TEACHING MATHEMATICS TO YOUNG CHILDREN THROUGH THE USE OF CONCRETE AND VIRTUAL MANIPULATIVES

Dr. Frank D'Angelo, Nevin Iliev

Bloomsburg University of Pennsylvania, 400 East Second Street, Bloomsburg, Pennsylvania, 17815, USA

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Abstract: The use of manipulatives is an essential key to teaching mathematics to young children. Throughout history, different types of manipulatives have been used to aid in comprehension of mathematical concepts including quipu, abaci and pattern blocks. Today, concrete and virtual manipulatives are the tools that early childhood teachers are using in their mathematics classrooms across the country to instruct young learners. The concrete experience that these objects provide allows children to have a greater understanding of mathematical concepts which becomes the basis of their conceptual mathematical knowledge. Teachers who effectively implement concrete and virtual manipulatives create a mathematically rich environment in which young learners are able to examine ideas critically and solve problems. The use of these manipulatives fosters the beginning of life-long mathematical learning in inquisitive young learners.

Use of manipulatives in classrooms significantly helps young learners to build strong mathematical foundations. Concrete manipulatives such as counters, pattern blocks, base ten blocks, geoboards, Unifix cubes, fraction strips, and Cuisenaire Rods and virtual manipulatives are the tools that early childhood teachers are using in their mathematics classrooms across the country to instruct young learners. The experience that these objects provide allows children to have a greater understanding of mathematical concepts that become the basis of their conceptual mathematical knowledge. Since a child's development, especially in math, occurs in his or her early years of school, it is important for children to have multiple exposures to a variety of mathematical tools. Additionally, children need to be immersed in math experiences and have discussions about their experiences with their peers since listening to different perspectives on an idea can make a person see concepts in a diverse light. Students should have as many hands-on interactions with manipulatives as possible since the use of manipulatives enables students to clarify mathematical concepts. "Incorporating manipulatives into mathematics lessons in meaningful ways helps students grasp concepts with greater ease, making teaching most effective" (Boggan, Harper, Whitmire, n.d., p. 5). Research shows if children have the opportunity to create their own understanding about a mathematical theory, they are more likely to retain the information and apply it later (Boggan, et al., n.d.). Being able to touch and maneuver manipulatives connects learner's senses and accesses their spatial and kinesthetic intelligences, which in turn leads to a stronger retention of information. Since mathematical ideas are abstract and what students see are numbers, symbols, and words, concrete and virtual manipulatives help solidify abstract ideas into concrete concepts. As the introduction of new mathematical concepts may overwhelm some students, the incorporation of manipulatives into a lesson allows more children to grasp new concepts. Teachers will have more success in their mathematics classrooms when concrete and abstract ideas are linked and young students will have a stronger mathematical foundation when comprehension of these concepts becomes tangible.

OVERVIEW OF MANIPULTIVES

From the beginning of civilization many countries and cultures have used objects to solve problems that arose in their lives. The objects were as simple as clay boards and sand, in which they wrote with their fingers. Tools such as these then evolved to include the Inca quipu and the abacus, which allowed individuals to count and remember what they had counted. In the late 1800s, the first manipulatives were invented that showed promising outcomes for all learning styles and covered a variety of mathematical concepts. Two important educators who worked with the idea of manipulatives were Friedrich Froebel and Maria Montessori. Froebel, in 1837, created the

world's first kindergarten and put together "Froebel Gifts," which were two different types of moveable blocks. Maria Montessori continued to work with manipulatives and constructed manipulatives which targeted fundamental elementary math concepts. Today, manipulatives are still widely used in math classrooms across the country and educators highly advocate their use. "In fact, the National Council of Teachers of Mathematics (NCTM) has recommended the use of manipulatives in teaching mathematical concepts at all grade levels" (Boggan, et al., n.d., p. 2).

The five objectives that correspond to the NCTM standards are problem solving, communicating, reasoning, connections, and estimation. When using manipulatives, the goal is to "foster children's concepts of numbers and operations, patterns, geometry, measurement, data analysis, problem solving, reasoning, connections, and representations" (Boggan, et al., n.d., p. 3). Not only is the use of manipulatives supported by educational theory, but extensive research has been done on the topic, as well. Many students struggle with abstract reasoning. The use of manipulatives helps clarify these difficult concepts. A connection can be made from a mathematical idea to the input of knowledge in the brain. Boggan, Harper, and Whitmire (n.d.) conducted research that showed "mathematical achievement increases when manipulatives are put to good use and they improve children's long and short term retention of math" (p. 4). Tangible examples in any subject area will improve understanding of specific concepts or ideas. However, the use of manipulatives has to be in conjunction with a problem being solved. If the two processes are separate events, meaning is lost and no connection is made by students. Teacher support is vital in this case because teachers need to know when, why, and how to use manipulatives in the classroom in the most effective ways. They must also have strong content knowledge so that they are able to address questions when teachable moments occur. Utilizing these concrete objects merely because a teacher thinks he or she should does not benefit students. Children will see more success in their math classes if they have the opportunity to take advantage of proper manipulative use and if the tools furnished have been properly scaffolded for their ability levels. All students at any age will benefit from the use of manipulatives especially English Language Learners, low ability students, students with learning disabilities, etc.

