectively with one and other, and as we are able to demonstrate evidence of the inclusion of exciting new topics and technologies into our educational program. Are we, however, making progress fast enough? Are the standards successfully serving the different needs of different populations of students? Moreover, do the standards permit us to provide teachers with the necessary instructional guidance to permit them, if given adequate professional development opportunities, to improve instruction? Despite substantial, documented progress, the answer to these three questions is no.

In describing the evolution of the use of the term *standards*, I identified three components of standards: (a) a process of authorized expert advice and negotiation leading to consensus on a set of content standards, (b) expressing standards in forms suitable for related measurement and/ or documentation, and (c) the need to modulate change and restrict variation through ongoing evaluation of the empirical effects of standards. While we have made progress on the first, only by creating a system by which we can also *authorize* a standard-setting group and *charge them* with addressing these other two components can we significantly increase our rate of progress. We must create a coherent process of forging, authorizing, evaluating, and revising standards in order to fulfill our professional obligations to our children.

My own first encounter with the concept of “coherence” in education came in the context of reading James Stigler and Michelle Perry’s (1990) analytic comparison of American classroom instruction in mathematics to Japanese and Chinese approaches. They drew upon the use of coherence in text comprehension as “the extent to which it enables or allows the comprehender to infer relations between events” (Trabasso & van den Brock, 1985, as cited in Stigler & Perry, 1990, p. 345). They applied this term to the observations of mathematics classes, concluding that classes should consist of a sequence of events related to each other and to the goals of the class. They proceeded to analyze the frequency of events and the explicitness and sense made of transition points. They asked whether “students are given the opportunity to infer coherence across the episodes that constitute their experience in mathematics class” (Stigler & Perry, p. 349). American classrooms were woefully weak in relation to this construct.

We could equally validly ask if stakeholders can discern coherence, defined as the extent to which an event structure permits a comprehender to infer relations as linked to goals, in examining our standards-based system. Keep in mind that coherence is not a quality of the system; it is rather *a quality of the interaction between participants/ observers and that system*. The event structure in which standards are embedded links standards to curricular choices, to instructional activities, and to formative and then summative assessment. Feedback, obtained as the comparison of the students’ achievement in relation to the stated standards, should provide instructional guidance for the improvement of teaching and learning. For coherence, various stakeholders must be

able to interpret those findings, judge whether the goals are being met, and attach a system of incentives and sanctions to create accountability around the attainment of specified performance levels. Coherence across this event structure doesn't assume that standards succeed; rather it provides the ability to infer the *extent to which they are succeeding*, and to revise them in light of those findings. At this time, the system is not coherent, and it is certainly not transparent.

Examining the content standards in relation to coherence brings to the fore another criterion of standards: permeation. If content standards are successful, they should permeate instructional practice. In *Designing Coherent Educational Policy*, editor Susan Fuhrman (1993) treated the concept of coherence as related to Webster's definition, "having the quality of holding together as a firm mass" (p. xi) and here "firm" is used as a description of extensiveness, density, and connectedness. Content standards need to be examined in relation to their legal authority, how they compel compliance or empower change, and whether they are reaching deeply into improving everyday practice.

Much of the debate over fuzzy math vs. the new standards is red herring. Most instruction in the United States consists of heavy reliance on traditional textbooks (Ferrini-Mundy & Schram, 1997); the reform textbooks command only perhaps 10-12% of the market by the best indicators we could obtain in conducting our NRC report *On Evaluating Curricular Effectiveness* (National Research Council, 2004). As outlined in James Spillane's *Standards Deviation: How Schools Misunderstand Educational Policy* (2004), local practice actually depends most on human sense-making and often leads to tendencies to misconstrue the intention of policy makers. Inconsistent policies, such as unaligned standards and tests, increase the distortion. His conclusion was that it is "school districts [who] are the key players between the statehouse and the schoolhouse, and in their policymaking stance on instructional issues, they do not function chiefly as the implementation arm of state or federal agencies" (ibid., p. 170). The point here is that the policy mandate does not flow downward in our educational systems—or as Spillane wrote, "If the core intent of the mathematics and science standards was to fundamentally transform what students learn and how they learn, then [Michigan's] policy initiatives were not a success. The limited influence of the standards on what is counted as mathematics and science content and doing these subjects in classrooms is sobering" (ibid., p. 173). He suggested that while some aspects of reform penetrate more easily than others (problem solving, real world applications, multiple representations), "the academic task and discourse norms appear to be especially resilient" (p. 174). Spillane advocated for both a top-down and a bottom-up analysis. He emphasized that while strong accountability initiatives, rewards, incentives, and sanctions do get district leaders' attention, practice changes only when the goals and content are understood deeply by practicing teachers. Likewise, Elmore (2002) argued that standards will be realized only when we recognize we are asking new skills of teachers, and that until we create "a reciprocity of accountability to building capacity," standards-based reform will not be realized.

These comments by policy analysts hearken back to the predictions by Porter et al. (1991), cited above, that "control strategies will sustain, because of their political capital but that empowerment strategies are necessary for success." The *No Child Left Behind* Act has the theoretical appearance of a standards-based policy, consistent with the vision of O'Day and Smith (1993). However, I would argue that what it really did was to effect a shotgun wedding of two views of standards—"standards as statements of vision of what students should learn to do" and "standards as a means of holding the system

accountable.” However, because the testing models are simply imported from the accountability movement, and appended only loosely to the standards, the opportunity to stimulate coherence has been lost. While squeezing the system this way has produced some evidence of modest improvements, progress for most schools has stalled due to this system’s institutionalized incoherence. Furthermore, in the absence of both coherence and of a means to support empowerment of practitioners, the system has operated as predicted by Porter et al. (1991), namely, its control strategies are choking its empowerment strategies, especially as policy makers try to constrain the costs and consequently under-fund professional development. An apparent marriage, built on a substrate of incoherence in relation to the alignment of standards and tests, has left practitioners in vulnerable and dysfunctional situations, especially those in urban and rural settings where threats of schools becoming low-performing are especially pressing.

