Solving America's Mathematics Education Problem

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Executive Summary

American students test poorly in mathematics compared to those in other developed—and in some cases, less developed—countries. While we have seen some signs of improved performance in recent years, these improvements are not yet evident among high school students. And the proportion of new college graduates who majored in math-intensive subjects has declined by nearly half over the past sixty years. Will the United States lose its edge in innovation as the math skills of our elite students atrophy? Will the average worker possess the training necessary to take advantage of technically demanding twenty-first-century job opportunities? Most important, why has the United States lost ground, and what course must we follow to gain it back?

This report summarizes recent research that yields important insights into America's mathematics problem. Stated succinctly, the root of the problem is an excessive emphasis on equality in curriculum. Given the inherent variability in students' math aptitude, equity can be achieved only by delivering a suboptimal education to at least some students.

A recent policy initiative undertaken by one of the nation's largest and most successful school districts, Charlotte-Mecklenburg (North Carolina), illustrates the hazards of math acceleration. In 2002, the district joined a growing number of education agencies in promoting eighth grade algebra for a larger proportion of students. The push to accelerate algebra was based on a naïve interpretation of correlations between algebra timing and later success, ignoring the obvious counterargument that a propensity for future success drives early algebra taking, not the reverse. However ill-conceived the policy, though, the results are instructive:

- In the span of two years, Charlotte-Mecklenburg students performing below average in math witnessed threefold increases in the likelihood of taking Algebra I by eighth grade.
- Students subjected to algebra acceleration scored thirteen percentile points lower on a standardized end-of-course test than students permitted to take algebra on a regular schedule.
- Accelerated students were less likely to pass an end-of-course test in Geometry, despite receiving an extra year to do so. They were no more likely to pass an end-of-course test in Algebra II.

A more thorough review of curricular trends in high school mathematics over the twentieth century shows that the Charlotte-Mecklenburg experience is not a fluke. Since the beginning of the twentieth century, waves of reform, including the "new math" movement, have sought to improve the math achievement of moderate-performing students. The emphasis on the performance of lower-achieving students increased after the 1983 *A Nation At Risk* report and the 2001 passage of the No Child Left Behind Act. Recent studies have verified an obvious side effect of this focus: declining achievement among higher-performing students. The past thirty years have witnessed a twenty-point increase in average math SAT scores but a 25 percent drop in the proportion of college students who major in math-intensive subjects.

Altogether, the evidence suggests that America's math wounds have been self-inflicted, illustrating the hazards of a single-minded focus on relative rather than absolute performance. Closing the achievement gap by improving the performance of struggling students is hard; closing the gap by reducing the quality of education offered to high performers—for example, by eliminating tracking and promoting universal access to "rigorous" courses while reducing the definition of rigor—is easy. The thoughtless incentives often provided to close the gap make the path of least resistance even more tempting.

This report concludes with a series of prescriptions for ensuring forthcoming generations of American workers will include both innovators who create jobs in technically demanding industries and workers qualified to hold them:

- For several decades, the United States has counteracted its decline in math in part by importing highly talented immigrants. American immigration policy prioritizes family reunification over skills, in direct contrast with peer nations such as Australia and Canada. Any attempt at immigration reform should address this issue.
- Curricular fads such as Singapore math hold promise in many circles but may not be readily adaptable to American cultural and educational settings. Experimentation is warranted, but we must be mindful that the net effect of our past curricular tinkering has been negative.
- Pursuing equity in curriculum must harm some students, and evidence suggests that some past reforms have managed to harm all of them. American students are heterogeneous, and a rational strategy to improve math performance must begin with that premise.

Introduction

In the twenty-first-century workplace, mathematical ability is a key determinant of productivity. College graduates who majored in mathematically intensive subjects—math, engineering, and physical sciences—earn an average of 19 percent more than graduates who majored other fields, according to the American Community Surveys of 2009 and 2010.[1] Precollegiate mathematical ability matters as well: math SAT scores predict higher earnings among adults, while verbal SAT scores do not.[2]

The importance of mathematical ability and the relatively poor performance of American students relative to their counterparts around the developed world is not lost on US policymakers. The Obama administration's Educate to Innovate initiative seeks to "move American students from the middle of the pack to top in the next decade."[3] The initiative consists largely of a public relations campaign, exhorting the private sector to "mobilize," and sponsored games, science fairs, and "civic participation" events extolling the virtues of math and science.

The factors motivating this initiative are nothing new: concerns about the math training of the nation's youth date back a century or more. But somehow, in the face of these continuing concerns, the country has lost rather than gained ground. Between 1972 and 2011, real GDP per capita doubled in the United States, but the average math SAT score of college-bound high school seniors barely budged, as did the proportion of college graduates majoring in a mathematically intensive subject. American performance on the Programme for International Student Assessment (PISA) has slipped over the past decade, notwithstanding the No Child Left Behind movement.[4]

America's perpetual concern with youth math performance has spawned numerous initiatives that have, cumulatively, exacerbated rather than solved the problem. This policy brief begins by discussing a specific example of a widely espoused policy prescription with deleterious effects: the call to have most, if not all, students take an introductory algebra course as eighth graders. New evidence drawn from a 2002 initiative to expand middle school algebra enrollment in Charlotte, North Carolina, shows that accelerated students performed significantly worse in the course and were actually less likely to successfully complete the state's three-course college preparatory math curriculum (Algebra I, Geometry, and Algebra II). The net effect of an initiative to improve math-related college readiness was to decrease math-related college readiness.

A broader review of the historical record indicates that the Charlotte experience was no fluke. The root of America's math problem is the conflation of two goals: improving the absolute performance of American students and closing gaps between high and low performers. Following the failure of a significant initiative to accomplish both goals simultaneously—the "new math" movement of the mid-twentieth century—successive reforms have focused attention on bringing lower-performing students up to standards. In the process, the standards have been lowered, and the advancement of higher-performing students has been allowed to languish. Designers of the nation's mathematics curriculum, in short, have fallen into an "achievement-gap trap," raising the relative performance of average students in part by permitting the absolute performance of the best students to decline.

Of course, valuing this sort of trade-off is inherently subjective, and some in society might prefer to see more widespread proficiency rather than the promotion of new superstars. From an economic perspective, however, the performance of the superstars would seem to be more crucial to future economic growth. But there need not be a trade-off between promoting superstars and ensuring universal proficiency if we are willing to abandon a one-size-fits-all curricular strategy to avoid it.

The Charlotte-Mecklenburg Algebra Initiative

This section summarizes research I undertook with Duke University colleagues Charles Clotfelter and Helen Ladd. The complete report, available as a working paper from the National Bureau of Economic Research, studies the impact of an algebra acceleration initiative in one of North Carolina's largest school districts.[5]

North Carolina's Charlotte-Mecklenburg Schools (CMS) is generally regarded as a model district. The district serves over one hundred thousand students, ranking among the thirty largest in the United States. Among the eighteen large school districts identified in 2009 National Assessment of Educational Progress (NAEP) assessment results, CMS ranked first in terms of fourth grade math performance.[6] It was the only large district to post scores exceeding the national average. The district covers an entire county and incorporates both urban and suburban communities. Although it is more affluent than most large urban districts in its peer group, it has a higher student poverty rate than North Carolina as a whole. A majority of students in the district are either black or Hispanic.