THEORY SUPPORTING USE OF MANIPULATIVES

Since it is the goal of mathematics educators to facilitate concrete understanding of mathematical concepts in their students, teachers must follow a specific order of events in order for students to grasp and retain concepts.

Friedrich Frobel first recognized this need for order in his "series of Gifts and Occupations [which] were designed as part of a systematic method for children to learn through play" (Hewitt, 2001, pg.9). In 1973, Bruner observed children using concrete manipulatives and realized that there is a process by which students develop their knowledge of mathematical concepts. Young learners utilize concrete manipulatives through examples shown by the teacher, move towards drawn pictures of the concept and written explanation, and finally encompass the concept through a mental representation which locks in abstract thought. (Kosko and Wilkins, 2010, p. 79) This sequential order of events is significant to children's learning processes. According to Kosko and Wilkins (2010), "the last forms of representations are identified as critical for linking informal mathematical knowledge to abstract representations and understandings" (p. 80-81). The use of language through verbal and written forms takes the learner's understanding of the concept from informal to formal knowledge. Concrete manipulatives create a basis of understanding for children. Futhermore, virtual manipulatives can deepen or further this knowledge by giving students a different perspective with which to work and in doing so, understanding is created and stored for future reference.

Teachers who implement manipulatives in their mathematics lessons create constructivist based classrooms. Constructivist theory which involves top-down or student centered-processing of information is a theory in which children construct their own knowledge through interaction with the world around them. In constructivist based mathematical classrooms, "Mathematical ideas cannot be 'poured into' a passive learner. Children must be mentally active for learning to take place. In classrooms children must be encouraged to wrestle with new ideas, to work at fitting them into existing networks, and to challenge their own ideas and those of others" (Van de Walle 23). As children are given concrete manipulatives with which they can interact, they build new schema by making connections between old and new experiences. This schema combined with a teacher's carefully scaffolded lessons should be targeted toward students' zone of proximal development (ZPD). This zone of proximal development which was defined by Vygotsky as "those functions that have not yet matured but are in the process of maturation (86)" can be accessed through orientation to tangible objects such as manipulatives. Consequentially, manipulatives help bridge the gap between concrete and abstract mathematical ideas while helping young learners to mature in their mathematical knowledge.

Since the use of manipulatives is essential to comprehension of mathematical concepts, the earlier educators begin to use these objects, the stronger a students' knowledge base will become. The key to implementing these new objects into early childhood classrooms is giving students time to explore these new "toys" because "through play the child achieves a functional definition of concepts or objects" (Vygotsky 99). This type of exploration commonly referred to as play is developmentally appropriate for students of this age bracket. Friedrich Frobel, like Vygotsky, believed that "Through engaging with the world, understanding unfolds" (Smith 2011) and therefore through play, children make sense of the world and their place in it. Hence, children need time to play as time is important for exploration of objects given to them. The more students become comfortable with the options available to them, the more questions they will ask of each other and of the teacher. This critical discourse is an important component which must not be overlooked. Of course, some precautions must be taken when using concrete manipulatives as children may only think of objects such as geoboards and rubber bands as toys. Proper introduction of said toys by teachers is key. Teacher modeling allows students to physically see how and what to do with each manipulative. According to Durmus and Karakirik (2002), "lessons involving manipulative materials…will produce greater mathematical achievement than will lessons in which manipulative materials are not used" (p. 120). Careful planning and active teacher supervision of manipulatives is vital for the success of lessons. Furthermore, when students have multiple opportunities to interact with manipulatives as lessons progress and deepen, fundamental knowledge will be acquired.