Our first priority should be to fix this law so that it becomes a means to increase the permeation of content standards in instructional practice, and to seriously improve the coherence in the system. Only then will we have a system stable enough to promote progress on the other channels of influence for improvement (professional development, curriculum, formative assessment and diagnostic measures, etc.).

The success of this conference lies in the production of multiple sets of standards, each of which offers strengths and seeks to alleviate particular problems. However, at the same time, having a myriad of standards is confusing. School practitioners and state officials cannot help but be confused over what standards to use, and they do not have the time to shift through multiple national reports to decide which fit their situation. Continuous acrimonious debate on standards erodes the public trust and subjects at-risk districts to moving targets. In this final section, I outline four steps required to move the content standards to a fully realized policy initiative.

Step 1: Authorizing a National Committee. Authorize and charge a standing national committee to continue to examine and revise the standards as a means to address and adjudicate controversy in a reasoned way, respond to changes in the fields and technologies, weigh empirical evidence on effects, and identify targets that are in need of revision.⁵ The committee should be comprised of experts⁶ from the following areas of expertise: (a) the mathematical sciences and partner disciplines as described broadly by Steen (2007); (b) mathematics educators across the grade levels and with expertise on student learning and cognition, and teaching; and (c) experts on international comparisons, alternative standards, and on the links of standards to assessment systems, including innovative measures of performance. They cannot be strangers to schools. Membership would be by election or appointment from relevant professional groups. Terms would last five years. A National Mathematics Standards Committee (NMSC) would be the venue for disputes to be settled and would strengthen public trust.

⁵ Interestingly enough, a bill introduced January 7, after I first wrote the first draft of this paper, by Dodd and Ehlers ("Speak Act," 2007) argues for a similar position, the development of national standards. The Dodd-Ehlers' bill authorizes the board of NAGBY to be the agency charged with oversight. While this agency has the advantage of close ties to a national assessment, one must question if they will possess mathematics expertise demanded for a broad and balanced view of content, and for issues of pedagogy and content as discussed in previous sections.

⁶ Experts may come from the academy, business or practice. Because existing standards provide a valuable resource, they must be freed to work urgently and intensively. They must be widely read in research and in policy, represent their constituencies, and have open minds.

Such a standing national mathematics standards committee (NMSC) would be charged to begin with PSSM, the Achieve standards, the College Board standards, the ACT benchmarks, and any noteworthy state standards, etc., and devise a systematic means for holding hearings and gathering evidence on particular features of the standards over regular intervals. As with many other countries (Japan and France, to name two), the committee could cycle through close examination of certain elements of the standards (for example, by grade band, by major content area, in relation to assessments, by transitional periods [pre-K to K, elementary to middle, middle to high school, high school to college]).

While some might express concern that this Committee would not be broadly representative enough of the various stakeholders, I would argue that it should be comprised of experts in mathematics, mathematics education, assessment and evaluation of standards-based systems to maintain a consistent focus on mathematics standards, but that it should in turn report to a more broadly representative board, with a proven focus on mathematics, such as the Mathematics Sciences Education Board (MSEB),⁷ Conference Board of Mathematical Sciences, or the National Science Board. The governing board should be given direct input into the standard-making process and receive, review, and comment on the work of the committee determining if the charge to the National Mathematics Standards Board is being met.

The need for an authorized committee is urgent, as at the current time, at the secondary level, multiple non-governmental organizations have stepped in to fill the breach level by the lack of specificity of secondary standards at grade level. Their efforts are commendable to fill a documented need, whether it is to assist states in alignment or to define paths towards high levels of student success in postsecondary education. I would suggest, however, progress forward *as a nation* should not be propelled by individual organizations, nor even by professional organizations, despite the leadership shown thus far by NCTM, but by a collective determination by a nationally authorized group.

While creation of such a standing committee to stabilize the standards as a target for reform would permit progress, how the charge to the committee is written is a critical element. The charge, a key element for this proposal to succeed, to the committee should employ my more comprehensive definition of standards so that the authors take into account how to write standards with the precision required to incorporate the challenges of measurement, thus avoiding the reductive types of measures dominant today. I am not advocating a return to standards as behavioral objectives; rather I am asking that they be written with the kind of precision and specificity of meaning that points towards appropriate measurement and documentation. Moreover, the committee's work must anticipate the need to gather empirical evidence and feedback on the success of their standards and be prepared to make necessary adjustments to modulate changes and restrict unacceptable variation, on the basis of evidence.

Clearly the issues surrounding states' rights in education and the protection of local control will arise based on any proposal for national standards in mathematics. What is critical is for the public to recognize the inefficiency, waste of personpower, costs, and incoherence created by the current circumstances. States must see advantages to

⁷ The advantage of MSEB is its location in the Academy of Sciences and the process used by the National Research Council for report writing and revision.

participating in national standards, and those become clearer when one considers the next three steps, which can serve as incentives for support and participation by the states. At the same time, however, it is imperative to involve a political process, simultaneously⁸ to the development of standards, to gain purchase for the concept of national standards. Such an approach should involve the National Governor's Association, the Education Commission of the States, State Supervisors of Mathematics, and the Council of Chief State School Officers, working together to call on the government to initiate a form of national standards in the interests of the children of the country.

Step 2: Create Incentives for Better Assessments. We need to create incentives for innovation in the assessment of the content standards. With the variety of state and national standards, the lack of attention to the incoherence in the system, and virtually no accountability beyond the market and the courts in creating large scale tests, we have in fact created a very narrow, unresponsive, and expensive assessment array with too few players. Theoretical advances in testing are desperately needed (National Research Council, 2002c). Having studied the state tests from a content perspective in two states and related legal cases (Texas and Missouri), it is clear to me that the current technology of testing is not supporting reform. From a content perspective, there are problems with the lack of valid information at the level of subconstructs (when assessments are equated at the level of the whole test), with the choice of distracters (strong attractors, such as misconceptions, are viewed unfavorably in assessment development), with too much predictability of item type, and with narrow sampling of performance indicators. Disputes in the courts hinge on measures of reliability, but issues of validity and fairness (opportunity to learn) are neglected because they lack mathematical definition. The method for setting performance level cut points is pseudo-scientific at best, and our refusal to use sampling techniques instead of obtaining a score for each individual student leads to a narrowing of the curricula. Finally, while the phrase "data-driven decision making" resounds, the "data-based" reports are typically weak, most data are slow to reach the practitioners, and practitioners lack the capacity to use the data.