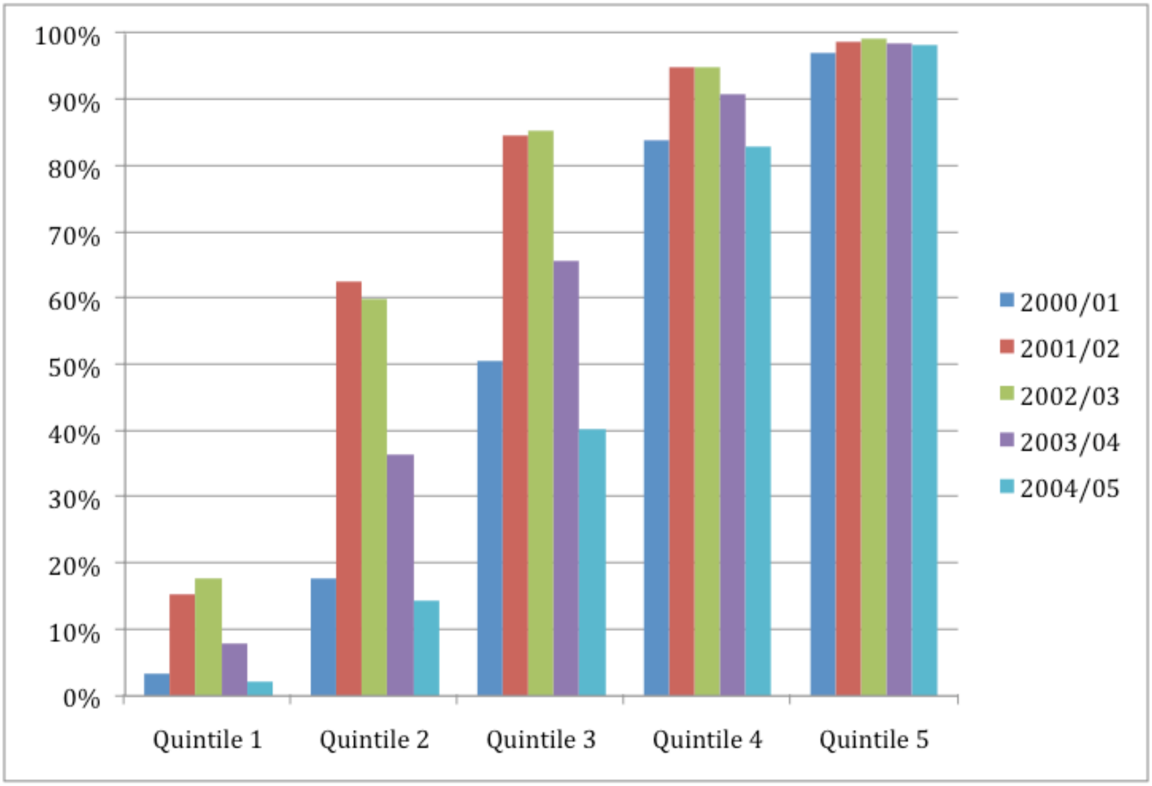
Beginning with the 2002–03 school year, CMS superintendent Eric Smith instructed middle school principals to enroll a larger proportion of students in Algebra I, the first course in the state's college-preparatory high school sequence. In a later interview, he cited evidence documenting higher rates of Advanced Placement course completion and better SAT scores among students who had taken Algebra I by eighth grade. In an interview with PBS, he referred to middle school math as "the definition of what the rest of the child's life is going to look like academically." His goal was to "make sure that kids were given that kind of access to upper-level math in middle school."[7]

Smith's goal rested on the presumption that what appears to be good for some students—in this case, early enrollment in Algebra I—is good for all of them. In fact, as I will explain, Smith's evidence does not prove that early enrollment in Algebra I changes lives, let alone that such a conclusion can be extrapolated to the entire population.

Figure 1 documents the impact of the policy initiative, using administrative data on CMS students from the North Carolina Education Research Data Center. It divides students into five groups of roughly equal size, based on their performance on the state's end-of-grade math assessment as sixth graders. Students are further divided into five age cohorts.

Students whose sixth grade test scores place them in the top quintile of the distribution are very likely to take Algebra I by eighth grade in all cohorts. For students closer to the middle of the sixth grade distribution, however, treatment varied considerably across cohorts. In the cohort entering seventh grade in 2000–01, before the initiative, about half of moderate-performing students (those between the 40th and 60th percentile) took Algebra I as eighth graders; low-performing students in the same cohort had virtually no chance of taking algebra in middle school.

Figure 1. Probability of Taking Algebra I by Eighth Grade, by Sixth Grade Math Test Score Quintile and Year Entering Seventh Grade, Charlotte-Mecklenburg Schools



Source: Charles Clotfelter, Helen Ladd, and Jacob Vigdor, "The Aftermath of Accelerating Algebra: Evidence from a District Policy Initiative" (National Bureau of Economic Research working paper #18161, Cambridge, MA, 2012).

Over the next two years, the effect of Smith's algebra policy can be readily observed. Moderate-performing students in the 2002–03 seventh grade cohort had an 85 percent chance of taking Algebra I as eighth graders; even the lowest-performing students had a one in six chance.

Just as quickly as the policy was introduced, data indicate the district returned to the status quo. The cohort of students entering seventh grade in 2004–05 took Algebra I in eighth grade at rates similar to, or even lower than, their counterparts in the first cohort. One might conclude from the rapid reversal that the policy did not have the anticipated effects.