IMPORTANCE OF DISCOURSE WHEN USING MANIPULATIVES

Additionally, communication facilitated through cooperative learning strengthens knowledge gained through the use of manipulatives. When paired with classroom discourse, mathematical tools are highly effective. "Social constructivism maintains that students can better build their knowledge when it is embedded in a social context. Thus, the interaction between teacher and students is enhanced when it involves a broader community of learners--that is, students working together. Students help one another create richer meanings for new mathematical content" (Stiff 2001). According to Lappan and Schram, "classroom discourse influences students' reasoning, problem solving competence, self-confidence, and social skills acquisition" (Varol and Farran, 2006, p. 383). In a study done by Moyer, (2001) discussion was a part of manipulative use and said to be an important part if manipulatives were to be effective. Research done by Moch (2001) showed that "manipulatives were used only 18 hours in seven weeks and improved test scores by an average of ten percentage points" (Kosko and Wilkins, 2010,

p. 82). Students had to summarize their thoughts through writing and discussion which occurred at some point in the lesson. Stein and Bovalino (2001) concluded that group and paired discussion along with writing was "linked" to the use of manipulatives. NCTM (2000) and Kenney (2005) wrote books inferring that the examples of students' written explanations and pictures in their texts lead to this goal of abstract thinking. Furthermore, Moch and Moyer concluded that "language is viewed as the key to making the leap to abstract understanding of a concept" (Kosko and Wilkins, 2010, p. 83) which illustrates how the two are linked.

CONCRETE MANIPULATIVES

An example of how to use mathematical manipulatives was cited by Rosen and Hoffman in their creation of a project about houses and homes for a first grade class. During this project, students were to be presented with a mixture of different manipulatives to gain experience with shapes. Use of manipulatives would allow students to make appropriate connections between shapes and the houses or homes in which they lived. Throughout the project, students could participate in various activities including learning centers, writing prompts, and discussions to reinforce their learning. In one learning center, students could use blocks and art materials to construct three dimensional and square houses. Children could then draw what they had created. The drawings would help connect students' understanding of the relationship between two-dimensional and three-dimensional structures. In another learning center, students could sort three-dimensional manipulatives based on their forms and attributes and then write about why they put certain blocks into each group. The additional integration of writing would help with the discrimination of objects, sorting, and grouping objects in this learning center. A third learning center was also suggested in which children would use shape tracing stencils to create houses of shapes. This center would support their knowledge of two-dimensional drawings symbolizing three-dimensional buildings. This example of how to integrate concrete manipulatives into an early childhood classroom by Rosen and Hoffman clearly indicates that the use of concrete manipulatives is key to early understanding of mathematics.

VIRTUAL MANIPULATIVES

Another path that can be taken in the constructional of mathematical knowledge is through the use of virtual manipulatives in the mathematics classroom. The National Council for Teacher of Mathematics in its Statement of Beliefs (2012) says that, "The widespread impact of technology on nearly every aspect of our lives requires changes in the content and nature of school mathematics programs. In keeping with these changes, students should be able to use calculators and computers to investigate mathematical concepts and increase their

mathematical understanding." Virtual manipulatives which are defined as "an interactive, Web-based virtual representation of a dynamic object that presents opportunities for constructing mathematical knowledge (Bolyard, Moyer and Spikell, 2002, pg. 373) fulfill this essential belief. Virtual manipulatives can function similarly to concrete manipulatives because the computer mouse works as an extension of a student's hand. Yet, they can be used for whole class, small group, or individual practice. Teachers can use virtual manipulatives when teaching a whole class lesson in conjunction with a smart board. They can differentiate instruction and target students' needs and ability levels through customizing access to various virtual manipulatives. Furthermore, virtual manipulatives appeal to students as they are a fun, engaging 21st century learning tool. Websites, like Utah State University's National Library of Virtual Manipulatives (http://nlvm.usu.edu/) and McGraw Hill Virtual Manipulatives (http://www.glencoe.com/sites/common_assets/mathematics/ebook_assets/vmf/VMF-Interface.html) offer an array of virtual manipulatives with which students can interact. Additionally, Illuminations (http://illuminations.nctm.org/), created by the National Council for Teacher of Mathematics, offers standards- based digital resources .Websites such as these focus on a variety of mathematical areas such as number operations, geometry, and probability, are available for kindergarten through twelfth grade. Finally, as many virtual manipulative websites are free and easy to access, students can continue their learning at home on their homes computers.