Without stabilizing standards and writing them with careful incorporation of these issues, we cannot hope to make marked, durable progress in our mathematics education system. Now is not the time for a national test. It is the time, however, to catalyze and reward creativity and innovation in testing.

However, as we create these incentives to create assessments for the standards, we need a correct balance of competition and collaboration. The tests need to take into consideration not only what to test, but how to test and how to deliver data in a timely and understandable way to school practitioners and the public. We need the kind of attention to assessment as a part of a work productivity toolkit for educators, not just narrowly conceiving of the assessment as a test score.

⁸ While it would be preferable to get political support first and then convene the committee, the urgency of the situation suggests a parallel process would be wiser.

Step 3: Invent Rigorous⁹ Independent Methodologies for Assessing Alignment.

Methods of independently analyzing alignment between content standards and assessment systems over time are a key element of creating accountability for test makers. The American Federation of Teachers (AFT, 2006) wrote, “Without strong, clear state content standards and tests aligned to them, state-level testing is compromised and the results suspect” (p. 2). According to their research, only 11 states have strong content standards and transparent alignment, 52% of state administered tests (n = 833 tests) are aligned to strong standards, and 15% of states failed in all grades and subjects. This raises the question of whether states lack motivation, capacity, or funding to undertake this critical task.

One proposal for assessing alignment (Rothman et al., 2002) has been suggested by Achieve consisting of four dimensions: content centrality, performance centrality, challenge, and balance and range. Another approach has been proposed by Webb (1999). Currently, the instability in the system makes it nearly fruitless to examine the alignment. Without the ability of knowledgeable practitioners to assess alignment, it is not possible to bring coherence to the mathematics educational system. There is a clear role for the NMSC in examining the quality of the alignment methodology; the benefits of doing so would flow bidirectionally. That is, an alignment analysis by the authorized experts in the standards-writing process would help ensure that the intentions of the standards are realized, and involvement in the process would help refine the standards. Furthermore, creating a rigorous means of assessing alignment would provide a way to assist the states in an educational activity that is currently being overlooked or, at best, practiced irregularly, in a way that does not infringe on the federal-state separation of educational responsibilities. One final point is that the activity of examining alignment is an excellent means to strengthen local capacity, because it provides a way for practitioners to learn about testing not for the purpose of “teaching to the test” but for the purpose of developing concrete understanding of the meaning of the standards as exemplified in assessments.

Step 4: Empirical Analysis of the Effects of Standards. Only once the first three steps are in place would it be possible to rigorously and systematically examine the effects of standards on student learning. In the end, to resolve many of the key controversies outlined in this paper, with regard to content dimensions, equity, and content and pedagogy, this final step of empirically analyzing the effects of the standards would be necessary. By stabilizing the standards, and linking them actively to better measurement, and a more coherent accountability system, we make progress in this system possible.

What is the scope of such an analysis? Findings would tell us if students are improving in learning what is articulated in the standards, and not necessarily identify other factors in the system that are associated, required, constraining, or facilitating those results. It would not show us whether the accountability system itself, setting performance standards, rewarding or sanctioning schools were effective, but it would tell us if the

⁹ While I previously argued that the term “rigor” is used too often to disenfranchise certain constituencies, I use it intentionally here to indicate a need to define rigor in the context of stabilized standards and innovations in assessment. I see it as a constructed methodology dependent on the involvement of the full chain of experts involved in delivering standards-based reform.

material is being learned, by whom, and to what extent. It would provide information to the NMSC on where performance is lagging and where it is progressing.

The charge to the NMSC would be to assess these issues, just as we now rely on NAEP to do so. By contrast with NAEP, however, the information would be far more detailed within the discipline of mathematics, and would extend the full range of the K-12 spectrum.

The first target of that should be to ensure that from the well-aligned system, it is possible to draw conclusions from the data that provide real instructional guidance to teachers. With a system thus aligned and coherent, we could assess the effectiveness of curricular materials and focus on the fundamental task of providing teachers the professional educational preparation and continuing education to do their jobs more effectively.

Part V: Conclusions

I have outlined three components of standards—

1. negotiation among authorized experts to identify the content students should learn and be able to do,
2. careful consideration of measurement and documentation needs, and
3. attention to how to modulate change and restrict the variation.

I tracked the history of the development of mathematics standards in the United States and linked it to other major initiatives, in particular the policy initiatives striving to achieve coherence in standards-based education. I have argued that we have made progress as evidenced by the decrease in intensity of the oscillation in approaches, forging agreement on numerous topics, and establishing consultative processes. A few key issues need further attention, particularly how to broaden the content dimensions of secondary education, how to improve our commitment to equity, and how to link content and pedagogy properly. Despite that progress, I have argued that we have not moved fast enough or comprehensively enough, based on data on international comparisons and a failure to be making enough progress on achievement gaps.

To increase the rate of progress, improve coherence in the system, and increase levels of permeation, I argued for a set of policy initiatives including the creation of a National Mathematics Standards Committee (NMSC) composed of mathematicians, inclusive of the partner disciplines, mathematics educators and assessment experts that would answer to a broader set of stakeholders such as MSEB. I further have advocated that this committee: (1) create a regular process for review of standards, (2) create incentives to improve assessment models, (3) develop rigorous methodologies for assessing alignment between national standards and those assessments, and (4) evaluate empirical data on student learning once the coherence of the system is established and revise standards as indicated. The reauthorization of NCLB provides an occasion to launch such an initiative. My hope is that the audience convened for this occasion might take up some such set of proposals, revise and improve them, and advocate strongly for what is needed to better serve the American children.

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**ANALYSIS OF CURRICULUM
RECOMMENDATIONS**

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with

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Some Common Themes and Notable Differences Across Recent National Mathematics Curriculum Documents

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Each of the national curriculum documents featured at this conference represent a major contribution to the treatment of content standards for school mathematics. The targeted content areas and grade level bands differed from document to document.