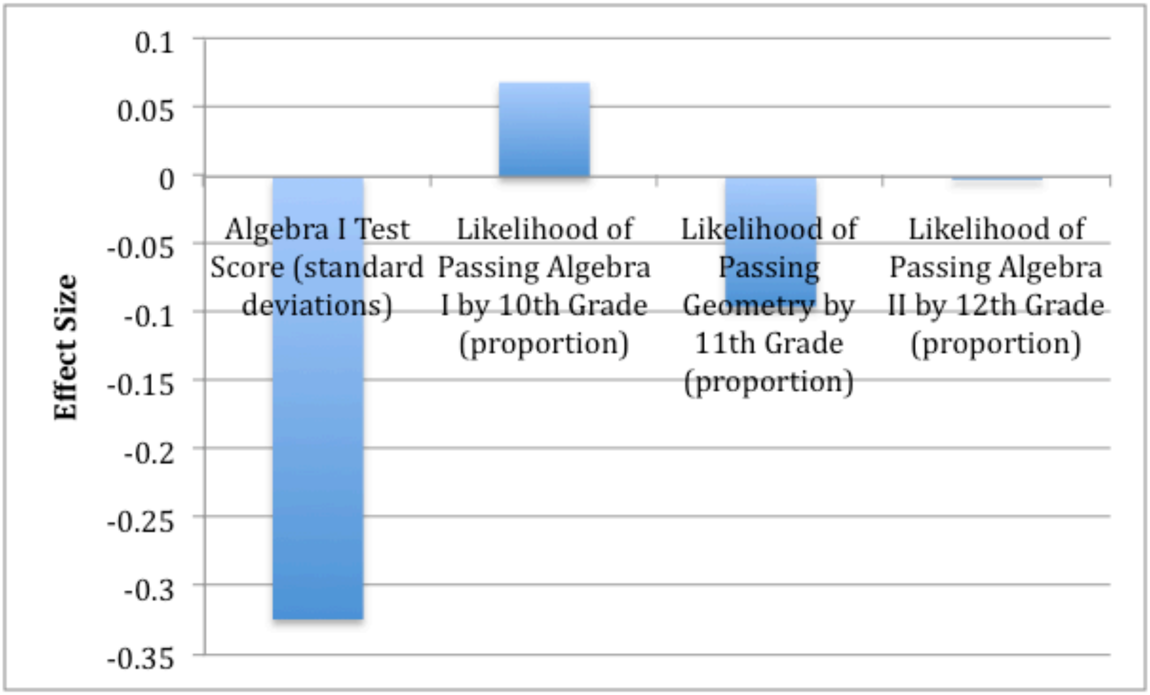
Smith's initiative was inspired by basic observational evidence. In the United States, as elsewhere, students observed taking advanced courses at an early age tend to accomplish better things later in life.[8] To infer from this that early entry benefits students, one must assume that the students in the advanced courses were no different from their counterparts, on average, before taking the course. clearly a misguided assumption. As figure 1 clearly shows, following a typical curriculum, students taking Algebra I in middle school tend to have much better math test scores in sixth grade—more than a year before they actually start their Algebra I coursework. Although it is possible that early progression to advanced coursework compounds this advantage, empirically it is very difficult to disentangle this marginal advantage from the profound baseline differences between early and late algebra takers.

The CMS policy initiative provides a rare opportunity to perform this disentangling. Moderate-performing students born just two years apart were subjected to radically different algebra placement policies. Were the students belonging to the accelerated cohort more likely to perform well in Algebra I or in the standard follow-up courses of Geometry and Algebra II? Figure 2 summarizes the evidence, based on student performance on North Carolina's standardized end-of-course tests in the three subjects. These estimated effects are derived using an instrumental variables estimation strategy.[9] They infer the impact of Algebra I acceleration by comparing the performance of otherwise-identical students who were subjected to different placement policies by virtue of belonging to different age cohorts.

Students perform significantly worse on the Algebra I end-of-course (EOC) test when they take the course earlier in their careers. The decline in performance is approximately one-third of a standard deviation, or thirteen percentile points for an average student. The course material forgone in the acceleration process and the additional maturity that comes with a year of age contribute positively to Algebra I performance.

The decline in EOC test performance suggests that students' risk of failing the course increases when they are accelerated, because the EOC grade is often incorporated as part of a student's course grade. One could adopt a relatively sanguine view, arguing that accelerated students who have to retake the course ultimately are not any worse off than those who were not accelerated in the first place. The second effect shown in figure 2 supports this view, showing that, in spite of their worse performance, accelerated students actually become a bit more likely to pass the course on a college-preparatory schedule—that is, no later than their tenth grade year. For most of these students, the acceleration provided three chances to pass the course—in eighth, ninth, and tenth grade—rather than two. The extra shot appears to have been beneficial.

Figure 2. Impact of Algebra I Acceleration on Accelerated Students



Source: Clotfelter, Ladd, and Vigdor, "The Aftermath of Accelerating Algebra: Evidence from a District Policy Initiative."

It is a different story when we consider the next outcome, whether students manage to pass the state's EOC test in Geometry by the end of their eleventh grade year. Accelerated students were ten percentage points less likely to meet this threshold, in spite of the fact that acceleration gives them two chances, rather than one, to retake either Algebra I or Geometry in the event they do not receive a passing grade.

By forgoing a year of pre-algebraic math, students miss an opportunity to receive some instruction in fundamental topics underlying geometry. Although certain topics in geometry flow naturally from algebra—translating an equation with two unknowns into a line in a two-dimensional plane, for example—others do not. In North Carolina's standard curriculum, geometry incorporates emphasis on area and volume calculations, trigonometric functions and proof-writing, topics with zero coverage in the standard Algebra I curriculum.

To complete the college-preparatory curriculum in North Carolina, students must at minimum pass the set of courses culminating in Algebra II. (The University of North Carolina system requires coursework beyond that, but the state's community colleges do not). The final effect reported in figure 2 shows that accelerated students were neither more nor less likely to clear this hurdle by the time they would ordinarily complete twelfth grade. What is not shown here is that many accelerated students who passed Algebra II did so without ever passing Geometry, so they had not quite completed the full college-preparatory math sequence.

Ultimately, then, the evidence from CMS indicates that the egalitarian impulse to help moderate-performing students by moving them more rapidly into advanced coursework did harm rather than good by reducing their chances of passing the full three-course college-preparatory sequence. The struggles of accelerated students undoubtedly explain why CMS so rapidly reversed course, returning to its initial placement policy after only two years of acceleration.

The CMS experience illustrates the hazards of combining good intentions with a naïve reading of empirical evidence. The district's willingness to reverse course in light of unexpectedly bad results is encouraging, and had other education agencies around the nation taken heed of the CMS experience, the value of this evidence might exceed the cost imposed on several thousand accelerated students. Unfortunately, word of the CMS policy reversal has not traveled swiftly. In 2008, four years after CMS reversed course on its algebra acceleration, the California State Board of Education voted to require 100 percent of eighth grade students to enroll in algebra. Fortunately for the state's students, the mandate was never fully implemented, although the state leads the nation in middle-school algebra enrollment.

The CMS and California initiatives are emblematic of a broader movement over the past three decades. In the mid-1980s, about one student in six took Algebra I in middle school.[10] In more recent years, the national average has been closer to one-third, doubling over the course of a generation.[11] This represents a significant departure from curricular strategies of the early twentieth century, when topics such as algebra, geometry, and trigonometry were considered "intellectual luxuries," worthy of instruction to a select few but of little to no relevance for the vast majority of the workforce—presumably, those destined to engage in manual labor.[12] From the 1930s through the mid-1950s, educational practice codified these beliefs. Less than a third of all high school students enrolled in algebra, substantially fewer in geometry, and only one in fifty in trigonometry.[13]

Can we really think of an algebra course offered to every eighth grade student as the intellectual equivalent of a course that was offered only to the top quarter of students, typically in tenth grade or later, sixty years ago? Figure 3 yields some insight by listing the tables of contents for two introductory algebra textbooks: George Chrystal's fifth edition of *Algebra: An Elementary Text-Book*, published in 1904, and *Algebra 1*, published by Prentice Hall exactly one century later. While there are similarities in the curricula outlined by these books—both, for example, cover quadratic equations late in the volume—the early book clearly covered more topics in greater detail. The later book does not mention series, logarithms, interest and annuities, complex numbers, or exponential functions beyond the quadratic. Ironically, the only topic covered in additional detail in the 2004 textbook is mathematical inequality.