Virtual and concrete manipulatives expand young students' knowledge of key math concepts in similar ways. Each one is just a different approach to teaching math; both are beneficial for diverse learners. In Rosen and Hoffman's project, students could additionally work together at computers. They could use computers to sort virtual blocks and then discuss their exploration of these concepts on the computer with a partner. During an activity such as this, an array of pattern blocks would be presented on the screen and students would have to come up with one specific answer. The answer a computer generated would be different from the numerous sorting options working with concrete manipulatives would produce. As children cooperated with each other, the opportunity to integrate math and language arts would arise. Students would be able to talk about their reasoning behind their answers and could include written statements and support for their answers. Additionally, another beneficial feature of virtual manipulatives is that students can check their answers at any time and as many times as they want. Websites will automatically remove incorrect part(s) and students can try again. This feature assists students in seeing what they

did wrong, and, through this, an understanding is formed. Young children have the opportunity to further their learning with virtual manipulatives because there is a larger "range" of problems available than with the use of concrete manipulatives. Virtual manipulatives have a wider variety of mathematical components so success and understanding can be furthered.

Indeed, the pairing of concrete and virtual manipulatives shows more progress in mathematical understanding and application than either one by itself. "NCTM's 2008 position statement states that technology is an essential tool for learning mathematics in the twenty-first century" (Rosen and Hoffman, 2009, p. 32). When teaching young students today, an educator cannot and should not teach only one way to one type of student. Using both kinds of manipulatives conveys the same mathematical concepts using an assortment of methods. If teachers are worried about their students being less engaged with virtual manipulatives, they should not be. According to Durmus and Karakirik (2006), "virtual manipulatives enable as much engagement as physical manipulatives do since they are actual models of physical manipulatives" (p. 121). This interactive option allows children to ask questions, make connections with mathematical concepts, and communicate with their peers, while getting immediate feedback. This immediate feedback forces students to reflect on their answers and as to why they were correct or incorrect. Since virtual manipulatives are not physical objects, learners must think in a more abstract way. Use of the computer increases students' technological knowledge and since we live in a technologically driven society, it is important that students become proficient in this type of abstract thinking. Virtual manipulatives which can be as diverse as those who use them are very promising when used in the proper way. Since no child should be limited by his or her classroom environment, virtual manipulatives furnish an entirely new world for children to explore. Furthermore, this world can continue afterschool with the use of a home computer. Teachers can explain to parents how these virtual tools function and students can then continue their learning at home.

CURRENT CLASSROOM APPLICATION

In a 2011 survey which the authors of this article conducted of kindergarten, first grade and second grade elementary school educators, teachers showed that they use a multitude of concrete manipulatives and are beginning to incorporate virtual manipulatives into their mathematics lessons. In a class of 20-25 students where the teacher spends 30-45 minutes per day on mathematics instruction, teachers showed that they begin their mathematics lessons with Direct Instruction, move to Guided Practice and conclude their lessons with Independent Practice. Of

this time, an average of 30 minutes is spent using manipulatives. Concrete manipulatives that are used include but are not limited to base ten blocks and base ten mats, affix cubes, money, rulers, counters, and a variety of learning centers. Popular virtual mathematics websites teachers who answered this survey are using include Harcourt Mathematics and Starfall Mathematics. One hundred percent positive response to the question as to whether teachers use manipulatives in their classroom indicates that educators commonly believe that young children need manipulatives in order to grasp mathematical concepts. Furthermore, a positive response to the question regarding use of virtual manipulatives in the classroom shows that educators value this type activity and find worth in their content. Conclusions drawn from this survey show that educators understand the need for students to be given designated time to work with manipulatives in order to further their comprehension of mathematical knowledge. Furthermore, teachers believe that when direct instruction is followed by guided practice and concludes with independent practice, a student grasp of concepts is greatly amplified.

CONCLUSION

The use of manipulatives is an essential key to teaching mathematics to young children. Mathematical tasks need to be appropriate for the grade level being taught. According to Hiebert and colleagues, "a meaningful task encourages reflection and communication to build mathematical understanding by enabling children to use skills and knowledge they already possess" (Varol and Farran, 2006). The earlier you start introducing concrete mathematics to a child, the higher the child's mathematical competence will be. Educators who understand their students' proficiency levels when teaching mathematics can scaffold material appropriately. To do so, they need to understand how and why their students think the way they do. Student- centered mathematics instruction is critical to mathematical knowledge attainment in young learners, therefore educators should work to create a mathematically rich environment in which students examine ideas critically, solve problems and openly share their ideas. Teachers who effectively implement concrete and virtual manipulatives into their classrooms can positively affect their young learners' mathematical competence. The benefits are endless when using manipulatives in the classroom, whether they are concrete or virtual. Use of these materials heightens learning experiences for all students, helps bridge the gap between the concrete and abstract and finally, helps foster life-long learning in inquisitive young learners.

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