Because of the differing nature and intent of the four documents, caution was used in making direct comparisons. In particular, unlike the other documents, NCTM's *Focal Points* is intended to suggest major topics or areas of concentration within a particular grade. These points of emphasis do not represent a full curriculum for each grade.

For this conference, the curriculum documents from Achieve, the American Statistical Association, the College Board, and the National Council of Teachers of Mathematics were examined and compared at several levels. We include here:

- A framework permitting global comparisons of the documents in terms of scope and sequence of recommended mathematics and/or statistics content to be learned
- A summary of global similarities and differences
- A brief summary of common themes and notable differences that emerged from the analysis

Global Comparisons

A framework for making broad comparisons of the documents articulating curriculum standards/learning expectations for school mathematics: *College Board Standards for College Success in Mathematics and Statistics* (College Board), *Curriculum Focal Points for Prekindergarten through Grade 8: A Quest for Coherence* (National Council of Teachers of Mathematics), *Guidelines for Assessment and Instruction in Statistics Education* (American Statistical Association), and *Secondary Mathematics Expectations* (Achieve, Inc.) is provided in Table 1. A visual scan of the framework reveals similarities and differences in grade band focus, organization, and scope and sequence of recommended mathematics and/or statistics content to be learned.

Examination of the documents by each member of the analysis group led to identification of global similarities and differences as reported in Table 2.

Table 1

A Global View of Recent National Curriculum Recommendations by Document, Strand, and Level

Document	Level	Number & Operations	Algebra	Geometry & Measurement	Statistics & Probability
Achieve ¹ 7-12	A	Rational numbers Absolute values Prime decomposition, factors and multiples Data Measurement Derived quantities and measures	Expressions Functions Linear functions Proportional functions Equations and identities Linear equations <i>Sets and Boolean algebra</i>	Angles and triangles Rigid motions and congruence Perimeter and area	Simple probability Relative frequency Linear trends
	B	Estimation and approximation Quantitative data Exponents and roots Real numbers	Quadratic functions Simple quadratic equations Linear equations in two and three variables Linear inequalities <i>Iteration and recursion</i>	Angles in the plane Coordinates and slope Pythagorean theorem Circles Similarity Visual representations Geometric constructions Length, area, volume <i>Discrete graphs</i>	Compound probability <i>Permutations and combinations</i> <i>Discrete graphs</i>
	C	Number bases <i>Algorithms</i>	Elementary functions Polynomial functions Polynomial and rational expressions and equations	Geometry of a circle Scaling, dilation, dimension Axioms, theorems, proofs <i>Mathematical reasoning</i> <i>Propositional logic</i>	Probability distributions Correlation and regression
	D	Complex numbers	General quadratic equations and inequalities Nonlinear equations and expressions	Triangle trigonometry Coordinates and transformations Three-dimensional geometry	Surveys and sampling Risk and decisions
	E	Argand diagrams <i>Quantitative applications</i> <i>Digital codes</i>	Trigonometric functions Matrices and linear equations Operations on functions Inverse functions Relations <i>Sequences and series</i> <i>Recursive equations</i> <i>Mathematical induction</i>	Spherical geometry <i>Vectors</i> <i>Conic sections</i> <i>Proof by contradiction</i>	Advanced probability Cross-classified data Statistical reasoning Statistical inference

¹The Achieve document includes a separate Discrete Mathematics strand. For comparison purposes, recommended discrete mathematics topics have been italicized and placed in the related cross-documents strand at the appropriate level. See, for example, *iteration and recursion*, in the Algebra strand at Level B.

Document	Level	Number & Operations	Algebra	Geometry & Measurement	Statistics & Probability
College Board 6-12	Middle School I	Nonnegative rational numbers and concepts of integers Ratios and rates	Linear patterns and relationships	Two-dimensional geometry and measurement	Univariate data analysis Experimental and theoretical probability
	Middle School II	Integers and rational numbers	Linear equations and inequalities	Two- and three-dimensional geometry Similarity and measurement	Bivariate data Probabilities in one-stage experiments*
			Patterns of change and algebraic representation Variables, expression, equations, and functions in linear settings Nonlinear expressions, equations, and functions		Surveys and random sampling
	Geometry			Geometric reasoning, proof and representations* Similarity and transformations Direct and indirect measurements	Two-stage experiments, conditional probability, and independence*
	Algebra II		Polynomial expressions, functions, and equations Exponential, logarithmic, and other functions Systems of equations and inequalities and matrices		Experiments, surveys, and observational studies Permutations, combinations, and probability distributions*
	PreCalculus		Properties of families of functions Trigonometric functions Conic sections and polar equations Structures of sequences and recursion* Vectors and parametric equations	Trigonometric functions Conic sections and polar equations	Bivariate data and trend-line models

* Standard incorporates discrete mathematics content

Document	Level	Number & Operations	Algebra	Geometry & Measurement	Statistics & Probability
Focal Points PreK-5	PreK	Developing an understanding of whole numbers, including concepts of correspondence, counting, cardinality, and comparison		Identifying shapes and describing spatial relationships Identifying measurable attributes and comparing objects by using these attributes.	
	K	Representing, comparing, and ordering whole numbers and joining and separating sets		Describing shapes and space	
	Grade 1	Developing understandings of addition and subtraction and strategies for basic addition facts and related subtraction facts Developing an understanding of whole number relationships, including grouping in tens and ones		Composing and decomposing geometric shapes	
	Grade 2	Developing an understanding of the base-ten numeration system and place-value concepts Developing quick recall of addition facts and related subtraction facts and fluency with multidigit addition and subtraction		Developing an understanding of linear measurement and facility in measuring lengths	
	Grade 3	Developing understandings of multiplication and division and strategies for basic multiplication facts and related division facts Developing an understanding of fractions and fraction equivalence		Describing and analyzing properties of two-dimensional shapes	
	Grade 4	Developing quick recall of multiplication facts and related division facts and fluency with whole number multiplication Developing an understanding of decimals, including the connections between fractions and decimal		Developing an understanding of area and determining the area of two-dimensional shapes	
	Grade 5	Developing an understanding of and fluency with division of whole numbers Developing an understanding of and fluency with addition and subtraction of fractions and decimals		Describing three-dimensional shapes and analyzing their properties, including volume and surface area	