Figure 3. Tables of Contents for Algebra Textbooks from 1904 and 2004

*Algebra: An Elementary Text-Book**	*Algebra I***
I. Fundamental Laws and Processes of Algebra	1. Tools of Algebra
II. Monomials – Laws of Indices – Degree	2. Solving Equations
III. Theory of Quotients – First Principles of Theory of Numbers	3. Solving Inequalities
IV. Distribution of Products – Elements of the Theory of Rational Integral Function	4. Solving and Applying Proportions
V. Transformation of the Quotient of Two Integral Functions	5. Graphs and Functions
VI. Greatest Common Measure and Least Common Multiplier	6. Linear Equations and their Graphs
VII. Factorisation of Integral Functions	7. Systems of Equations and Inequalities
VIII. Rational Fractions	8. Exponents and Exponential Functions
IX. Further Applications to the Theory of Numbers	9. Polynomials and Factoring
X. Irrational Functions	10. Quadratic Equations and Functions
XI. Arithmetical Theory of Surds	11. Radical Expressions and Equations
XII. Complex Numbers	12. Rational Expressions and Functions
XIII. Ratio and Proportion	

XIV. On Conditional Equations in General XV. Variation of a Function XVI. Equations and Functions of First Degree XVII. Equations of the Second Degree XVIII. General Theory of Integral Functions XIX. Solutions of Problems by Means of Equations XX. Arithmetic, Geometric, and Applied Series XXI. Logarithms XXII. Theory of Interest and Annuities	

Notes: *George Chrystal, *Algebra: An Elementary Text-Book for the Higher Classes of Secondary Schools and for Colleges* (London: A. & C. Black, 1904); **Allan E. Bellman, Sadie Chavis Bragg, and Randall I. Charles, *Algebra 1* (Upper Saddle River, NJ: Prentice Hall, 2004).

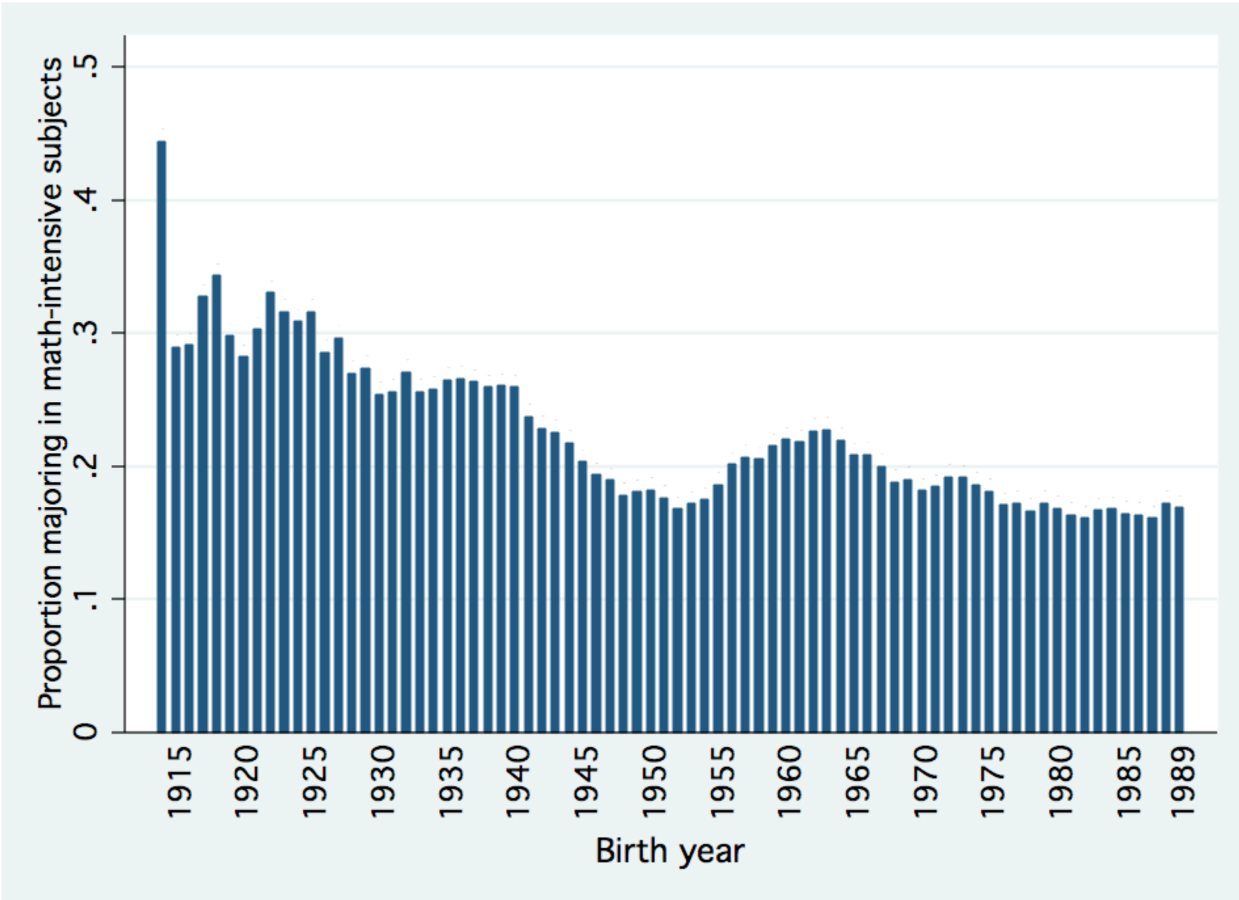
This textbook comparison sets the stage for a broader historical review. Over the past hundred years, generations of reformers have been motivated by the same basic concern as Eric Smith: to bring more students up to the level of the nation's top math performers. Over time, this focus on "leveling up" has shifted to a focus on "leveling," with predictable consequences.

The Bigger Picture: One Hundred Years of Hand-Wringing

Long-Run Trends

Figure 4 uses data from the American Community Surveys of 2009 and 2010 to track a basic indicator of math performance of elite students over a seventy-five-year span: the proportion of college graduates who majored in a math-intensive subject (math, statistics, engineering, or physical sciences) in each birth cohort. The sample here is restricted to male college graduates to address possible concerns about changing gender composition of the college graduate population, though the figure looks similar if females are included.

Figure 4. Math-Intensive Major Rates (by Birth Cohort)



Source: United States Census Bureau, *American Community Survey*, 2009 and 2010; author's calculations.

At one time, three college graduates in ten had majored in a math-intensive subject. The cohorts exhibiting this behavior would have been educated in the era when advanced math topics—algebra, geometry, and trigonometry—were considered irrelevant to the vast majority of high school students and reserved for a select few. As George Chrystal's textbook suggests, the courses were quite rigorous by modern standards.

Among cohorts educated in the post–World War II era, we have seen three distinct periods of decline in math performance, by this measure. The first decline was modest and occurred very soon after World War II. Students born in the 1930s majored in math-intensive subjects less often than those born in the early-to-mid-1920s. This decline pales in comparison to the second decline, which transpired between the birth cohorts of 1940 and 1952. The 1952 birth cohort, which commenced its formal education immediately following the Soviet launch of Sputnik in 1957, chose math-intensive majors at roughly half the rate of birth cohorts from the early 1920s.

Math-intensive majors enjoyed a comeback over the next ten years, peaking among early 1960s birth cohorts. The third period of decline occurred immediately after this decade of progress, gradually returning math-intensive major rates to levels at or below the 1952 cohort trough, where they have remained ever since.