Document	Level	Number & Operations	Algebra	Geometry & Measurement	Statistics & Probability
Focal Points 6-8	Grade 6	Developing an understanding of and fluency with multiplication and division of fractions and decimals Connecting ratio and rates to multiplication and division	Writing, interpreting, and using mathematical expression and equations		
	Grade 7	Developing an understanding of and applying proportionality, including similarity	Developing an understanding of operations on all rational numbers and solving linear equations	Developing an understanding of and using formulas to determine surface areas and volumes of three-dimensional shapes	
	Grade 8		Analyzing and representing linear functions and solving linear equations and systems of linear equations	Analyzing two- and three-dimensional space and figures by using distance and angle	Analyzing and summarizing data sets

Document	Process Component	Level A	Level B	Level C
GAISE ¹ PreK-12	Formulate Question	Beginning awareness of the statistics question distinction Teachers pose questions of interest Questions restricted to classroom	Increased awareness of the statistics question distinction Students begin to pose their own questions of interest Questions not restricted to classroom	Students can make the statistics question distinction Students pose their own questions of interest Questions seek generalization
	Collect Data	Do not yet design for differences Census of classroom Simple experiment	Beginning awareness of design for differences Sample surveys Begin to use random selection Comparative experiment Begin to use random allocation	Students make designs for differences Sampling designs with random selection Experimental designs with randomization
	Analyze Data	Use particular properties of distributions in context of specific example Display variability within a group Compare individual to individual Compare individual to group	Learn to use particular properties of distributions as tools of analysis Quantify variability within a group Compare group to group in displays Acknowledge sampling error Some quantification of association Simple models for association	Understand and use distributions in analysis as a global concept Measure variability within a group Measure variability between groups Compare group to group using displays and measures of variability Describe and quantify sampling error Quantification of association Fitting of models for association
	Interpret Results	Do not look beyond the data No generalization beyond the classroom Note differences between two individuals with different conditions Observe association in displays	Acknowledge that looking beyond the data is feasible Acknowledge that a sample may or may not be representative of larger population Note difference between two groups with different conditions Aware of distinction between observational study and experiment Note difference in strength of association Basic interpretation of models for association Aware of the distinction between association and cause and effect	Are able to look beyond the data in some contexts Generalize from sample to population Aware of the effect of randomization on the results of experiments Understand the difference between the observational studies and experiments Interpret measures of strength of association Interpret models for association Distinguishes between conclusions from association studies and experiments
	Nature of Variability	Measurement variability Natural variability Induced variability Variability within a group	Sampling variability Variability within a group and variability between groups Co-variability	Chance variability Variability in model fitting

¹The American Statistical Association GAISE document focuses only on the statistics and probability strand

Table 2

Global Similarities and Differences

	Achieve	College Board	Focal Points	GAISE
Grades	7–12	6–12	PreK–8	PreK–12
Organization	By content strand and levels (A–E) representing progressions of mathematical content by increasing degrees of complexity	By course and content clusters representing developmental progressions for student performance	By grade and strand representing developmental progressions	By process components of statistical practice (and variability) and levels (A–C) offering learning trajectories
Differentiation	Levels A–D for all students; Level E–advanced, elective topics	Six courses for all students; points to AP Statistics and/or AP Calculus as electives for accelerated students	All students	All students
Percent of Expectations by Strand	Number 18% (52) Algebra 36% (104) Geometry 28% (81) Statistics 18% (52) Total n = 289	Number 12% (36) Algebra 41% (118) Geometry 24% (71) Statistics 23% (65) Total n = 290	Number 53% (16) Algebra 10% (3) Geometry 33% (10) Statistics 4% (1) Total n = 30	Number 0% (0) Algebra 0% (0) Geometry 0% (0) Statistics 100% (44) Total n = 44
Most Frequent Verbs	Know (84) Identify (73) Explain (64) Total n = 1,116	Solve (50) Apply (44) Identify (39) Total n = 680	Solve (36) Understand (32) Apply (16) Total n = 326	Understand (10) Conduct (5) Analyze (4) Total n = 73
Technology Tools	Judicious use Technology tools cited in 16 expectations	Flexible use Technology tools cited in 18 expectations	No mention	Implicit use

Content Analysis

Because of the nature of the documents, the content analysis focused on the algebra, geometry, and statistics strands. The primary unit of analysis is given below for each document.

- Achieve—Performance expectation including bullets
- College Board—Performance expectation
- Focal Points—Description of focal point
- GAISE—Objectives of process levels A, B, and C

Within each strand, emerging categories were identified (see Table 3), performance expectations were coded by category, and subcategories were identified. Subcategories within the strands were analyzed to discern commonalities and differences across the documents. All language was considered.

Table 3

Content Analysis Categories

	Algebra	Geometry	Statistics & Probability
Categories	Variables & Algebraic Expressions	Shape	Univariate Data
	Equations, Inequalities & Systems	Relationships	Bivariate Data
	Functions	Transformations	Experiments
	Matrices	Coordinates	Surveys & Sampling
	Sequences & Recursion	Justification & Proof	Probability

Details of commonalities and differences across the documents that emerged from the analysis of these categories are available at <http://cltnet.org/cltnet/misc/csmcmath07/agenda.html> (Hirsch PowerPoint presentation).

Common Themes and Differences

Treatment of Algebra

Algebra is the dominant strand in the Achieve and College Board documents. In the Focal Points document, algebra is a “focal point” in grades 6 and 8 and a “connection” to the focal points in earlier grades.

Achieve focuses on the structure of algebra with an emphasis on developing students’ procedural fluency and their ability to characterize and understand quantitative relationships that can be modeled by functions.

College Board characterizes algebra as a way of using symbols to represent mathematical and real-world situations and functions as a way to model patterns of change. The focus is on symbol sense and symbolic reasoning. College Board makes explicit connections between functions and data analysis.

Focal Points highlights algebra as an extension of the study of properties of number and operation and as a tool for developing rules to represent quantitative relationships and describe number patterns and numerical relationships.

Treatment of Geometry

Geometry and measurement are prominent in the Focal Points, Achieve, and College Board documents. Each document reflects the continuing influence of Euclid’s organization of the subject.

Focal Points emphasizes the measurable aspects of shape, focusing on two-dimensional shapes prior to grade 5. Strategies for analyzing shape and relationships are closely integrated with the study of number. After grade 5, there is a stronger expectation that students will provide justification for statements that emerge from analysis.

Achieve emphasizes geometry as an axiomatic system, with focus on synthetic and transformation perspectives. Students are expected to know, understand, and apply definitions and relationships to solve problems and prove theorems.

College Board provides a balanced treatment of geometry from multiple perspectives, including synthetic, coordinate, transformations, and vectors. Students are expected to develop and apply properties and relationships involving shape while constructing geometric ways of thinking and justification in the context of semilocal axiomatics.

Treatment of Statistics and Probability

Statistics and Probability is strongly represented in all documents but Focal Points.

Achieve expects students to study standard statistical techniques and procedures, but powerful statistical ideas (e.g., categorical data analysis, experimental design) are reserved for a smaller number of advanced students at Level E. The document tends to focus on mathematical statistics over statistical inquiry.

College Board recommendations more closely reflect a statistical perspective. It is expected that all students experience powerful statistical concepts and tools across grades 6–12 with a focus on the development of concepts and sense-making rather than theory and procedures.

In Focal Points, data is primarily used as a vehicle to develop number sense as opposed to statistical reasoning, literacy, or problem solving. “Distribution” as an important construct is absent (although “see data as an aggregate” appears in 8c). Variability is inferred through the range and quartiles, but no language of within or between group variability is mentioned.

The ASA GAISE report offers a statistical inquiry perspective and provides tremendous content and pedagogical support for educators; understanding variability and distribution are major foci in GAISE.

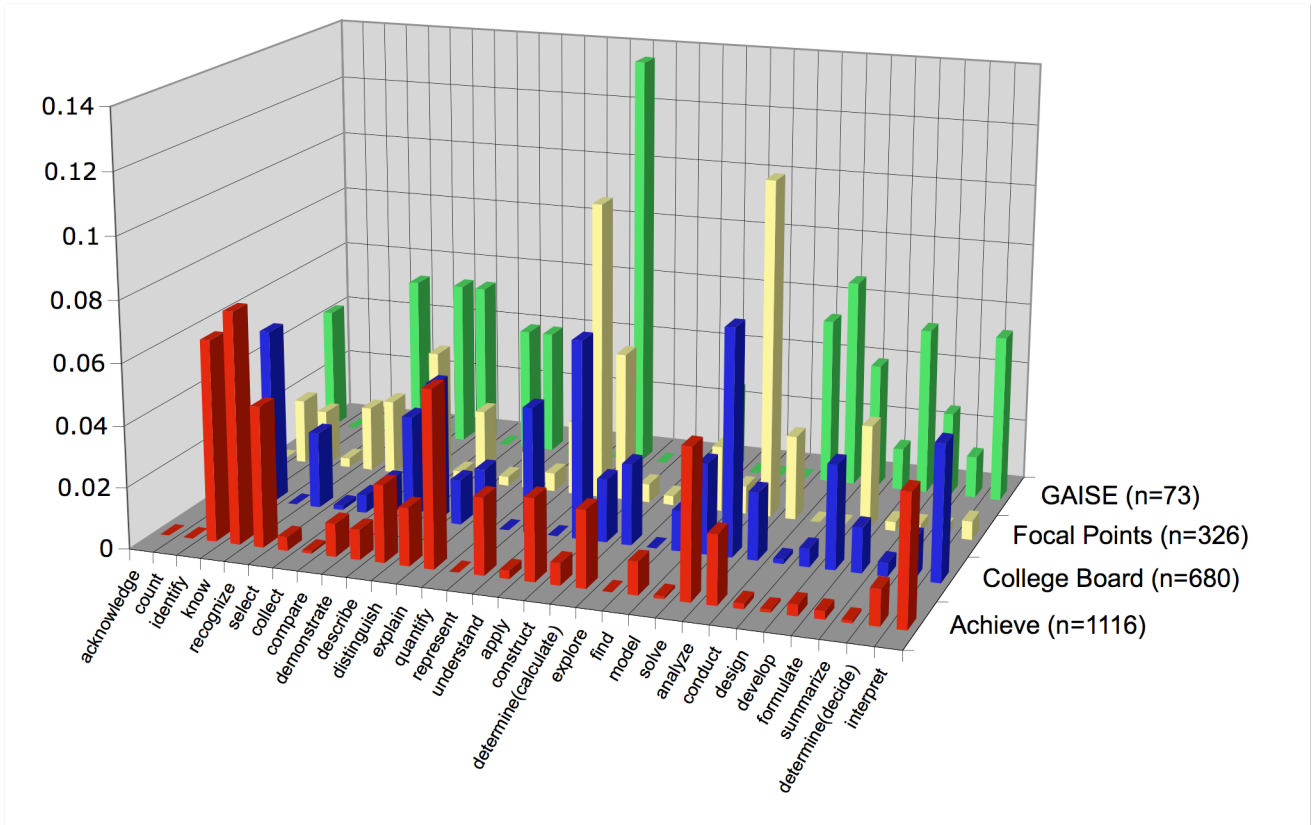
Verb Analysis

In addition to examining the documents in terms of what mathematics and statistics students should learn and when, we conducted a verb analysis as an indication of how the documents intended that students would “know” the recommended content.

For each document, student action verbs were counted using the primary unit of analysis. Following calculation of the relative frequency of verb use by document, commonly used verbs were identified. (See Figure 1.)