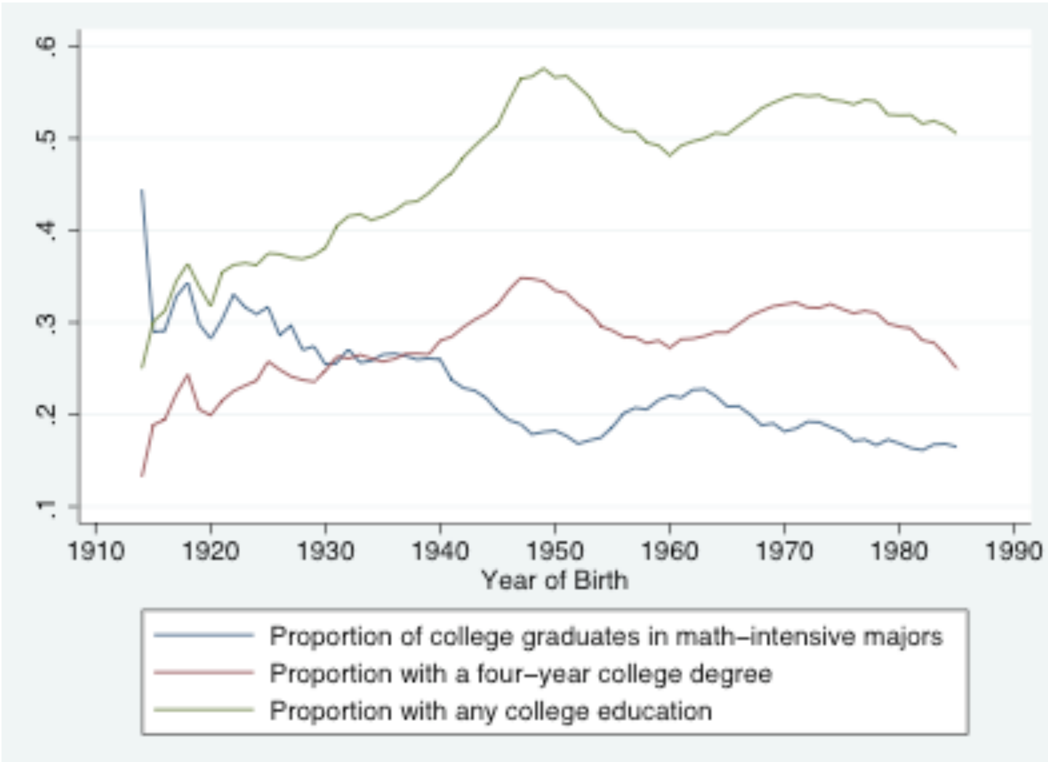
Why Have Math-Intensive Majors Declined?

Why do we observe such a broad decline in math-intensive study, despite the significant earnings premium awarded to those who complete such training? One might think that this reflects the expansion of college attendance over the twentieth century, which presumably added a large number of more marginal students to the ranks of college graduates.

Figure 5, which adds trends in college attendance and completion by birth cohort to the original plot, shows both some evidence to support this explanation and some glaring inconsistencies. The figure plots college-attendance rates, college-graduation rates, and math-intensive major rates by birth cohort. The early decline in math intensity corresponds with the GI Bill–associated run-up in college attendance and completion among 1920s birth cohorts. The second decline starts about the same time as attendance and completion rates start to rise, at or around the 1940 birth cohort. But the decline continues even after the attendance rate reverses course following the 1947 cohort. And the third period of decline has continued through both increases and decreases in attendance rates.

In addition to timing misalignment, the magnitude of changes in enrollment and completion rates is not sufficient to explain math intensity trends. In the 1940 birth cohort, 28 percent of males completed college and about a quarter of those graduates—7 percent of the entire cohort—majored in a math-intensive subject. Over the next dozen years, completion rates increased to 35 percent before receding. If none of the marginal college completers majored in a math-intensive subject but all others behaved the same way, we would expect the rate to decline to 20 percent (a 7 to 35 ratio). In fact, it falls below that level.

Figure 5. Math-Intensive Major Rates and College Attendance Rates (by Birth Cohort)



Source: United States Census Bureau, *American Community Survey*, 2009 and 2010; author's calculations.

A second possible explanation for these long-term trends is that math-intensive subjects are subject to random fluctuations in popularity. By this interpretation, the major-choice indicator would be a poor reflection of underlying mathematical ability of the college-bound. Note, however, that the second period of significant decline began after the birth cohort of 1940 and continued for a dozen years. These students would have entered college between 1958 and 1970—the "space race" period that began with Sputnik and culminated in lunar landings. It is difficult to imagine how engineering and other math-intensive subjects could have gone out of style in this period.

A third explanation for these long-run trends is that they reflect underlying variation in the math preparation of college-bound students. We have already noted the significant change in approach over the past century, which has transformed algebra and other subjects from luxuries for advanced students to necessities for all. But do the ebbs and flows of instructional practice match up well with the trend changes in figure 5? In fact, they do.

World War II has been cited as the root cause of a general rethinking of mathematics education in the early postwar era. The restriction of higher-order math to an elite group of students left many rank-and-file soldiers unable to calculate the trajectory of artillery shells, among other things, in an era when hand computation in the field was still a necessity.[14] The Cold War amplified many of these concerns. The impetus to reform the math curriculum, spearheaded by professional mathematicians rather than educators, led to the so-called "new-math" movement. Where "old math" was pragmatic, focusing students' efforts on mathematical tasks they were likely to perform in the course of their future careers, new math valued mastery of fundamental concepts, some of them quite abstract. During the new-math era, for example, calculus was introduced as a high school subject, albeit for only a select group of students.[15] The ambitious goal of the movement was to simultaneously raise the bar in math instruction and send more students over it.

The new-math movement may have succeeded in raising the bar, but students reacted by giving up rather than attempting to clear it. The implementation of new math in the 1950s associates with the marked decline in math-intensive majors: the birth cohorts of the late 1940s and early 1950s would have been exposed to this curriculum during their primary or secondary years. It is ironic that a curricular reform designed to introduce new rigor and bring higher-order subjects to more students in secondary school appears to have resulted in a strong movement away from math at the collegiate level.

Given that the substitution of rigor for practicality appears to have turned students off to math, it stands to reason that substitution in the reverse direction would undo the effect. And indeed, the wane of the new-math movement in the late 1960s and early 1970s helps to explain the resurgence of interest in math-intensive majors—the only such episode observed over a period of seventy-five years—among cohorts born in the late 1950s to early 1960s.

The resurgence was short-lived. From the 1962 birth cohort onward, the proportion of college graduates completing math-intensive majors dropped steadily. As we move forward from the 1962 birth cohort, we encounter students who spent a more significant proportion of their primary and secondary years in the 1980s, a decade when American policymakers focused increasingly on improving the performance of average students while not worrying much about those at the top. The alarm bells sounded by the influential *A Nation At Risk* report in 1983 pointed not to the performance of the elite, but rather to the prevalence of remedial education in colleges and universities.[16] It lamented the fact that a small fraction of high school students managed to complete calculus, in spite of the fact that most attended a school that offered the course.