Figure 1

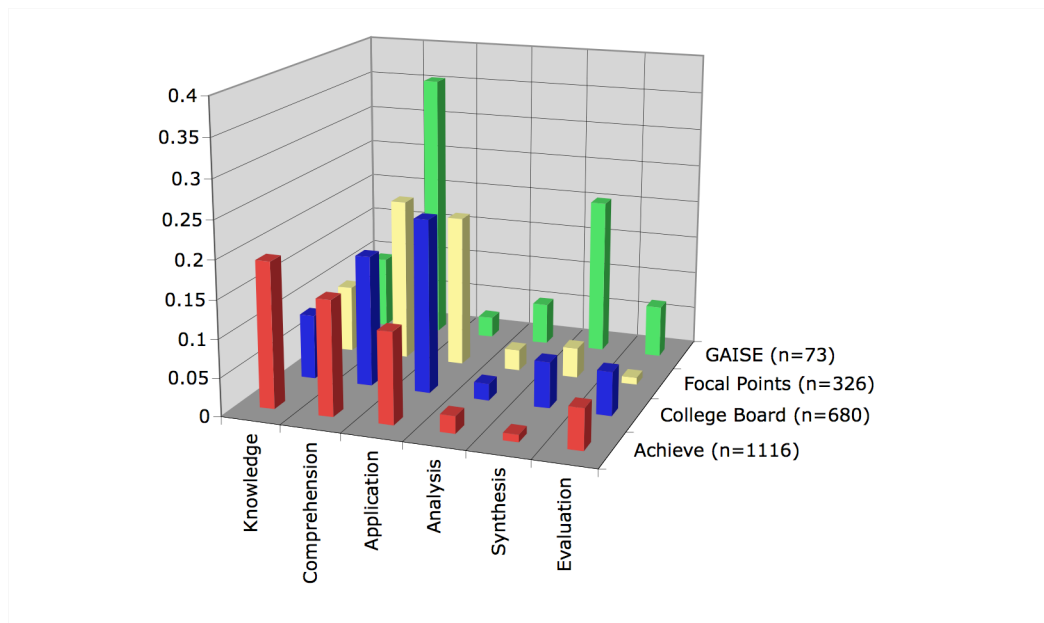
Most Frequently Used Verbs (≥ 0.02 relative frequency in any document)



Next, commonly used verbs were coded by cognitive demand based on Bloom's Taxonomy (consensus by committee). Results are summarized in Figure 2.

Figure 2

Cognitive Demand—Most Frequently Used Verbs



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CLOSING COMMENTS

Joan Ferrini-Mundy

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K-12 Mathematics: What should students learn and when should they learn it: *Challenges, Good Bets, and Context*

Joan Ferrini-Mundy

I'm delighted to be here. I've found this conference fascinating from lots of different vantage points, including my experience with a variety of standards efforts and from my twelve days of experience at the National Science Foundation. I've organized my comments around challenges, good bets, and context. For the challenges and good bets part, my comments are drawn from my professional experience and my own professional judgment rather than from my current role at NSF.

Challenges

There are at least four areas of what I describe as challenges related to the theme of this conference (capacity, consensus, connections, and competition). While these won't be surprises for most of the audience, I wanted to make explicit note of them.

Capacity. The amount of capacity (nationally) that has been and continues to be devoted to the development of standards is quite striking—not only related to the work described at this conference but at the state level as well. The expertise that is required and the time that is being devoted to generating standards is considerable. Wouldn't it be nice to have this done? Continuing this kind of work locally and also in a variety of national organizations takes many people away from other kinds of work that we might argue are equally important and equally compelling.

Consensus. In the examples of national standards-like documents highlighted at this meeting, although consensus was central to the work, it didn't overtake the work. That is to say, in the end, the experts that worked on these documents had the final word, they had to come together, but in relatively small groups without necessarily going through the work of trying to poll the entire field on all points and to try to get consensus. I think that's a strength of the process. A level of professional consensus is necessary in my view, but to assume that we could ever get to a place where we would have complete agreement would be a faulty way to go about the work.

Connections. There are a number of types of connections that were raised and discussed during this conference. Jere Confrey made an important point regarding the language of standards, arguing that content and pedagogy have a place. The example Jere shared underscored the notion that terse statements of what students should know really do mask an awful lot of important process and development, ideas leading up to a central concept. I think we need to pay careful attention to this message.

There's also a connection between standards writers and practitioners that I think is sometimes lost in these kinds of efforts. We've heard a number of speakers point to the importance of engaging folks who work day-to-day with children, with instructional

materials in classrooms, to be sure that there is a reality check on the kinds of work that we're doing.

And then finally, if standards documents include more terse, streamlined statements, we must figure out how to show relationships of core concepts to other concepts that connect to them in important ways, in ways that can motivate the mathematics and in ways that enrich students' learning.

Competition. I also worry that a type of "competition" may seep in and out of our conversations as we talk about creating standards. I hope that we can view the examples highlighted at this meeting as four complementary documents, in a sense, that can be used in some combined way rather than choosing particular organizations to pay attention to the most. I fear that we don't have the space to compete about whose standards will prevail.

Good Bets

Now some good bets regarding the work. It's one thing to have challenges, but we need to know where we can look and how we can think in a positive way about where we're headed. Jim Hiebert (1999) wrote, "How nice it would be if we could look at the research evidence and decide whether the Standards are right or wrong. That ability would make decisions simple and bring an end to the debates about the direction of mathematics education in the United States." This raises the question, how does research interface with the work that was described here? Jim goes on in that same paper to say what research cannot and can do. Research can't, in Jim's view, select the standards. It can't tell you what needs to be in the document and what doesn't, directly. Nor can it prove what is best, although I'm thinking that in the years since Jim wrote this paper we've actually made headway on both of those points. He does point out that research can prevent us from making unwitting mistakes and show us what's possible and what looks promising.

What role can the research community, including the National Science Foundation, serve in the work described in this conference? Let me illustrate a few areas where research can show us what's possible, what looks promising, and probably prevent us from making unwitting mistakes, maybe even help us select the standards and prove what's best.