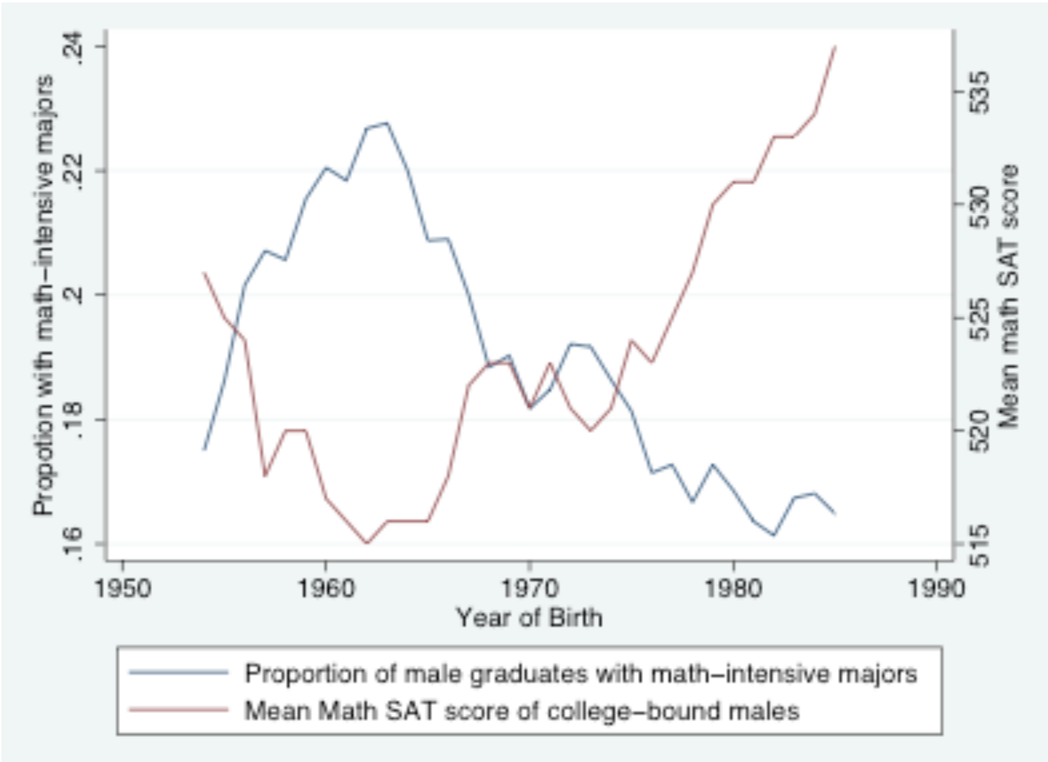
Six years after *A Nation At Risk*, the National Council of Teachers of Mathematics introduced new standards that favored calculators over pencil-and-paper computations, cooperative work over direct instruction, and intuition over solution algorithms.[17]

Educational rhetoric of the past decade, the No Child Left Behind era, has continued to prioritize the performance of average—or even below-average—students. The proficiency standards mandated by the No Child Left Behind Act impose sanctions on schools that fail to serve their worst-performing students but enact no penalty on those that react to the threat of sanctions by shifting resources away from their top performers. Studies have verified the predictable consequence: gains to students just below the proficiency level have been offset by losses among more advanced students.[18]

The net effect of these reforms on the math preparation of elite students appears in figure 5. In an era with very little change—and perhaps a decrease—in college completion rates, the proportion of college graduates in math-intensive majors declined and has remained at historic lows. Paradoxically, this trend has occurred even as the average math ability of college-bound high school seniors has improved. As shown in figure 6, the average math SAT score of college-bound male seniors has risen twenty points over the past twenty-five years, even as males' rate of completing math-intensive study has fallen by one-quarter.

The substantial rise in average SAT scores starting with mid-1970s birth cohorts suggests curricular reforms beginning in the 1980s were quite beneficial to the average student; however, they must have offered little to students who were proficient at computation and using algorithms to solve problems. The reforms succeeded in raising the mean by about a fifth of a standard deviation, but as the lower tail of the distribution has come up, the upper tail has come down.

Figure 6. Math-Intensive Major Rates and Math SAT Scores (by Birth Cohort)



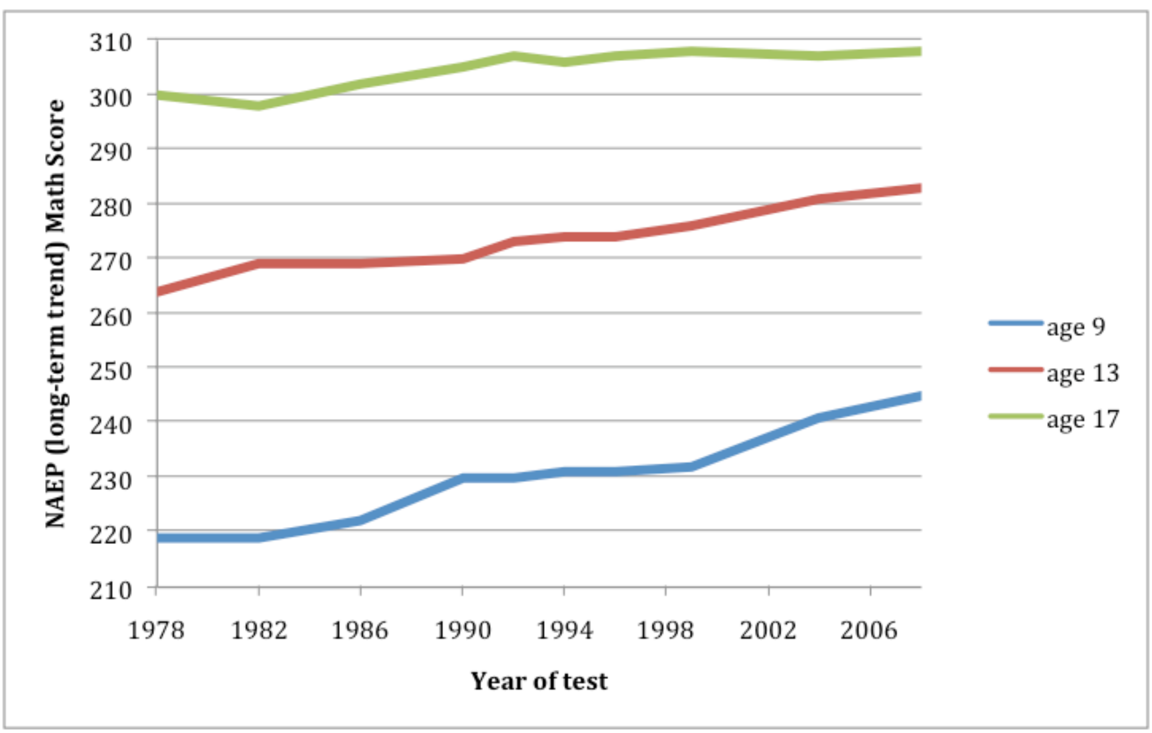
Source: United States Census Bureau, *American Community Survey*, 2009 and 2010; College Board *2011 SAT Trends,* Mean SAT Scores of College-Bound Seniors, 1972–2011; author's calculations.

Bear in mind that the "average" student, with a math SAT score in the mid-500 range, probably does not stand much of a chance in a major such as engineering. Three-quarters of MIT undergraduates have math SAT scores of 750 or better. At a more moderately ranked engineering-focused campus, such as Purdue University, a math SAT score of 530 would place a student at the 25th percentile of the distribution, not the middle. Only 70 percent of Purdue undergraduates receive a degree within six years of entering, so a student at the 25th percentile faces a significant risk of dropping out. The "average" student—even with a twenty-point advantage over the previous generation—takes on significant risk by proposing to pursue a math-intensive course of study.

The nation's successful math, science, and engineering majors are drawn from the upper tail of the distribution, not the middle. The inverse relationship between math-intensive majoring and average math SAT scores suggests that the nation faces a trade-off between offering moderately better math training to the average student and a rigorous training to students with greatest promise. With this in mind, the apparent paradox in figure 6 is not so puzzling. It is the natural result of a series of policy shifts over the past quarter century.

More evidence suggests that the reforms of the past generation have benefited low performers at the expense of high performers. Figure 7 shows that average NAEP math scores have risen. Interestingly, though, the gains are most pronounced among young students and appear to dissipate as students age. The set of students tested as nine-year-olds in 1990, for example, outperform the nine-year-olds of 1986. Their performance advantage is muted but still visible four years later, but has effectively disappeared by the time they are tested as seventeen-year-olds. This might have occurred because gains were concentrated among lower-performing students, who by the age of seventeen have reached the legal dropout age in many states. It might also reflect particular failings of the math curriculum at the high school level, as the Charlotte-Mecklenburg experience exemplifies.

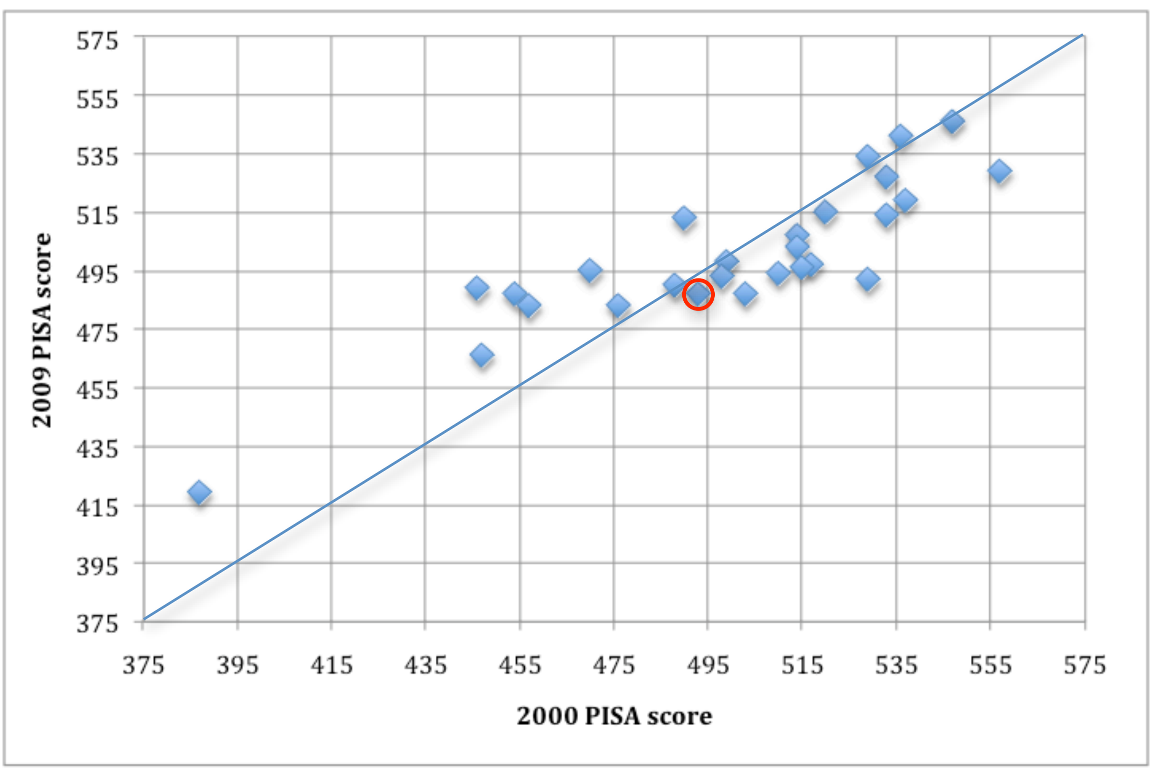
Figure 7. Trends in Math Performance



Source: National Assessment of Educational Progress, 1978–2008. Data available through the National Center for Education Statistics at http://nces.ed.gov/nationsreportcard/naepdata/.

Further evidence that math progress among young students has failed to carry forward to the high school years appears in figure 8, which compares the performance of American fifteen-year-olds on the OECD's PISA exam in 2000 and 2009. The figure depicts a forty-five-degree line; points below that line correspond to countries where performance worsened over this period. The United States lies below the line, indicating that math proficiency declined in an absolute sense. American students also fell behind those from several other countries: Luxembourg, Hungary, Poland, and Germany. The fifteen-year-olds tested in the PISA program thus follow a trend closer to that of the seventeen-year-old NAEP participants, suggesting that curricular problems—and not the removal of high school dropouts from the sample—explain the recent lack of progress among high school students. International comparisons focused on younger students, such as the Trends in Mathematics and Science Study, show more signs of progress in the United States relative to other nations.

Figure 8. PISA Scores in 2000 and 2009



Note: Points below diagonal line correspond to countries with worsening absolute performance. United States is circled in red.

Source: National Center for Education Statistics, *Highlights from PISA 2009*.

Policy Implications

The preceding analysis suggests that the United States has made a clear trade over the past few decades. With the twin goals of improving the math performance of the average student and promoting equality, it has made math curriculum more accessible. We can see the drawback to this more accessible curriculum among the nation's top-performing students, who find themselves either less willing or less able to follow career paths in math, science, and engineering that are the key to innovation and job creation. In the name of preparing more of the workforce to take those jobs, we have harmed the skills of those who might have created them.

To some extent, the nation has reduced the costs of this movement through immigration. Foreign students account for more than half of all doctorate recipients in science and engineering and two-thirds of those in engineering alone.[19] Many of these degree recipients leave the United States when they finish their studies, however, limiting their potential benefit to native-born Americans. Immigration policy reform that emphasizes skills over traditional family reunification criteria, much like the policies in place in Australia, Canada, and other developed nations, could change this pattern.

A second possible policy reform would be to implement a curricular reform more radical than tinkering with the timing of existing courses. Many schools have adopted the Singapore math model, which emphasizes in-depth coverage of a limited set of topics. There are concerns, however, regarding the applicability in the United States of a curriculum developed in a different cultural and educational context. Singapore's public schools, for example, use a year-round calendar, obviating the need to review basic subjects after a summer spent out of the classroom. Evidence also indicates that Singapore's teachers have a firmer grasp of math than their American counterparts.[20]

The United States need not import its science and engineering innovators, however. It need not borrow a faddish curriculum from a foreign context. And it need not sacrifice the math achievement of the average student to cater to superstars. It need only recognize that equalizing the curriculum for all students can be accomplished only by imposing significant lifelong costs on some—perhaps all—students.