Learning Progressions. One line of work that is being promoted in NSF's DRK12 solicitation is the area of learning progressions. Learning progressions are defined by a group of science educators as "descriptions of successively more sophisticated ways of reasoning within a content domain, based on research syntheses and conceptual analyses." Ongoing work in this area blends an understanding of alternative logical sequences within the discipline, when there isn't one certain way that concepts unfold, coupled with empirical work about students and about learning. These ideas are then tested out with sequences of instructional materials or design where we can get a look at the ways in which student learning might grow across a particular trajectory or a particular progression, based on interactions among students, teachers, and materials. This work is promising and is crucial in the on-going work developing standards.

The *Curriculum Focal Points* from NCTM try to suggest the ways in which concepts grow over the grades. In the development of *Principles and Standards for School* there are a

series of charts that show, by grade band and then by sub-topic within the standards, how mathematical ideas might progress across the grades. Those charts, however, are confined only to what we call the content standards. The writers couldn't figure out how to develop progressions across the grade bands for the process standards. We couldn't figure out how the sub-components of reasoning at grades PreK–2 might then progress to something more complicated at grades 3–5 and so forth, in part because there wasn't a research base that would adequately guide the work. So, there is certainly work to be done here so that guidance is provided for better specification of performance levels, or developmental levels, as we saw in the GAISE document. The notion that we can look across the grades or across a concept and specify quite particularly how mathematical ideas can grow, given appropriate instructional attention. My guess is that there are alternate progressions, even within major areas of mathematics.

Back-mapping. A second kind of work that was described by both the College Board and Achieve, is the notion of back-mapping, of taking a look at the mathematical demands somewhere downstream and trying to think about what it would take to get to those levels and how they can be articulated within standards. That seems to me to be a crucial way to continue to build our work and where we need help from research. Finally, I think alignment studies with sophisticated tools that let us compare different types of documents—instructional materials, standards, assessments—can help us see where there might be problems of incoherence and lack of focus.

What is still needed? So suppose we, or someone, decides what students should learn and when they should learn it. Suppose there is something created like national standards. The last panel, I think, did an admirable job of raising for us the issues on all sides of that question, so I wanted to just sort of jump over it a bit and say, "Then what?" Where would we be? What might we still need?

My own personal, professional view (not necessarily the view of NSF) is that we would still need research and theory-based tools and materials for learning. There will be a continuing need to innovate, to work with learning progressions, to test out sequences of ideas and materials and learn from them and continue to refine them. There will also be a need for research and theory-based guidance for teacher development. We've heard a bit about the concerns about teachers' engagements with standards and with instructional materials and we continue to need attention to these questions. This work should be tightly tied to particular areas of the subject matter, to the content so that we can understand in some depth what's involved when teachers work with children and with materials to advance mathematical ideas.

I mentioned earlier the need for more sophisticated tools for studying alignment. Checking against various sorts of lists of curricular topics is crucial and seems to me to be a first step. There are folks who are working on frameworks that look at performance expectations, or aspects of conceptual development, or other pedagogical matters and I think that having these kinds of tools available will help as things move forward. I think improved assessments are needed to measure the depth of understanding that is either expressed or implied in standards. It is crucial that assessments keep pace with the thinking that underlies the standards documents highlighted at this conference.

And then, finally, we need to watch and understand what happens when we try to use standards documents with teachers, school administrators and curriculum leaders. How

are they interpreted? What is made of them? Randy Charles' example of teachers "unpacking" standards is of particular concern. This needs to be taken into consideration as we continue our efforts to develop standards and to design them so they are useful and communicate accurately to practitioners.

Context

There are a few things about context that I would also like to mention. The role and impact of standards need to be considered in the context of other activities that bear upon the work. For example, we need to stay tuned to the discussions around the reauthorization of No Child Left Behind. As noted in this conference, proposed legislation is calling for national standards. You should also be aware of the work of the National Mathematics Advisory Panel. The website is kept current and you might take a look at work they are considering as they formulate recommendations. The group is broken into four sub-groups working on are: conceptual knowledge and skills, learning processes, instructional practices, and teachers and teaching. The committee is reviewing research and beginning the work of synthesizing and drawing conclusions from that research. So, watching the outcomes of that activity, I think, will be important for the kinds of things that have been discussed here.

It's also important to keep an eye on the discussions about higher education. The Spellings Commission has released its report and there are other reports and activities underway—the National Science Board-generated commission on the 21st century STEM education. As these commissions, committees, and task forces come forward with their recommendations, they are likely to have implications for K–12. Discussion of standards and accountability for higher education are underway. Someone recently commented on this general higher ed discussion, "You know, if there's a strong move towards national assessment or national standards in higher ed in mathematics, that would freeze the curriculum at K–12 in interesting ways." Again, all worth watching, but it's part of the context that I think is really important for the sorts of work that we are doing together.

About the Center for the Study of Mathematics Curriculum (CSMC)

The Center for the Study of Mathematics Curriculum, funded by the National Science Foundation in 2004, is engaged in a coordinated plan of scholarly inquiry and professional development around mathematics curriculum, examining and characterizing the role of curriculum materials and their influence on both teaching and student learning. The goal is to engage in systemic research to illuminate the essential features and characteristics of curriculum materials and related teacher support that contribute to increased student learning.

Major areas of CSMC work include understanding the influence and potential of mathematics curriculum materials, enabling teacher learning through curriculum material investigation and implementation, and building capacity for developing, implementing, and studying the impact of mathematics curriculum materials.

Principles that guide the work of CSMC:

A well-articulated, coherent, and comprehensive set of K-12 mathematics learning goals/standards is necessary to large-scale improvement of school mathematics.

Mathematics curriculum materials play a central role in any effort to improve school mathematics and that their development is a scholarly process involving a continual cycle of research-based design, field-testing, evidence gathering, and revision.

Teaching and curriculum materials are highly interdependent and increasing opportunities for student learning rests on better understanding the relationship between curriculum and instruction.

Research addressing mathematics curriculum can inform policy and practice and in so doing narrow the gap between the ideal and the achieved curriculum.

Center partners:

Michigan State University University of Missouri Western Michigan University University of Chicago	Horizon Research, Inc. Grand Ledge MI Public Schools Kalamazoo MI Public Schools Columbia MO Public Schools
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Website:

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