As with many policy choices, equality is a questionable goal when it involves reducing the well-being of the poorer-performing group. Math curriculum acceleration appears to be exactly such a policy. Differentiation might exacerbate test score gaps between moderate and high performers. A narrow-minded focus on the magnitude of the gap, however, can lead to scenarios where the gap is closed primarily by worsening the performance of high-achieving students without raising the performance of low-achieving students. Society's overriding goal should be to improve the status of low-performing students in absolute rather than relative terms. A growing body of evidence suggests that this type of improvement is best achieved by sorting students, even at a young age, into relatively homogenous groups to better enable curricular specialization.[21]

From a different perspective, the key error of equality-minded reformers has been to focus on a short-term indicator of equality under the assumption that reductions in long-term inequality will follow. Faced with evidence that those who take algebra early in life do better later on, they presume that equalizing algebra timing will equalize those later-life outcomes. In fact, this is a gross misreading of the evidence, and the resulting actions are verifiably harmful to vulnerable students.

Not all children are equally prepared to embark on a rigorous math curriculum on the first day of kindergarten, and no realistic policy alternatives can change this simple fact. Rather than wish these differences away, a rational policy for the twenty-first century will respond to them, tailoring lessons to children's needs. America's public schools are already demonstrating the benefits of adaptation. Evidence indicates that many American charter schools have found methods of raising the performance of disadvantaged students that do not translate well to a more advantaged student body.[22] A recent study of a remedial math program in Chicago's public high schools has shown significant benefits of extra time for lower-performing students.[23]

These types of strategies—which maintain high aspirations for disadvantaged students but recognize that they may need to find alternate paths to achieve them—promise to provide the next generation of prospective scientists and engineers with the training they need to create jobs and the next generation of workers with the skills they need to qualify for them.

Notes

[1] US Census Bureau, 2009 and 2010 American Community Survey—United States [machine-readable data files], various years.
[2] P. Arcidiacono, and J. Vigdor, “Does the River Spill Over? Estimating the Economic Returns to Attending a Racially Diverse College,” *Economic Inquiry* 48, no. 3 (2010): 537–57.
[3] See the White House, “Educate to Innovate,” www.whitehouse.gov/issues/education/educate-innovate (accessed August 6, 2012).
[4] National Center for Education Statistics, *Highlights from PISA 2009: Performance of US 15-Year-Old Students in Reading, Mathematics, and Science Literacy in an International Context* (NCES publication 2011-004, US Department of Education, 2010).
[5] Charles Clotfelter, Helen Ladd, and Jacob Vigdor, “The Aftermath of Accelerating Algebra: Evidence from a District Policy Initiative” (National Bureau of Economic Research working paper #18161, Cambridge, MA, 2012).
[6] Data accessed through the NAEP Data Explorer, http://nces.ed.gov/nationsreportcard/naepdata/dataset.aspx.
[7] See “Eric Smith Interview” [transcript], *Making Schools Work with Hedrick Smith*, n.d., www.pbs.org/makingschoolswork/dwr/nc/smith.html (accessed August 15, 2012).
[8] J. Smith, “Does an Extra Year Make Any Difference? The Impact of Early Access to Algebra on Long-Term Gains in Mathematics Attainment,” *Educational Evaluation and Policy Analysis* 18, no. 2 (1996): 141–53; J. A. Dossey, I. V. S Mullis, M. M. Lindquist, and D. L. Chambers, *The Mathematics Report Card. Are We Measuring Up? Trends and Achievement Based on the 1986 National Assessment* (Princeton, NJ: Educational Testing Service, 1988); A. Gamoran and E. Hannigan, “Algebra for Everyone? Benefits of College Preparatory Mathematics for Students with Diverse Abilities in Early Secondary School,” *Educational Evaluation and Policy Analysis* 22, no. 3 (2000): 241–54; X. Ma, “Early Acceleration of Students in Mathematics: Does It Promote Growth and Stability of Growth in Achievement Across Mathematical Areas?” *Contemporary Educational Psychology* 30, no. 4 (2005): 439–60; X. Ma, “A Longitudinal Assessment of Early Acceleration of Students in Mathematics on Growth in Mathematics Achievement,” *Developmental Review* 25, no. 1 (2005): 104–31.
[9] Clotfelter, Ladd, and Vigdor, “The Aftermath of Accelerating Algebra.”
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[11] J. Walston, and J. C. McCarroll, *Eighth Grade Algebra: Findings from the Eighth-Grade Round of the Early Childhood Longitudinal Study, Kindergarten Class of 1998-99 (ECLS-K)* (Washington, DC: National Center for Education Statistics, 2010).
[12] Samuel Tennenbaum, *William Heard Kilpatrick* (New York: Harper & Brothers, 1951); A. Osborne and J. Crosswhite, “Forces and Issues Related to Curriculum and Instruction, 7–12,” in *A History of Mathematics Education in the United States and Canada*, ed. Philip Jones (Reston, VA: National Council of Teachers of Mathematics, 1970); D. Klein, “A Brief History of American K–12 Mathematics Education in the 20th Century,” in *Mathematical Cognition*, ed. James Royer (Charlotte, NC: Information Age Publishing, 2003).
[13] P. Jones and A. Coxford Jr., “Mathematics in the Evolving Schools,” in *A History of Mathematics Education in the United States and Canada*, ed. Philip Jones (Reston, VA: National Council of Teachers of Mathematics, 1970).
[14] R. Raimi, “Judging Standards for K–12 Mathematics,” in *What’s at Stake in the K–12 Standards Wars: A Primer for Educational Policy Makers*, ed. Sandra Stotsky (Bern, Switzerland: Peter Lang Publishing, 2000).
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[16] Klein, “A Brief History.”
[17] Ibid.
[18] D. Neal, and D. Schanzenbach, “Left Behind by Design: Proficiency Counts and Test-Based Accountability,” *Review of Economics and Statistics* 92, no. 2 (2010): 263–83; C. Clotfelter, H. Ladd, and J. Vigdor, “The Academic Achievment Gap in Grades 3 to 8,” *Review of Economics and Statistics* 91, no. 2 (2009): 398–419.
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[20] A. Ginsburg et al., *What the United States Can Learn from Singapore's World-Class Mathematics System (and What Singapore Can Learn from the United States): An Exploratory Study* (Washington, DC: American Institutes for Research, 2005).

[21] E. Duflo, P. Dupas, and M. Kremer, "Peer Effects, Teacher Incentives, and the Impact of Tracking: Evidence from a Randomized Evaluation in Kenya" *American Economic Review* 101, no. 5 (2011): 1739–74.

[22] P. Gleason et al., *The Evaluation of Charter School Impacts: Final Report* (US Department of Education, National Center for Education Evaluation and Regional Assistance, NCEE 2010-4029, 2010).

[23] K. Cortes, J. Goodman, and T. Nomi, "Doubling Up: The Long Run Impacts of Remedial Algebra on High School Graduation and College Enrollment," forthcoming, *Education Next*.